#### Lesson 17 **Understand Division with Unit Fractions OVERVIEW**

#### **Lesson Objectives**

#### **Content Objectives**

LESSON

- · Identify situations that involve dividing a unit fraction by a whole number.
- Identify situations that involve dividing a whole number by a unit fraction.
- Use a visual fraction model to find the quotient of a unit fraction divided by a whole number or the quotient of a whole number divided by a unit fraction.
- For a given division equation with a unit fraction and a whole number, use the relationship between multiplication and division to write a related multiplication equation.

#### Language Objectives

- Divide a whole number by a unit fraction using common denominators.
- · Draw a model to illustrate using multiplication to find the quotient of a whole number and a unit fraction.
- Draw a visual model to illustrate dividing a unit fraction by a whole number.

#### Standards for Mathematical Practice (SMP)

- 1 Make sense of problems and persevere in solving them.
- 2 Reason abstractly and quantitatively.
- 3 Construct viable arguments and critique the reasoning of others.
- 4 Model with mathematics.
- 5 Use appropriate tools strategically.
- 6 Attend to precision.
- 7 Look for and make use of structure.
- 8 Look for and express regularity in repeated reasoning.

#### Learning Progression

Since Grade 3, students have used visual models such as arrays and area models to build foundational understanding of multiplication and division of whole numbers and the inverse relationship between the two operations.

Now in Grade 5, students apply these understandings to dividing with unit fractions (fractions with a numerator of 1). Students have multiplied with fractions in Grade 4 and in previous Grade 5 lessons. Students build on the understanding of fractions as quotients that they acquired earlier in Grade 5 when they learned that dividing a whole number by a whole number is

#### **Prerequisite Skills**

- Divide whole numbers.
- Multiply whole numbers by unit fractions.
- Understand that multiplication and division are inverse operations.
- Understand that  $a \div b = a \times \frac{1}{b}$ .

#### Lesson Vocabulary

There is no new vocabulary. Review the following key term.

• unit fractions a fraction with a numerator of 1; other fractions are built from unit fractions.

#### equivalent to multiplying the whole number by a related unit fraction. In this lesson, students use the relationship between multiplication and division to gain a conceptual understanding of dividing with unit fractions. They divide whole numbers by unit fractions and divide unit fractions by whole numbers, using area models, number lines, and pictures to represent the division.

In the next lesson, students will apply their understanding of the concepts in this lesson when they divide with unit fractions in order to solve word problems. In Grade 6 students solve mathematical and real-world problems involving division of fractions by fractions.

# **Lesson Pacing Guide**

# **Whole Class Instruction**

Day 1 45–60 minutes

Introduction	
• Think It Through Question 5 r	nin
• Think 10 min	

**Toolbox: Interactive Tutorial\*** 

Understand Division with Unit Fractions

• Think 15 min

 Compare 10 min Analyze 10 min • Justify 10 min

**Unit Fractions** 

Lesson 17 Quiz

Independent Practice

• Put It Together 25 min

**Toolbox: Lesson Quiz** 

**Apply Ideas About Dividing with** 

• On-Level Class Activity 15 min

• On-Level, Intervention, or Challenge Activity 20 min

• Reflect 5 min

#### Day 2 45–60 minutes

Day 3

Day 4

45–60 minutes

**Guided Instruction Think About Using Unit Fractions in** Division • Let's Explore the Idea 20 min • Let's Talk About It 15 min • Try It Another Way 10 min **Guided Practice** 45–60 minutes **Connect Ideas About Dividing with Unit Fractions** 

**Practice and Problem Solving** Assign pages 165–166.

# **Practice and**

**Practice and** 

**Problem Solving** 

Assign pages 167–168.

**Problem Solving** Assign pages 169–170.

# **Small Group Differentiation**

#### **Teacher-Toolbox.com**

#### Reteach

Ready Prerequisite Lessons 45–90 min

#### Grade 4

• Lesson 18 Understand Fraction **Multiplication** 

• Lesson 19 Multiply Fractions

#### **Student-led Activities** Math Center Activities 30–40 min

Grade 4 (Lessons 18 and 19) • 4.33 Multiplying with Fractions • 4.34 Fraction Word Problems

- Grade 5 (Lesson 17) • 5.33 Quotients—Greater than 1 or Less
- than 1?

#### **Personalized Learning**

#### i-Ready.com

Independent i-Ready Lessons\* 10-20 min

Grade 4 (Lessons 18 and 19) Understand Fraction Multiplication

\*We continually update the Interactive Tutorials. Check the Teacher Toolbox for the most up-to-date offerings for this lesson.

#### Introduction

#### At A Glance

Students explore the meaning of division with unit fractions. They base their exploration on what they already know about the inverse relationship of multiplication and division and their knowledge of multiplying with fractions. An area model is used to illustrate a real-world situation that requires dividing a unit fraction by a whole number. Then students explore dividing a whole number by a unit fraction. They use a number line to visualize the situation and look at the relationship to multiplication.

#### **Step By Step**

- Introduce the question at the top of the page.
- Use the example at the top of the page to remind students of what they already know about dividing whole numbers and multiplying a whole number by a unit fraction.
- Read the **Think** question and read the problem to introduce the context.

#### **SMP TIP** Make Sense of Problems

Help students understand the problem context in order to understand the meaning of division with fractions. To highlight the need for division, pose a simpler problem: *What equation would you write if Mrs. Cook wanted to share 6 pounds of fish equally among 3 cats*? [6 ÷ 3 = 2] Have students draw a picture of  $\frac{1}{4}$  pound of fish and 3 cats, and discuss what happens when the food is divided equally among the cats. *(SMP 1)* 

- Introduce the area model. Have students identify  $\frac{1}{4}$  pound and the amount that each cat gets  $(\frac{1}{12} \text{ pound})$ .
- Read the explanation for using multiplication and discuss.
- Mathematical Discourse 1 and 2

Lesson 17 & Introduction Understand Division with Unit Fractions

#### 😽 Think It Through



You know that multiplication and division are related. Dividing 8 by 4, for example, gives the same result as multiplying 8 by  $\frac{1}{d}$ .

 $8 \div 4 = 2$ 

 $8 \times \frac{1}{4} = 2$ 

Dividing with unit fractions works the same way. You can solve a division problem involving fractions by multiplication.

Think What does dividing a unit fraction by a whole number mean?

Mrs. Cook wants to share  $\frac{1}{4}$  pound of fish equally among 3 cats.

That means she needs to divide  $\frac{1}{4}$  into 3 equal parts. You can draw an area model to represent the problem.



If  $\frac{1}{4}$  pound of fish is divided into 3 equal parts, each cat will receive  $\frac{1}{3}$  of the  $\frac{1}{4}$  pound of fish.

Circle the multiplication equation that solves the division situation.

#### Mathematical Discourse

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 $\frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$ 

- 1 Explain how dividing with fractions is related to multiplying with fractions. Division is the inverse of multiplication; you can solve  $8 \div 4$  by solving  $\_\_ \times 4 = 8$ . It is the same with fractions:  $\frac{1}{4} \div 3$  can be solved using  $\_\_ \times 3 = \frac{1}{4}$ .
- **2** How does the area model show the division? How does it show multiplication?

It shows  $\frac{1}{4}$  being divided into 3 equal parts,  $\frac{1}{4} \div 3$ . The shaded part represents  $\frac{1}{3} \times \frac{1}{4}$ .

**3** How does the number line help you understand dividing a whole number by a unit fraction?

The number line shows the wholes. You can divide each whole into the number of parts that the unit fraction represents. Then you can see all the parts that are in the given number of wholes.

**4** When you divide a whole number by a unit fraction, what does the quotient mean?

It tells you how many of the unit fraction make up that whole number.

#### Think What does dividing a whole number by a unit fraction mean?

Mr. Putnam wants to cut a 3-foot rope into  $\frac{1}{4}$ -foot sections.

To figure out how many sections he will get, Mr. Putnam thinks, "How many fourths are in 3?"



this division problem. It

is greater than 3, the number I started with!

You can draw a number line to represent the 3 feet of rope. There are three 1-foot sections.



You can mark fourths on the number line to represent  $\frac{1}{4}$  foot. You can see there are twelve  $\frac{1}{4}$ -foot sections in 3 feet.

$$3 \div \frac{1}{4} = 12$$

$$4 + \frac{1}{4}$$

$$4 + \frac{1}{$$

You can also write a multiplication equation to show how many fourths are in 3. There are 4 fourths in each whole foot. To find the number of fourths in 3 feet, you can multiply.

 $3 \times 4 = 12$ 

When you divide 3 by  $\frac{1}{4}$ , you are dividing 3 into parts smaller than 1. So there will be more than 3 of those parts.

#### Reflect

**1** Explain what it means to divide 5 by  $\frac{1}{4}$ .

Possible answer: It means finding out how many fourths are in 5 wholes.

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#### Hands-On Activity

# Use fraction circles to divide by a unit fraction.

#### Materials: fraction circles

- Use 8 whole fraction circles to represent whole sandwiches.
- Set up the problem. Say: Suppose we wanted to give each child half a sandwich. How many children can we feed with these 8 sandwiches?
- Have students cover the 8 whole circles with halves to represent cutting each sandwich in half.
- Ask: *How many half-sandwiches are there*? [16, so you can give 16 children each half a sandwich]

• Write "8  $\div \frac{1}{2} = 16."$ 

 Repeat the activity with other whole numbers and unit fractions.

#### English Language Learners

Help students understand the question, "How many fourths are in 3?" "3" means 3 wholes. There are 4 fourths in 1 whole. So, how many are in 3 wholes? (4 + 4 + 4, or 12)

#### **Step By Step**

- Read the **Think** section with students. Discuss the meaning of the problem: cutting a rope into some number of sections that are each a given equal size.
- Have a volunteer describe the first number line.
- Discuss the second number line and have students count the  $\frac{1}{4}$  sections. [There are 12 sections.]
- English Language Learners
- Mathematical Discourse 3 and 4
- Have students read and discuss the last sentence in the **Think** section.
- Have students read and reply to the **Reflect** directive.
- Hands-On Activity

#### **Ready** Mathematics PRACTICE AND PROBLEM SOLVING

Assign *Practice and Problem Solving* **pages 165–166** after students have completed this section.

Lesson 17 Understand Division with Unit Fractions

**Guided Instruction** 

## At A Glance

Students continue their investigation of dividing a whole number by a unit fraction. They draw a visual model of a real-world problem and use reasoning and mathematical knowledge to solve. Then students work in small groups to solve problems involving division of a unit fraction by a whole number. They first use a visual model and multiplication, and then they explore using a common denominator to divide.

#### Step By Step

#### Let's Explore the Idea

- Tell students that they will have time to work individually on the problems on this page and then share their responses in groups. You may choose to work through the first problem together as a class.
- As students work individually, circulate among them. This is an opportunity to assess student understanding and address student misconceptions. Use the Hands-On Activity to help redirect thinking of students who struggle.

#### Hands-On Activity

#### **Student Misconception Alert**

Students may believe that the quotient in division is always smaller than the dividend and the divisor. Point out that this problem situation is not about something getting smaller. It is about asking how many groups there are of a certain size. Ask students to think, for example, about what happens to the number of pieces if they cut a stalk of celery into 20 equal pieces. [There are more pieces of celery.] Explain that this is the meaning of dividing by a fraction.

- Have students discuss their work in groups. Listen for explanations that include the idea of 3 thirds in one whole cup.
- Guide students to recognize that  $5 \times 3$  and  $5 \div \frac{1}{3}$  are two ways to describe the same situation. Students should recognize that the answer is the same, either way.

Lesson 17 🍪 Guided Instruction

#### Think About Using Unit Fractions in Division

Let's Explore the Idea Explore dividing a whole number by a unit fraction with the problem below.



Jemma made 5 cups of pancake batter. She uses a scoop measuring  $\frac{1}{3}$  cup to pour batter onto the skillet to make large pancakes. How many pancakes can Jemma make?

The 5 rectangles below represent the 5 cups of pancake batter.



#### Hands-On Activity Act out the problem.

**Materials:** 1-cup measuring cups,  $\frac{1}{3}$ -cup measuring cups, sand, chart

- Give each group of students five 1-cup measuring cups, one  $\frac{1}{3}$ -cup measuring cup, and sand.
- Create a chart with 1 column for each cup and one row labeled "number of  $\frac{1}{3}$ -cups."
- Ask: How many  $\frac{1}{3}$ -cup scoops are in 1 cup? Students fill their  $\frac{1}{3}$ -cup measure and pour into a 1-cup measure. Repeat to fill the cup (3 scoops). Record that 3 scoops filled 1 cup.
- Ask: How many  $\frac{1}{3}$ -cup scoops are in 2 cups? Students count scoops as they fill another cup. Record that 6 scoops filled 2 cups.
- Students continue scooping, counting, recording.
- Discuss the pattern. [number of cups × 3 scoops in each cup = total number of scoops]

#### Lesson 17



#### Mathematical Discourse

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**1** Explain how the overlapping section shows the solution.

 $\frac{1}{3}$  of the whole is shaded horizontally. This represents how much batter Jemma has,  $\frac{1}{3}$  cup. The vertical lines divide the batter into 4 equal parts to make 4 pancakes. The overlapping section shows how much batter goes into each pancake. It is  $\frac{1}{4}$  of  $\frac{1}{3}$  cup, or  $\frac{1}{12}$  cup.

2 Why do you think you can use a multiplication equation to solve a division problem?

Because multiplication and division are inverse operations. They are different ways to show the same relationship among quantities.

#### **Step By Step**

#### Let's Talk About It

- Organize students into pairs or groups. You may choose to work through problem 9 together as a class.
- Walk around to each group, listen to, and join in on discussions at different points.

#### Mathematical Discourse 1 and 2

#### **Try It Another Way**

- Direct the group's attention to Try It Another Way. Have a volunteer from each group come to the board to explain the group's solutions to problems 16-18.
- To help students better understand division with fractions, encourage them to use models to represent the problems 16-18.

#### **SMP TIP** Model with Mathematics

To help students develop appropriate models when dividing with fractions, describe the two basic problem scenarios. Explain that, with division, sometimes you know the size of the groups you want to make and you divide to find out how many groups there will be. Other times, you know the number of groups and you divide to find out what size the groups will be. Students can use this understanding to determine how to model a problem visually and mathematically. (SMP 4)

**Ready** Mathematics PRACTICE AND PROBLEM SOLVING

Assign Practice and Problem Solving pages 167–168 after students have completed this section.

#### **Guided Practice**

## At A Glance

Students demonstrate their understanding of dividing with unit fractions by discussing several mathematical problems. Students use models to justify their conclusions. Then students choose a problem to model both visually and mathematically, using both division and multiplication.

#### Step By Step

• Discuss each problem as a class using the discussion points outlined below.

#### Compare

- You may choose to have students work in pairs to encourage sharing ideas. Have one partner draw a model for  $\frac{1}{4} \div 4$  and the other draw a model for  $\frac{1}{4} \times \frac{1}{4}$ .
- Have students share and explain their models. Use the following questions to lead the class discussion:
  - What is the same about your two models? What is different?

How does one model show division (or multiplication)?

#### Analyze

- The second problem focuses on the importance of estimating to check the reasonableness of an answer. Multiplying a whole number by a fraction less than one results in a number greater than the whole number.
- Ask: How does your model show where Helen's mistake was?
- If students struggle, ask specific questions, such as: How many thirds are there in 12? 12 divided by what equals 4?

Lesson 17 🍰 Guided Practice

#### **Connect** Ideas About Dividing with Unit Fractions

#### Talk through these problems as a class. Then write your answers below.

**[9 Compare** Draw a model to represent  $\frac{1}{4} \div 4$  and a model to represent  $\frac{1}{4} \times \frac{1}{4}$ . Explain the relationship between the two expressions.

Possible student model.



Possible explanation: The expressions are equivalent. They both have a



**20** Analyze Helena said that  $12 \div \frac{1}{3}$  is 4. Draw a model and use words to explain why Helena's statement is not reasonable.

Possible student model:





**21** Justify Show that  $\frac{1}{2} \div 3 = \frac{1}{6}$  by using a model. Explain why the result is less than the number you started with,  $\frac{1}{2}$ .

Possible student model:



Possible explanation: When you divide a number into parts, each part will be

smaller than the number you started with.

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#### Justify

- Students use a number line to model a fraction divided by a whole number.
- Have students explain their models. Ask: How does your model show  $\frac{1}{2}$ ? How does it show 3? Guide students to make a general statement to explain why the quotient is less than the number they started with.

#### **SMP TIP** Reason Quantitatively

Until now, students' work with fraction division has been grounded in real-world situations. Now, they begin to decontextualize and use quantitative reasoning to explain their solutions. Encourage students to use visual models as a bridge between numbers as symbols and the actual quantities that the numbers represent. (SMP 2) Lesson 17 🔓 Independent Practice

#### Apply Ideas About Dividing with Unit Fractions

22 Put It Together Use what you have learned to complete this task.

Choose one of the following problems to solve. Circle the problem you choose.

Drew wants to run at least 6 miles this month. He plans to run  $\frac{1}{4}$  mile each day. How many days will it take Drew to run 6 miles?

Maya made  $\frac{1}{2}$  quart of strawberry jam. She plans to share it equally among 4 friends. How much jam will each friend get?

Part A Draw a model to represent the problem.

Models will vary. A model for the first problem should show  $6 \div \frac{1}{4} = 24$ , and a model for the second problem should show  $\frac{1}{2} \div 4 = \frac{1}{8}$ .

**Part B** Write a division equation and a multiplication equation that represent the problem.

Possible equations for first problem:  $6 \div \frac{1}{4} = 24$ ;  $6 \times 4 = 24$ 

Possible equations for second problem:  $\frac{1}{2} \div 4 = \frac{1}{8}; \frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$ 

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#### **Scoring Rubrics**

Part A						
Points	Expectations					
2	The model correctly shows the dividend being divided into sections specified by the divisor (each whole into fourths, or the half into 4 equal parts). The quotient is correct and is clearly indicated.					
1	The model correctly shows either the dividend or the divisor but not both. The quotient is correct but may not be clearly indicated.					
0	Of the dividend, divisor, and quotient, at least two quantities are represented incorrectly or not at all.					

Part B							
Points	Expectations						
2	Both equations are correct. The multiplication equation can be considered correct if the factors and the product are correct, regardless of the order in which the factors are written.						
1	One equation is correct but the other is not. The multiplication equation can be considered correct if the factors and the product are correct, regardless of the order in which the factors are written.						
0	Neither equation is correct. (The multiplication equation can be considered correct if the factors and the product are correct, regardless of the order in which the factors are written.)						

# Independent Practice

Lesson 17

#### Step By Step

#### **Put It Together**

- Direct students to complete the **Put It Together** task on their own.
- Remind them to read the problem carefully and be able to describe what it means.
- As students work on their own, walk around to assess their progress and understanding, to answer their questions, and to give additional support, if needed.
- Students may use area models or number lines to represent the problem.
- If time permits, have students share their visual and mathematical models. Students can compare visual models to see different ways to approach each problem.

**Ready** Mathematics PRACTICE AND PROBLEM SOLVING

Assign *Practice and Problem Solving* **pages 169–170** after students have completed Guided Practice.

# Lesson 17 Understand Division with Unit Fractions

# **Differentiated Instruction**

#### Intervention Activity

# Model a fraction division problem three different ways.

Give this fraction division problem: "It takes  $\frac{1}{4}$  meter of lumber to make a birdhouse. Alex has 3 meters of lumber. How many birdhouses can Alex build?" Have students model 3-meter long pieces of lumber by drawing 3 rectangles (see the pancake model in **Let's Explore the Idea**). They should label each rectangle's length as "1 meter". Then, have them mark off fourths in each rectangle and count the number of birdhouses that can be made. [12] Next, have students make a number line to model the problem. They draw and label a number line from 0 to 3 and show tick marks between each whole number to represent fourths. They count the fourths in 3 wholes. [12] Finally, have them write a division equation.  $[3 \div \frac{1}{4} = 12]$ 

Discuss how the three models are alike and different and how they represent the quantities in the problem.

#### Challenge Activity

#### Brainstorm fraction division situations.

Have students brainstorm situations that involve the division of whole numbers by unit fractions. Have students describe a specific situation and provide a division problem that models the situation. Students should solve the problem.

Repeat this activity, but this time have the situations involve dividing a unit fraction by a whole number.

#### On-Level Activity

Use fraction circles to model dividing a unit fraction by a whole number.

#### Materials: fraction circles

Organize students in small groups and distribute fraction circles. Have students model  $\frac{1}{2}$ . Say: *Let's model*  $\frac{1}{2} \div 3$ . Ask students what this means. [dividing  $\frac{1}{2}$ into 3 equal groups] To model  $\frac{1}{2} \div 3$  with fraction circles, tell students to find three equal-sized pieces that exactly cover  $\frac{1}{2}$ . [3 one-sixths pieces] Explain that the quotient tells the size of each piece. Write " $\frac{1}{2} \div 3 = \frac{1}{6}$ ." Repeat with other problems, such as  $\frac{1}{4} \div 2$  or  $\frac{1}{3} \div 4$ .

Teacher Notes						

#### Lesson 17 LESSON **Understand Division with Unit Fractions**

# **Teacher-Toolbox.com**

#### **Overview**

OUIZ

Assign the Lesson 17 Quiz and have students work independently to complete it.

Use the results of the quiz to assess students' understanding of the content of the lesson and to identify areas for reteaching. See the Lesson Pacing Guide at the beginning of the lesson for suggested instructional resources and the Differentiated Instruction activities that follow.

## **Tested Skills**

Problems on this assessment form require students to be able to identify situations that require dividing a whole number by a unit fraction or a unit fraction by a whole number, to use visual fraction models to find quotients, and to write related multiplication equations for given fraction division equations. Students will also need to be familiar with basic multiplication and division facts, multiplying whole numbers by unit fractions, and comparing fractions.



# Common Misconceptions and Errors

Errors may result if students:

- confuse the divisor and the dividend.
- count tick marks, instead of parts, on a number line.
- confuse when to multiply by the reciprocal of a number and when to multiply the numerators and multiply the denominators.

<i>Ready</i> <sup>®</sup> Mathematics							
Le	sson 1	7 Qui	z Answe	er Key			
1.	B <b>DOK 2</b>						
2.	1 3 1 3 1 3 ÷ 3 1 9 is not Lee <b>DOK 3</b>						
3.	18 <b>DOK 1</b>						
4.	Part A: B DOK 2 Part B: $3 \div \frac{1}{8} =$ $3 \times 8 =$ 24 DOK 2	24 24					