

Lesson 9

Ratios Involving Complex Fractions

Name: _____

Prerequisite: Find Unit Rates

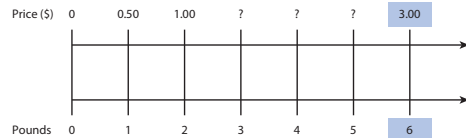
Study the example problem showing how to find a unit rate. Then solve problems 1–6.

Example

Six pounds of grapefruit cost \$3. What is the cost per pound?

You can use a double number line to help you find rate and unit rate. Start with the ratio you know: \$3 for 6 pounds.

Divide both \$3 and 6 pounds by 3 to get \$1 for 2 pounds. Then divide \$1 and 2 pounds by 2 to find the cost per pound, which is \$0.50 for 1 pound.



Notice that you can also divide \$3 by 6 to get the cost per pound directly.

- B 1** In the example, how much would it cost to buy 3, 4, and 5 pounds of grapefruit? Explain how you found your answers.

\$1.50, \$2.00, \$2.50; Possible explanation: I multiplied the cost per pound by 3, 4, and 5 to find each cost.

- B 2** Leila uses 5 cups of grated zucchini to make 2 loaves of zucchini bread. How many cups of zucchini does she need to make 1 loaf?

$2\frac{1}{2}$ cups

- B 3** Jenna walks 4 miles in 1 hour. At this rate, how many miles can she walk in 3.5 hours?

14 miles

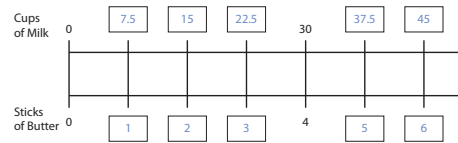
Vocabulary

rate the ratio that compares the first quantity to one unit of the second quantity. For a ratio of 2 dollars for every 5 apples, the rate is $\frac{2}{5}$ dollars per apple.

unit rate the numerical part of the rate, without the units. For a rate of $\frac{2}{5}$ dollar per apple, the unit rate is $\frac{2}{5}$.

Solve.

- M 4** It takes 30 cups of milk to make 4 sticks of butter. Use this ratio to complete the double number line. Describe the unit rate.



It takes 7.5 cups of milk to make each stick of butter.

- M 5** Rashid is paid by the hour. He earned \$50 for a 4-hour workday. How much does he earn for a $5\frac{1}{2}$ -hour workday?

Show your work.

Possible work: $\$50 \div 4 \text{ hours} = \12.50 per hour

$\$12.50 \text{ per hour} \times 5.5 \text{ hours} = \68.75

Solution: Rashid earns \$68.75 for a $5\frac{1}{2}$ -hour workday.

- C 6** Ace Bike Rentals rents bikes for \$28 per day. Renters can keep the bike for 8 hours. Bart's Bikes rents bikes for \$30 per day. Renters can keep the bike for 10 hours. Which company charges a lower hourly rate? How much lower?

Show your work.

Possible work:

Ace Bike Rentals: $\$28 \text{ per day} \div 8 \text{ hours} = \3.50 per hour

Bart's Bikes: $\$30 \text{ per day} \div 10 \text{ hours} = \3.00 per hour

Solution: Ace Bike Rentals charges a rate of \$3.50 per hour. Bart's Bikes charges a rate of \$3.00 per hour. Bart's Bikes charges an hourly rate that is \$0.50 less per hour.

Key

B Basic

M Medium

C Challenge



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Find Unit Rates with Fractions

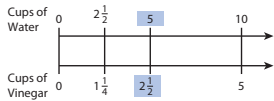
Study the example problem showing how to find a unit rate. Then solve problems 1–8.

Example

Helena uses this recipe to make homemade fabric softener. How much water and vinegar does she use for a half batch? How much water and vinegar does she use for a double batch?

You can use a double number line to help you solve this problem.

You can use the recipe to see that you need 5 cups of water for $2\frac{1}{2}$ cups of vinegar. Label these on the number line. Find half of each quantity for a half batch. Double each quantity for a double batch.



Helena needs $2\frac{1}{2}$ cups water and $1\frac{1}{4}$ cup vinegar for a half batch. She needs 10 cups of water and 5 cups of vinegar for a double batch.

Recipe for
Fabric Softener

Ingredients:

5 cups hot water

 $2\frac{1}{2}$ cups white vinegar $1\frac{1}{2}$ cups hair conditioner

Directions:

Mix all ingredients together.

- B 1** If Helena uses $1\frac{1}{4}$ cups of water to make fabric softener, how much vinegar does she need?
 $\frac{5}{8}$ cup of vinegar
- B 2** Use the recipe to find how much hair conditioner Helena needs to make a half batch and a double batch.
 $\frac{3}{4}$ cup of conditioner for a half batch and 3 cups for a double batch
- M 3** Explain how to find the unit rate of water to vinegar. What does the unit rate mean in this context?
Possible explanation: Use the ratio 10 cups of water to 5 cups of vinegar. The unit rate is $10 \div 5$, or 2. The unit rate means that you need 2 cups of water for each cup of vinegar.

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Solve.

Use the following recipe for granola bars to solve problems 4–8.

2 cups of oats	$\frac{1}{4}$ cup of flaxseed
$\frac{1}{2}$ cup of quinoa	$\frac{2}{3}$ cup of soy protein

- B 4** Write ratios for the amount of oats to the amount of each of the other grains.
oats to quinoa: $\frac{2}{\frac{1}{2}}$; oats to flaxseed: $\frac{2}{\frac{1}{4}}$; oats to soy protein: $\frac{2}{\frac{2}{3}}$
- M 5** Show how to find the unit rate of oats to quinoa. Tell what the unit rate means.
Possible answer: oats to quinoa = $2 \div \frac{1}{2} = 4$. The unit rate is 4. The recipe uses 4 times the amount of oats compared to quinoa.
- M 6** Find unit rates for the other two ratios that you wrote in problem 4.
The unit rate for oats to flaxseed is $2 \div \frac{1}{4} = 8$. The unit rate for oats to soy protein is $2 \div \frac{2}{3} = 3$.
- C 7** Suppose you are making granola bars with 3 cups of oats. How much quinoa, flaxseed, and soy protein would you need to use? Explain.
Possible answer: The unit rates show how many cups of oats you need per cup of the other ingredients. Divide the amount of oats by each unit rate. Quinoa: $3 \text{ cups} \div 4 = \frac{3}{4}$ cup. Flaxseed: $3 \text{ cups} \div 8 = \frac{3}{8}$ cup. Soy protein: $3 \text{ cups} \div 3 = 1$ cup.
- C 8** In addition to the grains listed above, the recipe also includes $1\frac{1}{4}$ cups of dried fruit. Show how to find the amount of dried fruit needed if you use 3 cups of oats.
The ratio of oats to fruit is 2 to $1\frac{1}{4}$, which is a unit rate of $\frac{8}{5}$. For 3 cups of oats: $3 \div \frac{8}{5}$ is $1\frac{7}{8}$. Use $1\frac{7}{8}$ cups of fruit for 3 cups of oats.

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Compare Unit Rates

Study the example problem showing how to compare unit rates. Then solve problems 1–6.

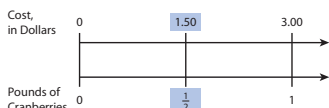
Example

The Produce Place sells cranberries in bulk for \$2.40 per pound. The store also sells an 8-ounce package of cranberries for \$1.50. Which is the better buy?

Use a double number line to find the cost per pound for the package of cranberries. Remember that 8 ounces = $\frac{1}{2}$ pound.

Multiply both \$1.50 and $\frac{1}{2}$ pound by 2 to find the price for 1 pound.

The bulk price is \$2.40 per pound, and the package price is \$3.00 per pound. Because $\$2.40 < \3.00 , the bulk cranberries are a better buy.



- B 1** Use division to find the price per ounce for the bulk cranberries and the package of cranberries. Round to the nearest cent. (1 pound = 16 ounces)

Bulk: $\frac{\$2.40}{16} = \0.15 and **Package:** $\frac{\$1.50}{8} \approx \0.19

- B 2** Compare the prices per ounce for packaged and bulk cranberries. Which price is less? How much less?

Possible answer: The price per ounce is less for the bulk cranberries. The bulk cranberries are about \$0.04 less per ounce.

- B 3** The Produce Place sells a 12-ounce package of cranberries for the same unit price as the 8-ounce package. What is the price of the 12-ounce package? Explain.

Possible answer: 12 ounces = $\frac{3}{4}$ pound and $\frac{3}{4}$ of \$3.00 = \$2.25. The price of the 12-ounce package is \$2.25.

Solve.

- M 4** Brandi jogged $2\frac{1}{2}$ miles in 30 minutes. Sharna jogged $3\frac{1}{2}$ miles in $\frac{3}{4}$ hour. Who jogged at a slower rate?

Show your work.

$$30 \text{ minutes} = \frac{1}{2} \text{ hour}$$

$$\text{Brandi: } \frac{2\frac{1}{2} \text{ miles}}{\frac{1}{2} \text{ hour}} = 5 \text{ miles per hour}$$

$$\text{Sharna: } \frac{3\frac{1}{2} \text{ miles}}{\frac{3}{4} \text{ hour}} = 4\frac{2}{3} \text{ miles per hour}$$

Solution: Sharna jogged at a slower rate.

- M 5** Amhed pays \$2.10 for $5\frac{1}{4}$ pounds of bananas. Janel pays \$3.25 for $6\frac{1}{2}$ pounds of bananas. Which is the better buy? Explain why.

$$\text{Amhed: } \frac{\$2.10}{5\frac{1}{4} \text{ lb}} = \$0.40 \text{ per pound; Janel: } \frac{\$3.25}{6\frac{1}{2} \text{ lb}} = \$0.50 \text{ per pound. Amhed paid less per}$$

pound for the bananas, so he got the better buy.

- C 6** The directions for Clean-All say to mix $1\frac{1}{2}$ cups of cleaner with 2 quarts of water. The directions for Mega-Clean say to mix $3\frac{1}{2}$ cups of cleaner with 1 gallon of water. Which solution has more cleaner per quart of water? (1 gallon = 4 quarts)

Show your work.

$$\text{Clean-All: } \frac{1\frac{1}{2} \text{ cups}}{2 \text{ quarts}} = \frac{3}{2} \div 2 = \frac{3}{2} \times \frac{1}{2}, \text{ or } \frac{3}{4}$$

$$\text{Mega-Clean: } \frac{3\frac{1}{2} \text{ cups}}{1 \text{ gallon}} = \frac{3\frac{1}{2} \text{ cups}}{4 \text{ quarts}} = \frac{7}{2} \div 4 = \frac{7}{2} \times \frac{1}{4}, \text{ or } \frac{7}{8}$$

Solution: Mega-Clean has $\frac{7}{8}$ cup of cleaner to 1 quart of water. Clean-All has $\frac{3}{4}$ cup of cleaner to 1 quart of water. Mega-Clean has more cleaner per quart of water.



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Solve the problems.

M

- 1 A road sign gives drivers information about traffic on busy highways. One sign shows "15 miles in 20 minutes." What is this speed in miles per hour?

Show your work.

Possible work: $20 \text{ minutes} = \frac{1}{3} \text{ hour}$

$$\frac{15}{\frac{1}{3}} = 15 \times 3, \text{ or } 45.$$

Solution: The speed is 45 miles per hour.

What fraction of an hour is 20 minutes?



B

- 2 During a heavy rainstorm, a city in Florida received $12\frac{1}{4}$ inches of rain in $25\frac{1}{2}$ hours. What is the approximate rainfall rate in inches per hour?

- A about 2 inches per hour
B about $\frac{1}{4}$ inch per hour
C about 1 inch per hour
D about $\frac{1}{2}$ inch per hour

You can write a ratio and estimate the unit rate.



C

- 3 Jeremy swims $5\frac{3}{5}$ kilometers in a 7-day period. He swims the same distance each day. What distance does he swim in a day?

- A $39\frac{1}{5}$ km C $\frac{4}{5}$ km
B $1\frac{1}{5}$ km D $\frac{4}{35}$ km

What rate do you need to find?



Eli chose A as the correct answer. How did he get that answer?

Possible answer: Eli multiplied the total distance by 7 days. He should have divided the distance by 7 days.

Solve.

M

- 4 Mrs. Cain's coleslaw recipe calls for $\frac{1}{3}$ cup of oil, $\frac{1}{2}$ cup of vinegar, and $\frac{1}{4}$ cup of sugar. Tell whether each statement is True or False.

Make sure the terms in each ratio are in the correct order.



- a. The recipe uses $1\frac{1}{2}$ cups of oil for each cup of vinegar. ☐ True ☒ False
b. The recipe uses 2 cups of sugar for each cup of vinegar. ☐ True ☒ False
c. The recipe uses 2 cups of vinegar for each cup of sugar. ☒ True ☐ False
d. The recipe uses $\frac{2}{3}$ cup of oil for each cup of vinegar. ☒ True ☐ False

M

- 5 Fill in the table to show the amount of each ingredient needed to make different-size batches of coleslaw. Use the information from problem 4.

How can you use unit rates to solve the problem?



	Batch 1	Batch 2	Batch 3
Vinegar (cups)	1	$1\frac{1}{2}$	3
Oil (cups)	$\frac{2}{3}$	1	2
Sugar (cups)	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{3}{2}$

C

- 6 Rida and Elisa are each paid by the hour. Rida worked $5\frac{1}{2}$ hours and earned \$77. Elisa worked $3\frac{3}{4}$ hours and earned \$60. Whose hourly rate is greater?

In this problem, hourly rate means how much the person is paid per hour.



Show your work.

Possible work: Rida: $77 \div 5\frac{1}{2} = 77 \div \left(\frac{11}{2}\right) = 77 \times \frac{2}{11} = 14$

Elisa: $60 \div 3\frac{3}{4} = 60 \div \left(\frac{15}{4}\right) = 60 \times \frac{4}{15} = 16$

Solution: Elisa has the greater hourly rate.