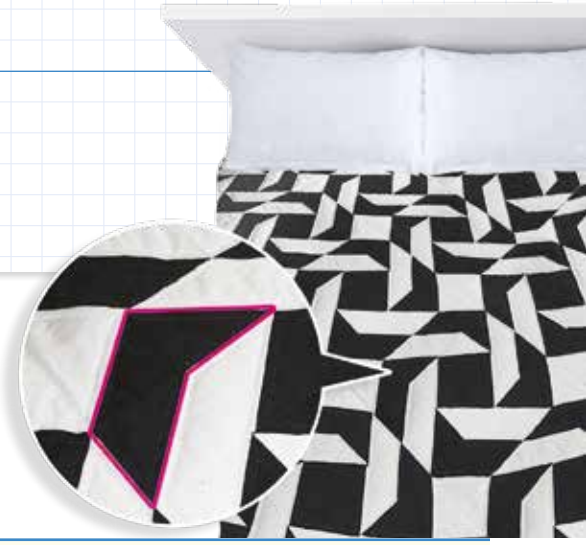
Explore Sequences of Rigid Transformations and Congruence



Previously, you learned how to perform and describe rigid transformations. In this lesson, you will learn what happens when you perform sequences of one or more rigid transformations.

➤ **Use what you know to try to solve the problem below.**

Arachne's Quilt is an Amish quilt pattern made of polygons. You can use a coordinate plane to plan a quilt. Graph a pentagon with the vertices $A(8, 0)$, $B(6, -2)$, $C(6, -6)$, $D(4, -4)$, and $E(4, 0)$. Then to graph another pentagon, follow these steps.

- Translate pentagon $ABCDE$ 4 units to the left to form pentagon $A'B'C'D'E'$.
- Rotate pentagon $A'B'C'D'E'$ 90° counterclockwise around the origin to form pentagon $A''B''C''D''E''$.

Is pentagon $A''B''C''D''E''$ the same size and shape as pentagon $ABCDE$?

TRY IT

Math Toolkit graph paper, protractors, rulers, tracing paper

DISCUSS IT

Ask: How did you decide whether pentagons $ABCDE$ and $A''B''C''D''E''$ are the same size and shape?

Share: The strategy I used was . . .

Learning Target **SMP 1, SMP 2, SMP 3, SMP 4, SMP 5, SMP 6, SMP 7**

Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.

CONNECT IT

1 Look Back Is pentagon $A''B''C''D''E''$ the same size and shape as pentagon $ABCDE$? Explain.

2 Look Ahead You already know that you can transform a figure. You can also transform the image of a figure. This is called performing a **sequence of transformations** on a figure.

a. You can add a prime mark to the vertex labels for each transformation you perform to name an image resulting from a sequence of transformations. How many transformations were performed to get image $X''''Y''''Z''''$? Explain.

b. Figures that are the same size and shape are **congruent**. When a figure is translated, reflected, or rotated, the image is always congruent to the original figure. That means pentagons $ABCDE$ and $A'B'C'D'E'$ are congruent.

pentagon $ABCDE \cong$ pentagon $A'B'C'D'E'$

Read $\cong$ as *is congruent to*.

Write a congruence statement for pentagons $A''B''C''D''E''$ and $A'B'C'D'E'$.

c. Enrico translates figure Q to get figure Q'. Then he rotates figure Q' to get figure Q''. Explain why figure Q and figure Q'' are congruent.

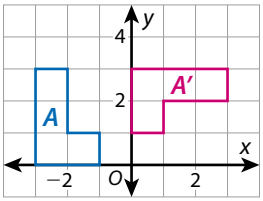
3 Reflect How can you use a sequence of transformations to show that one figure is congruent to another?

Prepare for Sequences of Rigid Transformations and Congruence

1 Think about what you know about transformations. Fill in each box. Use words, numbers, and pictures. Show as many ideas as you can.

Word	In My Own Words	Examples
translation		
reflection		
rotation		

2 **a.** Tell which type of transformation maps figure *A* onto figure *A′*. Then draw the line of reflection, the center of rotation, or a direction arrow to show the type of transformation on the graph.



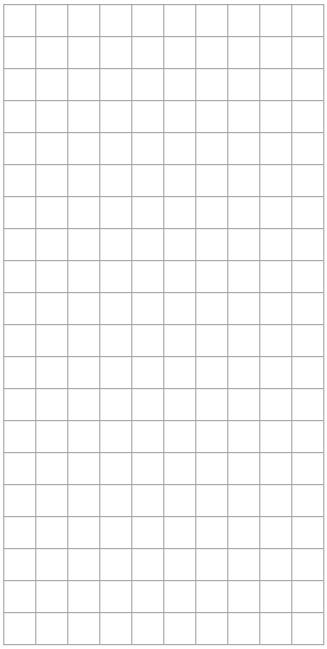
b. Describe the transformation in more detail.

3 The quilt shown has a border made of right triangles and trapezoids.

a. Graph one triangle with vertices at $D(-4, 0)$, $E(0, 0)$, and $F(0, -8)$. To locate another triangle, follow these steps:

- Reflect $\triangle DEF$ across the y-axis to form $\triangle D'E'F'$.
- Translate $\triangle D'E'F'$ 8 units up to form $\triangle D''E''F''$.

Is $\triangle DEF$ the same size and shape as $\triangle D''E''F''$? Show your work.



SOLUTION

b. Check your answer to problem 3a. Show your work.

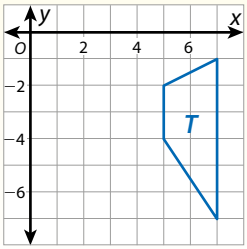
Develop Performing Sequences of Rigid Transformations

➤ **Read and try to solve the problem below.**

Kimani and Jake each perform the same two transformations on figure *T*, but not in the same order.

- **Kimani's sequence:** A reflection across the *y*-axis, followed by a translation 4 units to the left.
- **Jake's sequence:** A translation 4 units to the left, followed by a reflection across the *y*-axis.

Will both sequences map figure *T* onto the same final image?



TRY IT

Math Toolkit graph paper, tracing paper, transparency sheets

DISCUSS IT

Ask: How did you decide whether both sequences map figure *T* onto the same final image?

Share: The strategy I used was . . .

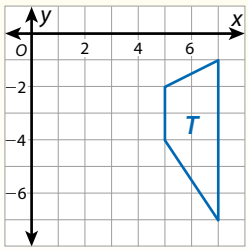
➤ **Explore different ways to perform sequences of transformations.**

Kimani and Jake each perform the same two transformations on figure *T*, but not in the same order.

Kimani's sequence: A reflection across the *y*-axis, followed by a translation 4 units to the left.

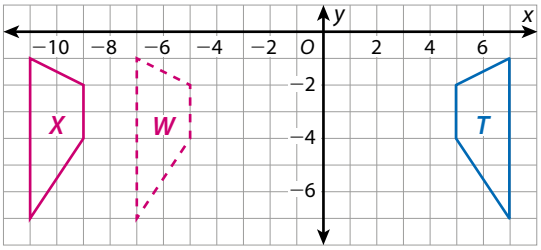
Jake's sequence: A translation 4 units to the left, followed by a reflection across the *y*-axis.

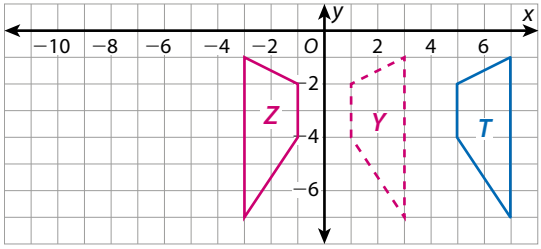
Will both sequences map figure *T* onto the same final image?



Picture It

You can draw each sequence in separate coordinate planes.





Model It

You can think about how the coordinates of each vertex change when you perform each transformation.

Kimani's Sequence		
Original coordinate	**Reflected across *y*-axis**	**Translated 4 units left**
(5, −2)	(−5, −2)	(−9, −2)
(7, −1)	(−7, −1)	(−11, −1)
(7, −7)	(−7, −7)	(−11, −7)
(5, −4)	(−5, −4)	(−9, −4)

Jake's Sequence		
Original coordinate	**Translated 4 units left**	**Reflected across *y*-axis**
(5, −2)	(1, −2)	(−1, −2)
(7, −1)	(3, −1)	(−3, −1)
(7, −7)	(3, −7)	(−3, −7)
(5, −4)	(1, −4)	(−1, −4)

CONNECT IT

Use the problem from the previous page to help you understand how an image formed by a sequence of transformations compares to the original figure.

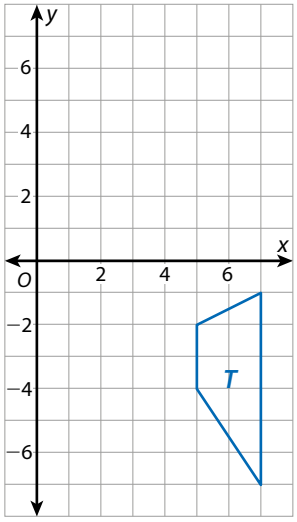
1 **a.** Did both sequences map figure *T* onto the same final image? How do **Picture It** and **Model It** each show this?

b. How do you know that both final images are congruent to the original figure *T*?

2 **a.** Translate figure *T* 3 units to the left. Then reflect the image across the *x*-axis. Draw the final image.

b. Then perform these same transformations in reverse order. Draw the final image.

c. Do both sequences map figure *T* onto the same final image? How do you know?



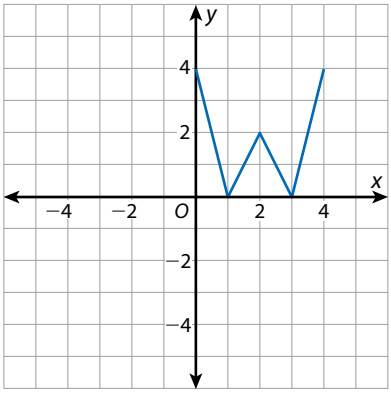
3 What can be different about a figure and any of its images after a sequence of translations, reflections, and rotations? What always stays the same? Why?

4 **Reflect** Think about all the models and strategies you have discussed today. Describe how one of them helped you better understand how to perform sequences of transformations.

Apply It

Use what you learned to solve these problems.

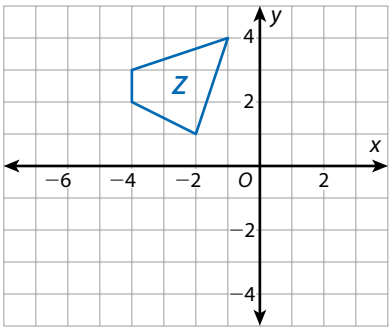
5 **a.** Rotate the letter W 180° around the origin. Then translate the image up 4 units. Draw the final image. What new letter did you form?



b. Is the new letter congruent to the original letter? Explain.

6 A sequence of transformations is performed on a figure. Then a different sequence is performed on the same figure. The resulting images are not in the same position. Can they be congruent? Explain.

7 **a.** Translate figure *Z* 3 units to the left. Then translate the image 5 units down. Draw the image after each transformation.



b. Now perform the transformations in reverse order. Draw the image after each transformation.

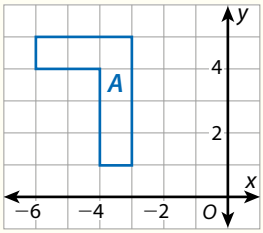
c. Do both sequences map figure *Z* onto the same final image? Explain.

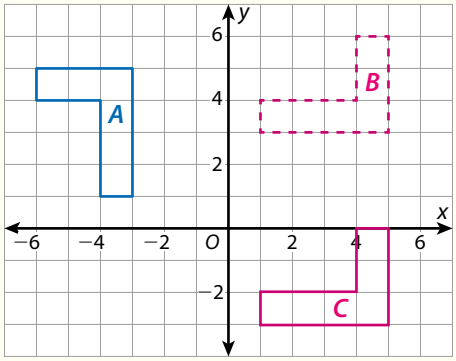
Practice Performing Sequences of Rigid Transformations

➤ **Study the Example showing how to perform a sequence of transformations. Then solve problems 1–4.**

Example

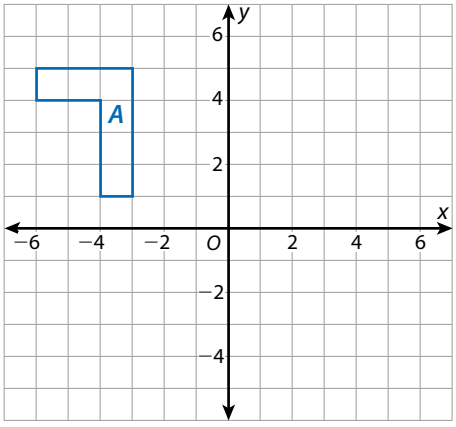
Rotate figure *A* 90° clockwise around the origin to form figure *B*. Then translate figure *B* 6 units down to form figure *C*. Is figure *C* congruent to figure *A*? Explain.





Yes, figure *C* ≅ figure *A* because figure *C* is the result of a sequence of rigid transformations on figure *A*.

1 Translate figure *A* from the Example 6 units down to form figure *D*. Then rotate figure *D* 90° clockwise around the origin to form figure *E*. Does this sequence of transformations map figure *A* onto figure *C* in the Example? Explain.

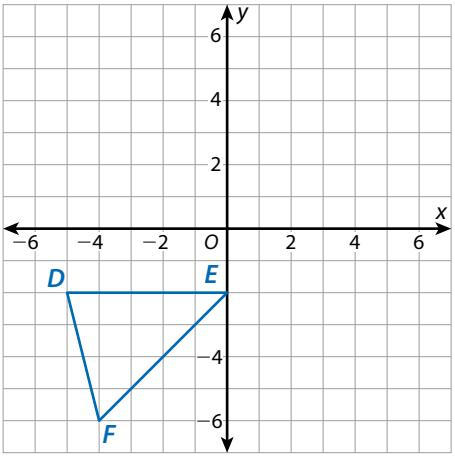


Vocabulary

congruent
same size and shape.

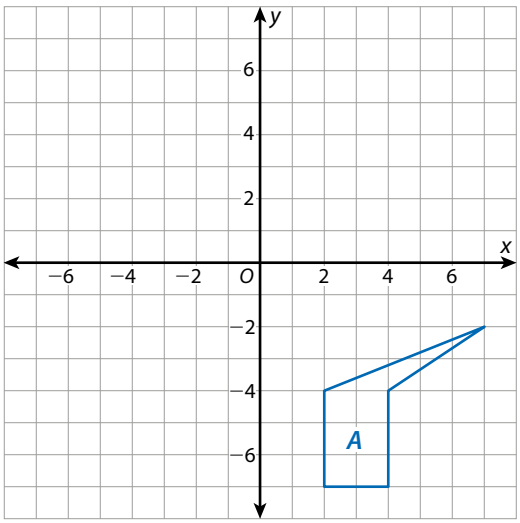
sequence of transformations
one or more transformations performed in a certain order.

2 Reflect $\triangle DEF$ across the y-axis. Then rotate $\triangle D'E'F'$ 90° counterclockwise around the origin. What are the coordinates of the vertices of $\triangle D''E''F''$? Show your work.



SOLUTION

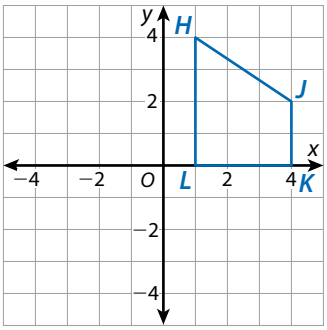
3 Translate figure A 8 units to the left to form figure A'. Then reflect figure A' across the x-axis to form figure A''. Draw figures A' and A''. Is figure A congruent to figure A''? How do you know?



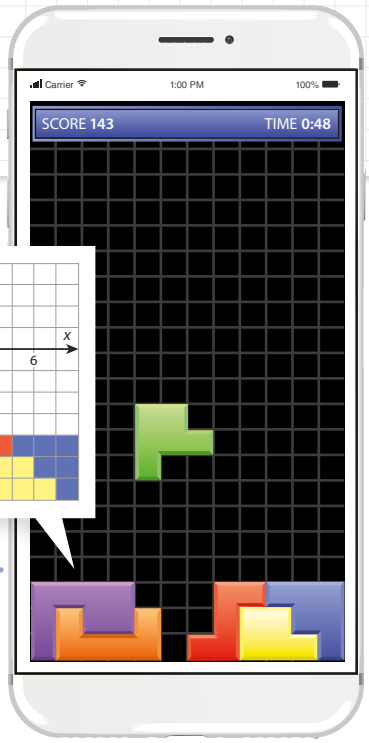
4 Perform the following sequence of transformations on figure $HJKL$:

- Rotate figure $HJKL$ 180° around the origin to form figure $H'J'K'L'$.
- Reflect figure $H'J'K'L'$ across the y-axis to form figure $H''J''K''L''$.
- Reflect figure $H''J''K''L''$ across the x-axis to form figure $H'''J'''K'''L'''$.

What are the coordinates of the vertices of figure $H'''J'''K'''L'''$? How do the vertices of figure $H'''J'''K'''L'''$ compare to the corresponding vertices of figure $HJKL$?

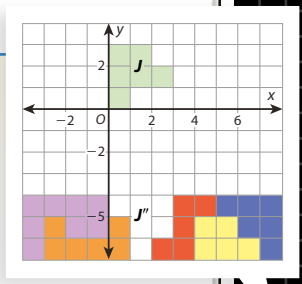


Develop Describing Sequences of Rigid Transformations



➤ **Read and try to solve the problem below.**

Paula plays a game on her phone where she needs to fit shapes together. Figure *J* will fit into space *J″* only if the figure and the space are congruent. Describe a sequence of two transformations that moves the shape into its space.



TRY IT

Math Toolkit graph paper, unit tiles, tracing paper, transparency sheets

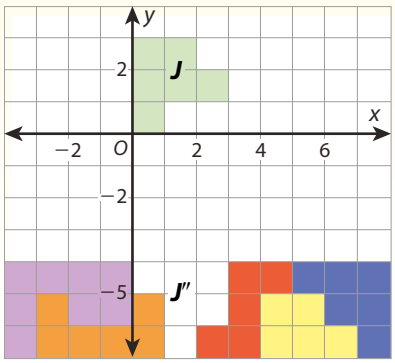
DISCUSS IT

Ask: How did you decide which transformation to do first?

Share: I started by . . .

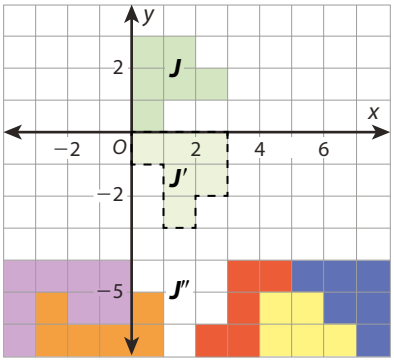
➤ **Explore different ways to map one figure onto another.**

Paula plays a game on her phone where she needs to fit shapes together. Figure *J* will fit into space *J″* only if the figure and the space are congruent. Describe a sequence of two transformations that moves the shape into its space.



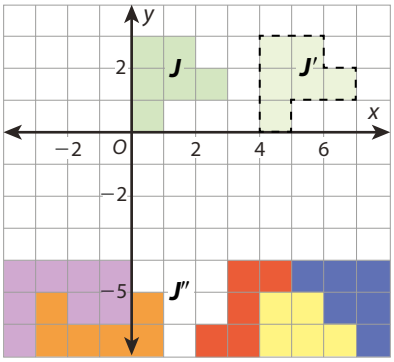
Picture It

You can start by rotating. Then you can translate.



Picture It

You can start by translating. Then you can rotate around the origin.



CONNECT IT

➤ **Use the problem from the previous page to help you understand how to describe sequences of transformations.**

1. Look at both **Picture Its**. Explain how the figure was transformed in each.

2. Look at the longest side of figure *J* and the corresponding side of figure *J*″. Are the sides horizontal or vertical? How could this help you decide whether or not to include a rotation in the sequence?

3. Why is one rotation around the origin not enough? What else is needed?

4. Is it possible to map figure *J* onto figure *J*″ if one or both of the two transformations in the sequence is a reflection across an axis? Explain.

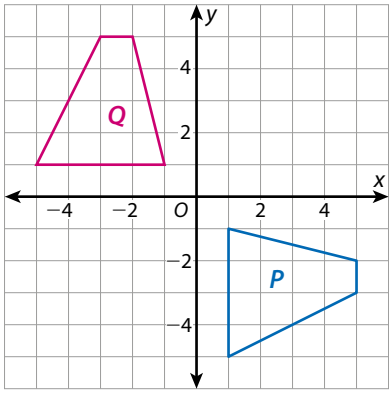
5. How can comparing a figure and its image help you describe a sequence of transformations that shows the two figures are congruent?

6. **Reflect** Think about all the models and strategies you have discussed today. Describe how one of them helped you better understand how to solve the **Try It** problem.

Apply It

Use what you learned to solve these problems.

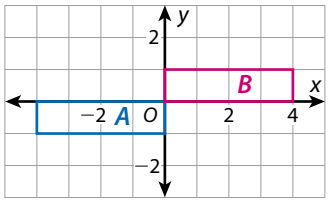
7 Figure *P* is congruent to figure *Q*. Describe a sequence of transformations that shows this. Show your work.



SOLUTION _____

_____

8 Figure *A* and figure *B* are congruent rectangles. Which sequences of transformations show this? Select all that apply.



A Rotate figure *A* 90° counterclockwise around the origin. Then translate the image 1 unit up.

B Translate figure *A* 4 units right. Then translate the image 1 unit up.

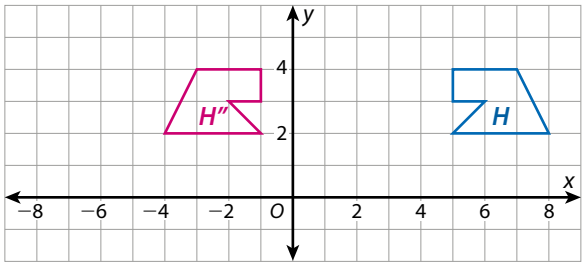
C Translate figure *A* 1 unit up. Then reflect the image across the *x*-axis.

D Rotate figure *A* 180° around the origin.

E Reflect figure *A* across the *x*-axis. Then reflect the image across the *y*-axis.

F Reflect figure *A* across the *y*-axis. Then translate the image 1 unit up.

9 Figure *H* is congruent to figure *H*″. Describe two different sequences of transformations you can use to show this.



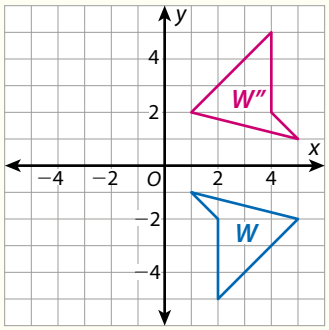
Practice Describing Sequences of Rigid Transformations

➤ **Study the Example showing how to describe a sequence of transformations. Then solve problems 1–3.**

Example

Describe a sequence of transformations you could use to show that figure *W* is congruent to figure *W″*.

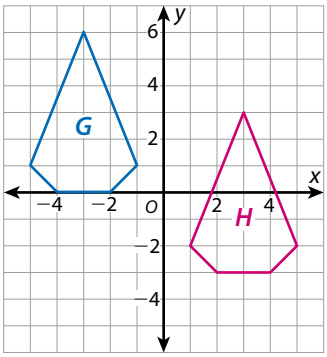
Rotate figure *W* 180° around the origin. Then translate the image 6 units to the right.



1 **a.** Look at the Example. Perform the same sequence of transformations described on figure *W″*. How does the image compare to figure *W*?

b. Describe a different sequence of transformations that you can perform on figure *W* to show that figure *W* ≅ figure *W″*.

2 **a.** Describe a sequence of transformations that you can perform on figure *G* to show that figure *G* ≅ figure *H*. Use only one type of transformation.



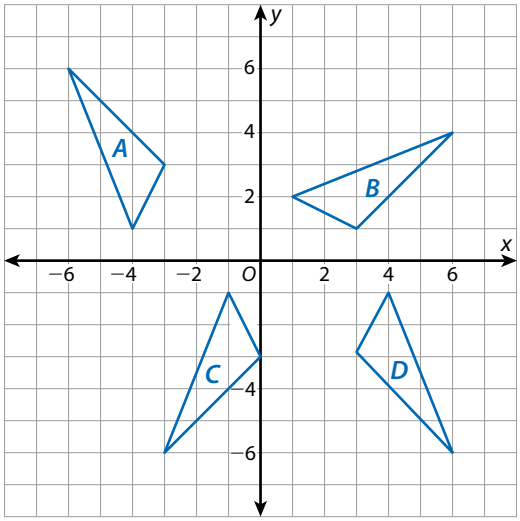
b. Describe a sequence of two different types of transformations that you can perform on figure *G* to show that figure *G* ≅ figure *H*.

Vocabulary

congruent
same size and shape.

sequence of transformations
one or more transformations performed in a certain order.

3 Triangles *A, B, C,* and *D* are all congruent.



a. Describe a sequence of transformations that you can perform on $\triangle B$ to show that $\triangle B \cong \triangle D$.

b. Describe a sequence of transformations that you can perform on $\triangle A$ to show that $\triangle A \cong \triangle C$.

c. Describe a sequence of transformations that you can perform on $\triangle C$ to show that $\triangle C \cong \triangle D$.

d. Use your answers to problems 3b and 3c to describe a four-step sequence of transformations that you can perform on $\triangle A$ to show that $\triangle A \cong \triangle D$.

e. Describe a sequence of two transformations that you can perform on $\triangle A$ to show that $\triangle A \cong \triangle D$.

f. What single transformation could you perform on $\triangle A$ to show that $\triangle A \cong \triangle D$?

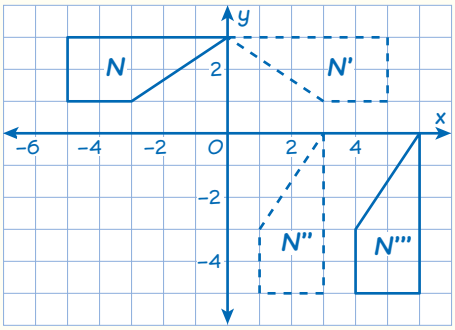
Refine Working with Sequences of Transformations and Congruence

➤ **Complete the Example below. Then solve problems 1–9.**

Example

Figure *N* has vertices at (−5, 3), (0, 3), (−3, 1), and (−5, 1). Figure *N‴* has vertices at (6, −5), (6, 0), (4, −3), and (4, −5). Describe a sequence of three transformations that you can perform on figure *N* to show that figure *N* ≅ figure *N‴*.

Look at how you could show your work in the coordinate plane.



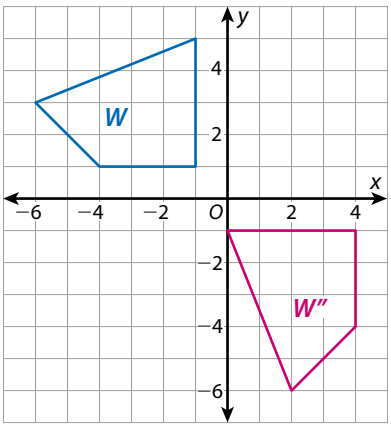
SOLUTION _____

CONSIDER THIS . . .
You can use triple prime notation to name the final image after three transformations.

PAIR/SHARE
Could you start with a rotation instead of a reflection?

Apply It

1 Figure *W* is congruent to figure *W″*. What sequence of transformations can you use to show this?



CONSIDER THIS . . .
Rotations, translations, and reflections are types of transformations.

PAIR/SHARE
Will the sequence you described map figure *W″* onto figure *W*?

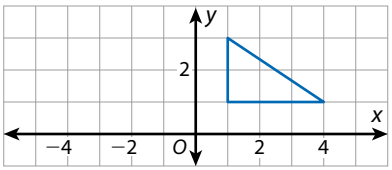
2 $\triangle PQR$ has vertices $P(-4, 4)$, $Q(-2, 3)$, and $R(-5, 2)$. $\triangle PQR$ is reflected across the x-axis and rotated 180° around the origin. What are the coordinates of $\triangle P''Q''R''$? Show your work.

CONSIDER THIS . . .
You can think of the reflection as forming $\triangle P'Q'R'$.

PAIR/SHARE
If you performed the transformations in reverse order, would the sequence map $\triangle PQR$ onto the same final image?

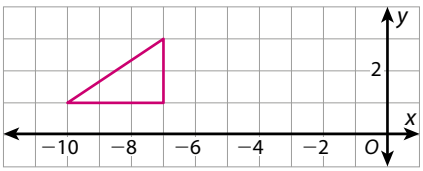
SOLUTION

3 Which graph shows the image of the triangle after a translation 6 units to the left, followed by a reflection across the y-axis?

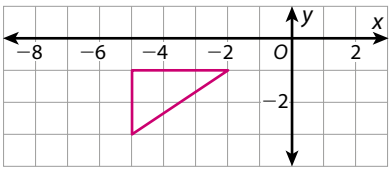


CONSIDER THIS . . .
Could making a sketch help you find the answer?

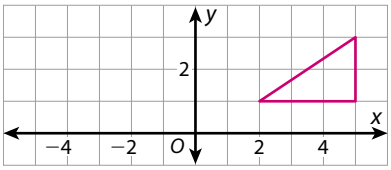
A



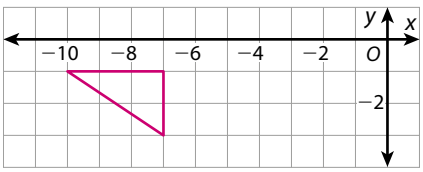
B



C



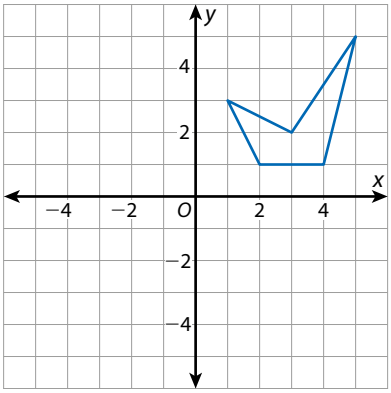
D



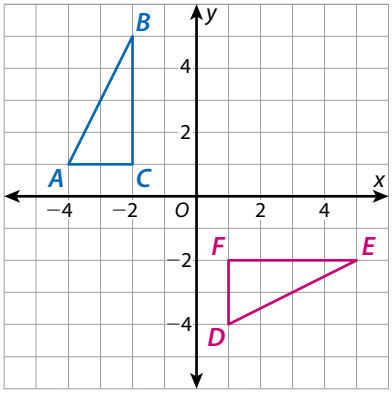
Kamal chose A as the correct answer. How might he have gotten that answer?

PAIR/SHARE
How can you tell whether all of the triangles shown are congruent to the original triangle?

4 Reflect the figure at the right across the *y*-axis. Then rotate the image 180° around the origin. Draw the image after each transformation. What single transformation could you perform on the figure to get the same final image?



5 $\triangle ABC \cong \triangle DEF$. Which sequences of transformations can you perform on $\triangle ABC$ to show this? Select all that apply.



A Rotate $\triangle ABC$ 180° around the origin.

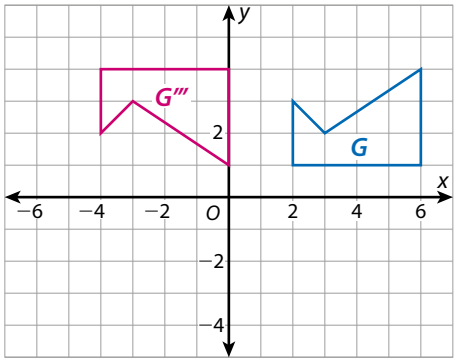
B Reflect $\triangle ABC$ across the *x*-axis. Then rotate the image 90° counterclockwise around the origin.

C Rotate $\triangle ABC$ 90° clockwise around the origin. Then reflect the image across the *x*-axis.

D Rotate $\triangle ABC$ 90° counterclockwise around the origin. Then rotate the image 90° counterclockwise around the origin.

E Reflect $\triangle ABC$ across the *y*-axis. Then rotate the image 90° clockwise around the origin.

6 Figures *G* and *G*‴ are congruent. Which sequence of transformations can you perform on figure *G* to show this?



A Reflect figure *G* across the *x*-axis. Then translate figure *G*′ 3 units up. Then translate figure *G*″ 6 units to the left.

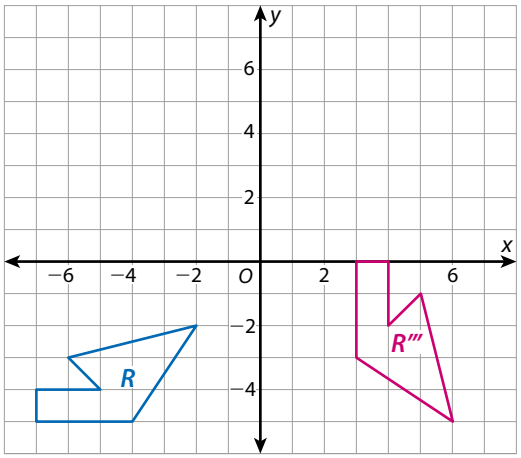
B Reflect figure *G* across the *x*-axis. Then translate figure *G*′ 6 units to the left. Then translate figure *G*″ 5 units up.

C Rotate figure *G* 90° counterclockwise around the origin. Then translate figure *G*′ 3 units up. Then translate figure *G*″ 2 units to the right.

D Rotate figure *G* 180° around the origin. Then translate figure *G*′ 5 units up. Then translate figure *G*″ 2 units to the right.

7 $\triangle XYZ$ is in Quadrant I. It is rotated 90° counterclockwise around the origin. Then $\triangle X'Y'Z'$ is reflected across the y-axis. Then $\triangle X''Y''Z''$ is translated 5 units to the right. In which Quadrant is $\triangle X'''Y'''Z'''$? How do you know?

8 Figure $R \cong$ figure R'''. Describe a sequence of three transformations you can perform on figure R to show this. Show your work.

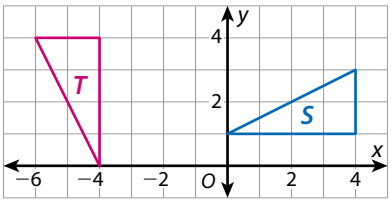


SOLUTION _____

_____

_____

9 **Math Journal** Describe two ways to use transformations to show that $\triangle S \cong \triangle T$.



End of Lesson Checklist

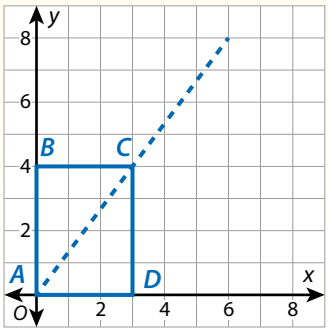
- ☐ **INTERACTIVE GLOSSARY** Find the entry for *congruent*. Draw two pairs of congruent figures. Explain how you know they are congruent.
- ☐ **SELF CHECK** Go back to the Unit 1 Opener and see what you can check off.

Explore Dilations in the Coordinate Plane

Previously, you learned about rigid transformations and dilations. In this lesson, you will learn about sequences of transformations in the coordinate plane involving both rigid transformations and dilations.

➤ **Use what you know to try to solve the problem below.**

Malcolm uses a program on his computer to resize photos for the yearbook. He drags the top right corner of the photo, represented by rectangle *ABCD*, along the dashed line. This forms the image rectangle $A'B'C'D'$. Malcolm uses a scale factor of 2. The center of dilation is at the origin. What are the coordinates of the vertices of image rectangle $A'B'C'D'$?



TRY IT

Math Toolkit graph paper, tracing paper, transparency sheets

DISCUSS IT

Ask: How did you get started finding the coordinates of rectangle $A'B'C'D'$?

Share: At first I thought . . .

Learning Target **SMP 1, SMP 2, SMP 3, SMP 4, SMP 5, SMP 6, SMP 7, SMP 8**
Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

CONNECT IT

1 Look Back What are the coordinates of the vertices of rectangle $A'B'C'D'$? Describe your strategy for finding them.

2 Look Ahead You know that when you dilate a figure, the image is similar to the original figure. In similar figures, corresponding angle measures are congruent and corresponding side lengths are proportional. When the center of dilation is at the origin, the coordinates of the corresponding vertices of a figure and its image are also proportional.

a. Compare the coordinates of the vertices of rectangles $ABCD$ and $A'B'C'D'$ in the **Try It**. What can you multiply each vertex coordinate of figure $ABCD$ by to get the corresponding vertex coordinate of figure $A'B'C'D'$?

b. How could you use the scale factor to find the coordinates of the vertices of rectangle $A'B'C'D'$?

c. The table shows the coordinates of the vertices of $\triangle DEF$ and its dilated image $\triangle D'E'F'$. The center of dilation is the origin. What is the scale factor? How do you know?

DEF	$D(2, 3)$	$E(6, 4)$	$F(4, 8)$
D′E′F′	$D'(1, \frac{3}{2})$	$E'(3, 2)$	$F'(2, 4)$

3 Reflect You have the coordinates of a figure and a scale factor for a dilation. How can you find the coordinates of the image if the center of dilation is at the origin?

 Name:

Prepare for Transformations Involving Dilations

1 Think about what you know about ratios and proportional relationships. Fill in each box. Use words, numbers, and pictures. Show as many ideas as you can.

What Is It?

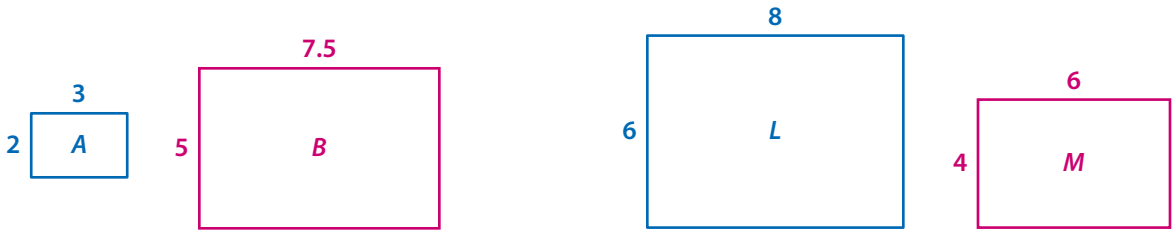
What I Know About It

proportional relationship

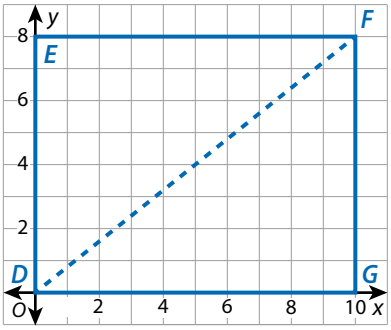
Examples

Examples

2 Is there a proportional relationship between the corresponding sides of each pair of rectangles?



3 Cai uses a computer program to resize photos for the school newspaper. She drags the top right corner of the photo, represented by rectangle *DEFG*, along the dashed line to form the image rectangle $D'E'F'G'$. The center of dilation is at the origin. The scale factor is $\frac{1}{2}$.

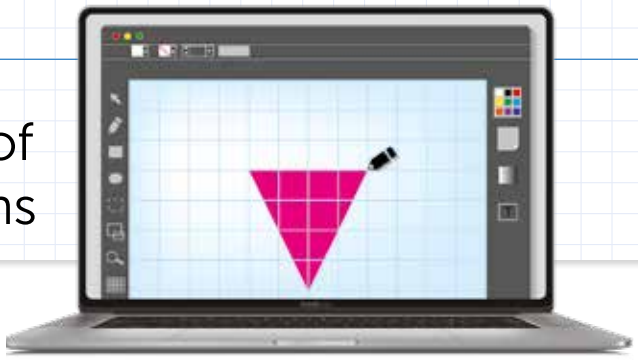


a. What are the coordinates of the vertices of image rectangle $D'E'F'G'$? Show your work.

SOLUTION

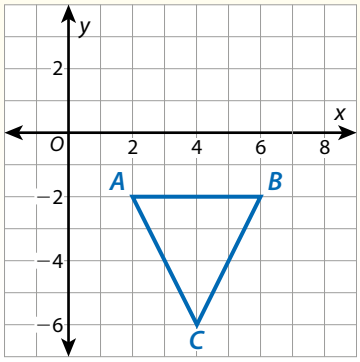
b. Check your answer to problem 3a. Show your work.

Develop Performing Sequences of Transformations Involving Dilations



➤ **Read and try to solve the problem below.**

Abran is learning to program graphics on a computer. He draws $\triangle ABC$ and reflects it across the x-axis to form $\triangle A'B'C'$. Then he dilates $\triangle A'B'C'$ using a scale factor of $\frac{1}{2}$ with the center of dilation at the origin. The final image is $\triangle A''B''C''$. What are the coordinates of the vertices of $\triangle A''B''C''$?



TRY IT

Math Toolkit graph paper, tracing paper, transparency sheets

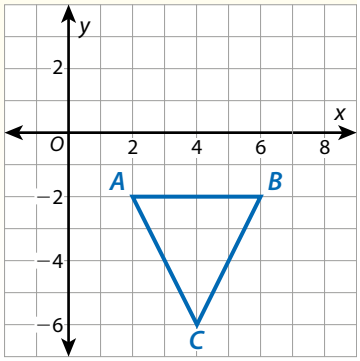
DISCUSS IT

Ask: Why did you choose that strategy to find the coordinates of $\triangle A''B''C''$?

Share: I knew . . . so I . . .

➤ **Explore different ways to perform a sequence of transformations involving dilations.**

Abran is learning to program graphics on a computer. He draws $\triangle ABC$ and reflects it across the x-axis to form $\triangle A'B'C'$. Then he dilates $\triangle A'B'C'$ using a scale factor of $\frac{1}{2}$ with the center of dilation at the origin. The final image is $\triangle A''B''C''$. What are the coordinates of the vertices of $\triangle A''B''C''$?



Model It

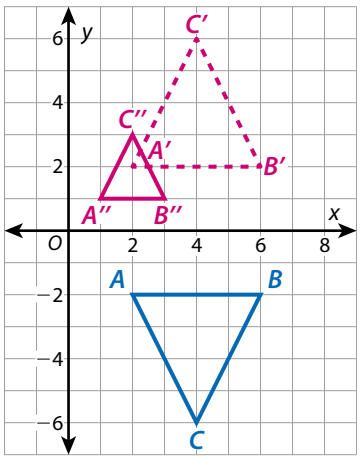
You can use what you know about how coordinates change for different transformations.

Original	After reflection	After dilation
(2, −2)	(2, 2)	(1, 1)
(6, −2)	(6, 2)	(3, 1)
(4, −6)	(4, 6)	(2, 3)

Picture It

You can show both transformations in the coordinate plane.

Use the coordinates from your table to draw each transformation.



CONNECT IT

Use the problem from the previous page to help you understand how to perform a sequence of transformations involving dilations.

1 **a.** Look at **Model It**. Describe how the coordinates of the vertices change from $\triangle ABC$ to the reflected figure $\triangle A'B'C'$.

b. Describe how the coordinates of the vertices change from $\triangle A'B'C'$ to the dilated figure $\triangle A''B''C''$. What are the coordinates of the vertices of $\triangle A''B''C''$?

2 **a.** Is $\triangle A'B'C'$ similar to $\triangle ABC$? Is $\triangle A''B''C''$ similar to $\triangle A'B'C'$? Explain.

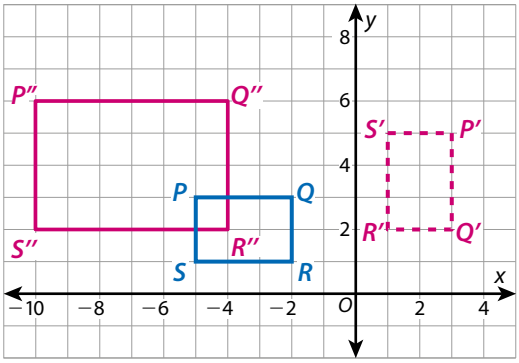
b. Is $\triangle A''B''C''$ similar to $\triangle ABC$? Explain.

3 **Reflect** Think about all the models and strategies you have discussed today. Describe how one of them helped you better understand how to perform sequences of transformations involving dilations.

Apply It

Use what you learned to solve these problems.

4 Grace wants to rotate $PQRS$ 90° clockwise around the origin to form figure $P'Q'R'S'$. Then she wants to dilate the image using a scale factor of 2 and the origin as the center of dilation to form figure $P''Q''R''S''$. Grace's graph is shown. What mistake did Grace make? What are the correct coordinates of the vertices of figure $P''Q''R''S''$?



5 Two transformations are performed on $\triangle X$ in the coordinate plane. First, $\triangle X$ is rotated 90° counterclockwise around the origin to form image $\triangle X'$. Then $\triangle X'$ is dilated using a scale factor of $\frac{3}{4}$ with the center of dilation at the origin to form image $\triangle X''$. Which statement about $\triangle X$ and $\triangle X''$ is true?

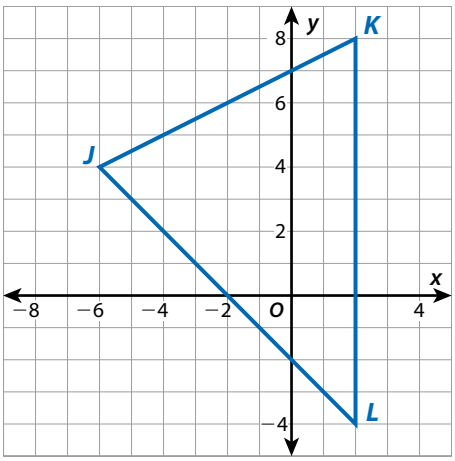
A $\triangle X$ is similar to $\triangle X''$.

B $\triangle X$ is congruent to $\triangle X''$.

C $\triangle X$ is both congruent and similar to $\triangle X''$.

D $\triangle X$ is neither congruent nor similar to $\triangle X''$.

6 Dario dilates $\triangle JKL$ in the coordinate plane using a scale factor of 0.5 and a center of dilation at the origin. He labels the image $\triangle J'K'L'$. Then he translates the image 3 units up to form $\triangle J''K''L''$. What are the coordinates of the vertices of $\triangle J''K''L''$? Show your work.



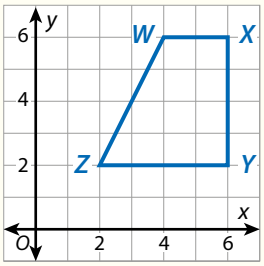
SOLUTION

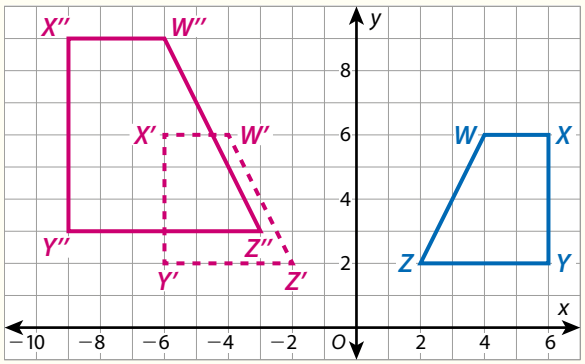
Practice Performing Sequences of Transformations Involving Dilations

➤ **Study the Example showing how to perform a sequence of transformations involving dilations. Then solve problems 1–4.**

Example

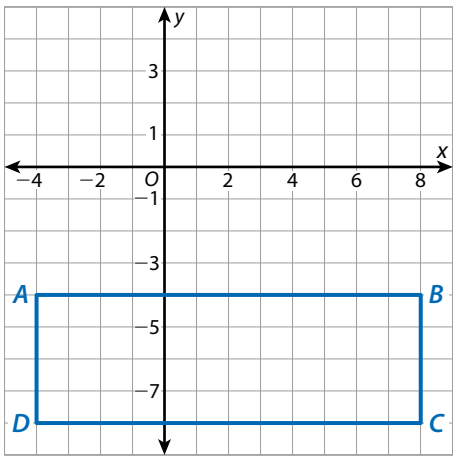
Lamont designs obstacles for a skate park. He draws trapezoid *WXYZ* in the coordinate plane. Then he reflects trapezoid *WXYZ* across the *y*-axis to form image *W′X′Y′Z′*. Then he dilates the image to form trapezoid *W″X″Y″Z″*. He uses a scale factor of 1.5 and a center of dilation at the origin. What are the coordinates of the vertices of trapezoid *W″X″Y″Z″*?





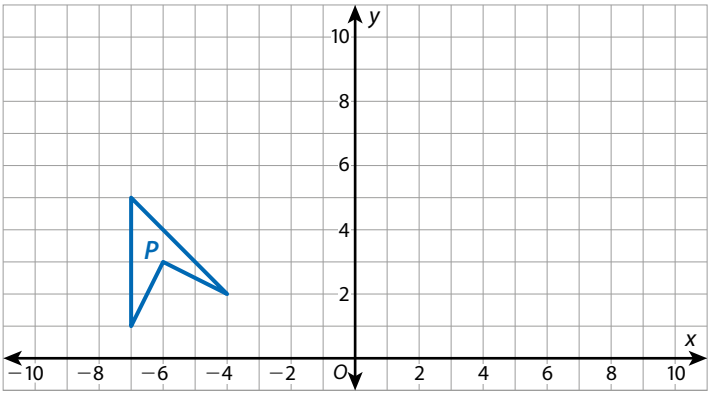
The coordinates are *W″*(−6, 9); *X″*(−9, 9); *Y″*(−9, 3); *Z″*(−3, 3).

1 Rani dilates figure *ABCD* to form figure *A′B′C′D′*. She uses a scale factor of $\frac{3}{4}$ and a center of dilation at the origin. Then she translates the image up 7 units to form figure *A″B″C″D″*. What are the coordinates of the vertices of figure *A″B″C″D″*? Show your work.



SOLUTION _____

2 Cyrus plans a design to paint on his bass drum. He starts with figure P and translates it 3 units to the right to form figure P'. Then he reflects figure P' across the y-axis to form figure P''. Then Cyrus dilates figure P'' using a scale factor of 2 to form figure P'''. The center of dilation is at the origin. Draw figures P', P'', and P''' in the coordinate plane.



3 $\triangle EFG$ is translated 3 units to the left. Then the image is dilated by a scale factor of 0.5 to form $\triangle E''F''G''$. The center of dilation is the origin. Vertex E is located at $(5, -6)$. What are the coordinates of vertex E''? Show your work.

SOLUTION

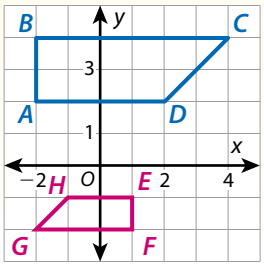
4 Indira uses an app to add a photo to a poster. She rotates the photo. Then she dilates the image using a scale factor of 3 and a center of dilation at the origin. Tell whether each statement is *True* or *False*.

	True	False
a. The original photo is congruent to the final image.	○	○
b. The original photo is congruent to the rotated photo.	○	○
c. The original figure is similar to the rotated photo.	○	○
d. The original photo is similar to the final image.	○	○

Develop Describing Sequences of Transformations Involving Dilations

➤ **Read and try to solve the problem below.**

For the Challenge of the Week, Ryan's teacher draws figures *ABCD* and *EFGH* in the coordinate plane. The challenge is to show that the two figures are similar using three or fewer transformations. What is one sequence of transformations that Ryan could use?



TRY IT

Math Toolkit graph paper, tracing paper, transparency sheets

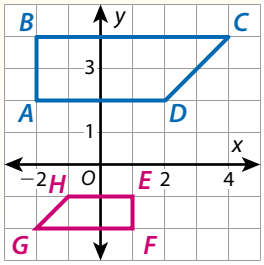
DISCUSS IT

Ask: Why did you choose that strategy to show that the two figures are similar?

Share: I started this way because . . .

➤ **Explore different ways to map one figure onto another.**

For the Challenge of the Week, Ryan's teacher draws figures *ABCD* and *EFGH* in the coordinate plane. The challenge is to show that the two figures are similar using three or fewer transformations. What is one sequence of transformations that Ryan could use?

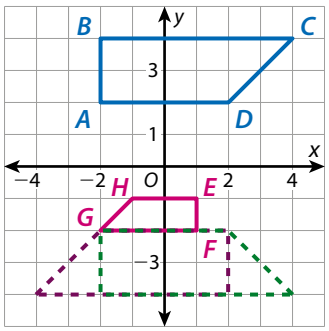


Picture It

You can graph the possible transformations.

You can map figure *ABCD* onto figure *EFGH* using three transformations.

- reflection
- reflection
- dilation



Model It

You can use what you know about coordinates of transformations.

You can map figure *ABCD* onto figure *EFGH* using two transformations.

A rotation faces figure *ABCD* in the same direction as figure *EFGH*.
A dilation reduces the image.

ABCD	After rotation	*EFGH*
$A(-2, 2)$	$(2, -2)$	$E(1, -1)$
$B(-2, 4)$	$(2, -4)$	$F(1, -2)$
$C(4, 4)$	$(-4, -4)$	$G(-2, -2)$
$D(2, 2)$	$(-2, -2)$	$H(-1, -1)$

CONNECT IT

Use the problem from the previous page to help you understand how to describe sequences of transformations involving dilations.

1. Look at **Picture It** and **Model It**. Give the details of the transformations used in each sequence.

2. Look at **Model It**. Does it matter whether figure *ABCD* is dilated or rotated first in the sequence? Explain.

3. What is another way you could map figure *ABCD* onto figure *EFGH*?

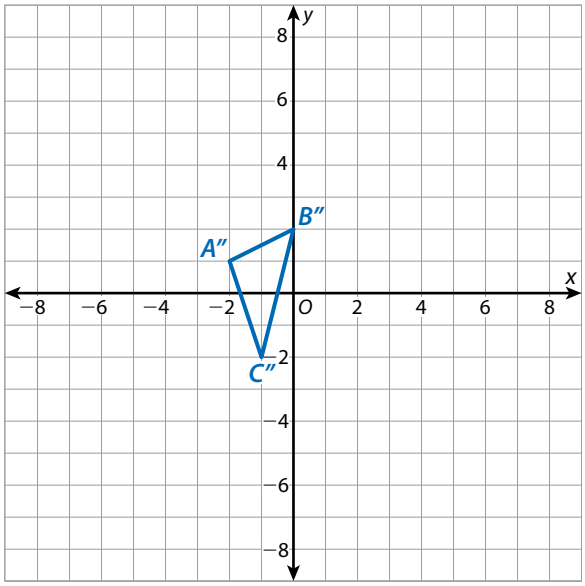
4. Suppose figure *EFGH* is the original figure and figure *ABCD* is the final image. What two transformations could you perform to map figure *EFGH* onto figure *ABCD*?

5. **Reflect** Think about all the models and strategies you have discussed today. Describe how one of them helped you better understand how to solve the **Try It** problem.

Apply It

Use what you learned to solve these problems.

6 The vertices of $\triangle ABC$ are located at $A(4, 8)$, $B(8, 0)$, and $C(-8, 4)$. Which sequence of transformations of triangle $\triangle ABC$ results in $\triangle A''B''C''$?



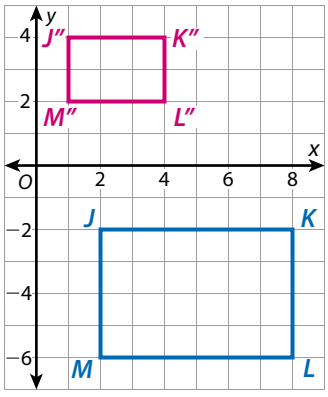
A Rotate $\triangle ABC$ 90° clockwise around the origin. Dilate the image by a scale factor of 4 with a center of dilation at the origin.

B Rotate $\triangle ABC$ 90° counterclockwise around the origin. Then dilate the image by a scale factor of 4 with a center of dilation at the origin.

C Rotate $\triangle ABC$ 90° clockwise around the origin. Then dilate the image by a scale factor of $\frac{1}{4}$ with a center of dilation at the origin.

D Rotate $\triangle ABC$ 90° counterclockwise around the origin. Then dilate the image by a scale factor of $\frac{1}{4}$ with a center of dilation at the origin.

7 Pilar draws rectangle *JKLM* in the coordinate plane. She performs two transformations on rectangle *JKLM* to form rectangle *J″K″L″M″*.



a. What sequence of transformations could Pilar have performed?

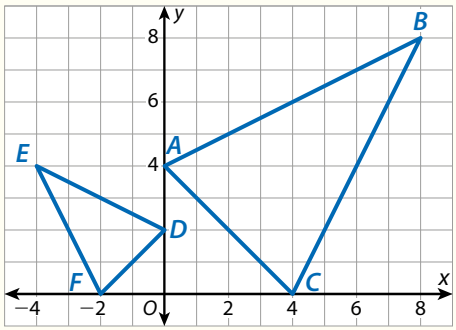
b. Could Pilar have performed the same two transformations in the reverse order to form rectangle *J″K″L″M″*? Explain.

Practice Describing Sequences of Transformations Involving Dilations

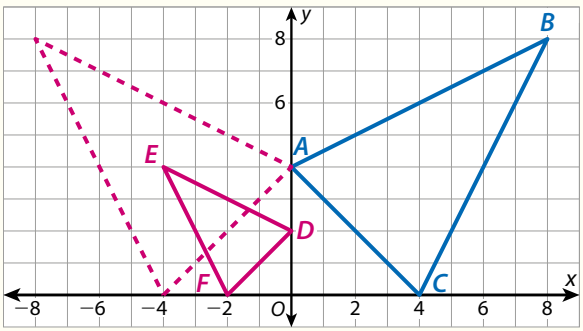
➤ **Study the Example showing how to describe a sequence of transformations involving dilations. Then solve problems 1–5.**

Example

What is one possible sequence of transformations that shows that $\triangle ABC \sim \triangle DEF$?

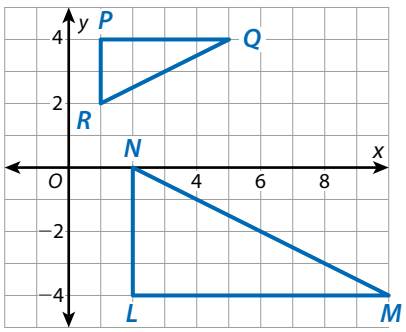


Reflect $\triangle ABC$ across the y-axis. Then dilate the image using a scale factor of $\frac{1}{2}$ with a center of dilation at the origin.



1 Look at the Example. What is one possible sequence of transformations you could perform to map $\triangle DEF$ onto $\triangle ABC$?

2 Describe a sequence of transformations that maps $\triangle PQR$ onto $\triangle LMN$.

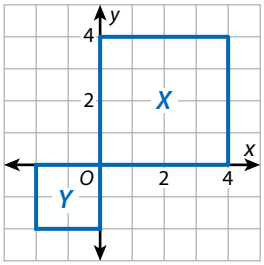


Vocabulary

dilation
a transformation that makes a scale copy of a figure.

sequence of transformations
one or more transformations performed in a certain order.

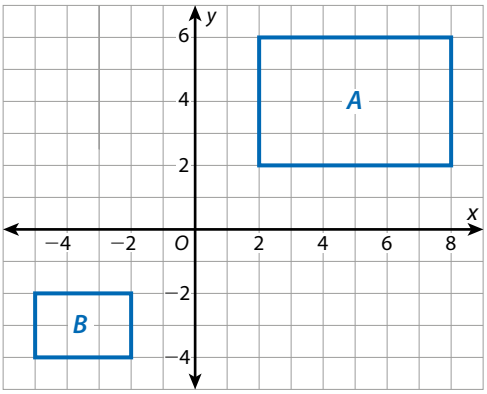
3 Notah draws figure *X* in the coordinate plane. Then he performs a sequence of transformations on figure *X* to form figure *Y*.



a. What sequence of transformations could Notah have performed to map figure *X* onto figure *Y*?

b. What is another sequence of transformations Notah could have performed to map figure *X* onto figure *Y*?

4 Which sequences of transformations could map rectangle *A* onto rectangle *B*? Select all that apply.



A A 90° clockwise rotation around the origin, followed by a translation to the left

B A dilation with a center of dilation at the origin, followed by a translation down and a translation to the left

C A reflection across the *x*-axis, followed by a dilation with a center of dilation at the origin and a translation to the left

D A reflection across the *y*-axis, followed by a translation down and a dilation with a center of dilation at the origin

E A 180° rotation around the origin, followed by a dilation with a center of dilation at the origin, a translation down, and a translation to the left

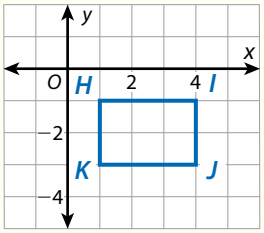
5 Chantel graphs $\triangle S$ and $\triangle T$. The corresponding angles are congruent. The side lengths of $\triangle T$ are 5 times as long as the corresponding side lengths of $\triangle S$. Can you give a sequence of transformations that will map $\triangle S$ onto $\triangle T$? Explain.

Refine Performing and Describing Transformations Involving Dilations

➤ **Complete the Example below. Then solve problems 1–8.**

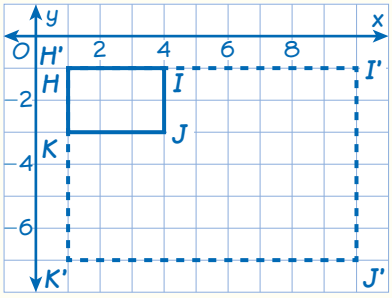
Example

Dylan dilates figure *HIJK* to form figure *H′I′J′K′*. He uses a scale factor of 3 with a center of dilation at vertex *H*. What are the coordinates of figure *H′I′J′K′*?



Look at how you could use the coordinate plane.

Find the length of each side of figure *HIJK*. Multiply each side by 3 to find the corresponding lengths of figure *H′I′J′K′*. Count from *H* to find the corresponding vertices *I′*, *J′*, and *K′*.



SOLUTION _____

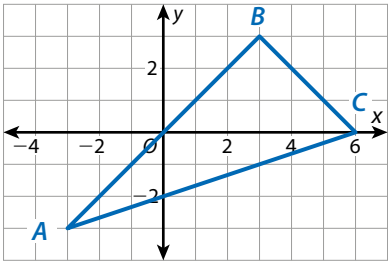
CONSIDER THIS . . .
Could the quotient of corresponding side lengths help you find the coordinates of figure *H′I′J′K′*?

PAIR/SHARE
How would the coordinates of figure *H′I′J′K′* change if the center of dilation were the origin?

Apply It

1 Khadija draws $\triangle ABC$ in the coordinate plane. She dilates $\triangle ABC$ using a scale factor of $\frac{2}{3}$ with the center of dilation at the origin to form $\triangle A'B'C'$. Then she dilates $\triangle A'B'C'$ using a scale factor of $\frac{1}{2}$ with a center of dilation at the origin to form $\triangle A''B''C''$.

What are the coordinates of $\triangle A''B''C''$? Show your work.

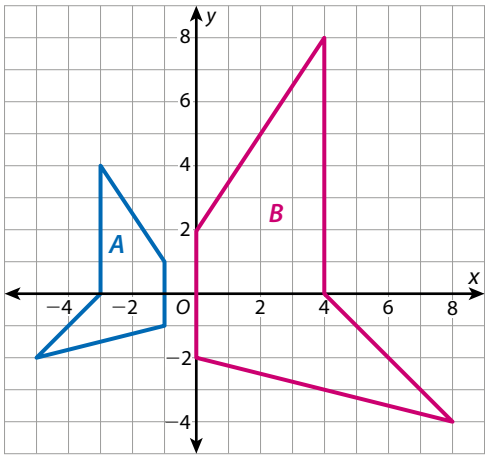


SOLUTION _____

CONSIDER THIS . . .
Will $\triangle A'B'C'$ be larger or smaller than $\triangle ABC$? Will $\triangle A''B''C''$ be larger or smaller than $\triangle A'B'C'$?

PAIR/SHARE
Does it matter in which order you perform the dilations?

2 Caleb wants to prove that figures *A* and *B* are similar. What sequence of transformations could Caleb perform to prove that the figures are similar? Show your work.

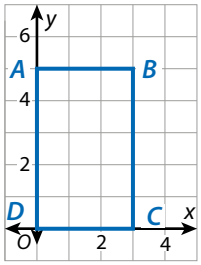


SOLUTION _____

CONSIDER THIS . . .
How can you determine the scale factor of a dilation?

PAIR/SHARE
What is the fewest number of transformations you can perform to map figure *A* onto figure *B*?

3 Yolanda designs a holiday card that measures 3 inches by 5 inches. The card is represented by figure *ABCD*. She enlarges the card to form image $A'B'C'D'$. She uses a scale factor of 2 with a center of dilation at the origin. What are the coordinates of B'?



A (3, 5)

B (5, 7)

C (6, 5)

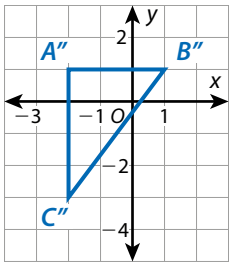
D (6, 10)

Wyatt chose B as the correct answer. How might he have gotten that answer?

CONSIDER THIS . . .
How are the dimensions of Yolanda's card represented in the coordinate plane?

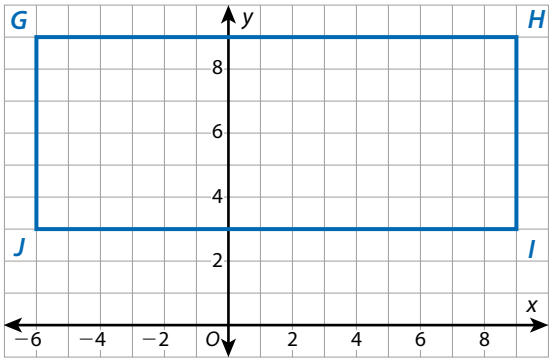
PAIR/SHARE
How can you use the scale factor to find the coordinates of a dilated figure?

4 $\triangle ABC$ was rotated 90° counterclockwise around the origin to form $\triangle A'B'C'$. $\triangle A'B'C'$ was dilated using a scale factor of $\frac{1}{3}$ with a center of dilation at the origin. The result is $\triangle A''B''C''$, shown in the coordinate plane. What are the coordinates of $\triangle ABC$? Show your work.



SOLUTION

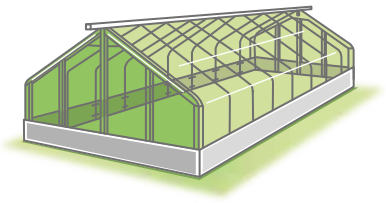
5 Gabe is designing a greenhouse. The original plan for the floor is represented by figure *GHIJ*. Gabe dilates the floor by a scale factor of $\frac{2}{3}$ with a center of dilation at the origin to form figure $G'H'I'J'$. Then he translates figure $G'H'I'J'$ 3 units to the right and 3 units up. The final figure is labeled $G''H''I''J''$.



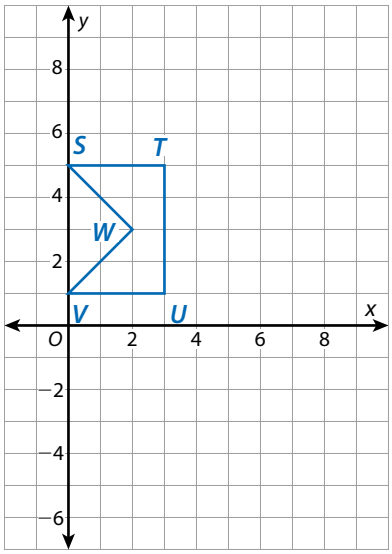
a. Draw Gabe's transformations in the coordinate plane.

b. What single transformation could map figure *GHIJ* onto figure $G''H''I''J''$?

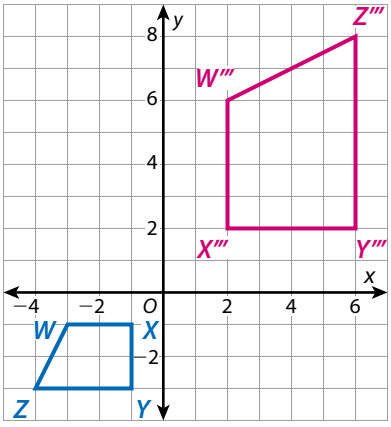
c. Why do you think the parallel sides in figure *GHIJ* remain parallel to the corresponding sides of figure $G'H'I'J'$ after the dilation?



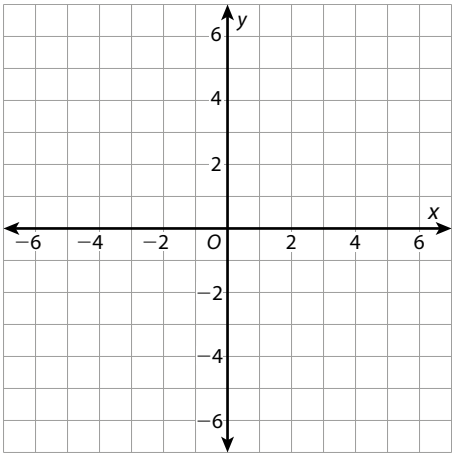
6 Leon designs a fabric pattern using figure *STUVW*. He dilates figure *STUVW* using a scale factor of 2 with a center of dilation at vertex *V* to form figure $S'T'U'V'W'$. Then Leon rotates this image 90° clockwise around the origin to form figure $S''T''U''V''W''$. What are the coordinates of figure $S''T''U''V''W''$?



7 Maya performs three transformations on figure *WXYZ* to show that figures *WXYZ* and $W'''X'''Y'''Z'''$ are similar. She dilates the figure by a scale factor of 2 with center of dilation at the origin, rotates the image 90° counterclockwise around the origin, and then reflects that image across the *x*-axis. Does the order of transformations matter? Explain.



8 **Math Journal** Draw a triangle in the coordinate plane and label it $\triangle ABC$. Transform $\triangle ABC$ using one rigid transformation and one dilation. Draw and label the final image $\triangle A''B''C''$.



End of Lesson Checklist

- ☐ **INTERACTIVE GLOSSARY** Find the entry for *dilation*. Add two important things you learned about dilations in this lesson.
- ☐ **SELF CHECK** Go back to the Unit 2 Opener and see what you can check off.