LESSON OVERVIEW

Lesson 10
Understand Proportional Relationships

CCSS Focus

Domain
Ratios and Proportional Relationships

Cluster
A. Analyze proportional relationships and use them to solve real-world and mathematical problems.

Standards
7.RP.A.2 Recognize and represent proportional relationships between quantities.

a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

Standards for Mathematical Practice (SMP)
3 Construct viable arguments and critique the reasoning of others.

4 Model with mathematics.

Lesson Objectives

Content Objectives
- Determine whether two quantities are in a proportional relationship by looking at values in a table, a line in the coordinate plane, and an equation. (Use equivalent fraction relationships and multiplication/division to find proportional ratios.)
- Identify the constant of proportionality (unit rate) in a table and represented by an equation.

Language Objectives
- Summarize information from text, tables, and graphs to identify proportional relationships and determine constant of proportionality.
- Identify equations that represent proportional relationships and defend reasoning.
- Explain proportional relationships represented by tables and graphs.
- Use *proportional relationship* and *constant of proportionality* accurately in speaking and writing.

Prerequisite Skills
- Understand ratios, unit rate, and proportions.
- Use ratio and unit rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios or equations.
- Graph ordered pairs from a table on a coordinate grid.
- Recognize and generate simple equivalent fractions, including writing whole numbers as fractions.

Lesson Vocabulary
- **proportional relationship** the relationship among a group of ratios that are equivalent.
- **constant of proportionality** the unit rate in a proportional relationship.

Review the following key terms.
- **coordinate plane** a two-dimensional space formed by two perpendicular number lines called *axes*.
- **ordered pair** a pair of numbers (x, y) that describes the location of a point on a coordinate plane.
- **origin** the point (0, 0) where the x-axis and y-axis intersect on a coordinate plane.

Learning Progression

In Grade 6 students learned about proportional relationships using tables and equivalent ratios.

In this lesson students learn that a graph of the proportional relationship is a straight line that passes through the origin. They learn that another name for the unit rate is the constant of proportionality. They use these concepts to analyze relationships that may or may not be proportional. They write equations to describe proportional relationships in the form of $y = mx$, in which m is the constant of proportionality.

In Grade 8 students will connect proportional relationships to linear and nonlinear functions.

Lesson Pacing Guide

Whole Class Instruction

Day	Lesson	Practice
Day 1 *45–60 minutes*	**Toolbox: Interactive Tutorial*** *Recognizing Proportional Relationships* **Introduction** • Think It Through Question *10 min* • Think *15 min* • Think *15 min* • Reflect *5 min*	**Practice and Problem Solving** Assign pages 97–98.
Day 2 *45–60 minutes*	**Guided Instruction** **Think About Identifying Proportional Relationships** • Let's Explore the Idea *15 min* • Let's Talk About It *20 min* • Try It Another Way *10 min*	**Practice and Problem Solving** Assign pages 99–100.
Day 3 *45–60 minutes*	**Guided Practice** **Connect Ideas About Identifying Proportional Relationships** • Compare *10 min* • Apply *10 min* • Analyze *5 min*	**Practice and Problem Solving** Assign pages 101–102.
	Independent Practice **Apply Ideas About Identifying Proportional Relationships** • Put It Together *10 min* • Intervention, On-Level, or Challenge Activity *10 min*	
	Toolbox: Lesson Quiz Lesson 10 Quiz	

*We continually update the Interactive Tutorials. Check the Teacher Toolbox for the most up-to-date offerings for this lesson.

Small Group Differentiation

Teacher-Toolbox.com

Reteach

Ready Prerequisite Lessons *45–90 min*

Grade 6
- Lesson 2 Understand Unit Rate
- Lesson 3 Equivalent Ratios

Teacher-led Activities

Tools for Instruction *15–20 min*

Grade 7
- Identifying Proportional Relationships

Personalized Learning

i-Ready.com

Independent

i-Ready Lessons* *15–20 min*

Grade 7
- Recognizing Proportional Relationships

ELL English Language Development

Prepare for Day 1: Use with *Think It Through*

Math Terms: The *x*-axis and *y*-axis intersect perpendicularly to form a two-dimensional space known as a *coordinate plane*. The point where the axes intersect is called the *origin* and is represented by the *ordered pair* (0, 0).

ELP Levels 1–3

Reading/Writing Display and review the Math Terms. Explain that the plural form of *axis* is *axes*, which is pronounced AX-ees. Post the terms with visuals for reference throughout the lesson.

Model previewing ***Think It Through*** by asking students to point out headings, bold words, tables, graphs and text boxes. Read the text aloud as students follow along. Have students highlight important information using yellow for definitions and green for big ideas. Pause after each section to paraphrase the text or summarize the model. Use the ***Act It Out*** routine to model the proportional relationship of the cost of the movie tickets.

ELP Levels 2–4

Reading/Writing Display and review the Math Terms. Explain that the plural form of *axis* is *axes*, which is pronounced AX-ees. Have students refer to definitions as they read. Have students partner-read ***Think It Through***. Remind them to preview the text to support comprehension. Ask students to highlight important information and then pause after each section to discuss the big ideas. Have students use the highlighted information to take notes in their Math Journals. Model taking notes on the first section by defining *proportional relationships* and *constant of proportionality*. Have partners decide what information from subsequent sections they will record.

ELP Levels 4–5

Reading/Writing Display and review the Math Terms. Ask if students know the plural form of *axis*. If they do not, explain that the plural of *axis* is *axes*, and it is pronounced AX-ees. Have students discuss the meaning of the vocabulary with a partner.

Have students read ***Think It Through*** and take notes in their Math Journals. Review characteristics of note-taking, such as recording big ideas and definitions and using bulleted lists and abbreviations. Pair students to compare their notes with a partner. Ask: *How are your notes and your partner's notes similar? How are they different?* Encourage students to discuss style and content but to focus on content.

Prepare for Day 2: Use with *Let's Talk About It*

Math Terms: An *ordered pair* uses two numbers to describe the location of a point in the coordinate plane. When you *plot a point*, you locate and graph a point on the coordinate plane using the coordinates from an ordered pair.

ELP Levels 1–3

Listening/Speaking Display and review the Math Terms. Model *plotting points* on a graph using *ordered pairs*.

Read the ***Let's Talk About It*** problems aloud. Display a concrete model such as the one described in the *Hands-On Activity* to help students visualize common ratios. Provide the following sentence frames to support discussion:

- The ratio for _____ is _____ to _____.
- Mixtures _____ and _____ are in a proportional relationship.
- The constant of proportionality is _____ per _____.

ELP Levels 2–4

Listening/Speaking Display and review the Math Terms. Support students as they use the terms in sample sentences.

Pair students to read and discuss the ***Let's Talk About It*** questions. Provide the following sentence frames to support discourse:

- The ratio for _____ is _____.
- _____ and _____ are in a proportional relationship.
- The table shows _____ so _____.
- The graph shows _____ so _____.

ELP Levels 4–5

Listening/Speaking Display the Math Terms and have partners discuss their meanings. Then have pairs read and discuss the ***Let's Talk About It*** questions. Provide a word bank with terms that support student discourse such as *ratio, constant of proportionality, proportional relationship, origin,* and *in common*. Challenge partners to ask questions that require them to defend and explain their answers, such as:

- How do you know _____?
- How did you find _____?
- Why did you _____?
- Can you explain _____?

Prepare for Day 3: Use with *Connect, Problem 12*

ELP Levels 1–3

Reading/Writing Adapt the ***Three Reads*** routine by reading ***Connect*** problem 12 aloud as students follow along. After each reading, call on students to identify the purpose of each reading. Point out that there are two questions to answer.

Organize students into small groups to interpret the graphs and explain how to solve the problem. Then display the following sentence frames for students to complete as a group:

- Games A and C have a proportional relationship between level and number of points.
- In Level 2, you earn 100 points in Game A, 150 in Game B, and 200 in Game C.

ELP Levels 2–4

Reading/Writing Pair students to solve ***Connect*** problem 12. Adapt the ***Three Reads*** routine by reading the text aloud to students, pausing between readings to allow time for partners to discuss each purpose for reading.

Have partners interpret the graphs and use the information to write a response to the problem that addresses both questions.

Encourage partners to incorporate the following terms into their written answers: *proportional relationship, line,* and *origin*.

ELP Levels 4–5

Reading/Writing Pair students to solve the ***Connect*** problem 12. Have them use the ***Three Reads*** routine to make sense of what the problem is asking them to do.

Before writing, adapt the ***Co-constructed Word Bank*** routine by having partners list terms they will likely include in their responses. Remind them to use a series of connected sentences to answer the questions in the problem. Ask partners to draft their responses independently, then have them trade and compare their responses.

After reading their partner's response, allow students the opportunity to revise their answers if necessary.

Introduction

At A Glance

Students review the idea that data displayed in a table show a proportional relationship if all the ratios formed are equivalent. They learn that the ratio expressed as the unit rate is called the constant of proportionality. Then they explore how to use graphs to determine whether relationships are proportional.

Step By Step

Think It Through

- Introduce the Question at the top of the page.
- Reinforce the definitions of *proportional relationship* and *constant of proportionality.* Have a volunteer explain what a unit rate is and relate it to the constant of proportionality.

▶ **Concept Extension**

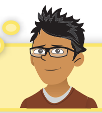
- Read the first part of **Think** with students. Make sure students can connect the data in the first table with the ratios and the equations. Relate all the representations to the context of movie tickets.

▶ **Mathematical Discourse 1–3**

- Read the second part of **Think** with students. Ask how the simplified ratios formed by the data in the second table are different from those formed by the data in the first. Emphasize that when the ratios are not equivalent, the data do not show a proportional relationship.

Understand Proportional Relationships

Think It Through



What is a proportional relationship?

Suppose you and some friends plan to go to a movie and the tickets cost $8 each.

You will pay $8 for 1 ticket, $16 for 2 tickets, $24 for 3 tickets, $32 for 4 tickets, and so on. The ratios of the total cost of the tickets to the number of tickets are all equivalent.

A group of ratios that are equivalent are in a **proportional relationship**. When ratios are equivalent, they all have the same unit rate. In a proportional relationship, the unit rate is called the **constant of proportionality**.

Think How can you use a table to tell if a relationship is proportional?

The table below shows the total cost of movie tickets based on the number of tickets you buy.

Total Cost of Tickets ($)	8	16	24	32
Number of Tickets	1	2	3	4

Circle the ratio in the table that shows the constant of proportionality.

The ratios of the total cost of tickets to the number of tickets are equivalent. The ratios all simplify to $\frac{8}{1}$ or 8, so **the ratios are in a proportional relationship**.

$\frac{8}{1} = 8$ $\quad \frac{16}{2} = 8$ $\quad \frac{24}{3} = 8$ $\quad \frac{32}{4} = 8$

The unit rate is 8, so the constant of proportionality is 8. The equation $c = 8t$, where c is the total cost and t is the number of tickets, represents this relationship. The total cost is always 8 times the number of tickets.

The table below shows the cost to play in the town soccer tournament.

Total Cost ($)	7	8	9	10
Number of Family Members	1	2	3	4

You can find and simplify the ratios of the total cost to the number of family members.

$\frac{7}{1} = 7$ $\quad \frac{8}{2} = 4$ $\quad \frac{9}{3} = 3$ $\quad \frac{10}{4} = 2\frac{1}{2}$

The ratios are not equivalent, so **the quantities are not in a proportional relationship**.

▶ **Mathematical Discourse**

1 *Relationships can be described in equations and in words. The relationship of total cost to tickets is shown in the equation $c = 8t$. How could you describe the relationship in a "word equation"?*

Responses should convey the idea that the total cost of the tickets is 8 times the number of tickets.

2 *How would your word equation be different if the situation were about teams and players?*

The total number of players is 8 times the number of teams.

3 *Think of something in our class or school that $c = 8t$ could describe and use it in a word equation.*

Students might suggest desks in a group, students at lunch tables, or weeks in a semester.

▶ **Concept Extension**

Reinforce the connection between constant of variation and unit rate.

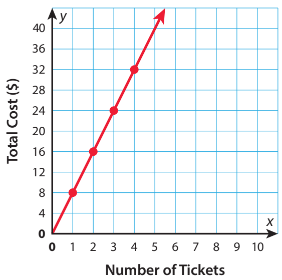
Materials: dictionary

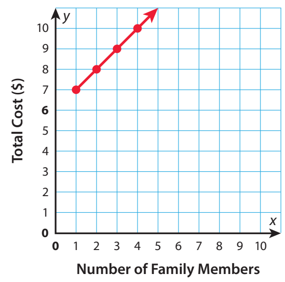
- Write *constant of variation* on the board. Say that variation means change.
- Have students look up the word *constant* in the dictionary. Have them read the various definitions and decide which definition best applies to the term *constant of variation*.
- Have volunteers describe the meaning of constant of variation in their own words.
- Have students explain why a unit rate expresses a constant rate of change and therefore can be called the constant of variation.

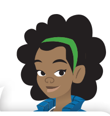
Think **How can you use a graph to tell if a relationship is proportional?**

You can use a graph to determine if a relationship is proportional.

The data for the cost of movie tickets and the cost to participate in the soccer tournament can be modeled by the graphs below.







Compare the two graphs. How are they alike? How are they different?

The points on the graphs are on a straight line for both sets of data, but only the data for the cost of movie tickets goes through the origin. This means that only the total cost of the movie tickets compared to the number of tickets is a proportional relationship.

Proportional Relationship	Non-Proportional Relationship
• The graph can be represented by a **straight line**. • The straight line **goes through the origin**.	• The graph may or may not be represented by a straight line. • If the graph is a straight line, it does not go through the origin.

Reflect

1 Look at the graph that compares the total cost to the number of movie tickets you buy. How can you use the graph to find the cost of 5 movie tickets?

Possible answer: The point (1, 8) shows that $8 represents the cost of 1 ticket.

Therefore, multiply 5 tickets by $8 to get $40 for 5 tickets.

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Step By Step

- Read **Think** with the students. Ask students how they can represent the data in a table using a graph.

▶ English Language Learners

- Have students compare and contrast the two graphs. Discuss why the first graph shows a proportional relationship but the second graph does not.
- After students have read the information in the table, have them restate each statement in their own words.
- Have students read and reply to the **Reflect** directive.

▶ Mathematical Discourse 4 and 5

SMP TIP Model with Mathematics
Using graphs to determine whether a relationship is proportional helps students see how they can model real-world situations with mathematics. *(SMP 4)*

Ready® Mathematics PRACTICE AND PROBLEM SOLVING

Assign *Practice and Problem Solving* **pages 97–98** after students have completed this section.

▶ English Language Learners

- Sketch examples and non-examples of *straight line* and *through the origin* on the board. Model the correct language such as: *This graph goes through the origin, but it is not a straight line* or *This is a straight line that does not go through the origin.*
- Have a volunteer go to the board and draw an example or non-example on a coordinate plane. The volunteer will call on classmates to describe the graph using *straight line* and *through the origin*. Repeat with other volunteers.
- Once students are comfortable with the vocabulary, tie the terms to the graphs of proportional and non-proportional relationships.

▶ Mathematical Discourse

Extend the discussion of the **Reflect** directive with these questions.

4 *How can you use the movie ticket graph to find the constant of proportionality?*

Responses should describe how the student could find the constant of proportionality from the graph.

5 *Is there another way to find the constant of proportionality?*

Responses could include making a table of ratios from the points on the line, using the y-coordinate of the point where $x = 1$, or recognizing that each point is 8 units higher on the y-axis.

Guided Instruction

At A Glance

Students examine data in tables to see if they represent proportional relationships. Then they graph data from a table to see if there is a proportional relationship.

Step By Step

Let's Explore the Idea

- Tell students that they will have time to work individually on the problems on this page and then share their responses in groups.
- As students work individually, circulate among them. This is an opportunity to assess student understanding and address student misconceptions. Use the Mathematical Discourse questions to engage student thinking.

▶ Mathematical Discourse 1–3

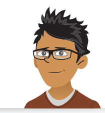
- For the second table, suggest to students that they can use either equivalent ratios or graphs to determine if the relationships are proportional.
- Help students understand what they are being asked to find in the last problem. Help them connect their answer to the idea of equivalent ratios.
- Take note of students who are still having difficulty and wait to see if their understanding progresses as they work in their groups during the next part of the lesson.

Student Misconception Alert
Some students may find the ratios but not remember that **all** the ratios must be the same for the data to be proportional and have a constant of proportionality. Have students find and simplify the ratios for each problem. Then note that there can be only one constant of proportionality. If the simplified ratios are not equivalent, ask students why they cannot pick one of them to be the constant of proportionality. Then reinforce the idea that the relationship is not proportional.

Think About Identifying Proportional Relationships

Let's Explore the Idea Use the table below to analyze the cost of downloading applications to a phone.

Number of Downloads	2	4	5	6	10
Total Cost ($)	6	12	15	18	30



2 How can you find the ratio of the total cost to the number of downloads?
Divide the total cost by the corresponding number of downloads.

3 What is the ratio of the total cost to the number of downloads when you download
2 applications? $\frac{6}{2}$ 4 applications? $\frac{12}{4}$ 5 applications? $\frac{15}{5}$
6 applications? $\frac{18}{6}$ 10 applications? $\frac{30}{10}$

4 Are the data in the table in a proportional relationship? If so, what is the constant of proportionality? Yes; all the ratios are equivalent so the data are in a proportional relationship. The constant of proportionality is 3.

Now try these problems.

5 The table shows the number of hours needed for different numbers of people to clean up after a school dance.

Hours Needed to Clean Up	12	9	8	6
Number of People Cleaning	2	3	4	6

Are the quantities in the table in a proportional relationship? Explain your reasoning.
No. The ratios are not equivalent, so the quantities are not proportional.

6 The students in the Service Club are mixing paint to make a mural. The table below shows the different parts of paint that the students mix together.

	A	B	C	D	E
Parts of Red Paint	1	2	4	2	3
Parts of White Paint	3	4	8	6	9

Two mixtures of paint will be the same shade if the red paint and the white paint are in the same ratio. How many different shades of paint did the students make? Explain.
2; The ratio of white paint to red paint is $\frac{3}{1}$ for A, D, and E and is $\frac{2}{1}$ for B and C.

▶ Mathematical Discourse

1 *How can you tell if the data in the table form equivalent ratios?*
Responses might indicate that if they all simplify to the same ratio, then they are equivalent.

2 *Do you think you should check every ratio before you decide if the relationship is proportional or not? Why or why not?*
Responses might include that you can recognize a non-proportional relationship with the first non-equivalent ratio.

3 *If the relationship is proportional, how do you find the constant of proportionality? Could you do it another way?*
Responses might use the term "unit rate" or indicate that it is the ratio with the denominator of 1.

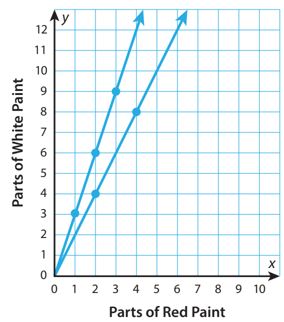
Let's Talk About It

Solve the problems below as a group.

7 Refer to the situation in Problem 6. Which shades of paint are the most red? Why?

Mixtures B and C; Possible explanation: For Mixtures B and C, the ratio of white paint to red paint is 2 to 1. For Mixtures A, D, and E, the ratio of white paint to red paint is 3 to 1, so there is more red paint in Mixtures B and C.

8 Use the table in Problem 6. Plot a point for each ordered pair. After you plot each point, draw a line connecting the point to (0, 0).



9 Based on the graph, what do the mixtures that are the same shade have in common? What does this tell you about their relationship?

The points lie on a line that goes through the point (0, 0) so they are in a proportional relationship.

Try It Another Way

Work with your group to determine whether the equation represents a proportional relationship. Explain your choice. You may want to make a table similar to those in Problems 5 and 6 or a graph similar to that in problem 8 on separate sheet of paper to support your reasoning.

10 $y = 2x + 4$ **No; the graph is a line but it does not go through the origin.**

11 $y = 2x$ **Yes; the graph is a line and goes through the origin.**

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Step By Step

Let's Talk About It

- Organize students into pairs or groups. You may choose to work through the first **Let's Talk About It** problem together as a class.
- Walk around to each group. Listen to and join in on discussions at different points. Use questions to help support or extend students' thinking.
- If students need more support, have them use the Hands-On-Activity to help them visualize the common ratios.

▶ **Hands-On Activity**

Try It Another Way

- Direct the group's attention to **Try It Another Way**. Have a volunteer from each group come to the board to present a table or graph that illustrates the group's solutions to problems 10 and 11.

▶ **Mathematical Discourse 4–6**

Ready Mathematics PRACTICE AND PROBLEM SOLVING

Assign *Practice and Problem Solving* **pages 99–100** after students have completed this section.

▶ Hands-On Activity

Use concrete materials to model ratios.

Materials: red paper, white paper, scissors, drawing paper, glue sticks

- Have students cut 12 small squares from red paper and 30 from white paper.
- Have them divide a sheet of drawing paper into 5 sections.
- Students should use glue and the small squares to illustrate the following ratios of red paint to white paint: 1 to 3, 2 to 4, 4 to 8, 2 to 6, and 3 to 9.
- Direct students to write 2 or 3 sentences that identify the two sets of equivalent ratios and explain why they are equivalent.

▶ Mathematical Discourse

4 *For the Try It Another Way problem, what did your group do to get started with the questions?*

Responses may include making a table of values and graphing or checking equivalent ratios.

5 *Did other groups use a different way to decide which relationship is proportional?*

Listen for responses that show students have connected the form of the equation to proportional and non-proportional relationships and encourage explanation as a preview to upcoming lessons.

6 *How can you use your method to decide if $y = 3x + 6$ is proportional?*

Responses should indicate understanding of the method.

Guided Practice

At A Glance

Students demonstrate their understanding of proportional relationships as they examine relationships represented using both graphs and tables. Then students generate one table of data that compares side length and perimeter and another that compares side length and area. They analyze the data using both ratios and graphs to determine if the data are proportional.

Step By Step

- Discuss each **Connect** problem as a class using the discussion points outlined below.

Compare

- As students evaluate each graph, have them identify the two features that show whether a graph shows a proportional relationship.
- Lead a class discussion that relates the idea of a constant ratio to the graphs. Ask: *What is the number of points possible for Level 1 of each game?* [A: 50; B: 100, C: 100] *Do you think the ratio of points per level will remain constant for all the levels of each game?* [Only for Games A and C]

Apply

- The second problem focuses on the idea of the unit rate or constant of proportionality.
- Once students find the unit rate, have them explain how they can use it to find the total cost of other amounts of yogurt.

Analyze

- This problem requires students to focus on the multiplicative aspects of proportional relationships.
- Have students suggest different ways to show whether the data are proportional. If they use ratios, discuss why there is no constant of proportionality. If students chose to use a graph, have them explain why it does not display a proportional relationship.
- After students have determined that the data are not proportional, have them examine the table. Ask: *How do the boys' ages compare when you go from one column to the next?* [Raj's age is always 5 more than Dani's.] *Is the ratio of Raj's age to Dani's age a constant?* [No] *How does this confirm that the data are not proportional?* [The ages are related by addition, not multiplication.]

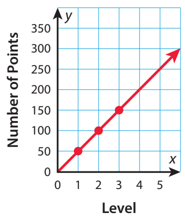
SMP TIP Construct Arguments
Encourage students to support their answers by referring to the characteristics of the graph or the idea of equivalent common ratios. This helps them practice constructing viable arguments and critique the reasoning of others as they explain whether the relationships are proportional. *(SMP 3)*

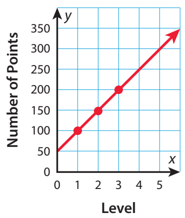
Lesson 10 Guided Practice

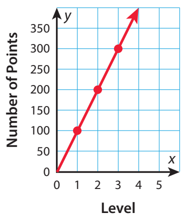
Connect Identifying Proportional Relationships

Talk through these problems as a class. Then write your answers below.

12 Compare The graphs below show the number of points you earn in each level of a game. Which games, if any, have a proportional relationship between the number of points you earn and the level of the game? In which game can you earn the most points in Level 2? Explain your answer.







Games A and C have a proportional relationship. The points are on straight lines that go through the origin; You earn the most points in Game C. For Level 2, you earn 100 points in Game A, 150 points in Game B, and 200 points in Game C.

13 Apply Servers at a snack shop use the table below to find the total cost for frozen yogurt, but some of the numbers have worn off. The total cost is proportional to the number of cups of frozen yogurt. Find the missing numbers in the table.

Number of Cups of Frozen Yogurt	1	2	3	4
Total Cost ($)	4.50	9.00	13.50	18.00

14 Analyze Michael says that the difference between Dani's and Raj's ages is always the same, so Raj's age is proportional to Dani's age. Is Michael correct? Explain.

	2010	2015	2020	2025
Dani's Age	5	10	15	20
Raj's Age	10	15	20	25

No, Michael is not correct. Possible explanation: The difference between their ages is always the same, but none of the ratios $\frac{10}{5}$, $\frac{15}{10}$, $\frac{20}{15}$, or $\frac{25}{20}$ are equivalent, so the ages are not in a proportional relationship.

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Apply Identifying Proportional Relationships

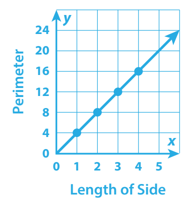
15 Put It Together Use what you know to complete this task.

Paige works in an art store that sells square pieces of canvas. There are 5 different squares to choose from.

Canvas	A	B	C	D	E
Length of side (in feet)	1	2	3	4	5

Part A Add a row to the table to show the perimeter for each square piece of canvas. Then draw a graph to compare the length of a side of each square to its perimeter. Use your table and graph to explain whether this is a proportional relationship.

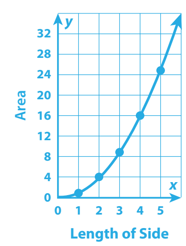
Canvas	A	B	C	D	E
Length of side (in feet)	1	2	3	4	5
Perimeter	4	8	12	16	20



Yes; the ratios are all equivalent to $\frac{4}{1}$. The graph is a line through the origin.

Part B Add a row to the table to show the area for each square piece of canvas. Then draw a graph to compare the length of a side of each square to its area. Use your table and graph to explain whether this is a proportional relationship.

Canvas	A	B	C	D	E
Length of side (in feet)	1	2	3	4	5
Perimeter	4	8	12	16	20
Area	1	4	9	16	25



No; Possible explanations: The ratios of the areas to the side lengths are not all equal; The graph of the ratios of the areas to the side lengths is not a straight line.

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Step By Step

Put It Together

- Direct students to complete **Put It Together** on their own.
- As students work on their own, walk around to assess their progress and understanding, to answer their questions, and to give additional support, if needed.
- If time permits, have students share their tables and graphs and explain why they do or do not show a proportional relationship.

Ready Mathematics PRACTICE AND PROBLEM SOLVING

Assign *Practice and Problem Solving* **pages 101–102** after students have completed this section.

Scoring Rubrics

See student facsimile page for possible student answers.

Part A

Points	Expectations
2	The response demonstrates the student's mathematical understanding of how to show that a relationship is proportional using both • a table of equivalent ratios and • a graph of a straight line passing through the origin.
1	The student was able to show that the data are proportional using either a table of equivalent ratios or a graph, but not both.
0	There is no response or the response does not demonstrate that the data are proportional.

Part B

Points	Expectations
2	The response demonstrates the student's mathematical understanding of how to show that a relationship is not proportional because • the ratios formed by the data in the table are not equivalent and • the graph formed by the data is not a straight line.
1	The student was able to show that the data are not proportional by showing that the ratios formed are not equivalent or the graph formed is not a straight line, but not both.
0	There is no response or the response does not demonstrate that the data are not proportional.

Differentiated Instruction

▶ Intervention Activity

Use graphs to model proportional and non-proportional relationships.

Materials: graph paper

Students will connect graphs, ratios, and proportional relationships.

Students should label the left half of a sheet of graph paper "Proportional" and draw and label a coordinate plane. They should plot 5 points that lie on a line passing through the origin. Beneath the graph, have them record the data in a table with rows labeled x and y. Have them find and simplify the ratios, x:y. Review the idea of the constant of proportionality and have them record their constant of proportionality below the table.

Have students label the right half "Not Proportional" and repeat the process with 5 points that are not part of a straight line passing through the origin. After they find and simplify the ratios, x:y, discuss why the data do not have a constant of proportionality.

▶ On-Level Activity

Analyze real-world situations to see if they are proportional.

Materials: graph paper

Students will generate data from real-world situations and then analyze the relationships to see if they are proportional.

Write the following information on the board.

Video Plan A: $2 for each video you rent

Video Plan B: $1 for each video you rent plus a $10 monthly fee

Have students make a table of data for each plan to show the amount it would cost to rent various numbers of videos in one month. After they have generated the data, ask students to describe two methods they can use to tell whether or not either plan represents a proportional relationship. Then have them work in pairs to analyze each set of data using both ratios and a graph. They should then explain why Plan A is a proportional relationship and name the constant of proportionality.

▶ Challenge Activity

Develop and interpret a proportional relationship.

Materials: graph paper

Students will develop and interpret a proportional relationship from a point on a coordinate plane.

Have students plot one point such as (3, 6) or (5, 2) on a coordinate plane. They should connect the origin and their point and extend the line to the edge of the paper. Have them identify several other points on the line and enter the coordinates in a table with rows labeled x and y.

Have students work individually to find the following:

- the ratio of x to y in simplest form for each point
- the constant of proportionality
- an equation that relates x and y
- a real-world situation that could be modeled by their data

Have students share their work in small groups. They should explain how the graph, the table, the equation, and the real-world situation are related.

Teacher Notes

LESSON QUIZ

Lesson 10 *Understand* Proportional Relationships

Teacher-Toolbox.com

Overview

Assign the Lesson 10 Quiz and have students work independently to complete it.

Use the results of the quiz to assess students' understanding of the content of the lesson and to identify areas for reteaching. See the Lesson Pacing Guide at the beginning of the lesson for suggested instructional resources.

Context and Vocabulary

Review the meaning of the term *substance* in the context of problem 1. Ensure that students understand that the academic term *assume* in problem 3 means "understand to be true."

Tested Skills

Assesses 7.RP.A.2a, 7.RP.A.2b

Problems on this assessment form require students to be able to decide whether two quantities are in a proportional relationship using tables, graphs, equations, and verbal descriptions; and identify the constant of proportionality (unit rate). Students will also need to be familiar with concepts of ratio, unit rate, and proportions to solve real-world problems.

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Lesson 10 Quiz

Solve the problems.

1 Cal is measuring the temperature changes in four substances over different time periods as part of a school project. He records his data in the four tables shown.

Which data show a proportional relationship? Choose all that apply.

A

Time (hours)	3	5	6	9
Temperature (°F)	12	25	36	81

B

Time (min)	3	4	7	9
Temperature (°F)	22.5	30	52.5	67.5

C

Time (hours)	2	3	5	8
Temperature (°F)	10	15	25	40

D

Time (min)	2	5	7	8
Temperature (°F)	8.5	22.5	28.5	36

2 A water truck is filling a swimming pool. The equation that represents this relationship is $y = 19.75x$ where y is the number of gallons of water in the pool and x is the number of minutes the truck has been filling the pool.

Choose *True* or *False* for each statement.

a. The unit rate for this relationship is 1 gallon per 19.75 minutes. ☐ True ☐ False

b. This is a proportional relationship because the graph of the equation is a straight line through the origin. ☐ True ☐ False

c. The water in the swimming pool increases by about 100 gallons every 5 minutes. ☐ True ☐ False

d. A table of related x- and y-values for this relationship would show equivalent ratios. ☐ True ☐ False

e. The point (8, 150) could be a point on this graph. ☐ True ☐ False

Lesson 10 Quiz continued

3 Jessica reads a story about a turtle that swam 16 feet in 24 seconds. She assumes that the time, t, that it takes the turtle to swim a distance of d feet is a proportional relationship. Jessica starts to create the table shown for other times and distances the turtle swims.

Enter the missing numbers in the table to represent the proportional relationship.

Time (seconds)	1	______	24	45
Distance (feet)	______	10	16	______

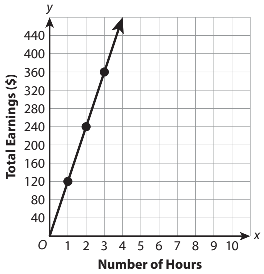
4 Two different crews earn different amounts for different hours they work.

Part A

The graph shows the proportional relationship between the number of hours Crew A worked and their total earnings.

What is the constant of proportionality?

Answer: ______________



Part B

Crew B earns \$315 after 3 hours, \$525 after 5 hours, and \$735 after 7 hours of work.

Explain whether or not Crew B's data show a proportional relationship. If the data are proportional, identify the constant of proportionality.

____

____

____

____

____

Common Misconceptions and Errors

Errors may result if students:

- use the wrong values to determine the ratios for a set of data.
- find the ratio for only one data point instead of all of them in a set of data.
- misinterpret the values when a relationship is written in the form $y = mx$.
- misread the values plotted on each axis of a graph.

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Lesson 10 Quiz Answer Key

1. B, C
DOK 2

2. **a.** False
b. True
c. True
d. True
e. False
DOK 2

3.

Time (seconds)	1	**15**	24	45
Distance (feet)	$\frac{2}{3}$	10	16	**30**

DOK 2

4. ***Part A:***
120
DOK 1

Part B:
Yes, Crew B's data show a proportional relationship. Possible explanation: The data are proportional if the ratios are equivalent. $\frac{315}{3} = \frac{525}{5} = \frac{735}{7} = 105$. The data show a proportional relationship with a constant of proportionality of 105.
DOK 2