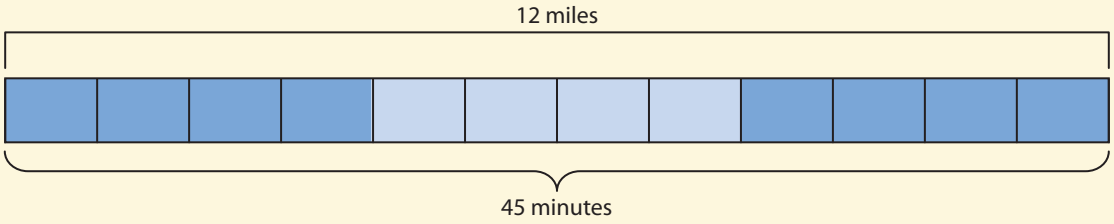
Ratios Involving Complex Fractions

Use What You Know

In Grade 6, you learned about unit rates. Take a look at this problem.

Jana is training for a triathlon that includes a 112-mile bike ride. Today, she rode her bike 12 miles in 45 minutes. What is Jana's rate in miles per hour?



Use the math you already know to solve the problem.

a. If Jana biked at a constant rate, how many miles did she bike in the first 15 minutes?

_____

b. At the same rate, how many miles did she bike in the next 15 minutes? _____

c. At the same rate, how many miles did she bike in the last 15 minutes? _____

d. How many more minutes would Jana need to bike to total one hour? _____

e. At the same rate, how many miles would she bike in that amount of time? _____

f. Explain how you could find the number of miles Jana bikes in one hour.

_____

_____

Find Out More

The number of miles Jana bikes in one hour is a **unit rate**. A unit rate compares two quantities where the second quantity is 1. A unit rate tells you how many units of the first quantity correspond to one unit of the second quantity.

The units in this problem are miles and hours. The problem tells you that Jana bikes **12 miles** in **45 minutes**. That's the same as 12 miles in $\frac{3}{4}$ **hour**.

$$\frac{\text{number of miles}}{\text{number of hours}} = \frac{12}{\frac{3}{4}}$$

The fraction $\frac{12}{\frac{3}{4}}$ is a **complex fraction**. A complex fraction is a fraction where the numerator is a fraction, the denominator is a fraction, or both the numerator and the denominator are fractions. You can simplify a complex fraction by dividing, just as you would do if the numerator and denominator were whole numbers.

The fraction bar represents division, so you can think of $\frac{6 \text{ miles}}{2 \text{ hours}}$ as $6 \div 2 = 3$ miles per hour.

You can think about $\frac{12 \text{ miles}}{\frac{3}{4} \text{ hour}}$ in the same way.

$$\frac{12}{\frac{3}{4}} = \frac{12}{1} \div \frac{3}{4}$$

$$= \frac{12}{1} \times \frac{4}{3}$$

$$= \frac{48}{3} \text{ or } 16 \text{ miles per hour}$$

The unit rate is 16. The number of miles Jana bikes is 16 times the number of hours.

Reflect

1 On another training ride, Jana bikes 15 miles in 50 minutes. Explain how you could find the number of miles she bikes in 1 hour.

Learn About Finding Unit Rates with Fractions

Read the problem below. Then explore different ways to understand how to find a unit rate.

Max's favorite recipe for oatmeal raisin cookies makes 48 servings. He wants to make some cookies but only has one egg. Max has to adjust the amounts of the other ingredients. How much flour will he need?

RECIPE

OATMEAL RAISIN COOKIES

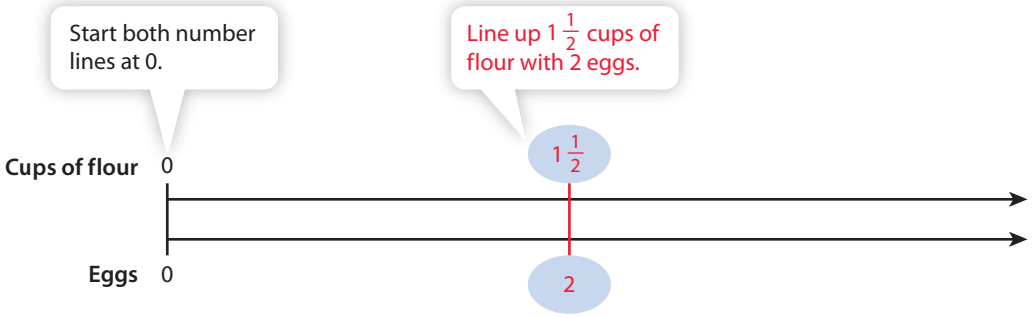
Ingredients

- $\frac{3}{4}$ cup butter
- 2 eggs
- $1\frac{1}{2}$ cups flour
- $1\frac{1}{2}$ cups brown sugar
- 1 teaspoon vanilla
- 1 teaspoon baking soda
- $\frac{3}{4}$ teaspoon cinnamon
- $2\frac{3}{4}$ cups oats
- 1 cup raisins

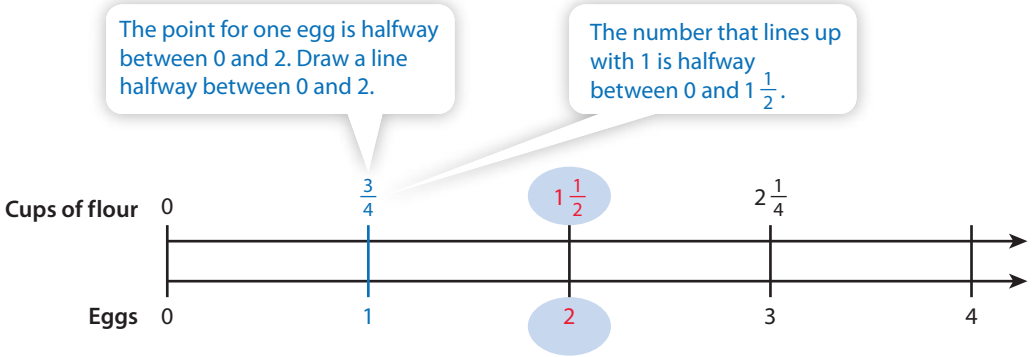
Model It

You can draw a double number line to show the relationship described in the problem.

The units you need to compare are cups of flour and eggs.



You need to find the unit rate, the number of cups of flour needed for **1** egg.



Connect It

Now you will see how to solve the problem from the previous page by writing a ratio.

2 Why do you need to find the number that is halfway between 0 and $1\frac{1}{2}$?

3 How could you find the number that is between 0 and $1\frac{1}{2}$?

4 How many cups of flour does Max need to use if he has just 1 egg? Show your work.

5 Write the ratio that compares $1\frac{1}{2}$ cups of flour to 2 eggs.

6 Write and simplify a division expression to find the number of cups of flour Max needs to use if he has just 1 egg.

7 The unit rate is _____. The number of cups of flour is _____ times the number of eggs.

8 Explain how to find a unit rate. _____

Try It

Use what you just learned about finding a unit rate to solve these problems. Show your work on a separate sheet of paper.

Use the information in the recipe on the previous page.

9 If Max has only one egg, how much butter will he need? _____

10 If Max has only one cup of flour, how much vanilla will he need? _____

Learn About Comparing Unit Rates

Read the problem below. Then explore different ways to understand how to find and compare unit rates.

José's mother is trying to decide whether or not she should buy a 12-ounce package of coffee on sale for $7.50. She knows that she can buy the same coffee for $9.00 per pound. Which is the better buy?

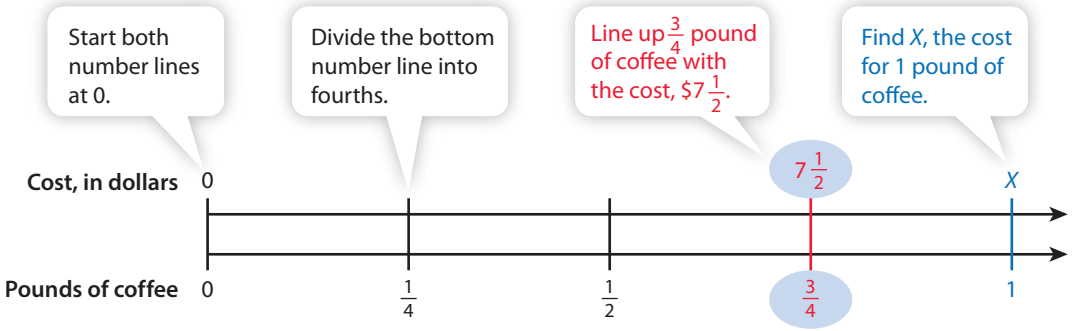
Model It **You can draw a double number line to show the relationship described in the problem.**

To find the better buy, compare the unit rate of each option.

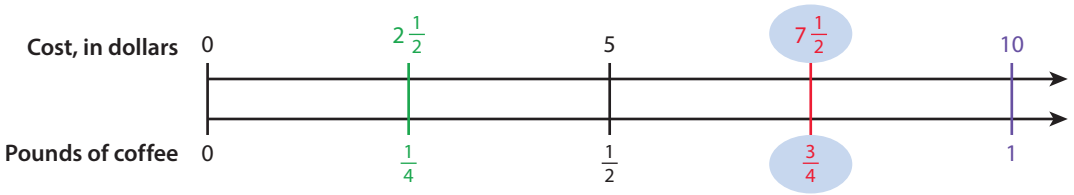
The problem gives you one unit rate: $9.00 per pound. To compare unit rates, the units you use must be the same. So, find the weight of the other coffee in pounds.

There are 16 ounces in 1 pound, so 12 ounces is $\frac{12}{16}$ or $\frac{3}{4}$ pound.

You can write $7.50 using fractions. $7.50 is the same as $7\frac{1}{2}$.



Find the **cost for each quarter-pound of coffee**. Then find the **unit cost**.



Connect It **Now you will see how to use a ratio to solve the problem.**

11 The top number line is divided into 3 equal parts from 0 to $7\frac{1}{2}$, and the bottom number line is divided into 3 equal parts from 0 to $\frac{3}{4}$. How can you use this to find the cost of 1 pound of coffee? _____

12 Write the ratio that compares $\$7\frac{1}{2}$ dollars to $\frac{3}{4}$ pound of coffee. _____

13 Write and simplify a division expression to find the cost of 1 pound of coffee.

14 Which is the better buy, 12 ounces for $7.50 or 1 pound for $9.00? Explain your reasoning.

15 If you started the problem by converting 1 pound to 16 ounces, would you get the same result? Justify your conclusion. _____

16 Can you compare any two unit rates? Explain. _____

Try It **Use what you just learned about unit rates to solve this problem. Show your work on a separate sheet of paper.**

17 Rina's recipe uses 2 cups of sugar to make $2\frac{1}{2}$ dozen cookies. Jonah's recipe uses $2\frac{1}{4}$ cups of sugar to make 3 dozen cookies. Which recipe uses more sugar for a dozen cookies? Why? _____

Practice Finding Ratios Involving Complex Fractions

Study the example below. Then solve problems 18–20.

Example

Oliver is training for a marathon. In practice, he runs 15 kilometers in 72 minutes. What is his speed in kilometers per hour?

Convert the time in minutes to hours to find kilometers per hour.

72 minutes = 1 hour 12 minutes

$= 1\frac{12}{60}$ hours or $1\frac{1}{5}$ hours

$\frac{\text{km}}{\text{hr}} = \frac{15}{1\frac{1}{5}}$

$= 15 \div 1\frac{1}{5}$

$= 15 \div \frac{6}{5}$

$= 15 \times \frac{5}{6}$

$= \frac{75}{6}$ or $12\frac{1}{2}$

Solution _____

The student knew that 60 minutes = 1 hour, so 72 minutes = 60 minutes + 12 minutes, or 1 hour 12 minutes.

Pair/Share
How did you decide how to write the ratio?

18 Alexis washes $10\frac{1}{2}$ windows in $\frac{3}{4}$ hour. At this rate, how many windows can she wash in one hour?

How do you evaluate a complex fraction?

Pair/Share
How can you tell if your answer is reasonable?

Solution _____

19 A restaurant uses $8\frac{1}{4}$ pounds of carrots to make 6 carrot cakes. Frank wants to use the same recipe. How many pounds of carrots does Frank need to make one carrot cake?

Show your work.

What is the ratio of pounds of carrots to cakes?

Solution _____

_____

Pair/Share

What steps did you take to find the unit rate?

20 It takes Zach 15 minutes to walk $7\frac{1}{2}$ blocks to the swimming pool. At this rate, how many blocks can he walk in one minute? Circle the letter of the correct answer.

A $\frac{1}{5}$ block

B $\frac{1}{2}$ block

C 2 blocks

D 5 blocks

What unit rate do you need to find?

Dee chose **C** as the correct answer. What was her error?

_____

_____

_____

_____

Pair/Share

Does Dee's answer make sense?

Practice Finding Ratios Involving Complex Fractions

Solve the problems.

1 One of the highest snowfall rates ever recorded was in Silver Lake, Colorado, in April 1921, when just over 7 feet of snow fell in $27\frac{1}{2}$ hours. What was that rate in inches per hour?

A $\frac{14}{55}$ inch per hour

B $\frac{55}{158}$ inch per hour

C $3\frac{3}{55}$ inches per hour

D $3\frac{13}{14}$ inches per hour

2 A grocery store sells different types of Trail Mix, as shown in the table below.

	Trail Mix A	Trail Mix B	Trail Mix C
Cost ($)	6	8.50	2.25
Weight	$\frac{3}{4}$ lb	1 lb	4 oz

1 lb = 16 oz

Which statement is correct?

A Trail Mix A is the best buy.

B Trail Mix B is the best buy.

C Trail Mix C is the best buy.

D They are all the same price.

3 A treadmill counts $\frac{1}{5}$ mile as one lap. The display of the treadmill indicates the number of laps already completed and highlights how much of the current lap has been completed. Create a display that shows a total of $\frac{13}{10}$ miles run.

4 A restaurant makes a special citrus dressing for its salads. Here is how the ingredients are mixed:

$\frac{1}{3}$ of the mixture is oil

$\frac{1}{4}$ of the mixture is orange juice

$\frac{1}{6}$ of the mixture is vinegar

$\frac{1}{4}$ of the mixture is lemon juice

When the ingredients are mixed in the same ratio as shown above, every batch of dressing tastes the same.

- **Batch 1:** If you have 1 cup of oil, how much vinegar will you need? How much lemon juice? Show your thinking.
- **Batch 2:** If you have 1 cup of vinegar, how much will you need of the other ingredients? Show your thinking.
- **Batch 3:** If you have 1 cup of orange juice, how much will you need of the other ingredients? Show your thinking.

5 Two friends worked out on treadmills at the gym.

- Alden walked 2 miles in $\frac{3}{4}$ hour.
- Kira walked $1\frac{3}{4}$ miles in 30 minutes.

Who walked at a faster rate? Explain your reasoning.

Show your work.

Answer _____

_____

Self Check **Go back and see what you can check off on the Self Check on page 79.**