LESSON OVERVIEW

Lesson 9
Ratios Involving Complex Fractions

CCSS Focus

Domain
Ratios and Proportional Relationships

Cluster
A. Analyze proportional relationships and use them to solve real-world and mathematical problems.

Standard
7.RP.A.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks* $\frac{1}{2}$ *mile in each* $\frac{1}{4}$ *hour, compute the unit rate as the complex fraction* $\frac{\frac{1}{2}}{\frac{1}{4}}$ *miles per hour, equivalently 2 miles per hour.*

Standards for Mathematical Practice (SMP)
1 Make sense of problems and persevere in solving them.

6 Attend to precision.

7 Look for and make use of structure.

Lesson Objectives

Content Objectives
- Compute unit rates involving ratios with a fraction in the denominator.
- Compute unit rates involving ratios with a fraction in the numerator.
- Compute unit rates involving ratios with a fraction in both the numerator and denominator.

Language Objectives
- Read about finding unit rates that involve complex fractions and confirm understanding by paraphrasing.
- Explain how to find a unit rate by using number lines and simplifying ratios involving complex fractions.
- Use the terms *unit rate, complex fraction, per, adjust, convert,* and *justify* accurately in speaking and writing.
- Discuss the steps for simplifying complex fractions to find a unit rate.
- Interpret data in tables, recipes, and illustrations to solve word problems involving unit rate.

Prerequisite Skills
- Compute unit rates involving ratios with whole numbers.
- Find equivalent fractions.
- Divide fractions.
- Write whole numbers as fractions.

Lesson Vocabulary
- **unit rate** a rate in which the first quantity is compared to 1 unit of the second quantity.
- **complex fraction** a fraction where either the numerator is a fraction, the denominator is a fraction, or both the numerator and the denominator are fractions.

Review the following key terms.
- **per** for each or in each.
- **adjust** to change or modify.
- **convert** to change from one measurement unit to another.
- **justify** to prove something is right or correct by giving reasons.

Learning Progression

In Grade 6 students learned the concept of unit rates.

In this lesson students focus on solving unit-rate problems involving complex fractions. Students model real-world situations that involve ratios with fractions in the numerator and/or denominator. They also learn to connect the process of simplifying complex fractions with the algorithm for division of fractions. Students also learn how to interpret simplified ratios as unit rates to solve real-world problems.

Later in Grade 7 students will extend the concepts of unit rate that they learned in Grade 6.

Lesson Pacing Guide

Whole Class Instruction

Use the Ratios Involving Complex Fractions slides in the Teacher Toolbox for the Think-Share-Compare routine.

Day 1
45–60 minutes

Toolbox: Interactive Tutorial*
Ratios involving Complex Fractions

Introduction
- Use What You Know *20 min*
- Find Out More *15 min*
- Reflect *10 min*

Practice and Problem Solving
Assign pages 87–88.

Day 2
45–60 minutes

Modeled and Guided Instruction

Learn About Finding Unit Rates with Fractions
- Model It *15 min*
- Connect It *20 min*
- Try It *10 min*

Practice and Problem Solving
Assign pages 89–90.

Day 3
45–60 minutes

Modeled and Guided Instruction

Learn About Comparing Unit Rates
- Model It *20 min*
- Connect It *15 min*
- Try It *10 min*

Practice and Problem Solving
Assign pages 91–92.

Day 4
45–60 minutes

Guided Practice

Practice Finding Ratios Involving Complex Fractions
- Example *5 min*
- Problems 18–20 *15 min*
- Pair/Share *15 min*
- Solutions *10 min*

Practice and Problem Solving
Assign pages 93–94.

Day 5
45–60 minutes

Independent Practice

Practice Finding Ratios Involving Complex Fractions
- Problems 1–5 *25 min*
- Quick Check and Remediation *10 min*
- Hands-On or Challenge Activity *10 min*

Toolbox: Lesson Quiz
Lesson 9 Quiz

*We continually update the Interactive Tutorials. Check the Teacher Toolbox for the most up-to-date offerings for this lesson.

Small Group Differentiation

Teacher-Toolbox.com

Reteach
Ready Prerequisite Lessons *45–90 min*

Grade 6
- Lesson 1 Ratios
- Lesson 2 Understand Unit Rate

Teacher-led Activities
Tools for Instruction *15–20 min*

Grade 7
- Divide Positive and Negative Rational Numbers
- Fractions, Division, and Unit Rates

Personalized Learning

i-Ready.com

Independent
i-Ready Lessons* *15–20 min*

Grade 7
- Ratios involving Complex Fractions

ELL English Language Development

Prepare for Day 1: Use with *Find Out More*

Academic Vocabulary: *Per* can mean "for each" and "in each."

ELP Levels 1–3

Reading/Speaking Display and review the Academic Vocabulary. Display examples of the term in context, such as *3 notebooks per student, 20 miles per hour, 4 chairs per table*. Sketch the meanings of each phrase.

Read ***Find Out More*** aloud as students follow along. Adapt the ***In Your Own Words*** routine by modeling paraphrasing after each paragraph. Pair students to orally complete the sentence frames:

- The unit rate is 16, or 16 miles per hour.
- The fraction $\frac{12}{\frac{3}{4}}$ is a complex fraction because the denominator is a fraction.

ELP Levels 2–4

Reading/Speaking Display and review the Academic Vocabulary. Encourage students to scan ***Find Out More*** to locate *per* in a sentence. Have partners substitute *for each* and *in each* for *per* to decide which makes more sense. [in each]

Have partners preview ***Find Out More*** and then read it. Ask them to underline important information. After each paragraph, have partners use the underlined information to paraphrase. Have partners study the equation model and retell the steps for dividing by a fraction.

ELP Levels 4–5

Reading/Speaking Display the Academic Vocabulary. Support students as they share examples aloud of sentences with *per*.

Have students partner-read ***Find Out More***, using the ***In Your Own Words*** routine to paraphrase information after each paragraph. After reading, have students retell the steps for dividing by a fraction. Ask them to use precise math and academic vocabulary as they restate the steps, including *unit rate, per, complex fraction, denominator, numerator,* and *quotient*.

Call on students to share their descriptions of the modeled equation with the class.

Prepare for Day 2: Use with *Learn About* and *Model It*

Academic Vocabulary: *Adjust* means "to change or modify something."

ELP Levels 1–3

Reading/Speaking Display and review the Academic Vocabulary. Read the ***Learn About*** problem aloud as students follow along. Point out the word *adjust* and explain that Max must adjust the amount of the other ingredients so that the ratio of other ingredients to eggs stays the same.

Work with small groups to interpret ***Model It***. Provide the following sentence frames:

- Start both number lines at _____.
- Line up _____ with 2 eggs.
- Find the point for 1 egg halfway between _____ and _____.
- The number that lines up with 1 is _____.

ELP Levels 2–4

Reading/Speaking Display and review the Academic Vocabulary. Read the ***Learn About*** problem aloud. Call on a student to explain why Max needs to adjust the amounts of the other ingredients.

Pair students to interpret ***Model It*** by studying the double number lines and reading the labels. Have them cover the labels with a sticky note and explain the models. Provide the following sentence starters:

- Start _____.
- Line up _____.
- Find _____.

ELP Levels 4–5

Reading/Speaking Display the Academic Vocabulary. Ask partners to locate the term *adjust* in the ***Learn About*** problem and think of a synonym for it. Possible answers include *change*, *modify*, and *alter*.

Pair students to interpret ***Model It***. Have partners take turns covering the labels with sticky notes and explaining how to use the double number lines to find the unit rate. Then have students explain how to find the unit rate of brown sugar for 1 egg using double number lines. Ask: *How would finding the unit rate for vanilla be different?*

Prepare for Day 3: Use with *Connect It*

Academic Vocabulary: *Convert* means "to change from one unit of measurement to another." *Justify* means "to give reasons explaining why an answer is correct."

ELP Levels 1–3

Speaking/Writing Display and review the Academic Vocabulary. Ask students to locate the terms in ***Connect It***.

Read ***Connect It*** aloud as students follow along. After each question, use the ***Turn and Talk*** routine for partners to share ideas. Call on students to share with the whole group. Use the ***Revoicing*** routine as appropriate to clarify ideas and model correct grammar and sentence structure. For problem 11, have students use the models from the previous page to support their explanations. For problem 14, review the term *reasoning*. Point out the academic vocabulary in problem 15 and solve it as a group.

ELP Levels 2–4

Speaking/Writing Display and review the Academic Vocabulary. Support students as they practice using the words in sentences. Point out the use of the terms in ***Connect It***.

Read the ***Connect It*** problems aloud. After each question, use the ***Turn and Talk*** routine for partners to share ideas. Call on students to share with the whole group. Validate understanding and clarify misconceptions as students share. Provide a sentence starter to help students frame their answers, such as: Use the number lines to find the cost of 1 pound of coffee by _____. Allow time for students to write their answers independently.

ELP Levels 4–5

Speaking/Writing Display the Academic Vocabulary. Ask students to read ***Connect It*** problem 15, underline the vocabulary, and discuss it with a partner. Ask: *What is the problem asking you to do? What do the terms mean in context?*

Have students solve the ***Connect It*** problems independently. Point out problems 14 and 15, which require students to explain reasoning and justify conclusions. Ask students to generate ideas about how they might justify their answers. [explanations, models, equations] Adapt the ***Turn and Talk*** routine to allow students to debrief and compare answers with a partner.

Prepare for Day 4: Use with *Practice: Pair/Share*

ELP Levels 1–3

Listening/Speaking Organize students into small groups to discuss their solutions to the ***Practice*** problems and respond to the ***Pair/Share*** questions.

Encourage students to use *per* as they describe their answers (e.g., *per hour, per cake, per block*). Refer to the example and say: *Oliver runs* $12\frac{1}{2}$ *kilometers per hour.*

Read each ***Pair/Share*** question aloud as students follow along. Provide sentence starters to support discourse:

- I know my answer makes sense because _____.
- First, I divided _____.
- Dee's answer _____.

ELP Levels 2–4

Listening/Speaking Pair students to discuss their solutions to the ***Practice*** problems and respond to the ***Pair/Share*** questions.

Discuss the steps for evaluating a complex fraction to find the unit rate in problem 18 using sequencing words such as *first, next,* and *then*. Have students take turns explaining the steps for finding how many pounds of carrots are needed in problem 19 using sequencing words.

Provide a word bank with the following terms to support discourse: *complex fraction, numerator, denominator, divide, multiply, simplify the expression,* and *mixed number*.

ELP Levels 4–5

Listening/Speaking Pair students to discuss their solutions to the ***Practice*** problems and respond to the ***Pair/Share*** questions.

Point to the last ***Pair/Share*** question and have students substitute their partner's name for Dee's in a sentence frame:

- Does _____'s answer make sense?

Ask students to keep that question in mind as they listen to their partner's responses. If an answer does not make sense, have them challenge their partner to justify the answer.

Encourage students to brainstorm ways to justify their answers, such as explaining solutions, creating models, and using equations.

Introduction

At A Glance

Students read a word problem and answer a series of questions designed to help them find a unit rate when one of the given quantities is a fraction. They learn how to model the problem using a ratio written as a complex fraction. Then they simplify the complex fraction by dividing.

Step By Step

- Work through **Use What You Know** as a class.
- Tell students that this page models how to use a diagram to find a rate in miles per hour when the time is given as a number of minutes less than an hour.
- Have students read the problem at the top of the page.

▶ Mathematical Discourse 1

- Have students look at the diagram and explain how to figure out how many rectangles are needed to represent 15 minutes.
- Help students understand that in the diagram, 4 rectangles represent the ratio 4 miles : 15 minutes.

▶ Visual Model

- Ask student pairs or groups to explain their answers for the remaining questions.

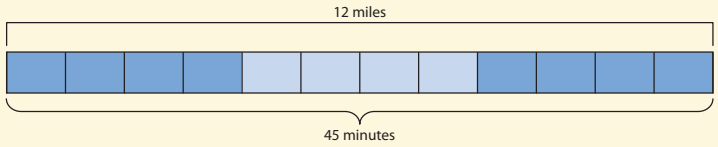
SMP TIP Make Sense of Problems
Help students make sense of problems and persevere in solving them by asking them to explain what they are asked to find and to identify the needed information. Allow plenty of wait time. *(SMP 1)*

▶ Mathematical Discourse 2

Use What You Know

In Grade 6, you learned about unit rates. Take a look at this problem.

Jana is training for a triathlon that includes a 112-mile bike ride. Today, she rode her bike 12 miles in 45 minutes. What is Jana's rate in miles per hour?



Use the math you already know to solve the problem.

a. If Jana biked at a constant rate, how many miles did she bike in the first 15 minutes?
4

b. At the same rate, how many miles did she bike in the next 15 minutes? 4

c. At the same rate, how many miles did she bike in the last 15 minutes? 4

d. How many more minutes would Jana need to bike to total one hour? 15

e. At the same rate, how many miles would she bike in that amount of time? 4

f. Explain how you could find the number of miles Jana bikes in one hour.
Possible answers: I could add 4 + 4 + 4 + 4 to get 16;
I could multiply 4 × 4 to get 16.

▶ Mathematical Discourse

1 *Why is it important that the first question says, "If Jana biked at a constant rate"?*
Listen for responses that indicate that a constant rate means the distance traveled is the same during each minute, so the problem can be solved with multiplication or division.

2 *The information is given in miles and minutes. Why might Jana want to know her rate in miles per hour instead of miles per minute?*
Listen for responses that note that she only rides a small part of a mile in one minute.

▶ Visual Model

Extend a diagram.

- Tell students that you will extend the diagram to show the number of miles per hour.
- Sketch the diagram on the board. Ask a volunteer to explain how many more rectangles you would need to draw to show 60 minutes instead of 45. [4] Add them to the diagram.
- Ask another volunteer to explain how to use the extended diagram to solve the problem.

Find Out More

The number of miles Jana bikes in one hour is a **unit rate**. A unit rate compares two quantities where the second quantity is 1. A unit rate tells you how many units of the first quantity correspond to one unit of the second quantity.

The units in this problem are miles and hours. The problem tells you that Jana bikes **12 miles** in **45 minutes**. That's the same as 12 miles in $\frac{3}{4}$ **hour**.

$$\frac{\text{number of miles}}{\text{number of hours}} = \frac{12}{\frac{3}{4}}$$

The fraction $\frac{12}{\frac{3}{4}}$ is a **complex fraction**. A complex fraction is a fraction where the numerator is a fraction, the denominator is a fraction, or both the numerator and the denominator are fractions. You can simplify a complex fraction by dividing, just as you would do if the numerator and denominator were whole numbers.

The fraction bar represents division, so you can think of $\frac{6 \text{ miles}}{2 \text{ hours}}$ as $6 \div 2 = 3$ miles per hour.

You can think about $\frac{12 \text{ miles}}{\frac{3}{4} \text{ hour}}$ in the same way.

$$\frac{12}{\frac{3}{4}} = \frac{12}{1} \div \frac{3}{4}$$

$$= \frac{12}{1} \times \frac{4}{3}$$

$$= \frac{48}{3} \text{ or } 16 \text{ miles per hour}$$

The unit rate is 16. The number of miles Jana bikes is 16 times the number of hours.

Reflect

1. On another training ride, Jana bikes 15 miles in 50 minutes. Explain how you could find the number of miles she bikes in 1 hour.

Possible answer: Jana bikes 3 miles every 10 minutes, so she would bike 18 miles in 60 minutes or 1 hour. Write the ratio $\frac{15}{\frac{5}{6}}$ and then divide to get 18 miles per hour.

Step By Step

Real-World Connection

- Read **Find Out More** as a class.
- Review the meaning of *unit rate*.

English Language Learners

- Have students look at the ratio $\frac{12}{\frac{3}{4}}$.

  Ask: *Why is it not a unit rate?* [The number of hours must be one.]
- Have students describe how the ratio looks different from other fractions they have seen. Discuss the definition of a complex fraction. Ask students to give examples of complex fractions.
- Reinforce the idea that the fraction bar can mean division. Give other examples such as $\frac{15}{3}$ and $\frac{20}{5}$.
- Work through the steps used to divide $12 \div \frac{3}{4}$.
- Have students assess the reasonableness of the answer. Note that 1 hour is slightly more than 45 minutes and 16 miles is slightly more than 12 miles.

Ready Mathematics PRACTICE AND PROBLEM SOLVING

Assign *Practice and Problem Solving* **pages 87–88** after students have completed this section.

English Language Learners

Write the word *per* on the board. Next to it, write *for each* and *in each*. Give examples such as "5 crayons *for each* student" means "5 crayons *per* student" and "driving 50 miles *in each* hour" means "50 miles *per* hour." Give other examples and such as "$1.50 for each pound of peaches" or "3 cups of flour in each loaf of bread." Have students restate each example, using the word *per*.

Then write *unit rate* on the board. Circle the word *unit* and write a 1 above it. Say that in 50 miles per hour, the unit rate is 50 because it tells the number of miles in 1 hour. The word *per* can mean *in one* or *for one*. Give more examples. Have students restate each ratio using the word *per* and then give the unit rate.

Real-World Connection

Identify fraction measurements.

Encourage students to think of everyday situations in which measurements are given as fractions. Have volunteers share their ideas.

Examples: Cooking $\left(\frac{3}{4} \text{ cup}, \frac{1}{2} \text{ dozen}\right)$; sewing $\left(\frac{5}{8} \text{ yard}, 2\frac{1}{2} \text{ feet}\right)$; traveling $\left(12\frac{1}{2} \text{ miles in } \frac{1}{4} \text{ hour}, 3\frac{1}{2} \text{ blocks in } 7\frac{1}{2} \text{ minutes}\right)$

Modeled and Guided Instruction

At A Glance

Students solve problems that require them to find unit rates by using number lines and simplifying ratios involving complex fractions.

Step By Step

- Read the problem at the top of the page as a class.
- Ask students to look at the recipe to find the number of eggs and cups of flour needed.
- Have students use their own words to explain what they are trying to find in order to solve this problem.

▶ Mathematical Discourse 1 and 2

Model It

- Have students read **Model It**. Call students' attention to the first double number line. Have them read the information and note the labels. Ask how the number line is related to the problem.

▶ Hands-On Activity

- Read the information above the second double number line. Discuss how to find the number that is halfway between 0 and $1\frac{1}{2}$. Guide students to see how they can use the unit rate to find the other numbers on the top number line.

Learn About Finding Unit Rates with Fractions

Read the problem below. Then explore different ways to understand how to find a unit rate.

Max's favorite recipe for oatmeal raisin cookies makes 48 servings. He wants to make some cookies but only has one egg. Max has to adjust the amounts of the other ingredients. How much flour will he need?

RECIPE

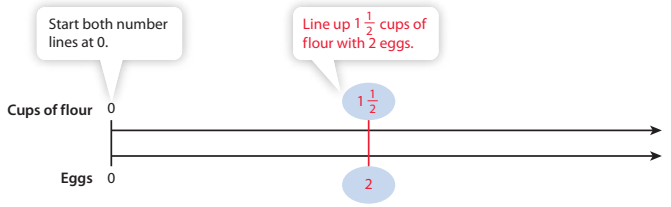
OATMEAL RAISIN COOKIES

Ingredients

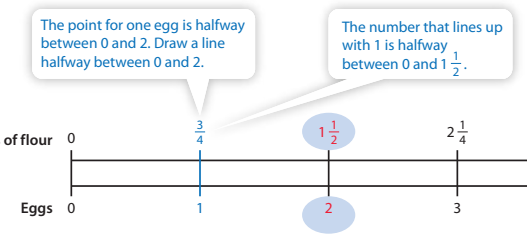
- $\frac{3}{4}$ cup butter
- 2 eggs
- $1\frac{1}{2}$ cups flour
- $1\frac{1}{2}$ cups brown sugar
- 1 teaspoon vanilla
- 1 teaspoon baking soda
- $\frac{3}{4}$ teaspoon cinnamon
- $2\frac{3}{4}$ cups oats
- 1 cup raisins

Model It **You can draw a double number line to show the relationship described in the problem.**

The units you need to compare are cups of flour and eggs.



You need to find the unit rate, the number of cups of flour needed for **1** egg.



▶ Mathematical Discourse

1 *Why is it helpful to know a unit rate when shopping?*

Student responses may include that unit rates allow shoppers to compare similar products of different sizes.

2 *Does the problem ask you to find a unit rate? Explain why or why not.*

Students should explain that it does ask for a unit rate because it asks for the amount of flour needed for 1 egg.

▶ Hands-On Activity

Fold paper strips to model unit rate.

Materials: strips of paper, scissors, markers, rulers

- Have students cut a strip of paper so that it is $1\frac{1}{2}$ inches long.
- Direct students to draw a horizontal line across the entire length of the paper then divide it into $\frac{1}{4}$-inch segments.
- Have students fold the paper in half vertically and then determine the length of each half.
- On the board write $1\frac{1}{2} \div 2 = \frac{3}{4}$ and $\frac{3}{4} \times 2 = 1\frac{1}{2}$.
- Have students relate the result to the number line used to model the problem.

Connect It **Now you will see how to solve the problem from the previous page by writing a ratio.**

2 Why do you need to find the number that is halfway between 0 and $1\frac{1}{2}$?

That is the amount of flour to use if you have just one egg.

3 How could you find the number that is between 0 and $1\frac{1}{2}$?

Divide $1\frac{1}{2}$ by 2 or multiply $1\frac{1}{2}$ by $\frac{1}{2}$.

4 How many cups of flour does Max need to use if he has just 1 egg? Show your work.

$\frac{3}{4}$ cup of flour; $1\frac{1}{2} \times \frac{1}{2} = \frac{3}{2} \times \frac{1}{2} = \frac{3}{4}$

5 Write the ratio that compares $1\frac{1}{2}$ cups of flour to 2 eggs.

$\frac{1\frac{1}{2}}{2}$

6 Write and simplify a division expression to find the number of cups of flour Max needs to use if he has just 1 egg.

$1\frac{1}{2} \div 2 = \frac{3}{2} \div \frac{2}{1}$; $\frac{3}{2} \div \frac{2}{1} = \frac{3}{2} \times \frac{1}{2} = \frac{3}{4}$; $\frac{3}{4}$ cup of flour

7 The unit rate is $\frac{3}{4}$. The number of cups of flour is $\frac{3}{4}$ times the number of eggs.

8 Explain how to find a unit rate. Possible answer: Write a ratio that compares the quantities described in the problem. Then divide the first quantity by the second quantity.

Try It **Use what you just learned about finding a unit rate to solve these problems. Show your work on a separate sheet of paper.**

Use the information in the recipe on the previous page.

9 If Max has only one egg, how much butter will he need? $\frac{3}{8}$ cup

10 If Max has only one cup of flour, how much vanilla will he need? $\frac{2}{3}$ teaspoon

Concept Extension

Help students see how the unit rate helps them find equivalent ratios.

- Draw a ratio table on the board. Label the first row *Cups of flour* and the second row *Eggs*.
- Fill in the first two columns with information from **Connect It**.
- Have students fill in two more columns by multiplying the number of eggs by $\frac{3}{4}$.
- Compare the results with the number line.
- Ask students to explain how to show that each ratio of flour to eggs is equal to $\frac{3}{4}$.

Step By Step

Connect It

- Read **Connect It** as a class. Be sure to point out that the questions refer to the problem on the previous page.
- Emphasize the idea that since 1 egg is halfway between 0 and 2 eggs, the amount of flour must be halfway between 0 and $1\frac{1}{2}$ cups.
- Once students have written the ratio, ask them to check that they have written the ratio in the same order as it is expressed in the problem. Then have them explain why it is a complex fraction.
- Have students simplify $\frac{1\frac{1}{2}}{2}$. Have them compare the steps they use to the steps used to find $\frac{1}{2}$ of $1\frac{1}{2}$ on the number line.

SMP TIP Use Structure
Students use structure when they explain how dividing $1\frac{1}{2}$ by 2 is the same as multiplying $1\frac{1}{2}$ by $\frac{1}{2}$ because division by a number and multiplication by its reciprocal are equivalent operations. *(SMP 7)*

Concept Extension

Try It

9 Solution

$\frac{3}{8}$ cup; Students may draw a number line to show that $\frac{3}{8}$ is halfway between 0 and $\frac{3}{4}$. They may write and simplify the ratio $\frac{\frac{3}{4}}{2}$.

10 Solution

$\frac{2}{3}$ teaspoon; Students may write and simplify the ratio $\frac{1}{1\frac{1}{2}}$. They may also draw a double number line to show that 1 cup is $\frac{2}{3}$ of $1\frac{1}{2}$ cups and then show $\frac{2}{3}$ of 1 teaspoon.

Error Alert Students who wrote $\frac{1}{2}$ found the amount of vanilla needed for 1 egg instead of for 1 cup of flour.

Ready Mathematics PRACTICE AND PROBLEM SOLVING

Assign *Practice and Problem Solving* **pages 89–90** after students have completed this section.

Modeled and Guided Instruction

At A Glance

Students use double number lines to find a unit rate. Then they solve a problem by simplifying ratios and comparing unit rates.

Step By Step

- Read the problem at the top of the page as a class.

▶ English Language Learners

- Ask: *Why can't you just say that $7.50 is less than $9.00 so it is a better buy?* [The packages are different weights so they do not contain the same amount.]

Model It

- Read **Model It** as a class. Reinforce that when comparing unit rates, the units must be the same. Make sure students understand why 12 ounces is equivalent to $\frac{3}{4}$ pound.
- Have students study the first double number line. Go over the steps used to draw the number line accurately.
- Direct students' attention to the second number line. Ask: *How do we know that $2\frac{1}{2}$ dollars lines up with $\frac{1}{4}$ pounds?* [$\frac{1}{4}$ is one third of the way from 0 to $\frac{3}{4}$, and $2\frac{1}{2}$ is one third of the way from 0 to $7\frac{1}{2}$.]

▶ Mathematical Discourse 1 and 2

Learn About Comparing Unit Rates

Read the problem below. Then explore different ways to understand how to find and compare unit rates.

José's mother is trying to decide whether or not she should buy a 12-ounce package of coffee on sale for $7.50. She knows that she can buy the same coffee for $9.00 per pound. Which is the better buy?

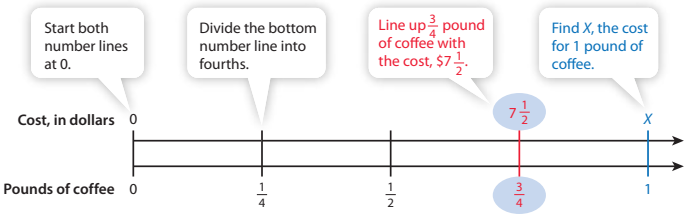
Model It **You can draw a double number line to show the relationship described in the problem.**

To find the better buy, compare the unit rate of each option.

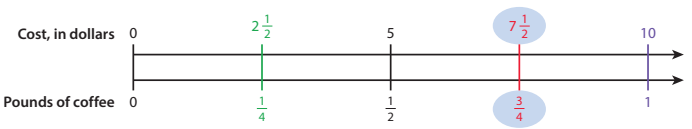
The problem gives you one unit rate: $9.00 per pound. To compare unit rates, the units you use must be the same. So, find the weight of the other coffee in pounds.

There are 16 ounces in 1 pound, so 12 ounces is $\frac{12}{16}$ or $\frac{3}{4}$ pound.

You can write $7.50 using fractions. $7.50 is the same as $7\frac{1}{2}$.



Find the **cost for each quarter-pound of coffee**. Then find the **unit cost**.



▶ Mathematical Discourse

1 *What are some equivalent ratios shown by the number line?*

Students may list $\frac{2\frac{1}{2}}{\frac{1}{4}}$, $\frac{5}{\frac{1}{2}}$, $\frac{7\frac{1}{2}}{\frac{3}{4}}$, and $\frac{10}{1}$.

2 *Using the number line, how can you tell the ratios are equivalent? Can you explain it another way?*

Students may note that they are the same distances apart on the number line. They may also explain that when you double $2\frac{1}{2}$ and $\frac{1}{4}$ you get 5 and $\frac{1}{2}$, and when you triple them you get $7\frac{1}{2}$ and $\frac{3}{4}$. They may also explain that when you simplify each ratio, the result is 10 to 1.

▶ English Language Learners

- Write *1 pound* on the board. Ask students to describe what we measure with pounds. Pounds are a unit of measure to find *weight* or *how heavy* an object is.
- Ask students how we would measure the weight of something less than a pound. Accept the idea that we could use a fraction of a pound. If no one mentions the term *ounce*, introduce it as a unit of measure less than 1 pound.
- Write *1 pound = 16 ounces* on the board. Discuss the equivalency in concrete terms. *Dora has 16 ounces of grapes. That is the same as 1 pound of grapes.*

Connect It **Now you will see how to use a ratio to solve the problem.**

11 The top number line is divided into 3 equal parts from 0 to $7\frac{1}{2}$, and the bottom number line is divided into 3 equal parts from 0 to $\frac{3}{4}$. How can you use this to find the cost of 1 pound of coffee? **Divide $7\frac{1}{2}$ by 3 to find the length of each part. $7\frac{1}{2} \div 3 = 2\frac{1}{2}$, so each part is $2\frac{1}{2}$. To get to the number that lines up with 1, you need 4 of these parts.**

12 Write the ratio that compares $\$7\frac{1}{2}$ dollars to $\frac{3}{4}$ pound of coffee. $\frac{7\frac{1}{2}}{\frac{3}{4}}$

13 Write and simplify a division expression to find the cost of 1 pound of coffee.
$7\frac{1}{2} \div \frac{3}{4} = \frac{15}{2} \div \frac{3}{4} = \frac{15}{2} \times \frac{4}{3} = \frac{60}{6} = 10$; $10

14 Which is the better buy, 12 ounces for $7.50 or 1 pound for $9.00? Explain your reasoning.
The 12-ounce package of coffee is $10.00 per pound and the 16-ounce package is $9.00 per pound. The better buy is 1 pound for $9.00.

15 If you started the problem by converting 1 pound to 16 ounces, would you get the same result? Justify your conclusion. **Yes. Students may draw a double number line or write and solve a proportion.**

16 Can you compare any two unit rates? Explain. **No, the rates must use the same units to be able to compare them.**

Try It **Use what you just learned about unit rates to solve this problem. Show your work on a separate sheet of paper.**

17 Rina's recipe uses 2 cups of sugar to make $2\frac{1}{2}$ dozen cookies. Jonah's recipe uses $2\frac{1}{4}$ cups of sugar to make 3 dozen cookies. Which recipe uses more sugar for a dozen cookies? Why? **Rina's recipe; Rina's ratio is $\frac{4}{5}$, which is greater than Jonah's ratio, $\frac{3}{4}$.**

Teacher Notes

Step By Step

Connect It

- Be sure to point out that **Connect It** refers to the problem on the previous page.
- Ask: *Once you know that each $\frac{1}{4}$ pound costs $2.50, how can you figure out how much a full pound costs?* [There are 4 fourths in a whole, so you would multiply $2.50 by 4.]
- Have students explain why you would divide to simplify a ratio involving a complex fraction. Have students complete the division process and review it as a class.
- Ask whether finding cost per ounce or the cost per pound is easier in this situation.

SMP TIP Attend to Precision
Students should realize that it is important to specify cost as *per pound* or *per ounce* when writing and simplifying ratios. Be sure to model this language as you attend to precision. *(SMP 6)*.

Try It

17 Solution

Rina's recipe.; Students may simplify ratios to find the unit rate. Rina: $\frac{2}{2\frac{1}{2}} = 2 \div 2\frac{1}{2} = \frac{4}{5}$; Jonah: $\frac{2\frac{1}{4}}{3} = 2\frac{1}{4} \div 3 = \frac{3}{4}$

Each dozen of Rina's cookies contains $\frac{4}{5}$ cup sugar. Each dozen of Jonah's contains $\frac{3}{4}$ cup of sugar. Rina's cookies use more sugar per dozen. $\frac{4}{5}$ is greater than $\frac{3}{4}$.

Error Alert Students who wrote Jonah may have found the rate of dozens of cookies per cup of sugar.

Rina: $\frac{2\frac{1}{2}}{2} = 1\frac{1}{4}$; Jonah: $\frac{3}{2\frac{1}{4}} = 1\frac{1}{3}$ However, that means Jonah's recipe has more cookies per cup of sugar, not more sugar per dozen cookies.

Ready Mathematics PRACTICE AND PROBLEM SOLVING

Assign *Practice and Problem Solving* **pages 91–92** after students have completed this section.

Guided Practice

At A Glance

Students write and simplify ratios to solve word problems involving unit rate. They may also use double number lines to find the solution.

Step By Step

- Ask students to solve the problems individually and interpret their answers in the context of the problems.
- **Pair/Share** When students have completed each problem, have them discuss their solutions with a partner or in a group.

Solutions

Example The example shows how to write and simplify a ratio as one way to solve the problem. Students could also use a double number line.

18 Solution

14; Students could solve the problem by simplifying $\frac{10\frac{1}{2}}{\frac{3}{4}}$ or use a double number line.

DOK 1

Practice Finding Ratios Involving Complex Fractions

Study the example below. Then solve problems 18–20.

Example

Oliver is training for a marathon. In practice, he runs 15 kilometers in 72 minutes. What is his speed in kilometers per hour?

Convert the time in minutes to hours to find kilometers per hour.

$72 \text{ minutes} = 1 \text{ hour } 12 \text{ minutes}$

$= 1\frac{12}{60} \text{ hours or } 1\frac{1}{5} \text{ hours}$

$\frac{\text{km}}{\text{hr}} = \frac{15}{1\frac{1}{5}}$

$= 15 \div 1\frac{1}{5}$

$= 15 \div \frac{6}{5}$

$= 15 \times \frac{5}{6}$

$= \frac{75}{6} \text{ or } 12\frac{1}{2}$

Solution Oliver runs $12\frac{1}{2}$ kilometers per hour.

The student knew that 60 minutes = 1 hour, so 72 minutes = 60 minutes + 12 minutes, or 1 hour 12 minutes.

Pair/Share
How did you decide how to write the ratio?

18 Alexis washes $10\frac{1}{2}$ windows in $\frac{3}{4}$ hour. At this rate, how many windows can she wash in one hour?

$\frac{\text{number of windows}}{\text{number of hours}} = \frac{10\frac{1}{2}}{\frac{3}{4}}$

$10\frac{1}{2} \div \frac{3}{4} = \frac{21}{2} \div \frac{3}{4}$

$= \frac{21}{2} \times \frac{4}{3}$

$= \frac{84}{6}$

$= 14$

Solution Alexis can wash 14 windows in one hour.

How do you evaluate a complex fraction?

Pair/Share
How can you tell if your answer is reasonable?

Teacher Notes

19 A restaurant uses $8\frac{1}{4}$ pounds of carrots to make 6 carrot cakes. Frank wants to use the same recipe. How many pounds of carrots does Frank need to make one carrot cake?

Show your work.

What is the ratio of pounds of carrots to cakes?

$$\frac{\text{pounds of carrots}}{\text{cakes}} = \frac{8\frac{1}{4}}{6}$$

$$8\frac{1}{4} \div 6 = \frac{33}{4} \div 6$$

$$= \frac{33}{4} \times \frac{1}{6}$$

$$= \frac{33}{24}$$

$$= 1\frac{9}{24} \text{ or } 1\frac{3}{8}$$

Solution Frank will need $1\frac{3}{8}$ pounds of carrots for each cake.

Pair/Share
What steps did you take to find the unit rate?

20 It takes Zach 15 minutes to walk $7\frac{1}{2}$ blocks to the swimming pool. At this rate, how many blocks can he walk in one minute? Circle the letter of the correct answer.

What unit rate do you need to find?

A $\frac{1}{5}$ block

B $\frac{1}{2}$ block

C 2 blocks

D 5 blocks

Dee chose **C** as the correct answer. What was her error?

She found the number of minutes per block instead of the number of blocks per minute.

Pair/Share
Does Dee's answer make sense?

Teacher Notes

Solutions

19 Solution

$1\frac{3}{8}$; Students could solve the problem by simplifying $\frac{8\frac{1}{4}}{6}$ or use a double number line.

DOK 1

20 Solution

B; Divide $7\frac{1}{2}$ by 15 to find the number of blocks per minute.

Explain to students why the other two answer choices are not correct:

A is not correct because $7\frac{1}{2} \div 15 = 0.5$, which is $\frac{1}{2}$, not $\frac{1}{5}$.

D is not correct because it does not make sense for him to walk 5 blocks in one minute if it takes him 15 minutes to walk $7\frac{1}{2}$ blocks.

DOK 3

Ready Mathematics PRACTICE AND PROBLEM SOLVING

Assign *Practice and Problem Solving* **pages 93–94** after students have completed this section.

Independent Practice

At A Glance

Students find unit rates to solve word problems that might appear on a mathematics test.

Solutions

1 Solution

C; Rewrite 7 feet as 84 inches and then write and simplify the ratio of inches to hours, $\frac{84}{27\frac{1}{2}}$.

DOK 1

2 Solution

A; Find the cost per pound for each brand. (Trail Mix A: \$8/pound, B: \$8.50/pound, C: \$9/pound.) Then find the lowest unit rate.

DOK 2

3 Solution

6 laps completed, and students shade in 5 sections of the display; Divide $\frac{13}{10}$ by $\frac{1}{5}$ to get $6\frac{1}{2}$ laps, which is 6 full laps and 5 of 10 sections of the display shaded.

DOK 2

Practice Finding Ratios Involving Complex Fractions

Solve the problems.

1 One of the highest snowfall rates ever recorded was in Silver Lake, Colorado, in April 1921, when just over 7 feet of snow fell in $27\frac{1}{2}$ hours. What was that rate in inches per hour?

A $\frac{14}{55}$ inch per hour

B $\frac{55}{158}$ inch per hour

C $3\frac{3}{55}$ inches per hour

D $3\frac{13}{14}$ inches per hour

2 A grocery store sells different types of Trail Mix, as shown in the table below.

	Trail Mix A	Trail Mix B	Trail Mix C
Cost (\$)	6	8.50	2.25
Weight	$\frac{3}{4}$ lb	1 lb	4 oz

1 lb = 16 oz

Which statement is correct?

A Trail Mix A is the best buy.

B Trail Mix B is the best buy.

C Trail Mix C is the best buy.

D They are all the same price.

3 A treadmill counts $\frac{1}{5}$ mile as one lap. The display of the treadmill indicates the number of laps already completed and highlights how much of the current lap has been completed. Create a display that shows a total of $\frac{13}{10}$ miles run.

Quick Check and Remediation

- A recipe calls for $2\frac{1}{4}$ cups of sugar for $1\frac{1}{2}$ dozen cookies. Have students find the amount of sugar per dozen cookies. $\left[1\frac{1}{2} \text{ cups}\right]$
- For students who are struggling, use the chart to guide remediation.
- After providing remediation, check students' understanding. Have students find Carlos' rate in laps per minute if he runs $6\frac{1}{4}$ laps in 10 minutes. $\left[\frac{5}{8}\right]$
- If a student is still having difficulty, use *Ready Instruction, Grade 7,* Lesson 6.

If the error is . . .	Students may . . .	To remediate . . .
$\frac{1}{8}$	have found the amount of sugar per cookie, not per dozen.	Have students reread the problem and state what they need to find. Have them explain why they do not need to convert $1\frac{1}{2}$ dozen to individual cookies.
$\frac{2}{3}$	have found the number of dozens per cup of sugar.	Write the ratio using words, $\frac{\text{sugar}}{\text{dozen}}$. Have students substitute numbers for words.
$3\frac{3}{8}$	have multiplied instead of divided.	Remind students that the fraction bar indicates division. Review the steps used to divide two fractions.
any other answer	have divided incorrectly.	Go over the student's work to make sure each step was done correctly.

4 A restaurant makes a special citrus dressing for its salads. Here is how the ingredients are mixed:

$\frac{1}{3}$ of the mixture is oil

$\frac{1}{4}$ of the mixture is orange juice

$\frac{1}{6}$ of the mixture is vinegar

$\frac{1}{4}$ of the mixture is lemon juice

When the ingredients are mixed in the same ratio as shown above, every batch of dressing tastes the same.

- **Batch 1:** If you have 1 cup of oil, how much vinegar will you need? How much lemon juice? Show your thinking.
- **Batch 2:** If you have 1 cup of vinegar, how much will you need of the other ingredients? Show your thinking.
- **Batch 3:** If you have 1 cup of orange juice, how much will you need of the other ingredients? Show your thinking.

5 Two friends worked out on treadmills at the gym.

- Alden walked 2 miles in $\frac{3}{4}$ hour.
- Kira walked $1\frac{3}{4}$ miles in 30 minutes.

Who walked at a faster rate? Explain your reasoning.

Show your work.

Alden's rate: $\frac{2}{\frac{3}{4}}$

$2 \div \frac{3}{4} = \frac{2}{1} \times \frac{4}{3}$

$= \frac{8}{3}$ or $2\frac{2}{3}$ miles per hour

Kira's rate: $\frac{1\frac{3}{4}}{\frac{1}{2}}$

$1\frac{3}{4} \div \frac{1}{2} = \frac{7}{4} \times \frac{2}{1}$

$= \frac{14}{4}$ or $3\frac{1}{2}$ miles per hour

Answer Kira walks at a faster rate.

Self Check Go back and see what you can check off on the Self Check on page 79.

Solutions

4 Solution

- 1 cup oil, $\frac{1}{2}$ cup vinegar, $\frac{3}{4}$ cup orange juice, $\frac{3}{4}$ cup lemon juice
- 2 cups oil, 1 cup vinegar, $1\frac{1}{2}$ cups orange juice, $1\frac{1}{2}$ cups lemon juice
- $1\frac{1}{3}$ cups oil, $\frac{2}{3}$ cup vinegar, 1 cup orange juice, 1 cup lemon juice

DOK 2

5 Solution

Alden's rate is $\frac{2}{\frac{3}{4}}$ or $2\frac{2}{3}$ miles per hour.

Kira's rate is $\frac{1\frac{3}{4}}{\frac{1}{2}}$ or $3\frac{1}{2}$ miles per hour.

Kira's rate is faster.

DOK 3

▶ Hands-On Activity

Use a paper model to find a unit rate.

Materials: small pieces of paper that are the same shape and size

On the board, write "Sheila buys $9\frac{1}{3}$ pounds of nuts for 4 gift baskets. How many pounds of nuts does Sheila buy per gift basket?" Distribute 10 pieces of paper to each student. Tell students that each piece represents a pound of nuts. Ask: *How can you represent $9\frac{1}{3}$ pounds using the paper?* [Tear one sheet in thirds and discard two of the thirds.] Direct students to distribute the paper into 4 piles so that there is the same amount of paper in each pile. It is acceptable to tear the paper into pieces that are the same size. When students have completed the task, write $\frac{9\frac{1}{3}}{4} = 2\frac{1}{3}$. Ask students what $2\frac{1}{3}$ represents.

▶ Challenge Activity

Extend the concept of unit rate to solve problems.

Tell students that when Ginger made applesauce using $2\frac{1}{4}$ pounds of apples, she used $1\frac{1}{2}$ tablespoons of sugar. She now has 8 pounds of apples and wonders how much sugar she should use. Ask students how they could find and use the unit rate to solve the problem. [Possible answer: Find the unit rate by simplifying $\frac{1\frac{1}{2}}{2\frac{1}{4}}$, which is $\frac{2}{3}$. Then either create a ratio table or multiply $8 \times \frac{2}{3}$ to show that Ginger should use $5\frac{1}{3}$ tablespoons of sugar for 8 pounds of apples.]

LESSON QUIZ

Lesson 9
Ratios Involving Complex Fractions

Teacher-Toolbox.com

Overview

Assign the Lesson 9 Quiz and have students work independently to complete it.

Use the results of the quiz to assess students' understanding of the content of the lesson and to identify areas for reteaching. See the Lesson Pacing Guide at the beginning of the lesson for suggested instructional resources.

Tested Skills

Assesses 7.RP.A.1

Problems on this assessment form require students to compute unit rates associated with ratios of fractions, including ratios of lengths and areas. Students will also need to be familiar with computing unit rates with whole numbers and dividing fractions.

Context and Vocabulary

Show a picture of people rock climbing, or explain that context for problem 1. If students are not familiar with the term *acre*, explain that an *acre* is a unit that measures land area.

***Ready*® Mathematics**

Lesson 9 Quiz

Solve the problems.

1 Martin and Alexia are rock climbing. Each person's rate stays the same for the first hour they climb.

- Martin climbs $3\frac{1}{3}$ meters in the first $\frac{1}{6}$ hour.
- Alexia climbs $17\frac{1}{2}$ meters in the first $\frac{5}{6}$ hour.

Which statements are true? Choose all that apply.

A Martin climbs at a rate of 20 meters per hour.

B Alexia takes $\frac{1}{21}$ of an hour to climb one meter.

C Martin climbs at a rate of $\frac{1}{20}$ meter per hour.

D Alexia climbs at a rate of 21 meters per hour.

E Martin takes $\frac{1}{20}$ of an hour to climb one meter.

F Alexia climbs at a rate of $\frac{1}{21}$ meter per hour.

2 Sarah is training for a bike race. She rides her bike $5\frac{3}{4}$ miles in $\frac{1}{3}$ hour. What is Sarah's rate in miles per hour? Express your answer as a mixed number.

Show your work.

Answer: _____ miles per hour

3 Donovan bought $5\frac{1}{2}$ kilograms of flour for $8.25.

Tell whether each statement is *True* or *False*.

a. The product $\left(\frac{33}{4}\right)\left(\frac{2}{11}\right)$ gives the price of one kilogram of flour. ☐ True ☐ False

b. Donovan paid less than $1.25 per kilogram of flour. ☐ True ☐ False

c. $1.00 can purchase $\frac{2}{3}$ kilogram of flour. ☐ True ☐ False

Lesson 9 Quiz continued

4 A stand at a farmer's market sells different types of apples, as shown in the table.

	Apple A	Apple B	Apple C
Cost ($)	6.90	0.90	0.45
Weight	6 lb	12 oz	$\frac{1}{4}$ lb

1 lb = 16 oz

Which type of apple costs the least per pound?

Show your work.

Answer: _____

5 Jami can mow $\frac{1}{6}$ acre in 8 minutes. If her rate is constant, can Jami mow $1\frac{1}{2}$ acres in one hour? Explain your reasoning.

_____

_____

_____

_____

_____

Common Misconceptions and Errors

Errors may result if students:

- switch the values used when computing a rate.
- ignore units of measurement when completing calculations.
- compare values in different units rather than converting to the same unit.
- do not convert mixed numbers to improper fractions.
- use multiplication instead of division, or vice versa.

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Lesson 9 Quiz Answer Key

1. A, B, D, E
DOK 2

2. $17\frac{1}{4}$
DOK 1

3. **a.** True
b. False
c. True
DOK 2

4. Apple A
DOK 2

5. No. Possible explanation: Jami can mow $\frac{1}{6}$ acre in 8 minutes, or $\frac{8}{60}$ hour. Her unit rate in acres per hour is $\frac{\frac{1}{6}}{\frac{8}{60}} = \frac{1}{6} \times \frac{60}{8} = \frac{5}{4}$. Jami can mow $1\frac{1}{4}$ acres in one hour. $1\frac{1}{4} < 1\frac{1}{2}$, so she cannot mow $1\frac{1}{2}$ acres in one hour.
DOK 3