LESSON 6

Overview | Describe Angle Relationships

STANDARDS FOR MATHEMATICAL PRACTICE (SMP)

SMP 1, 2, 3, 4, 5, and 6 are integrated into the Try-Discuss-Connect routine.*

This lesson provides additional support for:

3 Construct viable arguments and critique the reasoning of others.

* See page 1o to learn how every lesson includes these SMP.

Objectives

Content Objectives

- Identify corresponding angles, alternate interior angles, alternate exterior angles, same-side interior angles, and same-side exterior angles when given a pair of lines that is cut by a transversal.
- Informally establish and understand the angle relationships that exist when parallel lines are cut by a transversal.
- Use angle relationships to find unknown measures of angles formed by parallel lines cut by a transversal.
- Use angle relationships to determine whether two lines cut by a transversal are parallel.

Language Objectives

- Confirm understanding of lesson vocabulary by identifying examples of angle relationships and naming appropriate angles with geometric notation.
- Explain whether angles are congruent by analyzing geometric figures and using definitions during discussion and in writing.
- Justify angle measurements by naming the appropriate angle relationship.
- Understand and use lesson vocabulary and reasoning to explain why two lines cut by a transversal are parallel.
- Listen to and compare ideas with a partner and provide reasons for any disagreement.

Prior Knowledge

- Understand that vertical angles are congruent.
- Know that the measure of a straight angle is 180°.
- Know that the sum of the measures of supplementary angles is 180°.
- Recognize parallel lines.

Vocabulary

Math Vocabulary

alternate exterior angles when two lines are cut by a transversal, a pair of angles on opposite sides of the transversal and outside the two lines.

alternate interior angles when two lines are cut by a transversal, a pair of angles on opposite sides of the transversal and between the two lines.

corresponding angles angles in the same relative position when two lines are cut by a transversal.

linear pair two angles that are adjacent and supplementary.

same-side exterior angles when two lines are cut by a transversal, a pair of angles on the same side of the transversal and outside the two lines.

same-side interior angles when two lines are cut by a transversal, a pair of angles on the same side of the transversal and between the two lines.

transversal a line that cuts two or more lines. The lines cut by the transversal may or may not be parallel.

Review the following key terms.

adjacent angles two non-overlapping angles that share a vertex and a side.

supplementary angles two angles whose measures sum to 180°.

vertical angles opposite angles formed when two lines intersect. Vertical angles are congruent.

Academic Vocabulary

intersect to meet or cross. When two lines intersect, they cross at a common point.

Learning Progression

Earlier in Grade 8, students learned that both parallel lines and angle measures are preserved by rigid transformations. Students also used the relationships between vertical angles and linear pairs of angles to find angle measures.

➤

In this lesson, students establish facts about the relationships among the measures of angles formed by two parallel lines cut by a transversal. Students also learn how to find new angle relationships using the angle relationships they already know.

➤

Later in Grade 8, students will use the relationships discovered in this lesson to discover and show relationships among the interior and exterior angles of a triangle.

LESSON 6
Overview

Pacing Guide

Items marked with ↖ are available on the **Teacher Toolbox**.

	MATERIALS	DIFFERENTIATION
SESSION 1 **Explore** Angle Relationships (35–50 min)		
- **Start** (5 min) - **Try It** (5–10 min) - **Discuss It** (10–15 min) - **Connect It** (10–15 min) - **Close: Exit Ticket** (5 min) **Additional Practice** (pages 119–120)	Presentation Slides ↖	**PREPARE** Interactive Tutorial ↖ **RETEACH or REINFORCE** Hands-On Activity **Materials** For each student: protractor
SESSION 2 **Develop** Describing Congruent Angle Relationships (45–60 min)		
- **Start** (5 min) - **Try It** (10–15 min) - **Discuss It** (10–15 min) - **Connect It** (15–20 min) - **Close: Exit Ticket** (5 min) **Additional Practice** (pages 125–126)	**Math Toolkit** graph paper, tracing paper, transparencies Presentation Slides ↖	**RETEACH or REINFORCE** Hands-On Activity **Materials** For each student: 4 markers, index card, ruler, scissors **REINFORCE** Fluency & Skills Practice ↖ **EXTEND** Deepen Understanding
SESSION 3 **Develop** Describing Supplementary Angle Relationships (45–60 min)		
- **Start** (5 min) - **Try It** (10–15 min) - **Discuss It** (10–15 min) - **Connect It** (15–20 min) - **Close: Exit Ticket** (5 min) **Additional Practice** (pages 131–132)	Presentation Slides ↖	**RETEACH or REINFORCE** Hands-On Activity **Materials** For each student: protractor, ruler **REINFORCE** Fluency & Skills Practice ↖ **EXTEND** Deepen Understanding
SESSION 4 **Refine** Describing Angle Relationships (45–60 min)		
- **Start** (5 min) - **Monitor & Guide** (15–20 min) - **Group & Differentiate** (20–30 min) - **Close: Exit Ticket** (5 min)	**Math Toolkit** Have items from previous sessions available for students. Presentation Slides ↖	**RETEACH** Hands-On Activity **Materials** For each student: 2 colored pencils (1 blue, 1 red), centimeter ruler, scissors, tracing paper **REINFORCE** Problems 4–9 **EXTEND** Challenge **PERSONALIZE** i-Ready
Lesson 6 Quiz ↖ or **Digital Comprehension Check**		**RETEACH** Tools for Instruction ↖ **REINFORCE** Math Center Activity ↖ **EXTEND** Enrichment Activity ↖

LESSON 6

Overview | Describe Angle Relationships

Connect to Culture

➤ **Use these activities to connect with and leverage the diverse backgrounds and experiences of all students. Engage students in sharing what they know about contexts before you add the information given here.**

SESSION 1 ■ □ □ □

Try It Ask students to raise their hands if they have ever checked the status of a flight using a computer or phone app. Have some students share their experiences. It is important for anyone in the aviation industry to be able to track flights. Every airplane uses a computerized system to transmit information such as altitude, direction, speed, and precise latitude and longitude positions during any portion of its flight. There are several flight-tracking software systems available to those in the industry as well as the general public. Most flight trackers get their flight data from the United States Federal Aviation Administration, AirNav Systems, and Global Distribution Systems, as well as from direct airport data feeds. While the general public may use these tracking systems to see when a flight of a relative or friend may arrive, those in the industry use these systems to ensure the safety of passengers and accountability of those operating airplanes.

SESSION 2 ■ ■ □ □

Try It Ask if any students have ever been to a Native American wedding. The vase shown is a wedding vase from the Taos Pueblo, a federally recognized tribal nation of Native Americans in Taos, New Mexico. Native American wedding vases have two spouts that symbolize the two people getting married. The handle in the middle symbolizes the union of the couple. The open space between the handle and spouts represents the circle of life. As part of the Native American wedding ceremony, the couple drinks from their wedding vase at the same time. Parents of the groom in the ceremony typically provide the wedding vase, either by buying one or making one themselves. Married couples normally keep their wedding vase on display after the wedding. Ask students if they have been to a wedding in which a similar symbolic union of the couple occurred. Have them share their experiences with the class.

SESSION 3 ■ ■ ■ □

Try It Ask if any students have ladder shelves in their homes. Ladder shelves can be used for many purposes. Some are used to display plants, either indoors or outdoors. Other ladder shelves are used to display photographs, such as ones from a family vacation. Some ladder shelves are used as exercise apparatuses for pets. Cats can climb up and down the shelves and rest as needed. Have students draw a diagram of a ladder shelf they would design to have in their home. Select students to share their drawings.

CULTURAL CONNECTION

Alternate Notation You can use angle notation to write "angle *ABC*" as ∠*ABC*. In many Latin American countries, different notation is used where the angle symbol is placed above the letters instead of to the left. Encourage students who have experience with this angle notation to share what they know with the class.

$\angle ABC$

OR

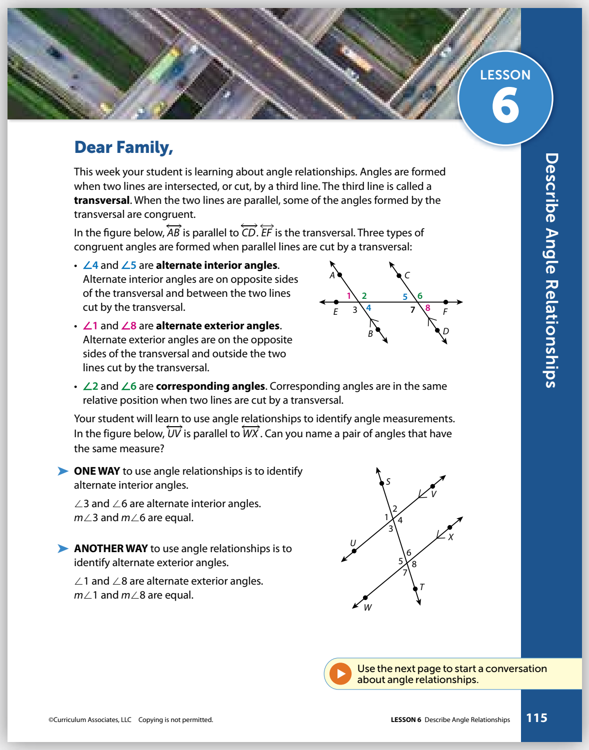
$\widehat{ABC}$

LESSON 6

Overview

Connect to Family and Community

After the Explore session, have students use the Family Letter to let their families know what they are learning and to encourage family involvement.



LESSON 6

Describe Angle Relationships

Dear Family,

This week your student is learning about angle relationships. Angles are formed when two lines are intersected, or cut, by a third line. The third line is called a **transversal**. When the two lines are parallel, some of the angles formed by the transversal are congruent.

In the figure below, $\overleftrightarrow{AB}$ is parallel to $\overleftrightarrow{CD}$. $\overleftrightarrow{EF}$ is the transversal. Three types of congruent angles are formed when parallel lines are cut by a transversal:

- ∠4 and ∠5 are **alternate interior angles**. Alternate interior angles are on opposite sides of the transversal and between the two lines cut by the transversal.
- ∠1 and ∠8 are **alternate exterior angles**. Alternate exterior angles are on the opposite sides of the transversal and outside the two lines cut by the transversal.
- ∠2 and ∠6 are **corresponding angles**. Corresponding angles are in the same relative position when two lines are cut by a transversal.

Your student will learn to use angle relationships to identify angle measurements. In the figure below, $\overleftrightarrow{UV}$ is parallel to $\overleftrightarrow{WX}$. Can you name a pair of angles that have the same measure?

ONE WAY to use angle relationships is to identify alternate interior angles.

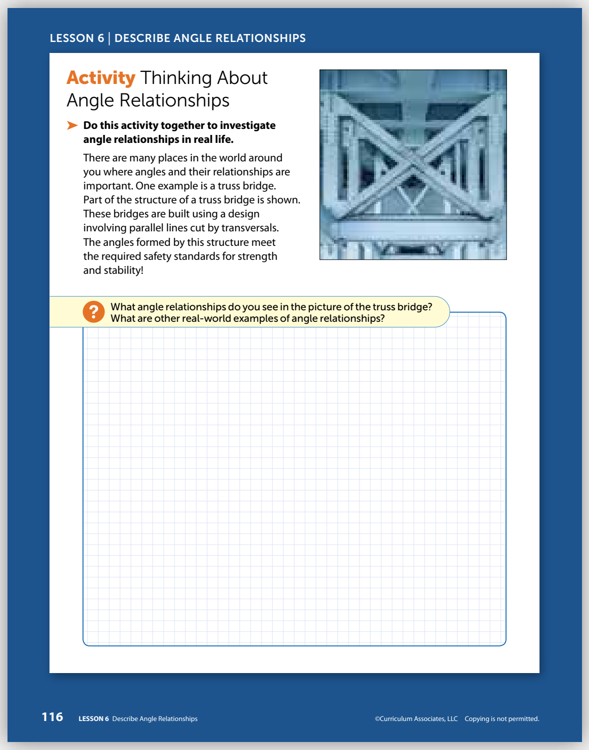
∠3 and ∠6 are alternate interior angles.
m∠3 and m∠6 are equal.

ANOTHER WAY to use angle relationships is to identify alternate exterior angles.

∠1 and ∠8 are alternate exterior angles.
m∠1 and m∠8 are equal.

Use the next page to start a conversation about angle relationships.

©Curriculum Associates, LLC Copying is not permitted. **LESSON 6** Describe Angle Relationships 115



LESSON 6 | DESCRIBE ANGLE RELATIONSHIPS

Activity Thinking About Angle Relationships

Do this activity together to investigate angle relationships in real life.

There are many places in the world around you where angles and their relationships are important. One example is a truss bridge. Part of the structure of a truss bridge is shown. These bridges are built using a design involving parallel lines cut by transversals. The angles formed by this structure meet the required safety standards for strength and stability!

What angle relationships do you see in the picture of the truss bridge? What are other real-world examples of angle relationships?

116 **LESSON 6** Describe Angle Relationships ©Curriculum Associates, LLC Copying is not permitted.

Connect to Language

For English language learners, use the Differentiation chart to scaffold the language in each session. Use the Academic Vocabulary routine for academic terms before Session 1.

DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Use with **Session 1** **Connect It**

Levels 1–3: Listening/Writing

Help students prepare to respond to Connect It problem 2. Read the problem aloud as students follow along. Use a **Co-Constructed Word Bank** to help students understand the language in this problem. Have students circle words that they are unsure of. Point out positional words such as *opposite, inside, between, outside,* and *relative to*.

Write these words on the board and have students listen as you represent each word with a diagram using simple symbols such as stars and hearts to model which symbols are *between* two lines, *outside* of the lines, and in the same position *relative to* the lines. Have students add these words and the diagrams to their word banks, along with any other words they circled.

Levels 2–4: Listening/Writing

Help students prepare to solve Connect It problem 2. Read the problem with students. Use a **Co-Constructed Word Bank** to help students write about pairs of angles. Have students circle words that name and describe pairs of angles. Use the words to start a word bank on the board.

Point out positional words in the problem such as *opposite, inside, between, outside,* and *relative to* that describe the angles. Model how to draw a diagram of the pair of angles as they listen to the positional terms. Guide students to represent each pair of angles in their word bank with diagrams.

Levels 3–5: Listening/Writing

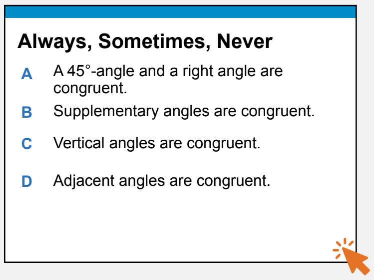
Prepare students to write responses to Connect It problem 2. Have students read the problem. Use a **Co-Constructed Word Bank** to help students represent pairs of angles. Have students circle words that name and describe pairs of angles. Then have them use the words to create a word bank. Have students add diagrams to represent each word. Ask students to turn and talk to a partner to review the word banks and make sure they used relevant words and sketched accurate diagrams. Have them refer to the word banks as they write about pairs of angles in this lesson.

Explore Angle Relationships

Purpose

- **Explore** the idea that angle relationships can be used to find unknown angle measures in a given figure.
- **Understand** the different types of related angle pairs that are formed when a pair of lines is cut by a transversal.

START CONNECT TO PRIOR KNOWLEDGE



Always, Sometimes, Never

A A 45°-angle and a right angle are congruent.

B Supplementary angles are congruent.

C Vertical angles are congruent.

D Adjacent angles are congruent.

Solutions

A is never true.

B is sometimes true.

C is always true.

D is sometimes true.

WHY? Support students' understanding of angle relationships.

TRY IT

SMP 1, 2, 4, 5, 6

Make Sense of the Problem

See ***Connect to Culture*** to support student engagement. Before students work on Try It, use **Co-Craft Questions** to help them make sense of the problem, showing only the diagram. Students will likely suggest finding the value of *x* as a possible question but may also mention determining the measure of other specific angles in the figure.

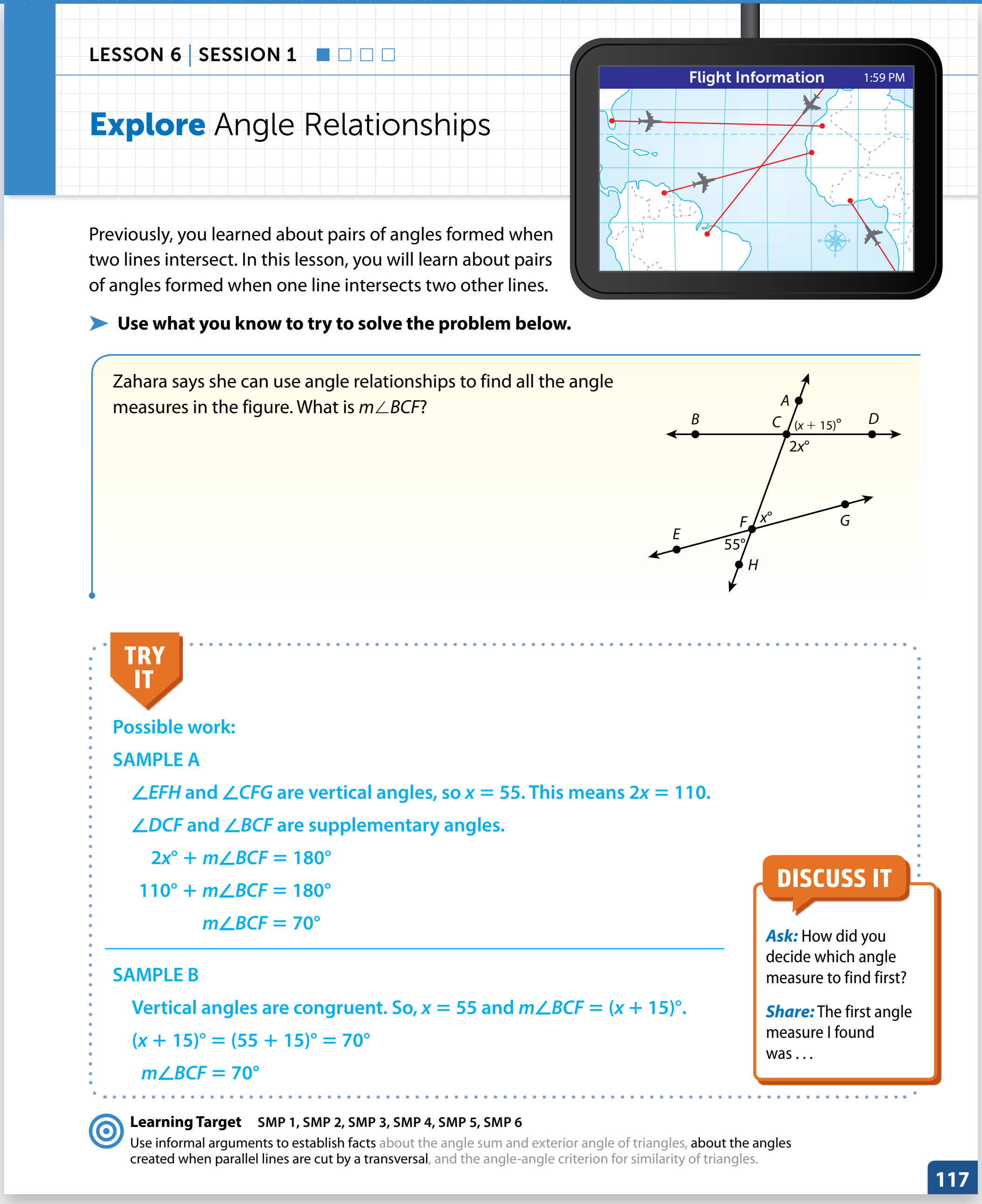
DISCUSS IT

SMP 2, 3, 6

Support Partner Discussion

After students work on Try It, have them respond to Discuss It with a partner. Listen for understanding that:

- $\angle CFG$ is a vertical angle with the 55° angle, so the value of x can be found.
- $\angle ACD$ and $\angle DCF$ are supplementary angles, so an equation can show the sum of those angle measures is 180°, and the value of x can be found.



LESSON 6 | SESSION 1 ■ □ □ □

Explore Angle Relationships

Previously, you learned about pairs of angles formed when two lines intersect. In this lesson, you will learn about pairs of angles formed when one line intersects two other lines.

➤ **Use what you know to try to solve the problem below.**

Zahara says she can use angle relationships to find all the angle measures in the figure. What is $m\angle BCF$?

TRY IT

Possible work:

SAMPLE A

$\angle EFH$ and $\angle CFG$ are vertical angles, so $x = 55$. This means $2x = 110$.

$\angle DCF$ and $\angle BCF$ are supplementary angles.

$2x° + m\angle BCF = 180°$

$110° + m\angle BCF = 180°$

$m\angle BCF = 70°$

SAMPLE B

Vertical angles are congruent. So, $x = 55$ and $m\angle BCF = (x + 15)°$.

$(x + 15)° = (55 + 15)° = 70°$

$m\angle BCF = 70°$

DISCUSS IT

Ask: How did you decide which angle measure to find first?

Share: The first angle measure I found was . . .

Learning Target SMP 1, SMP 2, SMP 3, SMP 4, SMP 5, SMP 6
Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles.

117

Common Misconception Listen for students who think the 55° angle and $\angle ACD$ have the same measure. As students share their strategies, have them trace the 55° angle and place their tracing over $\angle ACD$ so they can see those angles are not congruent.

Select and Sequence Student Strategies

Select 2–3 samples that represent the range of student thinking in your classroom. Here is one possible order for class discussion:

- vertical angles used to find x, expression $x + 15$ used to find $m\angle ACD$, and vertical angles used to find $m\angle BCF$
- (misconception) incorrect congruent angles identified, leading to an incorrect solution
- vertical angles used to find x, expression $2x$ used to find $m\angle DCF$, and supplementary angles used to find $m\angle BCF$

Facilitate Whole Class Discussion

Call on students to share selected strategies. Remind students that one way to agree and build on ideas is to give reasons that explain why the strategy makes sense. Invite students to reword informal language with mathematical vocabulary.

Guide students to **Compare and Connect** the representations. Call on several students to rephrase important ideas so that everyone hears them more than once and in more than one way.

> **ASK** *What angle relationships do all of these strategies use?*
>
> **LISTEN FOR** The strategies use the fact that vertical angles have the same measure and that the sum of the measures of supplementary angles is 180°.

CONNECT IT

SMP 2, 4, 5

① **Look Back** Look for understanding that the relationship between vertical angles can be used to find x, and then one of the expressions, $x + 15$ or $2x$, and the relationship between vertical or supplementary angles can be used to find $m\angle BCF$.

DIFFERENTIATION | RETEACH or REINFORCE

Hands-On Activity

Verify angle measures with a protractor.

If students are unsure about how to use angle relationships to find measures of angles, then use this activity to help them gain confidence in their work.

Materials For each student: protractor

- Instruct students to use the diagram of the lines and transversal from Try It.
- After students have used angle relationships to find $m\angle BCF$, have them measure $\angle BCF$ with a protractor to verify their answer. (Students may need to extend the lines in the figure in order to measure the angles accurately.)
- Instruct students to use angle relationships to find $m\angle ACB$.
- Ask: *What is $m\angle ACB$?* [110°]
- Have them measure $\angle ACB$ with a protractor to verify their answer.
- Instruct students to use angle relationships to find $m\angle EFC$.
- Ask: *What is $m\angle EFC$?* [125°]
- Have students measure $\angle EFC$ with a protractor to verify their answer.

CONNECT IT

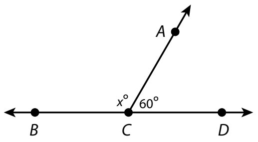
① **Look Back** What is $m\angle BCF$? What types of angle relationships did you use to find $m\angle BCF$?

70°; Possible answers: supplementary angles, vertical angles

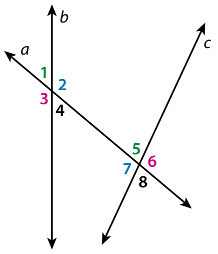
② **Look Ahead** The figure in the **Try It** problem shows pairs of angles you know, such as adjacent angles, supplementary angles, and vertical angles. The figure also shows pairs of angles that are new to you.

a. You know that supplementary angles are two angles whose measures have a sum of 180°. A **linear pair** is a pair of supplementary angles that are adjacent. What two angles form the linear pair shown? What is the value of x?

$\angle BCA$ and $\angle ACD$; $x = 120$



b. A **transversal** is a line that intersects or cuts two or more lines. Which line is the transversal in the figure at the right? line a



c. In the figure, $\angle 2$ and $\angle 7$ are **alternate interior angles**. These angles are on opposite sides of the transversal, and they are inside, or between, the other two lines. What is the other pair of alternate interior angles? $\angle 4$ and $\angle 5$

d. $\angle 3$ and $\angle 6$ are **alternate exterior angles**. These angles are on opposite sides of the transversal, but are on the outside of the other two lines. What is the other pair of alternate exterior angles?

$\angle 1$ and $\angle 8$

e. $\angle 1$ and $\angle 5$ are **corresponding angles**. These angles are in the same position relative to the lines and the transversal. $\angle 2$ and $\angle 6$ are also corresponding angles. What are the other two pairs of corresponding angles?

$\angle 3$ and $\angle 7$, $\angle 4$ and $\angle 8$

③ **Reflect** Is it possible for a pair of angles to be both corresponding angles and alternate interior angles? Explain.

No; Possible explanation: Pairs of corresponding angles are in different positions than pairs of alternate interior angles. It is possible for the same angle to be one of both a pair of corresponding angles and a different pair of alternate interior angles.

② **Look Ahead** Point out that there are many types of related pairs of angles formed when two lines are cut by a transversal. Students should recognize that there is more than one pair for each type.

Ask volunteers to rephrase the definitions of *linear pair*, *transversal*, *alternate interior angles*, *alternate exterior angles*, and *corresponding angles*. Support student understanding by drawing two lines cut by a transversal for students to analyze. Label the angles. Then have students point out examples of each term.

CLOSE EXIT TICKET

③ **Reflect** Look for understanding that a pair of angles cannot be both corresponding angles and alternate interior angles.

Error Alert If students think it is possible for a pair of angles to be both corresponding and alternate interior, then draw a diagram of two lines intersected by a transversal and have students point to pairs of alternate interior angles and pairs of corresponding angles. Point out that corresponding angles are on the same side of the transversal, while alternate interior angles are on opposite sides of the transversal.

Prepare for Describing Angle Relationships

Support Vocabulary Development

Assign **Prepare for Describing Angle Relationships** as extra practice in class or as homework.

If you have students complete this in class, then use the guidance below.

Ask students to consider the terms *adjacent angles*, *supplementary angles*, and *vertical angles*. They should define each term in their own words and include a labeled diagram supporting their understanding of the definition.

Have students work in pairs to complete the graphic organizer. Invite pairs to share their completed organizers and prompt a whole-class comparative discussion of the paraphrased definitions and diagrams of each term.

Have students look at the diagrams in problem 2 and discuss with a partner how the numbered angles relate to one another. Students should decide between *adjacent angles* and *vertical angles* and include the reasoning for the term they chose.

Problem Notes

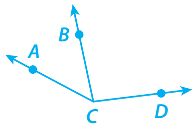
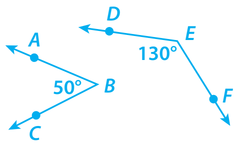
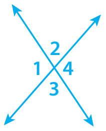
① Students should understand that adjacent angles are two angles that share a vertex and a side and do not overlap, that supplementary angles are two angles whose measures add up to 180°, and vertical angles are the congruent opposite angles formed when two lines intersect. Student responses might include those specific definitions or paraphrased versions of the definitions.

② Students should recognize that for the diagram in part a, the numbered angles are opposite angles formed by intersecting lines. They should recognize that for the diagram in part b, the numbered angles share a vertex and a side and do not overlap.

LESSON 6 | SESSION 1 Name:

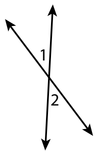
Prepare for Describing Angle Relationships

① Think about what you know about angles. Fill in each box. Use words, numbers, and pictures. Show as many ideas as you can. **Possible answers:**

Word	In My Own Words	Example
adjacent angles	two angles that share a vertex and a side and do not overlap	 ∠ACB and ∠BCD
supplementary angles	two angles whose measures add up to 180°	 ∠ABC and ∠DEF
vertical angles	the opposite angles formed when two lines intersect; vertical angles are congruent	 ∠1 and ∠4; ∠2 and ∠3

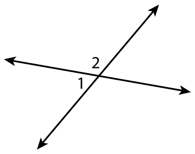
② For each figure, are ∠1 and ∠2 *adjacent angles* or *vertical angles*? Explain.

a.



vertical angles; they are formed by crossed lines and are not adjacent

b.



adjacent angles; they share a side and a vertex

REAL-WORLD CONNECTION

Civil engineers design roadways and need to analyze street intersections. They often use angle relationships to make calculations in their designs. They know that if one street intersects another at a certain angle, there is another corner at that intersection with an angle measure of 180° minus that measure. They use this information to decide whether a stop sign or traffic light will best serve this intersection based on visibility potential at those angles. Ask students to think of other real-world examples when using angle relationships might be useful.

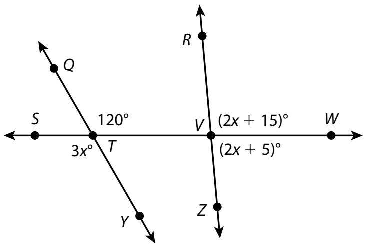
③ Problem 3 provides another look at using angle relationships to find unknown angle measures. This problem is similar to the problem about Zahara finding the measure of angle *BCF*. In both problems, one angle measure is known, and the measures of others are given in terms of a variable. This problem asks for the measure of angle *TVZ*.

Students may want to use the relationships between vertical angles and supplementary angles to solve.

Suggest that students use **Three Reads**, asking themselves one of the following questions each time:

- *What is this problem about?*
- *What is the question I am trying to answer?*
- *What information is important?*

③ **a.** What is $m\angle TVZ$? Show your work.



Possible work:

$m\angle RVW + m\angle WVZ = 180°$

$2x + 15 + 2x + 5 = 180$

$4x + 20 = 180$

$4x = 160$

$x = 40$

$m\angle RVW = (2x + 15)°$

$= (80 + 15)°$

$= 95°$

$m\angle TVZ = m\angle RVW$

$m\angle TVZ = 95°$

SOLUTION $m\angle TVZ = 95°$

b. Check your answer to problem 3a. Show your work.

Possible work:

$m\angle STY = m\angle QTV$

$3x = 120$

$x = 40$

$m\angle TVZ + m\angle ZVW = 180°$

$m\angle TVZ + (2x + 5)° = 180°$

$m\angle TVZ + 85° = 180°$

$m\angle TVZ = 95°$

DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Use with **Session 2** **Apply It**

Levels 1–3: Speaking/Writing

Help students interpret Apply It problem 5. Read the problem aloud. Support students in responding by asking them to name the transformations as you record them. Then help them define *corresponding angles*.

Have students color pairs of corresponding angles with a partner. Monitor as they share their work and explain any disagreements:

- *I don't think these are ____. They are not ____.*

Have students use a transparency or trace the lines on a sheet of paper so they can visualize the transformations. Different sequences are possible. Have partners discuss and write their responses using the terms from the board.

Levels 2–4: Speaking/Writing

Help students read and discuss Apply It problem 5. Have partners identify and color pairs of corresponding angles. If they disagree, have them use the definition of corresponding angles to explain their reasons:

- *Corresponding angles are ____, so I don't think ____.*

Encourage students to use the **Co-Constructed Word Bank** from Session 1 as they discuss transformations that can show the angles are congruent. Remind them that they can use transparencies or trace the lines on paper to perform the transformations. After partners discuss and model the transformations, have them respond to the problem in writing.

Levels 3–5: Speaking/Writing

Help students interpret Apply It problem 5. Have students read the problem and answer any questions they may have. Then ask students to solve the problem using colored pencils, transparencies, or rotating their text. Allow time for students to work independently with the model of their choice.

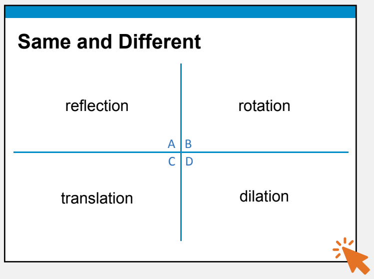
Have students turn and talk to a partner to compare models and share solutions. If partners used the same sequence of transformations, have them discuss another sequence that could achieve the same outcome. If their solutions differ, ask them to discuss whether one sequence was more efficient. Remind students to be respectful as they listen and explain.

Develop Describing Congruent Angle Relationships

Purpose

- **Develop** strategies, such as using transformations, to show that certain angles formed by parallel lines cut by a transversal are congruent.
- **Recognize** that when the lines cut by a transversal are parallel, then alternate interior angles are congruent, alternate exterior angles are congruent, and corresponding angles are congruent.

START CONNECT TO PRIOR KNOWLEDGE



Same and Different

reflection A	rotation B
translation C	dilation D

Possible Solutions

All are transformations.

D is the only one that can change size.

A and B can both change orientation.

B and D both need a center.

WHY? Support students' facility with the characteristics of transformations.

DEVELOP ACADEMIC LANGUAGE

WHY? Support students as they respectfully disagree with an idea during discussion.

HOW? Discuss with students how to disagree with an idea respectfully during discourse. Ask them to disagree with the idea, not the person. Model for students how understanding and working through disagreements is a way to learn. Suggest these sentence frames:

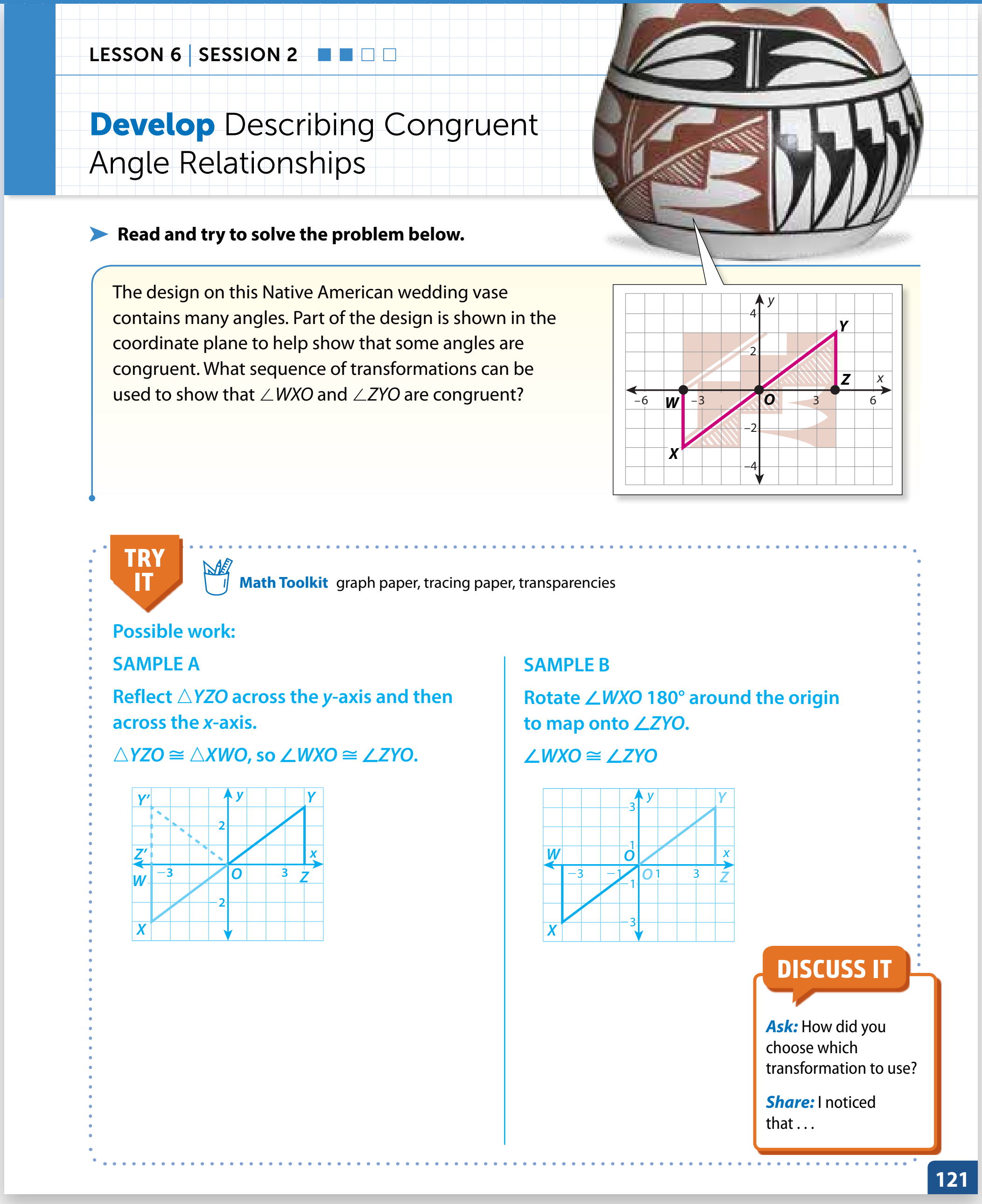
- _____ *said* _____. *I disagree because* _____.
- *I thought about this differently* _____.

TRY IT

SMP 1, 2, 4, 5, 6

Make Sense of the Problem

See ***Connect to Culture*** to support student engagement. Before students work on Try It, use **Say It Another Way** to help them make sense of the problem. Listen for understanding that in a sequence of transformations, there is one or more movements, including reflection, rotation, translation, or dilation.



LESSON 6 | SESSION 2 ■ ■ □ □

Develop Describing Congruent Angle Relationships

➤ **Read and try to solve the problem below.**

The design on this Native American wedding vase contains many angles. Part of the design is shown in the coordinate plane to help show that some angles are congruent. What sequence of transformations can be used to show that $\angle WXO$ and $\angle ZYO$ are congruent?

TRY IT Math Toolkit graph paper, tracing paper, transparencies

Possible work:

SAMPLE A

Reflect $\triangle YZO$ across the y-axis and then across the x-axis.

$\triangle YZO \cong \triangle XWO$, so $\angle WXO \cong \angle ZYO$.

SAMPLE B

Rotate $\angle WXO$ 180° around the origin to map onto $\angle ZYO$.

$\angle WXO \cong \angle ZYO$

DISCUSS IT

Ask: How did you choose which transformation to use?

Share: I noticed that . . .

121

DISCUSS IT

SMP 2, 3, 6

Support Partner Discussion

After students work on Try It, encourage them to respond to Discuss It with a partner. Listen for understanding that:

- a combination of two reflections could be used to arrive at the final image.
- because the image has a different orientation, a rotation could be used.

Common Misconception Listen for students who think that moving a figure from one quadrant to another requires a translation. As students share their strategies, have them consider the specific movements of a translation and compare those movements to the image to determine that a translation would not work in this instance.

Select and Sequence Student Strategies

Select 2–3 samples that represent the range of student thinking in your classroom. Here is one possible order for class discussion:

- tangram triangle demonstrating two reflections or one 180° rotation
- **(misconception)** strategy that incorrectly includes a translation
- tracing paper demonstrating two reflections or one 180° rotation
- graph to show two reflections or 180° rotation

Facilitate Whole Class Discussion

Call on students to share selected strategies. Remind students to be respectful when they disagree with another's ideas. There can be more than one strategy used to solve the problem.

Guide students to **Compare and Connect** the representations. After each strategy, allow think time for students to process the ideas.

> **ASK** *How did [student name] and [student name] use the position and orientation of the two angles to figure out which transformations to use?*
>
> **LISTEN FOR** The orientation indicates that a rotation or reflection is needed. The position indicates the degree of the rotation or that two reflections are needed instead of just one.

Model It

If students presented these models, have students connect these models to those presented in class.

If no student presented at least one of these models, have students first analyze key features of the models, and then connect them to the models presented in class.

> **ASK** *Why are two reflections needed, but only one rotation?*
>
> **LISTEN FOR** After reflecting over one axis, the angle is in Quadrant II or IV with the wrong orientation. It must be reflected over the other axis to match ∠ZYO. A 180° rotation takes the angle to Quadrant I, where it matches ∠ZYO.

For the reflection, prompt students to consider the reflections presented.

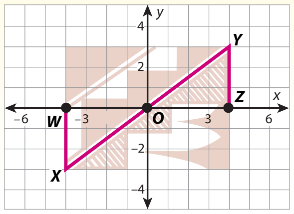
- *Is the order of the two reflections important?*
- *Could other reflections have the same result?*

For the rotation, prompt students to consider the degree of rotation of the figure.

- *Why is it important to rotate 180°?*
- *Why is a 90° or 270° rotation not a solution?*

➤ Explore different ways to find and describe congruent angle relationships.

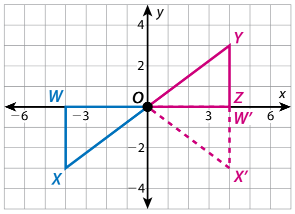
The design on this Native American wedding vase contains many angles. Part of the design is shown in the coordinate plane to help show that some angles are congruent. What sequence of transformations can be used to show that ∠WXO and ∠ZYO are congruent?



Model It

You can use reflections.

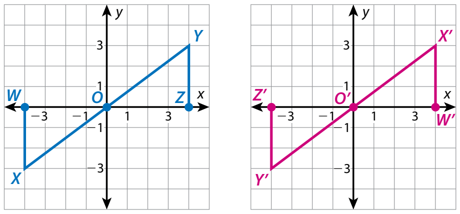
A sequence of two reflections maps ∠WXO onto ∠ZYO.



Model It

You can use a rotation.

Rotate the entire figure 180° around the origin to map ∠WXO onto ∠W′X′O′.



DIFFERENTIATION | EXTEND

SMP 3

Deepen Understanding

Using Coordinates to Prove That a Reflection Across Both Axes Is the Same As a Rotation 180° Around the Origin

Prompt students to consider how to use a general point (x, y) to prove that a sequence of two reflections over both axes is equivalent to a 180° rotation.

> **ASK** *Suppose you have a general point (x, y). What are the coordinates of the point after a reflection across the y-axis? Explain.*
>
> **LISTEN FOR** (−x, y); The x-coordinate is opposite and the y-coordinate is the same.

> **ASK** *Now suppose you reflected that point across the x-axis. What are the new coordinates of the point? Explain.*
>
> **LISTEN FOR** (−x, −y); The x-coordinate is the same and the y-coordinate is opposite.

> **ASK** *How do these new coordinates relate to the coordinates of a general point (x, y) after a rotation 180° around the origin?*
>
> **LISTEN FOR** After a rotation 180° around the origin, both the x- and y-coordinates are opposite, so the point is (−x, −y). The coordinates are the same.

Develop Describing Congruent Angle Relationships

CONNECT IT

SMP 2, 4, 5, 6

Remind students that the transformed images are the same in each representation. Explain that they will now use those representations to reason about congruent angle relationships.

Before students begin to record and expand on their work in Model It, tell them that problems 1 and 2 will prepare them to draw conclusions in problem 3. To help students better understand problem 1, you might have them copy the diagram from Try It onto grid paper and extend the parallel lines and transversal.

Monitor and Confirm Understanding ①–②

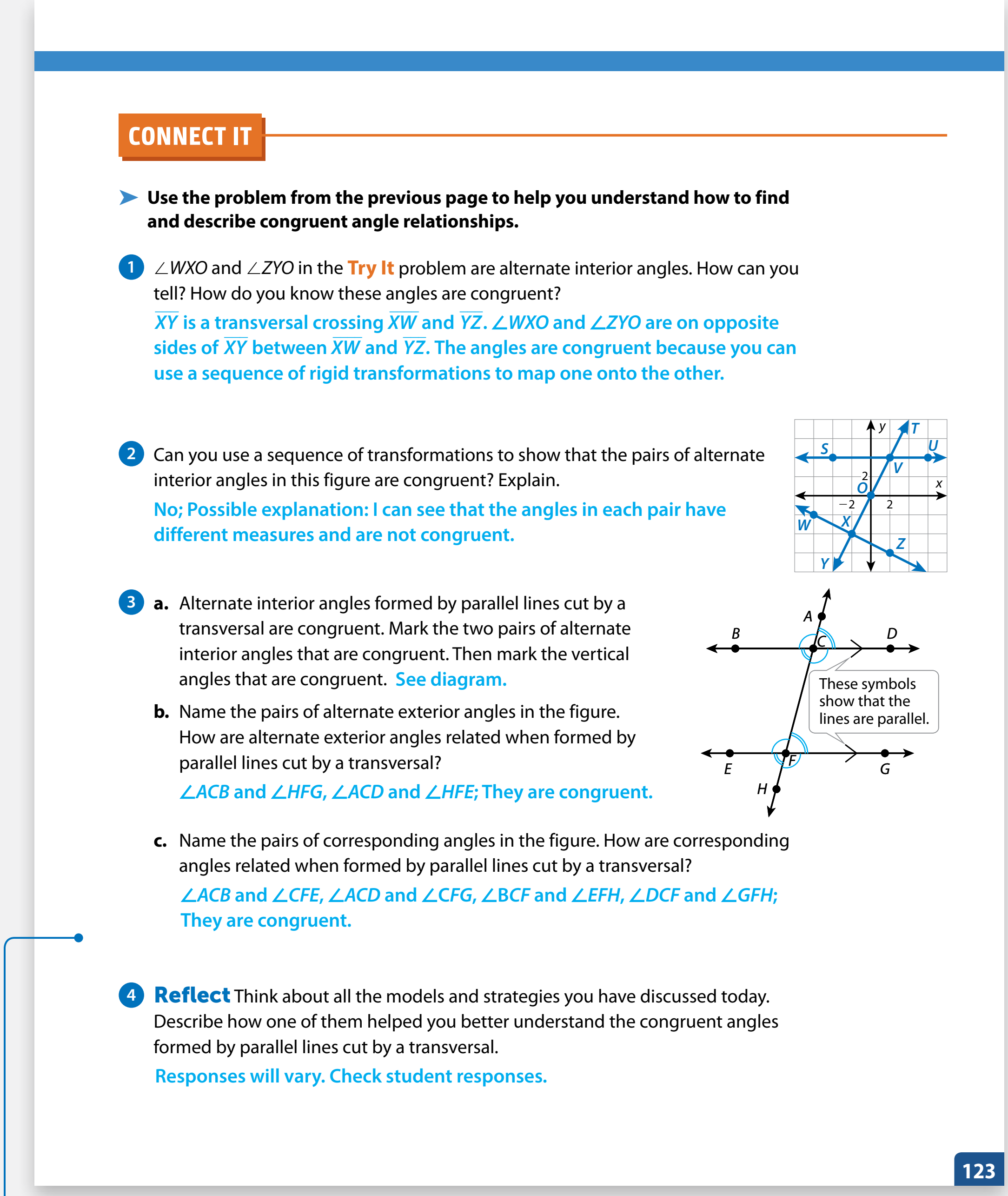
- ∠*WXO* and ∠*ZYO* are congruent because you can use rigid transformations to map one onto the other.
- The pairs of alternate interior angles in problem 2 are not congruent, so you cannot use rigid transformations to map one angle onto the other.

Facilitate Whole Class Discussion

③ Look for understanding that alternate interior, alternate exterior, and corresponding angles are congruent when formed by parallel lines cut by a transversal.

> **ASK** *Why is it significant that the lines are parallel?*
>
> **LISTEN FOR** If the lines are not parallel, you can still identify the angle relationships, but the angles will not be congruent.

④ **Reflect** Have all students focus on the strategies used to solve Try It. If time allows, have students discuss their ideas with a partner.



CONNECT IT

➤ **Use the problem from the previous page to help you understand how to find and describe congruent angle relationships.**

① ∠*WXO* and ∠*ZYO* in the **Try It** problem are alternate interior angles. How can you tell? How do you know these angles are congruent?

$\overline{XY}$ is a transversal crossing $\overline{XW}$ and $\overline{YZ}$. ∠*WXO* and ∠*ZYO* are on opposite sides of $\overline{XY}$ between $\overline{XW}$ and $\overline{YZ}$. The angles are congruent because you can use a sequence of rigid transformations to map one onto the other.

② Can you use a sequence of transformations to show that the pairs of alternate interior angles in this figure are congruent? Explain.

No; Possible explanation: I can see that the angles in each pair have different measures and are not congruent.

③ **a.** Alternate interior angles formed by parallel lines cut by a transversal are congruent. Mark the two pairs of alternate interior angles that are congruent. Then mark the vertical angles that are congruent. **See diagram.**

b. Name the pairs of alternate exterior angles in the figure. How are alternate exterior angles related when formed by parallel lines cut by a transversal?

∠*ACB* and ∠*HFG*, ∠*ACD* and ∠*HFE*; They are congruent.

These symbols show that the lines are parallel.

c. Name the pairs of corresponding angles in the figure. How are corresponding angles related when formed by parallel lines cut by a transversal?

∠*ACB* and ∠*CFE*, ∠*ACD* and ∠*CFG*, ∠BCF and ∠EFH, ∠DCF and ∠GFH; They are congruent.

④ **Reflect** Think about all the models and strategies you have discussed today. Describe how one of them helped you better understand the congruent angles formed by parallel lines cut by a transversal.

Responses will vary. Check student responses.

123

DIFFERENTIATION | RETEACH or REINFORCE

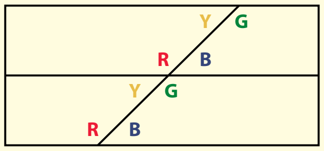
Hands-On Activity

Use a model to identify congruence in corresponding angles.

If students are unsure about the congruence of corresponding angles, then use this activity to help them visualize the connection.

Materials For each student: 4 markers, index card, ruler, scissors

- Have students draw a line on the index card, parallel to the top and near the middle.
- Instruct students that the top of the card, bottom of the card, and the drawn line represent three parallel lines. Have students use a ruler to draw a transversal from the top of the card to the bottom.
- Have students color each pair of corresponding angles a different color. An example of what this might look like is shown in the figure.



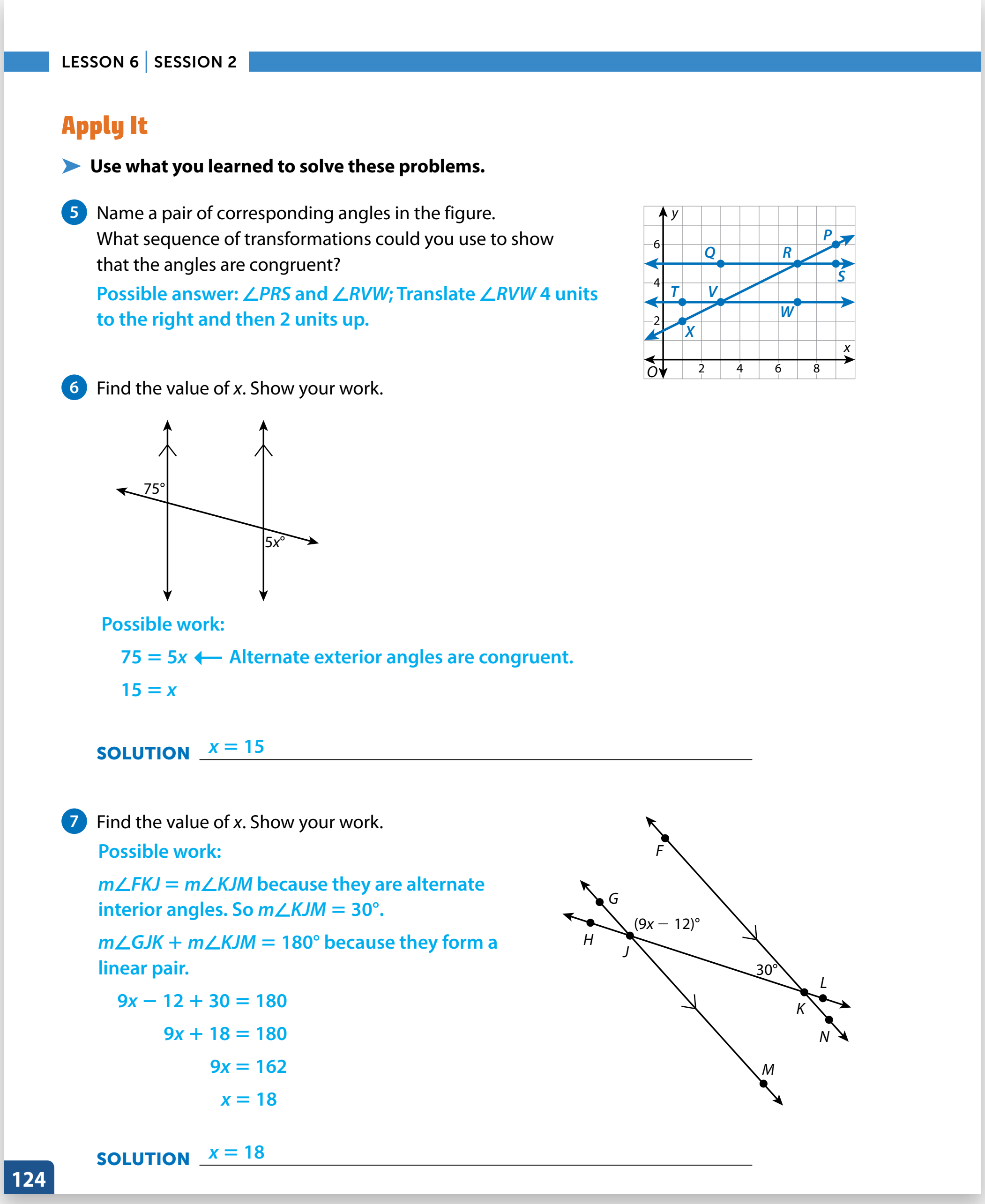
- Have students cut along the lines to make four trapezoids.
- Have students place trapezoids with same-color, corresponding angles on top of each other. Ask: *What do you notice?* [All pairs of corresponding angles are congruent.]

Apply It

For all problems, encourage students to use geometric notation and equations to record their work.

5 Students may color angles, rotate their worktext, or use a transparency or tracing paper to help visualize appropriate transformations.

6 Students should recognize that the lines are marked as parallel. The labeled angles are alternate exterior angles.



LESSON 6 | SESSION 2

Apply It

➤ **Use what you learned to solve these problems.**

5 Name a pair of corresponding angles in the figure. What sequence of transformations could you use to show that the angles are congruent?

Possible answer: ∠*PRS* and ∠*RVW*; Translate ∠*RVW* 4 units to the right and then 2 units up.

6 Find the value of *x*. Show your work.

Possible work:

$75 = 5x$ ⟵ Alternate exterior angles are congruent.

$15 = x$

SOLUTION $x = 15$

7 Find the value of *x*. Show your work.

Possible work:

$m\angle FKJ = m\angle KJM$ because they are alternate interior angles. So $m\angle KJM = 30°$.

$m\angle GJK + m\angle KJM = 180°$ because they form a linear pair.

$9x - 12 + 30 = 180$

$9x + 18 = 180$

$9x = 162$

$x = 18$

SOLUTION $x = 18$

124

CLOSE EXIT TICKET

7 Students' solutions should show an understanding that:

- when lines are parallel and cut by a transversal, alternate interior angles are congruent.
- linear pairs of angles have measures that add to 180°.

Error Alert If students get $x = 4\frac{2}{3}$, then they are assuming the labeled angles are congruent and solving $9x - 12 = 30$. Review with students the various types of angle pairs that are congruent when parallel lines are cut by a transversal. Ask students if the labeled angle pair is one of those types.

Practice Describing Congruent Angle Relationships

Problem Notes

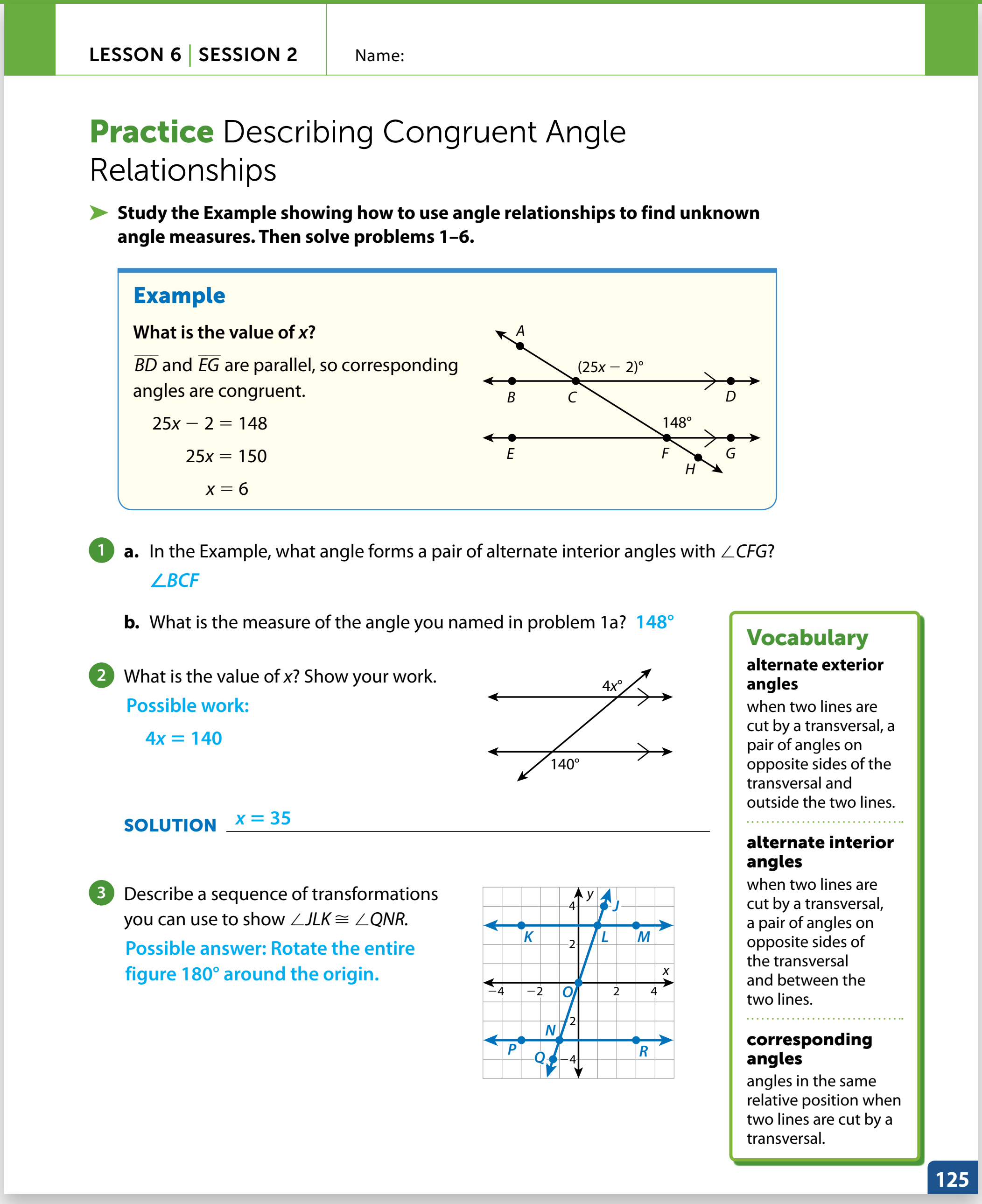
Assign **Practice Describing Congruent Angle Relationship** as extra practice in class or as homework.

① **a.** Students should understand that alternate interior angles are between the parallel lines and on opposite sides of the transversal. ***Basic***

b. Students should recognize that when parallel lines are cut by a transversal, alternate interior angles are congruent. ***Basic***

② Students should recognize that the marked angles are alternate exterior angles, so they have the same measure. ***Medium***

③ Students may also use a series of two reflections, one over the *x*-axis and one over the *y*-axis, to demonstrate congruence. ***Medium***



LESSON 6 | SESSION 2 Name:

Practice Describing Congruent Angle Relationships

➤ **Study the Example showing how to use angle relationships to find unknown angle measures. Then solve problems 1–6.**

Example

What is the value of *x*?

$\overline{BD}$ and $\overline{EG}$ are parallel, so corresponding angles are congruent.

$25x - 2 = 148$

$25x = 150$

$x = 6$

① **a.** In the Example, what angle forms a pair of alternate interior angles with ∠*CFG*?
∠*BCF*

b. What is the measure of the angle you named in problem 1a? 148°

② What is the value of *x*? Show your work.

Possible work:

$4x = 140$

SOLUTION $x = 35$

③ Describe a sequence of transformations you can use to show ∠*JLK* ≅ ∠*QNR*.

Possible answer: Rotate the entire figure 180° around the origin.

Vocabulary

alternate exterior angles
when two lines are cut by a transversal, a pair of angles on opposite sides of the transversal and outside the two lines.

alternate interior angles
when two lines are cut by a transversal, a pair of angles on opposite sides of the transversal and between the two lines.

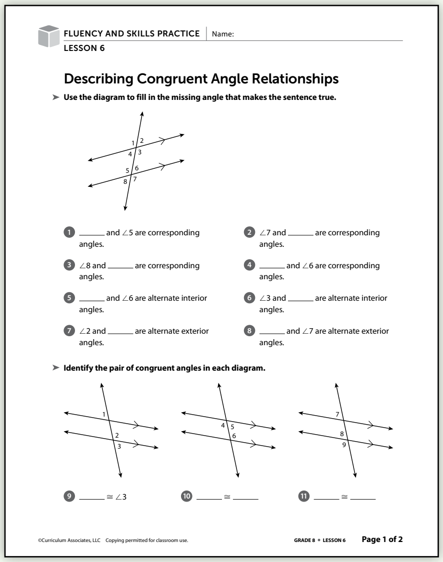
corresponding angles
angles in the same relative position when two lines are cut by a transversal.

125

Fluency & Skills Practice

Describing Congruent Angle Relationships

In this activity, students use information about angles formed when parallel lines are cut by a transversal to identify corresponding angles, alternate interior angles, and alternate exterior angles. They use these relationships to identify congruent angles and to solve for missing values.



FLUENCY AND SKILLS PRACTICE | Name:
LESSON 6

Describing Congruent Angle Relationships

➤ **Use the diagram to fill in the missing angle that makes the sentence true.**

① ______ and ∠5 are corresponding angles.

② ∠7 and ______ are corresponding angles.

③ ∠8 and ______ are corresponding angles.

④ ______ and ∠6 are corresponding angles.

⑤ ______ and ∠6 are alternate interior angles.

⑥ ∠3 and ______ are alternate interior angles.

⑦ ∠2 and ______ are alternate exterior angles.

⑧ ______ and ∠7 are alternate exterior angles.

➤ **Identify the pair of congruent angles in each diagram.**

⑨ ______ ≅ ∠3

⑩ ______ ≅ ______

⑪ ______ ≅ ______

©Curriculum Associates, LLC Copying permitted for classroom use.

GRADE 8 • LESSON 6 Page 1 of 2

LESSON 6 | SESSION 2

Additional Practice

④ **A, E, and F are correct.** Students may solve the problem by first identifying whether the angles marked are corresponding, alternate interior, or alternate exterior. Then if the lines are parallel, the angles are congruent.

B is incorrect. ∠1 and ∠2 are supplementary.

C is incorrect. The angles are corresponding angles, but the lines are not parallel.

D is incorrect. The angles are alternate interior angles, but the lines are not parallel.

Medium

⑤ **a.** Students may cover the middle line and recognize both angles are in the same relative position.

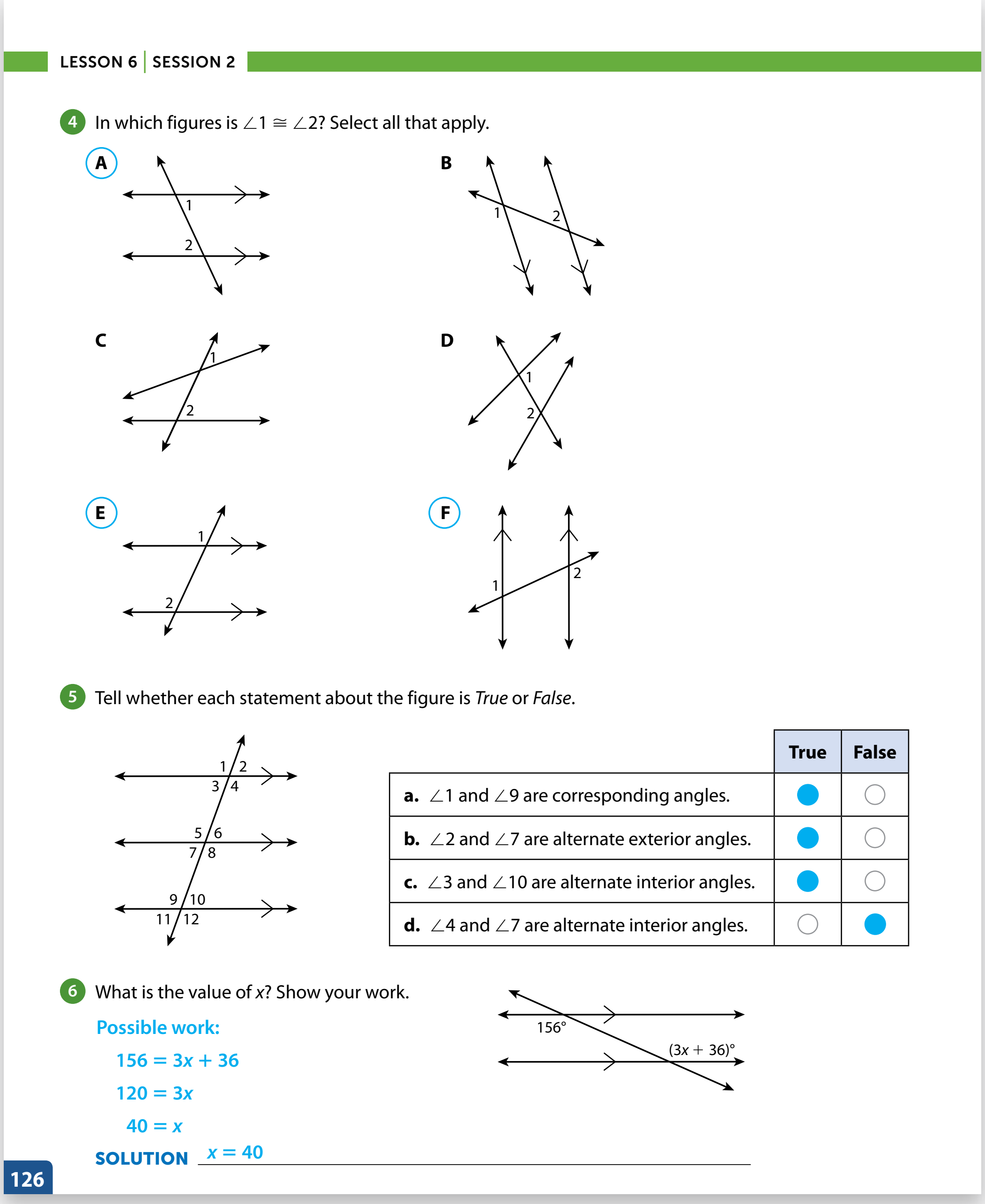
b. Students may cover the bottom line and recognize the angles are exterior to the parallel lines and on opposite sides.

c. Students may cover the middle line and recognize the angles are interior to the parallel lines and on opposite sides.

d. One angle is an interior angle and one is exterior to the parallel lines.

Challenge

⑥ Students should recognize the lines are parallel and the marked angles are alternate interior angles, so they are congruent. ***Medium***



LESSON 6 | SESSION 2

④ In which figures is ∠1 ≅ ∠2? Select all that apply.

⑤ Tell whether each statement about the figure is *True* or *False*.

	True	False
a. ∠1 and ∠9 are corresponding angles.	●	○
b. ∠2 and ∠7 are alternate exterior angles.	●	○
c. ∠3 and ∠10 are alternate interior angles.	●	○
d. ∠4 and ∠7 are alternate interior angles.	○	●

⑥ What is the value of *x*? Show your work.

Possible work:

$156 = 3x + 36$

$120 = 3x$

$40 = x$

SOLUTION $x = 40$

DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Use with **Session 3** **Apply It**

Levels 1–3: Reading/Speaking

Prepare students to respond to Apply It problem 6. Read the problem as students follow along. Point out that *m*∠*MPR* is read *the measure of angle MPR*. Then have students read it aloud chorally. Confirm understanding by asking students to complete this sentence frame:

- *∠LPR and ∠NLP are _____ angles. So, the sum of m∠LPR and m∠NLP is _____.*

Then have students work in pairs to solve the problem. Monitor to make sure they find the value of *x* and use that value to find *m*∠*LPR*. Call on volunteers to share their answers. Repeat and reword responses as needed to model using mathematical terms in fluent English.

Levels 2–4: Reading/Speaking

Prepare students to answer Apply It problem 6. Have students read and discuss the problem with a partner. Model how to read *m*∠*MPR*.

Have partners use the graphic to decide what types of angles are represented with values in the diagram. Provide terms such as *supplementary, same, side, interior,* and *equal*.

Encourage students to use the terms to talk about the measurements needed to solve the problem. Ask them what they need to find first and how they are going to use that value. Call on volunteers to explain their solutions. Encourage them to use mathematical terminology and the word *so* in their sentences.

Levels 3–5: Reading/Speaking

Prepare students to solve Apply It problem 6. Use **Notice and Wonder** to have students talk about the graphic. Have students turn and talk to a partner about what they notice. Then have them tell what they wonder. Ask: *What is missing in the graphic?* Encourage students to use mathematical terms as they notice and wonder. Then have them read and work on the problem.

Have students compare answers with their partners. If they did not get the same answer, encourage them to discuss the steps they took and explain their solutions.

Develop Describing Supplementary Angle Relationships

Purpose

- **Develop** strategies for showing that certain angles formed by parallel lines cut by a transversal are supplementary.
- **Recognize** that when the lines cut by a transversal are parallel, then same-side interior angles and same-side exterior angles are supplementary.

START CONNECT TO PRIOR KNOWLEDGE

Same and Different

corresponding angles	alternate interior angles
A	B
C	D
alternate exterior angles	vertical angles

Possible Solutions

All are related pairs of angles.

A, B, and C form congruent pairs when parallel lines are cut by a transversal.

D is the only pair that is always congruent.

B and C are pairs on opposite sides of the transversal when two lines are cut by a transversal.

WHY? Support students' facility in identifying angle relationships.

DEVELOP ACADEMIC LANGUAGE

WHY? Support students in using precise mathematical language.

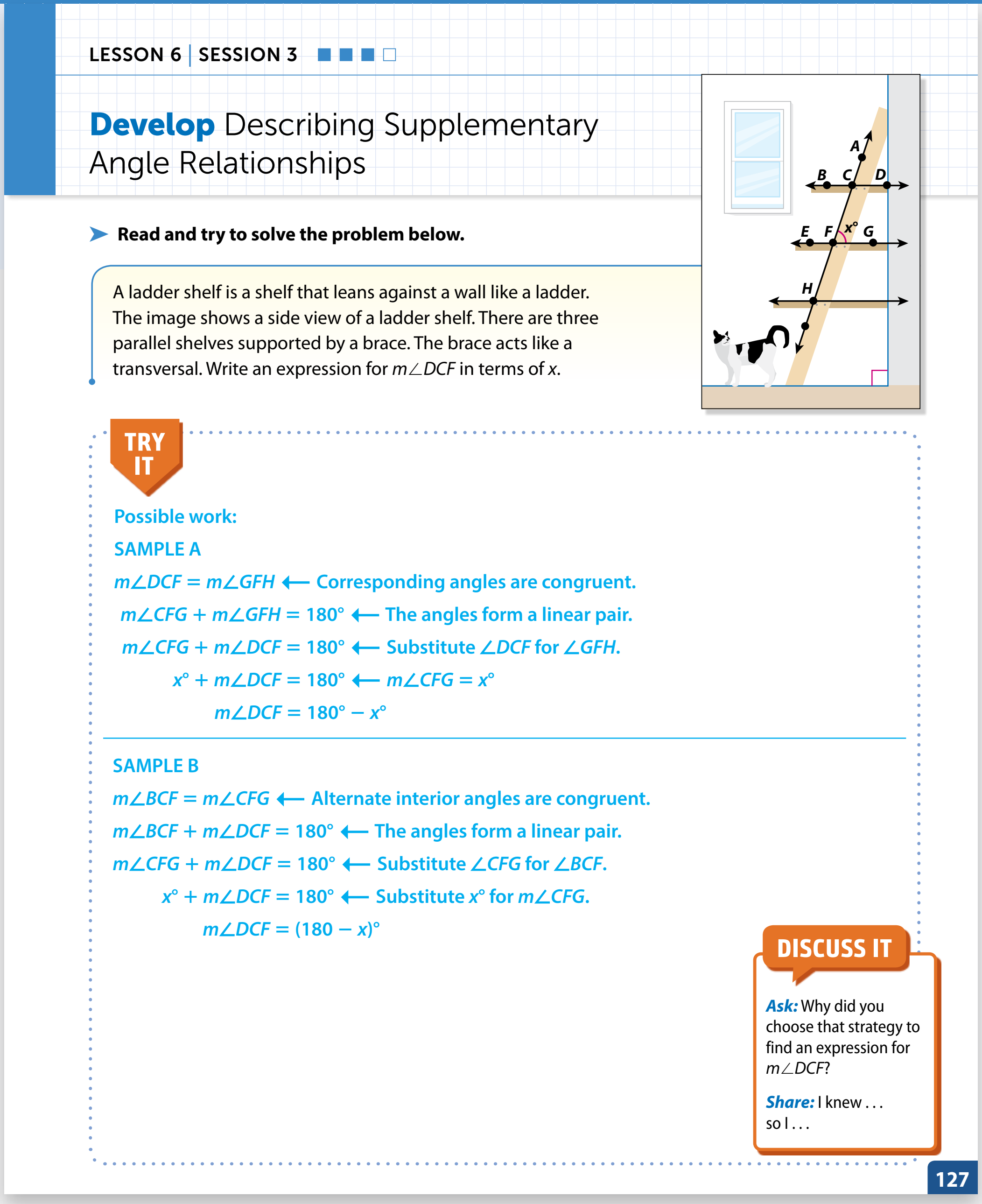
HOW? Using precise mathematical language is a skill that develops over time. Encourage students to use *congruent*, *intersect*, *corresponding*, and the names of angle relationships as they discuss the Try It problem. Notice and highlight times when students use lesson vocabulary accurately.

TRY IT

SMP 1, 2, 4, 5, 6

Make Sense of the Problem

See ***Connect to Culture*** to support student engagement. Before students work on Try It, use **Co-Craft Questions** to help them make sense of the problem. Encourage students to use details in the figure to suggest possible questions. If time allows, return to the students' questions at the end of the session and select one or two to answer.



LESSON 6 | SESSION 3 ■■■□

Develop Describing Supplementary Angle Relationships

➤ **Read and try to solve the problem below.**

A ladder shelf is a shelf that leans against a wall like a ladder. The image shows a side view of a ladder shelf. There are three parallel shelves supported by a brace. The brace acts like a transversal. Write an expression for $m\angle DCF$ in terms of x.

TRY IT

Possible work:

SAMPLE A

$m\angle DCF = m\angle GFH$ ⟵ Corresponding angles are congruent.

$m\angle CFG + m\angle GFH = 180°$ ⟵ The angles form a linear pair.

$m\angle CFG + m\angle DCF = 180°$ ⟵ Substitute $\angle DCF$ for $\angle GFH$.

$x° + m\angle DCF = 180°$ ⟵ $m\angle CFG = x°$

$m\angle DCF = 180° - x°$

SAMPLE B

$m\angle BCF = m\angle CFG$ ⟵ Alternate interior angles are congruent.

$m\angle BCF + m\angle DCF = 180°$ ⟵ The angles form a linear pair.

$m\angle CFG + m\angle DCF = 180°$ ⟵ Substitute $\angle CFG$ for $\angle BCF$.

$x° + m\angle DCF = 180°$ ⟵ Substitute $x°$ for $m\angle CFG$.

$m\angle DCF = (180 - x)°$

DISCUSS IT

Ask: Why did you choose that strategy to find an expression for $m\angle DCF$?

Share: I knew . . . so I . . .

127

DISCUSS IT

SMP 2, 3, 6

Support Partner Discussion

After students work on Try It, have them explain their work and then respond to Discuss It with a partner. Remind students to check that their responses are clear by pausing and asking their partner questions.

Common Misconception Listen for students who confuse the different angle relationships. As students share their strategies, have them use diagrams to emphasize the locations of the related angles they are referring to.

Select and Sequence Student Strategies

Select 2–3 samples that represent the range of student thinking in your classroom. Here is one possible order for class discussion:

- corresponding angles and a linear pair used to write the expression
- **(misconception)** confused angle relationships used to write the expression
- alternate interior angles and a linear pair used to write the expression

Facilitate Whole Class Discussion

Call on students to share selected strategies. After each strategy, allow individual think time for students to process the ideas.

Guide students to **Compare and Connect** the representations. Remind students that one way to agree and build on ideas is to give reasons that explain why the strategy makes sense.

> **ASK** *How did the angle relationships used in the different strategies compare?*
>
> **LISTEN FOR** The strategies used different pairs of related congruent angles, but all of them used a linear pair, with angle measures that add to 180°.

Model It

If students presented these models, have students connect these models to those presented in class.

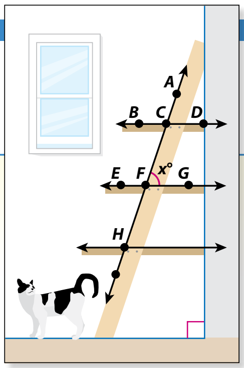
If no student presented at least one of these models, have students first analyze key features of the models, and then connect them to the models presented in class.

> **ASK** *What connections, if any, do you notice between the strategies used in the two Model Its?*
>
> **LISTEN FOR** Both strategies use a congruent-angle relationship related to parallel lines cut by a transversal, and both use a linear pair. Both end with a statement that the sum of $m\angle CFG$ and $m\angle DCF$ is 180°.

For the model about alternate interior angles, prompt students to consider the structure of the solution. Ask: *What consistencies or patterns do you notice between this model and the strategies presented by students?*

For the model about corresponding angles, prompt students to identify what is unique about this model. Ask: *What is different about the solutions in the first and second model?*

➤ **Explore different ways to find and describe supplementary angle relationships.**



A ladder shelf is a shelf that leans against a wall like a ladder. The image shows a side view of a ladder shelf. There are three parallel shelves supported by a brace. The brace acts like a transversal. Write an expression for $m\angle DCF$ in terms of x.

Model It

You can use what you know about alternate interior angles.

$\overleftrightarrow{BD} \parallel \overleftrightarrow{EG}$, so $\angle EFC$ and $\angle DCF$ are congruent alternate interior angles.

$\angle EFC$ and $\angle CFG$ form a linear pair, so $m\angle EFC + m\angle CFG = 180°$.

$m\angle EFC = m\angle DCF$, so you can substitute $m\angle DCF$ for $m\angle EFC$:

$m\angle DCF + m\angle CFG = 180°$

Model It

You can use what you know about corresponding angles.

$\overleftrightarrow{BD} \parallel \overleftrightarrow{EG}$, so $\angle ACD$ and $\angle CFG$ are congruent corresponding angles.

$\angle ACD$ and $\angle DCF$ form a linear pair, so $m\angle ACD + m\angle DCF = 180°$.

$m\angle ACD = m\angle CFG$, so you can substitute $m\angle CFG$ for $m\angle ACD$:

$m\angle CFG + m\angle DCF = 180°$

DIFFERENTIATION | EXTEND

SMP 3

Deepen Understanding

Starting with a Linear Pair or Vertical Angles to Find $m\angle DCF$

Point out that the strategies in the two Model Its each start with an angle pair related to parallel lines. Ask students to consider other angle relationships they could start with.

> **ASK** *$\angle CFG$ and $\angle EFC$ are a linear pair. How can you start with this fact to find an expression for $m\angle DCF$ in terms of x?*
>
> **LISTEN FOR** $\angle CFG$ and $\angle EFC$ are supplementary, so $m\angle EFC + x° = 180°$. So, $m\angle EFC = 180° - x°$. $\angle EFC$ and $\angle DCF$ are alternate interior angles, so they are congruent. So $m\angle DCF$ is also $180° - x°$.

> **ASK** *$\angle CFG$ and $\angle EFH$ are vertical angles. How can you start with this fact to find an expression for $m\angle DCF$?*
>
> **LISTEN FOR** Vertical angles are congruent, so $m\angle CFG = m\angle EFH = x°$. $\angle EFH$ and $\angle ACD$ are alternate exterior angles, so they are congruent. So $m\angle ACD = x°$. $\angle ACD$ and $\angle DCF$ are a linear pair, so $m\angle DCF + x° = 180°$. So, $m\angle DCF = 180° - x°$.

Develop Describing Supplementary Angle Relationships

CONNECT IT

SMP 2, 4, 5, 6

Remind students that the angles and their relationships are the same in each representation. Explain that they will now use those relationships to reason about supplementary angle relationships when two parallel lines are cut by a transversal.

Before students begin to record and expand on their work in Model It, tell them that problem 3 will prepare them to provide the explanation asked for in problem 4. Use turn and talk to help students think through their responses before sharing with the group.

Monitor and Confirm Understanding ①–②

- The expression for $m\angle DCF$ is $180^\circ - x^\circ$.
- The expression is dependent on angle relationships that exist only if the lines are parallel.

Facilitate Whole Class Discussion

③ **a.** Students should recognize the same-side interior angles are supplementary.

> **ASK** *In problem 1, you found $m\angle CFG = x^\circ$ and $m\angle DCF = 180^\circ - x^\circ$. What do you notice about the relationship between these angles?*
>
> **LISTEN FOR** The angles are supplementary because adding them results in a sum of 180°.

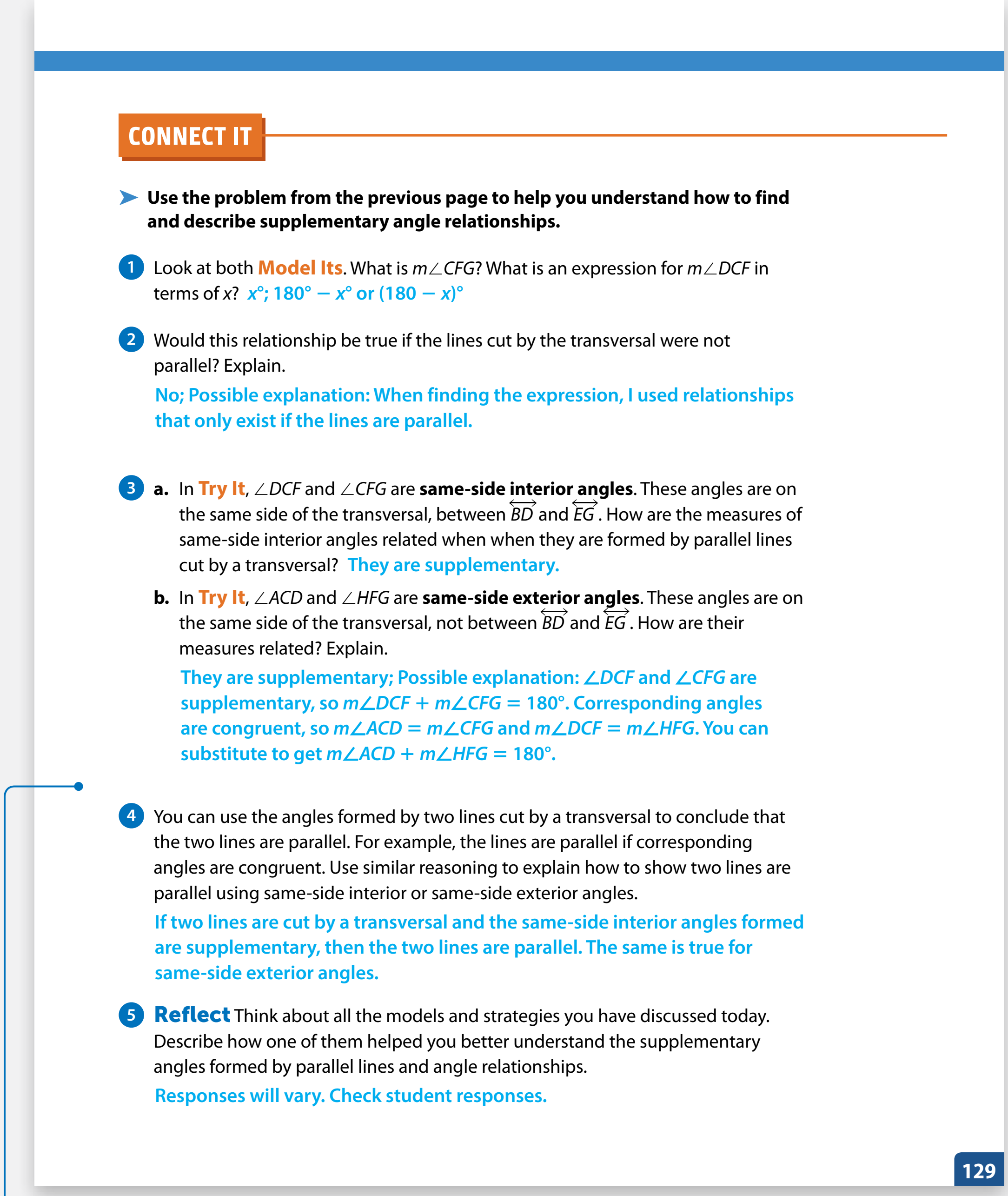
b. Students should understand the same-side exterior angles are supplementary.

> **ASK** *How can you use the expressions found in problem 1 and corresponding angles to write expressions for $m\angle ACD$ and $m\angle HFG$?*
>
> **LISTEN FOR** Because corresponding angles are congruent, $m\angle ACD = x^\circ$ and $m\angle HFG = 180^\circ - x^\circ$.

④ Look for the idea that given certain angle relationships, it can be concluded that lines are parallel.

> **ASK** *When two lines are cut by a transversal, what must be true about same-side interior or exterior angles for the lines to be parallel?*
>
> **LISTEN FOR** The pair of angles must be supplementary for the lines to be parallel.

⑤ **Reflect** Have all students focus on the strategies used to solve the Try It. If time allows, have students discuss their ideas with a partner.



CONNECT IT

➤ **Use the problem from the previous page to help you understand how to find and describe supplementary angle relationships.**

① Look at both **Model Its**. What is $m\angle CFG$? What is an expression for $m\angle DCF$ in terms of x? x°; $180^\circ - x^\circ$ or $(180 - x)^\circ$

② Would this relationship be true if the lines cut by the transversal were not parallel? Explain.
No; Possible explanation: When finding the expression, I used relationships that only exist if the lines are parallel.

③ **a.** In **Try It**, $\angle DCF$ and $\angle CFG$ are **same-side interior angles**. These angles are on the same side of the transversal, between $\overleftrightarrow{BD}$ and $\overleftrightarrow{EG}$. How are the measures of same-side interior angles related when when they are formed by parallel lines cut by a transversal? They are supplementary.

b. In **Try It**, $\angle ACD$ and $\angle HFG$ are **same-side exterior angles**. These angles are on the same side of the transversal, not between $\overleftrightarrow{BD}$ and $\overleftrightarrow{EG}$. How are their measures related? Explain.
They are supplementary; Possible explanation: $\angle DCF$ and $\angle CFG$ are supplementary, so $m\angle DCF + m\angle CFG = 180^\circ$. Corresponding angles are congruent, so $m\angle ACD = m\angle CFG$ and $m\angle DCF = m\angle HFG$. You can substitute to get $m\angle ACD + m\angle HFG = 180^\circ$.

④ You can use the angles formed by two lines cut by a transversal to conclude that the two lines are parallel. For example, the lines are parallel if corresponding angles are congruent. Use similar reasoning to explain how to show two lines are parallel using same-side interior or same-side exterior angles.
If two lines are cut by a transversal and the same-side interior angles formed are supplementary, then the two lines are parallel. The same is true for same-side exterior angles.

⑤ **Reflect** Think about all the models and strategies you have discussed today. Describe how one of them helped you better understand the supplementary angles formed by parallel lines and angle relationships.
Responses will vary. Check student responses.

129

DIFFERENTIATION | RETEACH or REINFORCE

Hands-On Activity

Use a model to understand supplementary relationships.

If students are unsure about same-side interior and same-side exterior angles being supplementary, then use this activity to help them understand the connections between the angle relationships when two parallel lines are cut by a transversal.

Materials For each student: protractor, ruler

- Have students draw two parallel lines and a transversal using their ruler.
- Have students use a protractor to measure and label each angle.
- Have students identify the pairs of same-side interior angles and same-side exterior angles with a letter or color. Each pair should get a different letter or color.
- Ask: *How did you identify the angle pairs?* [Interior angles are between the parallel lines and on the same side of the transversal. Exterior angles have one above and one below the parallel lines and are on the same side of the transversal.]
- Have students sum the angle measures of each pair of same-side interior angles and each pair of same-side exterior angles. Ask: *What is the sum of each pair of angles?* [Each sum is 180°, so each pair of angles is supplementary.]

Apply It

For all problems, encourage students to use a model to support their thinking. Remind students to use geometric notation and language in their work and explanations.

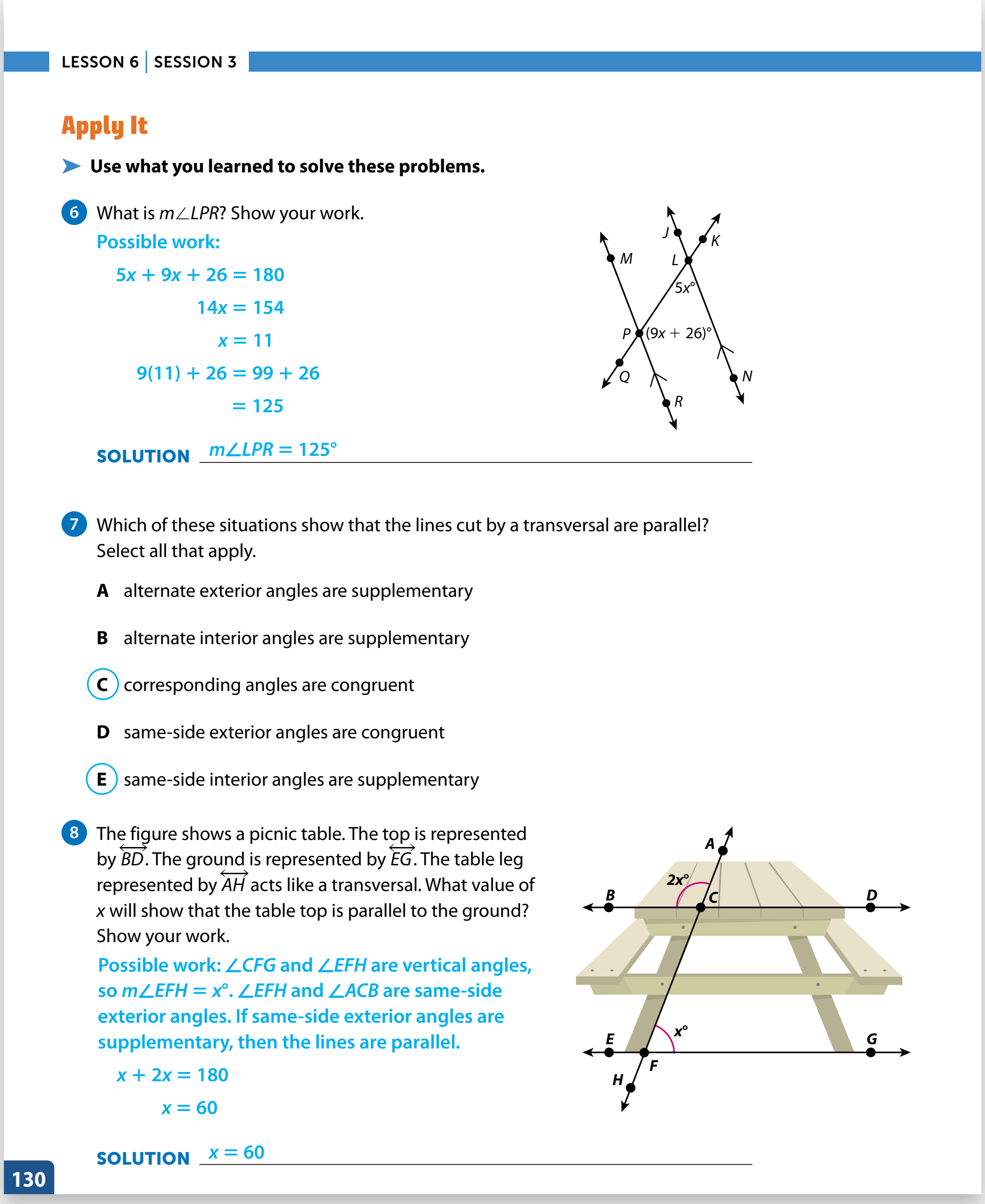
⑥ Students should recognize that the lines are parallel. The labeled angles are same-side interior angles, so they must be supplementary.

⑦ **C and E are correct.** When two parallel lines are cut by a transversal, corresponding angles are congruent, and same-side interior angles are supplementary.

A is incorrect. Alternate exterior angles are congruent.

B is incorrect. Alternate interior angles are congruent.

D is incorrect. Same-side exterior angles are supplementary.



LESSON 6 | SESSION 3

Apply It

➤ **Use what you learned to solve these problems.**

⑥ What is $m\angle LPR$? Show your work.

Possible work:

$5x + 9x + 26 = 180$

$14x = 154$

$x = 11$

$9(11) + 26 = 99 + 26$

$= 125$

SOLUTION $m\angle LPR = 125°$

⑦ Which of these situations show that the lines cut by a transversal are parallel? Select all that apply.

A alternate exterior angles are supplementary

B alternate interior angles are supplementary

[✓ **C**] corresponding angles are congruent

D same-side exterior angles are congruent

[✓ **E**] same-side interior angles are supplementary

⑧ The figure shows a picnic table. The top is represented by $\overleftrightarrow{BD}$. The ground is represented by $\overleftrightarrow{EG}$. The table leg represented by $\overleftrightarrow{AH}$ acts like a transversal. What value of x will show that the table top is parallel to the ground? Show your work.

Possible work: $\angle CFG$ and $\angle EFH$ are vertical angles, so $m\angle EFH = x°$. $\angle EFH$ and $\angle ACB$ are same-side exterior angles. If same-side exterior angles are supplementary, then the lines are parallel.

$x + 2x = 180$

$x = 60$

SOLUTION $x = 60$

130

CLOSE **EXIT TICKET**

⑧ Students' solutions should show an understanding that:

- same-side interior or same-side exterior angles are supplementary.
- vertical angles are congruent.

Error Alert If students confuse angle relationships, then use a diagram to review the names of related pairs of angles and whether they are congruent or supplementary.

Practice Describing Supplementary Angle Relationships

Problem Notes

Assign **Practice Describing Supplementary Angle Relationships** as extra practice in class or as homework.

① Students may also use corresponding angle relationships to identify the expression that relates ∠*DCF* and ∠*CFG* in the example. Then they can write an expression for supplementary angles, solve for *x*, and substitute to get the angle measures. ***Basic***

② Students should identify the labeled angles as same-side interior angles and the lines as parallel. Therefore, the angles are supplementary. ***Medium***

Practice Describing Supplementary Angle Relationships

➤ **Study the Example showing how to use angle relationships to solve problems. Then solve problems 1–5.**

Example

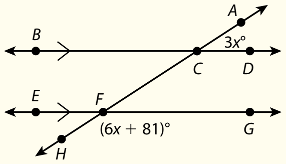
What is the value of *x*?

∠*ACD* and ∠*HFG* are same-side exterior angles. $\overleftrightarrow{BD}$ and $\overleftrightarrow{EG}$ are parallel, so $m\angle ACD + m\angle HFG = 180°$.

$3x + 6x + 81 = 180$

$9x = 99$

$x = 11$



① What is the angle relationship between ∠*DCF* and ∠*CFG* in the Example? What are the measures of these angles? Show your work.

same side interior angles; Possible work:

$m\angle ACD + m\angle DCF = 180°$ ⟵ The angles form a linear pair.

$33° + m\angle DCF = 180°$

$m\angle DCF = 147°$

$m\angle DCF + m\angle CFG = 180°$ ⟵ Same-side interior angles.

$147° + m\angle CFG = 180°$

$m\angle CFG = 33°$

SOLUTION $m\angle DCF = 147°$ and $m\angle CFG = 33°$

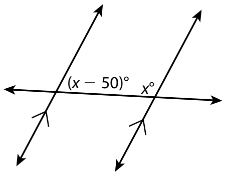
② Find the value of *x*. Show your work.

Possible work:

$x - 50 + x = 180$

$2x = 230$

$x = 115$



SOLUTION $x = 115$

Vocabulary

same-side exterior angles
when two lines are cut by a transversal, a pair of angles on the same side of the transversal and outside the two lines.

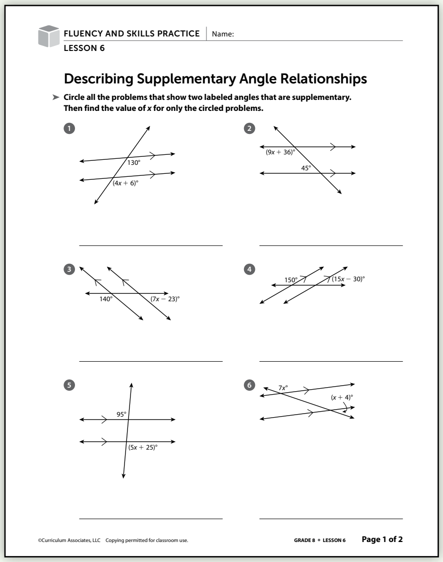
same-side interior angles
when two lines are cut by a transversal, a pair of angles on the same side of the transversal and between the two lines.

transversal
a line that cuts two or more lines.

Fluency & Skills Practice

Describing Supplementary Angle Relationships

In this activity, students use information about angles formed when parallel lines are cut by a transversal to identify pairs of supplementary angles. Then they use the measures of supplementary angles to solve for missing values.



FLUENCY AND SKILLS PRACTICE | Name:
LESSON 6

Describing Supplementary Angle Relationships

➤ **Circle all the problems that show two labeled angles that are supplementary. Then find the value of *x* for only the circled problems.**

©Curriculum Associates, LLC Copying permitted for classroom use.   GRADE 8 • LESSON 6   Page 1 of 2

LESSON 6 | SESSION 3

Additional Practice

③ Students may solve the problem by using vertical angles and either a same-side interior or same-side exterior relationship. Then they might use this supplementary relationship to write an equation and solve for x. ***Medium***

④ **C, D, and E are correct.** Students may solve the problem by covering the line that is not needed for each relationship, then verifying the angle relationships given.

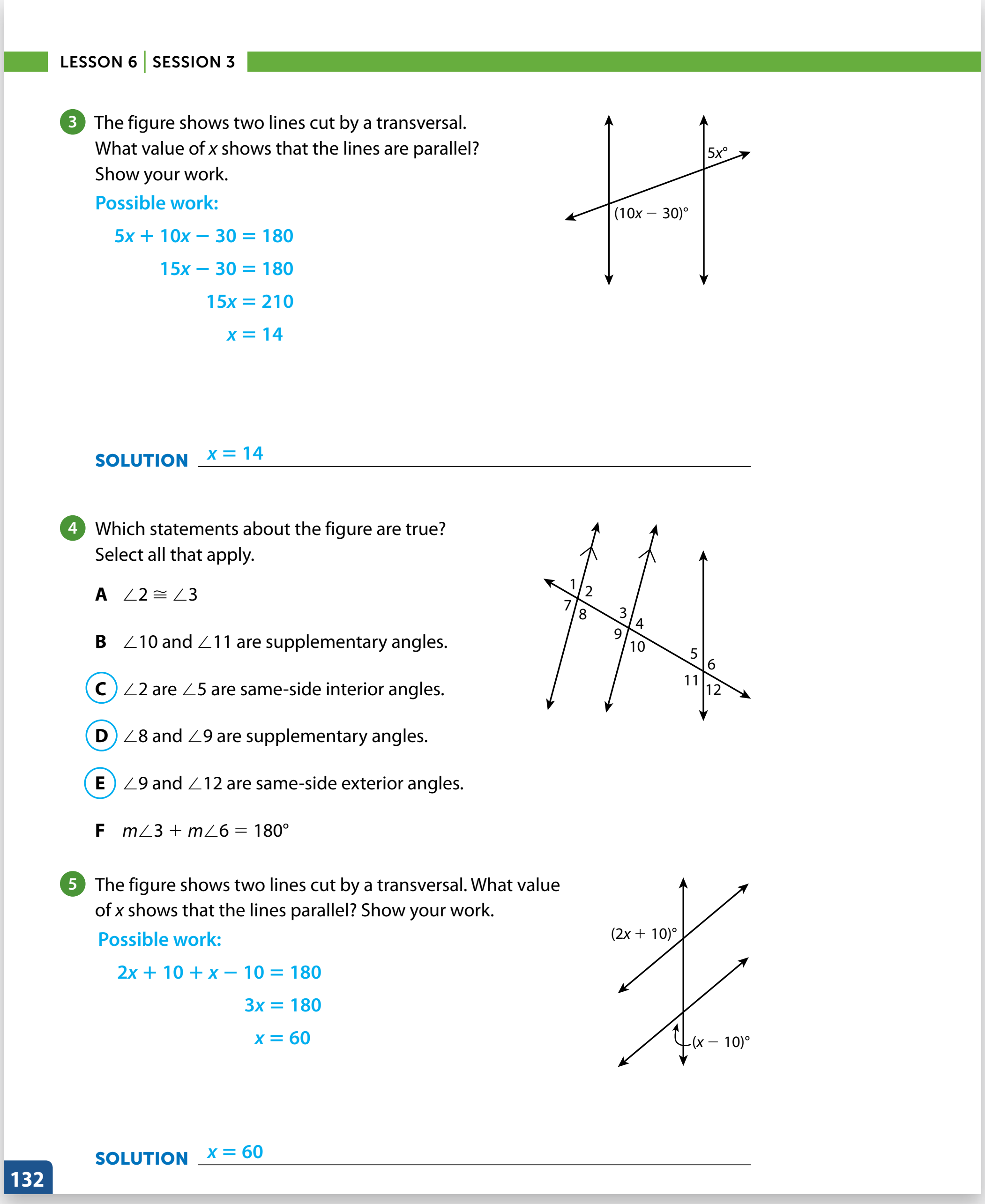
A is incorrect. $\angle 2$ and $\angle 3$ are same-side interior angles and the lines are parallel, so the angles are supplementary.

B is incorrect. $\angle 10$ and $\angle 11$ are same-side interior angles, but the lines are not parallel, so the angles are not supplementary.

F is incorrect. $\angle 3$ and $\angle 6$ are same-side exterior angles, but the lines are not parallel, so the angles are not supplementary.

Challenge

⑤ Students should recognize the labeled angles are same-side exterior angles. For the lines to be parallel, these angles must be supplementary. ***Medium***



LESSON 6 | SESSION 3

③ The figure shows two lines cut by a transversal. What value of x shows that the lines are parallel? Show your work.

Possible work:

$5x + 10x - 30 = 180$

$15x - 30 = 180$

$15x = 210$

$x = 14$

SOLUTION $x = 14$

④ Which statements about the figure are true? Select all that apply.

A $\angle 2 \cong \angle 3$

B $\angle 10$ and $\angle 11$ are supplementary angles.

[✓ **C**] $\angle 2$ are $\angle 5$ are same-side interior angles.

[✓ **D**] $\angle 8$ and $\angle 9$ are supplementary angles.

[✓ **E**] $\angle 9$ and $\angle 12$ are same-side exterior angles.

F $m\angle 3 + m\angle 6 = 180°$

⑤ The figure shows two lines cut by a transversal. What value of x shows that the lines parallel? Show your work.

Possible work:

$2x + 10 + x - 10 = 180$

$3x = 180$

$x = 60$

SOLUTION $x = 60$

DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Use with **Session 4** **Apply It**

Levels 1–3: Speaking/Writing

Prepare students to solve problem 5. Read the problem as students follow along. Call out different angle pairs and have students identify them. Have partners sort the angles in a T-chart: *Congruent* and *Supplementary.* As partners sort, ask questions to help them state how they know. Provide sentence frames to help students verbalize their ideas:

- *Angle ____ and angle ____ are congruent. They are ____ angles.*
- *Angle ____ and angle ____ are supplementary. They are ____.*

Then have students answer the first question in writing:

- *Angles ____ are congruent to* $\angle 5$*.*

Levels 2–4: Speaking/Writing

Prepare students to write responses to Apply It problem 5. Have students read the problem with a partner. Prompt them to make a two-column chart with the headings *Congruent* and *Supplementary*. Have them observe the angle pairs and sort them in the chart. Encourage them to tell how they know the angles are congruent or supplementary:

- *Angles ____ and ____ are ____ because ____.*

Allow think time for partners to review their charts. Then have them write the answer to the problem. Guide them to use words and symbols and the phrase *because they* in their explanations.

Levels 3–5: Speaking/Writing

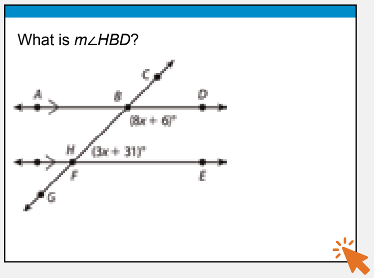
Prepare students to write responses to problem 5. Have partners read the problem. Prompt them to make a two-column chart with the headings *Congruent* and *Supplementary*. Have them observe the angle pairs and sort them in the chart. Encourage them to tell how they know the angles are congruent or supplementary. Allow think time for partners to review their charts. Then have them answer the problem using words, symbols, and the word *because.* To reinforce the use of plural versus singular pronouns, have students decide whether to use *because it* or *because they* in their explanations. Then call on volunteers to explain their choices.

Refine Describing Angle Relationships

Purpose

- **Refine** understanding of the relationships between pairs of angles formed when parallel lines are cut by a transversal.
- **Refine** strategies for finding unknown angle measures in diagrams and situations where parallel lines are cut by a transversal.

START CHECK FOR UNDERSTANDING



What is m∠HBD?

Solution

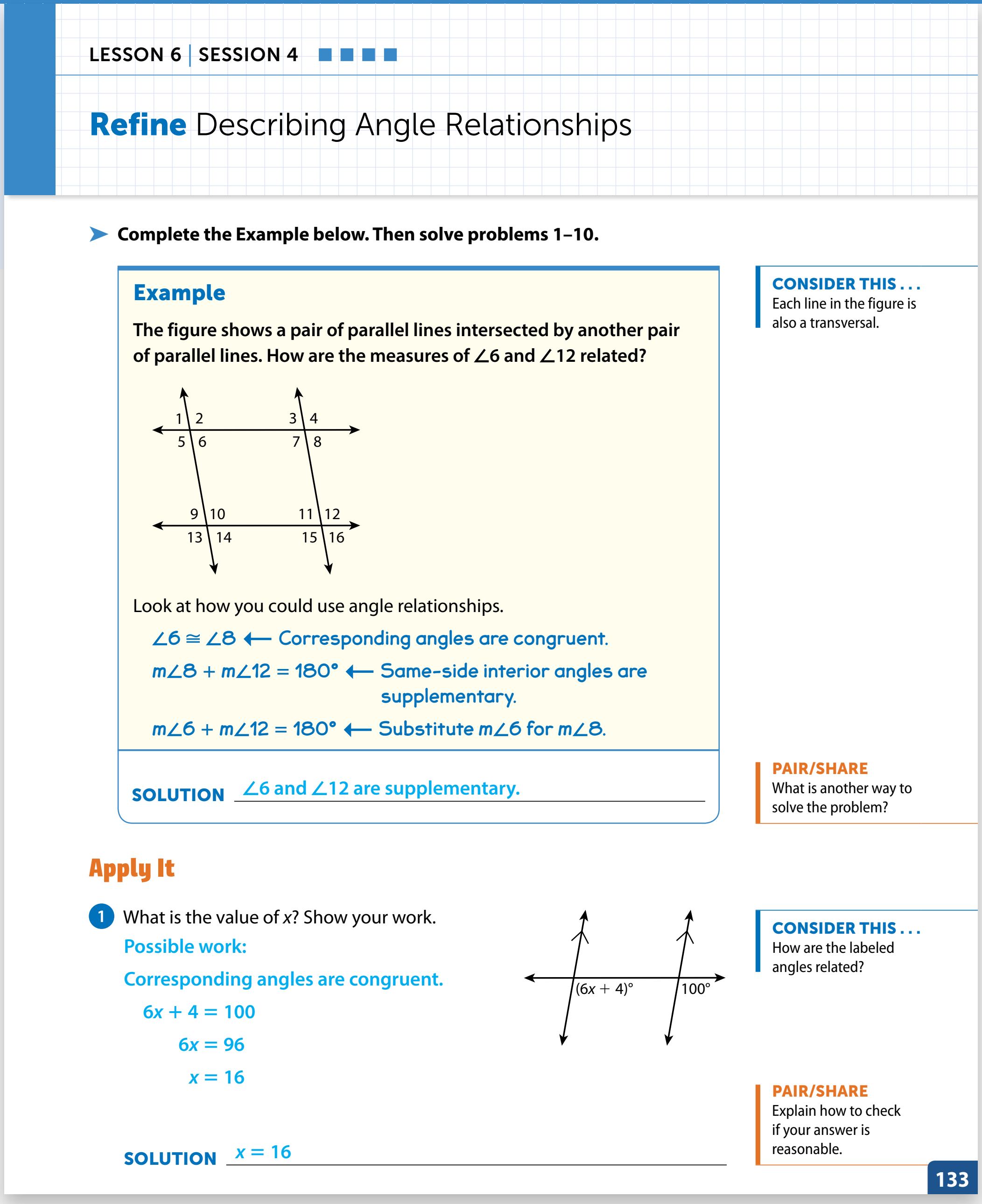
110°

WHY? Confirm students' understanding of angles formed when parallel lines are cut by a transversal, identifying common errors to address as needed.

MONITOR & GUIDE

Before students begin to work, use their responses to the **Start** to determine those who will benefit from additional support. Use the **Error Analysis** table below to guide remediation.

Have all students complete the Example and problems 1–3, using Consider This and Pair/Share as appropriate. Observe and monitor their reasoning and guide or redirect students as needed.



LESSON 6 | SESSION 4 ■ ■ ■ ■

Refine Describing Angle Relationships

➤ **Complete the Example below. Then solve problems 1–10.**

Example

The figure shows a pair of parallel lines intersected by another pair of parallel lines. How are the measures of ∠6 and ∠12 related?

Look at how you could use angle relationships.

∠6 ≅ ∠8 ⟵ Corresponding angles are congruent.

m∠8 + m∠12 = 180° ⟵ Same-side interior angles are supplementary.

m∠6 + m∠12 = 180° ⟵ Substitute m∠6 for m∠8.

SOLUTION ∠6 and ∠12 are supplementary.

CONSIDER THIS . . . Each line in the figure is also a transversal.

PAIR/SHARE What is another way to solve the problem?

Apply It

① What is the value of x? Show your work.

Possible work:

Corresponding angles are congruent.

$6x + 4 = 100$

$6x = 96$

$x = 16$

SOLUTION $x = 16$

CONSIDER THIS . . . How are the labeled angles related?

PAIR/SHARE Explain how to check if your answer is reasonable.

133

START ERROR ANALYSIS

If the error is . . .	Students may . . .	To support understanding . . .
13°	have found the value of x and stopped.	Remind students that the value of x should be used to calculate m∠HBD.
70°	have found m∠BHE.	Ask students to carefully read the question and calculate the correct angle measure.
46°	have thought ∠HBD and ∠BHE are congruent angles.	Elicit that the marked angles are same-side interior angles. Prompt students to review the relationship of same-side interior angles when two parallel lines are cut by a transversal.

Example

Guide students in understanding the Example. Ask:

- *How is ∠6 related to ∠8?*
- *How is ∠8 related to ∠12?*
- *Why can you substitute m∠6 for m∠8?*

Help all students focus on the Example and responses to the questions by prompting them to check that their explanations are clear by pausing and asking classmates for questions or comments.

Look for understanding that corresponding angles and same-side interior angles formed when parallel lines are cut by a transversal are used to identify the relationship.

Apply It

① Students may solve the problem by writing an equation to relate corresponding angles and then using the equation to calculate the value of x. *DOK 1*

② Students may solve the problem by writing an equation to relate alternate interior angles. Let x represent the measure of one of the angles. Because alternate interior angles are congruent, the measure of the alternate interior angle to the angle measured x is also x. So, students can solve the equation $2x = 180$ to find the measure of each alternate interior angle, and then use other angle relationships to label other angles. *DOK 2*

③ **D** **is correct.** Students may solve the problem by writing an equation showing that the sum of the measures of the two labeled same-side exterior angles is 180°.

A is not correct. This is the result of setting the two given expressions equal to each other. The two labeled angles are not congruent, so the expressions are not equal.

B is not correct. This is the result of setting the expression $x + (2x - 36)$ equal to 90 rather than 180.

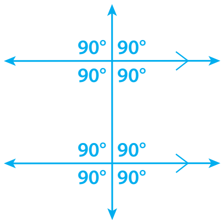
C is not correct. This is the result of setting the expression $x + (2x + 36)$ equal to 180.

DOK 3

LESSON 6 | SESSION 4

② Draw the lines described. Then label any angle measures that you can determine.

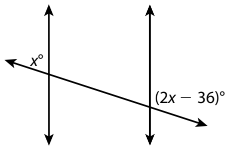
Two parallel lines are cut by a transversal. The alternate interior angles are supplementary.



CONSIDER THIS . . .
How are alternate interior angles related when formed by parallel lines cut by a transversal?

PAIR/SHARE
Suppose the alternate interior angles were congruent instead of supplementary. What angle measures can you calculate now?

③ The figure shows two lines cut by a transversal. Which value of x shows that the lines are parallel?



A $x = 36$

B $x = 42$

C $x = 48$

[✓ **D**] $x = 72$

Noah chose A as the correct answer. How might he have gotten that answer?

Possible answer: He may have thought that the angles would be congruent instead of supplementary when the lines are parallel. If so, he might have set the two expressions equal to each other instead of adding them and setting the sum equal to 180.

CONSIDER THIS . . .
What needs to be true about the angles labeled for the lines to be parallel?

PAIR/SHARE
If the lines are parallel, what are the measures of the eight angles formed where the transversal crosses?

134

GROUP & DIFFERENTIATE

Identify groupings for differentiation based on the **Start** and problems 1–3. A recommended sequence of activities for each group is suggested below. Use the resources on the next page to differentiate and close the lesson.

Approaching Proficiency

- **RETEACH** Hands-On Activity
- **REINFORCE** Problems 4, 5, 7

Meeting Proficiency

- **REINFORCE** Problems 4–9

Extending Beyond Proficiency

- **REINFORCE** Problems 4–9
- **EXTEND** Challenge

Have all students complete the **Close: Exit Ticket**.

Resources for Differentiation are found on the next page.

Refine Describing Angle Relationships

Apply It

④ **A** **is correct.** When two lines are cut by a transversal and corresponding angles are congruent, the lines are parallel.

D **is correct.** When two lines are cut by a transversal and alternate exterior angles are congruent, the lines are parallel.

F **is correct.** When two lines are cut by a transversal and same-side exterior angles are supplementary, the lines are parallel.

B is not correct. The same-side interior angles have a sum greater than 180°, so the lines are not parallel.

C is not correct. The corresponding angles are not congruent, so the lines are not parallel.

E is not correct. The alternate interior angles are not congruent, so the lines are not parallel.

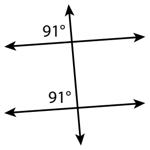
DOK 1

⑤ Students may first identify how each angle relates to ∠5. When two parallel lines are cut by a transversal, vertical angles, corresponding angles, and alternate interior angles are congruent. *DOK 2*

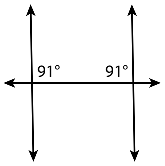
⑥ The lines are parallel when the labeled corresponding angles are congruent. So, students may solve the equation $6x + 9 = 63$ to find that the x value 9 makes the angle measures equal. *DOK 1*

④ Which figures show parallel lines? Select all that apply.

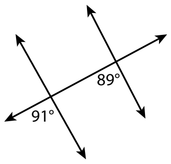
Ⓐ



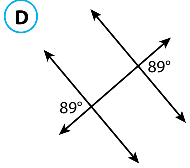
B



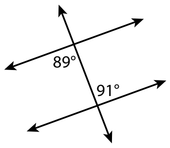
C



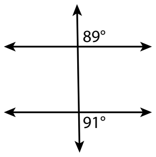
Ⓓ



E

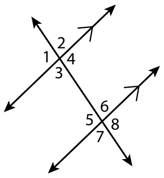


Ⓕ



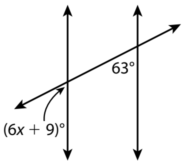
⑤ Which angles are congruent to ∠5? How do you know?

∠1, ∠4, ∠8; Possible answer: ∠8 ≅ ∠5 because they are vertical angles. The lines cut by the transversal are parallel, so ∠1 ≅ ∠5 because they are corresponding angles and ∠4 ≅ ∠5 because they are alternate interior angles.



⑥ The figure shows two lines cut by a transversal. Are the lines parallel if $x = 9$? Explain.

Yes; If $x = 9$, then $6x + 9 = 54 + 9 = 63$, which means the labeled angles are congruent. These angles are corresponding angles. When corresponding angles are congruent, the lines cut by the transversal are parallel.



135

DIFFERENTIATION

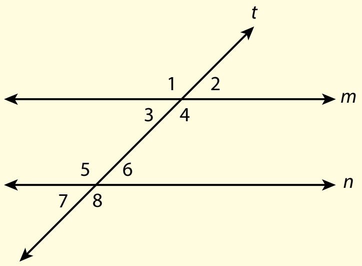
RETEACH

Hands-On Activity

Make a model to visualize the relationship between alternate interior angles and alternate exterior angles.

Students approaching proficiency with alternate interior angles and alternate exterior angles will benefit from drawing a pair of parallel lines cut by a transversal and identifying relationships.

Materials For each student: 2 colored pencils (1 blue, 1 red), centimeter ruler, scissors, tracing paper



- Have students use a centimeter ruler and tracing paper to draw two parallel lines cut by a transversal. Have them label the angles as shown in the diagram.
- Ask: *Which pairs of angles are alternate interior angles?* [∠3 and ∠6, ∠4 and ∠5]
- Have students color these angles blue.
- Ask: *Which pairs of angles are alternate exterior angles?* [∠1 and ∠8, ∠2 and ∠7]
- Have students color these angles red.
- Have students cut their paper along a horizontal line midway between line m and line n.
- Have students place line n on top of line m and rotate the page so that ∠8 is aligned with ∠1.
- Ask: *What do you notice about the two pairs of blue alternate interior angles?* [Alternate interior angles are congruent.]
- Ask: *What do you notice about the two pairs of red alternate exterior angles?* [Alternate exterior angles are congruent.]

Refine

⑦ *DOK 1*

⑧ Students can write and solve an equation relating the measures of the alternate interior angles. *DOK 2*

⑨ Students can use substitution to check their answers. *DOK 2*

CLOSE EXIT TICKET

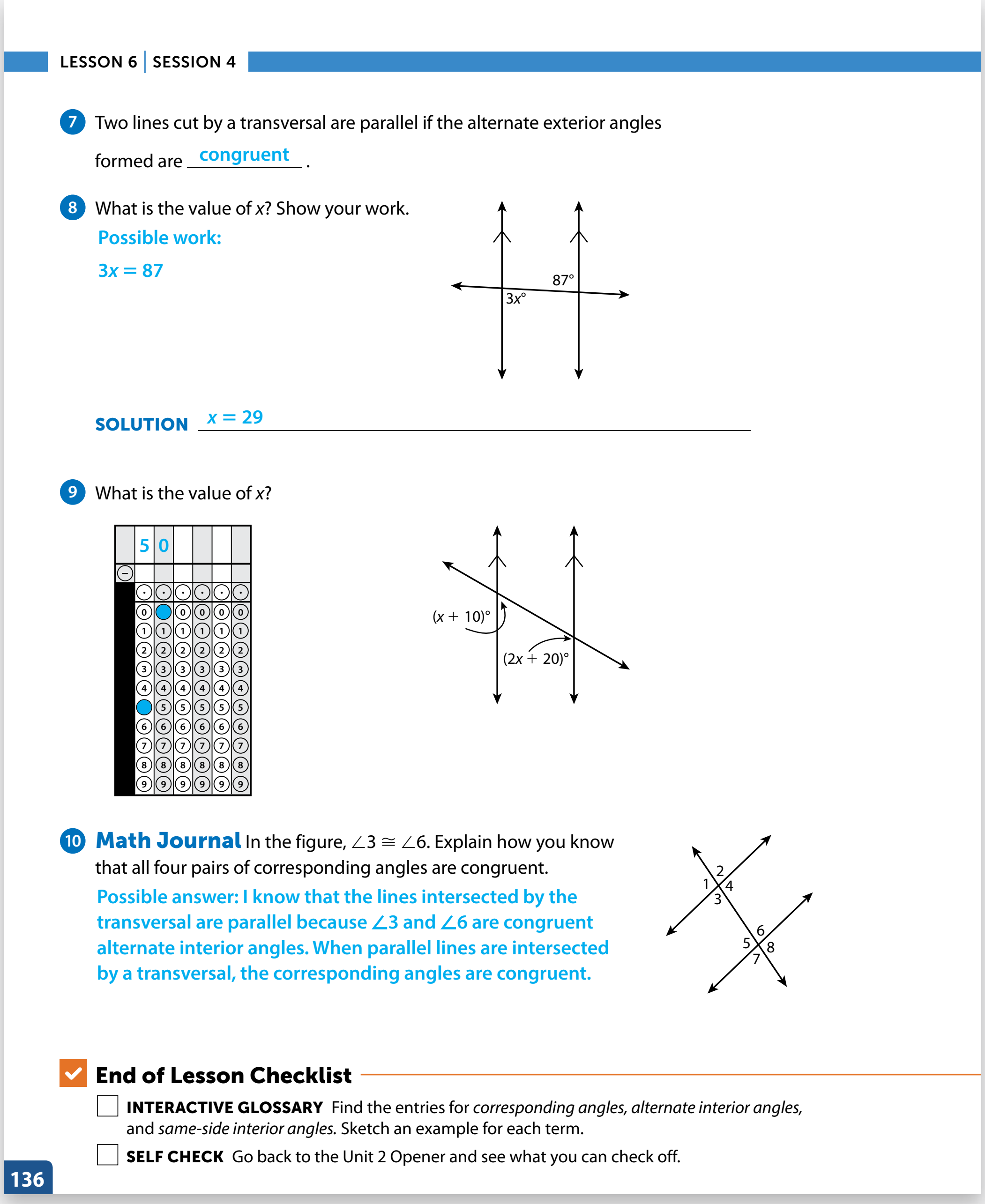
⑩ **Math Journal** Look for understanding that when two lines are cut by a transversal and one pair of alternate interior angles are congruent, then the lines are parallel. And, if the lines are parallel, all the pairs of corresponding angles are congruent.

Error Alert If students do not know that the lines cut by the transversal are parallel, then remind them the only way all four pairs of corresponding angles are congruent is if the two lines cut by the transversal are parallel.

✓ End of Lesson Checklist

INTERACTIVE GLOSSARY Support students by suggesting they draw parallel lines cut by a transversal for each term. On each diagram, encourage them to identify at least two examples for each angle relationship.

SELF CHECK Have students review and check off any new skills on the Unit 2 Opener.



LESSON 6 | SESSION 4

⑦ Two lines cut by a transversal are parallel if the alternate exterior angles formed are congruent.

⑧ What is the value of x? Show your work.

Possible work:

$3x = 87$

SOLUTION $x = 29$

⑨ What is the value of x?

⑩ **Math Journal** In the figure, ∠3 ≅ ∠6. Explain how you know that all four pairs of corresponding angles are congruent.

Possible answer: I know that the lines intersected by the transversal are parallel because ∠3 and ∠6 are congruent alternate interior angles. When parallel lines are intersected by a transversal, the corresponding angles are congruent.

✓ **End of Lesson Checklist**

☐ **INTERACTIVE GLOSSARY** Find the entries for *corresponding angles, alternate interior angles,* and *same-side interior angles.* Sketch an example for each term.

☐ **SELF CHECK** Go back to the Unit 2 Opener and see what you can check off.

136

REINFORCE

Problems 4–9

Solve angle relationship problems.

Students meeting proficiency will benefit from additional work with angles, formed when parallel lines are cut by a transversal by solving problems in a variety of formats.

- Have students work on their own or with a partner to solve the problems.
- Encourage students to show their work.

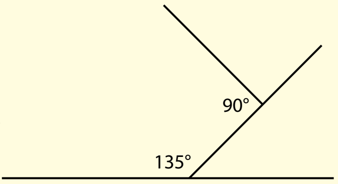
EXTEND

Challenge

Solve angle relationship problems in a geometric figure.

Students extending beyond proficiency will benefit from understanding how to form parallel lines.

- Have students work with a partner to solve this problem: *In the diagram, how many degrees must the upper line be rotated down so that the upper and lower lines are parallel?*



- Some students may find it helpful to draw a dashed line to show the upper line parallel to the lower line.

PERSONALIZE

i-Ready®

Provide students with opportunities to work on their personalized instruction path with *i-Ready* Online Instruction to:

- fill prerequisite gaps.
- build up grade-level skills.

LESSON 7

Overview | Describe Angle Relationships in Triangles

STANDARDS FOR MATHEMATICAL PRACTICE (SMP)

SMP 1, 2, 3, 4, 5, and 6 are integrated into the Try-Discuss-Connect routine.*

This lesson provides additional support for:

3 Construct viable arguments and critique the reasoning of others.

6 Attend to precision.

* See page 1o to learn how every lesson includes these SMP.

Objectives

Content Objectives

- Reason that the sum of the measures of the interior angles of a triangle is 180°.
- Reason that the sum of the measures of the exterior angles of a triangle is 360°.
- Reason that the measure of an exterior angle of a triangle is equal to the sum of the measures of the two nonadjacent interior angles.
- Reason that two triangles are similar if two pairs of corresponding angles are congruent.
- Solve problems by reasoning about angle relationships in triangles.

Language Objectives

- Use angle relationships to explain why the sum of the measures of the interior angles of a triangle is 180°.
- Explain the relationship of an exterior angle of a triangle to its adjacent interior angle.
- Communicate the steps to show that the measure of an exterior angle of a triangle is equivalent to the sum of the measures of the nonadjacent interior angles.
- Use an If/then statement to express the idea that two triangles are similar when two pairs of corresponding angles are congruent.
- Use lesson vocabulary involving angle relationships to explain solutions to problems involving geometric figures.

Prior Knowledge

- Understand angle relationships, such as vertical angles, linear pairs, corresponding angles, alternate interior angles, and same-side interior angles formed by parallel lines cut by a transversal.
- Combine like terms and solve equations with one unknown.

Vocabulary

Math Vocabulary

exterior angle when you extend one side of a polygon, the angle between the extended side and the adjacent side. This angle forms a linear pair with the adjacent interior angle of the polygon.

Review the following key terms.

linear pair two angles that are adjacent and supplementary.

similar triangles triangles that are scale drawings of one another. Similar triangles have the same shape but may have a different size.

Academic Vocabulary

nonadjacent not touching or being next to. Nonadjacent angles do not share a vertex or side.

related to connect one thing with one or more other things.

Learning Progression

Earlier in Grade 8, students learned that when parallel lines are cut by a transversal, the angles formed have certain relationships (any two are either congruent or supplementary). They used these relationships to find unknown angle measures. They learned about similarity and how to check whether two figures are similar.

➤

In this lesson, students discover relationships among the measures of the interior and exterior angles of a triangle. They learn how to use angle measures to determine whether two triangles are similar.

➤

Later in Grade 8, students will use similar triangles to understand slope.

In later grades, students will study similar triangles in greater depth and use this understanding to learn about trigonometric ratios.

LESSON 7

Overview

Pacing Guide

Items marked with ↖ are available on the **Teacher Toolbox**.

	MATERIALS	DIFFERENTIATION
SESSION 1 **Explore** The Sum of the Angle Measures in a Triangle (35–50 min)		
• **Start** (5 min) • **Try It** (5–10 min) • **Discuss It** (10–15 min) • **Connect It** (10–15 min) • **Close: Exit Ticket** (5 min) **Additional Practice** (pages 141–142)	**Math Toolkit** grid paper, straightedges Presentation Slides ↖	**PREPARE** Interactive Tutorial ↖ **RETEACH or REINFORCE** Hands-On Activity **Materials** For each student: construction paper, scissors, straightedge
SESSION 2 **Develop** Describing the Exterior Angles of a Triangle (45–60 min)		
• **Start** (5 min) • **Try It** (10–15 min) • **Discuss It** (10–15 min) • **Connect It** (15–20 min) • **Close: Exit Ticket** (5 min) **Additional Practice** (pages 147–148)	Presentation Slides ↖	**RETEACH or REINFORCE** Hands-On Activity **Materials** For each student: scissors, straightedge **REINFORCE** Fluency & Skills Practice ↖ **EXTEND** Deepen Understanding
SESSION 3 **Develop** Using Angles to Determine Similar Triangles (45–60 min)		
• **Start** (5 min) • **Try It** (10–15 min) • **Discuss It** (10–15 min) • **Connect It** (15–20 min) • **Close: Exit Ticket** (5 min) **Additional Practice** (pages 153–154)	**Math Toolkit** grid paper, protractors, rulers Presentation Slides ↖	**RETEACH or REINFORCE** Hands-On Activity **Materials** For each student: protractor, scissors, straightedge **REINFORCE** Fluency & Skills Practice ↖ **EXTEND** Deepen Understanding
SESSION 4 **Refine** Describing Angle Relationships in Triangles (45–60 min)		
• **Start** (5 min) • **Monitor & Guide** (15–20 min) • **Group & Differentiate** (20–30 min) • **Close: Exit Ticket** (5 min)	**Math Toolkit** Have items from previous sessions available for students. Presentation Slides ↖	**RETEACH** Hands-On Activity **Materials** For each student: scissors, straightedge, tracing paper **REINFORCE** Problems 4–9 **EXTEND** Challenge **PERSONALIZE** i-Ready
Lesson 7 Quiz ↖ or **Digital Comprehension Check**		**RETEACH** Tools for Instruction ↖ **REINFORCE** Math Center Activity ↖ **EXTEND** Enrichment Activity ↖

LESSON 7

Overview | Describe Angle Relationships in Triangles

Connect to Culture

➤ **Use these activities to connect with and leverage the diverse backgrounds and experiences of all students. Engage students in sharing what they know about contexts before you add the information given here.**

SESSION 1 ■ □ □ □

Try It Ask students to think about roofs. Many of them are slanted. Do they know why? The slope of a roof has a direct impact on how well rainwater drains off of it. There are recommended materials for coverings of roofs based on the slope of the roof. For example, steeper roofs can be covered with asphalt shingles, while flatter roofs can be made of concrete. Many concrete roofs have drainage systems associated with them to remove standing water. Some parts of the world have thatched roofs. Thatching uses straw or other vegetation to cover a roof. This method has a surprisingly long lifespan. Ask students to tell about the steepness of roofs on homes and buildings in their area. Have students describe unique roof structures they have seen.

SESSION 2 ■ ■ □ □

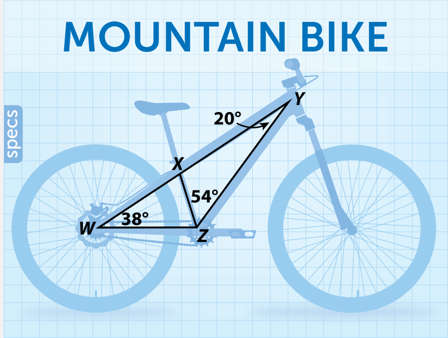
Try It Invite students to tell the class about buildings and bridges they have seen with triangular features. The Aula Medica conference center, located on the Karolinska Institute's Solna Campus in Sweden, holds a 1,000-seat auditorium and approximately 90 offices. Its triangular building elements, along with insulated glass panels and leaning walls, make it an extremely energy-efficient building. The facility is used for many scientific conferences, gala receptions, and Nobel lectures.

SESSION 3 ■ ■ ■ □

Apply It **Problem 9** Invite students to share their experiences with ironing or ironing boards. Ironing boards have been around since the 9th century, when Vikings would press their clothes into shape using heated rocks. In the 19th century, irons were cast-iron devices that were heated on a stove, but most people used their kitchen table or another flat surface, perhaps padded with a blanket, as an ironing surface. Then, in 1892, an African American woman named Sarah Boone patented an adjustable folding ironing board with a narrow, curved shape to make it easier to iron shirts. Modern ironing boards look a lot like her design.

SESSION 4 ■ ■ ■ ■

Apply It **Problem 4** Ask students why they think bicycle frames are designed with triangles. When you look at a bicycle frame, you will often see two triangles positioned with a side in common to form a diamond. This design has been around since the 1880s. Triangles are a very strong design element because it is very difficult to make a triangle distort or collapse. A triangle distributes force efficiently to all of its sides. Encourage students to describe other structures they have seen that use triangles.

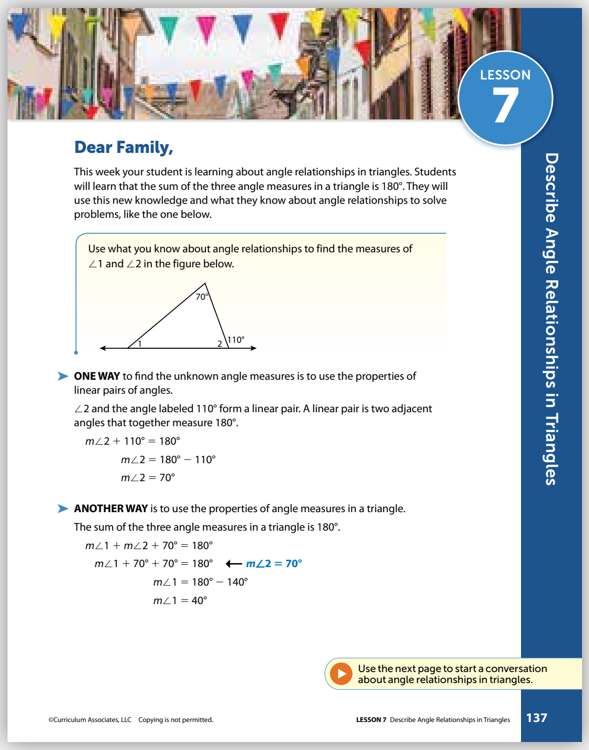


LESSON 7

Overview

Connect to Family and Community

After the Explore session, have students use the Family Letter to let their families know what they are learning and to encourage family involvement.



LESSON 7

Describe Angle Relationships in Triangles

Dear Family,

This week your student is learning about angle relationships in triangles. Students will learn that the sum of the three angle measures in a triangle is 180°. They will use this new knowledge and what they know about angle relationships to solve problems, like the one below.

Use what you know about angle relationships to find the measures of ∠1 and ∠2 in the figure below.

ONE WAY to find the unknown angle measures is to use the properties of linear pairs of angles.

∠2 and the angle labeled 110° form a linear pair. A linear pair is two adjacent angles that together measure 180°.

$$m\angle 2 + 110° = 180°$$
$$m\angle 2 = 180° - 110°$$
$$m\angle 2 = 70°$$

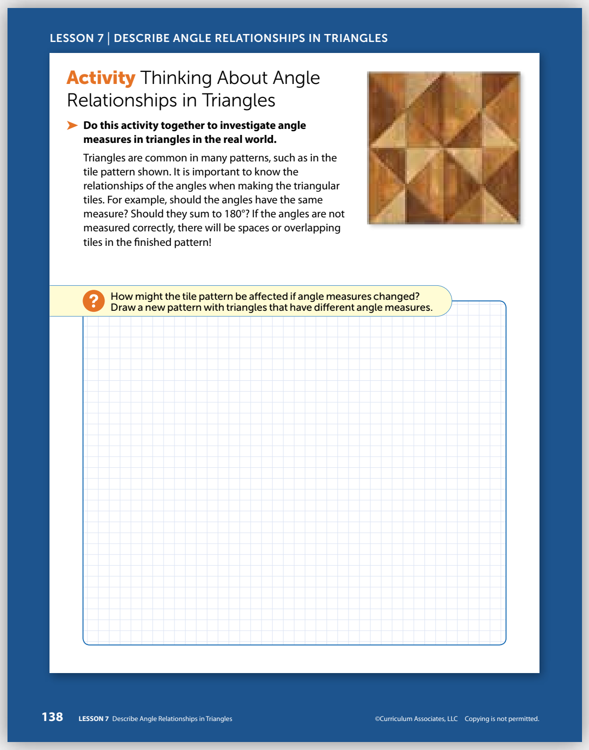
ANOTHER WAY is to use the properties of angle measures in a triangle.

The sum of the three angle measures in a triangle is 180°.

$$m\angle 1 + m\angle 2 + 70° = 180°$$
$$m\angle 1 + 70° + 70° = 180° \longleftarrow m\angle 2 = 70°$$
$$m\angle 1 = 180° - 140°$$
$$m\angle 1 = 40°$$

Use the next page to start a conversation about angle relationships in triangles.

©Curriculum Associates, LLC Copying is not permitted. **LESSON 7** Describe Angle Relationships in Triangles 137



LESSON 7 | DESCRIBE ANGLE RELATIONSHIPS IN TRIANGLES

Activity Thinking About Angle Relationships in Triangles

Do this activity together to investigate angle measures in triangles in the real world.

Triangles are common in many patterns, such as in the tile pattern shown. It is important to know the relationships of the angles when making the triangular tiles. For example, should the angles have the same measure? Should they sum to 180°? If the angles are not measured correctly, there will be spaces or overlapping tiles in the finished pattern!

How might the tile pattern be affected if angle measures changed? Draw a new pattern with triangles that have different angle measures.

138 **LESSON 7** Describe Angle Relationships in Triangles ©Curriculum Associates, LLC Copying is not permitted.

Connect to Language

For English language learners, use the Differentiation chart to scaffold the language in each session. Use the Academic Language routine for academic terms before Session 1.

DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Use with **Session 1** **Connect It**

Levels 1–3: Reading/Speaking

Help students interpret and discuss Connect It problem 3. Read the problem aloud as students follow along. Ask a volunteer to state the sum of the angles in problem 2c. Ask another volunteer to state the sum they found in Try It. Then have partners draw a triangle and measure its angles. Encourage them to use math terms, such as *measure* and *result* as they work to verify the conjecture. Allow time for partners to work before sharing with the group. Call on partners to share the sum of the measures of the angles of their triangles and complete this sentence frame: *The sum of the measures of the angles of my triangle is* _____. Ask: *Does the size and shape of the triangle change the sum?*

Levels 2–4: Reading/Speaking

Prepare students to discuss and solve Connect It problem 3. Read the problem with students. Ask students to turn to a partner to state the measures of the angles in problem 2c and in Try It. Have them discuss the methods they used in solving each of those problems and predict whether the sum of the angle measures for any triangle is 180°.

Have partners work together to draw another triangle to test the conjecture that the sum of the measures of the angles will be 180°. Provide a protractor to measure the angles.

Call on partners to share their answers, encouraging them to use precise language and complete sentences. Have them explain whether the size and shape of the triangle affect the sum.

Levels 3–5: Reading/Speaking

Support students to discuss and solve Connect It problem 3. Have students read the problem with a partner. Have them share their answers for problem 2c and for the Try It problem. Have them each draw another triangle to test their answer and share with their partner the method they used. Encourage one partner to draw an obtuse triangle and the other to draw an acute triangle.

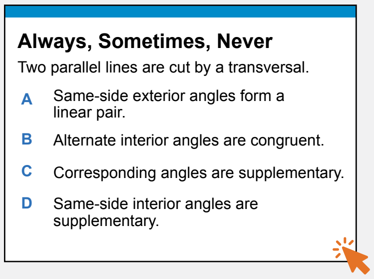
Have them use a two-person version of **Stronger and Clearer Each Time**. Partners should share their method of calculations orally with their partner and give each other feedback about what would make the response stronger or clearer. Encourage them to use *shape* and *size* in their explanations.

Explore The Sum of the Angle Measures in a Triangle

Purpose

- **Explore** the idea that angle relationships learned in the previous lesson can be used to determine how the angle measures of a triangle are related.
- **Understand** that the sum of the angle measures of a triangle is 180°.

START CONNECT TO PRIOR KNOWLEDGE



Always, Sometimes, Never

Two parallel lines are cut by a transversal.

A Same-side exterior angles form a linear pair.

B Alternate interior angles are congruent.

C Corresponding angles are supplementary.

D Same-side interior angles are supplementary.

Possible Solution

A is never true.

B is always true.

C is sometimes true.

D is always true.

WHY? Support students' understanding of angle relationships.

TRY IT

SMP 1, 2, 4, 5, 6

Make Sense of the Problem

See ***Connect to Culture*** to support student engagement. Before students work on Try It, use **Notice and Wonder** to help them make sense of the problem. Students should understand that they need to determine the measures of angles 2, 4, and 5 and then see that they can add those measures together to answer the question.

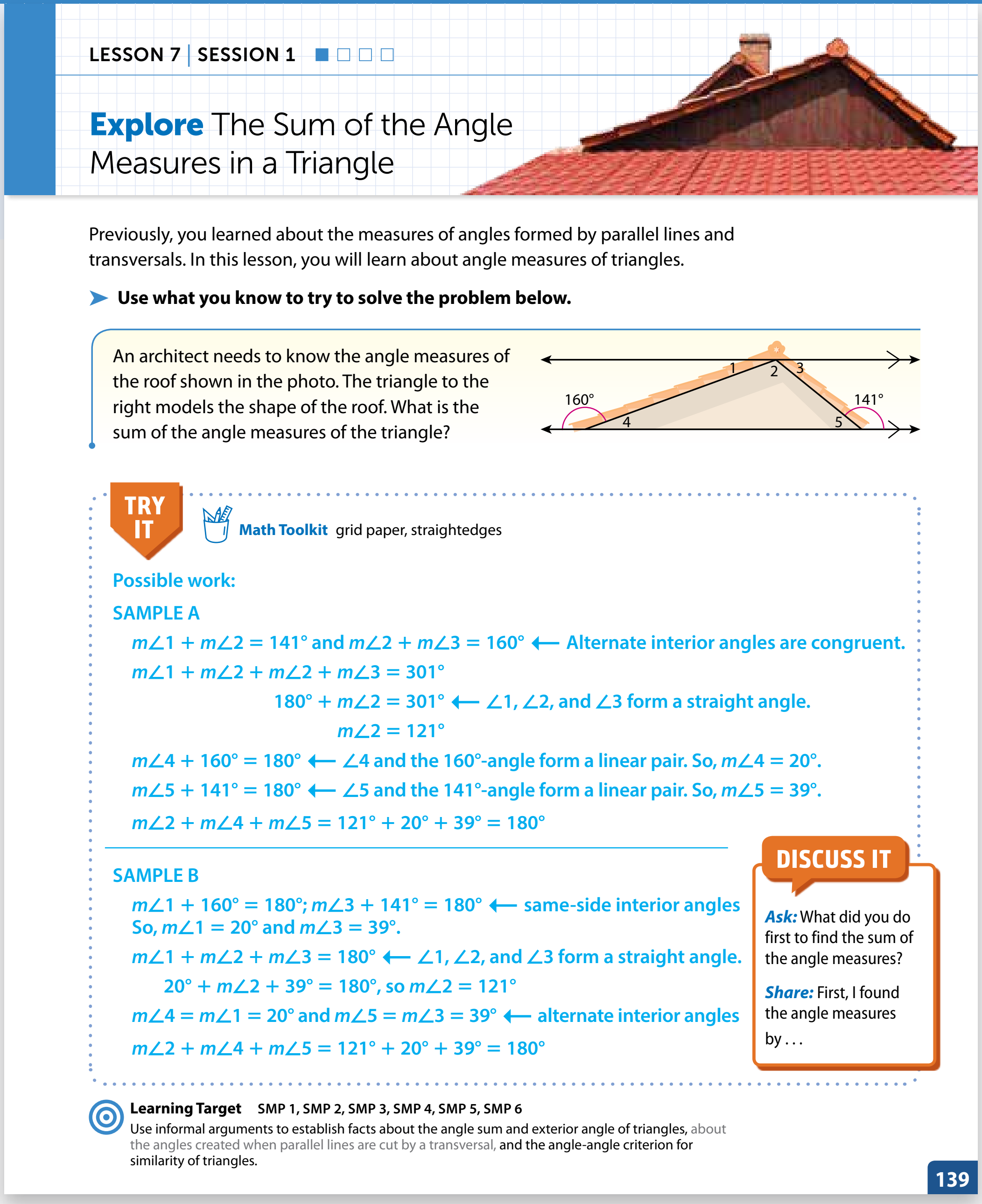
DISCUSS IT

SMP 2, 3, 6

Support Partner Discussion

After students work on Try It, have them respond to Discuss It with a partner. Listen for understanding of:

- using alternate interior angles and/or same-side interior angles to find measures of certain angles.
- using supplementary angle relationships to find measures of certain angles.
- substituting known angle measures into equations to find measures of unknown angles.



LESSON 7 | SESSION 1 ■ □ □ □

Explore The Sum of the Angle Measures in a Triangle

Previously, you learned about the measures of angles formed by parallel lines and transversals. In this lesson, you will learn about angle measures of triangles.

➤ **Use what you know to try to solve the problem below.**

An architect needs to know the angle measures of the roof shown in the photo. The triangle to the right models the shape of the roof. What is the sum of the angle measures of the triangle?

TRY IT **Math Toolkit** grid paper, straightedges

Possible work:

SAMPLE A

$m\angle 1 + m\angle 2 = 141^\circ$ and $m\angle 2 + m\angle 3 = 160^\circ$ ⟵ Alternate interior angles are congruent.

$m\angle 1 + m\angle 2 + m\angle 2 + m\angle 3 = 301^\circ$

$180^\circ + m\angle 2 = 301^\circ$ ⟵ $\angle 1$, $\angle 2$, and $\angle 3$ form a straight angle.

$m\angle 2 = 121^\circ$

$m\angle 4 + 160^\circ = 180^\circ$ ⟵ $\angle 4$ and the 160°-angle form a linear pair. So, $m\angle 4 = 20^\circ$.

$m\angle 5 + 141^\circ = 180^\circ$ ⟵ $\angle 5$ and the 141°-angle form a linear pair. So, $m\angle 5 = 39^\circ$.

$m\angle 2 + m\angle 4 + m\angle 5 = 121^\circ + 20^\circ + 39^\circ = 180^\circ$

SAMPLE B

$m\angle 1 + 160^\circ = 180^\circ$; $m\angle 3 + 141^\circ = 180^\circ$ ⟵ same-side interior angles So, $m\angle 1 = 20^\circ$ and $m\angle 3 = 39^\circ$.

$m\angle 1 + m\angle 2 + m\angle 3 = 180^\circ$ ⟵ $\angle 1$, $\angle 2$, and $\angle 3$ form a straight angle.

$20^\circ + m\angle 2 + 39^\circ = 180^\circ$, so $m\angle 2 = 121^\circ$

$m\angle 4 = m\angle 1 = 20^\circ$ and $m\angle 5 = m\angle 3 = 39^\circ$ ⟵ alternate interior angles

$m\angle 2 + m\angle 4 + m\angle 5 = 121^\circ + 20^\circ + 39^\circ = 180^\circ$

DISCUSS IT

Ask: What did you do first to find the sum of the angle measures?

Share: First, I found the angle measures by . . .

Learning Target SMP 1, SMP 2, SMP 3, SMP 4, SMP 5, SMP 6

Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles.

139

Common Misconception Listen for students who identify $\angle 3$ and the angle labeled 160° as alternate interior angles and think $m\angle 3$ is 160°. Point out that $\angle 3$ is acute and therefore cannot measure 160°. As students share their strategies, help them see that the larger angle made up of $\angle 2$ and $\angle 3$ and the 160°-angle are alternate interior angles.

Select and Sequence Student Strategies

Select 2–3 samples that represent the range of student thinking in your classroom. Here is one possible order for class discussion:

- strategies that start by using alternate interior angles to find that $m\angle 1 + m\angle 2 = 141^\circ$ and $m\angle 2 + m\angle 3 = 160^\circ$
- (misconception) strategies that misidentify alternate interior angles
- strategies that start by using same side interior angles to find $m\angle 1$ and $m\angle 3$
- strategies that use alternate interior angles to establish that $m\angle 1 = m\angle 4$ and $m\angle 3 = m\angle 5$ and then use the fact that $\angle 1$, $\angle 2$, and $\angle 3$ form a straight angle

Facilitate Whole Class Discussion

Call on students to share selected strategies. Before strategies are presented and discussed, remind students to be respectful when they disagree with another's idea.

Guide students to **Compare and Connect** the representations. Encourage students to speak clearly and loudly as they present their responses.

> **ASK** *How did the strategies use angle relationships related to parallel lines?*
>
> **LISTEN FOR** The strategies used pairs of angles that were either congruent or supplementary, along with combinations of angles that form a linear pair to find the unknown measures.

CONNECT IT

SMP 2, 4, 5

① **Look Back** Look for understanding that the measures of the interior angles of a triangle sum to 180°.

DIFFERENTIATION | RETEACH or REINFORCE

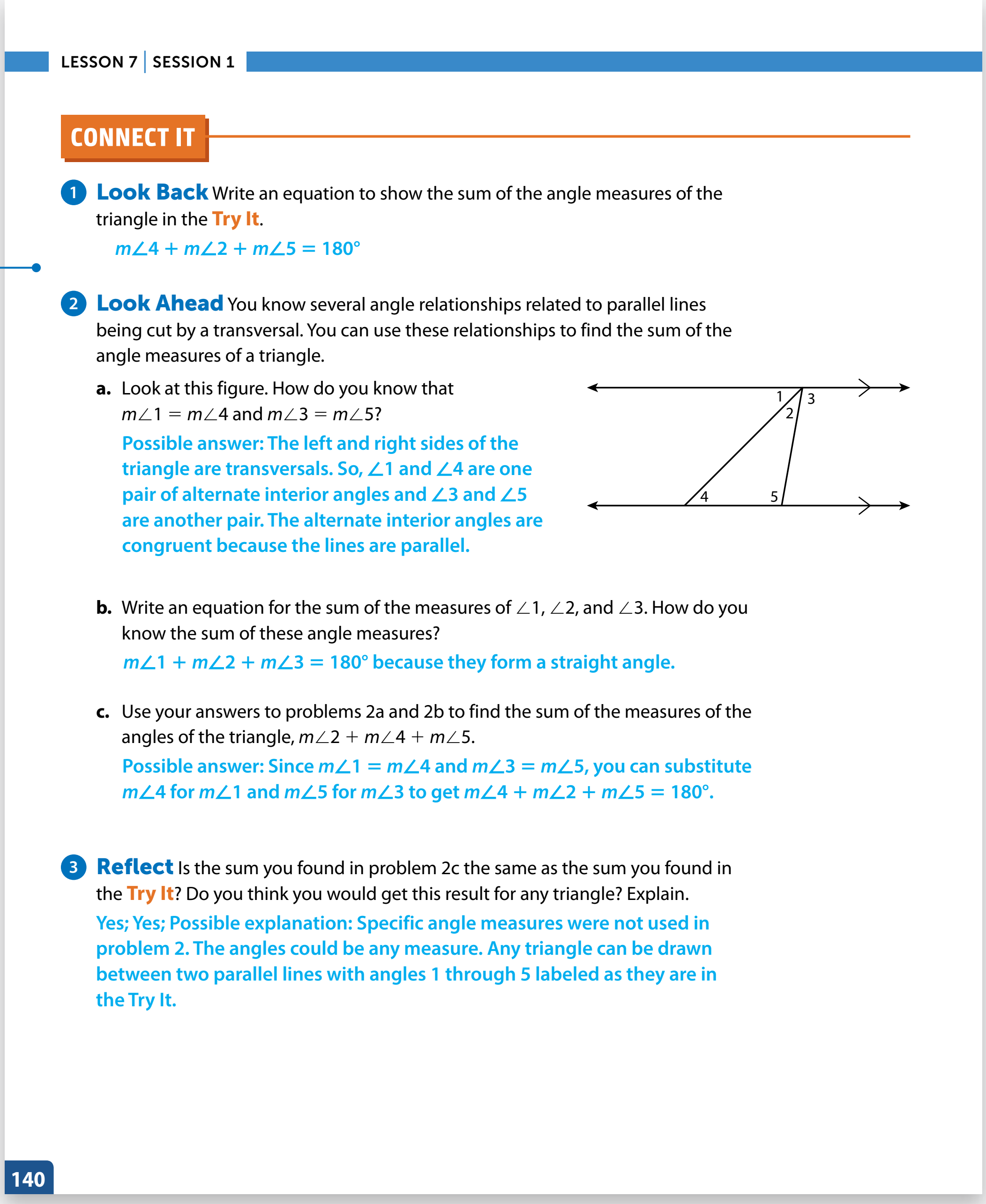
Hands-On Activity

View the angles of a triangle as a straight angle.

If students are unsure about the sum of the angle measures of a triangle, then use this activity to help them visualize the sum.

Materials For each student: construction paper, scissors, straightedge

- Instruct students to draw and cut out a triangle. Have students shade each interior angle.
- Have students cut off the three angles of their triangles.
- Tell students to place the three angles side-by-side with the vertices of the angles all at the same point. The sides of the angles should touch and not overlap.
- Ask: *What do you notice about the three angles?* [They form a straight angle.] *What does this tell you about the sum of the angle measures?* [It is 180°.]
- Have students compare their results with others'. Ask: *Did anyone have a triangle for which the three angles did not form a straight angle?* [No.]
- Ask: *How does this confirm that the sum of the angle measures for any triangle is 180°?* [There were no specific measures used to make the angles of the triangle. Yet, the three angles of every triangle formed a straight angle.]



LESSON 7 | SESSION 1

CONNECT IT

① **Look Back** Write an equation to show the sum of the angle measures of the triangle in the **Try It**.

$m\angle 4 + m\angle 2 + m\angle 5 = 180°$

② **Look Ahead** You know several angle relationships related to parallel lines being cut by a transversal. You can use these relationships to find the sum of the angle measures of a triangle.

a. Look at this figure. How do you know that $m\angle 1 = m\angle 4$ and $m\angle 3 = m\angle 5$?

Possible answer: The left and right sides of the triangle are transversals. So, ∠1 and ∠4 are one pair of alternate interior angles and ∠3 and ∠5 are another pair. The alternate interior angles are congruent because the lines are parallel.

b. Write an equation for the sum of the measures of ∠1, ∠2, and ∠3. How do you know the sum of these angle measures?

$m\angle 1 + m\angle 2 + m\angle 3 = 180°$ because they form a straight angle.

c. Use your answers to problems 2a and 2b to find the sum of the measures of the angles of the triangle, $m\angle 2 + m\angle 4 + m\angle 5$.

Possible answer: Since $m\angle 1 = m\angle 4$ and $m\angle 3 = m\angle 5$, you can substitute $m\angle 4$ for $m\angle 1$ and $m\angle 5$ for $m\angle 3$ to get $m\angle 4 + m\angle 2 + m\angle 5 = 180°$.

③ **Reflect** Is the sum you found in problem 2c the same as the sum you found in the **Try It**? Do you think you would get this result for any triangle? Explain.

Yes; Yes; Possible explanation: Specific angle measures were not used in problem 2. The angles could be any measure. Any triangle can be drawn between two parallel lines with angles 1 through 5 labeled as they are in the Try It.

140

② **Look Ahead** Point out that there are two transversals that intersect the parallel lines. Students should recognize that ∠1 and ∠4 are alternate interior angles and are congruent, and ∠3 and ∠5 are alternate interior angles and are congruent. Students write an equation showing that the sum of the measures of the three angles that form the straight angle (∠1, ∠2, and∠3) is 180° and then use substitution to find that the sum of the measures of the interior angles of the triangle is also 180°.

CLOSE **EXIT TICKET**

③ **Reflect** Look for understanding of the fact that the angle sum is the same regardless of the size or shape of the triangle.

Common Misconception If students do not believe that the sum of the measures of the interior angles of a triangle is 180°, then challenge them to come up with a counterexample. They can confirm each triangle they draw has a sum of 180° by using a protractor or by cutting the angles out and arranging them to form a straight angle. Once they realize they cannot produce a counterexample, then they should understand the concept.

Prepare for Angle Relationships in Triangles

Support Vocabulary Development

Assign **Prepare for Angle Relationships in Triangles** as extra practice in class or as homework.

If you have students complete this in class, then use the guidance below.

Ask students to consider the term *similar triangles*. Discuss the graphic organizer. Be sure students understand that they should define the term in their own words and include an illustration of the concept. They should include at least one example of triangles that are similar and one non-example, showing triangles that are not similar.

Have students work individually to complete the graphic organizer. Invite students to share their completed organizers, and prompt a whole-class comparative discussion of why the triangles in students' examples are similar and why the triangles in students' non-examples are not similar triangles.

Have students look at the diagram in problem 2 and discuss with a partner how to determine whether the two triangles that form the entire diagram are similar. Students should identify pairs of corresponding sides and understand how to use quotients to determine whether all pairs of corresponding sides are proportional.

Problem Notes

① Students should understand that similar triangles have the same shape, but not necessarily the same size. If two triangles are similar, then one can be mapped onto the other by a sequence of transformations. Student responses might include pairs of corresponding angles having the same measure and/or all pairs of corresponding sides being proportional.

② Students should recognize that the triangles are similar because each of the three angles in one triangle is congruent to a corresponding angle in the second triangle and because the quotients of the corresponding side lengths are equivalent.

LESSON 7 | SESSION 1 Name:

Prepare for Angle Relationships in Triangles

① Think about what you know about similarity and similar triangles. Fill in each box. Use words, numbers, and pictures. Show as many ideas as you can.

Possible answers:

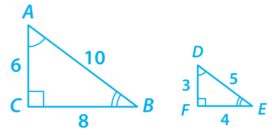
similar triangles

In My Own Words

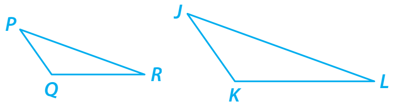
Triangles are similar if they have the same shape. They could be the same size or different sizes. If two triangles are similar, then one can be mapped onto the other by a sequence of transformations.

My Illustrations

$\triangle ABC \sim \triangle DEF$



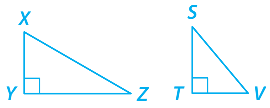
Examples



$\angle J \cong \angle P$ $\angle K \cong \angle Q$ $\angle L \cong \angle R$

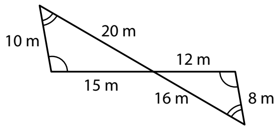
$\frac{JK}{PQ} = \frac{KL}{QR} = \frac{JL}{PR}$ $\triangle JKL \sim \triangle PQR$

Non-Examples



$\angle Y \cong \angle T$, but $\triangle XYZ$ is not similar to $\triangle STV$ because the other pairs of corresponding angles are not congruent.

② Are the triangles similar? Explain.



Yes; Corresponding angle measures are the same and the quotients of corresponding side lengths are equal.

$\frac{10}{8} = \frac{15}{12} = \frac{20}{16}$

REAL-WORLD CONNECTION

Construction installers take designs from architects and engineers and build physical structures. In various situations, these workers use the fact that the sum of the angle measures in a triangle is 180°. For example, suppose a support beam for a building rests against the building at the top and is a certain distance away from the building at the bottom. Suppose the installer wanted to know the angle at the top where the beam meets the building but cannot reach the angle to measure it. He could measure the two angles on the ground and then calculate the measure of the angle at the top. Ask students to think of other real-world examples when the fact that the sum of the angle measures of a triangle is 180˚ might be useful.

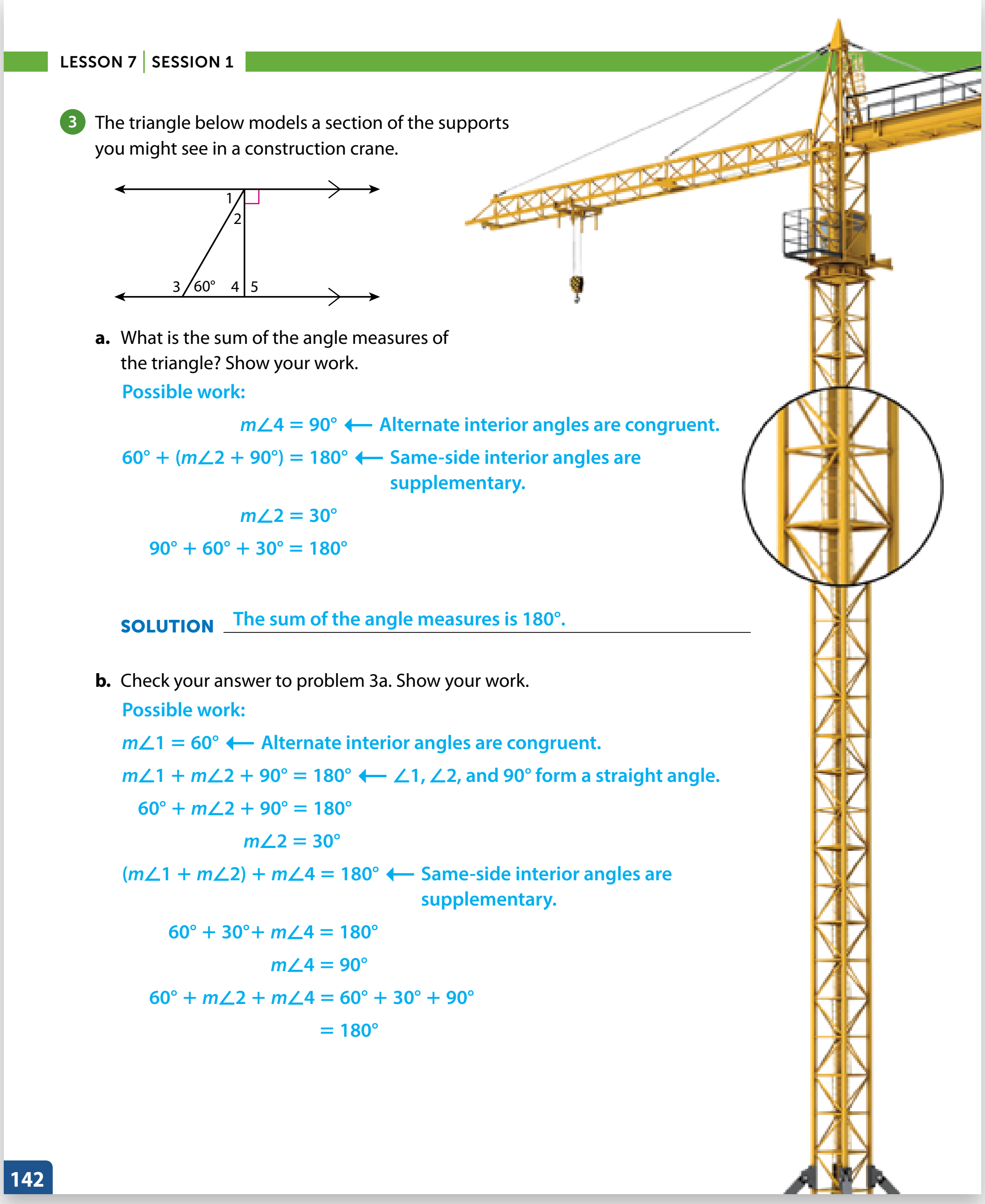
Additional Practice

③ Problem 3 provides another look at the sum of the angle measures in a triangle. This problem is similar to the problem about the architect needing to know the angle measures of a roof. In both problems, students use known angle measures and angle relationships to find other unknown angle measures. Then they find the sum of the angle measures in the triangle. This problem asks for the sum of the angle measures of a triangle that represents a support section of a crane.

Students may want to use grid paper and a straightedge to solve.

Suggest that students use **Three Reads**, asking themselves one of the following questions each time:

- *What is this problem about?*
- *What is the question I am trying to answer?*
- *What information is important?*



LESSON 7 | SESSION 1

③ The triangle below models a section of the supports you might see in a construction crane.

a. What is the sum of the angle measures of the triangle? Show your work.

Possible work:

$m\angle 4 = 90^\circ$ ← **Alternate interior angles are congruent.**

$60^\circ + (m\angle 2 + 90^\circ) = 180^\circ$ ← **Same-side interior angles are supplementary.**

$m\angle 2 = 30^\circ$

$90^\circ + 60^\circ + 30^\circ = 180^\circ$

SOLUTION The sum of the angle measures is 180°.

b. Check your answer to problem 3a. Show your work.

Possible work:

$m\angle 1 = 60^\circ$ ← **Alternate interior angles are congruent.**

$m\angle 1 + m\angle 2 + 90^\circ = 180^\circ$ ← **∠1, ∠2, and 90° form a straight angle.**

$60^\circ + m\angle 2 + 90^\circ = 180^\circ$

$m\angle 2 = 30^\circ$

$(m\angle 1 + m\angle 2) + m\angle 4 = 180^\circ$ ← **Same-side interior angles are supplementary.**

$60^\circ + 30^\circ + m\angle 4 = 180^\circ$

$m\angle 4 = 90^\circ$

$60^\circ + m\angle 2 + m\angle 4 = 60^\circ + 30^\circ + 90^\circ$

$= 180^\circ$

142

DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Use with **Session 2** **Connect It**

ACADEMIC VOCABULARY

To *extend* means to make something longer than it was.

Levels 1–3: Listening/Speaking

Help students interpret problem 1. Draw a triangle and extend one side. Have students use *extend* to describe what you did. Point to the two angles you formed and ask: *What do you know about these two angles?* Guide them to use *linear pairs* in their answers. Continue by extending the other side as shown in the second triangle. Help students explain why the two exterior angles at a vertex are congruent:

- *Each exterior angle forms ____.*
- *The interior angle is part of both ____. So the exterior angles are ____.*

Levels 2–4: Listening/Speaking

Prepare students to solve problem 1. Have students look at the diagrams and tell what they notice. Guide them to notice how the sides of the triangles extend. Read the problem aloud. Have students turn and talk to exchange ideas and to identify the measures of the interior angle, the straight angle, and the missing exterior angle. Then have them tell why the two exterior angles are congruent.

- *I can make two exterior angles by ____.*
- *The two exterior angles are congruent because ____.*

Levels 3–5: Listening/Speaking

Have students turn to a partner to read and discuss Connect It problem 1. Have them make connections between the exterior angles at the same vertex using the diagrams. Ask: *How were the two exterior angles formed?* Encourage students to use *extended* to answer.

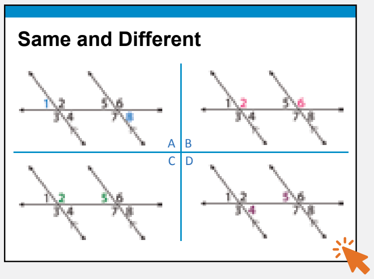
Encourage students to engage in the conversation by restating ideas, asking clarifying questions, and adding on to what their partner has said. For example, if a student says the exterior angle measures 80°, a partner might add: *That's correct because the interior angle measured 100° and both angles form a straight angle of 180°.*

Develop Describing the Exterior Angles of a Triangle

Purpose

- **Develop** strategies for describing the measure of an exterior angle of a triangle in terms of the measures of the interior angles and for finding the sum of the measures of the exterior angles.
- **Recognize** that the measure of an exterior angle of a triangle is equal to the sum of the measures of the two nonadjacent interior angles, and the sum of the measures of the exterior angles of a triangle is 360°.

START CONNECT TO PRIOR KNOWLEDGE



Possible Solutions

A, B, and D show pairs of congruent angles.

C shows a pair of supplementary angles.

C and D show pairs of interior angles.

A and B each show at least one exterior angle.

WHY? Support students' facility with recognizing angle relationships created by parallel lines cut by a transversal.

DEVELOP ACADEMIC LANGUAGE

WHY? Unpack the meaning of dense sentences.

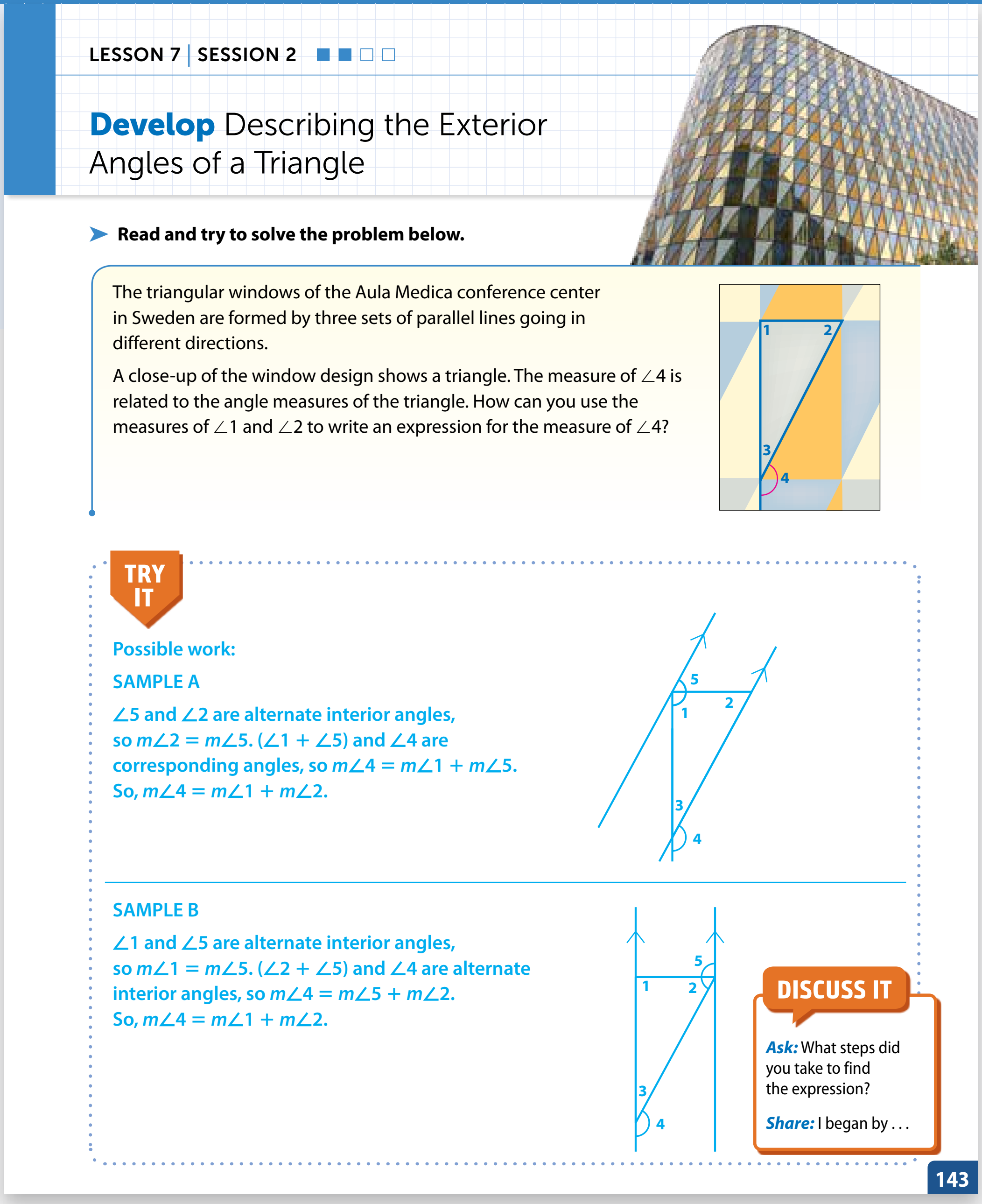
HOW? Display this sentence from Apply It problem 7: *Then/ use what you know/ about the interior angles of a triangle/ to show that/ the sum of the measures of the exterior angles of a triangle, / one at each vertex, / is 360°.* Use questions to help students unpack the sentence, for example: *What do you need to do? What are you going to show?* Have students turn and talk to rephrase the sentence.

TRY IT

SMP 1, 2, 4, 5, 6

Make Sense of the Problem

See ***Connect to Culture*** to support student engagement. Before students work on Try It, use **Notice and Wonder** to help them make sense of the problem. Emphasize the notice and wonder questions that are relevant to the problem.



LESSON 7 | SESSION 2 ■ ■ □ □

Develop Describing the Exterior Angles of a Triangle

➤ **Read and try to solve the problem below.**

The triangular windows of the Aula Medica conference center in Sweden are formed by three sets of parallel lines going in different directions.

A close-up of the window design shows a triangle. The measure of $\angle 4$ is related to the angle measures of the triangle. How can you use the measures of $\angle 1$ and $\angle 2$ to write an expression for the measure of $\angle 4$?

TRY IT

Possible work:

SAMPLE A

$\angle 5$ and $\angle 2$ are alternate interior angles, so $m\angle 2 = m\angle 5$. ($\angle 1 + \angle 5$) and $\angle 4$ are corresponding angles, so $m\angle 4 = m\angle 1 + m\angle 5$. So, $m\angle 4 = m\angle 1 + m\angle 2$.

SAMPLE B

$\angle 1$ and $\angle 5$ are alternate interior angles, so $m\angle 1 = m\angle 5$. ($\angle 2 + \angle 5$) and $\angle 4$ are alternate interior angles, so $m\angle 4 = m\angle 5 + m\angle 2$. So, $m\angle 4 = m\angle 1 + m\angle 2$.

DISCUSS IT

Ask: What steps did you take to find the expression?

Share: I began by . . .

143

DISCUSS IT

SMP 2, 3, 6

Support Partner Discussion

After students work on Try It, have them respond to Discuss It with a partner. If students need support in getting started, prompt them to ask each other questions such as:

- *What types of angle relationships are created by parallel lines?*
- *Did your strategy use any angles that are not labeled in the picture?*
- *What angles did you know right away were congruent to $\angle 4$?*

Common Misconception Listen for students who do not think $\angle 1$ and $\angle 2$ relate to $\angle 4$ because $\angle 4$ is not within the triangle. As students share their strategies, point out that they can use what they know about the angles formed when parallel lines are cut by a transversal to see how $\angle 4$ can relate to $\angle 1$ and $\angle 2$.

Select and Sequence Student Strategies

Select 2–3 samples that represent the range of student thinking in your classroom. Here is one possible order for class discussion:

- alternate interior angle relationship between ∠2 and an unmarked angle used to connect with ∠4
- (misconception) angles outside the triangle have no relationship
- alternate interior angle relationship between ∠1 and an unmarked angle used to connect with ∠4
- ∠4 decomposed into two angles and then alternate interior and corresponding angle relationships used to connect ∠4 to ∠1 and ∠2

Facilitate Whole Class Discussion

Call on students to share selected strategies. Prompt students to look at each speaker and try to understand that student's thinking.

Guide students to **Compare and Connect** the representations. To help students collect their ideas, ask them to turn and talk about the solutions and strategies that have been presented.

> **ASK** *Which angle relationships were used in the strategies to help write an expression for m∠4?*
>
> **LISTEN FOR** Pairs of corresponding and alternate interior angles are congruent. The sum of the angle measures of a triangle is 180°. The sum of the measures of the angles in a linear pair is 180°.

Picture It & Model It

If students presented these models, have students connect these models to those presented in class.

If no student presented at least one of these models, have students first analyze key features of the models, and then connect them to the models presented in class.

> **ASK** *What is different about the types of angle relationships used in the two representations?*
>
> **LISTEN FOR** Picture It involves parallel line relationships and Model It involves triangles and linear pairs.

For the model using the figure, prompt students to discuss related angle pairs.

- *What angle relationships are identified? Are there any that are not identified?*
- *Why are the angle relationships significant?*

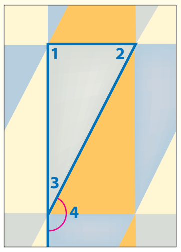
For the model using angle relationships, prompt students to think about how the equations relate.

- *Do either of the relationships relate to parallel lines?*
- *What conclusions stem from the final equation?*

➤ Explore different ways to describe angle relationships in triangles.

The triangular windows of the Aula Medica conference center in Sweden are formed by three sets of parallel lines going in different directions.

A close-up of the window design shows a triangle. The measure of ∠4 is related to the angle measures of the triangle. How can you use the measures of ∠1 and ∠2 to write an expression for the measure of ∠4?



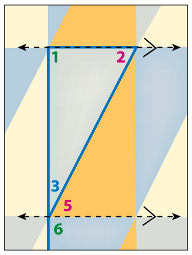
Picture It

You can look for pairs of congruent angles.

$m\angle 4 = m\angle 5 + m\angle 6$

$m\angle 5 = m\angle 2$ ⟵ ∠2 and ∠5 are alternate interior angles.

$m\angle 6 = m\angle 1$ ⟵ ∠1 and ∠6 are corresponding angles.



Model It

You can use angle relationships.

$m\angle 1 + m\angle 2 + m\angle 3 = 180^\circ$ ⟵ The sum of the angle measures of a triangle is 180°.

$m\angle 3 + m\angle 4 = 180^\circ$ ⟵ ∠3 and ∠4 form a linear pair.

$m\angle 1 + m\angle 2 + m\angle 3 = m\angle 3 + m\angle 4$

DIFFERENTIATION | EXTEND

SMP 3

Deepen Understanding

Constructing a Viable Argument When Using Angle Relationships to Write Expressions for Angle Measures

While it appears ∠1 is a right angle, the measure of ∠1 is not known. Prompt students to consider whether it matters if ∠1 is a right angle, acute angle, or obtuse angle.

> **ASK** *How would the angle relationships change if ∠1 were an obtuse angle?*
>
> **LISTEN FOR** They would not change. The two dashed lines would still be parallel and there is a transversal that cuts those lines. ∠2 and ∠5 would still be alternate interior angles. ∠1 and ∠6 would still be corresponding angles.

> **ASK** *How would the angle relationships change if ∠1 were an acute angle?*
>
> **LISTEN FOR** They would not change. The two dashed lines would still be parallel and there is a transversal that cuts those lines. ∠2 and ∠5 would still be alternate interior angles. ∠1 and ∠6 would still be corresponding angles.

> **ASK** *Would it matter if ∠1 were a right angle?*
>
> **LISTEN FOR** No, the angle relationships still hold true.

Develop Describing the Exterior Angles of a Triangle

CONNECT IT

SMP 2, 4, 5, 6

Remind students that angles 1, 2, 3, and 4 are the same in each representation, but the angle relationships used to connect angles 1, 2, and 4 are different. Explain that they will now use representations to reason about exterior angles of triangles.

Before students begin to record and expand on their work in Picture It & Model It, tell them that problems 2 and 4 will prepare them to provide the explanation asked for in problem 5.

Monitor and Confirm Understanding ①–②

- There are two exterior angles at each vertex of a triangle and their measures are the same.
- The measure of an exterior angle of a triangle is equal to the sum of the measures of the two nonadjacent interior angles.

Facilitate Whole Class Discussion

③ Look for understanding that the measure of an exterior angle is equal to the sum of the measures of the two nonadjacent interior angles.

④ Look for understanding that the measures of the exterior angles of the triangle sum to 360°.

> **ASK** *What pattern do you notice in the sums?*
>
> **LISTEN FOR** After substitution, each interior angle measure within the triangle is shown in the addition expression two times.

⑤ Look for understanding that the sum of the exterior angle measures of *any* triangle is 360°, and the measure of an exterior angle of *any* triangle is equal to the sum of the nonadjacent interior measures.

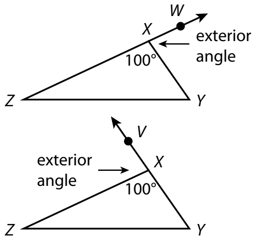
> **ASK** *Does the triangle to the right of problem 3 represent a particular triangle? Explain.*
>
> **LISTEN FOR** No; because no measurements have been given, the figure can represent any triangle.

⑥ **Reflect** Have all students focus on the strategies used to solve the Try It. If time allows, have students discuss their ideas with a partner.

CONNECT IT

➤ **Use the problem from the previous page to help you understand angle relationships in triangles.**

① An **exterior angle** of a triangle is formed by extending one side of a triangle. There are two exterior angles at each vertex of a triangle, as shown for $\angle ZXY$ in the figures at the right. Why are the two exterior angles at the same vertex congruent?

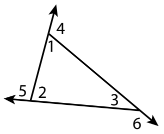


Possible answer: Since each exterior angle at a vertex forms a linear pair with the same interior angle, exterior angles on the same vertex must be equal in measure.

② Look at **Model It**. Simplify the last equation. What is the relationship between an exterior angle of a triangle and its nonadjacent interior angles?

$m\angle 4 = m\angle 1 + m\angle 2$; The measure of an exterior angle of a triangle is the sum of the measures of its two nonadjacent interior angles.

③ Look at the triangle to the right. Use the measures of the interior angles to write an equation for the measure of each exterior angle.



$m\angle 4 = m\angle 2 + m\angle 3$; $m\angle 5 = m\angle 1 + m\angle 3$; $m\angle 6 = m\angle 2 + m\angle 1$

④ Use the equations you wrote in problem 3 to find the sum of the measures of the exterior angles of a triangle, one at each vertex.

$m\angle 4 + m\angle 5 + m\angle 6 = (m\angle 2 + m\angle 3) + (m\angle 1 + m\angle 3) + (m\angle 2 + m\angle 1) = (m\angle 1 + m\angle 2 + m\angle 3) + (m\angle 1 + m\angle 2 + m\angle 3) = 180° + 180° = 360°$

⑤ Do you think your answers to problems 2 and 4 are true for all triangles? Explain.

Yes; Possible explanation: Specific angle measures were not used in either problem. The interior and exterior angles of any triangle can be labeled as they are in the figures.

⑥ **Reflect** Think about all the models and strategies you have discussed today. Describe how one of them helped you better understand how to solve the **Try It** problem.

Responses will vary. Check student responses.

DIFFERENTIATION | RETEACH or REINFORCE

Hands-On Activity

Use a model to show that the sum of the exterior angle measures of a triangle is 360°.

If students are unsure about exterior angles, then use this activity to help them see the connection between the interior and exterior angles of a triangle.

Materials For each student: scissors, straightedge

- Have students draw a scalene triangle and cut it out. Have them trace this triangle twice and cut out one more copy. Have them label each triangle's corresponding angles with the numbers 1, 2, and 3 and then cut off the angles of the two cut-out triangles.
- Instruct students to extend the sides of the last triangle and label the resulting exterior angles $\angle 4$, $\angle 5$, and $\angle 6$, paired with $\angle 1$, $\angle 2$, and $\angle 3$, respectively.
- Instruct students to use two of the torn angles and fit them perfectly into $\angle 4$. Have them repeat for $\angle 5$ and $\angle 6$. Ask: *Which two pieces fit in $\angle 4$?* [$\angle 2$ and $\angle 3$] *Which two pieces fit in $\angle 5$?* [$\angle 1$ and $\angle 3$] *Which two pieces fit in $\angle 6$?* [$\angle 1$ and $\angle 2$]
- Ask: *How does your model show that the sum of the measures of the exterior angles is 360°?* [The exterior angles are filled by the interior angles of two triangle copies.]

Apply It

For all problems, encourage students to use a model to support their thinking. Encourage students to use correct mathematical symbols and shorthand when representing concepts mathematically.

7 Students may use the first sentence of the problem to create an equation, then simplify that equation using properties and the information provided in the second sentence.

8 **C, D, and F are correct.** Students should use the ideas that the interior angle measures sum to 180°, the exterior angle measures sum to 360°, and the measures of the angles in a linear pair sum to 180° to check the validity of each statement.

A is not correct. This answer suggests that interior and exterior angle measures sum to 180°.

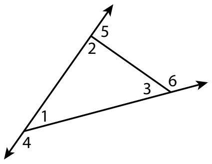
B is not correct. This answer shows a random equation from the three given values.

E is not correct. This answer suggests that interior and exterior angle measures sum to 180°.

Apply It

➤ **Use what you learned to solve these problems.**

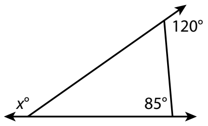
7 Use linear pairs to find the sum of all the angles labeled in the triangle. Then use what you know about the interior angles of a triangle to show that the sum of the measures of the exterior angles of a triangle, one at each vertex, is 360°.



Possible work:

$$(m\angle 1 + m\angle 4) + (m\angle 2 + m\angle 5) + (m\angle 3 + m\angle 6) = 180^\circ + 180^\circ + 180^\circ$$
$$(m\angle 1 + m\angle 2 + m\angle 3) + (m\angle 4 + m\angle 5 + m\angle 6) = 540^\circ$$
$$180^\circ + (m\angle 4 + m\angle 5 + m\angle 6) = 540^\circ$$
$$m\angle 4 + m\angle 5 + m\angle 6 = 360^\circ$$

8 Which equations can you use to find the value of x? Select all that apply.



A $x + 120 + 85 = 180$

B $x + 85 = 120$

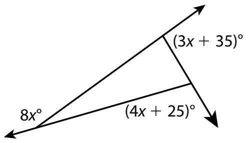
[✓ **C**] $x + 120 + 95 = 360$

[✓ **D**] $x = 60 + 85$

E $x + 60 + 85 = 180$

[✓ **F**] $x + 35 = 180$

9 What is the value of x? Show your work.



Possible work:

$$8x + 3x + 35 + 4x + 25 = 360$$
$$15x + 60 = 360$$
$$15x = 300$$
$$x = 20$$

SOLUTION $x = 20$

CLOSE **EXIT TICKET**

9 Students' solutions should show an understanding that the sum of the exterior angles of a triangle is 360°.

Error Alert If students set the sum of the given expressions equal to 180°, then they have confused the sum of the interior angle measures with the sum of the exterior angle measures. Ask students whether the labeled angles are inside or outside of the triangle and prompt them to reconsider the sum of the angle measures.

Practice Describing the Exterior Angles of a Triangle

Problem Notes

Assign **Practice Describing the Exterior Angles of a Triangle** as extra practice in class or as homework.

1. Students may substitute 18 for *x* to find the measure of each interior angle, and then use the fact that the sum of the angle measures in a linear pair is 180° to find the exterior angle measures. ***Basic***
2. Students may use the fact that ∠*CAB* is the other nonadjacent interior angle and the fact that the sum of the measures of the two nonadjacent interior angles is equal to the measure of the exterior angle. ***Basic***
3. Students may reason that a 90° exterior angle forms a linear pair with a 90° interior angle. There cannot be more than one 90° angle in a triangle. ***Challenge***

LESSON 7 | SESSION 2 Name:

Practice Describing the Exterior Angles of a Triangle

➤ **Study the Example showing how to use the relationship between exterior and interior angles of a triangle. Then solve problems 1–6.**

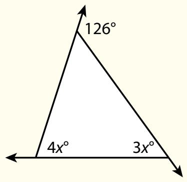
Example

Find the value of *x*.

$4x + 3x = 126$

$7x = 126$

$x = 18$



① What are the three exterior angle measures of the triangle in the Example?

108°, 126°, 126°

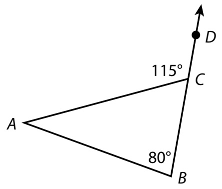
② What is $m\angle CAB$? Show your work.

Possible work:

$m\angle CAB + m\angle ABC = m\angle ACD$

$m\angle CAB + 80° = 115°$

$m\angle CAB = 35°$



SOLUTION $m\angle CAB = 35°$

③ Can a triangle have an exterior angle that measures 90° at two different vertices? Explain.

No; Possible explanation: An exterior angle forms a linear pair with an interior angle. Exterior angles measuring 90° at two different vertices means the triangle would have two interior angles measuring 90°. The sum of the interior angle measures of a triangle is 180°. If two interior angles measure 90°, then the third would have to measure 0°, which is impossible.

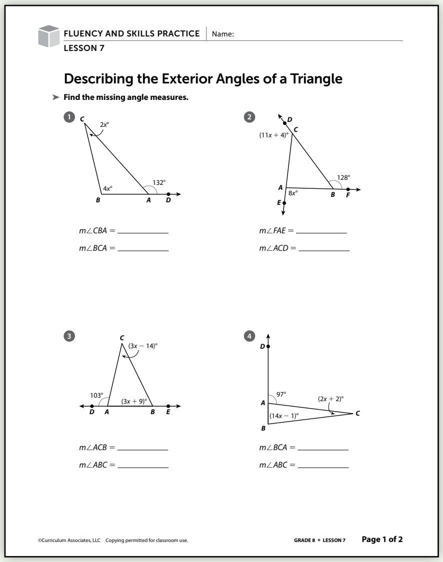
Vocabulary

exterior angle
when you extend one side of a polygon, the angle between the extended side and the adjacent side.

Fluency & Skills Practice

Describing the Exterior Angles of a Triangle

In this activity, students solve for missing values by using relationships involving exterior angles of a triangle. They use the sum of the measures of the exterior angles and the relationship between the measure of an exterior angle and the measures of the nonadjacent interior angles.


FLUENCY AND SKILLS PRACTICE | Name:
LESSON 7

Describing the Exterior Angles of a Triangle

➤ Find the missing angle measures.

① $m\angle CBA =$ ______
$m\angle BCA =$ ______

② $m\angle FAE =$ ______
$m\angle ACD =$ ______

③ $m\angle ACB =$ ______
$m\angle ABC =$ ______

④ $m\angle BCA =$ ______
$m\angle ABC =$ ______

©Curriculum Associates, LLC Copying permitted for classroom use. GRADE 8 • LESSON 7 Page 1 of 2

Additional Practice

④ **a.** Segment GF is an extension of segment HG and creates the exterior angle.

b. The sum of the measures of two exterior angles is not equal to the measure of a nonadjacent interior angle.

c. The sum of the measures of two nonadjacent interior angles is equal to the measure of the exterior angle.

d. $\angle EGL$ is not an exterior angle of $\triangle GDH$.

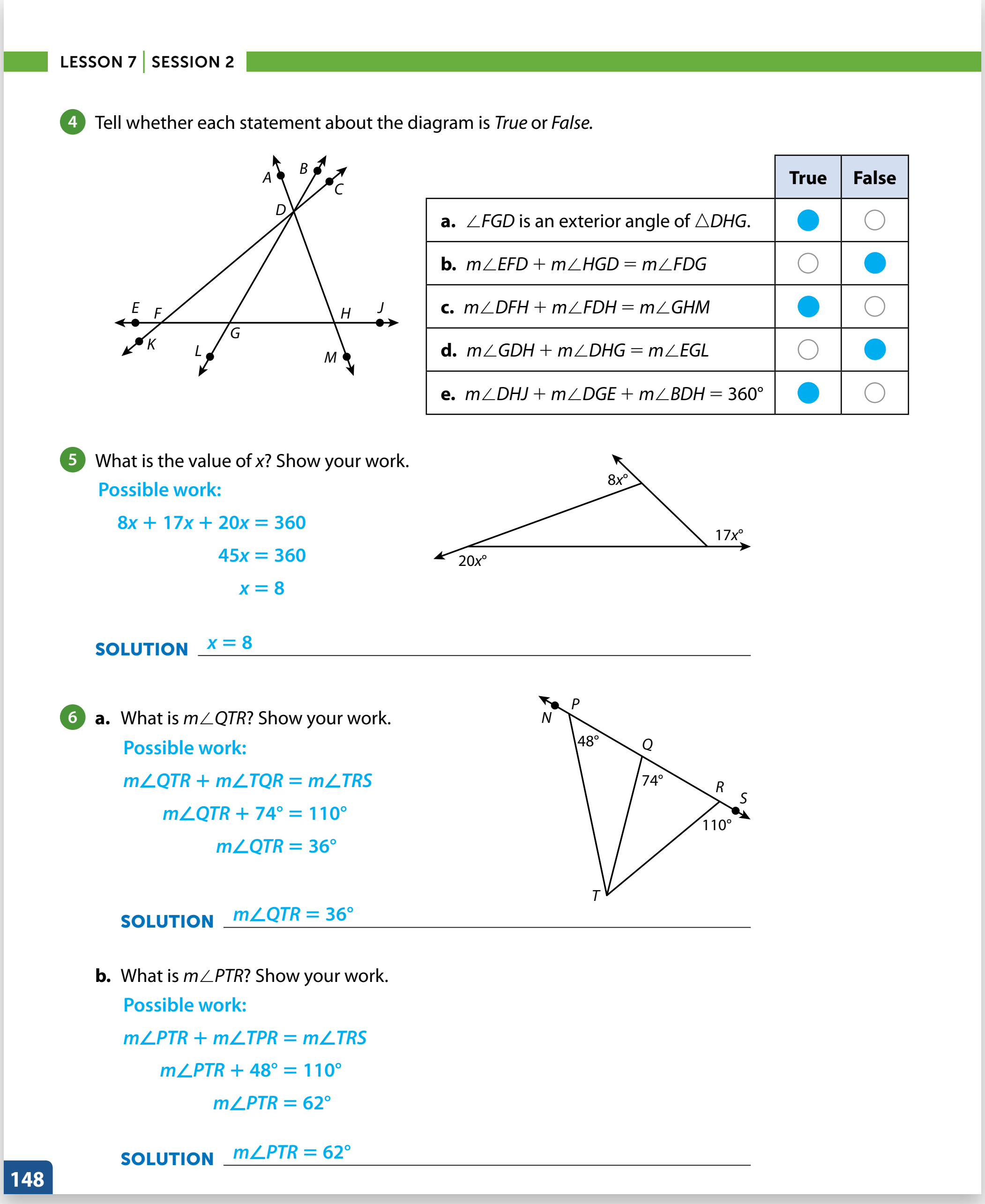
e. The sum of the measures of the exterior angles of $\triangle DGH$ is 360°.

Medium

⑤ Students should recognize the labeled angles are exterior angles, so the sum of their measures is 360°. ***Medium***

⑥ **a.** Students should see that $\angle TRS$ is an exterior angle of $\triangle RQT$, and $\angle QTR$ and $\angle RQT$ are the nonadjacent interior angles. ***Medium***

b. Students should see that $\angle TRS$ is an exterior angle of $\triangle PTR$, and $\angle RPT$ and $\angle PTR$ are the nonadjacent interior angles. ***Medium***



LESSON 7 | SESSION 2

④ Tell whether each statement about the diagram is *True* or *False*.

	True	False
a. $\angle FGD$ is an exterior angle of $\triangle DHG$.	●	○
b. $m\angle EFD + m\angle HGD = m\angle FDG$	○	●
c. $m\angle DFH + m\angle FDH = m\angle GHM$	●	○
d. $m\angle GDH + m\angle DHG = m\angle EGL$	○	●
e. $m\angle DHJ + m\angle DGE + m\angle BDH = 360°$	●	○

⑤ What is the value of x? Show your work.

Possible work:

$8x + 17x + 20x = 360$

$45x = 360$

$x = 8$

SOLUTION $x = 8$

⑥ **a.** What is $m\angle QTR$? Show your work.

Possible work:

$m\angle QTR + m\angle TQR = m\angle TRS$

$m\angle QTR + 74° = 110°$

$m\angle QTR = 36°$

SOLUTION $m\angle QTR = 36°$

b. What is $m\angle PTR$? Show your work.

Possible work:

$m\angle PTR + m\angle TPR = m\angle TRS$

$m\angle PTR + 48° = 110°$

$m\angle PTR = 62°$

SOLUTION $m\angle PTR = 62°$

148

DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Use with **Session 3** **Apply It**

Levels 1–3: Speaking/Writing

Prepare students to solve Apply It problem 7. Use **Notice and Wonder** to help students talk about the triangles in the problem. Ask: *What do you notice about the two triangles? What do you notice about the first triangle? And the second?* Encourage students to ask a question about the triangles. Then read the problem. Have partners share what they noticed and wondered. Then have them discuss if that can help them find the value of *x*. Review the lesson term *similar*. Have partners find the remaining angles and determine if the triangles are similar. Have students copy and complete the appropriate sentence:

- *The triangles are* _____.
- *The triangles are not* _____.

Levels 2–4: Speaking/Writing

Prepare students to write responses to Apply It problem 7. Use **Notice and Wonder** to help students talk about the triangles in the problem. Have them turn to a partner to tell what they notice and what they wonder. Then read the problem with students. Ask students which triangle provides the most information for solving for *x*. Then have them work on the problem independently. Have students write a complete sentence to tell if the triangles are similar. Then have them meet with their partners to share and explain their answers. Provide sentence frames:

- *I noticed that* _____.
- *I used that to* _____.
- *The triangles are* _____ *because* _____.

Levels 3–5: Speaking/Writing

Prepare students to write responses to Apply It problem 7. Use **Notice and Wonder** to have students talk about the triangles in the problem. Then have students work on the problem independently. Encourage them to show and explain their work to a partner. Remind them to justify their answer using precise language. Have students use sequence words, such as *first* and *next*, and the word *because* to help partners follow the explanations.

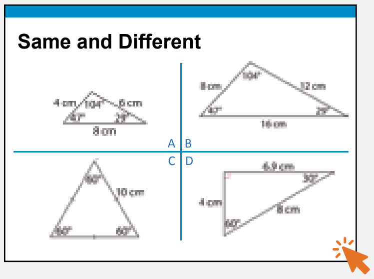
Allow think time for students to revise their work based on the conversation.

Develop Using Angles to Determine Similar Triangles

Purpose

- **Develop** strategies for using the relationship between corresponding angles to determine whether triangles are similar.
- **Recognize** that if two pairs of corresponding angles in two triangles are congruent, the triangles are similar.

START CONNECT TO PRIOR KNOWLEDGE



Possible Solutions

All are triangles.

A and B are scalene triangles with an obtuse angle. They are similar triangles.

C is an equilateral triangle.

D is a right triangle.

WHY? Support students' facility with classifying triangles.

DEVELOP ACADEMIC LANGUAGE

WHY? Guide students as they provide effective explanations.

HOW? Help students craft effective explanations by asking them to describe what they noticed about a particular problem. Then ask them to explain what they did and state reasons for what they decided to do. Provide a sentence starter to help students frame their explanations:

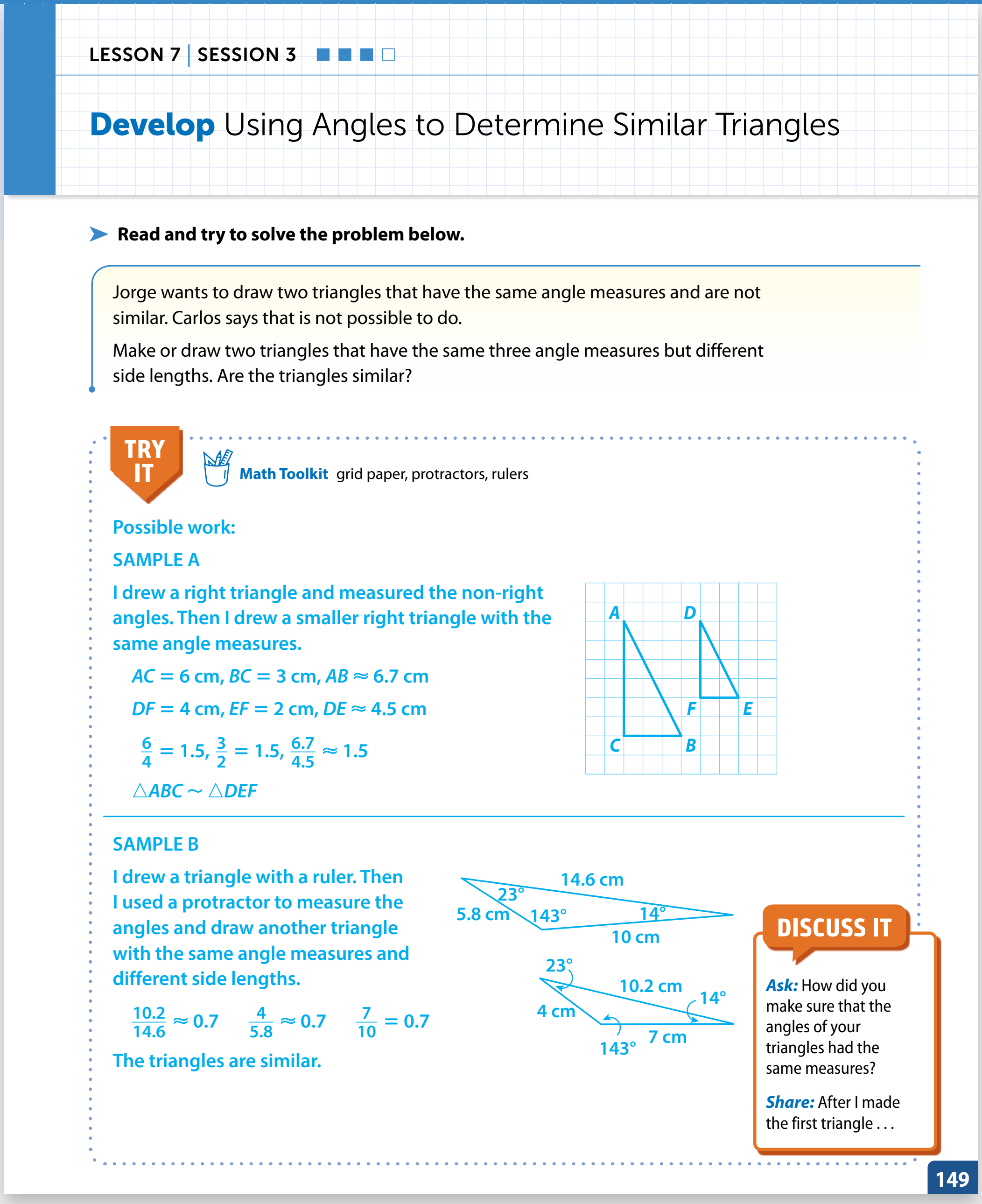
- *I noticed that* ____.
- *So, I decided to* ____ *because* ____.

TRY IT

SMP 1, 2, 4, 5, 6

Make Sense of the Problem

Before students work on Try It, use **Say It Another Way** to help them make sense of the problem. Have students read the problem with a partner. One student should paraphrase the problem and confirm understanding with his or her partner before beginning work. Partners should add detail or ask clarifying questions.



LESSON 7 | SESSION 3 ■ ■ ■ □

Develop Using Angles to Determine Similar Triangles

➤ **Read and try to solve the problem below.**

Jorge wants to draw two triangles that have the same angle measures and are not similar. Carlos says that is not possible to do.

Make or draw two triangles that have the same three angle measures but different side lengths. Are the triangles similar?

TRY IT **Math Toolkit** grid paper, protractors, rulers

Possible work:

SAMPLE A

I drew a right triangle and measured the non-right angles. Then I drew a smaller right triangle with the same angle measures.

$AC = 6$ cm, $BC = 3$ cm, $AB \approx 6.7$ cm

$DF = 4$ cm, $EF = 2$ cm, $DE \approx 4.5$ cm

$\frac{6}{4} = 1.5$, $\frac{3}{2} = 1.5$, $\frac{6.7}{4.5} \approx 1.5$

$\triangle ABC \sim \triangle DEF$

SAMPLE B

I drew a triangle with a ruler. Then I used a protractor to measure the angles and draw another triangle with the same angle measures and different side lengths.

$\frac{10.2}{14.6} \approx 0.7$ $\frac{4}{5.8} \approx 0.7$ $\frac{7}{10} = 0.7$

The triangles are similar.

DISCUSS IT

Ask: How did you make sure that the angles of your triangles had the same measures?

Share: After I made the first triangle . . .

149

DISCUSS IT

SMP 2, 3, 6

Support Partner Discussion

After students work on Try It, have them respond to Discuss It with a partner. If students need support in getting started, prompt them to ask each other questions such as:

- *What type of triangle did you draw? Do you think your method would work with any type of triangle?*
- *What tools did you use to make sure the angle measures of your triangles were accurate?*

Error Alert If students conclude that their triangles are not similar because the quotients of the side lengths are not exactly equal, then remind them that when they measure with tools such as protractors and rulers, their measurements may not be exact. If the three quotients they calculate are approximately equal, they may assume that the side lengths are proportional.

Select and Sequence Student Strategies

Select 2–3 samples that represent the range of student thinking in your classroom. Here is one possible order for class discussion:

- models drawn on grid paper
- models drawn using a ruler and protractor
- models created by drawing one triangle and then tracing its angles to create another triangle of a different size

Facilitate Whole Class Discussion

Call on students to share selected strategies. Prompt presenters to describe what they noticed or assumed about the problem and what they decided to do as a result.

Guide students to **Compare and Connect** the representations. If any student's explanation is unclear, you might ask another student to reword it so that others understand.

> **ASK** *How did [student name] and [student name] make sure the angles in the two triangles they drew were the same size?*
>
> **LISTEN FOR** Students used protractors to measure angles or traced the angles of one triangle to get the angles of the other.

Model It

If students presented these models, have students connect these models to those presented in class.

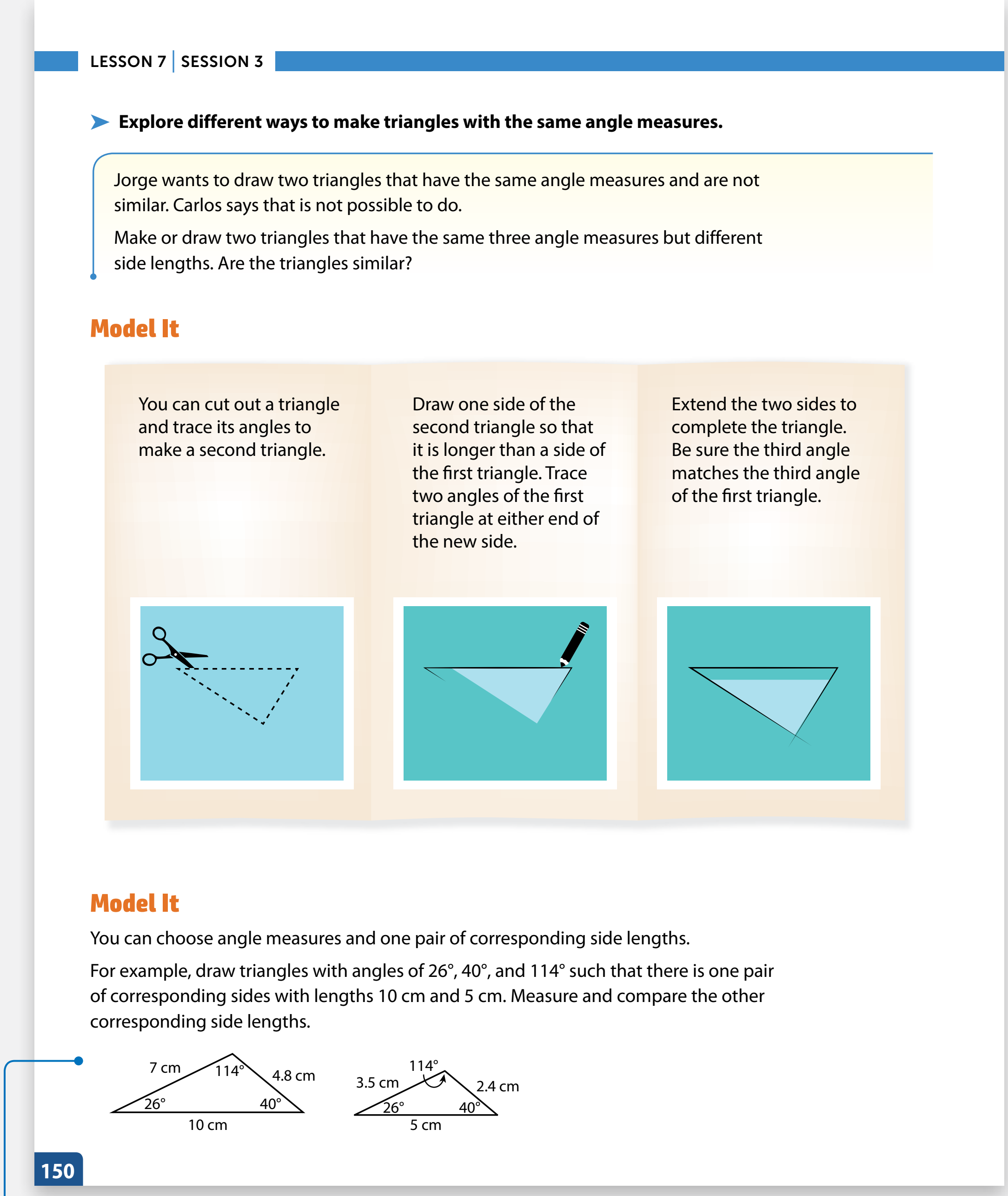
If no student presented at least one of these models, have students first analyze key features of the models, and then connect them to the models presented in class.

> **ASK** *What characteristics of the triangles are the same within each model? What characteristics of the triangles are different within each model?*
>
> **LISTEN FOR** Within each model, the angle measures of the two triangles are the same, but their side lengths are different.

For the model using scissors, prompt students to identify the strategy to ensure congruent angles.

- *Why do you think only one side was drawn before tracing two angles?*
- *Why is this side drawn to be longer than a side in the first triangle?*

For the model using a drawing and measurements, prompt students to consider the length of the first side drawn in the second triangle. Ask: *Is the length of the first side drawn in the second triangle important? Justify your response.*



LESSON 7 | SESSION 3

➤ **Explore different ways to make triangles with the same angle measures.**

Jorge wants to draw two triangles that have the same angle measures and are not similar. Carlos says that is not possible to do.

Make or draw two triangles that have the same three angle measures but different side lengths. Are the triangles similar?

Model It

You can cut out a triangle and trace its angles to make a second triangle.

Draw one side of the second triangle so that it is longer than a side of the first triangle. Trace two angles of the first triangle at either end of the new side.

Extend the two sides to complete the triangle. Be sure the third angle matches the third angle of the first triangle.

Model It

You can choose angle measures and one pair of corresponding side lengths.

For example, draw triangles with angles of 26°, 40°, and 114° such that there is one pair of corresponding sides with lengths 10 cm and 5 cm. Measure and compare the other corresponding side lengths.

150

DIFFERENTIATION | EXTEND

SMP 6

Deepen Understanding

Attending to Precision When Drawing Similar Triangles

Prompt students to compare the quotients of side lengths in the triangles drawn by several classmates and those provided in the second Model It.

> **ASK** *What do you notice about the quotients of side lengths in the model that may not be true for a hand-drawn pair of similar triangles?*
>
> **LISTEN FOR** The quotients of the side lengths are exactly equal in Model It but are not exactly equal in the hand-drawn models.

> **ASK** *Why might the quotients of side lengths not be exactly equal for hand-drawn models?*
>
> **LISTEN FOR** When using protractors and rulers, the tools can only measure to a certain degree of precision. So, while the measurements and the calculations may be close, they may not be exact.

Generalize Have students discuss ways they might decide whether an answer is "close enough" in a problem that involves using tools to make measurements.

Develop Using Angles to Determine Similar Triangles

CONNECT IT

SMP 2, 4, 5, 6

Remind students that in each representation, corresponding angle measures are equal. Explain that they will now use those representations to reason about how to use angle measures to tell whether two triangles are similar.

Before students begin to record and expand on their work in Model It, tell them that problems 3 and 4 will prepare them to provide the explanation asked for in problem 5. Prompt students to use individual think time to formulate an explanation for problems 3 and 4, then have them turn and talk with a partner before finalizing their responses to these two problems.

Monitor and Confirm Understanding ①–②

The triangles in each pair are similar because corresponding angles are congruent and corresponding side lengths are proportional.

Facilitate Whole Class Discussion

③ Students should recognize that triangles with three pairs of corresponding congruent angles are the same shape but not necessarily the same size. So, the triangles are similar.

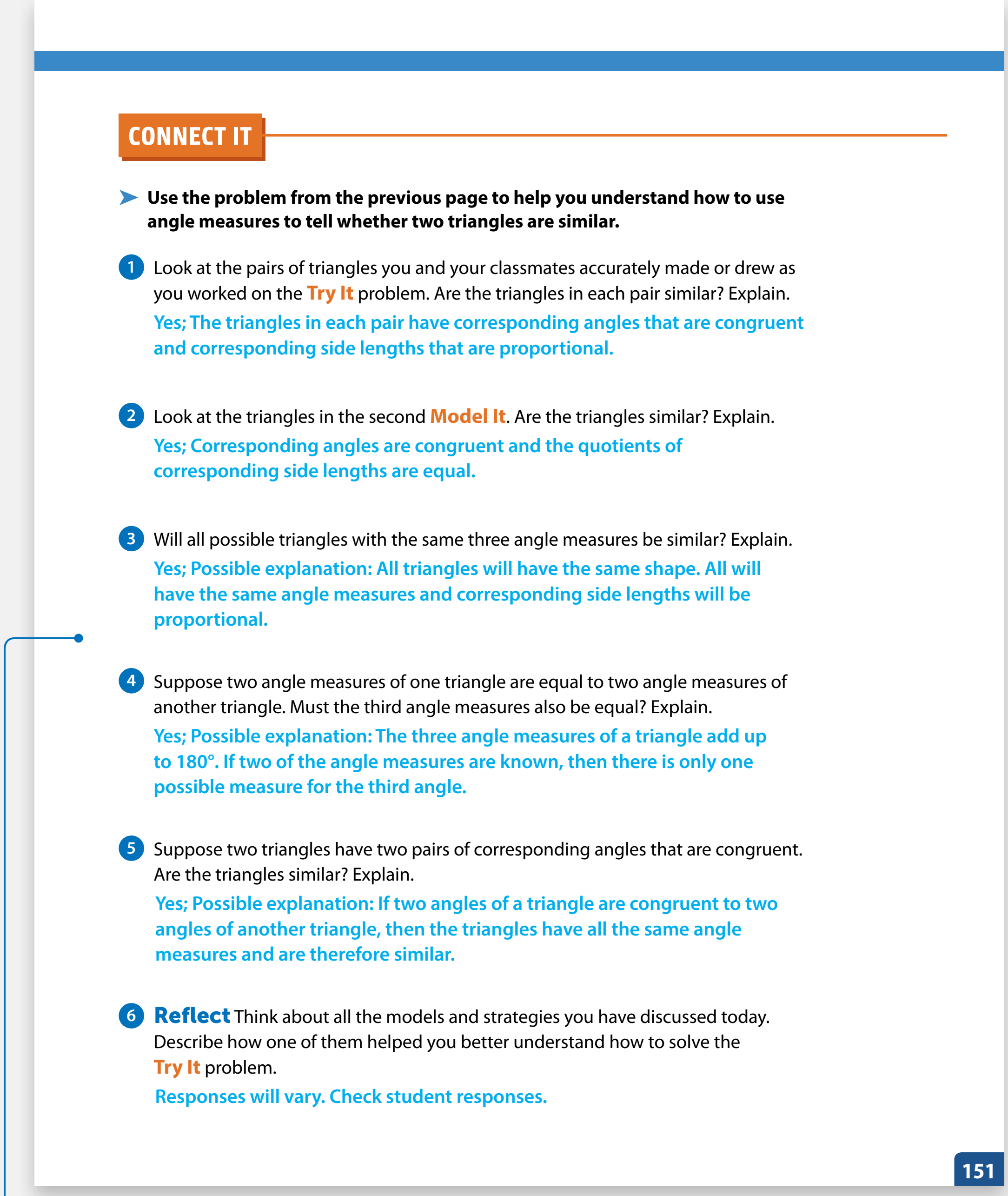
④ Since the interior angle measures of a triangle add to 180°, students should reason that if two angles of one triangle have the same measures as two angles of another, then the measures of the third angles are equal.

> **ASK** *If two angles of a triangle are known, how can the measure of the third angle be found?*
>
> **LISTEN FOR** Because the three angle measures of a triangle always add up to 180°, add the two known angle measures and subtract the sum from 180°.

⑤ Look for the idea that knowing two corresponding angle pairs are congruent is sufficient evidence to conclude similarity between two triangles.

> **ASK** *If two pairs of corresponding angles are congruent, what can you conclude about the third pair of corresponding angles? Can you conclude that the triangles are similar?*
>
> **LISTEN FOR** The third pair must also be congruent because the sum of the angle measures in both triangles is 180°. The triangles are similar.

⑥ **Reflect** Have students focus on the strategies used to solve the Try It. If time allows, have students discuss their ideas with a partner.



CONNECT IT

➤ **Use the problem from the previous page to help you understand how to use angle measures to tell whether two triangles are similar.**

① Look at the pairs of triangles you and your classmates accurately made or drew as you worked on the **Try It** problem. Are the triangles in each pair similar? Explain.

Yes; The triangles in each pair have corresponding angles that are congruent and corresponding side lengths that are proportional.

② Look at the triangles in the second **Model It**. Are the triangles similar? Explain.

Yes; Corresponding angles are congruent and the quotients of corresponding side lengths are equal.

③ Will all possible triangles with the same three angle measures be similar? Explain.

Yes; Possible explanation: All triangles will have the same shape. All will have the same angle measures and corresponding side lengths will be proportional.

④ Suppose two angle measures of one triangle are equal to two angle measures of another triangle. Must the third angle measures also be equal? Explain.

Yes; Possible explanation: The three angle measures of a triangle add up to 180°. If two of the angle measures are known, then there is only one possible measure for the third angle.

⑤ Suppose two triangles have two pairs of corresponding angles that are congruent. Are the triangles similar? Explain.

Yes; Possible explanation: If two angles of a triangle are congruent to two angles of another triangle, then the triangles have all the same angle measures and are therefore similar.

⑥ **Reflect** Think about all the models and strategies you have discussed today. Describe how one of them helped you better understand how to solve the **Try It** problem.

Responses will vary. Check student responses.

151

DIFFERENTIATION | RETEACH or REINFORCE

Hands-On Activity

Use appropriate tools to understand similar triangles.

If students are still struggling with understanding similar triangles, then have them use tools to measure angles and sides of similar triangles to see relationships.

Materials For each student: protractor, scissors, straightedge

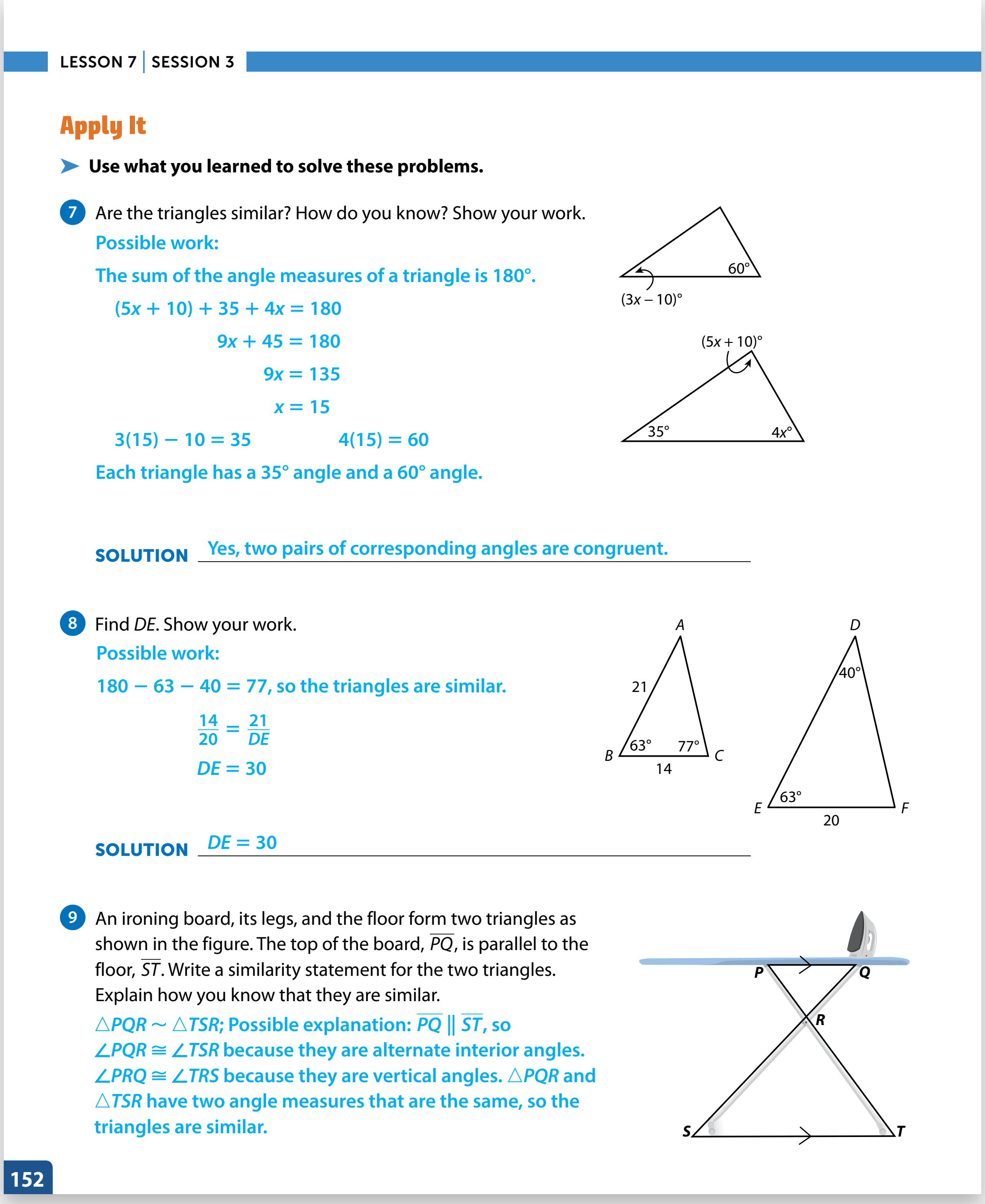
- Project a triangle onto a whiteboard or wall. Label the vertices *A*, *B*, and *C*.
- Ask volunteers to measure each angle. Record the measures.
- Have students use a protractor to draw a triangle with those same angle measures.
- Have students label the corresponding interior angles with *A*, *B*, and *C*.
- Instruct students to cut out the three angles of their triangles.
- Have students tape their angles on the projected triangle.
- Ask: *Are the angle measures the same?* [yes]
- Ask: *Was your triangle the same size as the one projected?* [no]
- Ask: *Is your triangle similar to the one projected? Explain.* [Yes, when the angles in two triangles are congruent, the triangles are similar.]

Apply It

For all problems, encourage students to use a model to support their thinking. Encourage students to use correct geometric notation and to make sure their work is clear. Some students may be able to combine simplification steps when solving equations, while others need the support of more detailed steps.

7 Students can use the second triangle to write an equation and solve for *x*, and then use the value of *x* to find the unknown angle measures in both triangles. Because two angles of one triangle have the same measures as two angles of the other, the triangles are similar.

8 Students should understand that similar triangles have pairs of sides with a proportional relationship so they can use a proportion to find *DE*.



LESSON 7 | SESSION 3

Apply It

➤ **Use what you learned to solve these problems.**

7 Are the triangles similar? How do you know? Show your work.

Possible work:

The sum of the angle measures of a triangle is 180°.

$(5x + 10) + 35 + 4x = 180$
$9x + 45 = 180$
$9x = 135$
$x = 15$

$3(15) - 10 = 35$ $4(15) = 60$

Each triangle has a 35° angle and a 60° angle.

SOLUTION Yes, two pairs of corresponding angles are congruent.

8 Find *DE*. Show your work.

Possible work:

$180 - 63 - 40 = 77$, so the triangles are similar.

$\frac{14}{20} = \frac{21}{DE}$

$DE = 30$

SOLUTION $DE = 30$

9 An ironing board, its legs, and the floor form two triangles as shown in the figure. The top of the board, $\overline{PQ}$, is parallel to the floor, $\overline{ST}$. Write a similarity statement for the two triangles. Explain how you know that they are similar.

$\triangle PQR \sim \triangle TSR$; Possible explanation: $\overline{PQ} \parallel \overline{ST}$, so $\angle PQR \cong \angle TSR$ because they are alternate interior angles. $\angle PRQ \cong \angle TRS$ because they are vertical angles. $\triangle PQR$ and $\triangle TSR$ have two angle measures that are the same, so the triangles are similar.

152

CLOSE EXIT TICKET

9 See ***Connect to Culture*** to support student engagement. Students' solutions should show an understanding of:

- angle relationships when parallel lines are cut by a transversal.
- the fact that two pairs of congruent corresponding angles of triangles ensures triangle similarity.

Error Alert If students write $\triangle PQR \sim \triangle STR$, then remind them that the order of the vertices in a similarity statement is important. Have them identify one transversal between the parallel lines and the resulting angle correspondence, and then repeat with the other transversal. They should see that $\angle P$ corresponds to $\angle T$ and $\angle Q$ corresponds to $\angle S$. So, one correct statement would be $\triangle PQR \sim \triangle TSR$, where *P* and *T* are in the same position and *Q* and *S* are in the same position in the triangle names.

Practice Using Angles to Determine Similar Triangles

Problem Notes

Assign **Practice Using Angles to Determine Similar Triangles** as extra practice in class or as homework.

① Students should recognize that the labeled angles are exterior angles. They must use the fact that the sum of the exterior angles is 360° to solve for x. They can then find the measure of each exterior and interior angle and compare the interior angle measures to those of the example triangles. ***Medium***

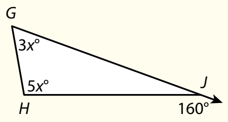
② Students should use the fact that the sum of the angle measures of a triangle is 180° to find the unknown angle measure in each triangle. Then, they can determine whether the triangles are similar. ***Challenge***

Practice Using Angles to Determine Similar Triangles

➤ **Study the Example showing how to use angle measures to identify similar triangles. Then solve problems 1–6.**

Example

Which triangles are similar?



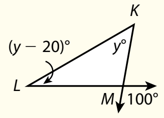
$3x + 5x = 160$

$8x = 160$

$x = 20$

$m\angle HGJ = 60°, m\angle GHJ = 100°$

$m\angle GJH = 180° - 160° = 20°$



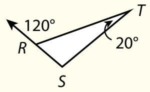
$100 + y + y - 20 = 180$

$2y + 80 = 180$

$2y = 100$

$y = 50$

$m\angle LKM = 50°, m\angle KLM = 30°$



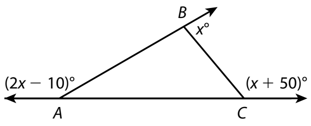
$m\angle SRT = 180° - 120°$

$= 60°$

$m\angle RTS = 20°$

$\angle HGJ \cong \angle SRT$ and $\angle GJH \cong \angle RTS$, so $\triangle GHJ \sim \triangle RST$.

① Is $\triangle ABC$ similar to any of the triangles in the Example? Explain.



Yes, $\triangle ABC \sim \triangle LMK$.

$2x - 10 + x + x + 50 = 360$

$4x + 40 = 360$

$4x = 320$

$x = 80$

$m\angle BAC = 180° - (160 - 10)° = 30°$

$m\angle ACB = 180° - (80 + 50)° = 50°$

$\angle BAC \cong \angle MLK$ and $\angle ACB \cong \angle LKM$, so $\triangle ABC \sim \triangle LMK$.

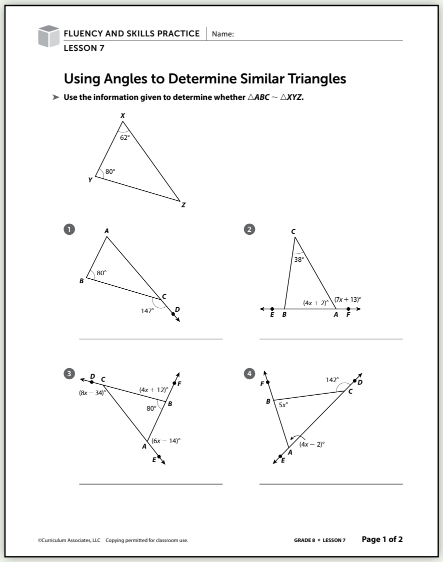
② Triangle X has two angles that measure 80° and 30°. Triangle Y has two angles that measure 80° and 70°. Hannah says that triangles X and Y are not similar. Jasmine says they are similar. Who is correct? Explain.

Jasmine; Possible explanation: The sum of the angle measures of a triangle is 180°. So the third angle of $\triangle X$ measures 70°. $\triangle X$ and $\triangle Y$ have two congruent angle pairs, so the triangles are similar.

Fluency & Skills Practice

Using Angles to Determine Similar Triangles

In this activity, students use relationships between angles to solve for missing values, find angle measures, and determine whether triangles are similar. They also apply understanding of parallel lines cut by a transversal to determine whether angles are congruent when determining whether triangles are similar.



FLUENCY AND SKILLS PRACTICE | Name:
LESSON 7

Using Angles to Determine Similar Triangles

➤ Use the information given to determine whether $\triangle ABC \sim \triangle XYZ$.

©Curriculum Associates, LLC Copying permitted for classroom use. GRADE 8 • LESSON 7 Page 1 of 2

Additional Practice

③ **a.** Students can determine that triangles *XYW* and *XWZ* each have a right angle. Then they should notice that both triangles include angle *X*. Then they can conclude the triangles are similar because they have two pairs of congruent angles. ***Medium***

b. Students can determine that triangles *WYZ* and *XWZ* each have a right angle. Then they should notice that both triangles include angle *Z*. Then they can conclude the triangles are similar because they have two pairs of congruent angles. ***Medium***

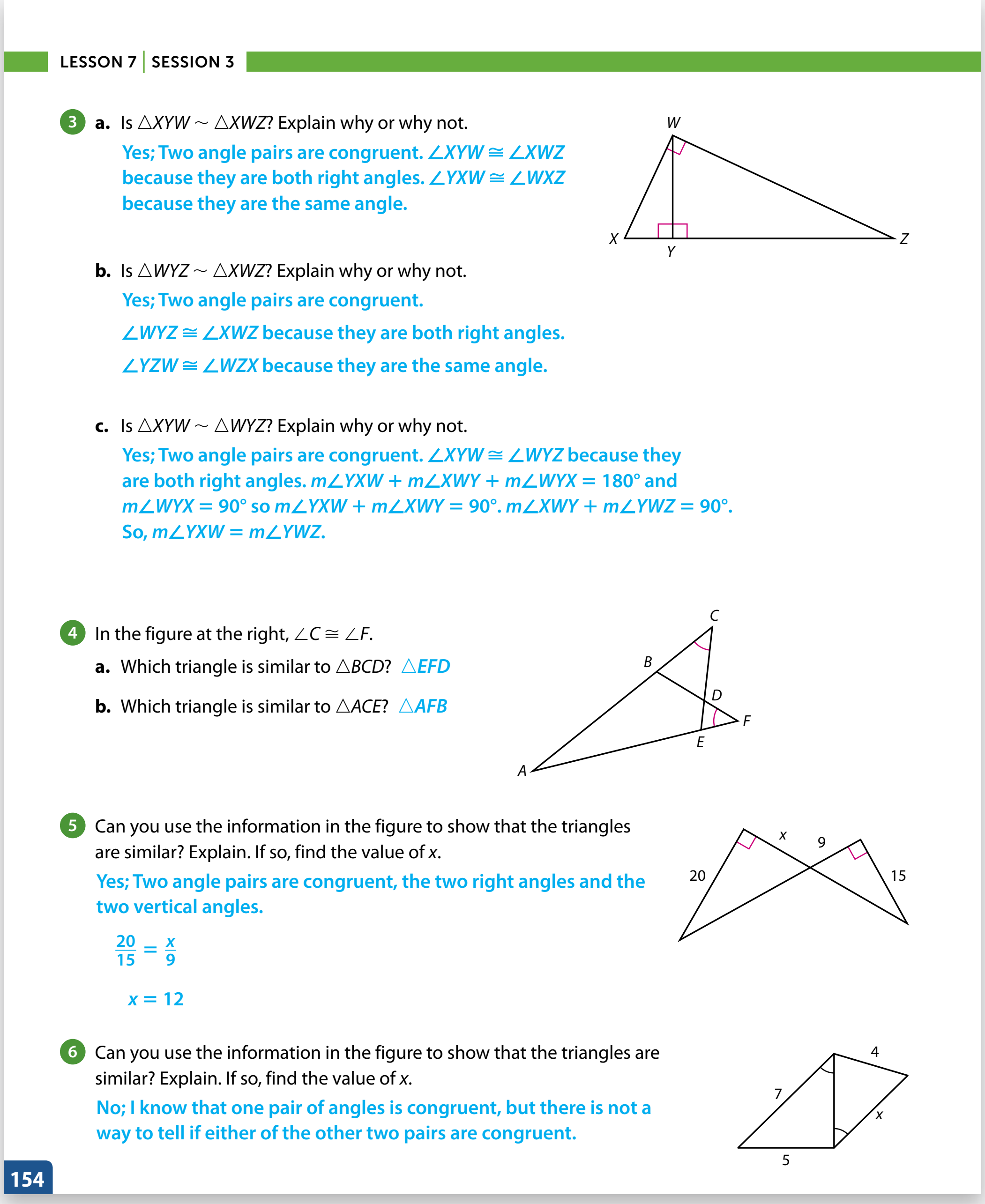
c. If both triangles *XYW* and *WYZ* are similar to triangle *XWZ,* then students can use reasoning to conclude that they must be similar to each other. ***Challenge***

④ **a.** Students may use vertical angles to identify a second pair of congruent angles in the two triangles. ***Basic***

b. Students may solve the problem using the fact that $\angle A$ is in both triangles, and it is congruent to itself. ***Basic***

⑤ Students should recognize that a pair of vertical angles can give them a second pair of congruent angles for similarity. Then they can use the fact that corresponding side lengths are proportional to find *x*. ***Medium***

⑥ Students should recognize that without parallel lines or a second pair of congruent angles given, there is not enough information to determine that the triangles are similar. ***Medium***



LESSON 7 | SESSION 3

③ **a.** Is $\triangle XYW \sim \triangle XWZ$? Explain why or why not.

Yes; Two angle pairs are congruent. $\angle XYW \cong \angle XWZ$ because they are both right angles. $\angle YXW \cong \angle WXZ$ because they are the same angle.

b. Is $\triangle WYZ \sim \triangle XWZ$? Explain why or why not.

Yes; Two angle pairs are congruent.

$\angle WYZ \cong \angle XWZ$ because they are both right angles.

$\angle YZW \cong \angle WZX$ because they are the same angle.

c. Is $\triangle XYW \sim \triangle WYZ$? Explain why or why not.

Yes; Two angle pairs are congruent. $\angle XYW \cong \angle WYZ$ because they are both right angles. $m\angle YXW + m\angle XWY + m\angle WYX = 180°$ and $m\angle WYX = 90°$ so $m\angle YXW + m\angle XWY = 90°$. $m\angle XWY + m\angle YWZ = 90°$. So, $m\angle YXW = m\angle YWZ$.

④ In the figure at the right, $\angle C \cong \angle F$.

a. Which triangle is similar to $\triangle BCD$? $\triangle EFD$

b. Which triangle is similar to $\triangle ACE$? $\triangle AFB$

⑤ Can you use the information in the figure to show that the triangles are similar? Explain. If so, find the value of *x*.

Yes; Two angle pairs are congruent, the two right angles and the two vertical angles.

$$\frac{20}{15} = \frac{x}{9}$$

$$x = 12$$

⑥ Can you use the information in the figure to show that the triangles are similar? Explain. If so, find the value of *x*.

No; I know that one pair of angles is congruent, but there is not a way to tell if either of the other two pairs are congruent.

154

DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Use with **Session 4** **Apply It**

Levels 1–3: Reading/Writing

Prepare students to respond to Apply It problem 2. Read the problem as students follow along. Have students work in pairs to read the labels and discuss what they need to do. Encourage them to use sequence words to write a list with the steps they plan to follow. Provide sentence starters such as:

- *First, I determine which triangles to ____.*
- *Next, I know that ____ are congruent because ____.*
- *Then, I know that ____ are congruent because ____.*
- *Last, I can explain that △MJK and △NLK are similar because ____.*

Levels 2–4: Reading/Writing

Prepare students to write responses to Apply It problem 2. Read the problem with students. Have students work in pairs to reread what they need to explain. Then have them discuss what they need to do. Encourage them to use sequence words, such as *first, next, then,* and *last*, to list the steps they plan to follow. Suggest students include what they need to identify, determine, compare, and decide.

Have students follow their lists to answer the problem and determine whether they need to add, delete, or change any steps.

Levels 3–5: Reading/Writing

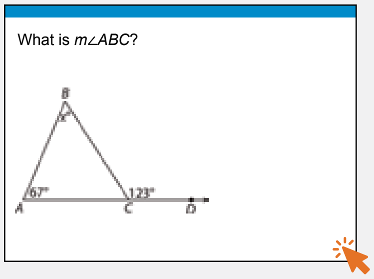
Prepare students to write responses to Apply It problem 2. Have students partner-read the problem and discuss what they need to do. Suggest students include what they need to identify, determine, compare, and decide. Have them brainstorm other verbs they can use to state what they need to do. Ask pairs to exchange lists with another pair who will follow the steps to work on the problem and provide feedback. Students tell whether the list is missing steps or has extra steps and whether any steps need to change. Have students write sentences to state if the triangles are similar. Ask students to compare sentences and decide if they state the same thing.

Refine Describing Angle Relationships in Triangles

Purpose

- **Refine** strategies for using relationships among angle measures of triangles to solve problems.
- **Refine** understanding of the angle-angle criterion for similarity of triangles.

START CHECK FOR UNDERSTANDING


What is $m\angle ABC$?

Solution

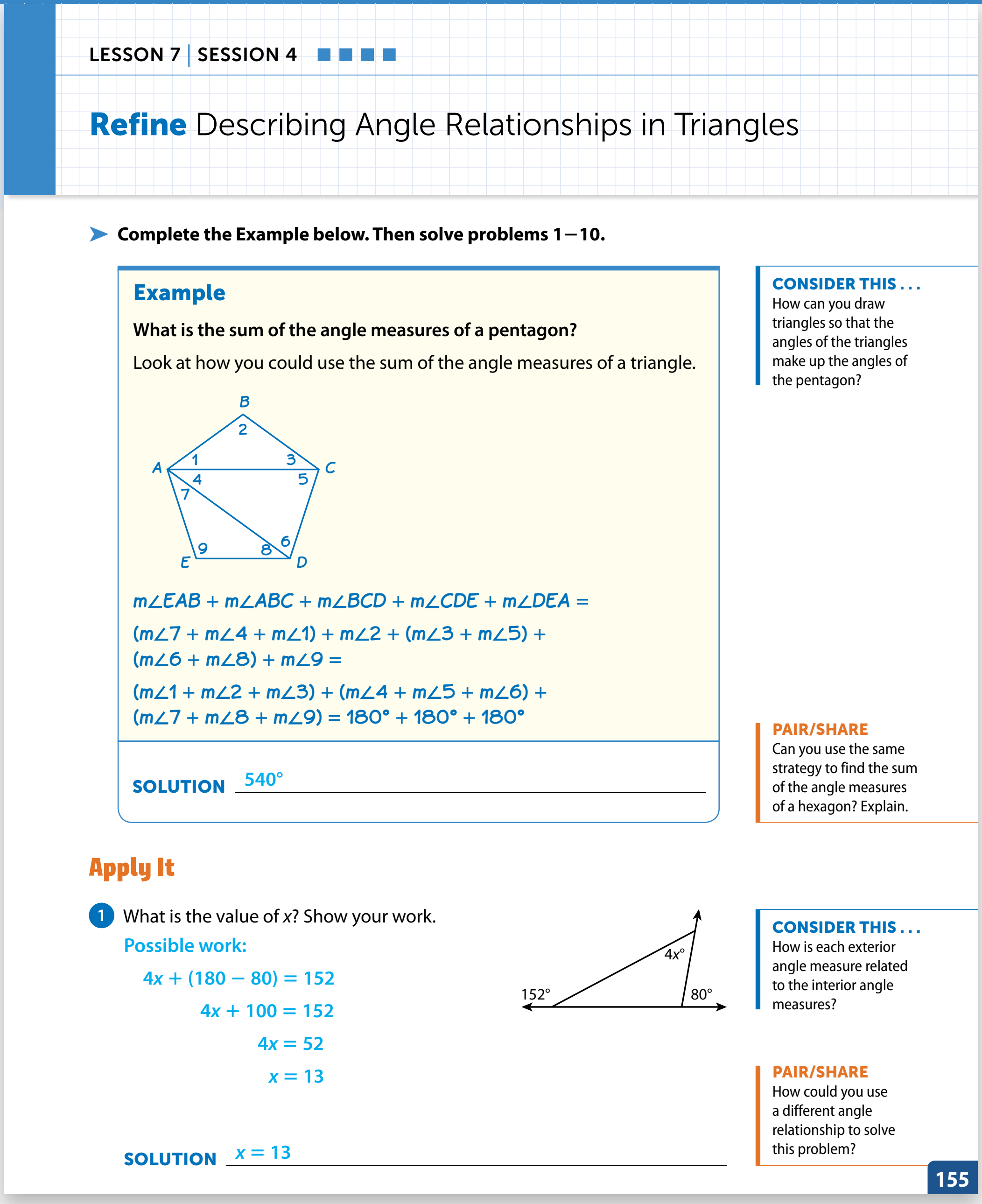
56°

WHY? Confirm students' understanding of the relationship between an exterior angle of a triangle and the nonadjacent interior angles, identifying common errors to address as needed.

MONITOR & GUIDE

Before students begin to work, use their responses to the **Start** to determine those who will benefit from additional support. Use the **Error Analysis** table below to guide remediation.

Have all students complete the Example and problems 1–3, using Consider This and Pair/Share as appropriate. Observe and monitor their reasoning and guide or redirect students as needed.


LESSON 7 | SESSION 4 ■ ■ ■ ■

Refine Describing Angle Relationships in Triangles

➤ **Complete the Example below. Then solve problems 1–10.**

Example

What is the sum of the angle measures of a pentagon?

Look at how you could use the sum of the angle measures of a triangle.

$m\angle EAB + m\angle ABC + m\angle BCD + m\angle CDE + m\angle DEA =$

$(m\angle 7 + m\angle 4 + m\angle 1) + m\angle 2 + (m\angle 3 + m\angle 5) + (m\angle 6 + m\angle 8) + m\angle 9 =$

$(m\angle 1 + m\angle 2 + m\angle 3) + (m\angle 4 + m\angle 5 + m\angle 6) + (m\angle 7 + m\angle 8 + m\angle 9) = 180° + 180° + 180°$

SOLUTION 540°

CONSIDER THIS . . .
How can you draw triangles so that the angles of the triangles make up the angles of the pentagon?

PAIR/SHARE
Can you use the same strategy to find the sum of the angle measures of a hexagon? Explain.

Apply It

① What is the value of x? Show your work.

Possible work:

$4x + (180 - 80) = 152$

$4x + 100 = 152$

$4x = 52$

$x = 13$

SOLUTION $x = 13$

CONSIDER THIS . . .
How is each exterior angle measure related to the interior angle measures?

PAIR/SHARE
How could you use a different angle relationship to solve this problem?

155

START ERROR ANALYSIS

If the error is . . .	Students may . . .	To support understanding . . .
57°	have found $m\angle BCA$.	Ask students to first describe the relationship between $\angle BCA$ and $\angle BCD$. Then ask students how they can use this relationship and the fact that the sum of the angle measures of a triangle is 180° to find $m\angle ABC$.
123°	have thought $\angle BCD$ and $\angle ABC$ are alternate interior angles.	Remind students that for alternate interior angles to be congruent, the angles must be on either side of a transversal that cuts two parallel lines. There are no parallel lines in the figure. Prompt students to find an angle relationship that does not use parallel lines to find $m\angle ABC$.
113°	have thought $\angle BAC$ and $\angle ABC$ are same-side interior angles.	Remind students that for same-side interior angles to be supplementary, the angles must be on the same side of a transversal that cuts two parallel lines. There are no parallel lines in the figure. Prompt students to use the relationship between an exterior angle of a triangle and the interior angles or the relationship between a linear pair and the fact that the sum of the angle measures of a triangle is 180° to find $m\angle ABC$.

Example

Guide students in understanding the Example. Ask:

- *Why is it helpful to decompose the pentagon into three triangles?*
- *What is the sum of the angle measures of a triangle?*
- *How does labeling each angle of the three triangles help you solve this problem?*

Help all students focus on the Example and responses to the questions. Remind students to be respectful when they disagree with another's ideas.

Look for understanding that the sum of the angle measures of the pentagon is equal to the sum of the angle measures for all three triangles.

Apply It

① Students may also solve this problem by first finding the measures of the interior angles adjacent to the 152° and 80° angles and then using the fact that the sum of the three interior angle measures must be 180°. ***DOK 2***

② Students may solve the problem by recognizing that two right angles ($\angle MJK$ and $\angle NLK$) are congruent and two vertical angles ($\angle MKJ$ and $\angle NKL$) are congruent. ***DOK 3***

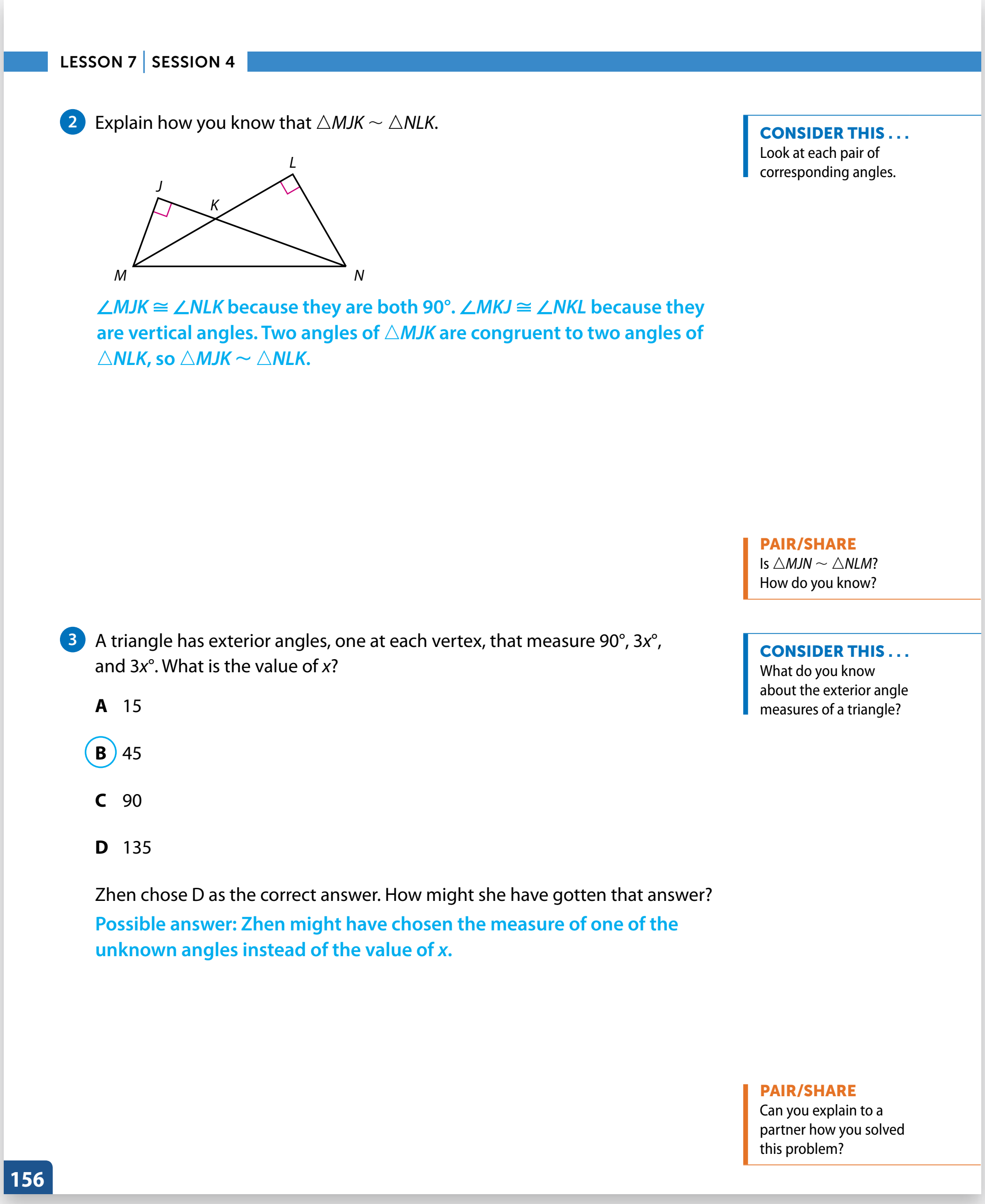
③ **B** **is correct.** Students may solve the problem by recalling that the sum of measures of the three exterior angles of a triangle is 360°.

A is not correct. This answer would be the value of x if the sum of measures of the three exterior angles of a triangle were 180°.

C is not correct. This is the measure of one of the exterior angles.

D is not correct. This is the measure of each exterior angle that measures $3x$°.

DOK 3



LESSON 7 | SESSION 4

② Explain how you know that $\triangle MJK \sim \triangle NLK$.

$\angle MJK \cong \angle NLK$ because they are both 90°. $\angle MKJ \cong \angle NKL$ because they are vertical angles. Two angles of $\triangle MJK$ are congruent to two angles of $\triangle NLK$, so $\triangle MJK \sim \triangle NLK$.

CONSIDER THIS . . .
Look at each pair of corresponding angles.

PAIR/SHARE
Is $\triangle MJN \sim \triangle NLM$? How do you know?

③ A triangle has exterior angles, one at each vertex, that measure 90°, $3x$°, and $3x$°. What is the value of x?

A 15

[✓ **B**] 45

C 90

D 135

Zhen chose D as the correct answer. How might she have gotten that answer?

Possible answer: Zhen might have chosen the measure of one of the unknown angles instead of the value of x.

CONSIDER THIS . . .
What do you know about the exterior angle measures of a triangle?

PAIR/SHARE
Can you explain to a partner how you solved this problem?

156

GROUP & DIFFERENTIATE

Identify groupings for differentiation based on the **Start** and problems 1–3. A recommended sequence of activities for each group is suggested below. Use the resources on the next page to differentiate and close the lesson.

Approaching Proficiency

- **RETEACH** Hands-On Activity
- **REINFORCE** Problems 4, 7, 9

Meeting Proficiency

- **REINFORCE** Problems 4–9

Extending Beyond Proficiency

- **REINFORCE** Problems 4–9
- **EXTEND** Challenge

Have all students complete the **Close: Exit Ticket**.

Resources for Differentiation are found on the next page.

Refine Describing Angle Relationships in Triangles

Apply It

4 **a.** See ***Connect to Culture*** to support student engagement. The measure of exterior angle $\angle WXZ$ is the sum of the measures of its two nonadjacent interior angles $\angle XYZ$ and $\angle YZX$.

b. The sum of the measures of the angles of a triangle is 180°.

DOK 1

5 The third angle measure of the given triangle is $180° - 45° - 55° = 80°$.

A is correct. The measure of the angle that forms a linear pair with the 135°-angle is 45°. Two pairs of angles are congruent.

B is correct. The sum of the measures of the exterior angles is 360°, so $x = 25$. The angle measures of the triangle are 45°, 55°, and 80°. Three pairs of angles are congruent.

D is correct. Solve $9x + 11x = 100$ to find that $x = 5$. The measures of the two angles are 45° and 55°. Two pairs of angles are congruent.

C is not correct. Two pairs of angles are not congruent.

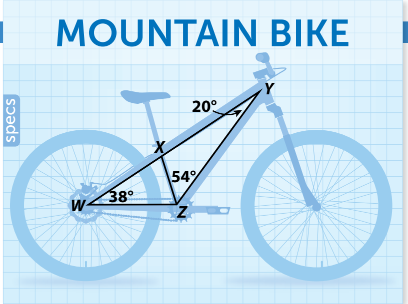
E is not correct. Two pairs of angles are not congruent.

F is not correct. Two pairs of angles are not congruent.

DOK 2

6 An obtuse exterior angle is adjacent to an acute interior angle as they form a linear pair. *DOK 3*

4 The angles of a mountain bike frame are shown.



a. What is $m\angle WXZ$? Show your work.

Possible work:

$$m\angle XYZ + m\angle YZX = m\angle WXZ$$
$$20° + 54° = m\angle WXZ$$
$$74° = m\angle WXZ$$

SOLUTION The measure of $\angle WXZ$ is 74°.

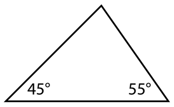
b. What is $m\angle XZW$? Show your work.

Possible work:

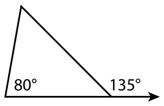
$$m\angle WXZ + m\angle XZW + m\angle ZWX = 180°$$
$$74° + m\angle XZW + 38° = 180°$$
$$m\angle XZW + 112° = 180°$$
$$m\angle XZW = 68°$$

SOLUTION The measure of $\angle XZW$ is 68°.

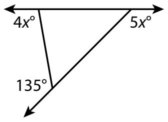
5 Which triangles are similar to the triangle at the right? Select all that apply.



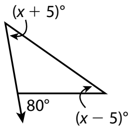
[✓ A]



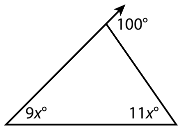
[✓ B]



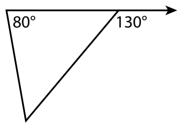
C



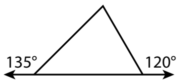
[✓ D]



E



F



6 Can a triangle have an exterior angle that is obtuse at two vertices? Explain.

Yes; Possible explanation: An obtuse exterior angle is adjacent to an acute interior angle. Every triangle has at least two acute interior angles, so every triangle has at least two obtuse exterior angles.

DIFFERENTIATION

RETEACH

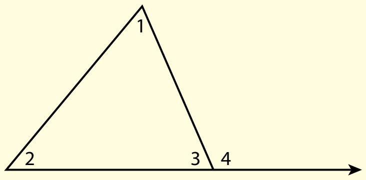
Hands-On Activity

Make a model to visualize the relationship between an exterior angle of a triangle and its two nonadjacent interior angles.

Students approaching proficiency with finding the measure of an exterior angle of a triangle will benefit from building a concrete model.

Materials For each student: scissors, straightedge, tracing paper

- Have students draw a triangle, such as the one shown, with an exterior angle and label each angle.



- Ask: *How are $\angle 1$ and $\angle 2$ related to $\angle 4$?* [$\angle 1$ and $\angle 2$ are the nonadjacent interior angles to exterior $\angle 4$.]
- Have students trace their triangle using tracing paper.
- Have students cut out $\angle 1$ and $\angle 2$ from their tracing paper.
- Have students arrange $\angle 1$ and $\angle 2$ on top of $\angle 4$. Ask: *What do you notice?* [The sum of $m\angle 1$ and $m\angle 2$ is equal to $m\angle 4$.]

7 **a.** If a triangle had more than one obtuse angle, its measures would have a sum greater than 180°.

b. If one angle measures 90°, the other two angles measure 180° − 90° = 90°.

c. This is true for every triangle.

d. The sum of the measures of the exterior angles of a triangle is 360°, not 180°.

DOK 2

8 Students may reason that because the sum of the measures of all three angles is 180°, each angle must measure 180° ÷ 3, or 60°. ***DOK 2***

9 Students may write $(6x - 10) + (4x - 5) + 5x = 360°$ to find the value of x. ***DOK 2***

CLOSE EXIT TICKET

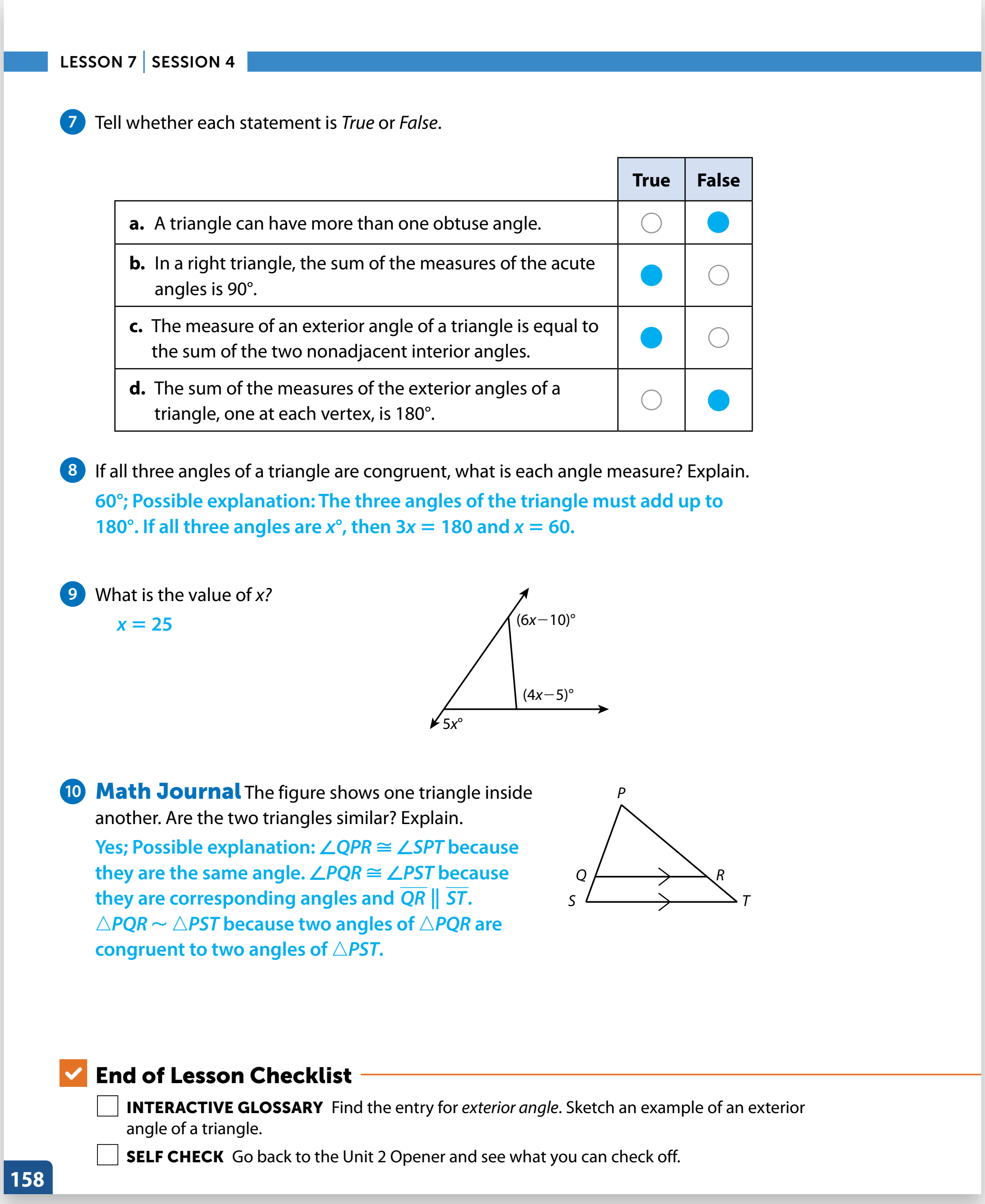
10 **Math Journal** Look for understanding that ∠*PQR* and ∠*PST* or ∠*PRQ* and ∠*PTS* are congruent corresponding angles.

Error Alert If students say the triangles are not similar, then be sure that they see that ∠*QPR* and ∠*SPT* are the same angle.

☑ End of Lesson Checklist

INTERACTIVE GLOSSARY Ask students to discuss the everyday meaning of *exterior* with a partner and relate this to an exterior angle of a triangle.

SELF CHECK Have students review and check off any new skills on the Unit 2 Opener.



LESSON 7 | SESSION 4

7 Tell whether each statement is *True* or *False*.

	True	False
a. A triangle can have more than one obtuse angle.	○	●
b. In a right triangle, the sum of the measures of the acute angles is 90°.	●	○
c. The measure of an exterior angle of a triangle is equal to the sum of the two nonadjacent interior angles.	●	○
d. The sum of the measures of the exterior angles of a triangle, one at each vertex, is 180°.	○	●

8 If all three angles of a triangle are congruent, what is each angle measure? Explain.

60°; Possible explanation: The three angles of the triangle must add up to 180°. If all three angles are $x°$, then $3x = 180$ and $x = 60$.

9 What is the value of x?

$x = 25$

10 **Math Journal** The figure shows one triangle inside another. Are the two triangles similar? Explain.

Yes; Possible explanation: ∠*QPR* ≅ ∠*SPT* because they are the same angle. ∠*PQR* ≅ ∠*PST* because they are corresponding angles and $\overline{QR} \parallel \overline{ST}$. △*PQR* ~ △*PST* because two angles of △*PQR* are congruent to two angles of △*PST*.

☑ **End of Lesson Checklist**

☐ **INTERACTIVE GLOSSARY** Find the entry for *exterior angle*. Sketch an example of an exterior angle of a triangle.

☐ **SELF CHECK** Go back to the Unit 2 Opener and see what you can check off.

158

REINFORCE

Problems 4–9

Solve angle sum and exterior angle problems.

Students meeting proficiency will benefit from additional work with angle sum and exterior angles of triangles by solving problems in a variety of formats.

- Have students work on their own or with a partner to solve the problems.
- Encourage students to show their work.

EXTEND

Challenge

Solve problems using the angle sum of triangles.

Students extending beyond proficiency will benefit from finding an expression for the sum of the angle measures of any polygon.

- Have students work with a partner to solve this problem: *What is the sum of the angle measures of a polygon with n sides?*
- Students may find the sum of the angle measures for a triangle, quadrilateral, pentagon, and hexagon and then look for a pattern they can use to write an expression for the angle sum for an *n*-sided polygon.

PERSONALIZE

i-Ready

Provide students with opportunities to work on their personalized instruction path with *i-Ready* Online Instruction to:

- fill prerequisite gaps.
- build up grade-level skills.

LESSON 27

Overview | Apply the Pythagorean Theorem

STANDARDS FOR MATHEMATICAL PRACTICE (SMP)

SMP 1, 2, 3, 4, 5, and 6 are integrated into the Try-Discuss-Connect routine.*

This lesson provides additional support for:

2 Reason abstractly and quantitatively.

4 Model with mathematics.

7 Look for and make use of structure.

* See page 1o to learn how every lesson includes these SMP.

Objectives

Content Objectives

- Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems, in two and three dimensions.
- Identify and use Pythagorean triples.
- Use the Pythagorean Theorem to find the distance between two points in the coordinate plane.

Language Objectives

- Provide reasons to explain in speaking and writing whether a set of numbers is a Pythagorean triple.
- Ask and answer questions about solution strategies for finding the distance between two points in the coordinate plane during partner and class discussion.
- Justify solutions to problems using the Pythagorean Theorem and the lesson vocabulary in speaking and in writing.

Prior Knowledge

- Find the square of a number.
- Find or estimate the square root of a number.
- Understand the Pythagorean Theorem and be able to explain geometric and algebraic proofs of it.

Vocabulary

Math Vocabulary

There is no new vocabulary. Review the following key terms.

multiple the product of a given number and any other whole number.

Pythagorean Theorem in any right triangle, the sum of the squares of the lengths of the legs, a and b, is equal to the square of the length of the hypotenuse, c. So, $a^2 + b^2 = c^2$.

right rectangular prism a right prism where the bases and other faces are all rectangles.

Academic Vocabulary

consider to think carefully.

Learning Progression

In Grades 6 and 7, students found areas of triangles and calculated distances between points on the same horizontal or vertical line in the coordinate plane.

Earlier in Grade 8, students evaluated expressions with exponents and found square roots of positive rational numbers.

In the previous lesson, students were introduced to the Pythagorean Theorem, and they explained proofs of the Pythagorean Theorem and its converse.

➤

In this lesson, students use the Pythagorean Theorem to find unknown lengths and distances in mathematical and real-world problems. The problems require them to find unknown side lengths in triangles, unknown diagonal lengths in rectangles and rectangular prisms, and distances between points in the coordinate plane.

➤

Later in Grade 8, students will apply the Pythagorean Theorem to find unknown dimensions in cylinders, cones, or spheres.

In high school, students will continue to use the Pythagorean Theorem to prove other theorems and solve applied problems involving triangles.

LESSON 27

Overview

Pacing Guide

Items marked with ↖ are available on the **Teacher Toolbox**.

	MATERIALS	DIFFERENTIATION
SESSION 1 **Explore** Applying the Pythagorean Theorem (35–50 min)		
• **Start** (5 min) • **Try It** (5–10 min) • **Discuss It** (10–15 min) • **Connect It** (10–15 min) • **Close: Exit Ticket** (5 min) **Additional Practice** (pages 633–634)	**Math Toolkit** grid paper, rulers, unit tiles Presentation Slides ↖	**PREPARE** Interactive Tutorial ↖ **RETEACH or REINFORCE** Hands-On Activity **Materials** For each group: 169 unit tiles
SESSION 2 **Develop** Finding an Unknown Length in a Right Triangle (45–60 min)		
• **Start** (5 min) • **Try It** (10–15 min) • **Discuss It** (10–15 min) • **Connect It** (15–20 min) • **Close: Exit Ticket** (5 min) **Additional Practice** (pages 639–640)	**Math Toolkit** centimeter grid paper, centimeter rulers, unit cubes Presentation Slides ↖	**RETEACH or REINFORCE** Hands-On Activity **Materials** For each pair: tape measure, yard stick, or meter stick **REINFORCE** Fluency & Skills Practice ↖ **EXTEND** Deepen Understanding
SESSION 3 **Develop** Finding an Unknown Length in a Three-Dimensional Figure (45–60 min)		
• **Start** (5 min) • **Try It** (10–15 min) • **Discuss It** (10–15 min) • **Connect It** (15–20 min) • **Close: Exit Ticket** (5 min) **Additional Practice** (pages 645–646)	**Math Toolkit** grid paper, rulers Presentation Slides ↖	**RETEACH or REINFORCE** Visual Model **Materials** For display: an open box, tape measure, string, tape **REINFORCE** Fluency & Skills Practice ↖ **EXTEND** Deepen Understanding
SESSION 4 **Develop** Finding Distance in the Coordinate Plane (45–60 min)		
• **Start** (5 min) • **Try It** (10–15 min) • **Discuss It** (10–15 min) • **Connect It** (15–20 min) • **Close: Exit Ticket** (5 min) **Additional Practice** (pages 651–652)	**Math Toolkit** compasses, graph paper, rulers, tracing paper Presentation Slides ↖	**RETEACH or REINFORCE** Visual Model **Materials** For display: large first-quadrant coordinate plane **REINFORCE** Fluency & Skills Practice ↖ **EXTEND** Deepen Understanding
SESSION 5 **Refine** Applying the Pythagorean Theorem (45–60 min)		
• **Start** (5 min) • **Monitor & Guide** (15–20 min) • **Group & Differentiate** (20–30 min) • **Close: Exit Ticket** (5 min)	**Math Toolkit** Have items from previous sessions available for students. Presentation Slides ↖	**RETEACH** Visual Model **Materials** For display: large four-quadrant coordinate plane **REINFORCE** Problems 4–7 **EXTEND** Challenge **PERSONALIZE** i-Ready
Lesson 27 Quiz ↖ or **Digital Comprehension Check**		**RETEACH** Tools for Instruction ↖ **REINFORCE** Math Center Activity ↖ **EXTEND** Enrichment Activity ↖

LESSON 27

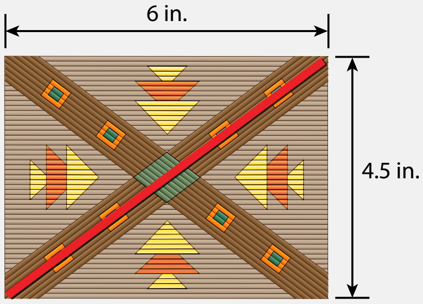
Overview | Apply the Pythagorean Theorem

Connect to Culture

➤ **Use these activities to connect with and leverage the diverse backgrounds and experiences of all students. Engage students in sharing what they know about contexts before you add the information given here.**

SESSION 1 ■ □ □ □ □

Try It Living primarily in the northeast United States and eastern Canada, the Mi’kmaq people are one of the largest groups of indigenous people in North America. Historically, the Mi’kmaq traveled from season to season as they hunted and fished for their food. Mi’kmaq clothing was made of deer and moose skin, with fur robes for winter. Clothing was often decorated with colored porcupine quills. Ask students to share anything else they know about the Mi’kmaq people.



SESSION 2 ■ ■ □ □ □

Try It Ask students whether they have ever seen a cat stuck in a tree. If so, how did the cat get down? Because their claws curve toward the back, cats are good at climbing up, but many cats do not know how to climb down. When a cat tries to climb down, its claws point up and cannot hold the cat’s body against the tree. This makes the cat think it is going to fall, so it stops trying. To be able to climb down, the cat must do so backward, with its tail to the ground.

SESSION 3 ■ ■ ■ □ □

Try It Ask students if they have ever used a paintbrush to paint a craft. Have students describe the different sizes of paintbrushes and explain why someone making a craft might want to use a larger or a smaller brush. Each paintbrush size is associated with a number, with greater numbers representing larger brushes. A paintbrush numbered *0* is very small and a paintbrush numbered *40* is large. The numbering system was invented before scale modeling became popular, so now brushes smaller than size *0* exist, down to size *00000*.

SESSION 5 ■ ■ ■ ■ ■

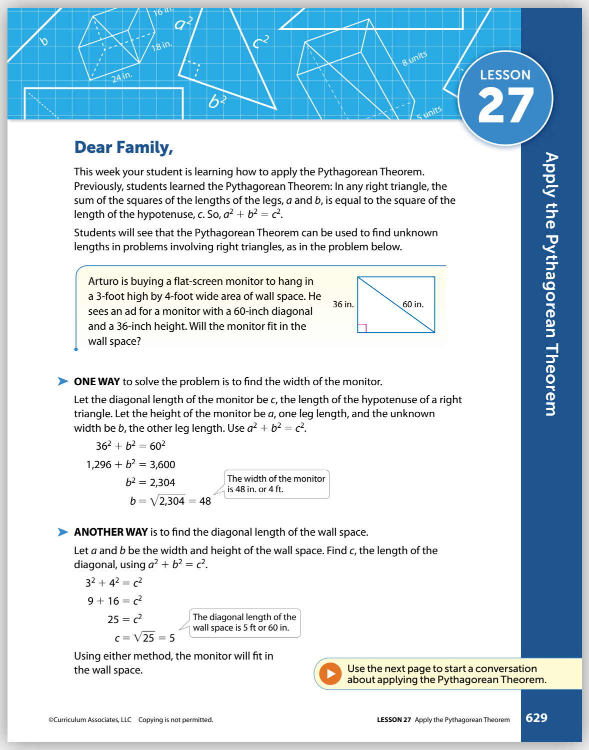
Apply It **Problem 5** Ask students who keep jewelry or other items in plastic storage cubes to describe what they like about these cubes and what they wish was different about the cubes. A storage cube with an edge length of 6 inches can also serve as a decoration on a nightstand or other shelf. Small items can fit in a storage cube this size, but putting too much in a small cube makes items hard to remove. Other advantages of a small storage cube are the ease of moving it to another location and packing it in luggage when needed.

LESSON 27

Overview

Connect to Family and Community

After the Explore session, have students use the Family Letter to let their families know what they are learning and to encourage family involvement.



LESSON 27

Apply the Pythagorean Theorem

Dear Family,

This week your student is learning how to apply the Pythagorean Theorem. Previously, students learned the Pythagorean Theorem: In any right triangle, the sum of the squares of the lengths of the legs, a and b, is equal to the square of the length of the hypotenuse, c. So, $a^2 + b^2 = c^2$.

Students will see that the Pythagorean Theorem can be used to find unknown lengths in problems involving right triangles, as in the problem below.

Arturo is buying a flat-screen monitor to hang in a 3-foot high by 4-foot wide area of wall space. He sees an ad for a monitor with a 60-inch diagonal and a 36-inch height. Will the monitor fit in the wall space?

➤ **ONE WAY** to solve the problem is to find the width of the monitor.

Let the diagonal length of the monitor be c, the length of the hypotenuse of a right triangle. Let the height of the monitor be a, one leg length, and the unknown width be b, the other leg length. Use $a^2 + b^2 = c^2$.

$$36^2 + b^2 = 60^2$$
$$1{,}296 + b^2 = 3{,}600$$
$$b^2 = 2{,}304$$
$$b = \sqrt{2{,}304} = 48$$

The width of the monitor is 48 in. or 4 ft.

➤ **ANOTHER WAY** is to find the diagonal length of the wall space.

Let a and b be the width and height of the wall space. Find c, the length of the diagonal, using $a^2 + b^2 = c^2$.

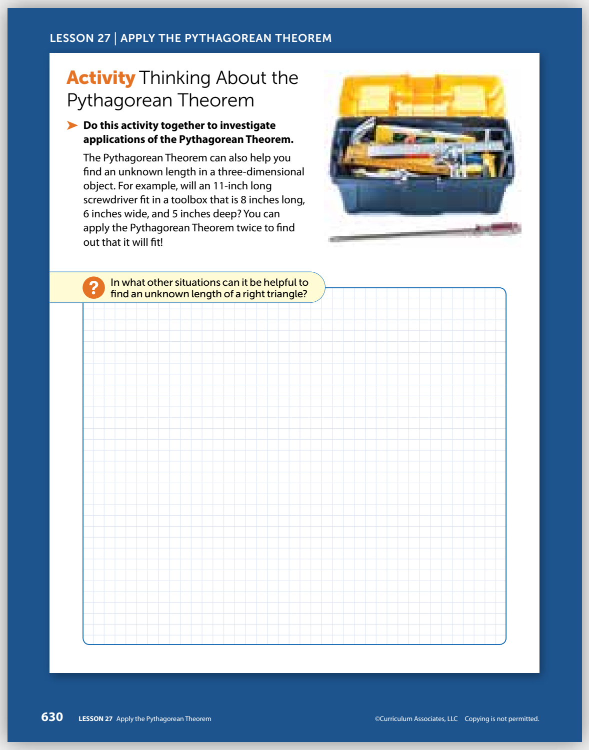
$$3^2 + 4^2 = c^2$$
$$9 + 16 = c^2$$
$$25 = c^2$$
$$c = \sqrt{25} = 5$$

The diagonal length of the wall space is 5 ft or 60 in.

Using either method, the monitor will fit in the wall space.

Use the next page to start a conversation about applying the Pythagorean Theorem.

©Curriculum Associates, LLC Copying is not permitted. **LESSON 27** Apply the Pythagorean Theorem 629



LESSON 27 | APPLY THE PYTHAGOREAN THEOREM

Activity Thinking About the Pythagorean Theorem

➤ **Do this activity together to investigate applications of the Pythagorean Theorem.**

The Pythagorean Theorem can also help you find an unknown length in a three-dimensional object. For example, will an 11-inch long screwdriver fit in a toolbox that is 8 inches long, 6 inches wide, and 5 inches deep? You can apply the Pythagorean Theorem twice to find out that it will fit!

In what other situations can it be helpful to find an unknown length of a right triangle?

630 **LESSON 27** Apply the Pythagorean Theorem ©Curriculum Associates, LLC Copying is not permitted.

Connect to Language

For English language learners, use the Differentiation chart to scaffold the language in each session. Use the Academic Vocabulary routine for academic terms before Session 1.

DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Use with **Session 1** **Connect It**

MATH TERM

A *theorem* is an idea that can be proved using a chain of logic and reasoning.

ACADEMIC VOCABULARY

A *set* is a group or collection of things.

A *triple* is made up of three units or things.

Levels 1–3: Reading/Speaking

Help students interpret Connect It problem 2. Read problem 2a with students. Guide them to circle the equation and *Pythagorean Theorem*. Display the theorem and say: *This theorem is named after the Greek mathematician Pythagoras.* Help students summarize the theorem:

- *The Pythagorean ____ describes properties of ____ triangles.*

Next, read problem 2b with students and have them circle *Pythagorean triple*. Then use **Act It Out** to guide partners to draw a 3, 4, 5 right triangle and show that the equation is true. Then ask students to identify the Pythagorean triples.

Levels 2–4: Reading/Speaking

Prepare students to interpret Connect It problem 2. Display *theorem, triple,* and *set,* and have students discuss meanings. Point out the Latin prefix *tri-*, meaning 3. Ask students to list related words, like *triangle* and *tripod,* and to tell the word for 3 in other languages (e.g., *tres, trois*).

Next, help students make sense of problems 2a and 2b. Clarify that the capital P in Pythagorean refers to the name of a mathematician. Have them use **Act It Out** to draw the 3, 4, 5 right triangle described in problem 2b and summarize:

- *Three whole-number_____of a right triangle are called a _____.*

Levels 3–5: Reading/Speaking

Have students prepare for Connect It problem 2 by reviewing the terms *theorem* and *triple*. Ask students to discuss meanings and generate words with the same prefixes (e.g., *theorem, theory, triple, triangle, tripod*).

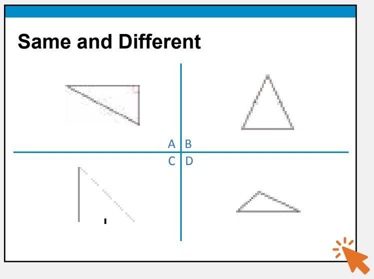
Next, have students work in pairs to make sense of problems 2a and 2b. Have partners discuss how Pythagorean triples relate to the theorem. Encourage students to identify math terms in both parts, like *lengths* and *right triangle.* For extra support, have students use **Act It Out** to draw the 3, 4, 5 right triangle described in problem 2b.

Explore Applying the Pythagorean Theorem

Purpose

- **Explore** the idea that you can use the Pythagorean Theorem to find the length of the hypotenuse of a right triangle when you are given the lengths of the legs.
- **Understand** that you can apply the Pythagorean Theorem in real-world situations to find unknown lengths in a right triangle.

START CONNECT TO PRIOR KNOWLEDGE


Same and Different

Possible Solutions

All are triangles.

A and D are scalene triangles.

A and C are right triangles.

B and C are isosceles triangles.

WHY? Support students' facility with recognizing types of triangles.

TRY IT

SMP 1, 2, 4, 5, 6

Make Sense of the Problem

See ***Connect to Culture*** to support student engagement. Before students work on Try It, use **Say It Another Way** to help them make sense of the problem.

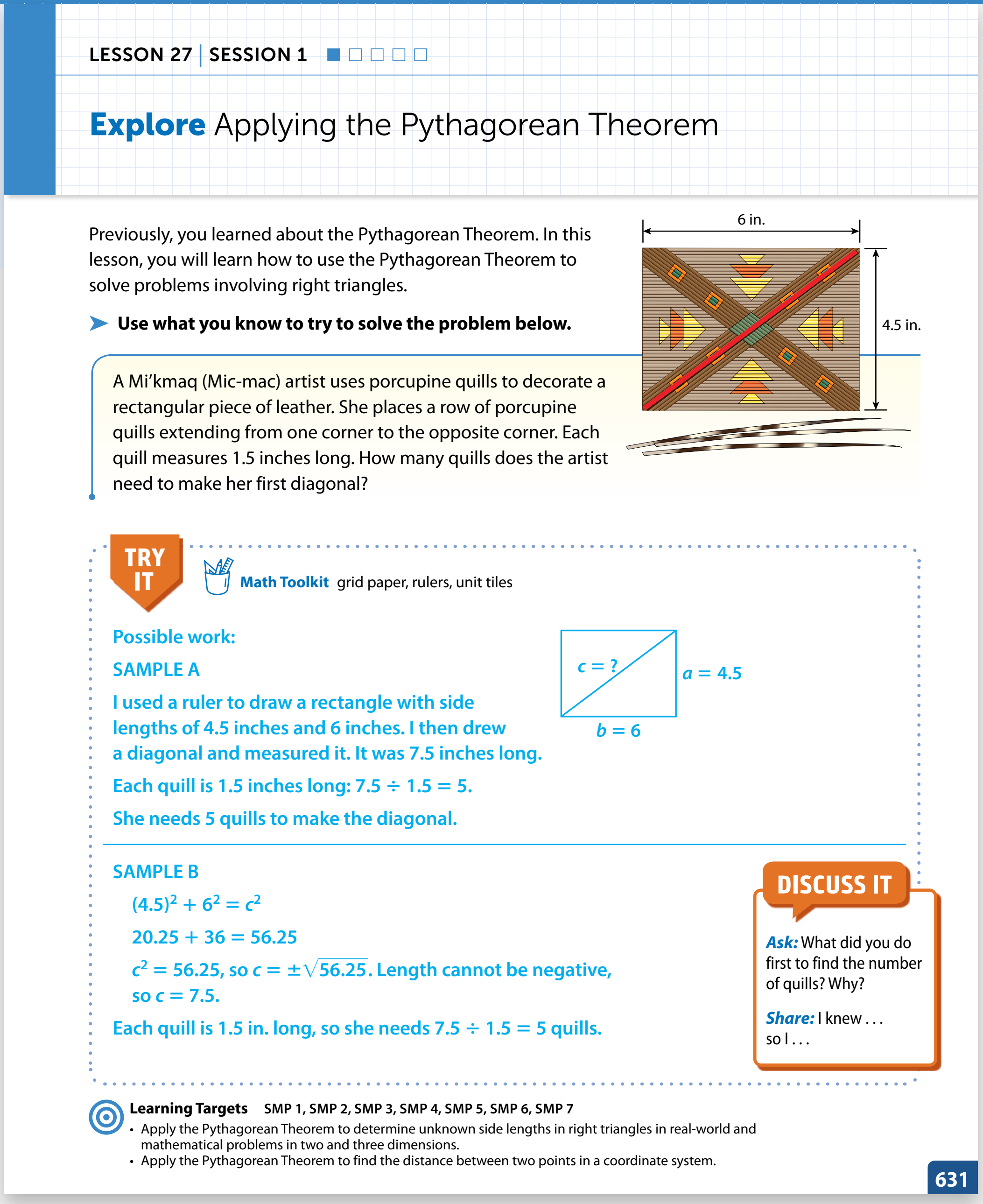
DISCUSS IT

SMP 1, 2, 3, 6

Support Partner Discussion

After students work on Try It, have them respond to Discuss It with a partner. Listen for understanding of:

- the relationship between the side lengths and the length of the diagonal of the piece of leather.
- the relationship between the length of the diagonal of the piece of leather and the length of each quill.


LESSON 27 | SESSION 1 ■□□□□

Explore Applying the Pythagorean Theorem

Previously, you learned about the Pythagorean Theorem. In this lesson, you will learn how to use the Pythagorean Theorem to solve problems involving right triangles.

➤ **Use what you know to try to solve the problem below.**

A Mi'kmaq (Mic-mac) artist uses porcupine quills to decorate a rectangular piece of leather. She places a row of porcupine quills extending from one corner to the opposite corner. Each quill measures 1.5 inches long. How many quills does the artist need to make her first diagonal?

TRY IT **Math Toolkit** grid paper, rulers, unit tiles

Possible work:

SAMPLE A

I used a ruler to draw a rectangle with side lengths of 4.5 inches and 6 inches. I then drew a diagonal and measured it. It was 7.5 inches long.

Each quill is 1.5 inches long: 7.5 ÷ 1.5 = 5.

She needs 5 quills to make the diagonal.

SAMPLE B

$(4.5)^2 + 6^2 = c^2$

$20.25 + 36 = 56.25$

$c^2 = 56.25$, so $c = \pm\sqrt{56.25}$. Length cannot be negative, so $c = 7.5$.

Each quill is 1.5 in. long, so she needs 7.5 ÷ 1.5 = 5 quills.

DISCUSS IT

Ask: What did you do first to find the number of quills? Why?

Share: I knew . . . so I . . .

Learning Targets SMP 1, SMP 2, SMP 3, SMP 4, SMP 5, SMP 6, SMP 7

- Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.
- Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

631

Error Alert If students think the number of quills needed is 7.5, then they neglected to divide the length of the diagonal by 1.5, the length of each quill. Make sure students understand that the problem is asking for the number of quills needed to make the diagonal, rather than the length of the diagonal in inches.

Select and Sequence Student Strategies

Select 2–3 samples that represent the range of student thinking in your classroom. Here is one possible order for class discussion:

- rectangle with side lengths 4.5 in. and 6 in. drawn and diagonal measured
- sides of drawing measured and proportion solved to find diagonal length of actual rectangle
- Pythagorean Theorem used to find diagonal length

Facilitate Whole Class Discussion

Call on students to share selected strategies. Encourage students to speak clearly and loudly so that everyone can hear.

Guide students to **Compare and Connect** the representations. Prompt students to ask for more information as needed during the discussion.

> **ASK** *How did these strategies use math that you learned in earlier lessons?*
>
> **LISTEN FOR** Some strategies used the Pythagorean Theorem, or squaring and finding square roots, and others used proportions.

CONNECT IT

SMP 2, 4, 5

① **Look Back** Look for understanding that the length of the diagonal is 7.5 inches, and it takes 5 quills to form that length.

DIFFERENTIATION | RETEACH

Hands-On Activity

Use unit tiles to find the hypotenuse of a right triangle, given the leg lengths.

If students are unsure about finding the length of the hypotenuse of a right triangle if they know the lengths of the legs, then use this activity to help them visualize the relationship.

Materials For each group: 169 unit tiles

- Display a triangle with legs of length 5 and 12.
- Tell students to model the 5-unit leg with 5 unit tiles. Then tell them to make a square out of tiles with side length 5 units.
- Ask: *How many unit tiles are in the square?* [25]
- Tell students to model the 12-unit leg with 12 unit tiles. Then tell them to make a square out of tiles with side length 12 units.
- Ask: *How many unit tiles are in the square?* [144]
- Ask: *How many unit tiles are used in all?* [169]
- Have students take apart their squares and use the combined tiles to make one larger square.
- Ask: *What is the side length of the new large square?* [13]
- Ask: *How is this length related to the triangle?* [13 units is the length of the hypotenuse.]
- Discuss with students how this activity shows the steps in finding the length of the hypotenuse when you know the lengths of the legs: 1) Square the length of each leg; 2) add the squares of the leg lengths; 3) take the square root of the sum.

CONNECT IT

① **Look Back** How many quills are in the diagonal? How do you know?

5; Possible answer: I drew a right triangle and used the Pythagorean Theorem to find the length of the hypotenuse. Then, I divided this length by 1.5 to find the number of quills.

② **Look Ahead**

a. Fill in the blanks to restate the Pythagorean Theorem.

In any __right__ triangle, the sum of the squares of the lengths of the __legs__ is equal to the square of the length of the __hypotenuse__.

If *a* and *b* are the lengths of the legs of a right triangle and *c* is the length of the hypotenuse, then the theorem can be represented by the equation __$a^2 + b^2 = c^2$__.

b. When the side lengths of a right triangle are whole numbers, the lengths are known as *Pythagorean triples*. For example, the set of lengths 3, 4, 5 is a Pythagorean triple because $3^2 + 4^2 = 5^2$. Look at each set of three numbers shown on the right. Circle each set that is a Pythagorean triple. See diagram.

2, 3, 4
4, 5, 6
[✓ 5, 12, 13]
[✓ 6, 8, 10]
[✓ 8, 15, 17]
9, 16, 25

c. Jason says that if you multiply each length in a Pythagorean triple by the same factor, you will get another Pythagorean triple. Explain why Jason is correct.

Possible explanation: Let the set of lengths *a*, *b*, *c* be a Pythagorean triple such that $a^2 + b^2 = c^2$. If $2a$, $2b$, $2c$ is also a Pythagorean triple, then $(2a)^2 + (2b)^2 = (2c)^2$ or $4a^2 + 4b^2 = 4c^2$. If you divide both sides of this equation by 4, you get $a^2 + b^2 = c^2$, so $2a$, $2b$, $2c$ is a Pythagorean triple. Any multiple will work the same way.

③ **Reflect** Give an example to show that a multiple of a Pythagorean triple is also a Pythagorean triple.

Possible answer: If you multiply the lengths in the Pythagorean triple 3, 4, 5 by 3, you get the lengths 9, 12, 15 which is also a Pythagorean triple: $9^2 + 12^2 = 15^2 \longrightarrow 81 + 144 = 225$.

② **Look Ahead** Students should recognize that the Pythagorean Theorem applies only to right triangles and that, in a right triangle, the legs are the two shorter sides and the hypotenuse is the longest side.

Ask a volunteer to rephrase the information about *Pythagorean triples*. Support student understanding by having a volunteer explain why 6, 8, 10 is a Pythagorean triple, while 4, 5, 6 is not.

CLOSE **EXIT TICKET**

③ **Reflect** Look for understanding of generating a new Pythagorean triple by multiplying each length in a Pythagorean triple by the same factor. Note that the new lengths must also be whole numbers.

Error Alert If students find an example where multiplying a Pythagorean triple by a factor does not produce another Pythagorean triple, then have them work with a partner to check their work. They should check their original Pythagorean triple, their multiplication, that they correctly squared each new number, and that they correctly compared the sum of the two smaller squares to the largest square.

Prepare for Applying the Pythagorean Theorem

Support Vocabulary Development

Assign **Prepare for Applying the Pythagorean Theorem** as extra practice in class or as homework.

If you have students complete this in class, then use the guidance below.

Ask students to consider the term *Pythagorean Theorem*. Ensure that students understand when the theorem can be used and when it cannot be used.

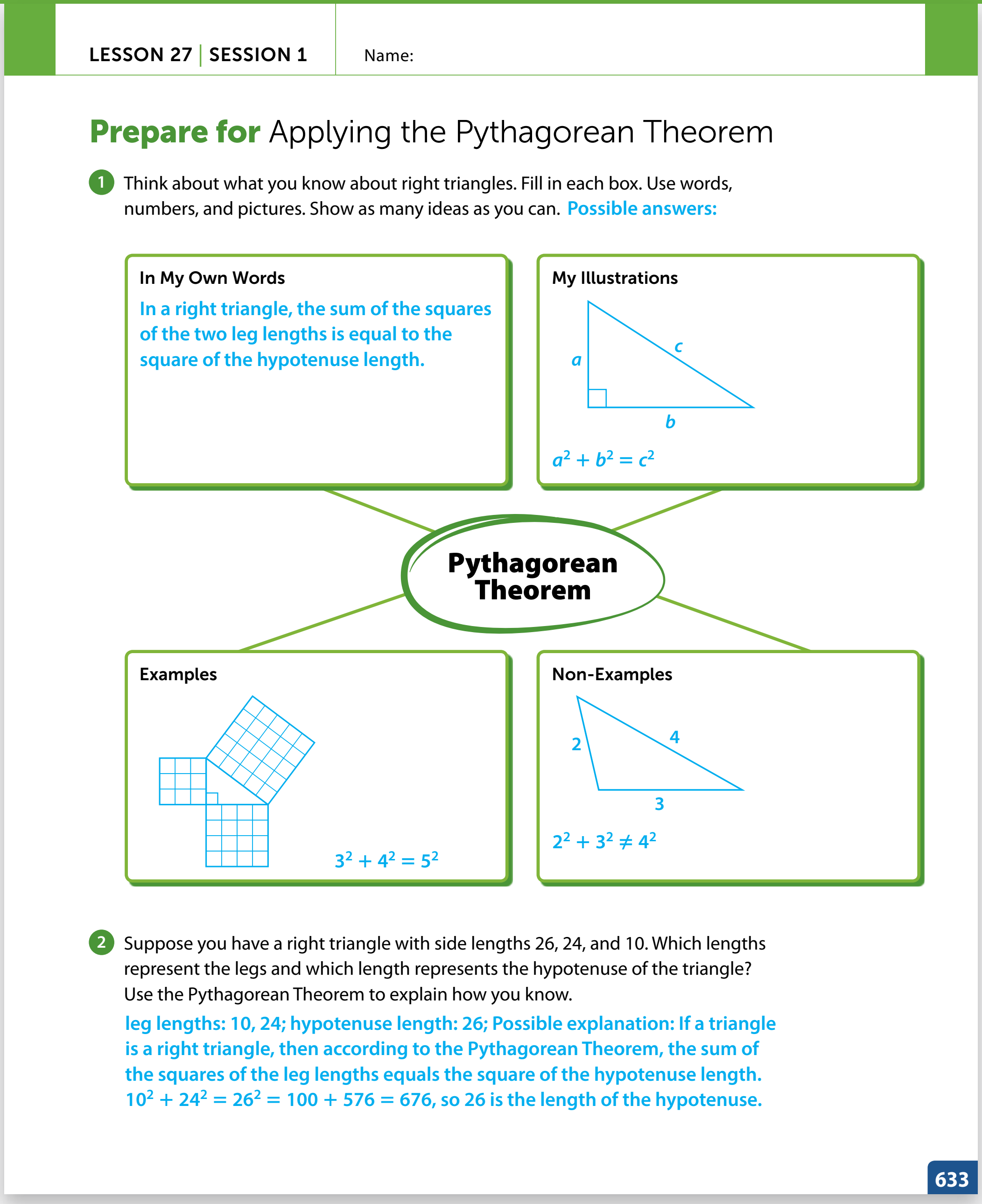
Have students work individually to complete the graphic organizer. Invite students to share their completed organizers, and prompt a whole-class comparative discussion of the definitions, illustrations, examples, and non-examples they used in their graphic organizers.

Have students look at the question in problem 2 and discuss with a partner what the triangle might look like. Have students draw a diagram to support their thinking.

Problem Notes

1. Students should understand that the Pythagorean Theorem states that in a right triangle, the sum of the squares of the two leg lengths is equal to the square of the hypotenuse length. Student illustrations might include an image of a right triangle with legs labeled *a* and *b* and hypotenuse labeled *c* and the equation $a^2 + b^2 = c^2$. Students should recognize this theorem applies to right triangles, so if a triangle is acute or obtuse, the Pythagorean Theorem cannot be used.

2. Students may reason that the hypotenuse is always the longest side in a right triangle.



LESSON 27 | SESSION 1 Name:

Prepare for Applying the Pythagorean Theorem

1. Think about what you know about right triangles. Fill in each box. Use words, numbers, and pictures. Show as many ideas as you can. **Possible answers:**

Pythagorean Theorem

In My Own Words

In a right triangle, the sum of the squares of the two leg lengths is equal to the square of the hypotenuse length.

My Illustrations

$a^2 + b^2 = c^2$

Examples

$3^2 + 4^2 = 5^2$

Non-Examples

$2^2 + 3^2 \neq 4^2$

2. Suppose you have a right triangle with side lengths 26, 24, and 10. Which lengths represent the legs and which length represents the hypotenuse of the triangle? Use the Pythagorean Theorem to explain how you know.

leg lengths: 10, 24; hypotenuse length: 26; Possible explanation: If a triangle is a right triangle, then according to the Pythagorean Theorem, the sum of the squares of the leg lengths equals the square of the hypotenuse length. $10^2 + 24^2 = 26^2 = 100 + 576 = 676$, so 26 is the length of the hypotenuse.

633

REAL-WORLD CONNECTION

Carpenters use the Pythagorean Theorem in their constructions. Suppose a shed is being built such that its front shape is a rectangle with two congruent right triangles on top. The right angles of the triangles are back to back, and the hypotenuses lie along the angles of the roof. To help calculate the amount of roofing material needed, a carpenter will need to know the length of the hypotenuse for each of the right triangles. She can use the height of the triangle and half of the width of the shed along with the Pythagorean Theorem to find the length of each triangle's hypotenuse. Ask students to think of other real-world examples when applying the Pythagorean Theorem might be useful.

③ Problem 3 provides another look at applying the Pythagorean Theorem. This problem is similar to the problem about an artist using porcupine quills to decorate a rectangular piece of leather. In both problems, two side lengths of a right triangle are given, and students use the Pythagorean Theorem to find the length of the hypotenuse. This problem asks for the length of ribbon needed to make two diagonals of a flag. Note that since we are finding an approximate length, there is no need to worry about the area of overlap of the ribbons.

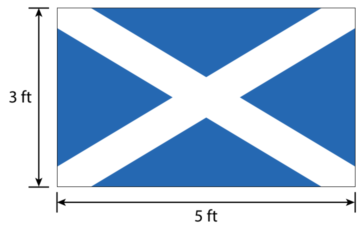
Students may want to use unit tiles or grid paper to solve.

Suggest that students use **Three Reads** to help them understand what the problem is about, what is being asked, and what information they are given.

LESSON 27 | SESSION 1

③ The flag of Scotland consists of a blue rectangular background with two white diagonals.

a. The dimensions of a Scottish flag are shown. If the flag's diagonals are made of white ribbon, what length of ribbon is needed for both diagonals? Show your work. Round your answer to the nearest foot.



Possible work:

$$3^2 + 5^2 = c^2$$
$$9 + 25 = c^2$$
$$34 = c^2$$
$$\pm\sqrt{34} = c$$

c must be positive, so $\sqrt{34} \approx 6$.

About 6 ft are needed for one ribbon, so about 12 ft are needed for two.

SOLUTION The total length of ribbon needed is about 12 ft.

b. Check your answer to problem 3a. Show your work.

Possible work:

Check that $\sqrt{34}$ is the correct diagonal length.

$$\left(\sqrt{34}\right)^2 = 3^2 + 5^2$$
$$34 = 9 + 25$$
$$34 = 34$$

$\sqrt{34} \approx 6$ is the length of one diagonal, so double it for two diagonals.

About 12 ft is correct.

634

DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Use with **Session 2** **Apply It**

Levels 1–3: Reading/Speaking

Help students make sense of Apply It problem 6. First, help students understand the problem context by reading aloud sentence 1 and using **Act It Out**. Guide students to label the banner, roof, and fence on the diagram. Then provide a string or rope and ask students to show what it means to string it from the top of one thing to another. Next, ask students to study the diagram and underline the question in the problem. Rephrase: *What is the height of the roof?* Have students trace the line on the diagram to show what they need to find. Encourage them to use math terms like *height, right triangle,* and *side length* to answer. Before having them solve the problem, read each sentence aloud and ask partners to find measurements on the diagram.

Levels 2–4: Reading/Speaking

Help students make sense of Apply It problem 6 by previewing the diagram. Give students time to study and label things in the diagram. [Possible terms: *roof, fence, banner, height, side lengths, distance, right triangle*] Then call on students to describe the diagram.

Next, help students read and make sense of the problem by discussing the meaning of compound and open compound words, such as *salesman, fence pole, banner rope,* and *rooftop.* Then have students use **Say It Another Way** to confirm understanding. Afterwards, ask students to identify the question and to tell how they can use the diagram to solve the problem.

Levels 3–5: Reading/Speaking

Have students read and interpret Apply it problem 6 using an adaptation of **Three Reads**. After each read, have students discuss the focus question with partners. Then prompt students to make connections between the problem and the diagram. Give students time to think about their answers before sharing. Provide sentence starters to guide discussions:

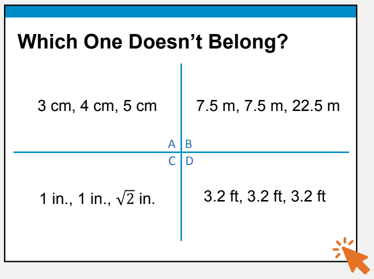
- *The diagram shows* _____.
- *The diagram helps me understand* _____.
- *How does* _____ *connect to the problem?*
- *Can you show me how* _____ *connects to the diagram?*
- *That makes sense. I also think* _____.

Develop Finding an Unknown Length in a Right Triangle

Purpose

- **Develop** strategies for finding an unknown leg length in a right triangle.
- **Recognize** that the Pythagorean Theorem can be used to find any unknown side length in a right triangle if the other two side lengths are known.

START CONNECT TO PRIOR KNOWLEDGE



Which One Doesn't Belong?

A 3 cm, 4 cm, 5 cm	B 7.5 m, 7.5 m, 22.5 m
C 1 in., 1 in., $\sqrt{2}$ in.	D 3.2 ft, 3.2 ft, 3.2 ft

Possible Solutions

A is the only set that involves a Pythagorean triple.

B is the only set that cannot be side lengths of a triangle.

C is the only set with an irrational length value.

D is the only set that can be side lengths of an equilateral triangle.

WHY? Support students' facility with recognizing side lengths that can represent different types of triangles, as well as those that cannot represent the side lengths of a triangle.

DEVELOP ACADEMIC LANGUAGE

WHY? Support understanding of the academic word *consider* with synonyms and antonyms.

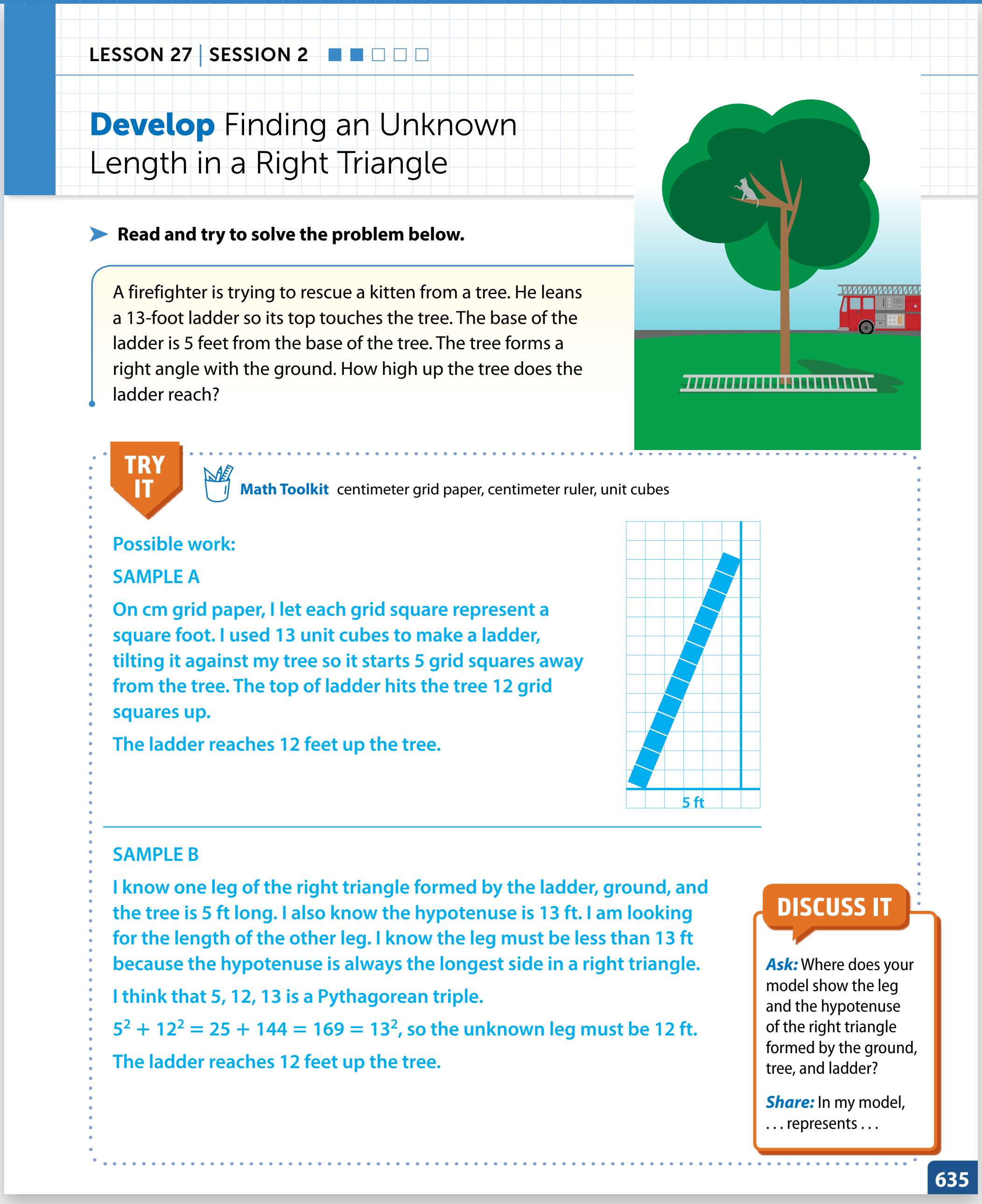
HOW? Write the following words and phrases in random order on the board: *disregard, mind, note, take into account, think about, ignore, neglect, examine, evaluate, pay no attention*. Have students think about the meaning of *consider* and tell how they can sort the words. Then read Connect It problem 2. Invite students to use synonyms or antonyms to talk about their answers. (Ex. *I can disregard . . . .*)

TRY IT

SMP 1, 2, 4, 5, 6

Make Sense of the Problem

See ***Connect to Culture*** to support student engagement. Before students work on Try It, use **Notice and Wonder** to help them make sense of the problem.



LESSON 27 | SESSION 2 ■■□□□

Develop Finding an Unknown Length in a Right Triangle

➤ **Read and try to solve the problem below.**

A firefighter is trying to rescue a kitten from a tree. He leans a 13-foot ladder so its top touches the tree. The base of the ladder is 5 feet from the base of the tree. The tree forms a right angle with the ground. How high up the tree does the ladder reach?

TRY IT **Math Toolkit** centimeter grid paper, centimeter ruler, unit cubes

Possible work:

SAMPLE A

On cm grid paper, I let each grid square represent a square foot. I used 13 unit cubes to make a ladder, tilting it against my tree so it starts 5 grid squares away from the tree. The top of ladder hits the tree 12 grid squares up.

The ladder reaches 12 feet up the tree.

SAMPLE B

I know one leg of the right triangle formed by the ladder, ground, and the tree is 5 ft long. I also know the hypotenuse is 13 ft. I am looking for the length of the other leg. I know the leg must be less than 13 ft because the hypotenuse is always the longest side in a right triangle.

I think that 5, 12, 13 is a Pythagorean triple.

$5^2 + 12^2 = 25 + 144 = 169 = 13^2$, so the unknown leg must be 12 ft.

The ladder reaches 12 feet up the tree.

DISCUSS IT

Ask: Where does your model show the leg and the hypotenuse of the right triangle formed by the ground, tree, and ladder?

Share: In my model, . . . represents . . .

635

DISCUSS IT

SMP 2, 3, 4, 5, 6

Support Partner Discussion

After students work on Try It, encourage them to respond to Discuss It with a partner. If students need support in getting started, prompt them to ask each other questions such as:

- *Where is the right angle in your model?*
- *What object represents the hypotenuse?*
- *What does the tree represent?*
- *Which lengths do you know, and which length is unknown?*

Common Misconception Listen for students who think that the unknown length can be found by adding the squares of the known lengths and taking the square root of the sum. As students share their strategies, have them identify the legs and the hypotenuse in a labeled drawing of the situation. Elicit the fact that the unknown length is a leg of a right triangle, not the hypotenuse, so it is the value of b in $5^2 + b^2 = 13^2$. Discuss how to solve this equation.

Select and Sequence Student Strategies

Select 2–3 samples that represent the range of student thinking in your classroom. Here is one possible order for class discussion:

- model drawn on grid paper and cubes or a ruler used to find unknown length
- (misconception) squares of known lengths added and square root of the sum taken to find the unknown length
- Pythagorean Theorem correctly used to find the unknown leg length

Facilitate Whole Class Discussion

Call on students to share selected strategies. Prompt students to describe what they assumed about the problem and what they decided to do as a result.

Guide students to **Compare and Connect** the representations. Allow individual think time for students to consider the different ideas.

> **ASK** *How do these strategies use right triangles?*
>
> **LISTEN FOR** The drawings show a right triangle with the ladder as the hypotenuse, the distance from the base of the ladder to the tree as one leg and the distance from the bottom of the tree to the top of the ladder as the other. The known and unknown lengths form a right triangle, so the Pythagorean Theorem can be used.

Picture It & Model It

If students presented these models, have students connect these models to those presented in class.

If no student presented at least one of these models, have students first analyze key features of the models, and then connect them to the models presented in class.

> **ASK** *How are the strategies in Picture It and Model It related?*
>
> **LISTEN FOR** The picture shows the right triangle that is formed and what the known and unknown lengths are. The model applies the Pythagorean Theorem to find the unknown length.

For the drawing, prompt students to compare the parts of the sketch to the key information provided in the problem. Ask: *How do you determine which length in the drawing is unknown?*

For the Pythagorean Theorem, prompt students to notice where the information from the description is substituted into the equation.

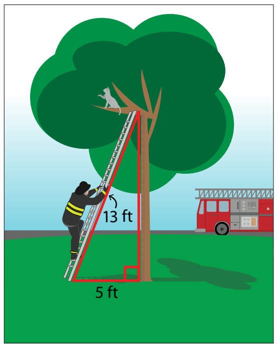
- *What does b represent in the drawing?*
- *Why is 13^2 the sum in the equation?*

➤ Explore different ways to find how to use the Pythagorean Theorem to find an unknown length.

A firefighter is trying to rescue a kitten from a tree. He leans a 13-foot ladder so its top touches the tree. The base of the ladder is 5 feet from the base of the tree. The tree forms a right angle with the ground. How high up the tree does the ladder reach?

Picture It

You can draw a picture to show the information you are given and what you still need to find.



Model It

You can use the Pythagorean Theorem.

The ladder represents the hypotenuse of the right triangle formed by the ladder, tree, and ground. Let *b* be the length you need to find.

$$5^2 + b^2 = 13^2$$
$$25 + b^2 = 169$$
$$b^2 = 169 - 25$$
$$b^2 = 144$$
$$b = \pm\sqrt{144}$$

DIFFERENTIATION | EXTEND

SMP 4

Deepen Understanding

Modeling a Problem to Help Understand How to Solve It

Prompt students to consider this scenario: *The firefighter knows the kitten in the tree is 12 feet from the ground. He wants to ensure that he can reach the kitten with the 13-foot ladder. How far from the base of the tree should he place the ladder?*

> **ASK** *How would the drawing for this scenario be different?*
>
> **LISTEN FOR** It would look the same, but the length of the vertical leg would be labeled 12 ft, and the length of the horizontal leg would be unknown.

> **ASK** *How would it change the computation?*
>
> **LISTEN FOR** If a is the unknown length, you need to solve $a^2 + 12^2 = 13^2$, or $a^2 + 144 = 169$. You would subtract 144 from both sides of the equation to get $a^2 = 25$, and then take the square root of both sides of the equation.

Ask students to consider another related scenario: *The firefighter has an extension ladder. The kitten in the tree is 12 feet from the ground. If the firefighter puts the base of the ladder 5 feet from the tree, how long will he need to make the ladder to reach the kitten?* Discuss how the drawing and computation would change. In this scenario, the unknown length is the hypotenuse and the equation is $5^2 + 12^2 = c^2$.

Develop Finding an Unknown Length in a Right Triangle

CONNECT IT

SMP 2, 3, 4, 5, 6

Remind students that the quantities and the relationships between them are the same in each representation. Explain that they will now use those representations to reason about how to use the Pythagorean Theorem to find an unknown length.

Before students begin to record and expand on their work in Picture It & Model It, tell them that problem 3 will prepare them to provide the explanation asked for in problem 4.

Monitor and Confirm Understanding ① – ②

- The tree, the ground, and the ladder form a right triangle.
- The length of the hypotenuse is 13 ft, the length of the ladder. The length of one leg is 5 ft, the distance from the base of the ladder to the base of the tree. The length of the other leg is the unknown distance, b ft.
- The unknown distance can be found by solving $5^2 + b^2 = 13^2$.
- The solution of the equation is $b = \pm 12$. Because b represents a length, only the positive solution, 12, makes sense.

Facilitate Whole Class Discussion

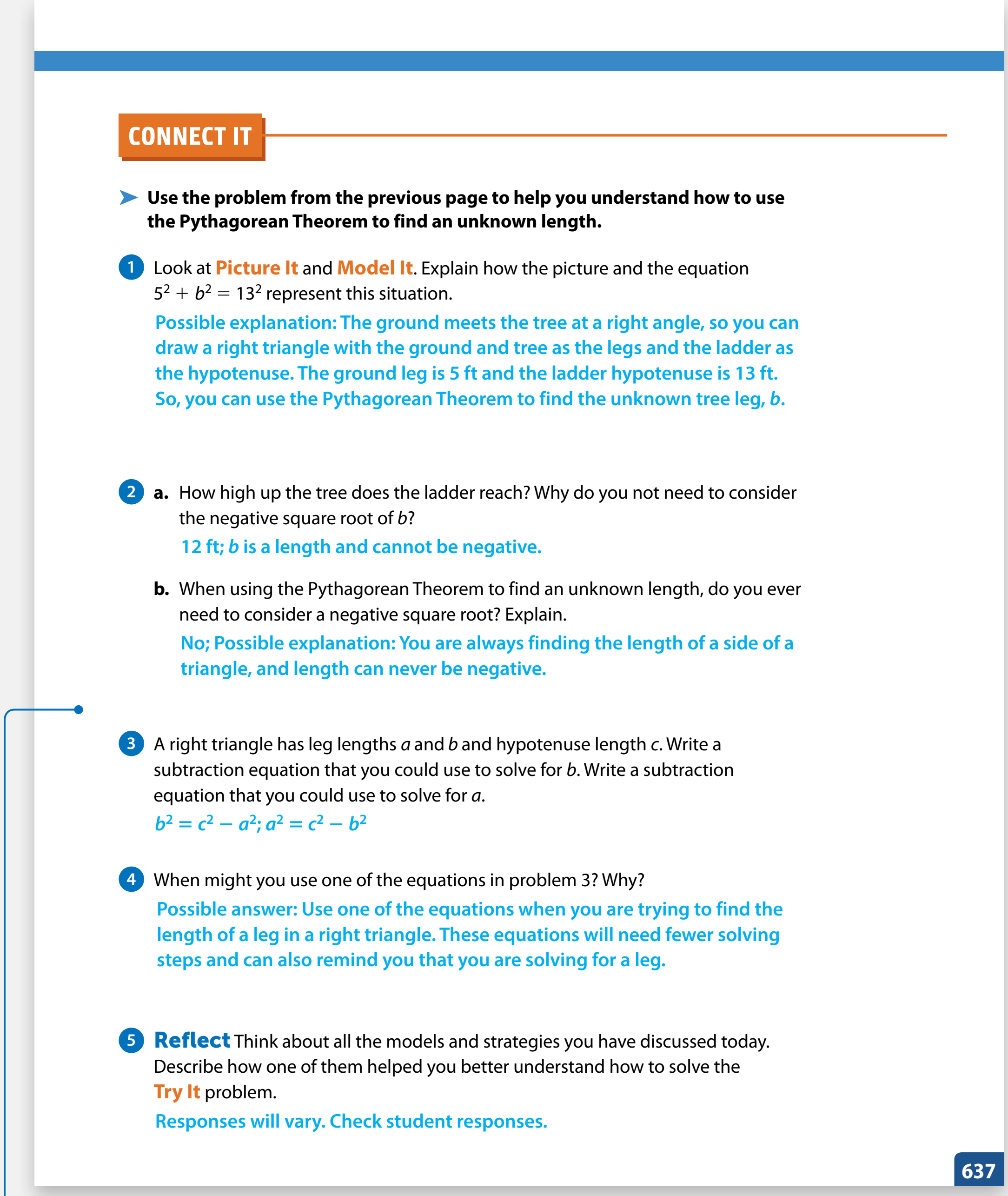
③ Students should begin with the formula for the Pythagorean Theorem, $a^2 + b^2 = c^2$, where a and b are the leg lengths and c is the hypotenuse length of a right triangle.

> **ASK** *Why does this problem specify subtraction equations?*
>
> **LISTEN FOR** To isolate either the a^2 or b^2 term, you have to subtract the other from each side of the equation.

④ Look for the idea that the Pythagorean Theorem can be used to find an unknown leg length when the lengths of the other leg and the hypotenuse are known.

> **ASK** *What do you need to know in order to apply the Pythagorean Theorem to find an unknown side length of a triangle?*
>
> **LISTEN FOR** You need to know that the triangle is a right triangle, and you need to know two of the side lengths.

⑤ **Reflect** Have all students focus on the strategies used to solve the Try It. If time allows, have students discuss their ideas with a partner.



CONNECT IT

➤ **Use the problem from the previous page to help you understand how to use the Pythagorean Theorem to find an unknown length.**

① Look at **Picture It** and **Model It**. Explain how the picture and the equation $5^2 + b^2 = 13^2$ represent this situation.

Possible explanation: The ground meets the tree at a right angle, so you can draw a right triangle with the ground and tree as the legs and the ladder as the hypotenuse. The ground leg is 5 ft and the ladder hypotenuse is 13 ft. So, you can use the Pythagorean Theorem to find the unknown tree leg, *b*.

② **a.** How high up the tree does the ladder reach? Why do you not need to consider the negative square root of b?

12 ft; *b* is a length and cannot be negative.

b. When using the Pythagorean Theorem to find an unknown length, do you ever need to consider a negative square root? Explain.

No; Possible explanation: You are always finding the length of a side of a triangle, and length can never be negative.

③ A right triangle has leg lengths a and b and hypotenuse length c. Write a subtraction equation that you could use to solve for b. Write a subtraction equation that you could use to solve for a.

$b^2 = c^2 - a^2$; $a^2 = c^2 - b^2$

④ When might you use one of the equations in problem 3? Why?

Possible answer: Use one of the equations when you are trying to find the length of a leg in a right triangle. These equations will need fewer solving steps and can also remind you that you are solving for a leg.

⑤ **Reflect** Think about all the models and strategies you have discussed today. Describe how one of them helped you better understand how to solve the **Try It** problem.

Responses will vary. Check student responses.

637

DIFFERENTIATION | RETEACH or REINFORCE

Hands-On Activity

Use the Pythagorean Theorem to find an unknown leg length.

If students are unsure about how to apply the Pythagorean Theorem to find an unknown leg length, then use this activity to reinforce the concept.

Materials For each pair: tape measure, yard stick, or meter stick

- Have each pair find a rectangular object in the room to measure. Some suggestions include a window, door, bookshelf, book, desktop, cabinet door, or drawer face.
- Encourage students to make a sketch of the object and label it as they work.
- Instruct students to measure and record the length and width of the object.
- Have students use the Pythagorean Theorem to calculate the length of the diagonal of the object to the nearest whole unit and then check by measuring its length.
- Ask students to choose another rectangular object and measure its width and the length of a diagonal.
- Have students use the Pythagorean Theorem to calculate the length of the object to the nearest whole unit, and then check by measuring its length.

Apply It

For all problems, encourage students to use a model to support their thinking.

6 First, students must use the Pythagorean Theorem to solve for the unknown length of the right triangle. Then they need to add this value to 8 ft, the height of the fence pole, to get the height of the building.

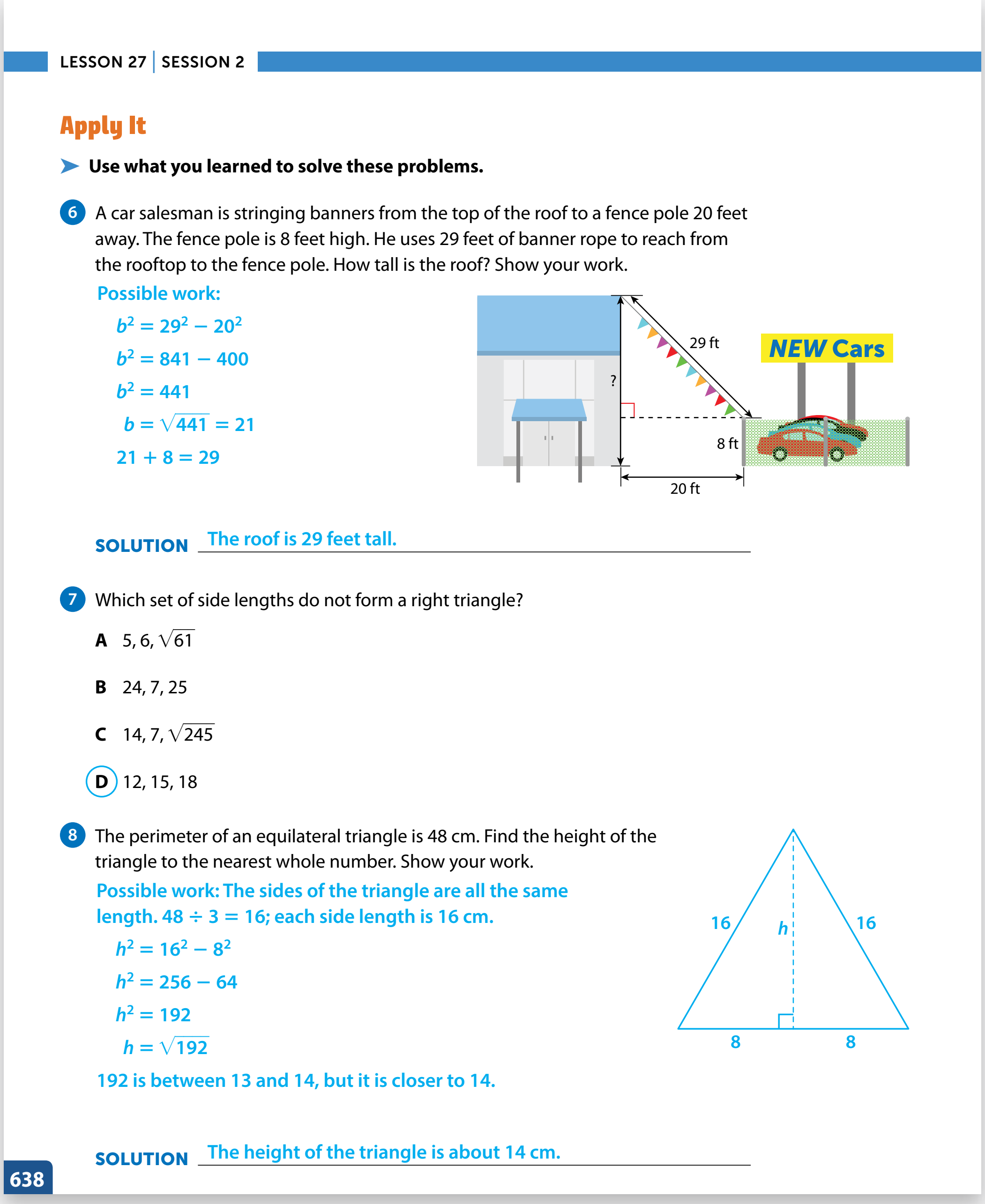
7 Students may substitute the given sets of numbers into $a^2 + b^2 = c^2$. The set of side lengths that do not make the equation true do not form a right triangle.

D **is correct.** $12^2 + 15^2 \neq 18^2$, or $144 + 225 \neq 324$.

A is not correct. $5^2 + 6^2 = (\sqrt{61})^2$, or $25 + 36 = 61$.

B is not correct. $24^2 + 7^2 = 25^2$, or $576 + 49 = 625$.

C is not correct. $14^2 + 7^2 = (\sqrt{245})^2$, or $196 + 49 = 245$.



Apply It

➤ **Use what you learned to solve these problems.**

6 A car salesman is stringing banners from the top of the roof to a fence pole 20 feet away. The fence pole is 8 feet high. He uses 29 feet of banner rope to reach from the rooftop to the fence pole. How tall is the roof? Show your work.

Possible work:

$b^2 = 29^2 - 20^2$

$b^2 = 841 - 400$

$b^2 = 441$

$b = \sqrt{441} = 21$

$21 + 8 = 29$

SOLUTION The roof is 29 feet tall.

7 Which set of side lengths do not form a right triangle?

A 5, 6, $\sqrt{61}$

B 24, 7, 25

C 14, 7, $\sqrt{245}$

[✓ **D**] 12, 15, 18

8 The perimeter of an equilateral triangle is 48 cm. Find the height of the triangle to the nearest whole number. Show your work.

Possible work: The sides of the triangle are all the same length. $48 \div 3 = 16$; each side length is 16 cm.

$h^2 = 16^2 - 8^2$

$h^2 = 256 - 64$

$h^2 = 192$

$h = \sqrt{192}$

192 is between 13 and 14, but it is closer to 14.

SOLUTION The height of the triangle is about 14 cm.

CLOSE EXIT TICKET

8 Students' solutions should show an understanding that:

- the side length of the equilateral triangle is 16 cm, one third of its perimeter.
- the height segment splits the equilateral triangle into two congruent right triangles.
- the length of the hypotenuse of each right triangle is 16 cm, and the lengths of the legs are 8 cm and the unknown height.
- the height can be found by solving $h^2 = 16^2 - 8^2$.
- the height, $\sqrt{192}$, is about 14 cm.

Error Alert If students find that the height is 18 cm, then they are mistakenly treating the unknown length as a hypotenuse and solving $8^2 + 16^2 = h^2$. Encourage them to draw a sketch of the situation. They should see that the unknown height is the length of a leg, not the hypotenuse.

LESSON 27 | SESSION 2 ■■□□□

Practice Finding an Unknown Length in a Right Triangle

Problem Notes

Assign **Practice Finding an Unknown Length in a Right Triangle** as extra practice in class or as homework.

① ***Basic***

② Students may just think that Kamal made a calculation error, so they may need to be encouraged to sketch the situation described and see whether Kamal wrote the correct equation to model the problem. ***Challenge***

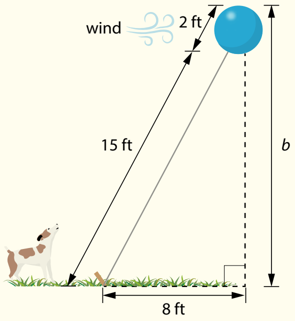
LESSON 27 | SESSION 2 Name:

Practice Finding an Unknown Length in a Right Triangle

➤ **Study the Example showing how to find an unknown length in a right triangle. Then solve problems 1–4.**

Example

A meteorologist ties a spherical balloon that is 2 feet in diameter to a stake in the ground. The string is 15 feet long. The wind blows the balloon so that the top of it is 8 feet to the right of the stake. What is the distance, *b*, from the top of the balloon to the ground?



Use a right triangle. The lengths of the hypotenuse and one leg are known. Use the Pythagorean Theorem to find the length of the other leg.

$b^2 = c^2 - a^2$

$b^2 = 17^2 - 8^2$

$b^2 = 289 - 64$

$b^2 = 225$

$b = \sqrt{225}$ or 15

The distance from the top of the balloon to the ground is 15 ft.

① Imani used the equation $8^2 + b^2 = 17^2$ to find the distance from the top of the balloon to the ground in the Example. Why does this equation also work?

Possible explanation: Imani's equation shows the same three terms, but the terms are rearranged. Both equations correctly use the Pythagorean Theorem, so either can be used to find the unknown distance.

② Kamal said the distance from the top of the balloon to the ground in the Example is $\sqrt{353}$ ft. What mistake might Kamal have made?

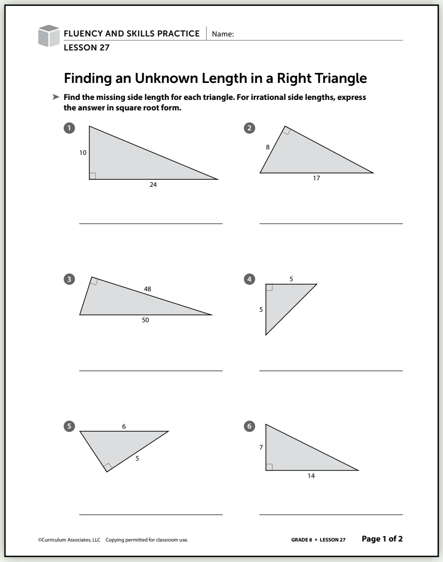
Possible answer: Kamal may have used 17 ft as a leg length of the right triangle instead of as the hypotenuse length when applying the Pythagorean Theorem.

639

Fluency & Skills Practice

Finding an Unknown Length in a Right Triangle

In this activity, students use the Pythagorean Theorem to find unknown side lengths of right triangles.



FLUENCY AND SKILLS PRACTICE | Name:
LESSON 27

Finding an Unknown Length in a Right Triangle

➤ **Find the missing side length for each triangle. For irrational side lengths, express the answer in square root form.**

©Curriculum Associates, LLC Copying permitted for classroom use. GRADE 8 • LESSON 27 Page 1 of 2

Additional Practice

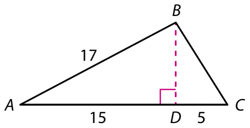
③ **a.** Students should recognize that the height of $\triangle ABC$ is a leg of right $\triangle ABD$. Substitute the known leg length, AD, and the known hypotenuse length, AB, into the Pythagorean Theorem to find the height of the triangle. Note, the height is the length of $\overline{BD}$, not the length of $\overline{BC}$. ***Medium***

b. To find BC, students apply the Pythagorean Theorem to find the hypotenuse length of right $\triangle BCD$ using the length of $\overline{BD}$ found in problem 3a. ***Medium***

④ Students might find $c = 1.5$ and record 1.5 yd as their final answer. Students must read carefully to see that the problem is asking for the perimeter of the deck, rather than just the side length, c. ***Medium***

LESSON 27 | SESSION 2

③ The diagram shows $\triangle ABC$.



a. What is the height of $\triangle ABC$? Show your work.

Possible work:

$a^2 = c^2 - b^2$

$a^2 = 17^2 - 15^2$

$a^2 = 289 - 225$

$a^2 = 64$

$a = \sqrt{64} = 8$

SOLUTION The height of $\triangle ABC$ is 8.

b. What is the length of $\overline{BC}$ in $\triangle ABC$? Show your work.

Possible work:

$5^2 + 8^2 = c^2$

$25 + 64 = c^2$

$89 = c^2$

$c = \sqrt{89}$

SOLUTION The length of $\overline{BC}$ is $\sqrt{89}$.

④ Alyssa is designing a square wooden deck with side length c yards. She will build the deck over her square patio, as shown in the diagram. Find the perimeter of the deck. Show your work.

Possible work: The length of one side of the deck is the hypotenuse of a right triangle with leg lengths of 0.9 yard and 1.2 yards.

$0.9^2 + 1.2^2 = c^2$

$0.81 + 1.44 = c^2$

$2.25 = c^2$

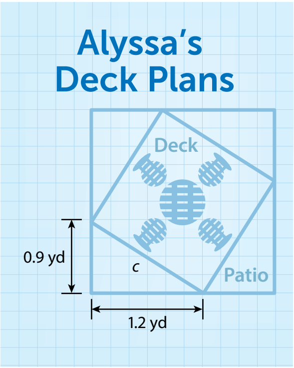
$\sqrt{2.25} = c$

$1.5 = c$

The perimeter of the deck is $4c$.

$4 \times 1.5 = 6$

SOLUTION The perimeter of the deck is 6 yards.



DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Use with **Session 3** **Connect It**

Levels 1–3: Reading/Writing

Prepare students for Connect It problem 4 by helping them generate a word bank of terms related to right rectangular prisms and right triangles. Provide right rectangular prisms and right triangles from the Math Toolkit. Guide students to point to and name different parts of the shapes as you record them. [Possible terms: *side, edge, face, base, diagonal, length, leg, hypotenuse, angle, right angle*]

Next, read the problem aloud and display the Pythagorean Theorem. Have partners identify a strategy that uses the Pythagorean Theorem. Ask: *What can you use the theorem to find? What do you need to know?* Allow partners to point to the models or shapes and help them write responses.

Levels 1–3: Reading/Writing

Read Connect It problem 4 with students. Prepare for written responses by helping students with key terms like *unknown*. Next, ask students to review the Pythagorean theorem and use **Co-Constructed Word Bank** to identify words and phrases they might use in their written responses. If needed, suggest *side length* and *right triangle*. Then ask students to add terms related to right rectangular prisms, like *face*, *base*, and *diagonal*.

Use **Stronger and Clearer Each Time** to help with written responses. Encourage students to use models or diagrams to develop their ideas and then explain to partners. After discussing, have students write responses using the Word Bank.

Levels 1–3: Reading/Writing

Have students read Connect It problem 4 and confirm understanding using **Say It Another Way.** If needed, help students make sense of *unknown* by identifying the prefix *un-* and discussing how it changes the meaning of the base.

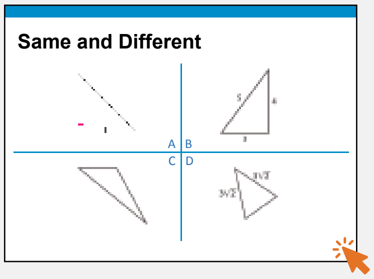
Next, have students use **Stronger and Clearer Each Time** to craft their written explanations. If needed, prompt students to develop their ideas using models or diagrams before drafting responses. Then have partners meet to explain their ideas. Encourage partner feedback on the use of precise language, like math terms describing a prism. After sufficient partner discussion, ask students to revise their writing.

Develop Finding an Unknown Length in a Three-Dimensional Figure

Purpose

- **Develop** strategies for finding unknown lengths in three-dimensional figures.
- **Recognize** that diagonal distances inside and on the faces of right rectangular prisms are hypotenuses of right triangles, so you can use the Pythagorean Theorem to find the measures.

START CONNECT TO PRIOR KNOWLEDGE



Possible Solutions

All of the figures are triangles.

A and B are right triangles.

B and C are scalene triangles.

A and D are isosceles triangles.

C is the only obtuse triangle and D is the only acute triangle.

WHY? Support students' facility with classifying triangles.

DEVELOP ACADEMIC LANGUAGE

WHY? Unpack dense sentences.

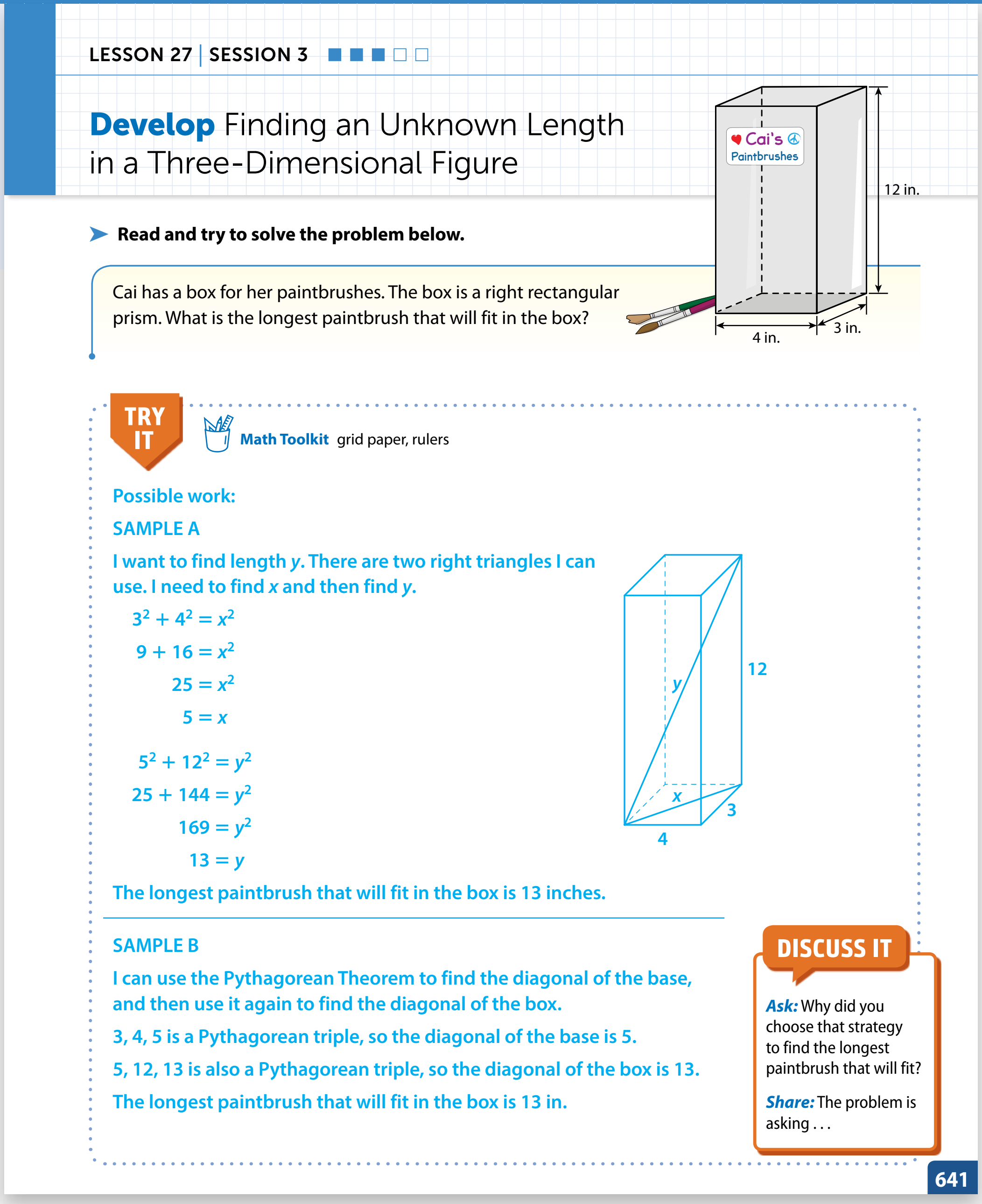
HOW? Ask a student to read Apply It problem 7 aloud. Point out that academic sentences can be long and dense in order to convey precise information. Have students consult the diagram and find parts of the sentence that describe the distance exactly. Encourage students to be precise as they identify parts of figures in other problems.

TRY IT

SMP 1, 2, 4, 5, 6

Make Sense of the Problem

See ***Connect to Culture*** to support student engagement. Before students work on Try It, use **Three Reads** to help them make sense of the problem. After the first read, ask: *What is the problem about?* After the second read, ask: *What are you trying to find?* After the third read, ask: *What are the important quantities and relationships in the problem?*



LESSON 27 | SESSION 3 ■ ■ ■ □ □

Develop Finding an Unknown Length in a Three-Dimensional Figure

Read and try to solve the problem below.

Cai has a box for her paintbrushes. The box is a right rectangular prism. What is the longest paintbrush that will fit in the box?

TRY IT

Math Toolkit grid paper, rulers

Possible work:

SAMPLE A

I want to find length y. There are two right triangles I can use. I need to find x and then find y.

$3^2 + 4^2 = x^2$
$9 + 16 = x^2$
$25 = x^2$
$5 = x$

$5^2 + 12^2 = y^2$
$25 + 144 = y^2$
$169 = y^2$
$13 = y$

The longest paintbrush that will fit in the box is 13 inches.

SAMPLE B

I can use the Pythagorean Theorem to find the diagonal of the base, and then use it again to find the diagonal of the box.

3, 4, 5 is a Pythagorean triple, so the diagonal of the base is 5.

5, 12, 13 is also a Pythagorean triple, so the diagonal of the box is 13.

The longest paintbrush that will fit in the box is 13 in.

DISCUSS IT

Ask: Why did you choose that strategy to find the longest paintbrush that will fit?

Share: The problem is asking . . .

641

DISCUSS IT

SMP 1, 2, 3, 4, 5, 6

Support Partner Discussion

After students work on Try It, encourage them to respond to Discuss It with a partner. If students need support in getting started, prompt them to ask each other questions such as:

- *How did you know where to begin?*
- *Did a sketch of the box help you visualize its dimensions?*
- *Why does the Pythagorean Theorem have to be applied twice?*

Common Misconception Listen for students who think the length of the longest paintbrush is the length of the diagonal of the 4 in.-by-12 in. face of the box. As students share their strategies, ask them to compare the length of the diagonal from one corner of the box to the opposite corner to the length of the diagonal of this face. If you have a real box available, you might have students measure and compare the length of the diagonal of the largest face to the length of the diagonal of the box. You could also use the classroom as an example.

Select and Sequence Student Strategies

Select 2–3 samples that represent the range of student thinking in your classroom. Here is one possible order for class discussion:

- draws diagram and applies Pythagorean Theorem twice to solve
- **(misconception)** thinks that length of diagonal of 4-by-12 face is the solution
- uses Pythagorean triples to find solution

Facilitate Whole Class Discussion

Call on students to share selected strategies. Prompt students to justify their solutions by providing reasons they are reasonable in the problem context.

Guide students to **Compare and Connect** the representations. Have students turn and talk with a partner to discuss ways they can check to be sure their answer is reasonable.

> **ASK** *How is the Pythagorean Theorem used in these strategies?*
>
> **LISTEN FOR** The theorem is used to find the length of the diagonal of the 3-by-4 face and then the diagonal of the box. Some strategies used Pythagorean triples.

Picture It & Model It

If students presented these models, have students connect these models to those presented in class.

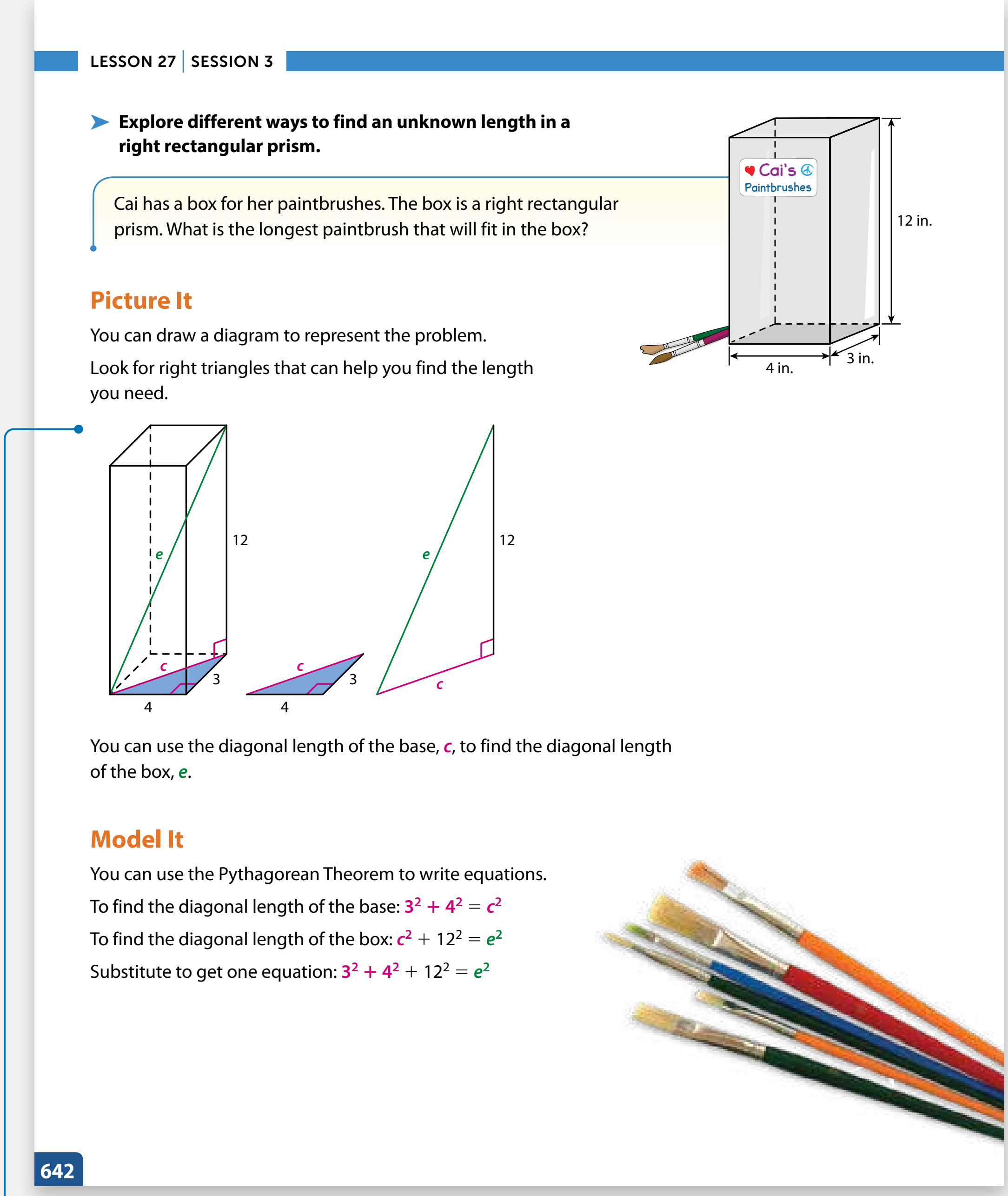
If no student presented at least one of these models, have students first analyze key features of the models, and then connect them to the models presented in class.

> **ASK** *How are the strategies in Picture It and Model It related?*
>
> **LISTEN FOR** Picture It shows labeled diagrams of the two right triangles involved in the problem. Model It applies the Pythagorean Theorem to those triangles to find the unknown length.

For the diagrams, prompt students to compare the parts of each diagram to the key information provided in the problem. Ask: *What do the variables c and e represent?*

For the equations, prompt students to note how the expression $3^2 + 4^2$ is substituted for c^2 in the last equation.

- *How is this strategy different from finding the diagonal of the 3-by-4 face first and then finding the diagonal of the box?*
- *Which approach do you prefer, doing two calculations or combining the steps and doing only one? Why?*



LESSON 27 | SESSION 3

➤ **Explore different ways to find an unknown length in a right rectangular prism.**

Cai has a box for her paintbrushes. The box is a right rectangular prism. What is the longest paintbrush that will fit in the box?

Picture It

You can draw a diagram to represent the problem.

Look for right triangles that can help you find the length you need.

You can use the diagonal length of the base, c, to find the diagonal length of the box, e.

Model It

You can use the Pythagorean Theorem to write equations.

To find the diagonal length of the base: $3^2 + 4^2 = c^2$

To find the diagonal length of the box: $c^2 + 12^2 = e^2$

Substitute to get one equation: $3^2 + 4^2 + 12^2 = e^2$

642

DIFFERENTIATION | EXTEND

SMP 7

Deepen Understanding

Applying the Structure of the Pythagorean Theorem to Cylinders

Prompt students to consider how the Pythagorean Theorem can be used to determine the diagonal length of a cylinder. Suppose Cai's box is a cylinder with height 12 inches and diameter 4 inches. What is the longest paintbrush that will fit in the cylinder?

> **ASK** *How would the diagram change?*
>
> **LISTEN FOR** There would be only one right triangle. The diameter of the base and the height of the cylinder are the two legs of a right triangle. The hypotenuse of the right triangle will be the diagonal of the cylinder.

> **ASK** *How will the solution change?*
>
> **LISTEN FOR** You would need only one equation. You would need to solve $4^2 + 12^2 = c^2$, where c is the diagonal of the cylinder.

> **ASK** *The dimension values are not a Pythagorean triple. How can you find c?*
>
> **LISTEN FOR** Evaluate the squares to get $16 + 144 = c^2$, simplify to get $160 = c^2$, and take the square root of both sides of the equation to get $c = \sqrt{160}$. Use a calculator or successive approximations to get $c \approx 12.6$ in.

Develop Finding an Unknown Length in a Three-Dimensional Figure

CONNECT IT

SMP 2, 3, 4, 5, 6

Remind students that the quantities and the relationships between them are the same in each representation. Explain that they will now use those representations to reason about how to find an unknown length in a right rectangular prism.

Before students begin to record and expand on their work in Picture It & Model It, tell them that problems 1–3 will prepare them to provide the explanation asked for in problem 4.

Monitor and Confirm Understanding ① – ②

- Finding the length of the diagonal of the prism, *e*, is dependent upon knowing *c*, the length of the diagonal of the 3-by-4 face.
- The value of *e* is 13, so the longest paintbrush that will fit has a length of 13 in.

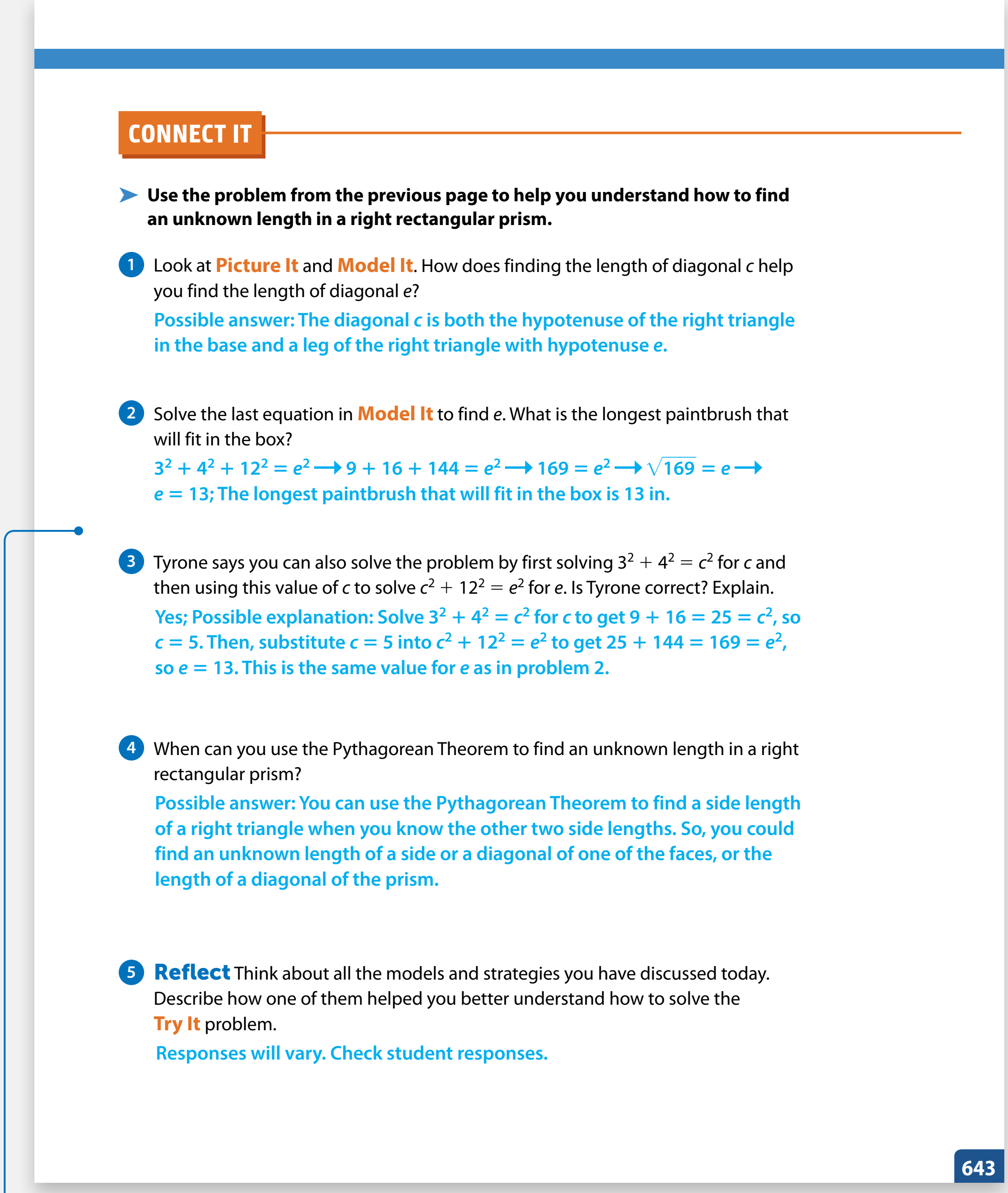
Facilitate Whole Class Discussion

③ Look for understanding that finding the value of *c* first and then using that value in the calculation of *e* gives the same solution as the method in Model It.

④ Look for understanding that you can use the Pythagorean Theorem to find a diagonal length in a right rectangular prism when the dimensions of the prism are given.

> **ASK** *If you knew only the height of the right rectangular prism and the length of the diagonal of the prism, would you be able to find the prism's dimensions?*
>
> **LISTEN FOR** No, the diagonal length and the height could be used to find the length of the diagonal of the base. However, you would need to know either the length or width of the base in order to find the other dimension.

⑤ **Reflect** Have all students focus on the strategies used to solve the Try It. If time allows, have students discuss their ideas with a partner.



CONNECT IT

➤ **Use the problem from the previous page to help you understand how to find an unknown length in a right rectangular prism.**

① Look at **Picture It** and **Model It**. How does finding the length of diagonal *c* help you find the length of diagonal *e*?

Possible answer: The diagonal *c* is both the hypotenuse of the right triangle in the base and a leg of the right triangle with hypotenuse *e*.

② Solve the last equation in **Model It** to find *e*. What is the longest paintbrush that will fit in the box?

$3^2 + 4^2 + 12^2 = e^2 \longrightarrow 9 + 16 + 144 = e^2 \longrightarrow 169 = e^2 \longrightarrow \sqrt{169} = e \longrightarrow$
$e = 13$; The longest paintbrush that will fit in the box is 13 in.

③ Tyrone says you can also solve the problem by first solving $3^2 + 4^2 = c^2$ for *c* and then using this value of *c* to solve $c^2 + 12^2 = e^2$ for *e*. Is Tyrone correct? Explain.

Yes; Possible explanation: Solve $3^2 + 4^2 = c^2$ for *c* to get $9 + 16 = 25 = c^2$, so $c = 5$. Then, substitute $c = 5$ into $c^2 + 12^2 = e^2$ to get $25 + 144 = 169 = e^2$, so $e = 13$. This is the same value for *e* as in problem 2.

④ When can you use the Pythagorean Theorem to find an unknown length in a right rectangular prism?

Possible answer: You can use the Pythagorean Theorem to find a side length of a right triangle when you know the other two side lengths. So, you could find an unknown length of a side or a diagonal of one of the faces, or the length of a diagonal of the prism.

⑤ **Reflect** Think about all the models and strategies you have discussed today. Describe how one of them helped you better understand how to solve the **Try It** problem.

Responses will vary. Check student responses.

643

DIFFERENTIATION | RETEACH or REINFORCE

Visual Model

Find the diagonal length of a rectangular prism.

If students are unsure about finding the length of a diagonal of a rectangular prism, then use this model to help them visualize the concept.

Materials For display: an open box, tape measure, string, tape

- Stretch a piece of string from one corner of the box to the opposite corner to form a diagonal, secure it with tape, and select a face of the box to be the base.
- Ask a student to point to the edge length that will represent the height.
- Invite a student to measure and record the length, width, and height of the box.
- Point out that the string is the diagonal and that you want to find its length. Discuss why it is necessary to first find the length of the diagonal of the base.
- Sketch the base and ask a student to calculate the length of its diagonal.
- Using the diagonal of the base as one leg, sketch a right triangle with the height of the box as the other leg length. Help students see how this triangle relates to the box.
- Invite a student to calculate the length of the diagonal of the box. Use a tape measure to verify the calculations.

Apply It

For all problems, encourage students to use a model to support their thinking.

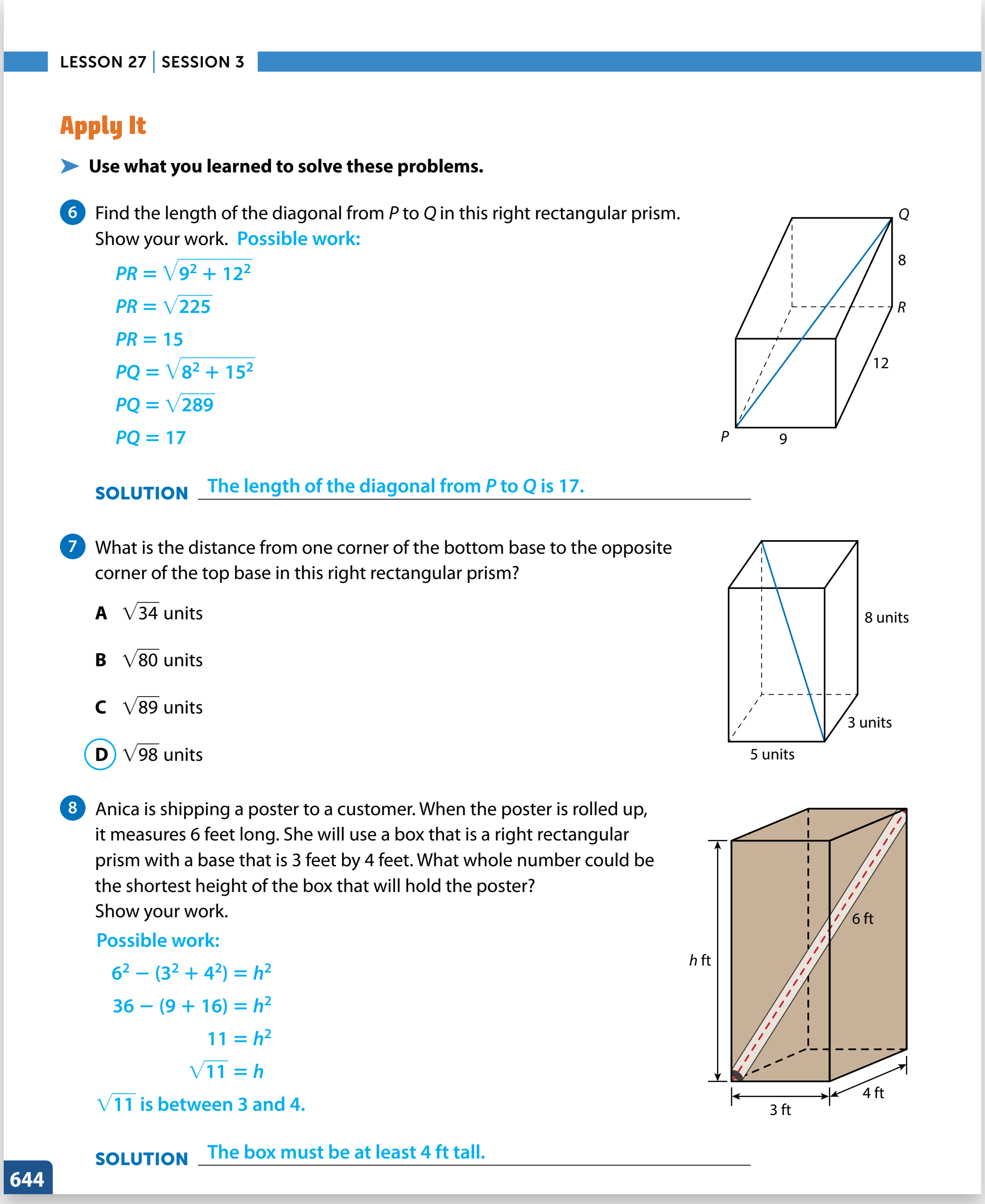
6 Students may use the expression $9^2 + 12^2$ for the length of one leg and use the Pythagorean Theorem: $a^2 + b^2 = PR^2 \rightarrow 9^2 + 12^2 + 8^2 = PR^2$.

7 **D is correct.** Students may solve the problem by first finding the hypotenuse of the base of the prism by substituting 3 for a and 5 for b, into $a^2 + b^2 = c^2$ to find $c = \sqrt{34}$. They may then find the diagonal of the prism by using $\sqrt{34}$ and 8 for leg lengths to find the hypotenuse length of $\sqrt{98}$ units.

A is not correct. This answer is the hypotenuse of the base of the rectangular prism.

B is not correct. This answer is the result of using 4 for the diagonal of the base to then calculate the diagonal of the prism as $4^2 + 8^2 = e^2$.

C is not correct. This answer is the hypotenuse of one of the 5-by-8 faces of the prism.



LESSON 27 | SESSION 3

Apply It

➤ **Use what you learned to solve these problems.**

6 Find the length of the diagonal from P to Q in this right rectangular prism. Show your work. Possible work:

$PR = \sqrt{9^2 + 12^2}$

$PR = \sqrt{225}$

$PR = 15$

$PQ = \sqrt{8^2 + 15^2}$

$PQ = \sqrt{289}$

$PQ = 17$

SOLUTION The length of the diagonal from P to Q is 17.

7 What is the distance from one corner of the bottom base to the opposite corner of the top base in this right rectangular prism?

A $\sqrt{34}$ units

B $\sqrt{80}$ units

C $\sqrt{89}$ units

[✓ **D**] $\sqrt{98}$ units

8 Anica is shipping a poster to a customer. When the poster is rolled up, it measures 6 feet long. She will use a box that is a right rectangular prism with a base that is 3 feet by 4 feet. What whole number could be the shortest height of the box that will hold the poster? Show your work.

Possible work:

$6^2 - (3^2 + 4^2) = h^2$

$36 - (9 + 16) = h^2$

$11 = h^2$

$\sqrt{11} = h$

$\sqrt{11}$ is between 3 and 4.

SOLUTION The box must be at least 4 ft tall.

644

CLOSE EXIT TICKET

8 Students' solutions should show an understanding that:

- the length of the rolled-up poster, 6 ft, is the shortest possible diagonal length of the prism.
- the unknown height is the length of a leg of a right triangle with hypotenuse length 6 ft and its other leg length as the diagonal of the base.
- the diagonal length of the base is $\sqrt{3^2 + 4^2}$ or 5.
- the unknown height can be found by solving $5^2 + h^2 = 6^2$ for h.

Error Alert If students mistakenly solve this problem as they did the previous examples, by using 6 ft to represent the height of the prism and calculating the diagonal length of the prism, then encourage them to reread the problem to see what they are being asked to find. Ask students to note which dimension is labeled with a variable.

Practice Finding an Unknown Length in a Three-Dimensional Figure

Problem Notes

Assign **Practice Finding an Unknown Length in a Three-Dimensional Figure** as extra practice in class or as homework.

① **a.** *Basic*

b. *Basic*

② Students may recognize 6 and 8 as leg lengths of the Pythagorean triple 6, 8, 10 and write $10^2 + 24^2 = d^2$ as a first step. ***Medium***

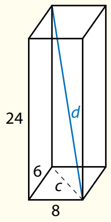
Practice Finding an Unknown Length in a Three-Dimensional Figure

➤ **Study the Example explaining how to find the length of a diagonal in a right rectangular prism. Then solve problems 1–4.**

Example

The diagram shows a diagonal drawn in a right rectangular prism. Explain how you can find *d*, the length of this diagonal.

Look for right triangles. Side length *d* is the hypotenuse of the right triangle with leg lengths *c* and 24. Side length *c* is the hypotenuse of the right triangle in the base of the prism with leg lengths 6 and 8. Use the Pythagorean Theorem to find *c*. Then use it again to find *d*.



① **a.** How do you know that the triangle with side lengths 6, 8, and *c* in the Example is a right triangle?

Possible answer: All the faces of a right rectangular prism have four right angles. The base is a face of the prism, so the angle between the sides of lengths 6 and 8 is a right angle.

b. How do you know that the triangle with side lengths *c*, 24, and *d* in the Example is a right triangle?

Possible answer: In a right rectangular prism, vertical faces meet the bases at right angles. Side length *c* is on a base and side length 24 is on a vertical face, so the angle between them is a right angle.

② What is the length of the diagonal *d* in the Example? Show your work.

Possible work:

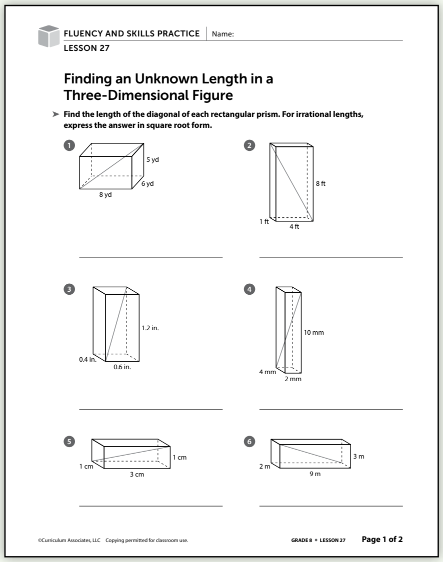
$$(6^2 + 8^2) + 24^2 = d^2$$
$$100 + 576 = d^2$$
$$676 = d^2$$
$$\sqrt{676} = d$$
$$d = 26$$

SOLUTION The length of the diagonal is 26.

Fluency & Skills Practice

Finding an Unknown Length in a Three-Dimensional Figure

In this activity, students use the Pythagorean Theorem to find the length of the diagonals of rectangular prisms.



FLUENCY AND SKILLS PRACTICE | Name:
LESSON 27

Finding an Unknown Length in a Three-Dimensional Figure

➤ **Find the length of the diagonal of each rectangular prism. For irrational lengths, express the answer in square root form.**

©Curriculum Associates, LLC Copying permitted for classroom use.

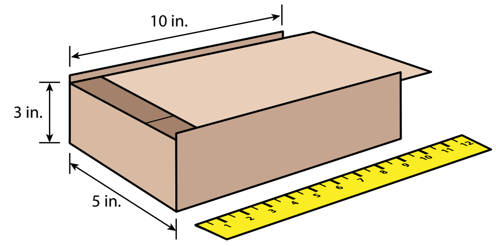
GRADE 8 • LESSON 27 Page 1 of 2

Additional Practice

③ Students may use a calculator to find the approximation of $\sqrt{134} \approx 11.6$ to determine that since $11.6 < 12$, the ruler will not fit. ***Medium***

④ Students may substitute the expression $0.8^2 + 1.2^2$ for d and write the equation $0.8^2 + 1.2^2 + 0.6^2 = d^2$. ***Challenge***

③ Will a 12-inch ruler fit in a box that is a right rectangular prism with a width of 5 inches, a length of 10 inches, and a height of 3 inches? Explain your answer.



No; Possible explanation:

Let c be the diagonal of the base.

$3^2 + 5^2 = c^2$

$9 + 25 = c^2$

$34 = c^2$

Let d be the diagonal of the box.

$34 + 10^2 = d^2$

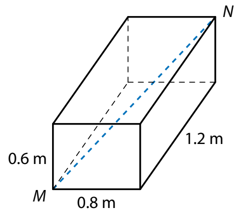
$34 + 100 = d^2$

$134 = d^2$

$\sqrt{134} = d$

$\sqrt{134}$ is less than $\sqrt{144}$, so it is less than 12. The ruler will not fit.

④ In the right rectangular prism, what is the length of the diagonal from M to N to the nearest tenth of a meter? Show your work.



Possible work:

Let c be the diagonal of the base.

$0.8^2 + 1.2^2 = c^2$

$0.64 + 1.44 = c^2$

$2.08 = c^2$

Let d be the diagonal from M to N.

$2.08 + 0.6^2 = d^2$

$2.08 + 0.36 = d^2$

$2.44 = d^2$

$d = \sqrt{2.44} \approx 1.562$

SOLUTION The diagonal from M to N is about 1.6 m.

DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Use with **Session 4** **Connect It**

ACADEMIC VOCABULARY

Distance is the linear space between two things.

Horizontal describes something that goes side to side instead of up and down.

Vertical describes something that goes up and down instead of side to side.

Levels 1–3: Speaking/Writing

Prepare for Connect It problem 5 by reviewing the Academic Vocabulary. Help students connect to other words in English or home languages, such as *horizon* or *horizonte* in Spanish. Ask students to draw a horizontal and a vertical line on a coordinate plane. Then have them draw one that is not horizontal or vertical. Help students read and interpret the problem using:

- *I want to find the ____ between____ points on a line. I know the line is not ____. The line is also not ____.*

Ask students to use graphs or models to show how to solve the problem. Help partners write responses.

Levels 2–4: Speaking/Writing

Help students read and interpret Connect It problem 5. Ask students to circle the Academic Vocabulary and use a coordinate plane to show their meanings.

Help students use **Say It Another Way** to paraphrase the problem. If needed, clarify that the verb phrase *to lie on*, in this context, means "to be (set) on."

Ask students to use **Stronger and Clearer Each Time** to craft explanations. Have students explain ideas to partners. Encourage partners to discuss the order of the steps and work together to revise their writing.

Levels 3–5: Speaking/Writing

Have students read and interpret Connect It problem 5. Ask students to plot possible points in a coordinate plane and then explain to partners how they know the points accurately represent the problem. Encourage students to use terms like *horizontal* and *vertical* in their explanations.

Next, have students draft responses by describing a possible solution strategy. Remind students to start with the first step and to explain why the strategy works. Have students use **Stronger and Clearer Each Time** to discuss their ideas with partners and then revise their writing.

Develop Finding Distance in the Coordinate Plane

Purpose

- **Develop** strategies for finding the shortest distance between two points in the coordinate plane.
- **Recognize** that you can use the Pythagorean Theorem to find nonhorizontal and nonvertical distances in the coordinate plane.

START CONNECT TO PRIOR KNOWLEDGE

Which One Doesn't Belong?

(2, 5) and (9, 5) A	B (–10, 3) and (–10, 3)
(2, –2) and (4, –8) C	D (–4, –3) and (–4, –8)

Possible Solutions

A is the only pair of points with the same *y*-coordinates but different *x*-coordinates.

B is the only pair of points that are the same.

C is the only pair of points where both the *x*-coordinates and the *y*-coordinates are different.

D is the only pair of points with the same *x*-coordinates but different *y*-coordinates.

WHY? Support students' facility with determining the similarities and differences for two ordered pairs.

DEVELOP ACADEMIC LANGUAGE

WHY? Reinforce understanding of *hypotenuse* through word origins.

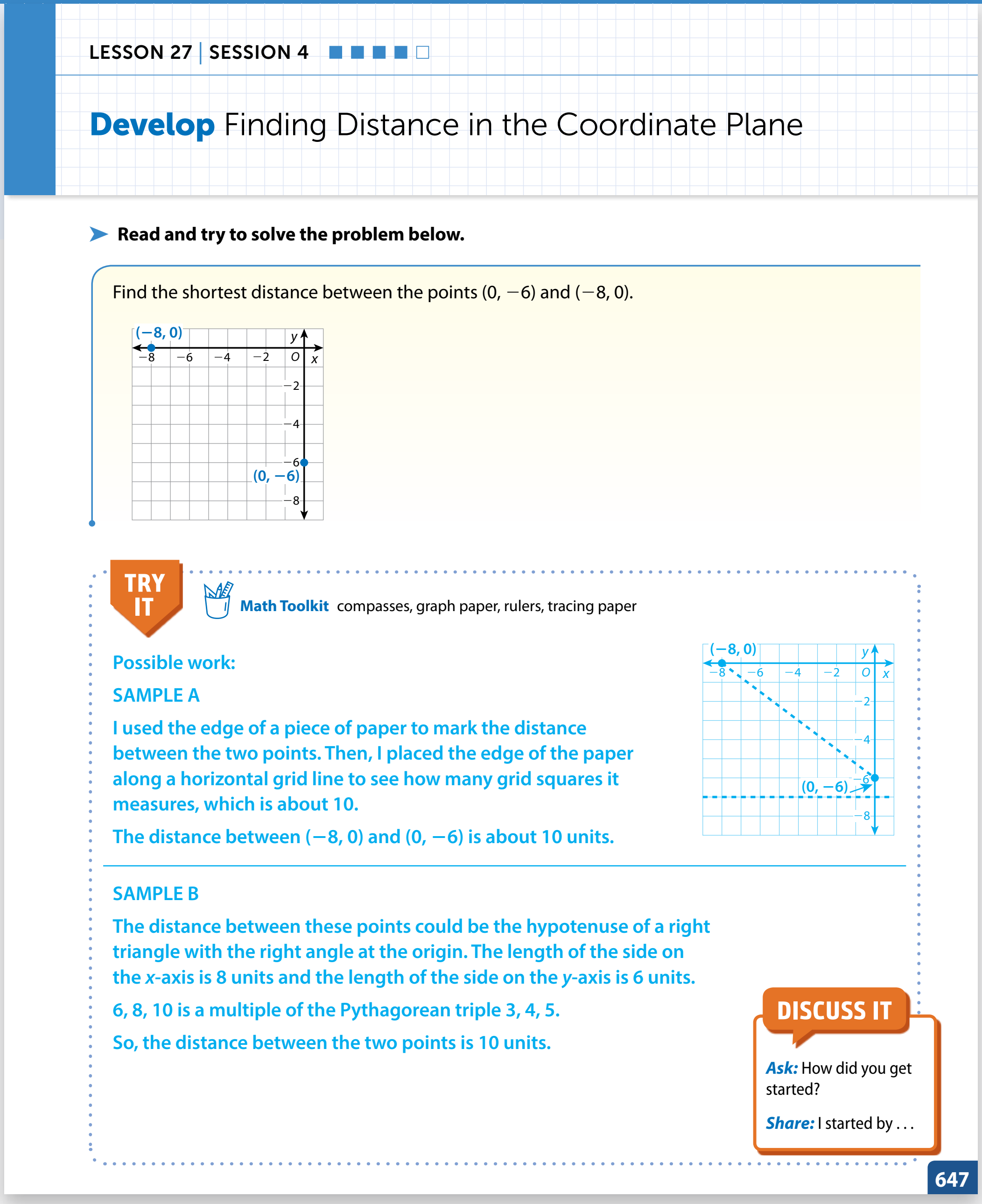
HOW? Explain that *hypotenuse* comes from the Greek word *hypoteinousa* that means "stretching under." Have students look at the triangle in Picture It. Ask: *Under what part does the hypotenuse stretch?* [the right angle] Next, review the math definition of the word. Have students discuss whether the hypotenuse of a right triangle always *stretches under* the right angle.

TRY IT

SMP 1, 2, 4, 5, 6

Make Sense of the Problem

Before students work on Try It, use **Co-Craft Questions** to help them make sense of the problem. Have students turn and talk with a partner to write questions they can ask about this problem before beginning to solve it.



LESSON 27 | SESSION 4 ■■■■□

Develop Finding Distance in the Coordinate Plane

➤ **Read and try to solve the problem below.**

Find the shortest distance between the points (0, −6) and (−8, 0).

TRY IT **Math Toolkit** compasses, graph paper, rulers, tracing paper

Possible work:

SAMPLE A

I used the edge of a piece of paper to mark the distance between the two points. Then, I placed the edge of the paper along a horizontal grid line to see how many grid squares it measures, which is about 10.

The distance between (−8, 0) and (0, −6) is about 10 units.

SAMPLE B

The distance between these points could be the hypotenuse of a right triangle with the right angle at the origin. The length of the side on the *x*-axis is 8 units and the length of the side on the *y*-axis is 6 units.

6, 8, 10 is a multiple of the Pythagorean triple 3, 4, 5.

So, the distance between the two points is 10 units.

DISCUSS IT

Ask: How did you get started?

Share: I started by . . .

647

DISCUSS IT

SMP 1, 2, 3, 4, 5, 6

Support Partner Discussion

After students work on Try It, encourage them to respond to Discuss It with a partner. If students need support in getting started, prompt them to ask each other questions such as:

- *Did you draw a segment connecting the points? How did this help you?*
- *Did you use a ruler or other tool?*

Common Misconception Listen for students who think that to find the distance, they can find the number of units each point is from the origin and then add those values. As students share their strategies, remind them that the shortest distance between the points is the distance along the segment that connects them.

Select and Sequence Student Strategies

Select 2–3 samples that represent the range of student thinking in your classroom. Here is one possible order for class discussion:

- marks distance between points on a piece of paper, then measures the distance in grid units
- (misconception) adds the horizontal distance between (−8, 0) and the origin to the vertical distance between (0, −6) and the origin
- draws segment to connect points, creating a right triangle with legs along the axes, and uses the Pythagorean Theorem to find distance

Facilitate Whole Class Discussion

Call on students to share selected strategies. As students share their strategies, remind them that one way to agree and build on ideas is to give reasons that explain why the strategy makes sense.

Guide students to **Compare and Connect** the representations. Remind students that good listeners ask questions to clarify ideas or ask for more information during math discussions.

> **ASK** *Which strategy would work best in cases where finding an exact distance between two points is important? Why?*
>
> **LISTEN FOR** A strategy that uses the Pythagorean Theorem will always give an exact answer if you find the exact horizontal and vertical distance. Measuring the distance only gives an estimate.

Picture It & Model It

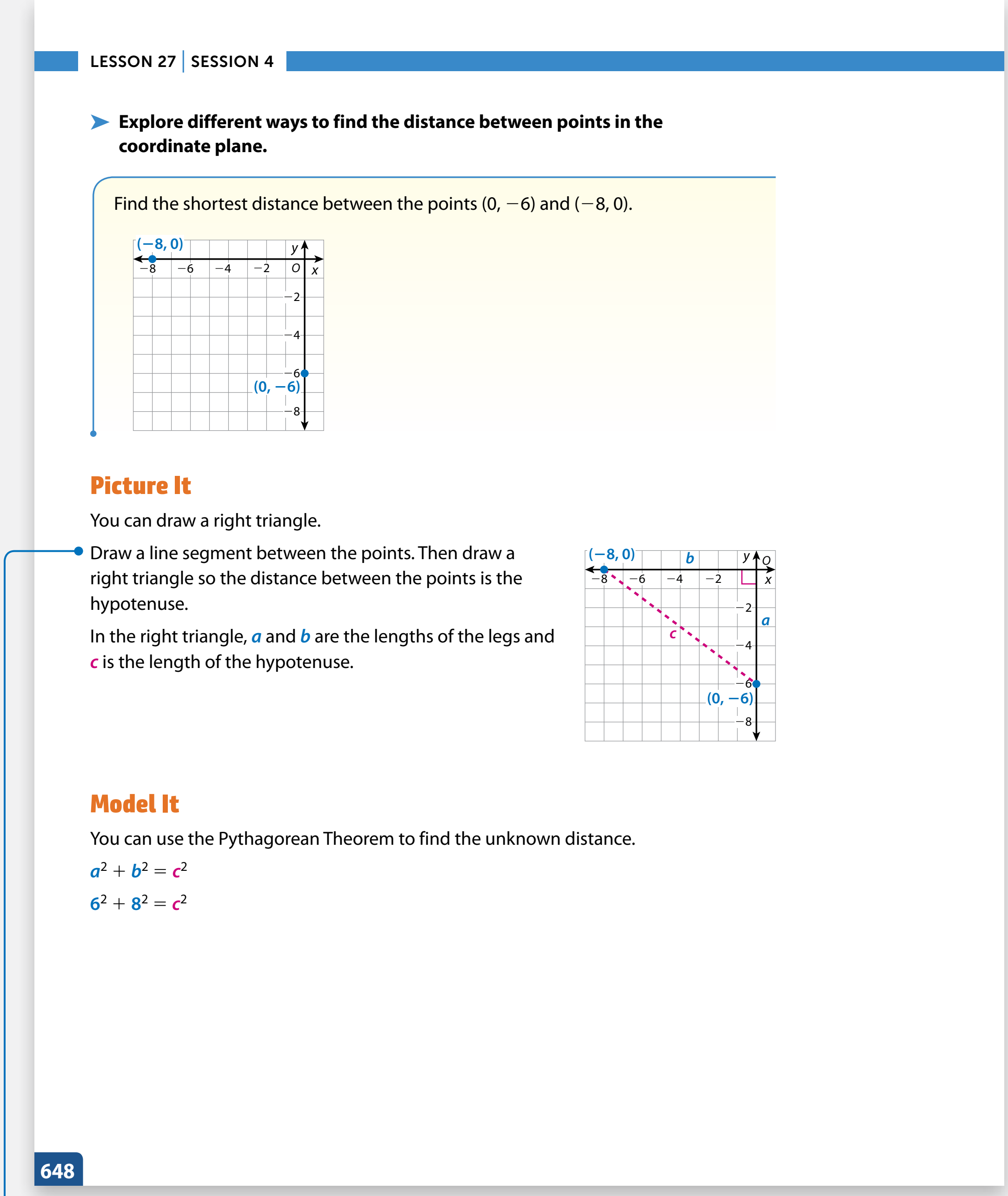
If students presented these models, have students connect these models to those presented in class.

If no student presented at least one of these models, have students first analyze key features of the models, and then connect them to the models presented in class.

> **ASK** *How are the strategies in Picture It and Model It related?*
>
> **LISTEN FOR** Picture It shows the right triangle formed when the points are connected and labels the legs *a* and *b* and the hypotenuse *c*. Model It applies the Pythagorean Theorem to the triangle.

For the drawing, prompt students to think about how the right triangle is formed. Ask: *If the two points were not on the axes, could you still draw a right triangle with the segment connecting the points as the hypotenuse?*

For the equation, prompt students to note the values each variable represents. Ask: *Do you think the values of a and b could be used interchangeably? Why or why not?*



LESSON 27 | SESSION 4

➤ **Explore different ways to find the distance between points in the coordinate plane.**

Find the shortest distance between the points (0, −6) and (−8, 0).

Picture It

You can draw a right triangle.

Draw a line segment between the points. Then draw a right triangle so the distance between the points is the hypotenuse.

In the right triangle, *a* and *b* are the lengths of the legs and *c* is the length of the hypotenuse.

Model It

You can use the Pythagorean Theorem to find the unknown distance.

$a^2 + b^2 = c^2$

$6^2 + 8^2 = c^2$

648

DIFFERENTIATION | EXTEND

SMP 2

Deepen Understanding

Generalizing the Distance from (*x*, 0) to (0, *y*)

Prompt students to think about general points (*x*, 0) and (0, *y*) on the *x*- and *y*-axes. Ask them to imagine connecting the points to form a right triangle with its legs on the axes, as in Picture It. Establish that the distance between the points is the hypotenuse.

> **ASK** *What would be the leg lengths? Remember that x and y can be positive or negative.*
>
> **LISTEN FOR** $|x|$ and $|y|$

> **ASK** *Apply the Pythagorean Theorem using $|x|$ and $|y|$ for the leg lengths and c for the hypotenuse length. What do you get when you solve for c?*
>
> **LISTEN FOR** $|x|^2 + |y|^2 = c^2$; You get $c = \sqrt{|x|^2 + |y|^2}$ because only the positive square root makes sense for the length.

> **ASK** *Are the absolute value symbols under the square root symbol needed? Explain.*
>
> **LISTEN FOR** No; whether *x* and *y* are positive or negative, you get a positive number when you square them.

Generalize Have students rewrite the equation as $c = \sqrt{x^2 + y^2}$. Discuss the fact that this is a formula for the distance between any two points of the form (*x*, 0) and (0, *y*).

Develop Finding Distance in the Coordinate Plane

CONNECT IT

SMP 2, 4, 5, 6

Remind students that the points and the distance between them are the same in each representation. Explain that they will now use those representations to reason about finding the distance between two points in the coordinate plane.

Before students begin to record and expand on their work in Picture It & Model It, tell them that problems 3 and 4 will prepare them to provide the explanation asked for in problem 5.

Monitor and Confirm Understanding ① – ③

- Because the two points lie on the *x*- and *y*-axes, the right angle of the triangle is at the origin where the two axes meet.
- Distance is never negative. The distance from the origin to (0, −6) is 6, and the distance from the origin to (−8, 0) is 8.
- The distance between (0, −6) and (−8, 0) is the hypotenuse of a right triangle with legs of length 6 and 8. So, the distance is $\sqrt{6^2 + 8^2} = \sqrt{100} = 10$.

Facilitate Whole Class Discussion

④ Look for an understanding that you can count each grid square on a horizontal or vertical line as 1 unit, but you cannot count grid squares on a diagonal line as 1 unit each.

> **ASK** *Why is it not possible to count the number of grid spaces/squares along the line segment between the two points?*
>
> **LISTEN FOR** The diagonal length or other interior slant-line lengths of a square with side length 1 do not also equal 1.

⑤ Look for the idea that you can use the Pythagorean Theorem to find nonvertical and nonhorizontal distances between points in the coordinate plane.

> **ASK** *Once you connect the given points to form the hypotenuse of the right triangle, how do you draw its legs?*
>
> **LISTEN FOR** Draw a horizontal line through one of the points and a vertical line through the other point. The point where the two lines intersect represents the vertex of the right angle.

⑥ **Reflect** Have all students focus on the strategies used to solve the Try It. If time allows, have students discuss their ideas with a partner.

CONNECT IT

➤ **Use the problem from the previous page to help you understand how to find the distance between two points in the coordinate plane.**

① Look at **Picture It**. How do you know the triangle formed by the two points and the origin is a right triangle?

Possible answer: The *x*-axis and the *y*-axis are perpendicular, so the angle formed at the origin is a right angle.

② Jessica said the lengths of the legs are −6 units and −8 units. What mistake did Jessica make? What are the correct lengths of the legs?

Distance, or length, is never negative; 6 units and 8 units

③ Look at **Model It**. What is the distance between (0, −6) and (−8, 0)? Why can you use the Pythagorean Theorem to find this distance?

10 units; Possible explanation: The distance between (0, −6) and (−8, 0) is the length of the hypotenuse of a right triangle with known leg lengths.

④ Why is it important that the distance between (0, −6) and (−8, 0) be the hypotenuse of the right triangle and not a leg?

Possible answer: If the distance is a leg, then the right angle must be at one end of this leg. This means at least one of the other sides will not be horizontal or vertical and so you cannot easily find its length.

⑤ Explain how to find the distance between any two points in the coordinate plane that do not lie on the same horizontal or vertical line.

Possible answer: Draw a line segment between the points. Draw horizontal and vertical legs so that this segment is the hypotenuse of a right triangle. Use the Pythagorean Theorem to find the length of the segment.

⑥ **Reflect** Think about all the models and strategies you have discussed today. Describe how one of them helped you better understand how to find the distance between two points in the coordinate plane.

Responses will vary. Check student responses.

649

DIFFERENTIATION | RETEACH or REINFORCE

Visual Model

Use the Pythagorean Theorem to find distance.

If students are unsure about how to use the Pythagorean Theorem to find the distance between two points on the coordinate plane, then use this activity as reinforcement.

Materials For display: large first-quadrant coordinate plane

- Invite students to plot points at (4, 6), (10, 6), and (10, 2) and label them *School, Park,* and *Home*, respectively. Connect the points. Tell students each grid unit is 1 mile.
- Explain that Raj plans to bike from school to the park, and then from the park home. Ask: *How far is it from school to the park? From the park to home?* [6 mi; 4 mi]
- Ask: *How did you find the distances?* [Subtract 4 from 10, or count over 6 units; Subtract 2 from 6, or count up 4 units] Label these lengths on the diagram.
- Ask: *What if Raj decides to bike straight home from school? How can you find that distance?* [Draw a right triangle and use the Pythagorean Theorem.]
- Ask volunteers to show their solution. [$4^2 + 6^2 = c^2$; $c = \sqrt{52} \approx 7.2$; 7.2 mi.]
- Ask: *How much farther is Raj's trip if he stops at the park?* [2.8 miles]

Apply It

For all problems, encourage students to use a model to support their thinking.

⑦ Students may plot each point to find the one that meets the requirement.

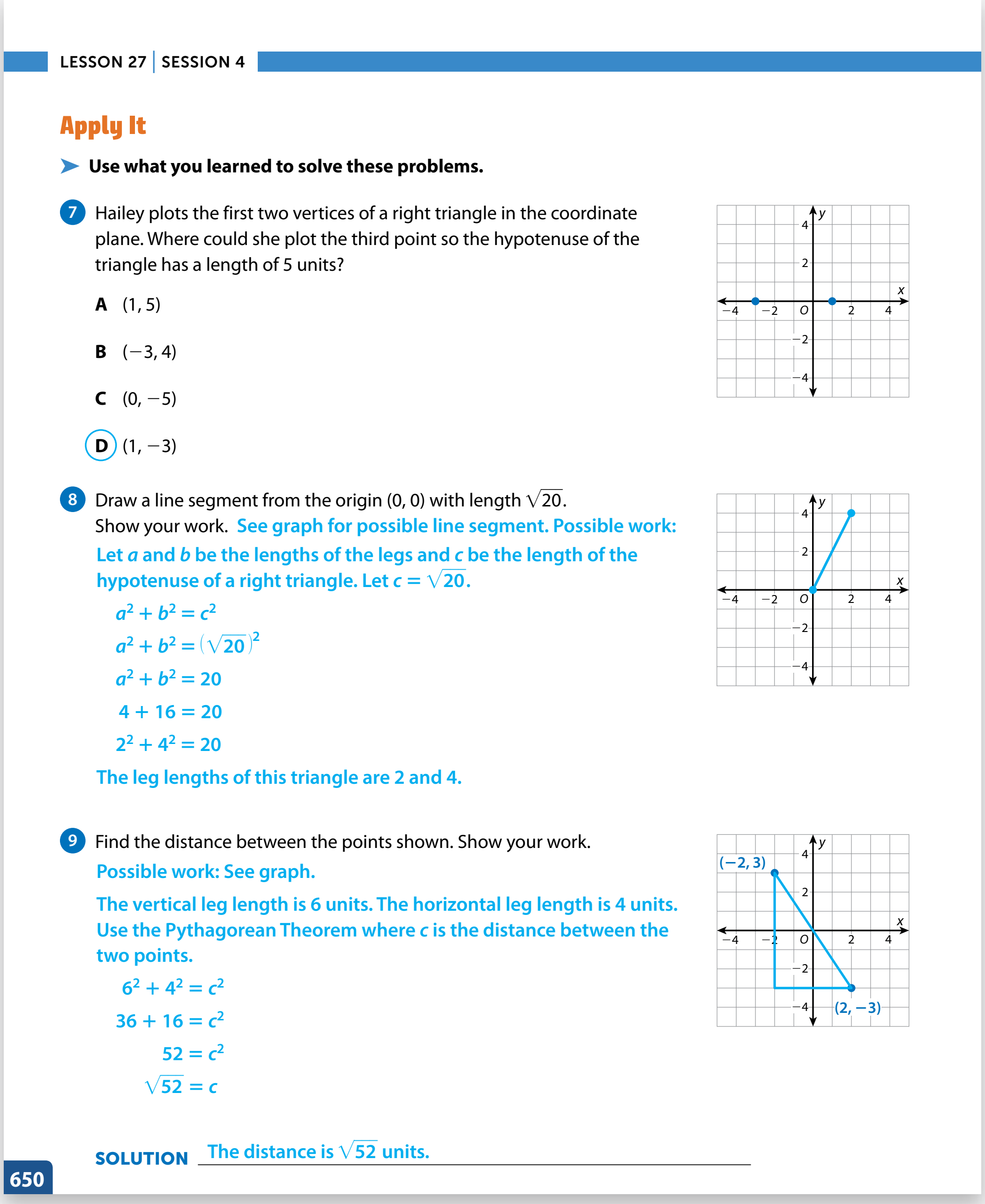
D **is correct.** One leg is already known to be 4 units. This point makes the other leg 3 units, so the side lengths of the right triangle are a 3, 4, 5 Pythagorean triple.

A is not correct. This answer will make one of the right triangle's legs 5 units long. The hypotenuse and a leg cannot have the same length.

B is not correct. This answer will make both legs 4 units long, so the hypotenuse would have a length of $4\sqrt{2}$ units rather than 5.

C is not correct. It is not possible to draw a right triangle using this point.

⑧ Students may work backwards to come up with leg lengths for which the hypotenuse length is 20. Students should look for two numbers whose squares add to 20 and draw legs with those lengths.



LESSON 27 | SESSION 4

Apply It

➤ **Use what you learned to solve these problems.**

⑦ Hailey plots the first two vertices of a right triangle in the coordinate plane. Where could she plot the third point so the hypotenuse of the triangle has a length of 5 units?

A (1, 5)

B (−3, 4)

C (0, −5)

[✓ **D**] (1, −3)

⑧ Draw a line segment from the origin (0, 0) with length $\sqrt{20}$. Show your work. See graph for possible line segment. Possible work: Let a and b be the lengths of the legs and c be the length of the hypotenuse of a right triangle. Let $c = \sqrt{20}$.

$a^2 + b^2 = c^2$

$a^2 + b^2 = (\sqrt{20})^2$

$a^2 + b^2 = 20$

$4 + 16 = 20$

$2^2 + 4^2 = 20$

The leg lengths of this triangle are 2 and 4.

⑨ Find the distance between the points shown. Show your work.

Possible work: See graph.

The vertical leg length is 6 units. The horizontal leg length is 4 units. Use the Pythagorean Theorem where c is the distance between the two points.

$6^2 + 4^2 = c^2$

$36 + 16 = c^2$

$52 = c^2$

$\sqrt{52} = c$

SOLUTION The distance is $\sqrt{52}$ units.

650

CLOSE EXIT TICKET

⑨ Students' solutions should show an understanding of:

- how to draw a right triangle with the segment between the given points as the hypotenuse.
- how to find the lengths of the horizontal and vertical legs.
- how to use the Pythagorean Theorem or a known Pythagorean triple to find the hypotenuse length, which is the distance between the points.

Error Alert If students try to find the length of the hypotenuse of the triangle by counting the number of intervals or grid squares through which the segment passes, then remind them that you can only count grid units when the segments are horizontal or vertical. You can use the Pythagorean Theorem to show that the diagonal of a square with side length 1 is not 1.

Practice Finding Distance in the Coordinate Plane

Problem Notes

Assign **Practice Finding Distance in the Coordinate Plane** as extra practice in class or as homework.

① **a.** *Basic*

b. Students may sketch this other triangle in order to see that the lengths of its legs are the same as the lengths of the legs in the Example. ***Medium***

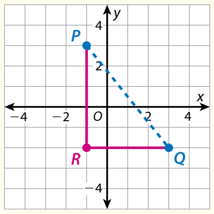
c. Students should understand that, if the points are not vertices of a right triangle, the Pythagorean Theorem cannot be used. ***Medium***

Practice Finding Distance in the Coordinate Plane

➤ **Study the Example showing how to find the distance between two points in the coordinate plane. Then solve problems 1–3.**

Example

What is the distance between $P(-1, 3)$ and $Q(3, -2)$ in the coordinate plane?



Draw a right triangle with $R(-1, -2)$ as the vertex of the right angle. Then use the Pythagorean Theorem to find the length of the hypotenuse, which is the distance between P and Q.

$QR = 4$ units, $RP = 5$ units, $PQ = c$ units

$$4^2 + 5^2 = c^2$$
$$16 + 25 = c^2$$
$$41 = c^2$$
$$\sqrt{41} = c$$

The distance between $P(-1, 3)$ and $Q(3, -2)$ is $\sqrt{41}$ units.

① Look at the Example.

a. Describe another right triangle you could draw from points $P(-1, 3)$ and $Q(3, -2)$.

Draw a right triangle with (3, 3) as the vertex of the right angle.

b. Will your triangle from problem 1a give you the same distance between points P and Q? Explain.

Yes; Possible explanation: The leg lengths I would use in the Pythagorean Theorem are the same: 4 units and 5 units.

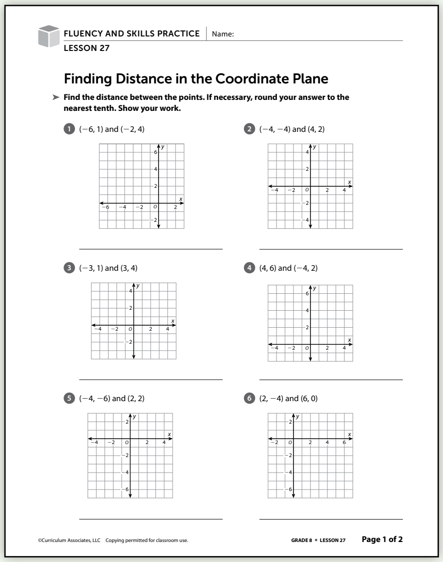
c. Would drawing a triangle with points $(-1, 3)$, $(3, -2)$, and $(3, 2)$ as vertices help you find the distance between the points? Explain.

No; Possible explanation: These vertices will not form a right triangle. You can only use the Pythagorean Theorem if you draw a right triangle.

Fluency & Skills Practice

Finding Distance in the Coordinate Plane

In this activity, students use the Pythagorean Theorem to find the distance between two points in the coordinate plane.



FLUENCY AND SKILLS PRACTICE | Name:
LESSON 27

Finding Distance in the Coordinate Plane

➤ Find the distance between the points. If necessary, round your answer to the nearest tenth. Show your work.

1 (−6, 1) and (−2, 4)

2 (−4, −4) and (4, 2)

3 (−3, 1) and (3, 4)

4 (4, 6) and (−4, 2)

5 (−4, −6) and (2, 2)

6 (2, −4) and (6, 0)

©Curriculum Associates, LLC Copying permitted for classroom use.

GRADE 8 • LESSON 27 Page 1 of 2

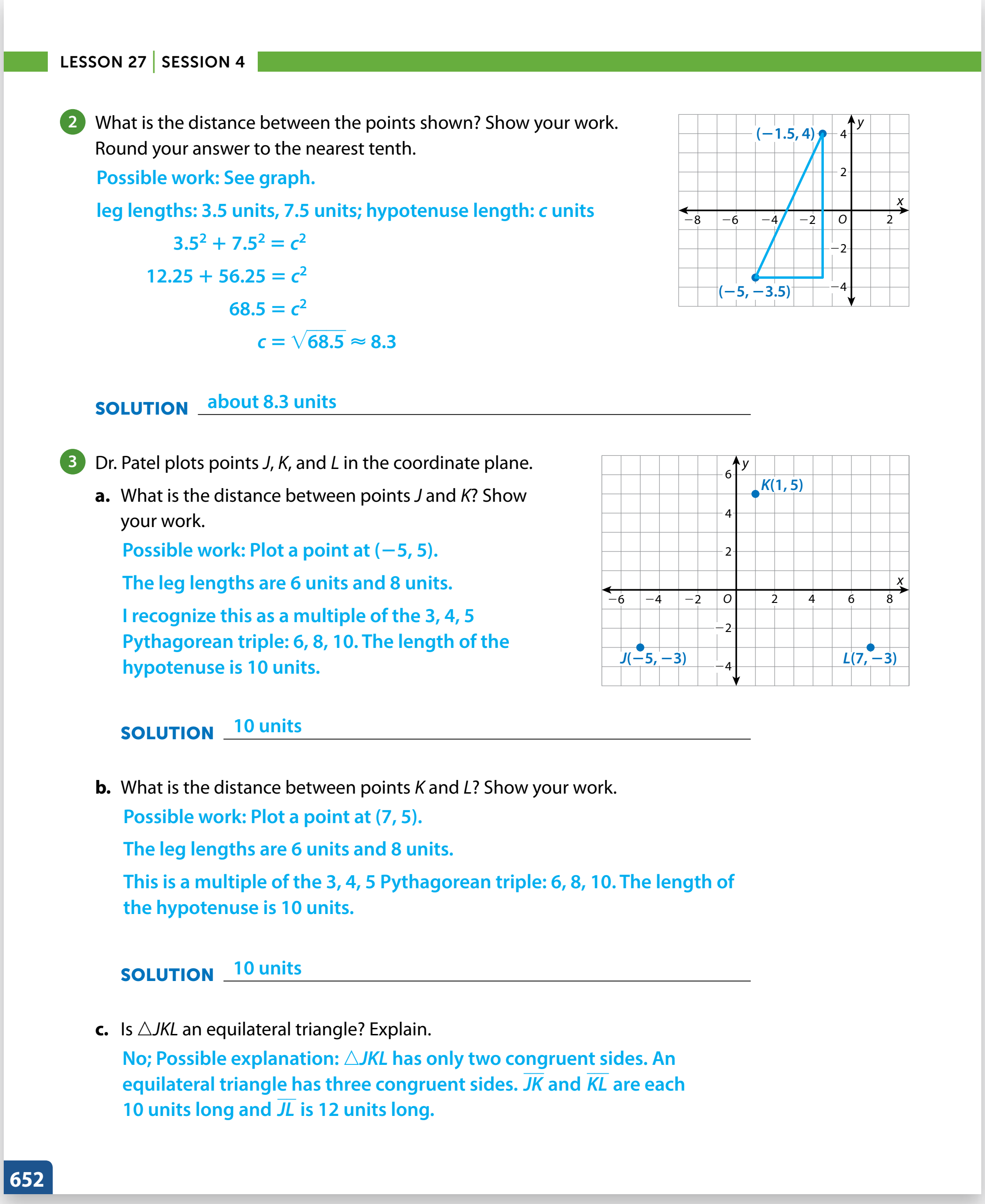
Additional Practice

② Students will need to subtract negative coordinates that are not integers or count partial units to find the side lengths. ***Challenge***

③ **a.** Students may recognize the side lengths are a multiple of a Pythagorean triple or may calculate the distance using the Pythagorean Theorem. Also, students may correctly choose the point (1, −3) instead of (−5, 5) to construct their right triangle. ***Medium***

b. Students may correctly use the point (1, −3) instead of (7, 5) to construct their right triangle. Also, students may sketch both triangles used to find the distances in each part of the problem in order to visualize their congruence. ***Medium***

c. Students need to find JL to answer this question: $JL = 7 - (-5) = 12$. ***Medium***



② What is the distance between the points shown? Show your work. Round your answer to the nearest tenth.

Possible work: See graph.

leg lengths: 3.5 units, 7.5 units; hypotenuse length: c units

$$3.5^2 + 7.5^2 = c^2$$
$$12.25 + 56.25 = c^2$$
$$68.5 = c^2$$
$$c = \sqrt{68.5} \approx 8.3$$

SOLUTION about 8.3 units

③ Dr. Patel plots points J, K, and L in the coordinate plane.

a. What is the distance between points J and K? Show your work.

Possible work: Plot a point at (−5, 5).

The leg lengths are 6 units and 8 units.

I recognize this as a multiple of the 3, 4, 5 Pythagorean triple: 6, 8, 10. The length of the hypotenuse is 10 units.

SOLUTION 10 units

b. What is the distance between points K and L? Show your work.

Possible work: Plot a point at (7, 5).

The leg lengths are 6 units and 8 units.

This is a multiple of the 3, 4, 5 Pythagorean triple: 6, 8, 10. The length of the hypotenuse is 10 units.

SOLUTION 10 units

c. Is $\triangle JKL$ an equilateral triangle? Explain.

No; Possible explanation: $\triangle JKL$ has only two congruent sides. An equilateral triangle has three congruent sides. $\overline{JK}$ and $\overline{KL}$ are each 10 units long and $\overline{JL}$ is 12 units long.

DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Use with **Session 5** **Apply It**

Levels 1–3: Speaking/Listening

To prepare for written responses, help students describe solution strategies for Apply It problem 3. Read Consider This aloud and prompt students to draw right triangles. Have partners describe the solution strategies:

- *This triangle can help us find* ____.
- *We can use the triangle to* ____.

Call on volunteers to share in their own words. Reword and record statements for reference. Then paraphrase Pair/Share: *We can find the solution using right triangles. Is there one or more than one possible triangle?* Encourage partners to draw other possibilities and use the recorded statements to describe the solution strategies.

Levels 2–4: Speaking/Listening

Have students read and interpret Apply It problem 3. Prompt students to tell what they need to solve. Ask students to read Consider This and discuss *useful* before they solve the problem.

To prepare for written responses, have students take turns describing their solution strategies to partners. Then ask partners to read and discuss Pair/Share and work together to describe other possible solution strategies. Ask: *What strategy do you think Ummi used to solve the problem? Do you think she used an incorrect strategy? Do you think she made a calculation error?* Give students time to think about Ummi's mistake before turning to discuss with partners.

Levels 3–5: Speaking/Listening

Have students make sense of Apply It problem 3 through partner discussion. Ask students to read the problem independently. Provide think time for students to consider solution strategies. Then have partners compare and discuss their strategies. If students need help getting started, have them read and discuss Consider This. Then ask students to solve the problem independently.

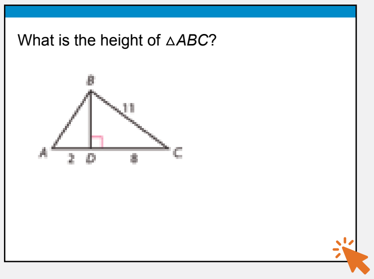
To prepare for written responses, ask students to read and discuss Pair/Share. Have partners describe other possible solution strategies. Ask: *What strategy do you think Ummi might have used?* Ask partners to discuss how Ummi might have gotten her answer and suggest how she might correct her mistake.

Refine Applying the Pythagorean Theorem

Purpose

- **Refine** strategies for applying the Pythagorean Theorem in real-world and mathematical contexts.
- **Refine** understanding of how the Pythagorean Theorem can be used to find unknown side lengths in a variety of problem situations.

START CONNECT TO PRIOR KNOWLEDGE

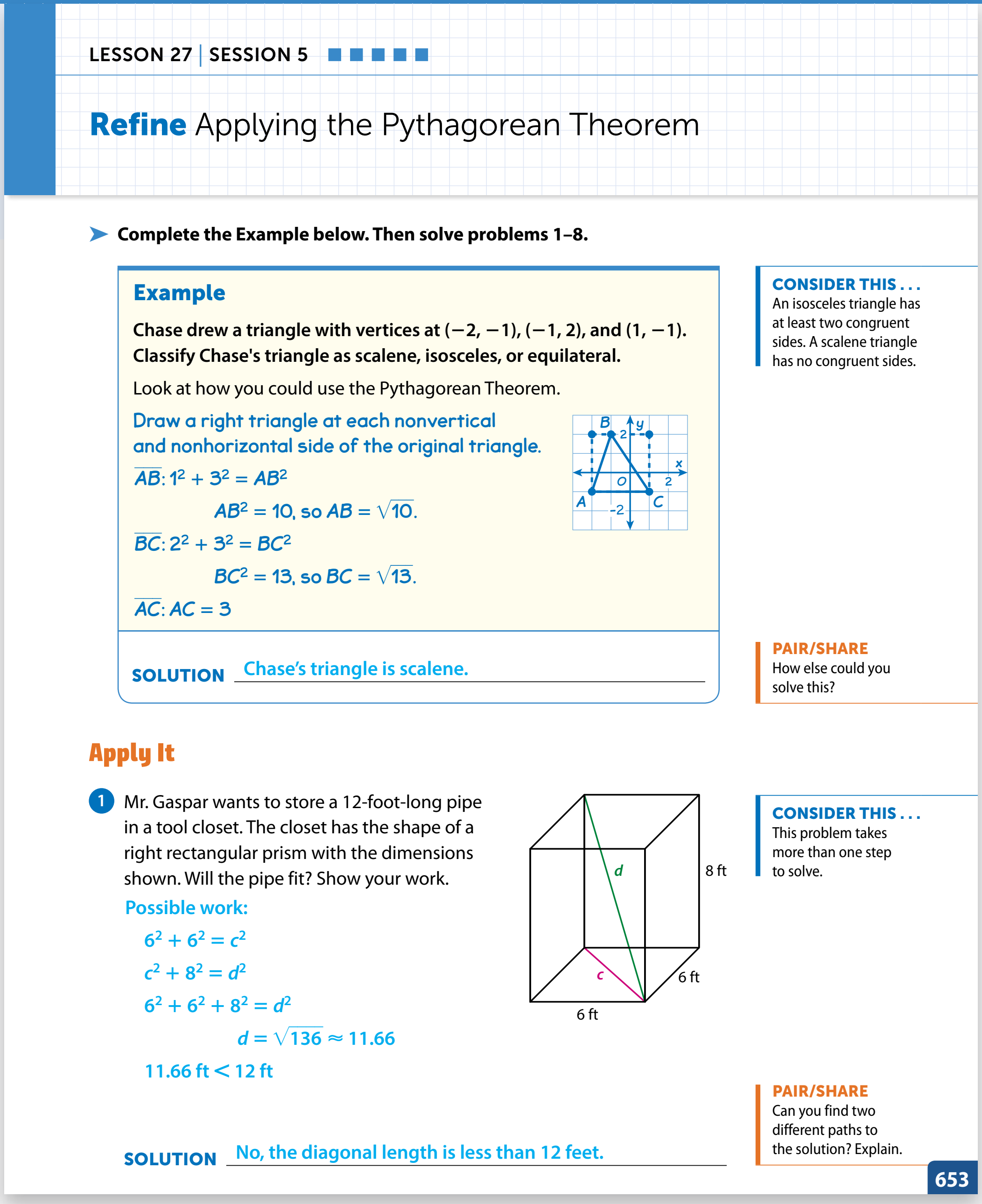


What is the height of $\triangle ABC$?

Solution

$\sqrt{57}$

WHY? Confirm students' understanding of applying the Pythagorean Theorem, identifying common errors to address as needed.



LESSON 27 | SESSION 5 ■■■■■

Refine Applying the Pythagorean Theorem

➤ **Complete the Example below. Then solve problems 1–8.**

Example

Chase drew a triangle with vertices at (−2, −1), (−1, 2), and (1, −1). Classify Chase's triangle as scalene, isosceles, or equilateral.

Look at how you could use the Pythagorean Theorem.

Draw a right triangle at each nonvertical and nonhorizontal side of the original triangle.

$\overline{AB}$: $1^2 + 3^2 = AB^2$

$AB^2 = 10$, so $AB = \sqrt{10}$.

$\overline{BC}$: $2^2 + 3^2 = BC^2$

$BC^2 = 13$, so $BC = \sqrt{13}$.

$\overline{AC}$: $AC = 3$

SOLUTION Chase's triangle is scalene.

CONSIDER THIS . . . An isosceles triangle has at least two congruent sides. A scalene triangle has no congruent sides.

PAIR/SHARE How else could you solve this?

Apply It

① Mr. Gaspar wants to store a 12-foot-long pipe in a tool closet. The closet has the shape of a right rectangular prism with the dimensions shown. Will the pipe fit? Show your work.

Possible work:

$6^2 + 6^2 = c^2$

$c^2 + 8^2 = d^2$

$6^2 + 6^2 + 8^2 = d^2$

$d = \sqrt{136} \approx 11.66$

$11.66 \text{ ft} < 12 \text{ ft}$

SOLUTION No, the diagonal length is less than 12 feet.

CONSIDER THIS . . . This problem takes more than one step to solve.

PAIR/SHARE Can you find two different paths to the solution? Explain.

653

MONITOR & GUIDE

Before students begin to work, use their responses to the **Start** to determine those who will benefit from additional support. Use the **Error Analysis** table below to guide remediation.

Have all students complete the Example and problems 1–3, using Consider This and Pair/Share as appropriate. Observe and monitor their reasoning and guide or redirect students as needed.

START ERROR ANALYSIS

If the error is . . .	Students may . . .	To support understanding . . .
57	have forgotten to take the square root of b^2 and used $c^2 - a^2 = b$.	Review with students that $c^2 - a^2 = b^2$ can be used to find the unknown leg length in a right triangle, not $c^2 - a^2 = b$.
$\sqrt{21}$	have used $2 + 8 = 10$ as the length of a leg in $\triangle BCD$ instead of using 8 as the leg length.	Prompt students to draw a picture of a triangle to show the information they are given and what they still need to find.
$\sqrt{185}$	have solved the equation $11^2 + 8^2 = c^2$.	Ask students to identify the legs and hypotenuse of $\triangle BCD$. They should realize that $\overline{CD}$ and $\overline{BD}$ are legs and $\overline{BC}$ is the hypotenuse of $\triangle BCD$.

Refine

Example

Guide students in understanding the Example. Ask:

- *What measurements do you need to classify triangles as scalene, isosceles, or equilateral?*
- *How can you draw a right triangle and use the Pythagorean Theorem to find the length of* $\overline{AB}$*?*
- *How can you draw a right triangle and use the Pythagorean Theorem to find the length of* $\overline{BC}$*?*
- *How can you find the length of a horizontal segment, such as* $\overline{AC}$*?*

Help all students focus on the Example and responses to the questions by prompting students to refer to a diagram as they answer the questions.

Look for understanding that the Pythagorean Theorem is used to find nonhorizontal and nonvertical distances in the coordinate plane.

Apply It

① Students may solve the problem by using the Pythagorean Theorem twice, first to find the diagonal of the base and then to find the diagonal of the prism. *DOK 2*

② Students may also draw a point *E* at (2, 3) to make a right triangle. Then from point *A*, they might go left 2 units and down 4 units to find point *C* at (−4, −1). From point *B*, they might go left 2 units and down 4 units to find point *D* at (0, −3). *DOK 3*

③ **B is correct.** This problem can be solved by drawing a right triangle with hypotenuse length *w* ft and leg lengths $30 - 14$, or 16, ft and $50 - 20$, or 30, ft. Applying the Pythagorean Theorem gives

$w = \sqrt{16^2 + 30^2} = \sqrt{1{,}156} = 34.$

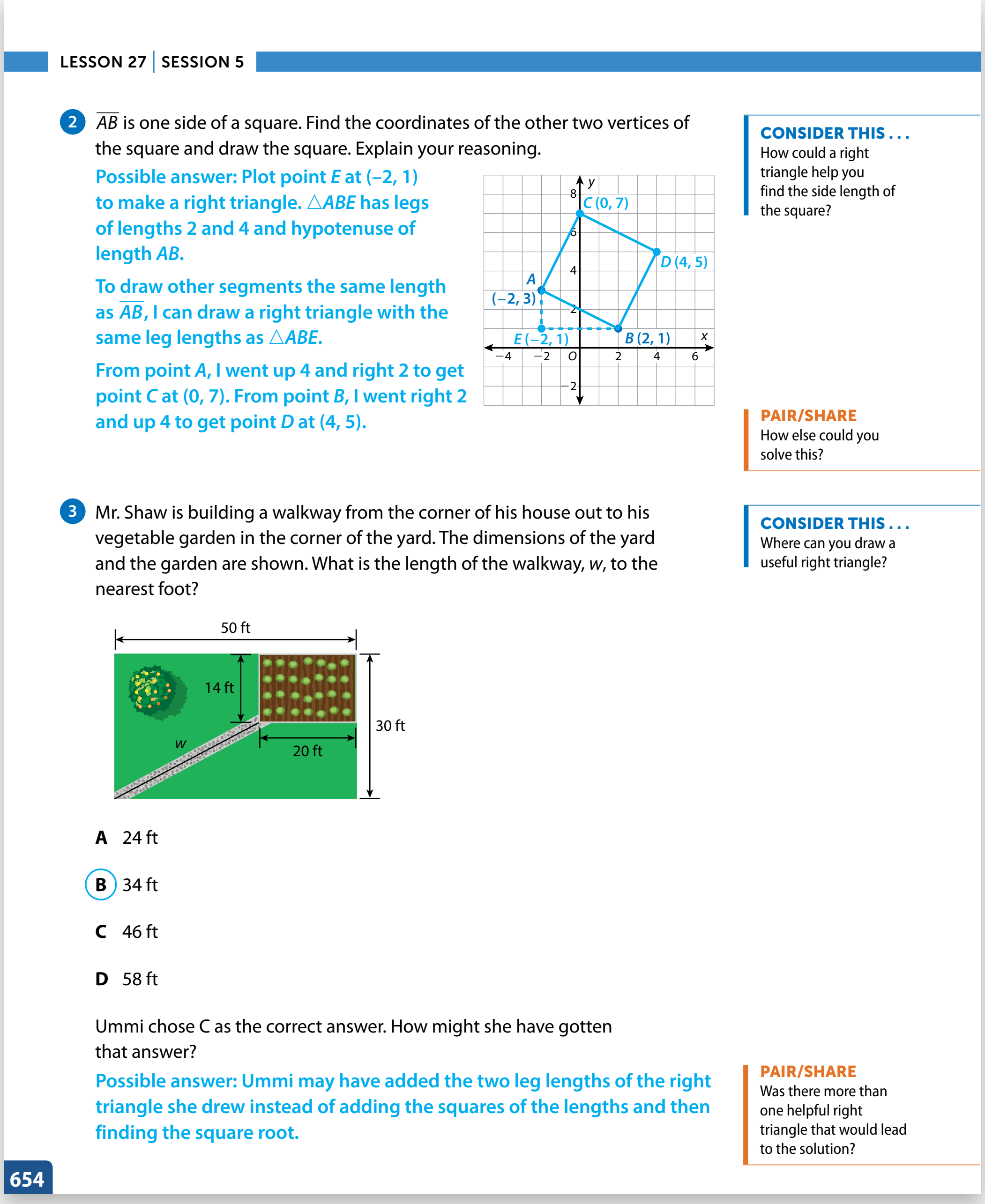
Or, students may find the length of the diagonal of the whole yard and subtract the length of the hypotenuse of the garden.

A is not correct. This answer is the approximate length of the diagonal of the garden.

C is not correct. In this answer, the two leg lengths were added instead of adding the squares of the leg lengths and then finding the square root.

D is not correct. This answer is the approximate length of the diagonal of the entire yard.

DOK 3



LESSON 27 | SESSION 5

② $\overline{AB}$ is one side of a square. Find the coordinates of the other two vertices of the square and draw the square. Explain your reasoning.

Possible answer: Plot point *E* at (–2, 1) to make a right triangle. △*ABE* has legs of lengths 2 and 4 and hypotenuse of length *AB*.

To draw other segments the same length as $\overline{AB}$, I can draw a right triangle with the same leg lengths as △*ABE*.

From point *A*, I went up 4 and right 2 to get point *C* at (0, 7). From point *B*, I went right 2 and up 4 to get point *D* at (4, 5).

CONSIDER THIS . . . How could a right triangle help you find the side length of the square?

PAIR/SHARE How else could you solve this?

③ Mr. Shaw is building a walkway from the corner of his house out to his vegetable garden in the corner of the yard. The dimensions of the yard and the garden are shown. What is the length of the walkway, *w*, to the nearest foot?

A 24 ft

[✓ **B**] 34 ft

C 46 ft

D 58 ft

Ummi chose C as the correct answer. How might she have gotten that answer?

Possible answer: Ummi may have added the two leg lengths of the right triangle she drew instead of adding the squares of the lengths and then finding the square root.

CONSIDER THIS . . . Where can you draw a useful right triangle?

PAIR/SHARE Was there more than one helpful right triangle that would lead to the solution?

654

GROUP & DIFFERENTIATE

Identify groupings for differentiation based on the **Start** and problems 1–3. A recommended sequence of activities for each group is suggested below. Use the resources on the next page to differentiate and close the lesson.

Approaching Proficiency

- **RETEACH** Visual Model
- **REINFORCE** Problems 5, 7

Meeting Proficiency

- **REINFORCE** Problems 4–7

Extending Beyond Proficiency

- **REINFORCE** Problems 4–7
- **EXTEND** Challenge

Have all students complete the **Close: Exit Ticket**.

Resources for Differentiation are found on the next page.

Refine Applying the Pythagorean Theorem

Apply It

④ After reflecting triangle *ABC*, students will need to find the side lengths of the image. For each of the nonhorizontal and nonvertical sides, they may draw a right triangle with the side as the hypotenuse, and then apply the Pythagorean Theorem. *DOK 3*

⑤ See ***Connect to Culture*** to support student engagement.

C is correct. The length of the diagonal of the base is $\sqrt{6^2 + 6^2}$, or $\sqrt{72}$. The length of the diagonal of the cube can be found using $d^2 = 6^2 + (\sqrt{72})^2 = 108$. $d = \sqrt{108} \approx 10.4$

A is not correct. This answer is the length of an edge of the cube.

B is not correct. This answer is the approximate length of a diagonal of the base.

D is not correct. This answer is the sum of two edge lengths.

DOK 2

⑥ **a.** $CA = \sqrt{7^2 + 24^2}$, or 25.

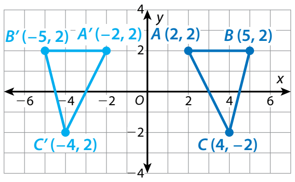
b. $CA = \sqrt{9^2 + 24^2}$, or $\sqrt{657}$ and $\sqrt{657}$ is not a whole-number length.

c. $CA = \sqrt{10^2 + 24^2}$, or 26.

d. $AB = \sqrt{25^2 - 24^2}$, or 7.

DOK 3

④ Draw the reflection of $\triangle ABC$ across the *y*-axis. Then show that the corresponding sides of the two triangles are congruent.



Possible work:

$AB = 3$

$A'B' = 3$

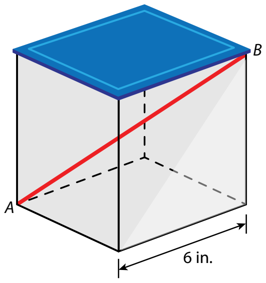
$AC = \sqrt{4^2 + 2^2} = \sqrt{20}$

$A'C' = \sqrt{4^2 + 2^2} = \sqrt{20}$

$BC = \sqrt{4^2 + 1^2} = \sqrt{17}$

$B'C' = \sqrt{4^2 + 1^2} = \sqrt{17}$

⑤ A small plastic storage cube has a side length of 6 inches. What is the length of $\overline{AB}$ to the nearest tenth of an inch?



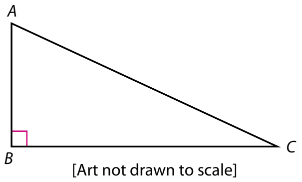
A 6 in.

B 8.5 in.

[✓ **C**] 10.4 in.

D 12 in.

⑥ In $\triangle ABC$, $\overline{BC}$ is longer than $\overline{AB}$, and $BC = 24$. $\overline{AB}$ and $\overline{CA}$ have whole-number lengths. Tell whether each side length is *Possible* or *Not Possible*.



[Art not drawn to scale]

	Possible	**Not Possible**
a. $AB = 7$	●	○
b. $AB = 9$	○	●
c. $AB = 10$	●	○
d. $CA = 25$	●	○

DIFFERENTIATION

RETEACH

Visual Model

Use the Pythagorean Theorem to find perimeter.

Students approaching proficiency with using the Pythagorean Theorem to find the distance between two points will benefit from this model.

Materials For display: large four-quadrant coordinate plane

- Tell students that the distance between grid lines represents 1 inch. Ask a student to plot the quadrilateral with vertices $A(-2, 6)$, $B(6, -1)$, $C(-4, -4)$, and $D(-6, 0)$.
- Ask: *Do you think it is possible to find the perimeter without a ruler or other measuring tool? If so, how?* [Yes; draw a right triangle along each of the 4 sides of the quadrilateral and find the length of each hypotenuse/side of the quadrilateral. Then add the side lengths.]
- Have a student show how to use horizontal and vertical grid lines to construct a right triangle with $\overline{AB}$ as the hypotenuse. Have another student state the leg lengths in inches.
- Ask a student to substitute these lengths into the Pythagorean Theorem to find *AB*. Ask: *What is the length of $\overline{AB}$?* [$\sqrt{8^2 + 7^2} = \sqrt{113}$; $\sqrt{113}$ in.]
- Have students work in pairs to find the lengths of the other sides.
- Have student volunteers demonstrate how they found the following: $BC = \sqrt{10^2 + 3^2}$, or $\sqrt{26}$, in.; $CD = \sqrt{2^2 + 4^2}$, or $\sqrt{20}$, in.; $DA = \sqrt{4^2 + 6^2}$, or $\sqrt{52}$, in.
- Ask: *What will you have to do next to find the perimeter of the quadrilateral?* [Add the four side lengths.]
- Ask: *What is the perimeter to the nearest inch?* [33 in.]

7 Students may count units to find the length of $\overline{AB}$, which is 6. The length of the opposite side, $\overline{CD}$, is also 6. Students may use the Pythagorean Theorem to find the length of $\overline{AD}$ by drawing a right triangle with $\overline{AD}$ as the hypotenuse and applying the Pythagorean Theorem. The length is 5. The length of the opposite side, $\overline{BC}$, is also 5. The perimeter is the sum of the four side lengths, which is 22. ***DOK 2***

CLOSE EXIT TICKET

8 **Math Journal** Look for understanding that the length and width of the base of the box and the Pythagorean Theorem can be used to find the diagonal length of the base of the box. Then the diagonal length of the base of the box, the height of the box, and the Pythagorean Theorem can be used to find the diagonal length of the box.

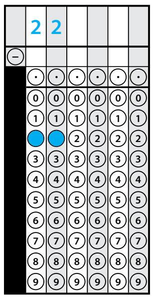
Error Alert If students do not think the baseball bat will fit in the box, then have them review Session 3 Picture It to reinforce using the Pythagorean Theorem twice to find the diagonal length of the box.

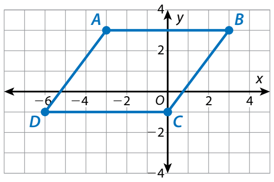
End of Lesson Checklist

INTERACTIVE GLOSSARY Support students by suggesting they make a list of things they will *consider* doing this upcoming weekend.

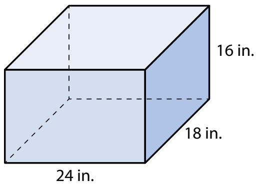
SELF CHECK Have students review and check off any new skills on the Unit 6 Opener.

7 What is the perimeter of parallelogram *ABCD*?





8 **Math Journal** José is mailing some baseball equipment to his cousin. He wants to include a baseball bat that is 33 in. long. Will the baseball bat fit completely in the box? Explain your reasoning.



Yes; Possible explanation: You can use the Pythagorean Theorem twice to find the maximum length that can fit along the longest diagonal of the box. The diagonal of the bottom face of the box is $\sqrt{24^2 + 18^2} = \sqrt{900} = 30$; 30 in. Then, the length of the longest diagonal is $\sqrt{30^2 + 16^2} = \sqrt{1{,}156} = 34$; 34 in. Because the bat is only 33 in. long, it will fit in the box.

End of Lesson Checklist

☐ **INTERACTIVE GLOSSARY** Write a new entry for *consider*. Tell what you do when you consider something.

☐ **SELF CHECK** Go back to the Unit 6 Opener and see what you can check off.

REINFORCE

Problems 4–7

Solve problems using the Pythagorean Theorem.

Students meeting proficiency will benefit from additional work with the Pythagorean Theorem by solving problems in a variety of formats.

- Have students work on their own or with a partner to solve the problems.
- Encourage students to show their work.

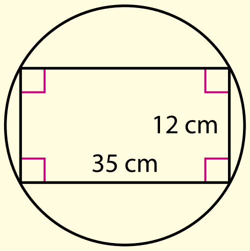
EXTEND

Challenge

Find the area of a circle using the Pythagorean Theorem.

Students extending beyond proficiency will benefit from using the Pythagorean Theorem to help them calculate the area of a circle from an inscribed rectangle.

- Have students work with a partner to solve this problem: *What is the area of the circle?*
- Students may first use the Pythagorean Theorem to find the diagonal of the rectangle, which is a diameter of the circle.



PERSONALIZE

i-Ready

Provide students with opportunities to work on their personalized instruction path with *i-Ready* Online Instruction to:

- fill prerequisite gaps.
- build up grade-level skills.