117

Explore Angle Relationships

measures in the figure. What is $m \angle BCF$?

LESSON 6 SESSION 1 🔳 🗆 🗆

Previously, you learned about pairs of angles formed when two lines intersect. In this lesson, you will learn about pairs of angles formed when one line intersects two other lines.

Use what you know to try to solve the problem below.

Zahara says she can use angle relationships to find all the angle





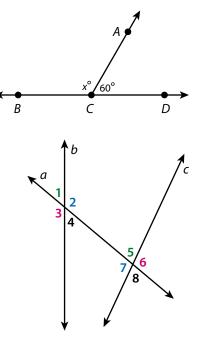
⊦ 15)°

LESSON 6 SESSION 1

CONNECT IT

- **1** Look Back What is $m \angle BCF$? What types of angle relationships did you use to find $m \angle BCF$?
- 2 Look Ahead The figure in the Try It problem shows pairs of angles you know, such as adjacent angles, supplementary angles, and vertical angles. The figure also shows pairs of angles that are new to you.
 - a. You know that supplementary angles are two angles whose measures have a sum of 180°. A **linear pair** is a pair of supplementary angles that are adjacent. What two angles form the linear pair shown? What is the value of *x*?
 - **b.** A **transversal** is a line that intersects or cuts two or more lines. Which line is the transversal in the figure at the right?
 - c. In the figure, ∠2 and ∠7 are alternate interior angles. These angles are on opposite sides of the transversal, and they are inside, or between, the other two lines. What is the other pair of alternate interior angles?
 - d. ∠3 and ∠6 are alternate exterior angles. These angles are on opposite sides of the transversal, but are on the outside of the other two lines. What is the other pair of alternate exterior angles?
 - e. ∠1 and ∠5 are corresponding angles. These angles are in the same position relative to the lines and the transversal. ∠2 and ∠6 are also corresponding angles. What are the other two pairs of corresponding angles?

3 Reflect Is it possible for a pair of angles to be both corresponding angles and alternate interior angles? Explain.



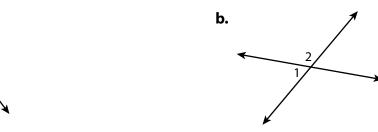
Prepare for Describing Angle Relationships

Name:

1 Think about what you know about angles. Fill in each box. Use words, numbers, and pictures. Show as many ideas as you can.

Word	In My Own Words	Example
adjacent angles		
supplementary angles		
vertical angles		

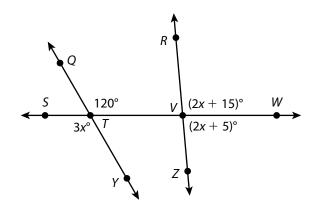
2 For each figure, are $\angle 1$ and $\angle 2$ adjacent angles or vertical angles? Explain.



a.



3 a. What is $m \angle TVZ$? Show your work.



SOLUTION _

b. Check your answer to problem 3a. Show your work.

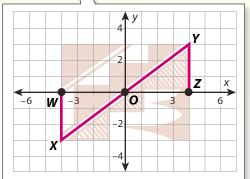
TRY

Develop Describing Congruent Angle Relationships

> Read and try to solve the problem below.

The design on this Native American wedding vase contains many angles. Part of the design is shown in the coordinate plane to help show that some angles are congruent. What sequence of transformations can be used to show that $\angle WXO$ and $\angle ZYO$ are congruent?





Math Toolkit graph paper, tracing paper, transparencies

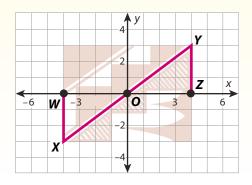


Ask: How did you choose which transformation to use?

Share: I noticed that . . .

> Explore different ways to find and describe congruent angle relationships.

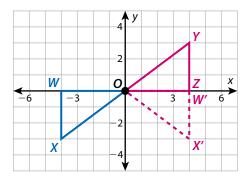
The design on this Native American wedding vase contains many angles. Part of the design is shown in the coordinate plane to help show that some angles are congruent. What sequence of transformations can be used to show that $\angle WXO$ and $\angle ZYO$ are congruent?



Model It

You can use reflections.

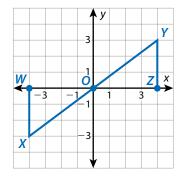
A sequence of two reflections maps $\angle WXO$ onto $\angle ZYO$.

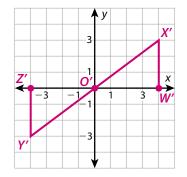


Model It

You can use a rotation.

Rotate the entire figure 180° around the origin to map $\angle WXO$ onto $\angle W'X'O'$.



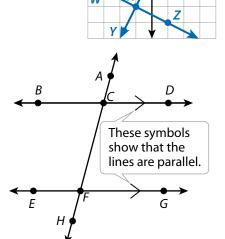




CONNECT IT

- Use the problem from the previous page to help you understand how to find and describe congruent angle relationships.
- 1 \angle WXO and \angle ZYO in the **Try It** problem are alternate interior angles. How can you tell? How do you know these angles are congruent?

- 2 Can you use a sequence of transformations to show that the pairs of alternate interior angles in this figure are congruent? Explain.
- **3 a.** Alternate interior angles formed by parallel lines cut by a transversal are congruent. Mark the two pairs of alternate interior angles that are congruent. Then mark the vertical angles that are congruent.
 - b. Name the pairs of alternate exterior angles in the figure. How are alternate exterior angles related when formed by parallel lines cut by a transversal?

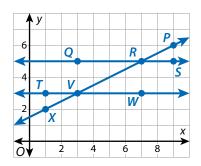


c. Name the pairs of corresponding angles in the figure. How are corresponding angles related when formed by parallel lines cut by a transversal?

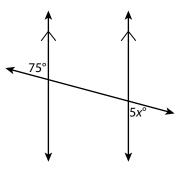
4 **Reflect** Think about all the models and strategies you have discussed today. Describe how one of them helped you better understand the congruent angles formed by parallel lines cut by a transversal.

Apply It

- > Use what you learned to solve these problems.
- 5 Name a pair of corresponding angles in the figure. What sequence of transformations could you use to show that the angles are congruent?

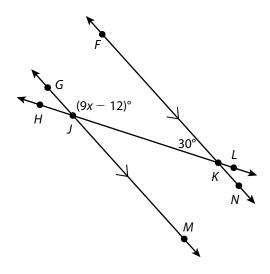


6 Find the value of *x*. Show your work.



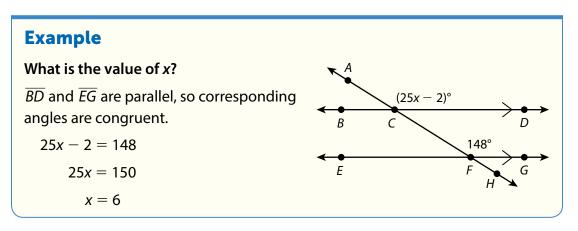
SOLUTION _

7 Find the value of *x*. Show your work.



Practice Describing Congruent Angle Relationships

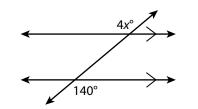
Study the Example showing how to use angle relationships to find unknown angle measures. Then solve problems 1–6.



1 a. In the Example, what angle forms a pair of alternate interior angles with $\angle CFG$?

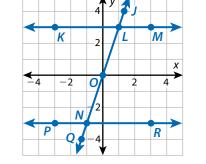
b. What is the measure of the angle you named in problem 1a?

What is the value of *x*? Show your work.



SOLUTION

3 Describe a sequence of transformations you can use to show $\angle JLK \cong \angle QNR$.



Vocabulary

alternate exterior angles

when two lines are cut by a transversal, a pair of angles on opposite sides of the transversal and outside the two lines.

alternate interior angles

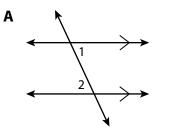
when two lines are cut by a transversal, a pair of angles on opposite sides of the transversal and between the two lines.

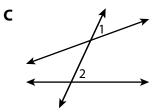
corresponding angles

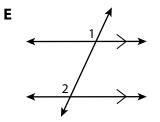
angles in the same relative position when two lines are cut by a transversal.

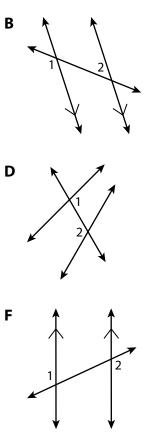
LESSON 6 SESSION 2

4 In which figures is $\angle 1 \cong \angle 2$? Select all that apply.

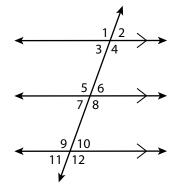






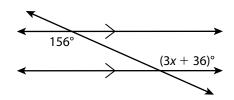


5 Tell whether each statement about the figure is *True* or *False*.



	Irue	False
a. $\angle 1$ and $\angle 9$ are corresponding angles.	\bigcirc	\bigcirc
b. $\angle 2$ and $\angle 7$ are alternate exterior angles.	\bigcirc	\bigcirc
c. $\angle 3$ and $\angle 10$ are alternate interior angles.	\bigcirc	\bigcirc
d. $\angle 4$ and $\angle 7$ are alternate interior angles.	\bigcirc	\bigcirc

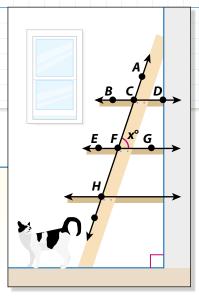
6 What is the value of x? Show your work.

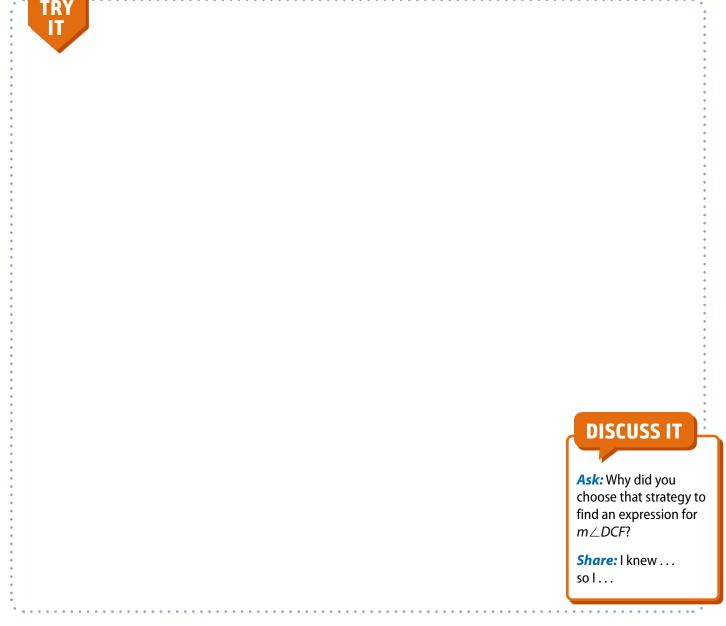


Develop Describing Supplementary Angle Relationships

Read and try to solve the problem below.

A ladder shelf is a shelf that leans against a wall like a ladder. The image shows a side view of a ladder shelf. There are three parallel shelves supported by a brace. The brace acts like a transversal. Write an expression for $m \angle DCF$ in terms of x.





Explore different ways to find and describe supplementary angle relationships.

A ladder shelf is a shelf that leans against a wall like a ladder. The image shows a side view of a ladder shelf. There are three parallel shelves supported by a brace. The brace acts like a transversal. Write an expression for $m \angle DCF$ in terms of x.

Model It

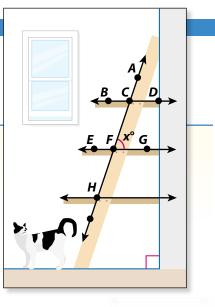
You can use what you know about alternate interior angles.

 $\overrightarrow{BD} \parallel \overrightarrow{EG}$, so $\angle EFC$ and $\angle DCF$ are congruent alternate interior angles. $\angle EFC$ and $\angle CFG$ form a linear pair, so $m \angle EFC + m \angle CFG = 180^\circ$. $m \angle EFC = m \angle DCF$, so you can substitute $m \angle DCF$ for $m \angle EFC$: $m \angle DCF + m \angle CFG = 180^\circ$

Model It

You can use what you know about corresponding angles.

 $\overrightarrow{BD} \parallel \overrightarrow{EG}$, so $\angle ACD$ and $\angle CFG$ are congruent corresponding angles. $\angle ACD$ and $\angle DCF$ form a linear pair, so $m \angle ACD + m \angle DCF = 180^\circ$. $m \angle ACD = m \angle CFG$, so you can substitute $m \angle CFG$ for $m \angle ACD$: $m \angle CFG + m \angle DCF = 180^\circ$





CONNECT IT

- Use the problem from the previous page to help you understand how to find and describe supplementary angle relationships.
- 1 Look at both **Model Its**. What is $m \angle CFG$? What is an expression for $m \angle DCF$ in terms of *x*?
- 2 Would this relationship be true if the lines cut by the transversal were not parallel? Explain.
- 3 a. In Try It, ∠DCF and ∠CFG are same-side interior angles. These angles are on the same side of the transversal, between BD and EG. How are the measures of same-side interior angles related when when they are formed by parallel lines cut by a transversal?
 - **b.** In **Try It**, $\angle ACD$ and $\angle HFG$ are **same-side exterior angles**. These angles are on the same side of the transversal, not between \overrightarrow{BD} and \overrightarrow{EG} . How are their measures related? Explain.

4 You can use the angles formed by two lines cut by a transversal to conclude that the two lines are parallel. For example, the lines are parallel if corresponding angles are congruent. Use similar reasoning to explain how to show two lines are parallel using same-side interior or same-side exterior angles.

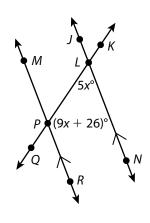
5 **Reflect** Think about all the models and strategies you have discussed today. Describe how one of them helped you better understand the supplementary angles formed by parallel lines and angle relationships.

Apply It

> Use what you learned to solve these problems.

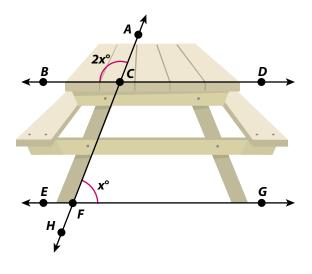


6 What is $m \angle LPR$? Show your work.



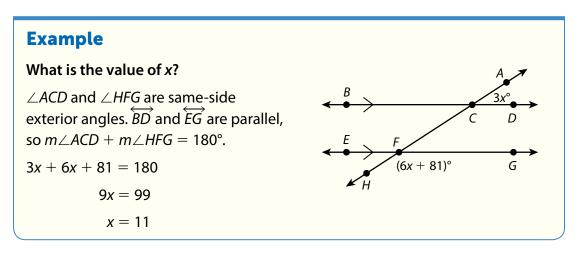
SOLUTION

- 7 Which of these situations show that the lines cut by a transversal are parallel? Select all that apply.
 - **A** alternate exterior angles are supplementary
 - **B** alternate interior angles are supplementary
 - **C** corresponding angles are congruent
 - **D** same-side exterior angles are congruent
 - **E** same-side interior angles are supplementary
- 8 The figure shows a picnic table. The top is represented by \overrightarrow{BD} . The ground is represented by \overrightarrow{EG} . The table leg represented by \overrightarrow{AH} acts like a transversal. What value of x will show that the table top is parallel to the ground? Show your work.



Practice Describing Supplementary Angle Relationships

Study the Example showing how to use angle relationships to solve problems. Then solve problems 1–5.



1 What is the angle relationship between $\angle DCF$ and $\angle CFG$ in the Example? What are the measures of these angles? Show your work.

SOLUTION

2 Find the value of *x*. Show your work.

50)° (x

Vocabulary

same-side exterior angles

when two lines are cut by a transversal, a pair of angles on the same side of the transversal and outside the two lines.

same-side interior angles

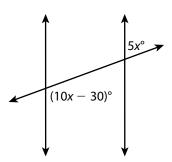
when two lines are cut by a transversal, a pair of angles on the same side of the transversal and between the two lines.

transversal

a line that cuts two or more lines.

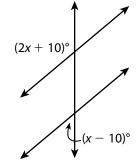
LESSON 6 SESSION 3

3 The figure shows two lines cut by a transversal. What value of *x* shows that the lines are parallel? Show your work.



SOLUTION

- Which statements about the figure are true? Select all that apply.
 - $\textbf{A} \quad \angle 2 \cong \angle 3$
 - **B** $\angle 10$ and $\angle 11$ are supplementary angles.
 - **C** $\angle 2$ are $\angle 5$ are same-side interior angles.
 - **D** $\angle 8$ and $\angle 9$ are supplementary angles.
 - **E** $\angle 9$ and $\angle 12$ are same-side exterior angles.
 - **F** $m \angle 3 + m \angle 6 = 180^{\circ}$
- 5 The figure shows two lines cut by a transversal. What value of *x* shows that the lines parallel? Show your work.



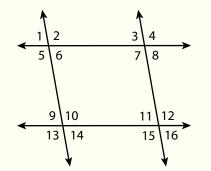
78 9 10 56 11 12

Refine Describing Angle Relationships

> Complete the Example below. Then solve problems 1–10.

Example

The figure shows a pair of parallel lines intersected by another pair of parallel lines. How are the measures of $\angle 6$ and $\angle 12$ related?



Look at how you could use angle relationships.

 $\angle 6 \cong \angle 8 \leftarrow$ Corresponding angles are congruent.

 $m \angle 8 + m \angle 12 = 180^{\circ}$ \leftarrow Same-side interior angles are supplementary.

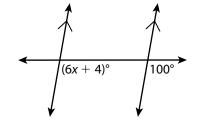
 $m \angle 6 + m \angle 12 = 180^{\circ} \leftarrow \text{Substitute } m \angle 6 \text{ for } m \angle 8.$

SOLUTION _

Apply It



1 What is the value of *x*? Show your work.



CONSIDER THIS... Each line in the figure is also a transversal.

PAIR/SHARE What is another way to solve the problem?

CONSIDER THIS... How are the labeled angles related?

```
PAIR/SHARE
Explain how to check
if your answer is
reasonable.
```

LESSON 6 SESSION 4

2 Draw the lines described. Then label any angle measures that you can determine.

Two parallel lines are cut by a transversal. The alternate interior angles are supplementary.

CONSIDER THIS ...

How are alternate interior angles related when formed by parallel lines cut by a transversal?

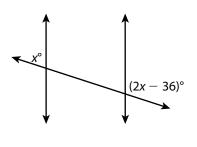
PAIR/SHARE

Suppose the alternate interior angles were congruent instead of supplementary. What angle measures can you calculate now?

CONSIDER THIS

What needs to be true about the angles labeled for the lines to be parallel?

3 The figure shows two lines cut by a transversal. Which value of *x* shows that the lines are parallel?





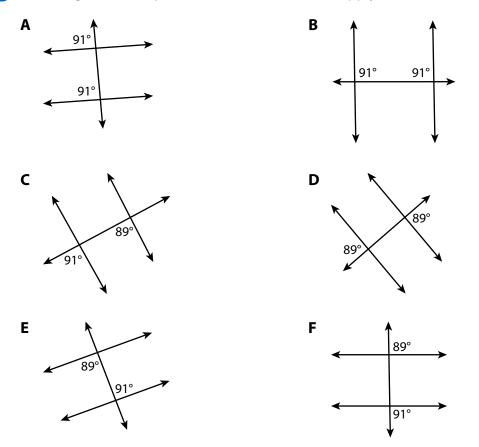
- **B** *x* = 42
- **C** *x* = 48
- **D** *x* = 72

Noah chose A as the correct answer. How might he have gotten that answer?

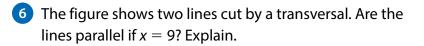
PAIR/SHARE

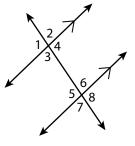
If the lines are parallel, what are the measures of the eight angles formed where the transversal crosses?

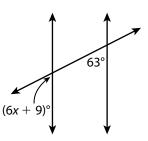
4 Which figures show parallel lines? Select all that apply.



5 Which angles are congruent to $\angle 5$? How do you know?





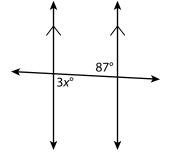


LESSON 6 SESSION 4

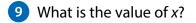
7 Two lines cut by a transversal are parallel if the alternate exterior angles

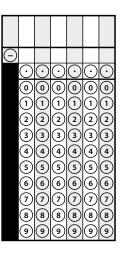
formed are _____.

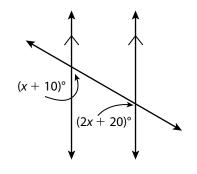
8 What is the value of x? Show your work.



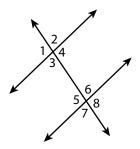
SOLUTION







10 Math Journal In the figure, $\angle 3 \cong \angle 6$. Explain how you know that all four pairs of corresponding angles are congruent.



End of Lesson Checklist

INTERACTIVE GLOSSARY Find the entries for *corresponding angles, alternate interior angles,* and *same-side interior angles.* Sketch an example for each term.

SELF CHECK Go back to the Unit 2 Opener and see what you can check off.

Explore The Sum of the Angle Measures in a Triangle

Previously, you learned about the measures of angles formed by parallel lines and transversals. In this lesson, you will learn about angle measures of triangles.

Use what you know to try to solve the problem below.

An architect needs to know the angle measures of the roof shown in the photo. The triangle to the right models the shape of the roof. What is the sum of the angle measures of the triangle?



Math Toolkit grid paper, straightedges



Ask: What did you do first to find the sum of the angle measures?

Share: First, I found the angle measures by . . .

Learning Target SMP 1, SMP 2, SMP 3, SMP 4, SMP 5, SMP 6

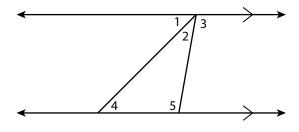
Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles.

 \bigcirc

LESSON 7 SESSION 1

CONNECT IT

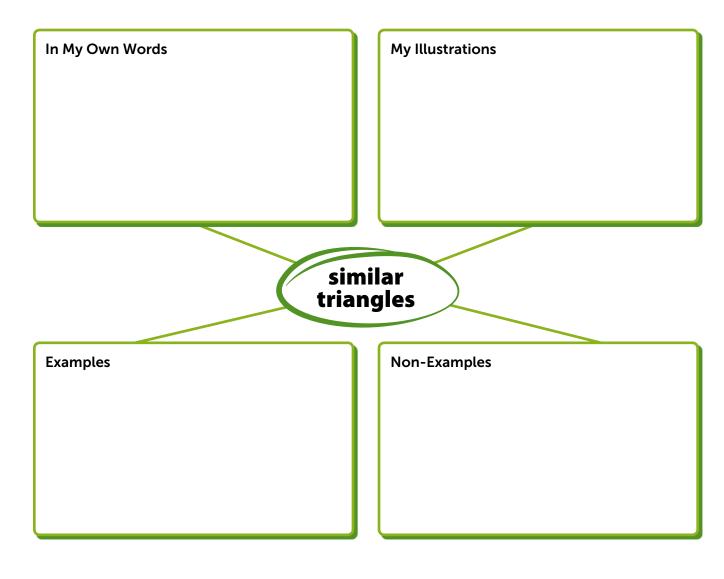
- 1 Look Back Write an equation to show the sum of the angle measures of the triangle in the Try It.
- 2 Look Ahead You know several angle relationships related to parallel lines being cut by a transversal. You can use these relationships to find the sum of the angle measures of a triangle.
 - **a.** Look at this figure. How do you know that $m \perp 1 = m \perp 4$ and $m \perp 3 = m \perp 5$?



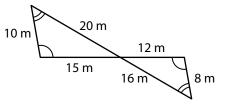
- **b.** Write an equation for the sum of the measures of $\angle 1$, $\angle 2$, and $\angle 3$. How do you know the sum of these angle measures?
- **c.** Use your answers to problems 2a and 2b to find the sum of the measures of the angles of the triangle, $m \angle 2 + m \angle 4 + m \angle 5$.
- 3 **Reflect** Is the sum you found in problem 2c the same as the sum you found in the **Try It**? Do you think you would get this result for any triangle? Explain.

Prepare for Angle Relationships in Triangles

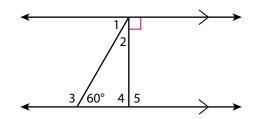
 Think about what you know about similarity and similar triangles. Fill in each box. Use words, numbers, and pictures. Show as many ideas as you can.



2 Are the triangles similar? Explain.



3 The triangle below models a section of the supports you might see in a construction crane.



a. What is the sum of the angle measures of the triangle? Show your work.

SOLUTION

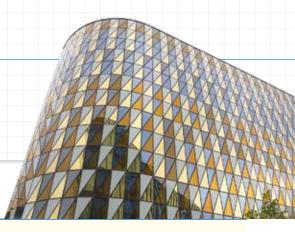
b. Check your answer to problem 3a. Show your work.

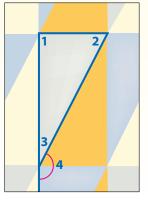
Develop Describing the Exterior Angles of a Triangle

> Read and try to solve the problem below.

The triangular windows of the Aula Medica conference center in Sweden are formed by three sets of parallel lines going in different directions.

A close-up of the window design shows a triangle. The measure of $\angle 4$ is related to the angle measures of the triangle. How can you use the measures of $\angle 1$ and $\angle 2$ to write an expression for the measure of $\angle 4$?





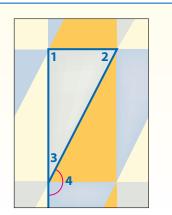


LESSON 7 SESSION 2

Explore different ways to describe angle relationships in triangles.

The triangular windows of the Aula Medica conference center in Sweden are formed by three sets of parallel lines going in different directions.

A close-up of the window design shows a triangle. The measure of $\angle 4$ is related to the angle measures of the triangle. How can you use the measures of $\angle 1$ and $\angle 2$ to write an expression for the measure of $\angle 4$?



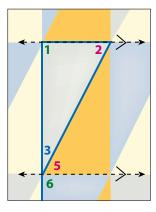
Picture It

You can look for pairs of congruent angles.

 $m \angle 4 = m \angle 5 + m \angle 6$

 $m \angle 5 = m \angle 2 \iff \angle 2$ and $\angle 5$ are alternate interior angles.

 $m \angle 6 = m \angle 1 \iff \angle 1$ and $\angle 6$ are corresponding angles.



Model It

You can use angle relationships.

 $m \perp 1 + m \perp 2 + m \perp 3 = 180^{\circ}$ \leftarrow The sum of the angle measures of a triangle is 180°.

 $m \angle 3 + m \angle 4 = 180^\circ \leftarrow \angle 3$ and $\angle 4$ form a linear pair.

 $m \angle 1 + m \angle 2 + m \angle 3 = m \angle 3 + m \angle 4$

CONNECT IT

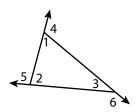
- Use the problem from the previous page to help you understand angle relationships in triangles.
- An exterior angle of a triangle is formed by extending one side of a triangle. There are two exterior angles at each vertex of a triangle, as shown for ∠ZXY in the figures at the right. Why are the two exterior angles at the same vertex congruent?

2 Look at **Model It**. Simplify the last equation. What is the relationship between an exterior angle of a triangle and its nonadjacent interior angles?

- 3 Look at the triangle to the right. Use the measures of the interior angles to write an equation for the measure of each exterior angle.
- 4 Use the equations you wrote in problem 3 to find the sum of the measures of the exterior angles of a triangle, one at each vertex.

5 Do you think your answers to problems 2 and 4 are true for all triangles? Explain.

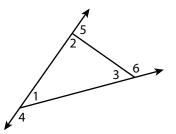
6 **Reflect** Think about all the models and strategies you have discussed today. Describe how one of them helped you better understand how to solve the **Try It** problem.



Apply It

> Use what you learned to solve these problems.

7 Use linear pairs to find the sum of all the angles labeled in the triangle. Then use what you know about the interior angles of a triangle to show that the sum of the measures of the exterior angles of a triangle, one at each vertex, is 360°.



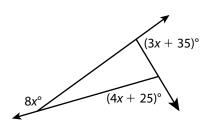
120°

85°

8	Which equations can you use to find the value of <i>x</i> ?
	Select all that apply.

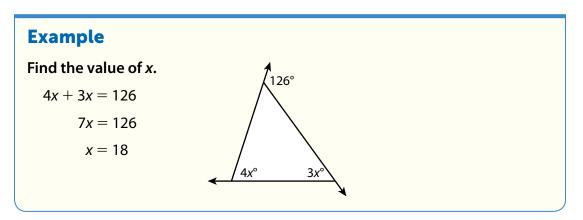
- **A** x + 120 + 85 = 180
- **B** *x* + 85 = 120
- **C** x + 120 + 95 = 360
- **D** x = 60 + 85
- **E** x + 60 + 85 = 180
- **F** x + 35 = 180

9 What is the value of x? Show your work.

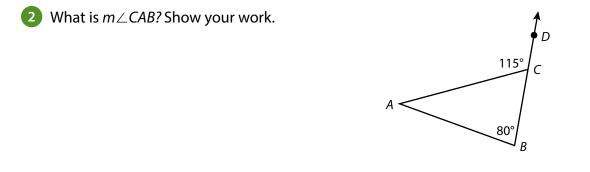


Practice Describing the Exterior Angles of a Triangle

Study the Example showing how to use the relationship between exterior and interior angles of a triangle. Then solve problems 1–6.



1 What are the three exterior angle measures of the triangle in the Example?

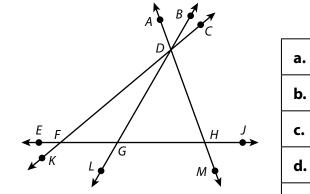


SOLUTION

3 Can a triangle have an exterior angle that measures 90° at two different vertices? Explain.

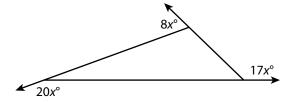
Vocabulary exterior angle

when you extend one side of a polygon, the angle between the extended side and the adjacent side. **4** Tell whether each statement about the diagram is *True* or *False*.

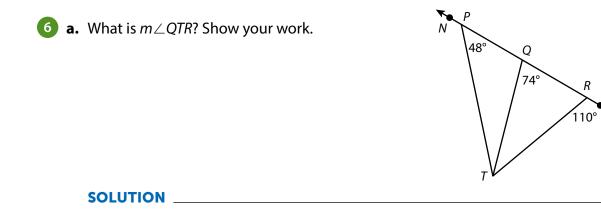


	True	False
a. \angle <i>FGD</i> is an exterior angle of \triangle <i>DHG</i> .	\bigcirc	\bigcirc
b. $m \angle EFD + m \angle HGD = m \angle FDG$	\bigcirc	\bigcirc
c. $m \angle DFH + m \angle FDH = m \angle GHM$	\bigcirc	\bigcirc
d. $m \angle GDH + m \angle DHG = m \angle EGL$	\bigcirc	\bigcirc
e. $m \angle DHJ + m \angle DGE + m \angle BDH = 360^{\circ}$	\bigcirc	\bigcirc

5 What is the value of x? Show your work.



SOLUTION _



b. What is $m \angle PTR$? Show your work.

Develop Using Angles to Determine Similar Triangles

Read and try to solve the problem below.

Jorge wants to draw two triangles that have the same angle measures and are not similar. Carlos says that is not possible to do.

Make or draw two triangles that have the same three angle measures but different side lengths. Are the triangles similar?



Math Toolkit grid paper, protractors, rulers



Ask: How did you make sure that the angles of your triangles had the same measures?

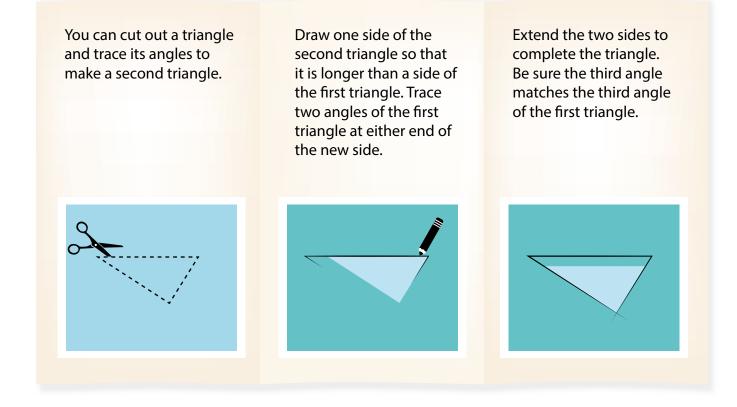
Share: After I made the first triangle . . .

Explore different ways to make triangles with the same angle measures.

Jorge wants to draw two triangles that have the same angle measures and are not similar. Carlos says that is not possible to do.

Make or draw two triangles that have the same three angle measures but different side lengths. Are the triangles similar?

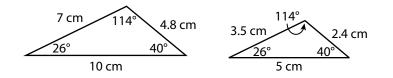
Model It



Model It

You can choose angle measures and one pair of corresponding side lengths.

For example, draw triangles with angles of 26°, 40°, and 114° such that there is one pair of corresponding sides with lengths 10 cm and 5 cm. Measure and compare the other corresponding side lengths.



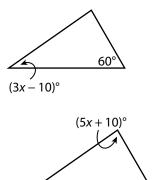
CONNECT IT

- Use the problem from the previous page to help you understand how to use angle measures to tell whether two triangles are similar.
 Look at the pairs of triangles you and your classmates accurately made or drew as you worked on the Try It problem. Are the triangles in each pair similar? Explain.
 Look at the triangles in the second Model It. Are the triangles similar? Explain.
 - 3 Will all possible triangles with the same three angle measures be similar? Explain.
 - 4 Suppose two angle measures of one triangle are equal to two angle measures of another triangle. Must the third angle measures also be equal? Explain.
- 5 Suppose two triangles have two pairs of corresponding angles that are congruent. Are the triangles similar? Explain.
- 6 **Reflect** Think about all the models and strategies you have discussed today. Describe how one of them helped you better understand how to solve the **Try It** problem.

Apply It

> Use what you learned to solve these problems.

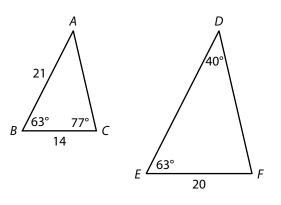
7 Are the triangles similar? How do you know? Show your work.



35°

SOLUTION

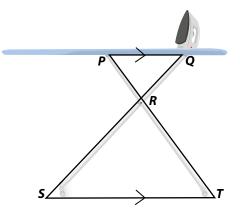
8 Find *DE*. Show your work.



4১

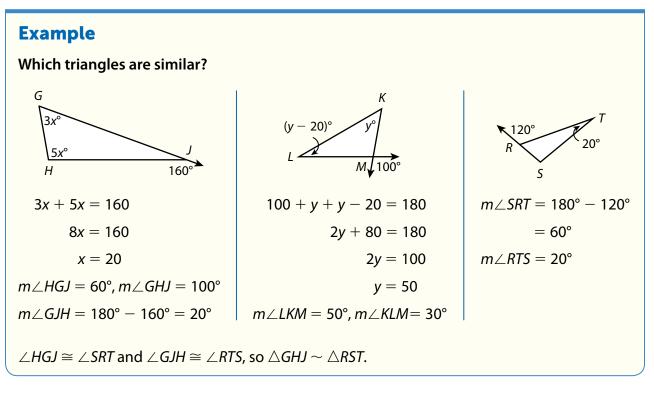
SOLUTION

9 An ironing board, its legs, and the floor form two triangles as shown in the figure. The top of the board, PQ, is parallel to the floor, ST. Write a similarity statement for the two triangles. Explain how you know that they are similar.

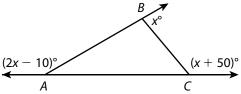


Practice Using Angles to Determine Similar Triangles

Study the Example showing how to use angle measures to identify similar triangles. Then solve problems 1–6.



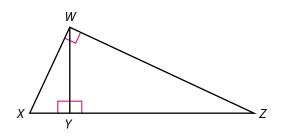
Is △ABC similar to any of the triangles in the Example? Explain.



2 Triangle X has two angles that measure 80° and 30°. Triangle Y has two angles that measure 80° and 70°. Hannah says that triangles X and Y are not similar. Jasmine says they are similar. Who is correct? Explain.

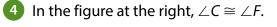
LESSON 7 SESSION 3

3 a. Is $\triangle XYW \sim \triangle XWZ$? Explain why or why not.

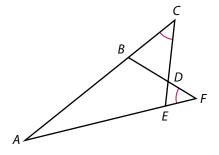


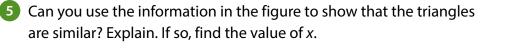
b. Is $\triangle WYZ \sim \triangle XWZ$? Explain why or why not.

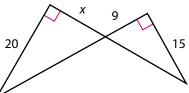
c. Is $\triangle XYW \sim \triangle WYZ$? Explain why or why not.



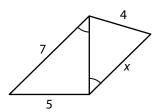
- **a.** Which triangle is similar to $\triangle BCD$?
- **b.** Which triangle is similar to $\triangle ACE$?







6 Can you use the information in the figure to show that the triangles are similar? Explain. If so, find the value of *x*.



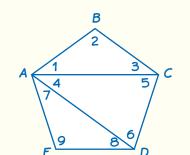
Refine Describing Angle Relationships in Triangles

Complete the Example below. Then solve problems 1–10.

Example

What is the sum of the angle measures of a pentagon?

Look at how you could use the sum of the angle measures of a triangle.



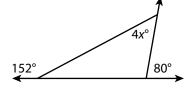
 $m \angle EAB + m \angle ABC + m \angle BCD + m \angle CDE + m \angle DEA =$ $(m \angle 7 + m \angle 4 + m \angle 1) + m \angle 2 + (m \angle 3 + m \angle 5) +$ $(m \angle 6 + m \angle 8) + m \angle 9 =$ $(m \angle 1 + m \angle 2 + m \angle 3) + (m \angle 4 + m \angle 5 + m \angle 6) +$ $(m \angle 7 + m \angle 8 + m \angle 9) = 180^{\circ} + 180^{\circ} + 180^{\circ}$

SOLUTION .

Apply It



1) What is the value of x? Show your work.



CONSIDER THIS

How can you draw triangles so that the angles of the triangles make up the angles of the pentagon?

PAIR/SHARE

Can you use the same strategy to find the sum of the angle measures of a hexagon? Explain.

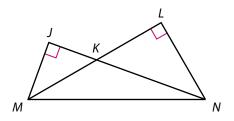
CONSIDER THIS

How is each exterior angle measure related to the interior angle measures?

FAIN/JUANE

How could you use a different angle relationship to solve this problem?

2 Explain how you know that $\triangle MJK \sim \triangle NLK$.



CONSIDER THIS... Look at each pair of corresponding angles.

PAIR/SHARE Is $\triangle MJN \sim \triangle NLM$? How do you know?

3 A triangle has exterior angles, one at each vertex, that measure 90°, 3x°, and 3x°. What is the value of x?

- **A** 15
- **B** 45
- **C** 90
- **D** 135

Zhen chose D as the correct answer. How might she have gotten that answer?

CONSIDER THIS

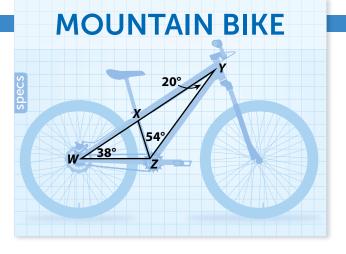
What do you know about the exterior angle measures of a triangle?

PAIR/SHARE

Can you explain to a partner how you solved this problem?



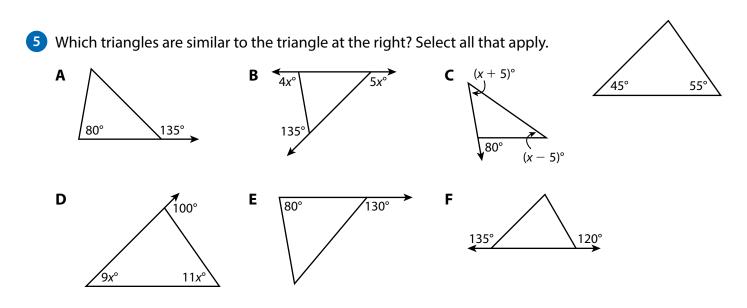
a. What is $m \angle WXZ$? Show your work.



SOLUTION _

b. What is $m \angle XZW$? Show your work.

SOLUTION



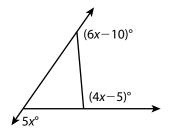
6 Can a triangle have an exterior angle that is obtuse at two vertices? Explain.

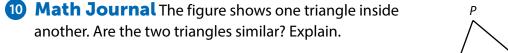
7 Tell whether each statement is *True* or *False*.

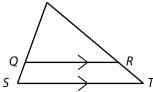
	True	False
a. A triangle can have more than one obtuse angle.	\bigcirc	\bigcirc
b. In a right triangle, the sum of the measures of the acute angles is 90°.	\bigcirc	\bigcirc
c. The measure of an exterior angle of a triangle is equal to the sum of the two nonadjacent interior angles.	\bigcirc	\bigcirc
d. The sum of the measures of the exterior angles of a triangle, one at each vertex, is 180°.	\bigcirc	\bigcirc

8 If all three angles of a triangle are congruent, what is each angle measure? Explain.

9 What is the value of x?







End of Lesson Checklist

INTERACTIVE GLOSSARY Find the entry for *exterior angle*. Sketch an example of an exterior angle of a triangle.

SELF CHECK Go back to the Unit 2 Opener and see what you can check off.

LESSON 27 | SESSION 1 🛛 🗆 🗆 🗆

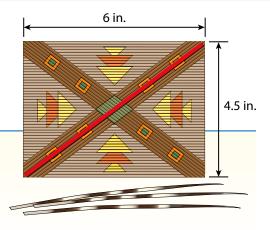
Explore Applying the Pythagorean Theorem

Previously, you learned about the Pythagorean Theorem. In this lesson, you will learn how to use the Pythagorean Theorem to solve problems involving right triangles.

Use what you know to try to solve the problem below.

A Mi'kmaq (Mic-mac) artist uses porcupine quills to decorate a rectangular piece of leather. She places a row of porcupine quills extending from one corner to the opposite corner. Each quill measures 1.5 inches long. How many quills does the artist need to make her first diagonal?

Math Toolkit grid paper, rulers, unit tiles



DISCUSS IT

Ask: What did you do first to find the number of quills? Why?

Share: I knew . . . so I . . .

Learning Targets SMP 1, SMP 2, SMP 3, SMP 4, SMP 5, SMP 6, SMP 7

Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

• Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

 \bigcirc

TR

CONNECT IT

1 Look Back How many quills are in the diagonal? How do you know?

2 Look Ahead

a. Fill in the blanks to restate the Pythagorean Theorem.

In any ______ triangle, the sum of the squares of the lengths of the

_____ is equal to the square of the length of the ______.

If *a* and *b* are the lengths of the legs of a right triangle and *c* is the length of the hypotenuse, then the theorem can be represented by the equation

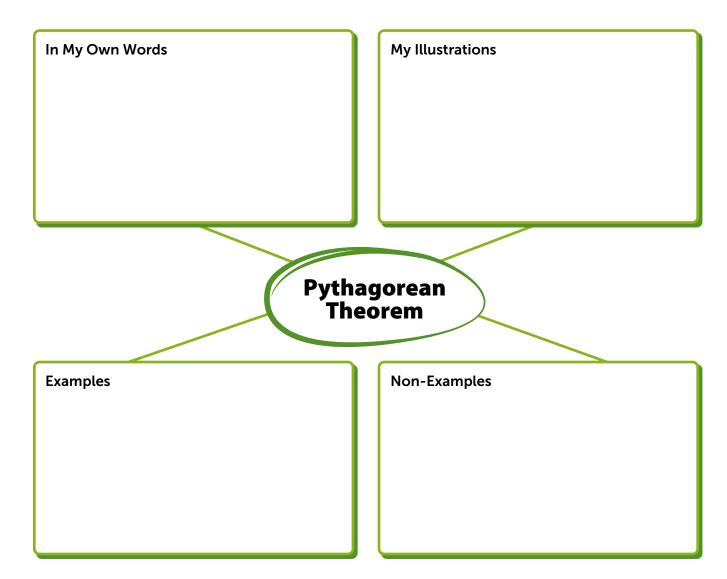
- **b.** When the side lengths of a right triangle are whole numbers, the lengths are known as *Pythagorean triples*. For example, the set of lengths 3, 4, 5 is a Pythagorean triple because $3^2 + 4^2 = 5^2$. Look at each set of three numbers shown on the right. Circle each set that is a Pythagorean triple.
- **c.** Jason says that if you multiply each length in a Pythagorean triple by the same factor, you will get another Pythagorean triple. Explain why Jason is correct.

2, 3, 4 4, 5, 6 5, 12, 13 6, 8, 10 8, 15, 17 9, 16, 25

3 Reflect Give an example to show that a multiple of a Pythagorean triple is also a Pythagorean triple.

Prepare for Applying the Pythagorean Theorem

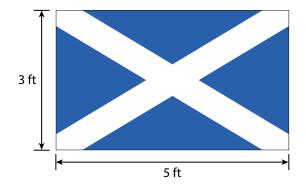
Think about what you know about right triangles. Fill in each box. Use words, numbers, and pictures. Show as many ideas as you can.



2 Suppose you have a right triangle with side lengths 26, 24, and 10. Which lengths represent the legs and which length represents the hypotenuse of the triangle? Use the Pythagorean Theorem to explain how you know.

LESSON 27 SESSION 1

- 3 The flag of Scotland consists of a blue rectangular background with two white diagonals.
 - a. The dimensions of a Scottish flag are shown. If the flag's diagonals are made of white ribbon, what length of ribbon is needed for both diagonals? Show your work. Round your answer to the nearest foot.



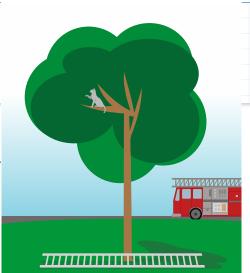
SOLUTION

b. Check your answer to problem 3a. Show your work.

Develop Finding an Unknown Length in a Right Triangle

Read and try to solve the problem below.

A firefighter is trying to rescue a kitten from a tree. He leans a 13-foot ladder so its top touches the tree. The base of the ladder is 5 feet from the base of the tree. The tree forms a right angle with the ground. How high up the tree does the ladder reach?





Math Toolkit centimeter grid paper, centimeter ruler, unit cubes

DISCUSS IT

Ask: Where does your model show the leg and the hypotenuse of the right triangle formed by the ground, tree, and ladder?

Share: In my model, ... represents ...

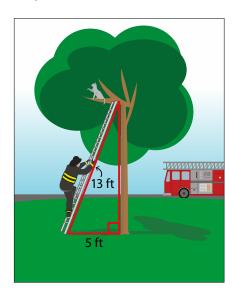
Explore different ways to find how to use the Pythagorean Theorem to find an unknown length.

A firefighter is trying to rescue a kitten from a tree. He leans a 13-foot ladder so its top touches the tree. The base of the ladder is 5 feet from the base of the tree. The tree forms a right angle with the ground. How high up the tree does the ladder reach?

Picture It

You can draw a picture to show the information you are given and what you still need to find.





Model It

You can use the Pythagorean Theorem.

The ladder represents the hypotenuse of the right triangle formed by the ladder, tree, and ground. Let *b* be the length you need to find.

$$5^{2} + b^{2} = 13^{2}$$

 $25 + b^{2} = 169$
 $b^{2} = 169 - 25$
 $b^{2} = 144$
 $b = \pm \sqrt{144}$

CONNECT IT

- Use the problem from the previous page to help you understand how to use the Pythagorean Theorem to find an unknown length.
- 1 Look at **Picture It** and **Model It**. Explain how the picture and the equation $5^2 + b^2 = 13^2$ represent this situation.

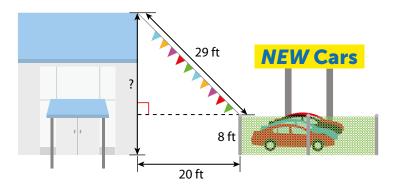
- 2 a. How high up the tree does the ladder reach? Why do you not need to consider the negative square root of b?
 - **b.** When using the Pythagorean Theorem to find an unknown length, do you ever need to consider a negative square root? Explain.
- 3 A right triangle has leg lengths *a* and *b* and hypotenuse length *c*. Write a subtraction equation that you could use to solve for *b*. Write a subtraction equation that you could use to solve for *a*.
- 4 When might you use one of the equations in problem 3? Why?

5 Reflect Think about all the models and strategies you have discussed today. Describe how one of them helped you better understand how to solve the Try It problem.

Apply It

> Use what you learned to solve these problems.

6 A car salesman is stringing banners from the top of the roof to a fence pole 20 feet away. The fence pole is 8 feet high. He uses 29 feet of banner rope to reach from the rooftop to the fence pole. How tall is the roof? Show your work.



SOLUTION _

7 Which set of side lengths do not form a right triangle?

- **A** 5, 6, √61
- **B** 24, 7, 25
- **C** 14, 7, $\sqrt{245}$
- **D** 12, 15, 18

8 The perimeter of an equilateral triangle is 48 cm. Find the height of the triangle to the nearest whole number. Show your work.

Practice Finding an Unknown Length in a Right Triangle

Study the Example showing how to find an unknown length in a right triangle. Then solve problems 1-4.



A meteorologist ties a spherical balloon that is 2 feet in diameter to a stake in the ground. The string is 15 feet long. The wind blows the balloon so that the top of it is 8 feet to the right of the stake. What is the distance, b, from the top of the balloon to the ground?

Use a right triangle. The lengths of the hypotenuse and one leg are known. Use the Pythagorean Theorem to find the length of the other leg.

$$b^{2} = c^{2} - a^{2}$$

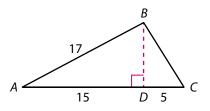
 $b^{2} = 17^{2} - 8^{2}$
 $b^{2} = 289 - 64$
 $b^{2} = 225$
 $b = \sqrt{225}$ or 15

wind 15 ft b V 8 ft The distance from the top of the balloon to the ground is 15 ft.

1 Imani used the equation $8^2 + b^2 = 17^2$ to find the distance from the top of the balloon to the ground in the Example. Why does this equation also work?

2 Kamal said the distance from the top of the balloon to the ground in the Example is $\sqrt{353}$ ft. What mistake might Kamal have made?

- 3 The diagram shows $\triangle ABC$.
 - **a.** What is the height of $\triangle ABC$? Show your work.

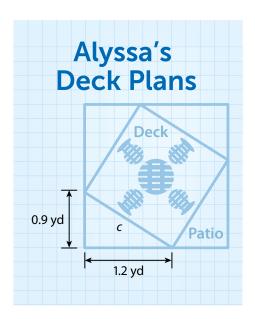


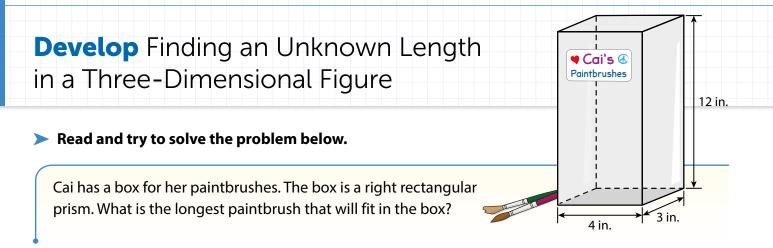
SOLUTION

b. What is the length of \overline{BC} in $\triangle ABC$? Show your work.

SOLUTION

Alyssa is designing a square wooden deck with side length c yards. She will build the deck over her square patio, as shown in the diagram. Find the perimeter of the deck. Show your work.







Math Toolkit grid paper, rulers



Ask: Why did you choose that strategy to find the longest paintbrush that will fit?

Share: The problem is asking . . .

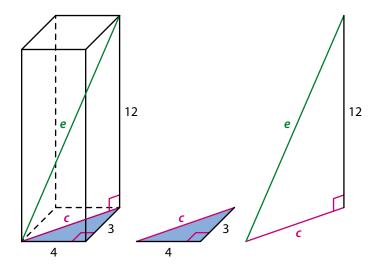
Explore different ways to find an unknown length in a right rectangular prism.

Cai has a box for her paintbrushes. The box is a right rectangular prism. What is the longest paintbrush that will fit in the box?

Picture It

You can draw a diagram to represent the problem.

Look for right triangles that can help you find the length you need.



Cai's Cai's Paintbrushes Paintbrushes 4 in.

You can use the diagonal length of the base, *c*, to find the diagonal length of the box, *e*.

Model It

You can use the Pythagorean Theorem to write equations. To find the diagonal length of the base: $3^2 + 4^2 = c^2$ To find the diagonal length of the box: $c^2 + 12^2 = e^2$ Substitute to get one equation: $3^2 + 4^2 + 12^2 = e^2$



CONNECT IT

- Use the problem from the previous page to help you understand how to find an unknown length in a right rectangular prism.
- 1 Look at **Picture It** and **Model It**. How does finding the length of diagonal *c* help you find the length of diagonal *e*?

2 Solve the last equation in **Model It** to find *e*. What is the longest paintbrush that will fit in the box?

3 Tyrone says you can also solve the problem by first solving $3^2 + 4^2 = c^2$ for c and then using this value of c to solve $c^2 + 12^2 = e^2$ for e. Is Tyrone correct? Explain.

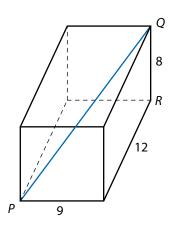
When can you use the Pythagorean Theorem to find an unknown length in a right rectangular prism?

5 Reflect Think about all the models and strategies you have discussed today. Describe how one of them helped you better understand how to solve the Try It problem.

Apply It

> Use what you learned to solve these problems.

6 Find the length of the diagonal from *P* to *Q* in this right rectangular prism. Show your work.

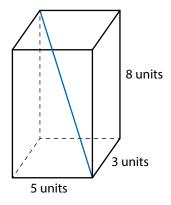


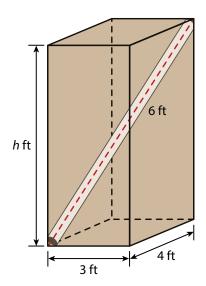
SOLUTION _

7 What is the distance from one corner of the bottom base to the opposite corner of the top base in this right rectangular prism?

- **A** $\sqrt{34}$ units
- **B** $\sqrt{80}$ units
- **C** $\sqrt{89}$ units
- **D** $\sqrt{98}$ units

8 Anica is shipping a poster to a customer. When the poster is rolled up, it measures 6 feet long. She will use a box that is a right rectangular prism with a base that is 3 feet by 4 feet. What whole number could be the shortest height of the box that will hold the poster? Show your work.





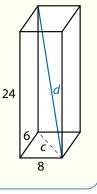
Practice Finding an Unknown Length in a Three-Dimensional Figure

Study the Example explaining how to find the length of a diagonal in a right rectangular prism. Then solve problems 1–4.

Example

The diagram shows a diagonal drawn in a right rectangular prism. Explain how you can find *d*, the length of this diagonal.

Look for right triangles. Side length *d* is the hypotenuse of the right triangle with leg lengths *c* and 24. Side length *c* is the hypotenuse of the right triangle in the base of the prism with leg lengths 6 and 8. Use the Pythagorean Theorem to find *c*. Then use it again to find *d*.

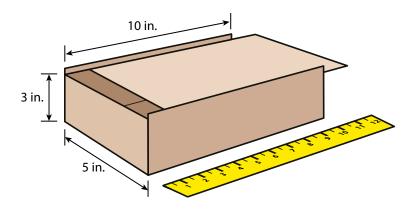


a. How do you know that the triangle with side lengths 6, 8, and *c* in the Example is a right triangle?

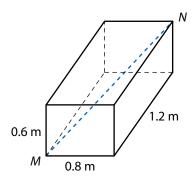
b. How do you know that the triangle with side lengths *c*, 24, and *d* in the Example is a right triangle?

2 What is the length of the diagonal *d* in the Example? Show your work.

Will a 12-inch ruler fit in a box that is a right rectangular prism with a width of 5 inches, a length of 10 inches, and a height of 3 inches? Explain your answer.



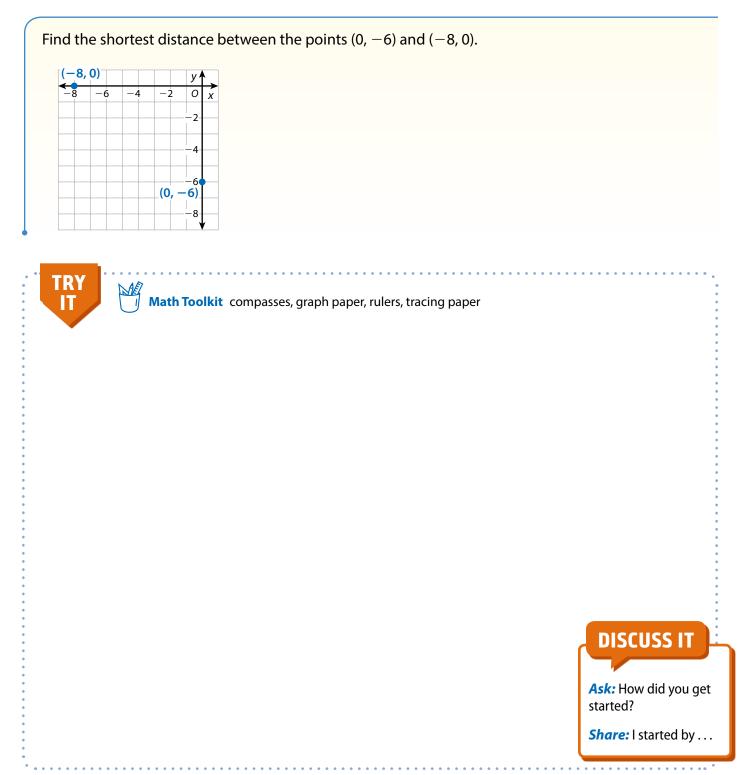
4 In the right rectangular prism, what is the length of the diagonal from *M* to *N* to the nearest tenth of a meter? Show your work.





Develop Finding Distance in the Coordinate Plane

> Read and try to solve the problem below.



Explore different ways to find the distance between points in the coordinate plane.

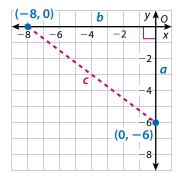
Find the shortest distance between the points (0, -6) and (-8, 0).

Picture It

You can draw a right triangle.

Draw a line segment between the points. Then draw a right triangle so the distance between the points is the hypotenuse.

In the right triangle, *a* and *b* are the lengths of the legs and *c* is the length of the hypotenuse.



Model It

You can use the Pythagorean Theorem to find the unknown distance.

 $a^2 + b^2 = c^2$ $b^2 + b^2 = c^2$

648 LESSON 27 Apply the Pythagorean Theorem

CONNECT IT

- Use the problem from the previous page to help you understand how to find the distance between two points in the coordinate plane.
 Look at Picture It. How do you know the triangle formed by the two points and the origin is a right triangle?
- 2 Jessica said the lengths of the legs are -6 units and -8 units. What mistake did Jessica make? What are the correct lengths of the legs?
- 3 Look at **Model It**. What is the distance between (0, −6) and (−8, 0)? Why can you use the Pythagorean Theorem to find this distance?
- 4 Why is it important that the distance between (0, −6) and (−8, 0) be the hypotenuse of the right triangle and not a leg?
- 5 Explain how to find the distance between any two points in the coordinate plane that do not lie on the same horizontal or vertical line.

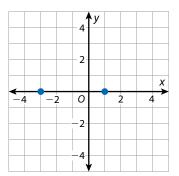
6 **Reflect** Think about all the models and strategies you have discussed today. Describe how one of them helped you better understand how to find the distance between two points in the coordinate plane.

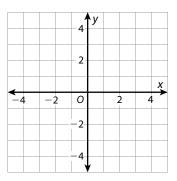
Apply It

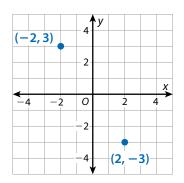
> Use what you learned to solve these problems.

- 7 Hailey plots the first two vertices of a right triangle in the coordinate plane. Where could she plot the third point so the hypotenuse of the triangle has a length of 5 units?
 - **A** (1, 5)
 - **B** (−3, 4)
 - **C** (0, −5)
 - **D** (1, −3)
- 8 Draw a line segment from the origin (0, 0) with length $\sqrt{20}$. Show your work.

9 Find the distance between the points shown. Show your work.

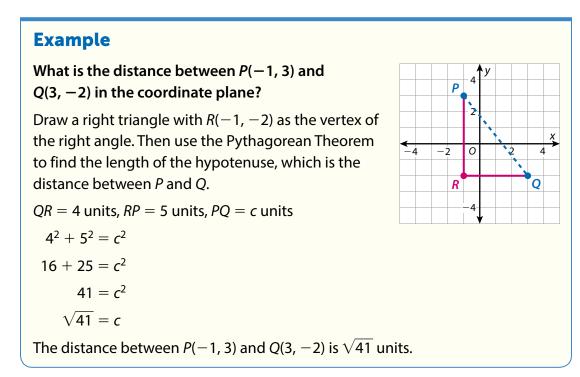






Practice Finding Distance in the Coordinate Plane

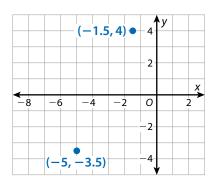
Study the Example showing how to find the distance between two points in the coordinate plane. Then solve problems 1–3.



1 Look at the Example.

- **a.** Describe another right triangle you could draw from points P(-1, 3) and Q(3, -2).
- **b.** Will your triangle from problem 1a give you the same distance between points *P* and *Q*? Explain.
- **c.** Would drawing a triangle with points (-1, 3), (3, -2), and (3, 2) as vertices help you find the distance between the points? Explain.

2 What is the distance between the points shown? Show your work. Round your answer to the nearest tenth.



SOLUTION _____

3 Dr. Patel plots points *J*, *K*, and *L* in the coordinate plane.

a. What is the distance between points *J* and *K*? Show your work.

	6 y	<i>K</i> (1, 5)	
	-4-		
	2		
← −6 −4 −2	0	2 4	6 8 ×
	-2		
J(-5, -3)	-4		L(7, -3)

SOLUTION _____

b. What is the distance between points *K* and *L*? Show your work.

SOLUTION _____

c. Is $\triangle JKL$ an equilateral triangle? Explain.

Refine Applying the Pythagorean Theorem

Complete the Example below. Then solve problems 1–8.

Example

Chase drew a triangle with vertices at (-2, -1), (-1, 2), and (1, -1). Classify Chase's triangle as scalene, isosceles, or equilateral.

Look at how you could use the Pythagorean Theorem.

Draw a right triangle at each nonvertical and nonhorizontal side of the original triangle.

$$\overline{AB}$$
: 1² + 3² = AB²
AB² = 10, so AB = $\sqrt{10}$.

$$\overline{BC}$$
: 2² + 3² = BC²

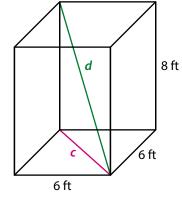
$$BC^{2} = 13$$
, so $BC = \sqrt{13}$.

 \overline{AC} : AC = 3

SOLUTION _

Apply It

 Mr. Gaspar wants to store a 12-foot-long pipe in a tool closet. The closet has the shape of a right rectangular prism with the dimensions shown. Will the pipe fit? Show your work.



CONSIDER THIS ...

An isosceles triangle has at least two congruent sides. A scalene triangle has no congruent sides.

PAIR/SHARE How else could you solve this?

CONSIDER THIS... This problem takes more than one step to solve.

PAIR/SHARE Can you find two different paths to

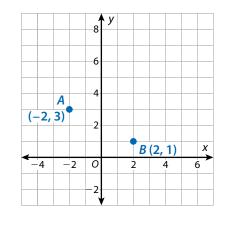
the solution? Explain.

2 \overline{AB} is one side of a square. Find the coordinates of the other two vertices of the square and draw the square. Explain your reasoning.

3 Mr. Shaw is building a walkway from the corner of his house out to his

vegetable garden in the corner of the yard. The dimensions of the yard

and the garden are shown. What is the length of the walkway, w, to the



CONSIDER THIS

How could a right triangle help you find the side length of the square?

PAIR/SHARE How else could you solve this?

CONSIDER THIS ...

Where can you draw a useful right triangle?

 $\begin{array}{c} 50 \text{ ft} \\ \hline \\ \hline \\ 14 \text{ ft} \\ \hline \\ 20 \text{ ft} \\ \hline \\ 20 \text{ ft} \\ \hline \\ \end{array} \right] 30 \text{ ft}$

A 24 ft

nearest foot?

- **B** 34 ft
- **C** 46 ft
- **D** 58 ft

Ummi chose C as the correct answer. How might she have gotten that answer?

PAIR/SHARE

Was there more than one helpful right triangle that would lead to the solution?

654 LESSON 27 Apply the Pythagorean Theorem

4 Draw the reflection of $\triangle ABC$ across the y-axis. Then show that the corresponding sides of the two triangles are congruent.

- 5 A small plastic storage cube has a side length of 6 inches. What is the length of \overline{AB} to the nearest tenth of an inch?
 - A 6 in.
 - **B** 8.5 in.
 - **C** 10.4 in.
 - **D** 12 in.

a. *AB* = 7

b. *AB* = 9

c. AB = 10

d. *CA* = 25

6 In $\triangle ABC$, \overline{BC} is longer than \overline{AB} , and BC = 24. \overline{AB} and \overline{CA} have whole-number lengths. Tell whether each side length is Possible or Not Possible.

Possible

()

()

 \bigcirc

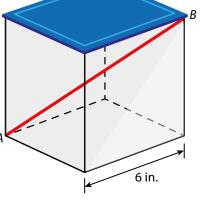
 \bigcirc

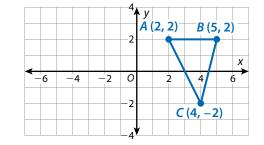
Not Possible

A		
В	 	

[Art not drawn to scale]

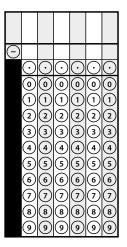
С



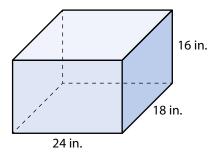


LESSON 27 SESSION 5

7 What is the perimeter of parallelogram ABCD?



8 Math Journal José is mailing some baseball equipment to his cousin. He wants to include a baseball bat that is 33 in. long. Will the baseball bat fit completely in the box? Explain your reasoning.



End of Lesson Checklist

INTERACTIVE GLOSSARY Write a new entry for *consider*. Tell what you do when you consider something.

SELF CHECK Go back to the Unit 6 Opener and see what you can check off.