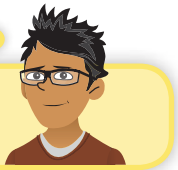
Understand Proportional Relationships

Think It Through

What is a proportional relationship?



Suppose you and some friends plan to go to a movie and the tickets cost $8 each.

You will pay $8 for 1 ticket, $16 for 2 tickets, $24 for 3 tickets, $32 for 4 tickets, and so on. The ratios of the total cost of the tickets to the number of tickets are all equivalent.

A group of ratios that are equivalent are in a **proportional relationship**. When ratios are equivalent, they all have the same unit rate. In a proportional relationship, the unit rate is called the **constant of proportionality**.

Think How can you use a table to tell if a relationship is proportional?

The table below shows the total cost of movie tickets based on the number of tickets you buy.

Total Cost of Tickets ($)	8	16	24	32
Number of Tickets	1	2	3	4

Circle the ratio in the table that shows the constant of proportionality.

The ratios of the total cost of tickets to the number of tickets are equivalent. The ratios all simplify to $\frac{8}{1}$ or 8, so **the ratios are in a proportional relationship**.

$\frac{8}{1} = \mathbf{8}$ $\quad \frac{16}{2} = \mathbf{8}$ $\quad \frac{24}{3} = \mathbf{8}$ $\quad \frac{32}{4} = \mathbf{8}$

The unit rate is 8, so the constant of proportionality is 8. The equation $c = 8t$, where c is the total cost and t is the number of tickets, represents this relationship. The total cost is always 8 times the number of tickets.

The table below shows the cost to play in the town soccer tournament.

Total Cost ($)	7	8	9	10
Number of Family Members	1	2	3	4

You can find and simplify the ratios of the total cost to the number of family members.

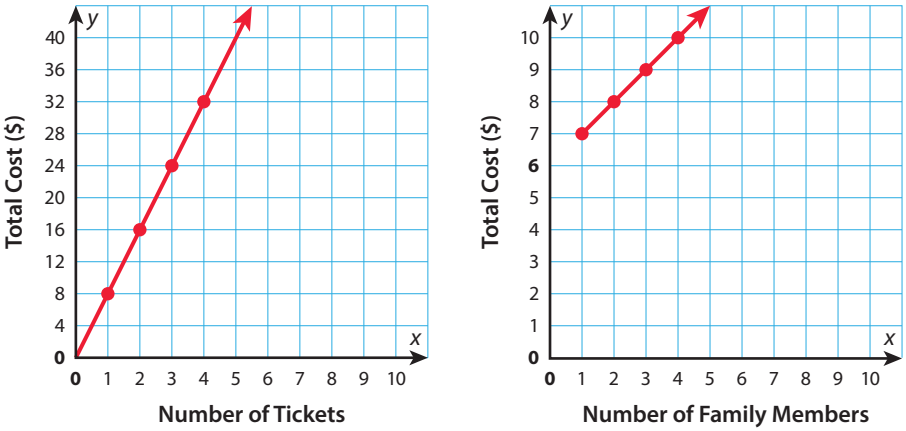
$\frac{7}{1} = \mathbf{7}$ $\quad \frac{8}{2} = \mathbf{4}$ $\quad \frac{9}{3} = \mathbf{3}$ $\quad \frac{10}{4} = \mathbf{2\frac{1}{2}}$

The ratios are not equivalent, so **the quantities are not in a proportional relationship**.

Think How can you use a graph to tell if a relationship is proportional?

You can use a graph to determine if a relationship is proportional.

The data for the cost of movie tickets and the cost to participate in the soccer tournament can be modeled by the graphs below.



Compare the two graphs. How are they alike? How are they different?

The points on the graphs are on a straight line for both sets of data, but only the data for the cost of movie tickets goes through the origin. This means that only the total cost of the movie tickets compared to the number of tickets is a proportional relationship.

Proportional Relationship	Non-Proportional Relationship
• The graph can be represented by a **straight line**. • The straight line **goes through the origin**.	• The graph may or may not be represented by a straight line. • If the graph is a straight line, it does not go through the origin.

Reflect

1 Look at the graph that compares the total cost to the number of movie tickets you buy. How can you use the graph to find the cost of 5 movie tickets?

Think About Identifying Proportional Relationships

Let's Explore the Idea

Use the table below to analyze the cost of downloading applications to a phone.

Number of Downloads	2	4	5	6	10
Total Cost ($)	6	12	15	18	30

2 How can you find the ratio of the total cost to the number of downloads?

3 What is the ratio of the total cost to the number of downloads when you download

2 applications? _____ 4 applications? _____ 5 applications? _____

6 applications? _____ 10 applications? _____

4 Are the data in the table in a proportional relationship? If so, what is the constant of proportionality? _____

Now try these problems.

5 The table shows the number of hours needed for different numbers of people to clean up after a school dance.

Hours Needed to Clean Up	12	9	8	6
Number of People Cleaning	2	3	4	6

Are the quantities in the table in a proportional relationship? Explain your reasoning.

6 The students in the Service Club are mixing paint to make a mural. The table below shows the different parts of paint that the students mix together.

	A	B	C	D	E
Parts of Red Paint	1	2	4	2	3
Parts of White Paint	3	4	8	6	9

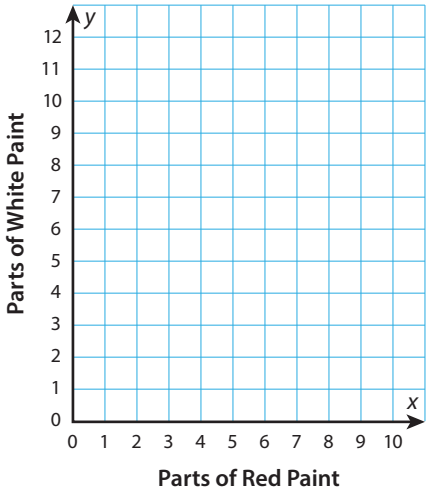
Two mixtures of paint will be the same shade if the red paint and the white paint are in the same ratio. How many different shades of paint did the students make? Explain.

Let's Talk About It

Solve the problems below as a group.

7 Refer to the situation in Problem 6. Which shades of paint are the most red? Why?

8 Use the table in Problem 6. Plot a point for each ordered pair. After you plot each point, draw a line connecting the point to (0, 0).



9 Based on the graph, what do the mixtures that are the same shade have in common? What does this tell you about their relationship?

Try It Another Way

Work with your group to determine whether the equation represents a proportional relationship. Explain your choice. You may want to make a table similar to those in Problems 5 and 6 or a graph similar to that in problem 8 on separate sheet of paper to support your reasoning.

10 $y = 2x + 4$

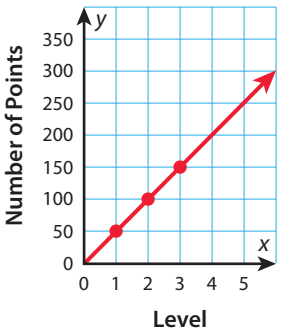
11 $y = 2x$

Connect Identifying Proportional Relationships

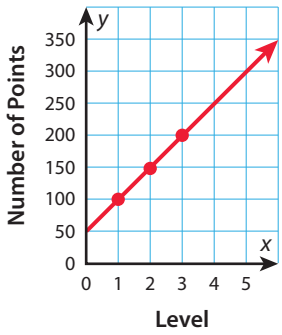
Talk through these problems as a class. Then write your answers below.

12 Compare The graphs below show the number of points you earn in each level of a game. Which games, if any, have a proportional relationship between the number of points you earn and the level of the game? In which game can you earn the most points in Level 2? Explain your answer.

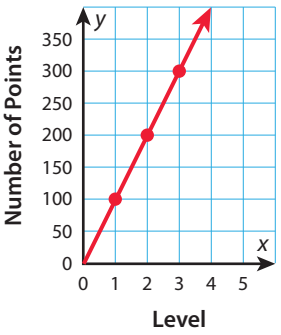
Game A



Game B



Game C



13 Apply Servers at a snack shop use the table below to find the total cost for frozen yogurt, but some of the numbers have worn off. The total cost is proportional to the number of cups of frozen yogurt. Find the missing numbers in the table.

Number of Cups of Frozen Yogurt	1	2	3	4
Total Cost ($)				18.00

14 Analyze Michael says that the difference between Dani's and Raj's ages is always the same, so Raj's age is proportional to Dani's age. Is Michael correct? Explain.

	2010	2015	2020	2025
Dani's Age	5	10	15	20
Raj's Age	10	15	20	25

Apply Identifying Proportional Relationships

15 Put It Together Use what you know to complete this task.

Paige works in an art store that sells square pieces of canvas. There are 5 different squares to choose from.

Canvas	A	B	C	D	E
Length of side (in feet)	1	2	3	4	5

Part A Add a row to the table to show the perimeter for each square piece of canvas. Then draw a graph to compare the length of a side of each square to its perimeter. Use your table and graph to explain whether this is a proportional relationship.

Part B Add a row to the table to show the area for each square piece of canvas. Then draw a graph to compare the length of a side of each square to its area. Use your table and graph to explain whether this is a proportional relationship.