Fractions as Sums

Objective Understand a fraction as a sum of other fractions.	**Materials** Strips of paper, scissors, crayons or markers

This activity builds on prior skills with expressing a number as the sum of other numbers, for example, 7 as the sum of 4 and 3; of 4, 1, 1, and 1; or of 2 and 5. It extends this understanding to fractions. The fundamental idea is that a fraction $\frac{a}{b}$ is not a random pairing of digits, but has a meaning. For example, $\frac{2}{b}$ means two of $\frac{1}{b}$, and $\frac{3}{b}$ means three of $\frac{1}{b}$, and in general $\frac{a}{b}$ means a of $\frac{1}{b}$. From this general understanding, students come to see that any fraction can be written as the sum of other fractions in various combinations. For example, $\frac{4}{6}$ is the sum of unit fractions ($\frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6}$) or of non-unit fractions ($\frac{2}{6} + \frac{2}{6}$), or a combination of the two ($\frac{1}{6} + \frac{3}{6}$). While students will later develop the skill of adding fractions, the focus here is on the concept of a fraction as representing a number of parts in a whole.

Step by Step 20–30 minutes

1 Make a model of fourths.

- Model and have the student fold a strip of paper into fourths.
- Ask the student to identify the fraction represented by one section. Have the student label each section as $\frac{1}{4}$.

2 See four fourths as one whole.

- Ask: *How many fourths are in one whole?* Have the student point to each fourth and write the sum as he adds: $\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{4}{4}$. Guide the student to conclude that there are four fourths in one whole.
- Emphasize that $\frac{4}{4}$ represents 1 whole, so $\frac{4}{4} = 1$.

3 Put fourths together.

- Have the student shade $\frac{2}{4}$. Ask him to identify how to add the two sections to make $\frac{2}{4}$. Write $\frac{1}{4} + \frac{1}{4} = \frac{2}{4}$ on the board.
- Write $\frac{2}{4} + \frac{1}{4}$ on the board. Instruct the student to shade another section in a different color and identify the fraction represented by the total shaded section. ($\frac{3}{4}$)
- Explain that $\frac{3}{4}$ can be thought of as the combination of $\frac{2}{4}$ and $\frac{1}{4}$, or as the sum of $\frac{1}{4} + \frac{1}{4} + \frac{1}{4}$.
- Ask the student to identify different ways to add fourths to make a whole. The student may give answers such as $\frac{2}{4} + \frac{2}{4} = \frac{4}{4}$, $\frac{2}{4} + \frac{1}{4} + \frac{1}{4}$, or $\frac{3}{4} + \frac{1}{4}$.

4 Practice decomposing fractions.

- Instruct the student to fold a few strips of paper into eighths.
- Ask the student to take one strip of eighths and use one color to shade and represent $\frac{5}{8}$ as a sum of fractions. Then have him write the addition shown.
- Have the student repeat with two or three models, using a different color and different representation of $\frac{5}{8}$ each time.

- Review the student's work and write the different expressions as equations equal to $\frac{5}{8}$. Remind the student that the sum can have more than 2 addends. Ask the student for other fraction sums of $\frac{5}{8}$.
- Have the student determine three ways to write $\frac{8}{8}$ as a sum of fractions.

Check for Understanding

Ask the student to write $\frac{4}{5}$ as a sum of only fractions equal to $\frac{1}{5}$, and as a sum of two or more other fractions. ($\frac{4}{5} = \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5}$; other answers will vary)

For the student who struggles, use the chart below to help pinpoint where extra help may be needed.

If you observe…	the student…	Then try…
the student cannot write an expression using only the fraction $\frac{1}{5}$	may not understand how to combine parts of a whole.	using a fraction model for fifths, labeling each section $\frac{1}{5}$. Have the student count one fifth, two fifths, . . . and then relate this to a sum of one fifths.
the student has trouble determining a different combination of fractions that totals $\frac{4}{5}$	may not understand that non-unit fractions are combinations of other fractions.	showing the student expressions such as $\frac{1}{5} + \frac{1}{5}$, $\frac{2}{5} + \frac{1}{5}$, and $\frac{2}{5} + \frac{2}{5}$. Ask the student to use a model like the one above to count and count on to find and write the sums.