

Equations for Proportional Relationships

Use What You Know

In Lesson 10, you learned about proportional relationships and how they are represented on a graph. In this lesson you will learn about equations that represent proportional relationships.

The table shows how the number of people who can ride a rollercoaster depends on the number of cars on the rollercoaster.

Number of Cars	Number of People
3	18
5	30
6	36
8	48

How many people can ride in 1 car? In 10 cars?

Use the math you already know to solve the problem.

- a. Find the ratio of the number of people to the number of rollercoaster cars for each set of data in the table.

- b. Are the ratios of the number of people to the number of rollercoaster cars in a proportional relationship? Explain your reasoning.

- c. How many people can ride in 1 car? _____

- d. What is the constant of proportionality for the data in the table? _____

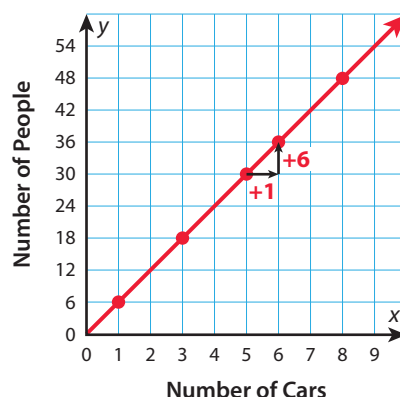
- e. Explain how you can find the number of people that can ride in a rollercoaster with 10 cars.

- f. Write an equation to represent the number of people, y , that can ride the rollercoaster with x cars. _____

Find Out More

The problem on the previous page is shown using a table. You can also represent this situation with both a graph and an equation. In each representation, you can identify the constant of proportionality.

The graph represents the problem on the previous page.



The **constant of proportionality** is the **unit rate**, or the **value of y when $x = 1$** , which is 6. In the graph, the value of y increases by 6 each time the value of x increases by 1.

The equation $y = 6x$, where y is the number of people who can ride the rollercoaster and x is the number of cars on the rollercoaster, represents this situation. Just as with the graph, the constant of proportionality is the value of y when $x = 1$.

If two quantities, x and y , are in a proportional relationship, then the **ratio $\frac{y}{x}$** equals the constant of proportionality. This means that you can represent any proportional relationship with the following equation:

$$y = \text{constant of proportionality} \cdot x$$

Reflect

- 1 Explain how the constant of proportionality is represented in the table, the equation, and the graph of the rollercoaster situation.

Learn About

Writing Equations for Proportional Relationships

Read the problem below. Then explore ways to represent a proportional relationship.

Jesse is making punch. For every 3 cups of juice, he needs 6 cups of seltzer. Represent this proportional relationship using a table, a graph, and an equation and identify the constant of proportionality. What does the constant of proportionality represent in this situation?

Model It You can use a table to represent the relationship.

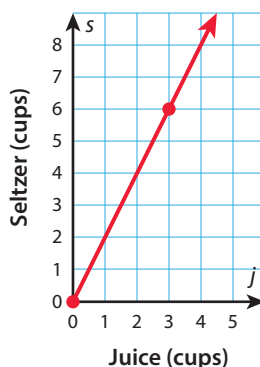
The ratio of seltzer to juice will be the same for all quantities.

All ratios will be equivalent to $\frac{6}{3} = \frac{2}{1}$.

Seltzer	2	3	4	5	6	7	8
Juice	1	1.5	2	2.5	3	3.5	4

Model It You can use a graph to represent the relationship.

Graph the point (3, 6) and connect it to the point (0, 0). Identify points on the line.



Model It You can use an equation to represent the relationship.

The ratio of seltzer to juice is $\frac{6}{3} = \frac{2}{1}$. This means that for every 1 cup of juice, you need 2 cups of seltzer.

amount of seltzer = constant of proportionality • amount of juice

Connect It Now you will solve the problem from the previous page.

- 2 In the table on the previous page, what does the ratio $\frac{2}{1}$ represent in terms of the problem? _____
- 3 What does the point (2, 4) represent on the graph? _____
- 4 Which point on the graph can be used to find the constant of proportionality? _____
- 5 Use the constant of proportionality to write an equation to represent s , the number of cups of seltzer you need for j cups of juice. _____
- 6 Use the equation you wrote in problem 5 to find the amount of seltzer you need if you have 15 cups of juice. _____
- 7 For a different punch recipe, the equation $s = 4c$ represents the number of cups of seltzer, s , that you need for c cups of cranberry juice. What is the constant of proportionality? Explain. _____

Try It Use what you just learned to solve these problems. Show your work on a separate sheet of paper.

- 8 Kelsey can buy 2 pounds of apples for \$7. Write an equation to represent the cost, c , for a pounds of apples. _____
- 9 The equation $m = 32 \cdot g$ represents the average number of miles, m , that a car can go on g gallons of gas. What is the constant of proportionality and what does it represent in this situation? _____

Practice Writing Equations for Proportional Relationships

Study the example below. Then solve problems 10–12.

Example

Julie is making bracelets. The table shows the number of bracelets she can make with different lengths of cord. Determine if the relationship is proportional. If so, find the constant of proportionality and write an equation to represent the situation.

Number of Bracelets (b)	Yards of Cord (c)
12	3
18	$4\frac{1}{2}$
24	6
30	$7\frac{1}{2}$

Show your work.

Possible student work:

$$\frac{3}{12} = \frac{1}{4} \qquad \frac{6}{24} = \frac{1}{4}$$

$$\frac{4\frac{1}{2}}{18} = \frac{\frac{9}{2}}{18} = \frac{9}{2} \div 18 = \frac{9}{2} \times \frac{1}{18} = \frac{1}{4} \qquad \frac{7\frac{1}{2}}{30} = \frac{\frac{15}{2}}{30} = \frac{15}{2} \times \frac{1}{30} = \frac{1}{4}$$

Solution $c = \frac{1}{4}b$; The constant of proportionality is $\frac{1}{4}$.



The student simplifies the ratios using the rules of division.

**Pair/Share**

How do you know that the relationship is proportional?

- 10** Michael reads 12 pages of a book in 18 minutes, 8 pages in 12 minutes, and 20 pages in 30 minutes. Is this a proportional relationship? If so, what is the constant of proportionality? What does it represent? Write an equation to represent this situation.

Show your work.

How can you find the constant of proportionality?

Solution _____

**Pair/Share**

Describe other ways you can find the constant of proportionality.

- 11** The equation $c = 2.5t$ represents the cost, c , for t tickets to the school play. Does a value of 3.5 for t make sense in this situation? Explain your reasoning.

Show your work.



What does t represent?

Solution _____

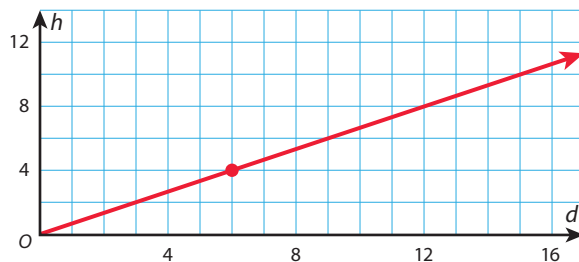


Pair/Share

Describe a situation in which the equation $y = 2.5x$ can have values for x that are mixed numbers.

- 12** The graph shows the height, h , in inches, of a plant after d days. The plant had a height of 4 inches after 6 days. Which equation can you use to represent the situation?

- A** $h = 4d$
- B** $h = 6d$
- C** $h = \frac{3}{2}d$
- D** $h = \frac{2}{3}d$



What point on the graph represents the constant of proportionality?

Stephen chose **C** as his answer. Explain his error.



Pair/Share

How do you know that the graph shows a proportional relationship?

Practice Writing Equations for Proportional Relationships

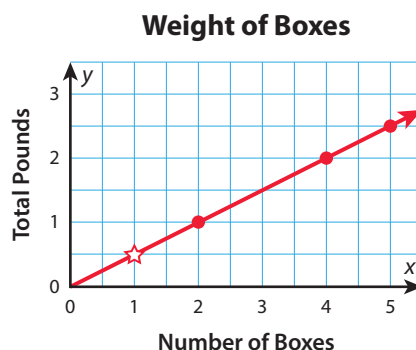
Solve the problems.

- 1** The equation $r = \frac{3}{4}b$ represents the number of cups of raisins, r , that you need to make b batches of trail mix. Which point would be on a graph that represents this proportional relationship?

- A $(\frac{3}{4}, 1)$ C $(3, 4)$
 B $(4, 3)$ D $(0, \frac{3}{4})$

- 2** Look at the graph. What is the meaning of the point shown with a star?

- A Half of a box weighs 1 pound.
 B Each box weighs $\frac{1}{2}$ pound.
 C Each box weighs 1 pound.
 D Each box weighs 2 pounds.



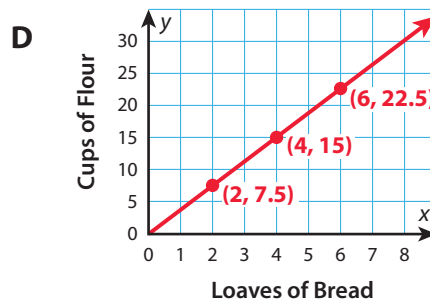
- 3** A baker is making dough for bread. The number of cups of flour he uses is proportional to the number of loaves he makes. The baker uses $18\frac{3}{4}$ cups of flour to make 5 loaves of bread. Which of the following correctly represents this proportional relationship? Select all that apply.

- A $f = 3\frac{3}{4}b$, where f is the number of cups of flour and b is the number of bread loaves.

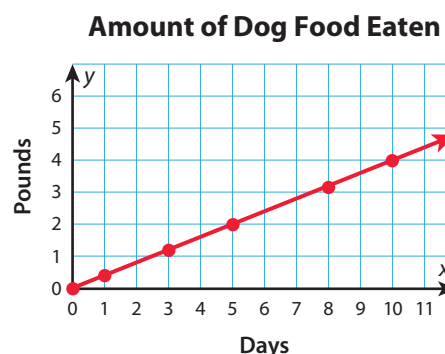
B

Cups of flour	$23\frac{3}{4}$	$28\frac{3}{4}$	$33\frac{3}{4}$
Number of loaves	10	15	20

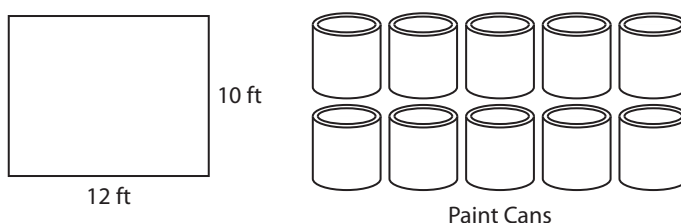
- C $b = 3\frac{3}{4}f$, where f is the number of cups of flour and b is the number of bread loaves.



- 4** The graph shows the total amount of dog food that Sophia's dog Nipper eats. Which statement is correct? Select all that apply.
- A** The data represents a proportional relationship.
 - B** The point (5, 2) means that Nipper eats 5 pounds of dog food in 2 days.
 - C** The point (0, 0) has no meaning in this situation.
 - D** The constant of proportionality is 0.4.



- 5** Len wants to paint a wall in his house. He knows that 1 can of paint covers an area of 32 square feet. The dimensions of the wall are shown. Shade in the correct number of paint cans he will need to buy.



- 6** Tom and Jeff studied the data in the table. They each wrote an equation to represent the relationship between the number of students and the number of pizzas ordered.

Tom's equation: $p = \frac{1}{4}s$

Jeff's equation: $s = 4p$

The teacher said that both equations were correct.

Explain why.

Students (s)	Pizzas (p)
28	7
12	3
32	8
16	4

✓ Self Check Go back and see what you can check off on the Self Check on page 79.