LESSON OVERVIEW

Lesson 30
Classify Two-Dimensional Figures

Lesson Objectives

Content Objectives

- Classify two-dimensional figures in a hierarchy based on properties of the figures.
- Draw and use flow charts, Venn diagrams, and tree diagrams to show the hierarchical relationship of two-dimensional figures.

Language Objectives

- Define the key term *hierarchy* and discuss its meaning with a partner.
- List relationships among two-dimensional figures shown by flow charts, Venn diagrams, and tree diagrams.

Standards for Mathematical Practice (SMP)

2 Reason abstractly and quantitatively

3 Construct viable arguments and critique the reasoning of others.

4 Model with mathematics.

5 Use appropriate tools strategically.

6 Attend to precision.

7 Look for and make use of structure.

Prerequisite Skills

- Recognize parallel and perpendicular lines.
- Recognize right, acute, and obtuse angles.
- Sort two-dimensional figures based on the kinds of sides they have and on the kinds of angles they have.

Lesson Vocabulary

- **hierarchy** a ranking of categories based on properties

Review the following key terms.

- **polygon** a closed two-dimensional shape made with three or more line segments
- **Venn diagram** a drawing that shows relationships among groups

Learning Progression

In Grade 4 students classified two-dimensional figures that included quadrilaterals, hexagons, trapezoids, and triangles. Students classified figures based on properties of sides and angles, such as parallel or perpendicular sides and right, acute, or obtuse angles. Students classified triangles based on lengths of sides and kinds of angles and named triangles as equilateral, isosceles, or scalene, as well as right, acute, or obtuse.

In this lesson students analyze categories of polygons based on their properties and relate the categories in a hierarchy. Students use the properties of figures to show categories of polygons in a hierarchical relationship from most general to most specific. They organize properties of figures in a table and classify figures in a hierarchy by using visual models such as Venn diagrams, flow charts, and tree diagrams. In this lesson, the hierarchical relationships between categories of figures have no overlap or are entirely contained within another category.

In the next lesson students will classify polygons in hierarchies with categories that have some overlap. Students will use more complex Venn diagrams to help them visualize properties that are shared by categories of polygons. For example, students will classify triangles based on both side lengths and angle measures. Students will continue to use visual models such as tables, Venn diagrams, flow charts, and tree diagrams to organize properties of polygons and to show more complex hierarchical relationships between categories of polygons.

Lesson Pacing Guide

Whole Class Instruction

Day 1
45–60 minutes

Toolbox: Interactive Tutorial*
Classify Two-Dimensional Figures

Introduction
- Use What You Know *10 min*
- Find Out More *10 min*
- Reflect *5 min*

Modeled and Guided Instruction

Learn About Ordering Shapes in a Hierarchy
- Model It/Model It *10 min*
- Connect It *10 min*
- Try It *5 min*

Practice and Problem Solving
Assign pages 323–326.

Day 2
45–60 minutes

Guided Practice

Practice Classifying Two-Dimensional Figures
- Example *5 min*
- Problems 7–9 *15 min*
- Pair/Share *15 min*
- Solutions *10 min*

Practice and Problem Solving
Assign pages 327–328.

Day 3
45–60 minutes

Independent Practice

Practice Classifying Two-Dimensional Figures
- Problems 1–3 *20 min*
- Quick Check and Remediation *10 min*
- Hands-On or Challenge Activity *15 min*

Toolbox: Lesson Quiz
Lesson 30 Quiz

*We continually update the Interactive Tutorials. Check the Teacher Toolbox for the most up-to-date offerings for this lesson.

Small Group Differentiation

Teacher-Toolbox.com

Reteach

Ready Prerequisite Lessons *45–90 min*

Grade 4
- Lesson 32 Classify Two-Dimensional Figures

Teacher-led Activities

Tools for Instruction *15–20 min*

Grade 5 *(Lesson 30)*
- Subcategories of Plane Figures

Student-led Activities

Math Center Activities *30–40 min*

Grade 4 *(Lesson 32)*
- 4.57 Triangle Vocabulary Match
- 4.58 Classifying Shapes

Grade 5 *(Lesson 30)*
- 5.51 Organize Polygons on a Venn Diagram
- 5.52 Organize Triangles on a Venn Diagram

Personalized Learning

i-Ready.com

Independent

i-Ready Lessons* *10–20 min*

Grade 4 *(Lesson 32)*
- Quadrilaterals
- Classifying Triangles

Introduction

At A Glance

Students identify the properties of three polygons and use the properties to determine the most specific name for each polygon. Then students arrange the polygons from most general to least general. Then students explore the hierarchy of the polygons from the previous page in a Venn diagram and a flow chart.

Step By Step

- Work through **Use What You Know** as a class.
- Tell students that this page models using properties of polygons to determine the most specific name for a polygon.
- Have students read the problem at the top of the page.
- Review the meaning of parallel lines. [Lines that never intersect and always remain the same distance apart.] Draw examples of parallel lines on the board.
- Ask students to describe a right angle. [An angle that looks like a square corner and measures 90°.] Draw an example of a right angle.
- Discuss the properties that each polygon has as students complete the table.
- Ask students what properties all three polygons have in common. [4 sides, 2 pairs of parallel sides, 2 pairs of sides of equal length] Ask: *Which polygon has only those properties?* [Polygon *B*] Ask: *Which polygon has more properties than Polygon B but does not have all the properties listed in the table?* [Polygon *A*] *Which polygon has all the properties listed in the table?* [Polygon *C*] Discuss the most specific name for each polygon and point out that students can arrange the polygons from most general to least general based on the polygons' properties.
- Ask students to explain their answers for the remaining problems.

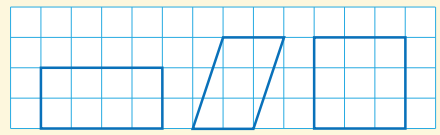
▶ Mathematical Discourse 1 and 2

Classify Two-Dimensional Figures

Use What You Know

In this lesson, you will classify polygons based on their properties. Take a look at this problem.

Arrange the polygons below so that a polygon can also be called by the name of the polygon before it. Order them from left to right.



A *B* *C*

a. Complete the table below. Put a check in each box if the polygon has the property listed.

Property	Polygon *A*	Polygon *B*	Polygon *C*
4 sides	X	X	X
2 pairs of parallel sides	X	X	X
2 pairs of sides of equal length	X	X	X
4 right angles	X		X
4 sides of equal length			X

b. Write the most specific name for each polygon from the list below.

quadrilateral parallelogram rectangle square

A: rectangle *B*: parallelogram *C*: square

c. How would you arrange the polygons so each shape has all the properties of the shape(s) before it? parallelogram, rectangle, square

▶ Mathematical Discourse

1 *Which is the most specific description of a breakfast menu item: a fried egg, an egg, or a fried egg over easy? Describe how this compares to determining the most specific name for polygons A, B, and C.*

Responses should indicate an understanding that the description that provides the most information is the most specific description.

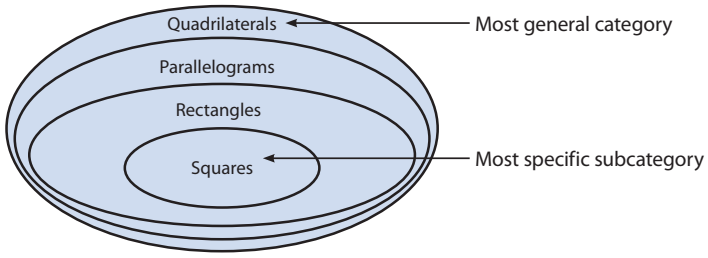
2 *Why might it make more sense to call polygon C a square when you could also call it a rectangle, a parallelogram, or a quadrilateral?*

Calling polygon *C* a square gives the most information possible about the figure.

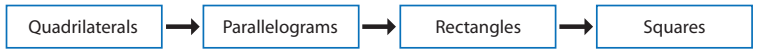
Find Out More

Shapes can be classified according to their properties. When you order categories of polygons by their properties, you put them in a **hierarchy**. A hierarchy organizes categories from the most to least general. One model you can use to show a hierarchy is a Venn diagram.

A Venn diagram can show categories and subcategories. This Venn diagram shows that squares have all the properties that rectangles have, plus more. This means all squares are also rectangles. A square is also a parallelogram and a quadrilateral.



You can also use a flow chart to show the hierarchy of quadrilaterals. The most general category is at the left, while the most specific is at the right. This means that a figure that belongs in one category also belongs in all categories to the left.



Reflect

1 How are the flow chart and the Venn diagram alike? How are they different?

Possible answer: They are alike because they order quadrilaterals first and squares last. They are different because the Venn diagram has a larger area for the largest category, and the flow chart shows each category in same-sized boxes.

Step By Step

- You may wish to review Venn diagrams with your class before discussing this page.
- Read **Find Out More** as a class.
- Have students look at the Venn diagram. Work from the outermost category to the innermost. Guide students to understand that a parallelogram is a quadrilateral because it has all of the properties of a quadrilateral plus some additional properties. Ask students to use the table on the previous page to name those additional properties. [2 pairs of parallel sides, 2 pairs of sides of equal length]
- Similarly, a rectangle is a parallelogram because it has all of the properties of a parallelogram plus an additional property. Ask students to use the table on the previous page to name the additional property. [4 right angles]
- Finally, have students explain why a square is a rectangle. [A square has all of the properties of a rectangle plus the property that it has 4 sides of equal length.]
- Point out that the flow chart shows the same relationships between the categories of quadrilaterals and that the hierarchy in the flow chart goes from the left, with the most general category, to the right, with the most specific category.

Concept Extension

Explore ordering three-dimensional figures.

Point out that just as two-dimensional figures can be ordered from general to most specific, so can three-dimensional figures. Show students an example of a generic prism [faces are parallelograms], a rectangular prism, and a cube. Point out that the faces of a generic prism are parallelograms, the faces of a rectangular prism are rectangles, and the faces of a cube are squares. Ask students to use what they learned about parallelograms, rectangles, and squares to order the three-dimensional figures from most general to most specific. [generic prism, rectangular prism, cube]

Real-World Connection

Encourage students to think about everyday places or situations in which people might see or talk about a hierarchy.

Example: A hierarchy can be applied to answer the question "Where do you live?" For example, you could answer based on your continent, country, state, city, town, neighborhood, or street name. All answers would be accurate, but the name of your street would be the most specific answer for where you live.

Real-World Connection

Concept Extension

Ready **Mathematics** PRACTICE AND PROBLEM SOLVING

Assign *Practice and Problem Solving* **pages 323–324** after students have completed this section.

Modeled and Guided Instruction

At A Glance

Students use a table to organize the properties of triangles. Then students use a tree diagram to arrange the triangles in a hierarchy. Then students revisit this problem to complete a tree diagram to understand the relationships among isosceles, scalene, and equilateral triangles and to order the triangles in a hierarchy.

Step By Step

- Read the problem at the top of the page as a class.

Model It

- Read **Model It**. Be sure that students understand that a triangle can be called isosceles if it has 2 or 3 sides of equal length. Point out that you could also say that an isosceles triangle has at least 2 sides of equal length.

▶ **Mathematical Discourse 1**

Model It

- Read **Model It**. Ask students to explain why Triangles is the more general category. [All of the other categories are specific types of triangles.]

▶ **English Language Learners**

SMP TIP Attend to Precision
Students attend to precision when they use clear and precise language to describe the properties of isosceles, scalene, and equilateral triangles. *(SMP 6)*

▶ **Mathematical Discourse 2**

Learn About Ordering Shapes in a Hierarchy

Read the problem below. Then explore different ways to classify figures in a hierarchy.

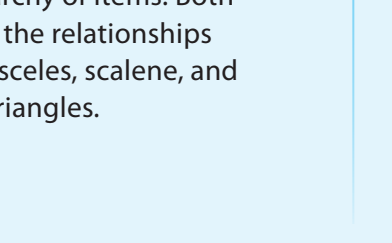
Classify the following triangles from the most general to the most specific: scalene triangle, isosceles triangle, and equilateral triangle. Use a tree diagram to classify them as types of triangles.

Model It **You can understand the problem by listing the properties of the triangles in a table before arranging them in a tree diagram.**

Types of Triangles	Properties of Sides
Isosceles	2 or 3 sides of equal length
Scalene	no sides of equal length
Equilateral	3 sides of equal length

Model It **You can represent the problem with a tree diagram.**

A tree diagram can also be used to show a hierarchy. Put the most general category as the top branch. Then put the more specific subcategories as the branches.



▶ Mathematical Discourse

1 *The table shows the properties of sides of isosceles, scalene, and equilateral triangles. What kinds of triangles could you put in the table if you wanted to show the properties of angles of triangles?*

You could put right triangles, acute triangles, and obtuse triangles in the table.

2 *How is a tree diagram similar to a Venn diagram?*

Responses may vary but should indicate an understanding that both show a hierarchy of items. Both would show the relationships between isosceles, scalene, and equilateral triangles.

▶ English Language Learners

Point out that a *tree diagram* gets its name because its shape resembles a tree with many branches.

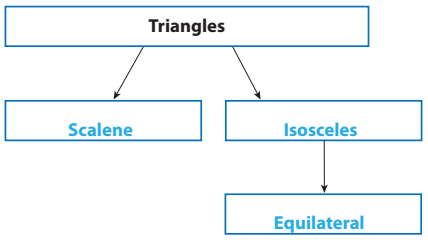
Connect It **Now you will solve the problem from the previous page by using the table to complete a tree diagram.**

2 Why is "Triangles" in the top row of the tree diagram? Possible answer: That's the most general category.

3 Write "Scalene" and "Isosceles" in the second row of the tree diagram at the right.

Why are those categories separate?

A scalene triangle cannot have sides of equal length.



4 Write "Equilateral" beneath "Isosceles."

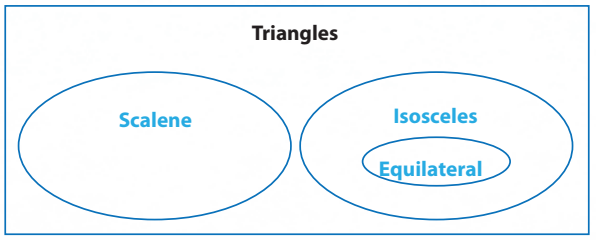
Why can all equilateral triangles be classified as isosceles triangles?

They have at least 2 sides of equal length.

5 How can you use a tree diagram to order figures? Possible answer: Place the most general category of shape at the top and more specific subcategories of shapes beneath.

Try It **Use what you learned about ordering figures in a hierarchy to solve this problem.**

6 Complete the Venn diagram below to show the hierarchy of isosceles, scalene, and equilateral triangles.



▶ Concept Extension

Explore Venn diagrams.

Point out that the Venn diagram for Quadrilaterals, Parallelograms, Rectangles, and Squares shown on the Introduction page uses an oval for the category Parallelograms. The category Parallelograms is set within an oval because there are figures that are quadrilaterals but not parallelograms. Ask students to name or draw such a shape. [students may name or draw a kite or another quadrilateral that is not a parallelogram]

Explain that when drawing a Venn diagram for triangles based on the properties of sides of triangles, you do need to set the category Triangles within an oval because Scalene and Isosceles triangles make up the entire category of Triangles. There are no triangles that are not Scalene or Isosceles. All triangles fit in one of these two categories.

Step By Step

Connect It

- Read **Connect It** as a class. Be sure to point out that the questions refer to the problem on the previous page.
- Guide students to understand that all triangles can be classified as either scalene or isosceles because scalene triangles have no sides of equal length and isosceles triangles have at least 2 sides of equal length. Since "at least 2" means 2 or more, equilateral triangles are also isosceles triangles.

SMP TIP **Model with Mathematics**
Students model the hierarchical order of triangles using a tree diagram. Point out that they could also use a flow chart or Venn diagram to model the order. *(SMP 4)*

▶ Concept Extension

Try It

- Ask students how they would show that all equilateral triangles are isosceles triangles using a Venn diagram. [The oval for Equilateral would be nested inside the oval for Isosceles.]

6 Solution
See completed Venn diagram on the Student Book page. Students may draw a Venn diagram with an outside category of triangles, two non-overlapping subcategories of scalene and isosceles, and a category of equilateral that is nested inside isosceles.

Error Alert Students who draw *equilateral* such that it overlaps both *scalene* and *isosceles* may not understand what overlapping categories of a Venn diagram represent.

Ready **Mathematics** PRACTICE AND PROBLEM SOLVING

Assign *Practice and Problem Solving* **pages 325–326** after students have completed this section.

Guided Practice

At A Glance

Students use tables, Venn diagrams, tree diagrams, and flow charts to classify plane figures in a hierarchy.

Step By Step

- Ask students to solve the problems individually and label categories in their diagrams.
- **Pair/Share** When students have completed each problem, have them Pair/Share to discuss their solutions with a partner or in a group.

Solutions

Example A Venn diagram illustrating the hierarchy is shown. Students may begin by creating a table or list of properties.

7 Solution

Students may say that all equilateral triangles are acute triangles. They may also say that some acute triangles are equilateral triangles.

DOK 3

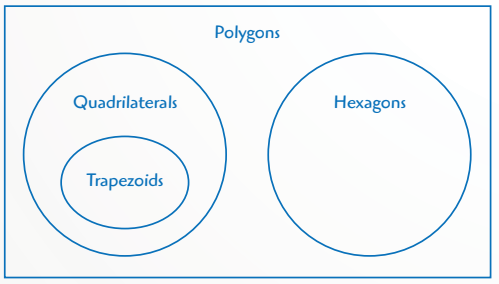
Practice Classifying Two-Dimensional Figures

Study the example below. Then solve problems 7–9.

Example

Create a Venn diagram to show the hierarchy of quadrilaterals, polygons, trapezoids, and hexagons.

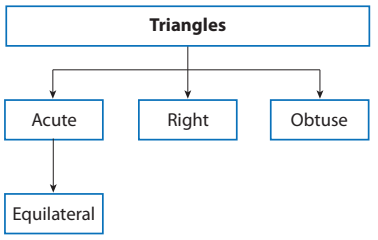
Look at how you could show your work using a Venn diagram.



A shape can never be both a hexagon and a quadrilateral. So these regions do not overlap.

Pair/Share
Recreate the hierarchy with a tree diagram.

7 Look at the tree diagram below. Write a statement about the relationship between acute triangles and equilateral triangles.



Which type of triangle is the most specific?

Pair/Share
Write a statement about the relationship between acute triangles and obtuse triangles.

Solution All equilateral triangles are acute triangles.

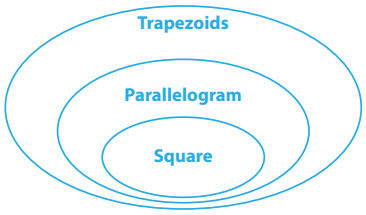
Teacher Notes

8 Create a Venn diagram to show the hierarchy of the polygons described in the chart.

Polygon	Description
Trapezoid	quadrilateral with at least 1 pair of parallel sides
Square	parallelogram with 4 sides of equal length
Parallelogram	quadrilateral with 2 pairs of parallel sides

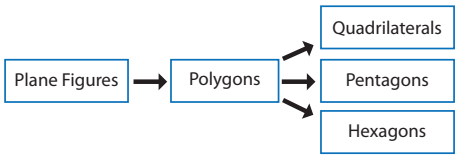
"At least 1" means 1 or more.

Possible Venn diagram:



Pair/Share
Draw one example of a polygon in each separate category of your Venn diagram.

9 Look at the flow chart below.



The flow chart is like a tree diagram. But the arrows show that the hierarchy moves from left to right instead of top to bottom.

Which statement is true? Circle the letter of the correct answer.

A A plane figure is always a polygon.

B All polygons are plane figures.

C All hexagons are also pentagons and quadrilaterals.

D A hexagon is not a plane figure.

Brad chose **C** as the correct answer. How did he get that answer?

Possible answer: Brad confused the flow chart with a tree diagram.

Pair/Share
Does Brad's answer make sense?

Teacher Notes

Solutions

8 Solution

See possible student work on the Student Book page; Students use the descriptions in the table to create a Venn diagram.

DOK 3

9 Solution

B: "Polygons" belong to the category "Plane Figures" because a figure that belongs in one category also belongs in all categories to the left.

Explain to students why the other two answer choices are not correct:

A is not correct because a figure that belongs in one category also belongs in all categories to the left (so a polygon is always a plane figure), but a figure that belongs in one category does not necessarily belong in all categories to the right (so a plane figure is not always a polygon).

D is not correct because a figure that is a hexagon also belongs in all categories to the left, so a hexagon is also a plane figure.

DOK 3

Ready Mathematics PRACTICE AND PROBLEM SOLVING

Assign *Practice and Problem Solving* **pages 327–328** after students have completed this section.

Independent Practice

At A Glance

Students classify plane figures in a hierarchy based on their properties to answer questions that might appear on a mathematics test.

Solutions

1 Solution

A; The most general, or least specific, name for the shape shown is polygon. The shape has 4 sides, so it is also a quadrilateral. The shape has 2 pairs of sides of equal length and 4 right angles, so it is also a rectangle.

DOK 2

2 Solution

See the Student Book page for the completed table; A scalene triangle has 3 sides of different lengths. An isosceles triangle has at least 2 sides of equal length. An obtuse triangle has an angle greater than 90°.

DOK 1

Practice Classifying Two-Dimensional Figures

Solve the problems.

1 Look at the shape below.

Which is a correct classification for this shape from LEAST specific to MOST specific?

A polygon, quadrilateral, rectangle

B quadrilateral, parallelogram, square

C polygon, quadrilateral, square

D quadrilateral, rectangle, square

2 Classify the triangles shown below as "scalene," "isosceles," or "obtuse." Sides that are the same length are marked with a slash. Draw the triangles in the correct column of the table. If a triangle fits more than one classification, draw it in all the columns that apply.

Scalene	Isosceles	Obtuse

Quick Check and Remediation

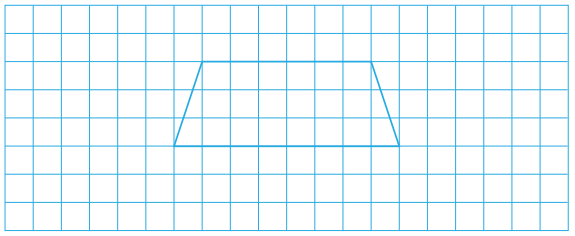
- Ask students to draw a Venn diagram to classify the following shapes in order from most general to most specific: parallelogram, polygon, rhombus, quadrilateral. [polygon, quadrilateral, parallelogram, rhombus] Remind students that a rhombus is a parallelogram with four sides of equal length.
- For students who are struggling, use the chart to guide remediation.
- After providing remediation, check students' understanding. Ask students to draw a Venn diagram to classify the following shapes in order from most general to most specific: parallelogram, square, rhombus, quadrilateral [quadrilateral, parallelogram, rhombus, square]
- If a student is still having difficulty, use *Ready Instruction*, Grade 4, Lesson 32.

If the error is . . .	Students may . . .	To remediate . . .
any order other than the correct order	not be able to identify all of the properties of the given shapes	Have students make a table with the names of the shapes as the headings of four columns. Work with students to list the properties of each of the shapes so that students can see, for example, that a rhombus has all the properties that parallelograms have plus one more.

3 Look at the flow chart below.

Part A Draw an example of a trapezoid that is not a parallelogram.

Possible trapezoid:



Part B Explain how trapezoids relate to parallelograms.

Possible answer: Trapezoids have at least 1 pair of parallel sides and parallelograms have 2 pairs of parallel sides. So all parallelograms are trapezoids but not all trapezoids are parallelograms.

Part C Can you use the term "parallelogram" to describe a rectangle? Explain your reasoning.

Yes. Possible explanation: All rectangles are parallelograms so you can use the term parallelogram to describe a rectangle.

Self Check Go back and see what you can check off on the Self Check on page 283.

Solutions

3 Part A Solution

Students draw a trapezoid that has 1 pair of parallel sides; see possible drawing on the Student Book page.

Part B Solution

See possible student explanation on the Student Book page.

Part C Solution

Yes; see possible student explanation on the Student Book page.

DOK 3

▶ Hands-On Activity

Build quadrilaterals that fit the given conditions.

Materials: geoboards and geobands

Have students make a shape that fits conditions you supply. Ask students to name the shape they made.

- four sides, opposite sides are parallel, no right angles [parallelogram (or rhombus)]
- four sides, opposite sides are parallel, four right angles [rectangle (or square)]
- four sides, opposite sides are parallel, four right angles, all sides are of equal length [square]

If time permits, provide conditions for students to build different triangles and have students build and name the triangles you have described.

▶ Challenge Activity

Name the figure in different ways.

Challenge students to provide as many different names as they can for figures that you draw.

Label important features such as sides of equal length and right angles in your drawings.

Include figures such as an equilateral triangle, a parallelogram, a square, a rectangle, and a rhombus. Have students justify why each of the names they use applies to the figure.

LESSON QUIZ

Lesson 30
Classify Two-Dimensional Figures

Teacher-Toolbox.com

Overview

Assign the Lesson 30 Quiz and have students work independently to complete it.

Use the results of the quiz to assess students' understanding of the content of the lesson and to identify areas for reteaching. See the Lesson Pacing Guide at the beginning of the lesson for suggested instructional resources.

Tested Skills

Problems on this assessment form require students to be able to categorize two-dimensional figures based on properties and attributes and to interpret and fill in flow charts to classify figures and organize them in a hierarchy from most general to least general. Students will also need to be familiar with acute, obtuse, and right angles, parallel and perpendicular lines, and how to sort figures based on side lengths and angle types.

***Ready*® Mathematics**

Lesson 30 Quiz

Solve the problems.

1 Which of the following most specifically describes a rhombus?

A a plane figure that is a polygon

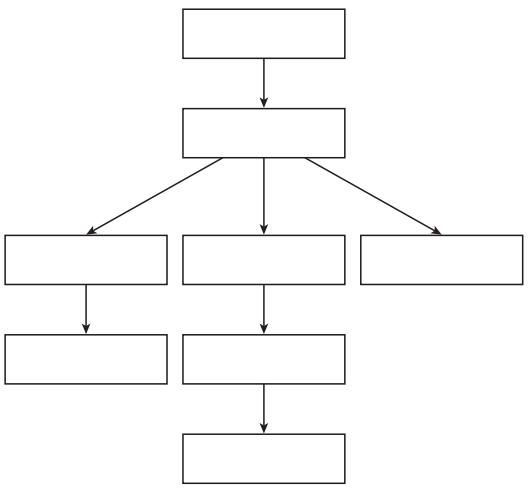
B a quadrilateral with 2 pairs of parallel sides

C a parallelogram with 4 sides of equal length

D a polygon with four sides

2 Create a hierarchy using the categories below. Write a category in each box to complete the tree diagram.

right triangles	pentagons	polygons	rhombuses
triangles	plane figures	quadrilaterals	parallelograms



Lesson 30 Quiz continued

3 Fill in the blanks to write true statements about the relationship between different types of triangles. Use the words in the box below. Answer choices may be used more than once.

always	sometimes	never

a. An equilateral triangle is _____ an obtuse triangle.

b. A right triangle is _____ an isosceles triangle.

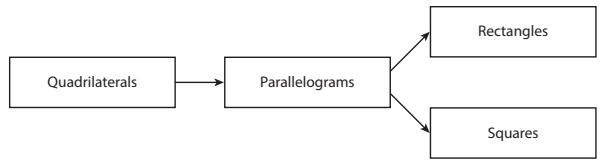
c. An equilateral triangle is _____ an isosceles triangle.

d. An isosceles triangle is _____ a scalene triangle.

e. An equilateral triangle is _____ an acute triangle.

f. An obtuse triangle is _____ an acute triangle.

4 Robert uses the flow chart below to order quadrilaterals in a hierarchy.



Which statement correctly explains whether Robert's flow chart is correct or not?

A It is not correct because rectangles and squares are both parallelograms, but *Rectangles* is a more specific subcategory of *Squares*.

B It is not correct because squares and rectangles are both parallelograms, but *Squares* is a more specific subcategory of *Rectangles*.

C It is correct because squares and rectangles are both parallelograms, and squares and rectangles do not share any other properties.

D It is correct because squares and rectangles are both parallelograms, and squares and rectangles both have 4 right angles.

Common Misconceptions and Errors

Errors may result if students:

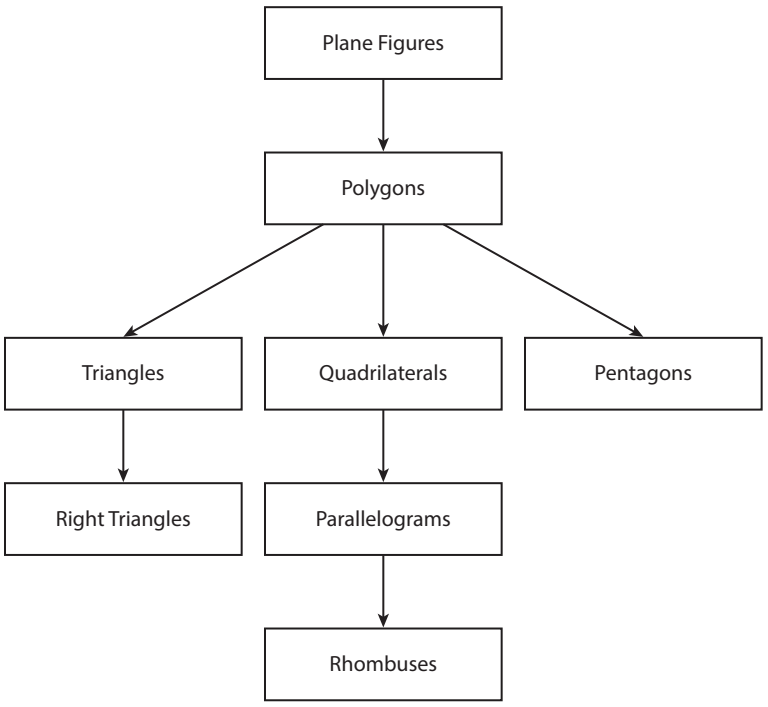
- confuse, incorrectly identify, or ignore some properties of different figures.
- choose a correct, but not the best/most specific, example or description.
- assume a relationship in one direction is also true in the other direction.
- cannot interpret or fill in a flow chart.

***Ready*® Mathematics**

Lesson 30 Quiz Answer Key

1. C
DOK 2

2.



DOK 2

3. **a.** never
b. sometimes
c. always
d. never
e. always
f. never
DOK 2

4. B
DOK 3