LESSON OVERVIEW

Lesson 11
Equations for Proportional Relationships

CCSS Focus

Domain
Ratios and Proportional Relationships

Cluster
A. Analyze proportional relationships and use them to solve real-world and mathematical problems.

Standards
7.RP.A.2 Recognize and represent proportional relationships between quantities.

c. Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as* $t = pn$.

d. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and $(1, r)$ where r is the unit rate.

Additional Standard
7.RP.A.2b (See page B3 for full text.)

Standards for Mathematical Practice (SMP)
1 Make sense of problems and persevere in solving them.
2 Reason abstractly and quantitatively.
4 Model with mathematics.
6 Attend to precision.
8 Look for and express regularity in repeated reasoning.

Lesson Objectives

Content Objectives
- Represent proportional relationships by equations.
- Graph proportional equations representing real-world situations on a coordinate grid.
- Explain what a given point (x, y) on the graph of the equation of a proportional relationship means in terms of a real-world situation.

Language Objectives
- Explain how a table, equation, and graph represent the constant of proportionality.
- Answer questions about equations and graphs that model proportional relationships.
- Evaluate an incorrect solution to determine why the error was made.
- Compare two equations that represent the same proportional relationship.

Prerequisite Skills
- Understand ratio, unit rate, and proportions.
- Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios or equations.
- Graph ordered pairs from a table on a coordinate grid.
- Write and solve a one-step equation.

Lesson Vocabulary
There is no new vocabulary.

Learning Progression

In Grade 6 students learned to write equations to express one quantity, the dependent variable, in terms of a second quantity, the independent variable. They studied graphs and tables and related them to the equations. **Earlier in Grade 7** students studied proportional relationships and how to identify the constant of proportionality using tables and graphs.

In this lesson students become more familiar with the characteristics of various representations. For example, they should know the graph looks like a straight line through the origin, and the equation will look like $y = mx$, where m is the constant of proportionality. Given one of the representations—equation, graph, or table of values—they should be able produce any of the others.

In Grade 8 this work will lead into scale drawings and will help introduce students to the concept of slope in graphing equations of lines.

Lesson Pacing Guide

Whole Class Instruction

Use the Equations for Proportional Relationships slides for the Think-Share-Compare routine.

Day	Activities	Practice and Problem Solving
Day 1 *45–60 minutes*	**Toolbox: Interactive Tutorial*** *Equations for Proportional Relationships* **Introduction** • Use What You Know *15 min* • Find Out More *20 min* • Reflect *10 min*	**Practice and Problem Solving** Assign pages 105–106.
Day 2 *45–60 minutes*	**Modeled and Guided Instruction** **Learn About Writing Equations for Proportional Relationships** • Model It/Model It/Model It *15 min* • Connect It *20 min* • Try It *10 min*	**Practice and Problem Solving** Assign pages 107–108.
Day 3 *45–60 minutes*	**Guided Practice** **Practice Writing Equations for Proportional Relationships** • Example *5 min* • Problems 10–12 *15 min* • Pair/Share *15 min* • Solutions *10 min*	**Practice and Problem Solving** Assign pages 109–110.
Day 4 *45–60 minutes*	**Independent Practice** **Practice Writing Equations for Proportional Relationships** • Problems 1–6 *25 min* • Quick Check and Remediation *10 min* • Hands-On or Challenge Activity *10 min* **Toolbox: Lesson Quiz** Lesson 11 Quiz	

*We continually update the Interactive Tutorials. Check the Teacher Toolbox for the most up-to-date offerings for this lesson.

Small Group Differentiation

Teacher-Toolbox.com

Reteach

Ready Prerequisite Lessons *45–90 min*

Grade 6
- Lesson 2 Understand Unit Rate
- Lesson 3 Equivalent Ratios

Grade 7
- Lesson 10 Understand Proportional Relationships

Teacher-led Activities

Tools for Instruction *15–20 min*

Grade 7
- Identifying Proportional Relationships

Personalized Learning

i-Ready.com

Independent

i-Ready Lessons* *15–20 min*

Grade 7
- Equations for Proportional Relationships

ELL English Language Development

Prepare for Day 1: Use with *Reflect*

ELP Levels 1–3

Speaking/Writing Read ***Reflect*** aloud. Ask partners to review ***Use What You Know*** and ***Find Out More*** and label each model named in ***Reflect.*** [table, graph, equation] Then, working as group, help students use these terms and sentence frames to draft an answer to ***Reflect:*** *value of y, unit rate, equivalent ratio, y-coordinate, when $x = 1$.*

- The constant of proportionality is the unit rate, or the value of y when $x = 1$.
- In a table, the constant of proportionality is _____.
- In a graph _____, the constant of proportionality is _____.
- In an equation, the constant of proportionality is _____.
- In a _____, the constant of proportionality is represented by _____.

ELP Levels 2–4

Speaking/Writing Have students work in pairs to write a response to the ***Reflect*** question. Ask partners to review ***Use What You Know*** and ***Find Out More*** to create a word bank using the ***Co-constructed Word Bank*** routine. Suggest words and phrases to include, such as *constant of proportionality, unit rate, ratio, equivalent, value of y,* and *$x = 1$.*

Provide sentence starters to support partners as they write a response:

- The constant of proportionality is _____.
- In a _____, the constant of proportionality is _____.

ELP Levels 4–5

Speaking/Writing Have students work in pairs to generate ideas for a response to the ***Reflect*** question. Ask partners to review ***Use What You Know*** and ***Find Out More*** to create a Word Bank using an adaptation of the ***Co-constructed Word Bank*** routine.

After gathering ideas with a partner, have students draft their answers independently, then trade answers and evaluate them.

Ask: *Did your partner explain how the constant of proportionality is represented for all three models? Did they use terms from the Word Bank? What other information could your partner add to clarify or improve the answer?*

Prepare for Day 2: Use with *Learn About*

ELP Levels 1–3

Speaking/Reading Organize students into small groups to better understand the ***Learn About*** problem by using the ***Act It Out*** routine. Use props, such as two colors of plastic cups, or manipulatives, such as red markers to represent the juice and white markers to represent the seltzer.

Provide the following sentence frames to support students as they discuss the models:

- If Jesse uses _____ cups of juice, how many cups of seltzer will he need?
- The ratio of seltzer to juice is 2 to 1.
- For every 1 cup of juice, Jesse needs 2 cups of seltzer.

ELP Levels 2–4

Speaking/Reading Organize students into small groups to make sense of the ***Learn About*** problem by using the ***Act It Out*** routine. Use manipulatives, such as red markers to represent the juice and white markers to represent the seltzer. Have students take turns using the manipulatives to explain how each model represents the constant of proportionality. Have them name the model, the constant of proportionality, and the ratio in their explanations. Model with the following example: *The graph shows that the constant of proportionality is 2, so if Jesse has 4 cups of seltzer he needs 2 cups of juice.*

Display 4 white markers and 2 red markers.

ELP Levels 4–5

Speaking/Reading Pair students to interpret and discuss the ***Learn About*** problem and models. Have students locate the information from the problem (3 cups of juice and 6 cups of seltzer) in the table and on the graph. Ask them to explain how the rest of the information was completed in the model. For each model, have students discuss the following questions:

- *How does the model show the constant of proportionality?*
- *How do you know the ratios are equivalent?*
- *How are the table and graph similar? How are they different?*

Prepare for Day 3: Use with *Practice: Pair/Share*

ELP Levels 1–3

Listening/Speaking Guide small groups of students through the ***Pair/Share*** prompts. Use the ***Revoicing*** routine to validate and extend understanding while modeling fluent speaking.

For problem 10, have the group explore other ways to find the constant of proportionality by challenging them to describe a table and graph that represents the information. For problem 11, suggest they think about recipes or prices to name situations in which x could have values that are mixed numbers. Provide examples as needed. For problem 12, provide this sentence frame:

- On a graph, a line passing through the origin shows a proportional relationship.

ELP Levels 2–4

Listening/Speaking Organize students into small groups to discuss their solutions to the ***Practice*** problems and respond to the ***Pair/Share*** prompts.

Provide the following sentence starters to support discussion:

- Another way to find the constant of proportionality is _____ .
- One situation in which the value of x could be a mixed number is _____ .
- A graph shows a proportional relationship when _____ .

ELP Levels 4–5

Listening/Speaking Pair students to discuss their solutions to the ***Practice*** problems and respond to the ***Pair/Share*** prompts.

Encourage students to speak in complete sentences by rearranging words from the question to form a sentence starter.

Have students ask their partners follow-up questions which require the speaker to elaborate on ideas, such as:

- Can you clarify _____ ?
- What did you mean by _____ ?
- Why did you _____ ?

Introduction

At A Glance

Students explore a proportional relationship between the number of cars on a rollercoaster and the number of people who can ride the rollercoaster. They review the term "constant of proportionality" and learn about graphs and equations of proportional relationships.

Step By Step

- Work through **Use What You Know** as a class.
- Tell students that this page models proportional relationships.
- Have students read the problem at the top of the page.

▶ Mathematical Discourse 1 and 2

- Remind students that a ratio is the division of two related numbers.
- Point out that, even though $\frac{6}{1} = 6$, it's helpful to know that the ratio represents 6 people to 1 car.

▶ English Language Learners

- Ask student pairs or groups to explain their answers for the last two questions.

Equations for Proportional Relationships

Use What You Know

In Lesson 10, you learned about proportional relationships and how they are represented on a graph. In this lesson you will learn about equations that represent proportional relationships.

The table shows how the number of people who can ride a rollercoaster depends on the number of cars on the rollercoaster.

Number of Cars	Number of People
3	18
5	30
6	36
8	48

How many people can ride in 1 car? In 10 cars?

Use the math you already know to solve the problem.

a. Find the ratio of the number of people to the number of rollercoaster cars for each set of data in the table.
$\frac{18}{3} = \frac{6}{1}$, $\frac{30}{5} = \frac{6}{1}$, $\frac{36}{6} = \frac{6}{1}$, $\frac{48}{8} = \frac{6}{1}$

b. Are the ratios of the number of people to the number of rollercoaster cars in a proportional relationship? Explain your reasoning.
Yes; Each ratio is equal to $\frac{6}{1}$ or 6, so the ratios are proportional.

c. How many people can ride in 1 car? 6

d. What is the constant of proportionality for the data in the table? 6

e. Explain how you can find the number of people that can ride in a rollercoaster with 10 cars.
Multiply the number of cars by 6, the constant of proportionality.

f. Write an equation to represent the number of people, y, that can ride the rollercoaster with x cars. $y = 6x$

▶ Mathematical Discourse

1 *Can you think of other places where you have heard the word* ratio *used? What did it mean?*

Ratios are used in many places where people want to give comparisons. Students may have heard it in news contexts, science classes, or numerous other places. Listen for answers that talk about comparing two quantities, such as the ratio of boys to girls in a class.

2 *How do you think ratios and proportions are related? Do you think they are similar or different, and in what ways?*

Students may think they are different words for the same thing, and they are often used in the same contexts. Listen to see if students understand that proportions involve more than one equivalent ratio.

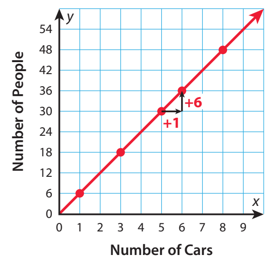
▶ English Language Learners

Remind students that a constant is something that does not change. While x can have different values in the expression $4x$, 4 can never be anything else.

Find Out More

The problem on the previous page is shown using a table. You can also represent this situation with both a graph and an equation. In each representation, you can identify the constant of proportionality.

The graph represents the problem on the previous page.



The **constant of proportionality** is **the unit rate**, or **the value of y when $x = 1$**, which is 6. In the graph, the value of y increases by 6 each time the value of x increases by 1.

The equation $y = 6x$, where y is the number of people who can ride the rollercoaster and x is the number of cars on the rollercoaster, represents this situation. Just as with the graph, the constant of proportionality is the value of y when $x = 1$.

If two quantities, x and y, are in a proportional relationship, then **the ratio $\frac{y}{x}$** equals the constant of proportionality. This means that you can represent any proportional relationship with the following equation:

y = **constant of proportionality** • x

Reflect

1 Explain how the constant of proportionality is represented in the table, the equation, and the graph of the rollercoaster situation.

Possible answer: The constant of proportionality is the unit rate, or the value of y when $x = 1$. For the table, find an equivalent ratio with a denominator of 1. On a graph, find the y-value for the point $(1, y)$. For an equation, find the value of y when $x = 1$.

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Step By Step

- Read **Find Out More** as a class.
- Ask students why the unit rate is the value of y when x is 1. Point out that "unit" means one.
- Ask students what the 6 represents in $y = 6x$. [It is the constant of proportionality.]
- Choose points on the graph. Prompt students to discuss what the coordinates represent.
- Have students work in pairs to write an answer to the **Reflect** problem. Encourage them to read their responses to the class. Discuss the answers with the class.

▶ **Real-World Connection**

SMP TIP Reason Abstractly and Quantitatively
Students contextualize the equation $y = 6x$ to understand that 6 represents 6 people per car. When working with the various models, reinforce this contextualization by asking students what different components represent in a problem. *(SMP 2)*

▶ **Visual Model**

Ready® Mathematics PRACTICE AND PROBLEM SOLVING

Assign *Practice and Problem Solving* **pages 105–106** after students have completed this section.

▶ Visual Model

Explore proportionality using rectangular objects.

Materials: ruler, two different sizes of rectangular objects such as large and small photograph frames or books

Ask students whether they think the dimensions of the two objects are proportional. Measure the length and width of both rectangles, and record them in a chart on the board. Again ask students whether they think the dimensions of the two objects are proportional and to explain their reasoning. Ask how they could check for sure. Then find the ratios of the lengths and widths of the rectangles. Ask whether they are the same. If so, they are proportional. If desired, work with students to find out whether the objects are proportional in the third dimension as well.

▶ Real-World Connection

Discuss how ratios are used in the real world.

Discuss with students how ratios and proportions may be used in real life. Ask them to think of examples. Share with them some examples. Rocket engineers talk about lift-to-drag ratios. Chemists talk about weight ratios of the materials they are working with. Proportions are especially important when things need to be scaled. If you were making several times the amount of a basic recipe, you would want to make sure you kept the amounts of all the ingredients proportional. If you wanted to make a model of a building, you would need to make all the dimensions proportionally smaller.

Modeled and Guided Instruction

At A Glance

Students see how tables, graphs, and equations model a proportional relationship between juice and seltzer in a punch recipe. They use the models given to answer questions about the proportional relationship.

Step By Step

- Read the problem at the top of the page as a class.

Model It

- In the first **Model It**, discuss the data in the table. Ask students what the 2 and 1 represent in the first column. [the ratio of seltzer to juice in simplest terms] Ask: *How was the data found to fill in the table?* [The seltzer row was filled with regularly spaced values. Then ratios of the same proportion as 2 : 1 were computed, or the constant of proportionality was used to find the corresponding juice values.]

SMP TIP Make Sense of Problems
When students understand what the data in a table represent they are making sense of the problem. Encourage students to question what different values in a representation mean, and use other models to check their understanding *(SMP 1)*.

Model It

- In the second **Model It**, discuss the graph. Ask students why the point (3, 6) was graphed. [It was the ratio given in the problem.] Ask why (0, 0) was used. [If 0 cups of juice are used, then 0 cups of seltzer are needed; there are 0 cups of punch.]

▶ **Mathematical Discourse 1**

Model It

- In the third **Model It**, discuss the word *equation*.

▶ **Mathematical Discourse 2**

Learn About Writing Equations for Proportional Relationships

Read the problem below. Then explore ways to represent a proportional relationship.

Jesse is making punch. For every 3 cups of juice, he needs 6 cups of seltzer. Represent this proportional relationship using a table, a graph, and an equation and identify the constant of proportionality. What does the constant of proportionality represent in this situation?

Model It **You can use a table to represent the relationship.**

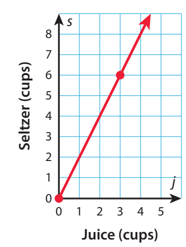
The ratio of seltzer to juice will be the same for all quantities.

All ratios will be equivalent to $\frac{6}{3} = \frac{2}{1}$.

Seltzer	2	3	4	5	6	7	8
Juice	1	1.5	2	2.5	3	3.5	4

Model It **You can use a graph to represent the relationship.**

Graph the point (3, 6) and connect it to the point (0, 0). Identify points on the line.



Model It **You can use an equation to represent the relationship.**

The ratio of seltzer to juice is $\frac{6}{3} = \frac{2}{1}$. This means that for every 1 cup of juice, you need 2 cups of seltzer.

amount of seltzer = constant of proportionality • amount of juice

▶ Mathematical Discourse

1 *How do you think the graph of this proportional relationship might be like other graphs? How might it be different? Explain.*

Students have seen two graphs in this lesson. They may conjecture that the graphs will be straight lines and that they go through (0, 0). Listen for answers that talk about how the constant of proportionality will affect the steepness of the graph.

2 *Which do you think is the best model of the proportional relationship for you—tables, graphs, or equations?*

Students will prefer different methods. Ask students to explain what they like about the method they choose. Visual learners may prefer tables or graphs, since they make things so easy to see. Other students may find the stated equation relationship easier to understand.

Connect It **Now you will solve the problem from the previous page.**

2 In the table on the previous page, what does the ratio $\frac{2}{1}$ represent in terms of the problem? **For every 1 cup of juice, you need 2 cups of seltzer.**

3 What does the point (2, 4) represent on the graph? **The point (2, 4) shows that if you have 2 cups of juice, you need 4 cups of seltzer.**

4 Which point on the graph can be used to find the constant of proportionality? **The constant of proportionality is the *y*-value of the point when the *x*-value is 1. The point (1, 2) is on the graph, so the constant of proportionality is 2.**

5 Use the constant of proportionality to write an equation to represent *s*, the number of cups of seltzer you need for *j* cups of juice. **$s = 2j$**

6 Use the equation you wrote in problem 5 to find the amount of seltzer you need if you have 15 cups of juice. **30 cups**

7 For a different punch recipe, the equation $s = 4c$ represents the number of cups of seltzer, *s*, that you need for *c* cups of cranberry juice. What is the constant of proportionality? Explain. **4; For each cup of cranberry juice, you need 4 cups of seltzer. The unit rate is 4, which is the constant of proportionality.**

Try It **Use what you just learned to solve these problems. Show your work on a separate sheet of paper.**

8 Kelsey can buy 2 pounds of apples for $7. Write an equation to represent the cost, *c*, for *a* pounds of apples. **$c = 3.5a$**

9 The equation $m = 32 \cdot g$ represents the average number of miles, *m*, that a car can go on *g* gallons of gas. What is the constant of proportionality and what does it represent in this situation? **32; the car can go 32 miles on 1 gallon of gas.**

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Step By Step

Connect It

- Work through **Connect It** with the class, referring to the problem on the previous page.
- In problem 5, point out that other variables can be used in place of *x* and *y*.
- In problem 6, ask students to explain how they used the equation to find the amount of seltzer.

▶ Hands-On Activity

- Have students complete **Try It** on their own.

Try It

8 Solution
$c = 3.5a$; Students may find the constant of proportionality by dividing 7 by 2.

Error Alert Students who wrote $c = 7a$ did not find the unit rate. Remind students that the constant of proportionality used in the equation must be the rate for 1 unit.

9 Solution
32; the car can go 32 miles on 1 gallon of gas.

Ready Mathematics PRACTICE AND PROBLEM SOLVING

Assign *Practice and Problem Solving* **pages 107–108** after students have completed this section.

▶ Hands-On Activity

Graph a proportional relationship.

Materials: colored paper, sheets of graphing paper, scissors, glue, pencils

- Form students into groups of 2 or 3. Using colored paper, have them cut 14 rectangles, each measuring 1 × 5 of their graph paper squares.
- With the graph paper in a vertical orientation, have students draw a coordinate grid with the horizontal axis near the bottom of the sheet and the vertical axis near the left side. Have them graph the ordered pair (2, 10) by gluing 2 rectangles, one above the other, in the second column to the right of the *y*-axis. Have them similarly graph the pairs (3, 15), (4, 20), and (5, 25).
- Tell students the graph represents the total cost of a given number of tickets to a school activity. Have them draw a line connecting the midpoints of the tops of the rectangles and extending to the *y*-axis. Ask: *What is the constant of proportionality, or what would be the y value when x is 1?* [5] *What would this mean in the context of the problem?* [One ticket costs $5.] *Does the graph go through the origin? Does this make sense?* [Yes; zero tickets would cost $0.]

At A Glance

Students study an example to decide if a situation is a proportional relationship and then solve problems about proportional relationships.

Step By Step

- Ask students to solve the problems individually.
- **Pair/Share** When students have completed each problem, have them discuss their solutions with a partner or in a group.

Solutions

Example Comparing fully simplified ratios is one way to see if it is a proportional relationship and to find the constant and insert it into equation form.

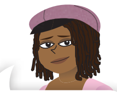
10 Solution

Yes; $\frac{3}{2}$; It means Michael can read 1 page in $1\frac{1}{2}$ minutes; $m = \frac{3}{2}p$; Students could solve the problem by finding $\frac{18}{12} = \frac{12}{8} = \frac{30}{20} = \frac{3}{2}$.

DOK 2

Practice Writing Equations for Proportional Relationships

Study the example below. Then solve problems 10–12.



The student simplifies the ratios using the rules of division.

Example

Julie is making bracelets. The table shows the number of bracelets she can make with different lengths of cord. Determine if the relationship is proportional. If so, find the constant of proportionality and write an equation to represent the situation.

Number of Bracelets (*b*)	Yards of Cord (*c*)
12	3
18	$4\frac{1}{2}$
24	6
30	$7\frac{1}{2}$

Show your work.

Possible student work:

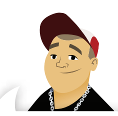
$\frac{3}{12} = \frac{1}{4}$ $\qquad \frac{6}{24} = \frac{1}{4}$

$\frac{4\frac{1}{2}}{18} = \frac{\frac{9}{2}}{18} = \frac{9}{2} \div 18 = \frac{9}{2} \times \frac{1}{18} = \frac{1}{4}$ $\qquad \frac{7\frac{1}{2}}{30} = \frac{\frac{15}{2}}{30} = \frac{15}{2} \times \frac{1}{30} = \frac{1}{4}$

Solution $c = \frac{1}{4}b$; The constant of proportionality is $\frac{1}{4}$.

Pair/Share How do you know that the relationship is proportional?

10 Michael reads 12 pages of a book in 18 minutes, 8 pages in 12 minutes, and 20 pages in 30 minutes. Is this a proportional relationship? If so, what is the constant of proportionality? What does it represent? Write an equation to represent this situation.



How can you find the constant of proportionality?

Show your work. **Possible student work:**

$\frac{18}{12} = \frac{3}{2}$ $\quad \frac{12}{8} = \frac{3}{2}$ $\quad \frac{30}{20} = \frac{3}{2}$

The constant of proportionality is $\frac{3}{2}$.

number of minutes $= \frac{3}{2} \cdot$ number of pages

Solution **The constant of proportionality is $\frac{3}{2}$, which means that Michael can read 1 page in $\frac{3}{2}$ or $1\frac{1}{2}$ minutes. The equation $m = \frac{3}{2} \cdot p$ represents this situation.**

Pair/Share Describe other ways you can find the constant of proportionality.

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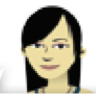
Teacher Notes

11 The equation $c = 2.5t$ represents the cost, c, for t tickets to the school play. Does a value of 3.5 for t make sense in this situation? Explain your reasoning.

Show your work.



What does t represent?

Solution **No; You cannot buy 3.5 tickets. You can either buy 3 tickets or 4 tickets, so t cannot equal 3.5.**

Pair/Share
Describe a situation in which the equation $y = 2.5x$ can have values for x that are mixed numbers.

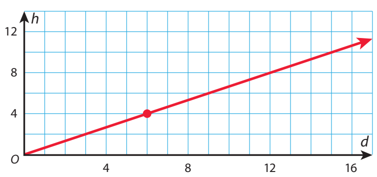
12 The graph shows the height, h, in inches, of a plant after d days. The plant had a height of 4 inches after 6 days. Which equation can you use to represent the situation?

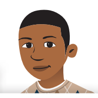
A $h = 4d$

B $h = 6d$

C $h = \frac{3}{2}d$

(D) $h = \frac{2}{3}d$





What point on the graph represents the constant of proportionality?

Stephen chose **C** as his answer. Explain his error.

Stephen divided both coordinates in (6, 4) by the y-coordinate instead of by the x-coordinate. He used the point $\left(\frac{3}{2}, 1\right)$ as the constant of proportionality instead of the point $\left(1, \frac{2}{3}\right)$.

Pair/Share
How do you know that the graph shows a proportional relationship?

Teacher Notes

Solutions

11 Solution
No; Half a ticket cannot be bought.
DOK 2

12 Solution
D; Divide both coordinates in the point (6, 4) by 6, the x-coordinate, in order to get the unit rate.

Explain to students why the other two answer choices are not correct:

A is not correct because students used the y-coordinate of (6, 4) as the unit rate.

B is not correct because students used the x-coordinate of (6, 4) as the unit rate.
DOK 3

Ready® Mathematics PRACTICE AND PROBLEM SOLVING

Assign *Practice and Problem Solving* **pages 109–110** after students have completed this section.

Independent Practice

At A Glance

Students use principles of proportional relationships to solve word problems that might appear on a mathematics test.

Solutions

1 Solution

B; Students could graph a line through (0, 0) and $\left(1, \frac{3}{4}\right)$ or substitute each x and y coordinate into $y = \frac{3}{4}x$ to find which is true after eliminating obvious wrong answers.

DOK 2

2 Solution

B; The meaning of each coordinate is given in the labels on the axes.

DOK 2

3 Solution

A; Divide $18\frac{3}{4}$ by 5 to find $3\frac{3}{4}$ cups of flour for each loaf of bread.

D; The line on the graph represents a ratio of $3\frac{3}{4}$ cups of flour for each loaf of bread.

DOK 2

Practice: Writing Equations for Proportional Relationships

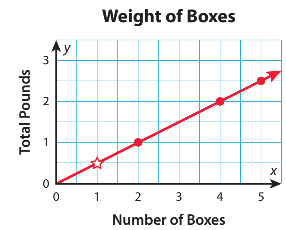
Solve the problems.

1 The equation $r = \frac{3}{4}b$ represents the number of cups of raisins, r, that you need to make b batches of trail mix. Which point would be on a graph that represents this proportional relationship?

- A $\left(\frac{3}{4}, 1\right)$
- **(B)** (4, 3)
- C (3, 4)
- D $\left(0, \frac{3}{4}\right)$

2 Look at the graph. What is the meaning of the point shown with a star?

- A Half of a box weighs 1 pound.
- **(B)** Each box weighs $\frac{1}{2}$ pound.
- C Each box weighs 1 pound.
- D Each box weighs 2 pounds.



3 A baker is making dough for bread. The number of cups of flour he uses is proportional to the number of loaves he makes. The baker uses $18\frac{3}{4}$ cups of flour to make 5 loaves of bread. Which of the following correctly represents this proportional relationship? Select all that apply.

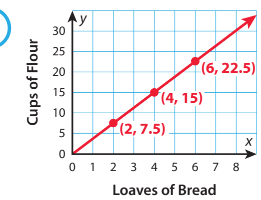
(A) $f = 3\frac{3}{4}b$, where f is the number of cups of flour and b is the number of bread loaves.

B

Cups of flour	$23\frac{3}{4}$	$28\frac{3}{4}$	$33\frac{3}{4}$
Number of loaves	10	15	20

C $b = 3\frac{3}{4}f$, where f is the number of cups of flour and b is the number of bread loaves.

(D)



Quick Check and Remediation

- Ask students to solve this problem:

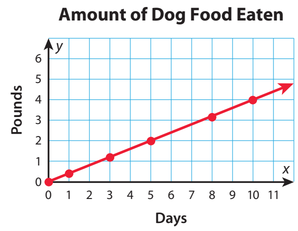
  Jeri ordered packets of beads. Three packets contained a total of 36 beads. Four packets had a total of 48 beads, 5 packets had a total of 60, and 6 packets had a total of 72. Is this a proportional relationship? [yes] If so, find the equation that models the relationship [$b = 12p$, or $p = \frac{1}{12}b$]
- For students who are struggling, use the chart to guide remediation.
- After providing remediation, check students' understanding. Ask students to find the equation that models the relationship shown in the table. [$y = \frac{3}{2}x$]

x	y
2	3
3	$\frac{9}{2}$
4	6
5	$\frac{15}{2}$

- If a student is still having difficulty, use *Ready Instruction, Grade 7,* Lesson 10.

If the error is . . .	Students may . . .	To remediate . . .
$b = 36p$	not have found the unit rate.	Remind students that the constant of proportionality is the unit rate, or the value of y when x is 1. Ask them the total number of beads that came in 3 packets. [36 beads] Then ask how they could find out how many beads were in each packet. [divide $\frac{36}{3} = 12$] Point out that 12 is the unit rate; it shows how many beads were in 1 packet. When you know how many are in 1 packet you can find out how many are in any number of packets.
$p = 12b$	have found the constant of proportionality, but confused the variables.	Tell students to check their answer. Replacing b with 36 would not result in $p = 3$ packets.
$b = p + 12$	have found the correct onstant of proportionality, but added it rather than multiplying.	Remind students that proportionality is a multiplication relationship. Also have students use data to check their equation. Replacing p with 3 results in $b = 15$, not 36.

4 The graph shows the total amount of dog food that Sophia's dog Nipper eats. Which statement is correct? Select all that apply.



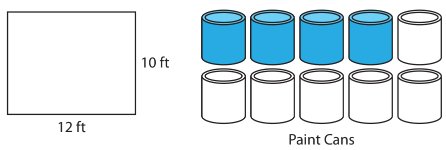
(A) The data represents a proportional relationship.

B The point (5, 2) means that Nipper eats 5 pounds of dog food in 2 days.

C The point (0, 0) has no meaning in this situation.

(D) The constant of proportionality is 0.4.

5 Len wants to paint a wall in his house. He knows that 1 can of paint covers an area of 32 square feet. The dimensions of the wall are shown. Shade in the correct number of paint cans he will need to buy.



6 Tom and Jeff studied the data in the table. They each wrote an equation to represent the relationship between the number of students and the number of pizzas ordered.

Tom's equation: $p = \frac{1}{4}s$

Jeff's equation: $s = 4p$

The teacher said that both equations were correct. Explain why.

Students (s)	Pizzas (p)
28	7
12	3
32	8
16	4

Tom's equation shows that each student eats $\frac{1}{4}$ of a pizza. Jeff's equation shows that each pizza will feed 4 students. The equations are two different ways of explaining the same relationship.

Self Check Go back and see what you can check off on the Self Check on page 79.

Solutions

4 Solution
A, D; The data is modeled by a line through the origin with slope 0.4.
DOK 2

5 Solution
Students shade 4 paint cans; Multiply the length of the wall by the height to find the area. Divide the area by 32 and round up to the nearest whole number.
DOK 2

6 Solution
Each equation represents the same relationship but from the perspective of different independent variables.
DOK 3

▶ Hands-On Activity

Make a model drawing.

Have students pick a large, simple object, such as a chair or desk. Tell students to measure the dimensions and record them in a table. Then have them find measurements of a model that would be scaled to perhaps $\frac{1}{4}$ the size and record those measurements in the table as well. Have students use the scaled measurements to make a drawing of their model.

▶ Challenge Activity

Write your own problem involving a proportional relationship.

Have students write a problem situation that describes a proportional relationship. On a separate paper, have them model the relationship by writing its equation, filling a table of 4 ordered pairs, and drawing its graph. Working in pairs, have students exchange their problem situations, solve each other's problems, and then correct them.

LESSON QUIZ

Lesson 11
Equations for Proportional Relationships

Overview

Assign the Lesson 11 Quiz and have students work independently to complete it.

Use the results of the quiz to assess students' understanding of the content of the lesson and to identify areas for reteaching. See the Lesson Pacing Guide at the beginning of the lesson for suggested instructional resources.

Tested Skills

Assesses 7.RP.A.2c, 7.RP.A.2d

Problems on this assessment form require students to be able to represent proportional relationships with equations and explain what a point (x, y) on the graph of an equation representing a proportional relationship means in terms of a real-world situation.

Context and Vocabulary

Depending on your students' knowledge about earning money at a job, you might explain the term *employee* and the relationship between an employee's hours and earnings.

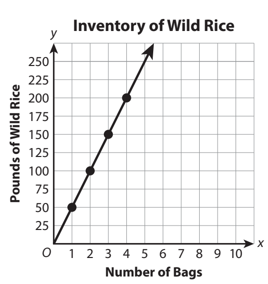
***Ready*® Mathematics**

Lesson 11 Quiz

Solve the problems.

1 The graph shows the relationship between the number of bags of wild rice in a restaurant and the total number of pounds of wild rice in those bags.

Which statement is NOT correct?



A The point (2, 100) means that there are 100 pounds of wild rice in 2 bags.

B The data represents a proportional relationship.

C The equation $x = 50y$ represents this situation.

D The constant of proportionality is 50.

2 The equation $y = 1\frac{1}{2}x$ represents the number of cups of dried fruit, y, needed to make x pounds of granola. Determine whether each point would be on the graph of this proportional relationship.

Choose *Yes* or *No* for each point.

a. $\left(1\frac{1}{2}, 1\right)$ ☐ Yes ☐ No

b. (4, 6) ☐ Yes ☐ No

c. (18, 12) ☐ Yes ☐ No

d. (0, 0) ☐ Yes ☐ No

e. $\left(2\frac{1}{2}, 3\frac{3}{4}\right)$ ☐ Yes ☐ No

Lesson 11 Quiz continued

3 Camille is making a graph that shows the proportional relationship between the amount of money collected, y, and the number of tickets sold, x. She correctly plots the point (300, 750).

Which of these statements are true? Choose all that apply.

A The point (180, 450) is on the graph because $450 dollars will be collected if 180 tickets are sold.

B The point (1, 0.4) is on the graph and identifies that the cost for each ticket is $0.40.

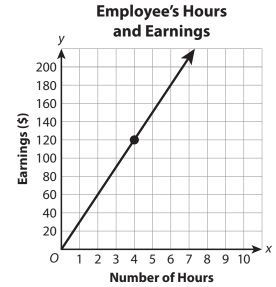
C The constant of proportionality is $\frac{750}{300}$, which is equal to 2.5.

D The point (0, 0) is on the graph because no money is collected if no tickets are sold.

E The point (5, 12.5) is not on the graph because it is impossible to sell 12.5 tickets.

F The equation $y = 2.5x$ represents this situation.

4 The graph shows a proportional relationship between the number of hours an employee works and the amount of money earned in that time.



Part A

Complete the statement to describe the meaning of the point shown on the graph.

Answer: The employee makes $__________ in __________ hour(s).

Part B

Complete the equation that represents this situation.

Answer: $y =$ __________

Common Misconceptions and Errors

Errors may result if students:

- substitute the wrong values into an equation.
- confuse the *x*- and *y*-coordinates in the ordered pair for a point.
- do not find the unit rate.

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Lesson 11 Quiz Answer Key

1. C
DOK 2

2. **a.** No
b. Yes
c. No
d. Yes
e. Yes
DOK 2

3. A, C, D, F
DOK 3

4. ***Part A:***
120, 4
DOK 1

Part B:
$30x$
DOK 2