LESSON 9

Overview | Derive and Graph Linear Equations of the Form $y = mx + b$

STANDARDS FOR MATHEMATICAL PRACTICE (SMP)

SMP 1, 2, 3, 4, 5, and 6 are integrated into the Try-Discuss-Connect routine.*

This lesson provides additional support for:

4 Model with mathematics.

5 Use appropriate tools strategically.

7 Look for and make use of structure.

* See page 1o to learn how every lesson includes these SMP.

Objectives

Content Objectives

- Derive the equations $y = mx$ for a line through the origin and $y = mx + b$ for a line that intercepts the *y*-axis at *b*.
- Understand that when the equation of a line is given in slope-intercept form $y = mx + b$, *m* is the slope and *b* is the *y*-intercept.
- Understand that slope can be positive, negative, 0, or undefined.
- Graph linear equations in any form.

Language Objectives

- Describe how to use the slope of a line in a proportional relationship to derive an equation in the form $y = mx$.
- Understand and use lesson vocabulary when describing equations and explaining what the slope and *y*-intercept represent in the context of the problem.
- Explain negative, zero, and undefined slopes using terms such as *decrease, horizontal,* and *vertical*.
- Interpret graphs of linear equations and make predictions based on the contextual situations represented by the graph.
- Explain reasoning and offer suggestions when disagreeing during discussion.

Prior Knowledge

- Graph proportional relationships.
- Determine the slope of a line given a graph or by using the slope formula.
- Write expressions to represent rate situations.

Vocabulary

Math Vocabulary

linear equation an equation whose graph is a straight line.

slope-intercept form a linear equation in the form $y = mx + b$, where *m* is the slope and *b* is the *y*-intercept.

***y*-intercept** the *y*-coordinate of the point where a line, or graph of a function, intersects the *y*-axis.

Review the following key terms.

slope for any two points on a line, the $\frac{\text{rise}}{\text{run}}$ or $\frac{\text{change in } y}{\text{change in } x}$. It is a measure of the steepness of a line. It is also called the rate of change of a linear function.

Academic Vocabulary

define to identify or explain the meaning of something.

derive to use reasoning and known information to create or generate something.

undefined without meaning.

Learning Progression

In Grade 7, students confirmed proportional relationships by graphing and checking whether the points formed a line through the origin. Students also learned that the unit rate of a proportional relationship determines the steepness of its graph.

Earlier in Grade 8, students found slopes of lines by using rise divided by run or the slope formula. Students realized that the slope of a line is constant.

➤

In this lesson, students derive the equations $y = mx$ and $y = mx + b$ and graph linear equations of these forms. They learn that graphs of lines do not have to go through the origin. They rewrite linear equations given in other forms in slope-intercept form ($y = mx + b$) and graph the equation of those lines as well.

➤

Later in Grade 8, students will solve linear equations in one variable and determine the number of solutions to one-variable linear equations.

Pacing Guide

Items marked with ↖ are available on the **Teacher Toolbox**.

	MATERIALS	DIFFERENTIATION
SESSION 1 **Explore** Deriving $y = mx$ (35–50 min)		
• **Start** (5 min) • **Try It** (5–10 min) • **Discuss It** (10–15 min) • **Connect It** (10–15 min) • **Close: Exit Ticket** (5 min) **Additional Practice** (pages 201–202)	**Math Toolkit** graph paper, straightedges Presentation Slides ↖	**PREPARE** Interactive Tutorial ↖ **RETEACH or REINFORCE** Visual Model **Materials** For display: large coordinate plane
SESSION 2 **Develop** Deriving $y = mx + b$ (45–60 min)		
• **Start** (5 min) • **Try It** (10–15 min) • **Discuss It** (10–15 min) • **Connect It** (15–20 min) • **Close: Exit Ticket** (5 min) **Additional Practice** (pages 207–208)	**Math Toolkit** graph paper, straightedges Presentation Slides ↖	**RETEACH or REINFORCE** Hands-On Activity **Materials** For each pair: 3 chenille stems, tape, Activity Sheet *Graph Paper* ↖ **REINFORCE** Fluency & Skills Practice ↖ **EXTEND** Deepen Understanding
SESSION 3 **Develop** Graphing a Linear Equation of the Form $y = mx + b$ (45–60 min)		
• **Start** (5 min) • **Try It** (10–15 min) • **Discuss It** (10–15 min) • **Connect It** (15–20 min) • **Close: Exit Ticket** (5 min) **Additional Practice** (pages 213–214)	**Math Toolkit** graph paper, straightedges Presentation Slides ↖	**RETEACH or REINFORCE** Visual Model **Materials** For display: large coordinate plane **REINFORCE** Fluency & Skills Practice ↖ **EXTEND** Deepen Understanding
SESSION 4 **Develop** Graphing a Linear Equation Given in Any Form (45–60 min)		
• **Start** (5 min) • **Try It** (10–15 min) • **Discuss It** (10–15 min) • **Connect It** (15–20 min) • **Close: Exit Ticket** (5 min) **Additional Practice** (pages 219–220)	**Math Toolkit** graph paper, straightedges Presentation Slides ↖	**RETEACH or REINFORCE** Visual Model **Materials** For display: large coordinate plane **REINFORCE** Fluency & Skills Practice ↖ **EXTEND** Deepen Understanding
SESSION 5 **Refine** Deriving and Graphing Linear Equations of the Form $y = mx + b$ (45–60 min)		
• **Start** (5 min) • **Monitor & Guide** (15–20 min) • **Group & Differentiate** (20–30 min) • **Close: Exit Ticket** (5 min)	**Math Toolkit** Have items from previous sessions available for students. Presentation Slides ↖	**RETEACH** Visual Model **Materials** For display: large coordinate plane **REINFORCE** Problems 4–7 **EXTEND** Challenge **PERSONALIZE** i-Ready
Lesson 9 Quiz ↖ or **Digital Comprehension Check**		**RETEACH** Tools for Instruction ↖ **REINFORCE** Math Center Activity ↖ **EXTEND** Enrichment Activity ↖

LESSON 9

Overview | Derive and Graph Linear Equations of the Form $y = mx + b$

Connect to Culture

➤ **Use these activities to connect with and leverage the diverse backgrounds and experiences of all students. Engage students in sharing what they know about contexts before you add the information given here.**

SESSION 1 ■ □ □ □ □

Try It Have students tell what they know about the Paralympics and have them share any experiences they have had participating in or watching them. Much like the traditional Olympic Games, the Paralympics and Junior Paralympics involve athletes from around the world competing in various sports. These competitions involve athletes with physical challenges. Some of the more unique sports are sitting volleyball, wheelchair tennis, and goalball.

SESSION 2 ■ ■ □ □ □

Apply It **Problem 6** Ask students to share whether they have seen a bamboo plant and to describe what it looks like. Bamboo plants are very strong plants. They grow thick and are relatively easy to grow. Bamboo plants tend to grow quickly and may need to be trimmed back so as not to get out of hand. Bamboo plants are so versatile that they can be used for decoration, in fabric and clothing, as building materials, or even as a food source.

SESSION 3 ■ ■ ■ □ □

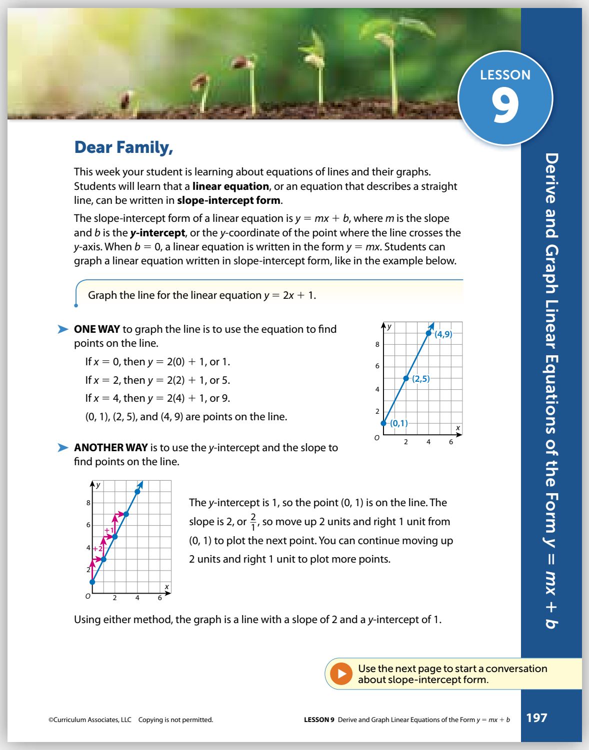
Try It Ask students whether they have seen rain barrels in private homes or businesses and have them describe what the barrels looked like. Rain barrels collect and store rainwater. They can include pumps, pipes, and barrels for storage, or they can be simple wooden or plastic containers. The collected water is used to water gardens or for other outdoor needs. The water is typically chemical-free and is a good source of nutrition for plants. The practice of collecting rainwater started in the Middle East around 2000 BCE.

SESSION 4 ■ ■ ■ ■ □

Try It Ask students about underwater sites that they would like to explore someday. Human beings have always had a drive to explore the planet, even below the surface of the ocean. Underwater exploration became much more accessible with the invention of scuba gear. *Scuba* is an acronym for Self-Contained Underwater Breathing Apparatus and was invented in 1942 by Jacques Cousteau and Emile Gagnan.

Connect to Family and Community

- **After the Explore session, have students use the Family Letter to let their families know what they are learning and to encourage family involvement.**



LESSON 9

Derive and Graph Linear Equations of the Form $y = mx + b$

Dear Family,

This week your student is learning about equations of lines and their graphs. Students will learn that a **linear equation**, or an equation that describes a straight line, can be written in **slope-intercept form**.

The slope-intercept form of a linear equation is $y = mx + b$, where m is the slope and b is the ***y*-intercept**, or the y-coordinate of the point where the line crosses the y-axis. When $b = 0$, a linear equation is written in the form $y = mx$. Students can graph a linear equation written in slope-intercept form, like in the example below.

Graph the line for the linear equation $y = 2x + 1$.

- **ONE WAY** to graph the line is to use the equation to find points on the line.

  If $x = 0$, then $y = 2(0) + 1$, or 1.
  If $x = 2$, then $y = 2(2) + 1$, or 5.
  If $x = 4$, then $y = 2(4) + 1$, or 9.
  (0, 1), (2, 5), and (4, 9) are points on the line.

- **ANOTHER WAY** is to use the y-intercept and the slope to find points on the line.

  The y-intercept is 1, so the point (0, 1) is on the line. The slope is 2, or $\frac{2}{1}$, so move up 2 units and right 1 unit from (0, 1) to plot the next point. You can continue moving up 2 units and right 1 unit to plot more points.

Using either method, the graph is a line with a slope of 2 and a y-intercept of 1.

Use the next page to start a conversation about slope-intercept form.

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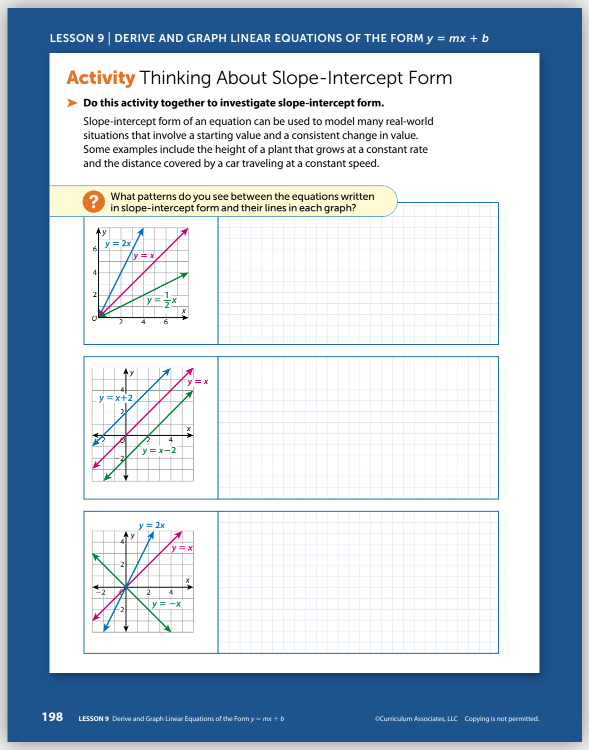

LESSON 9 | DERIVE AND GRAPH LINEAR EQUATIONS OF THE FORM $y = mx + b$

Activity Thinking About Slope-Intercept Form

- **Do this activity together to investigate slope-intercept form.**

  Slope-intercept form of an equation can be used to model many real-world situations that involve a starting value and a consistent change in value. Some examples include the height of a plant that grows at a constant rate and the distance covered by a car traveling at a constant speed.

What patterns do you see between the equations written in slope-intercept form and their lines in each graph?

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Connect to Language

- **For English language learners, use the Differentiation chart to scaffold the language in each session. Use the Academic Vocabulary routine for academic terms before Session 1.**

DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Use with **Session 1** **Connect It**

Levels 1–3: Speaking/Writing

Read Connect It problem 1 aloud. Break the problem into parts. Read the first part: *What is the slope and what is the equation of the line* and have students underline the two things they need to do. Then read: *representing Kendra's distance from the start in terms of time.* Explain that the phrase *in terms of time* means that the distance is determined by the amount of time.

Have students look at the Try It graph. Ask: *What two quantities are represented? What variable represents time? . . . distance?* Have students find Kendra's distance at 5 minutes. Have students work with a partner to write the slope and equation and explain how they found each, using terms *slope formula* and *points.*

Levels 2–4: Speaking/Writing

Read Connect It problem 1 aloud. To help students complete the task, have them underline what they need to do. Reread the second part of the sentence: *representing Kendra's distance from the start in terms of time.* Have partners turn and talk about what this part of the sentence means and then ask volunteers to explain. Clarify if needed that *in terms of time* means that the distance is determined by the amount of time.

Have students use the graph to find how far Kendra runs in 5 and 10 minutes and then predict how far she will run in 15 minutes. Then have students work independently to find the slope and write an equation. Then have them explain to a partner how they found each, using the terms *slope formula* and *points*.

Levels 3–5: Speaking/Writing

Have students read Connect It problem 1, underline what they need to do, and draft a response. Remind students to explain how they found the slope and the equation clearly by using precise language and providing details about the situation in the problem.

Form pairs and use **Stronger and Clearer Each Time** to help students refine their draft responses. Allow think time for students to revise their drafts based on the feedback they receive.

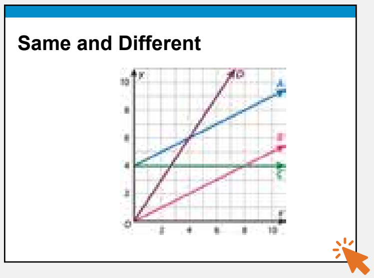
Invite students to share and explain their equations to the class.

Explore Deriving $y = mx$

Purpose

- **Explore** the idea that a line through the origin can be represented by an equation of the form $y = mx$.
- **Understand** that the equation of a line through the origin can always be written $y = mx$ and that m represents the slope of the line.

START CONNECT TO PRIOR KNOWLEDGE


Same and Different

Possible Solutions

Lines *A* and *B* have the same slope.

Lines *B* and *D* both represent proportional relationships.

Line *C* is the only horizontal line.

Lines *A* and *C* both cross the *y*-axis at (0, 4).

WHY? Support students' facility in recognizing characteristics of lines.

TRY IT

SMP 1, 2, 4, 5, 6

Make Sense of the Problem

See ***Connect to Culture*** to support student engagement. Before students work on Try It, use **Co-Craft Questions** to help them make sense of the problem. Students may develop many different questions about the graph and about Kendra's race. Encourage them to identify details in the problem statement and graph that would help them answer their questions.

DISCUSS IT

SMP 2, 3, 6

Support Partner Discussion

After students work on Try It, have them respond to Discuss It with a partner. Listen for understanding that:

- the relationship is proportional, and the slope is the unit rate, or the constant of proportionality.
- the slope, $\frac{1}{5}$, represents the change in Kendra's distance for each increase of 1 minute in time.
- multiplying the number of minutes Kendra runs, x, by the unit rate, $\frac{1}{5}$, gives the distance she runs, y.

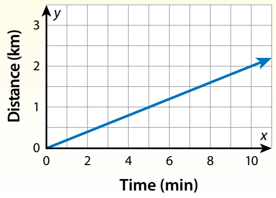
LESSON 9 | SESSION 1 ■ □ □ □ □

Explore Deriving $y = mx$

Previously, you learned about slope. In this lesson, you will learn about writing the equation of a line.

➤ **Use what you know to try to solve the problem below.**

Kendra is a blind marathon runner training for the Junior Paralympics. Kendra's coach graphs a line representing Kendra's distance from the start over the first 10 minutes of a practice 5K race. What is the slope of the line? What equation could you use to find y, Kendra's distance from the start after x minutes?

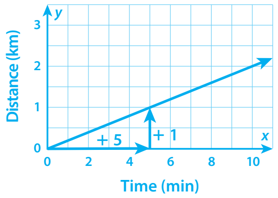


TRY IT

Math Toolkit graph paper, straightedges

Possible work:

SAMPLE A



$\frac{\text{rise}}{\text{run}} = \frac{1}{5}$

The slope is $\frac{1}{5}$.

Equation: $y = \frac{1}{5}x$

SAMPLE B

(5, 1) and (10, 2) are on the line.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 1}{10 - 5}$$
$$= \frac{1}{5}$$

The slope is $\frac{1}{5}$.

Equation: $y = \frac{1}{5}x$

DISCUSS IT

Ask: How might knowing what the slope represents help you write the equation?

Share: I knew . . . so I . . .

Learning Target SMP 1, SMP 2, SMP 3, SMP 4, SMP 5, SMP 6, SMP 7

Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.

Error Alert If students think the slope is 5, then ask them what this slope means in context. Running 5 km per minute would mean Kendra ran the entire 5K race in 1 minute. Once students realize this slope does not make sense, have them choose two points on the line and use the formula for slope. Encourage students to always check their answers for reasonableness.

Select and Sequence Student Strategies

Select 2–3 samples that represent the range of student thinking in your classroom. Here is one possible order for class discussion:

- table of (x, y) values used to find slope and write equation
- rise over run from graph used to find slope and proportional reasoning used to write equation
- slope formula used to find slope and proportional reasoning used to write equation

Facilitate Whole Class Discussion

Call on students to share selected strategies. Ask students to reword any unclear statements so that others understand. Confirm with the speaker that the rewording is accurate.

Guide students to **Compare and Connect** the representations. Remind students that good listeners use engaged body language, such as looking at the speaker and nodding to show understanding.

> **ASK** *Did everyone find the slope of the line in the same way? If not, how were the strategies different?*
>
> **LISTEN FOR** Some students chose two points and used the formula. Some counted to find the rise and run between two points and calculated rise over run.

CONNECT IT

SMP 2, 4, 5

① **Look Back** Look for understanding that to find slope, two points are needed and that the number of minutes, x, multiplied by the slope, or unit rate, is the distance, y.

DIFFERENTIATION | RETEACH or REINFORCE

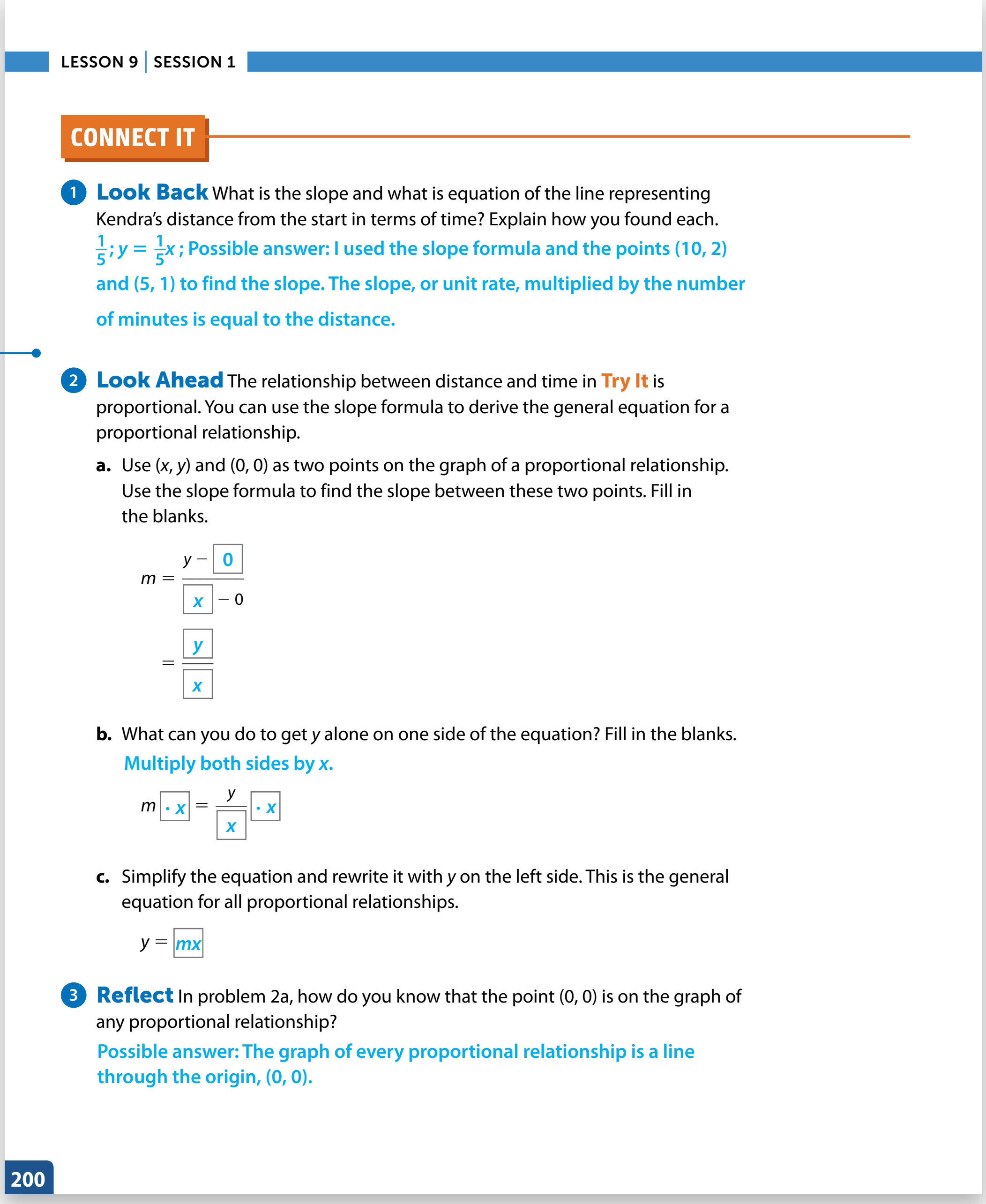
Visual Model

Use a graph to understand slope.

If students are unsure which two points to use to find slope, then use this activity to help them see that between any two points, the slope of a line is constant.

Materials For display: large coordinate plane

- Invite students to plot points at (0, 0), (5, 1), and (10, 2). Then invite a student to draw a line through the points.
- Ask: *From (0, 0) to (5, 1), what is the rise, or change in the vertical coordinates?* [1]
- Ask: *What is the run, or change in the horizontal coordinates?* [5]
- Ask: *What is the quotient of the rise and run?* $\left[\frac{1}{5}\right]$
- Ask: *What does this quotient represent?* [slope]
- Repeat the second through fourth steps with points (5, 1) and (10, 2). Repeat again with points (0, 0) and (10, 2).
- Ask: *Using the equation* $y = mx$*, what is the equation of the line through these points?* $\left[y = \frac{1}{5}x\right]$
- Remind students that any two points on a line can be used to find its slope. So, when choosing points, they might choose ones with integer coefficients or that allow easier calculations.



LESSON 9 | SESSION 1

CONNECT IT

① **Look Back** What is the slope and what is equation of the line representing Kendra's distance from the start in terms of time? Explain how you found each.

$\frac{1}{5}$; $y = \frac{1}{5}x$; Possible answer: I used the slope formula and the points (10, 2) and (5, 1) to find the slope. The slope, or unit rate, multiplied by the number of minutes is equal to the distance.

② **Look Ahead** The relationship between distance and time in **Try It** is proportional. You can use the slope formula to derive the general equation for a proportional relationship.

a. Use (x, y) and (0, 0) as two points on the graph of a proportional relationship. Use the slope formula to find the slope between these two points. Fill in the blanks.

$$m = \frac{y - \boxed{0}}{\boxed{x} - 0}$$

$$= \frac{\boxed{y}}{\boxed{x}}$$

b. What can you do to get y alone on one side of the equation? Fill in the blanks.

Multiply both sides by x.

$$m\boxed{\cdot x} = \frac{y}{\boxed{x}}\boxed{\cdot x}$$

c. Simplify the equation and rewrite it with y on the left side. This is the general equation for all proportional relationships.

$$y = \boxed{mx}$$

③ **Reflect** In problem 2a, how do you know that the point (0, 0) is on the graph of any proportional relationship?

Possible answer: The graph of every proportional relationship is a line through the origin, (0, 0).

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② **Look Ahead** Point out that because (0, 0) is on the graph of a proportional relationship, the point (0, 0) can be used in the slope formula with any other point (x, y) on the line. Students should recognize that when this is done and the equation is rewritten, the general equation for a proportional relationship, $y = mx$, is obtained.

CLOSE

EXIT TICKET

③ **Reflect** Look for understanding that the graph of any proportional relationship is a line through the origin.

Common Misconception If students do not believe that the graph of every proportional relationship is a line through the origin, then have them try to come up with a counterexample. For example, if you earn x dollars per hour, if you work 0 hours, you earn 0 dollars.

Prepare for Deriving and Graphing Linear Equations of the Form $y = mx + b$

Support Vocabulary Development

Assign **Prepare for Deriving and Graphing Linear Equations of the Form $y = mx + b$** as extra practice in class or as homework.

If you have students complete this in class, then use the guidance below.

Ask students to consider the term *slope*. Students should supply a definition of slope in their own words, both an example and a non-example of slope, and a drawing illustrating slope.

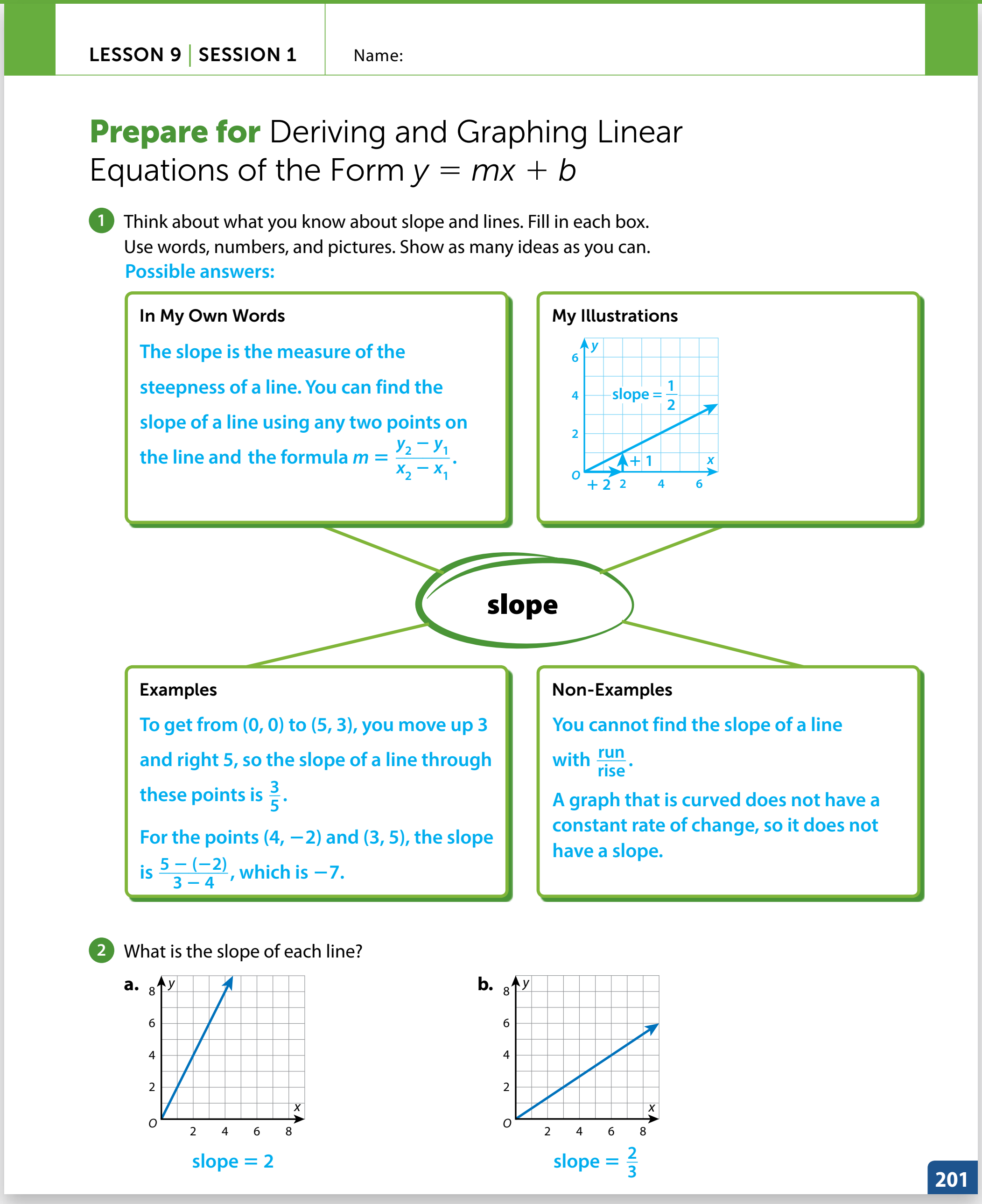
Have students work in pairs to complete the graphic organizer. Invite pairs to share their completed organizers and prompt a whole-class comparative discussion of the examples and non-examples that students supplied.

Have students look at the graphs in problem 2 and discuss with a partner how to find the values of the rise and run for each line. Students can find two points at intersections of gridlines and use them to find the rise and run by counting or by subtracting coordinates.

Problem Notes

① Students should understand that slope is the measure of the steepness of a line. Student responses might include that the slope can be found by counting to find the rise and run of the line and dividing them or by using the slope formula. Students should recognize that any two points on a line can be used to find the slope of the line.

② Students may either count units on the grid and divide the rise by the run, or they can use the slope formula, $m = \frac{y_2 - y_1}{x_2 - x_1}$.



LESSON 9 | SESSION 1 Name:

Prepare for Deriving and Graphing Linear Equations of the Form $y = mx + b$

① Think about what you know about slope and lines. Fill in each box. Use words, numbers, and pictures. Show as many ideas as you can.

Possible answers:

In My Own Words

The slope is the measure of the steepness of a line. You can find the slope of a line using any two points on the line and the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$.

My Illustrations

slope

Examples

To get from (0, 0) to (5, 3), you move up 3 and right 5, so the slope of a line through these points is $\frac{3}{5}$.

For the points (4, −2) and (3, 5), the slope is $\frac{5 - (-2)}{3 - 4}$, which is −7.

Non-Examples

You cannot find the slope of a line with $\frac{\text{run}}{\text{rise}}$.

A graph that is curved does not have a constant rate of change, so it does not have a slope.

② What is the slope of each line?

a.

slope = 2

b.

slope = $\frac{2}{3}$

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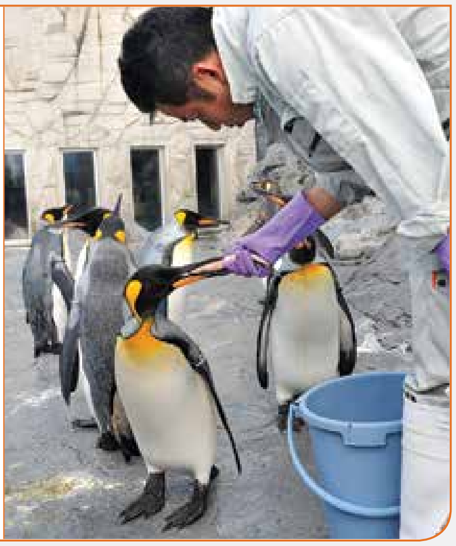
REAL-WORLD CONNECTION

Zookeepers use linear equations to help them know when a food supply for a group of animals needs to be reordered. Zookeepers can write and graph a linear equation using the initial amount in the food supply and the rate of change, based on how much food is given to the animals each day. Once the y-value of the graph reaches a certain point, the zookeeper can reason that more food must be ordered. Ask students to think of other real-world examples when writing an equation in $y = mx + b$ form, and graphing that equation might be useful.

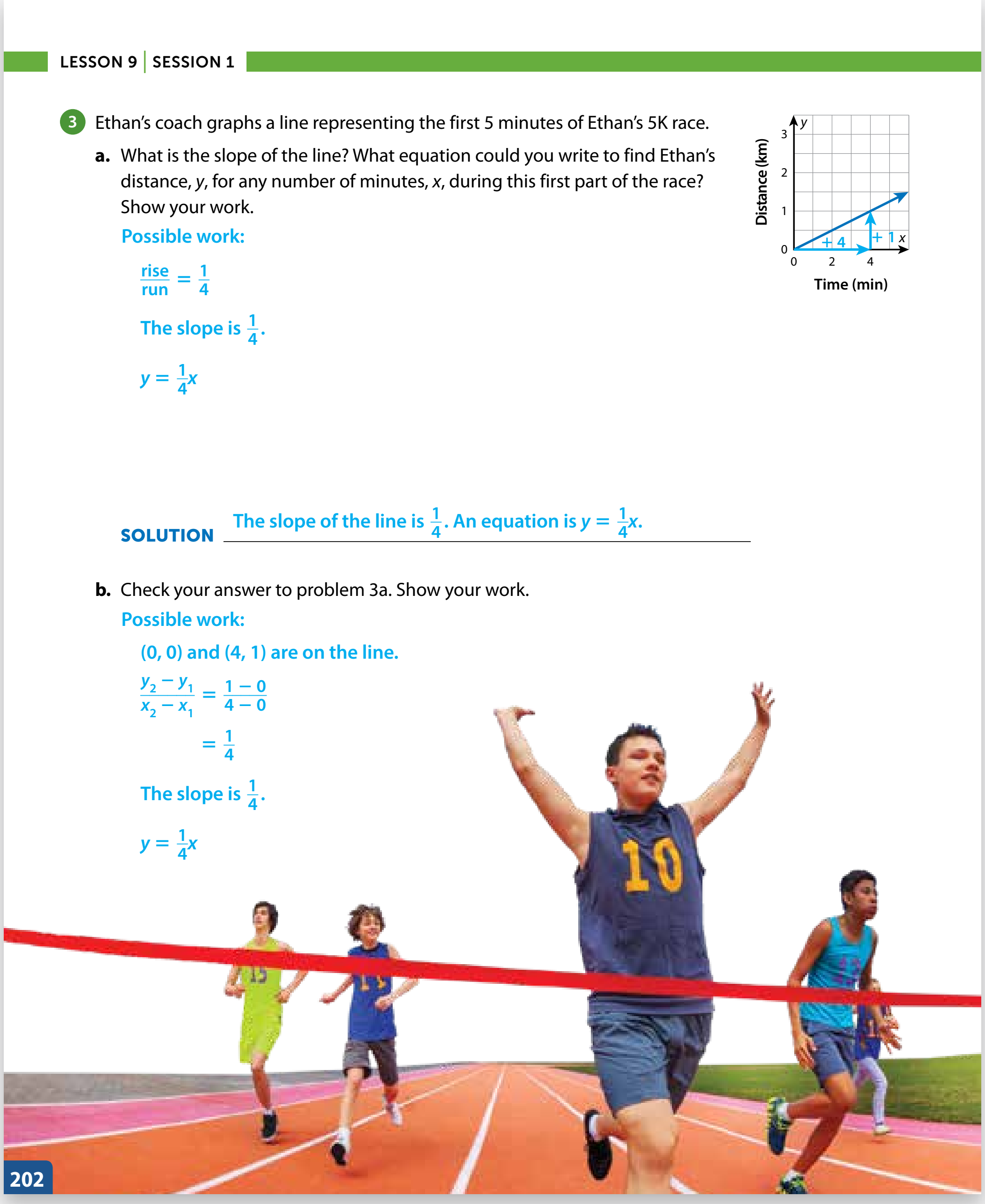


Additional Practice

③ Problem 3 provides another look at finding slope and writing an equation for a line that passes through the origin. This problem is similar to the problem about Kendra training for the Junior Paralympics. In both problems, students find the slope of a line and write an equation in the form $y = mx$. This problem asks for the slope and equation of a line of a different runner's times and distances.

Students may want to use a graph or an equation to solve.

Suggest that students use **Say It Another Way** to help them understand what the question is asking.



LESSON 9 | SESSION 1

③ Ethan's coach graphs a line representing the first 5 minutes of Ethan's 5K race.

a. What is the slope of the line? What equation could you write to find Ethan's distance, y, for any number of minutes, x, during this first part of the race? Show your work.

Possible work:

$\frac{\text{rise}}{\text{run}} = \frac{1}{4}$

The slope is $\frac{1}{4}$.

$y = \frac{1}{4}x$

SOLUTION The slope of the line is $\frac{1}{4}$. An equation is $y = \frac{1}{4}x$.

b. Check your answer to problem 3a. Show your work.

Possible work:

(0, 0) and (4, 1) are on the line.

$\frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 0}{4 - 0}$

$= \frac{1}{4}$

The slope is $\frac{1}{4}$.

$y = \frac{1}{4}x$

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DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Use with **Session 2** **Apply It**

Levels 1–3: Speaking/Writing

Read the first two sentences in Apply It problem 6. Paraphrase to simplify: *A class plants bamboo seedlings. The graph shows how the plant grows.* Then read the third sentence and explain: *To predict means to make a good guess based on information, a graph, or an equation. What does the class need to predict?*

Have students write an equation and discuss with partners. Provide sentence frames:

- *The class can predict the height of the bamboo after x days by using _____.*
- *The slope of my equation represents _____.*
- *The y-intercept represents _____.*

Levels 2–4: Speaking/Writing

Read Apply It problem 6 with students. Allow think time for students to look at the graph. Then have them write the coordinates of two points on the graph. Have students turn to partners to explain how to use the points to write an equation. Then have them explain what the slope and y-intercept mean in the situation.

Have partners discuss the meaning of *predict* and explain how the equation can help make a prediction. Provide sentence starters to help students respond to the problem:

- *In the problem, the slope means _____.*
- *The y-intercept is _____.*
- *The equation _____ will help the class _____.*

Levels 3–5: Speaking/Writing

Have students work in pairs to read Apply It problem 6. Use **Say It Another Way**. Monitor as students discuss words and phrases they can use to paraphrase. Ask questions to make sure students use the word *predict* and are including all relevant information. Then have students draft a response to the problem.

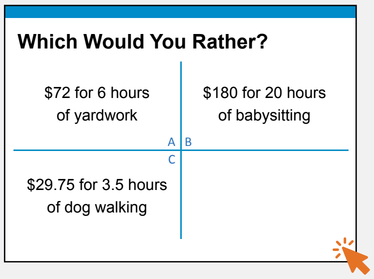
Have students compare equations with other partners and explain. Remind students to refer to the graph to support their explanations. Encourage students to pay attention as they listen and suggest specific math vocabulary partners could use to make their explanations clearer.

Develop Deriving $y = mx + b$

Purpose

- **Develop** strategies for deriving the equation $y = mx + b$ for a line not passing through the origin.
- **Recognize** that b represents the y-intercept of a graphed line.

START CONNECT TO PRIOR KNOWLEDGE



Which Would You Rather?

A: $72 for 6 hours of yardwork

B: $180 for 20 hours of babysitting

C: $29.75 for 3.5 hours of dog walking

Possible Solutions

A because it has the highest hourly pay rate.

B because it is the greatest total amount of money earned.

C because it is the fewest hours spent working.

WHY? Support students' understanding of comparing rates.

DEVELOP ACADEMIC LANGUAGE

WHY? Unpack the meaning of adjectival phrases in real-world problems.

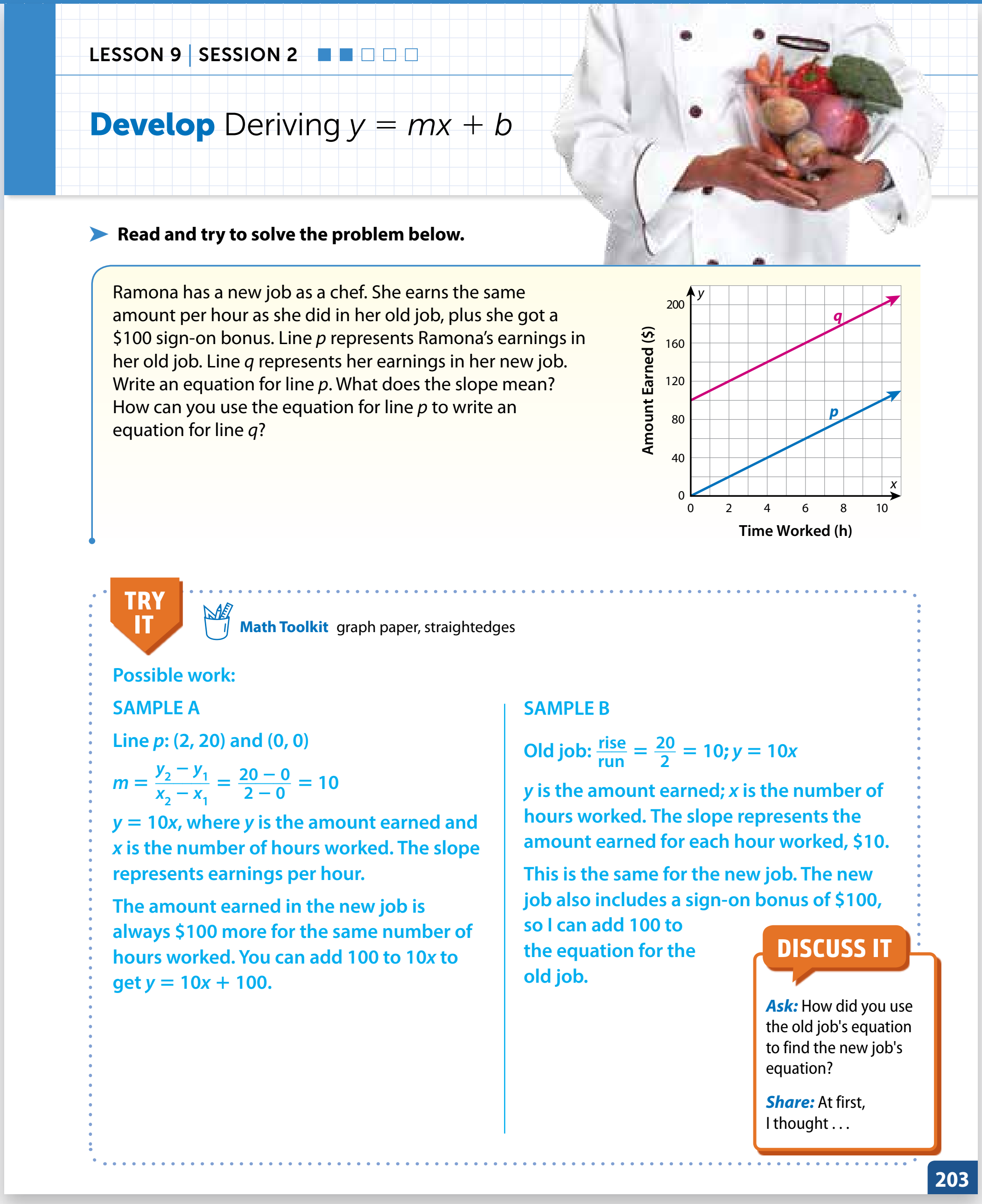
HOW? Read Apply It problem 7. Help students analyze the phrase *predicted outside temperature*. Circle *temperature* and point to each word as you ask: *What does this word say about the temperature? Where do people expect to have that temperature?* Discuss how *predicted outside* is an adjectival phrase. Then have students use their own words to tell what the graph represents.

TRY IT

SMP 1, 2, 4, 5, 6

Make Sense of the Problem

Before students work on Try It, use **Three Reads** to help them make sense of the problem. Draw their attention to the graph to ensure that students are noting and interpreting the information correctly.



LESSON 9 | SESSION 2 ■ ■ □ □ □

Develop Deriving $y = mx + b$

➤ **Read and try to solve the problem below.**

Ramona has a new job as a chef. She earns the same amount per hour as she did in her old job, plus she got a $100 sign-on bonus. Line p represents Ramona's earnings in her old job. Line q represents her earnings in her new job. Write an equation for line p. What does the slope mean? How can you use the equation for line p to write an equation for line q?

TRY IT

Math Toolkit graph paper, straightedges

Possible work:

SAMPLE A

Line p: (2, 20) and (0, 0)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{20 - 0}{2 - 0} = 10$$

$y = 10x$, where y is the amount earned and x is the number of hours worked. The slope represents earnings per hour.

The amount earned in the new job is always $100 more for the same number of hours worked. You can add 100 to $10x$ to get $y = 10x + 100$.

SAMPLE B

Old job: $\frac{\text{rise}}{\text{run}} = \frac{20}{2} = 10; y = 10x$

y is the amount earned; x is the number of hours worked. The slope represents the amount earned for each hour worked, $10.

This is the same for the new job. The new job also includes a sign-on bonus of $100, so I can add 100 to the equation for the old job.

DISCUSS IT

Ask: How did you use the old job's equation to find the new job's equation?

Share: At first, I thought . . .

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DISCUSS IT

SMP 1, 2, 3, 6, 7

Support Partner Discussion

After students work on Try It, have them respond to Discuss It with a partner. If students need support in getting started, prompt them to ask each other questions such as:

- *Why did you choose the strategy you used?*
- *How did the graph help you make sense of the problem?*
- *What do x and y represent in your equation(s)?*
- *What do the two graphs and situations have in common?*

Common Misconception Listen for students who use the equation $100 + y = 10x$ to model line q because line p moves up along the y-axis. As students share their strategies, encourage them to think about the situation and put themselves in Ramona's place. The pay she gets includes her hourly wages, $10x$, and the $100 sign-on bonus, so the $100 needs to be added to $10x$ to get her total pay, y.

Select and Sequence Student Strategies

Select 2–3 samples that represent the range of student thinking in your classroom. Here is one possible order for class discussion:

- table of values used to determine the equation for line *q*
- (misconception) incorrect reasoning used to write the equation $y + 100 = 10x$ for line *q*
- reasoning about situation used to determine the equation of line *q* from the equation for line *p*
- translation used to determine the equation for line *q* from the equation for line *p*

Facilitate Whole Class Discussion

Call on students to share selected strategies. Prompt students to check that their explanations are clear by pausing and asking classmates for questions or comments.

Guide students to **Compare and Connect** the representations. As students listen to the explanations, ask them to record their interpretations.

> **ASK** *What similarities do the strategies share?*
>
> **LISTEN FOR** Each finds the slope is equal to 10 and interprets it as the amount earned per hour.

Model It & Analyze It

If students presented these models, have students connect these models to those presented in class.

If no student presented at least one of these models, have students first analyze key features of the models, and then connect them to the models presented in class.

> **ASK** *What do you notice about the slopes of lines p and q? How are the graphed lines related to each other?*
>
> **LISTEN FOR** Both lines have the same slope. The lines are parallel.

For the translated graph, prompt students to examine the *y*-intercepts of lines *p* and *q*.

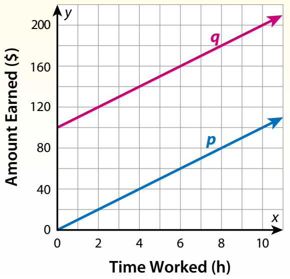
- *What does the y-intercept of line p represent?*
- *What does the y-intercept of line q represent?*
- *Why can line p be translated to map onto line q?*

For the equations, prompt students to compare the slope and *y*-intercept for each line.

- *What points are used to find the slope?*
- *What equation is used to represent line p?*
- *How does the equation of line q relate to the equation for line p?*

➤ **Explore different ways to derive $y = mx + b$.**

Ramona has a new job as a chef. She earns the same amount per hour as she did in her old job, plus she got a $100 sign-on bonus. Line *p* represents Ramona's earnings in her old job. Line *q* represents her earnings in her new job. Write an equation for line *p*. What does the slope mean? How can you use the equation for line *p* to write an equation for line *q*?

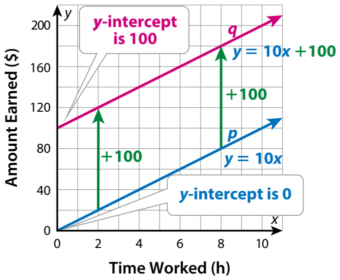


Model It

You can use a transformation to map line *p* onto line *q*.

The slopes of the lines are equal since the earnings per hour at each job are the same. The lines are parallel.

The *y*-coordinate of the point where a line meets or crosses the *y*-axis is called the ***y*-intercept**.



Analyze It

You can write the equation for line *p* in the form $y = mx$.

Line *p* represents earnings at the old job. (0, 0) and (2, 20) are on line *p*.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{20 - 0}{2 - 0} = 10$$

The equation for line *p* is $y = 10x$ where *y* is the amount earned and *x* is the number of hours worked.

The equation for line *q* should include the hourly earnings at the new job plus the sign-on bonus. The equation is $y = 10x + 100$.

DIFFERENTIATION | EXTEND

SMP 4

Deepen Understanding

Connecting Graphs and Equations to a Linear Situation

Prompt students to consider the relationship between line *p* and line *q*.

> **ASK** *What is the same about Ramona's old earnings and new earnings?*
>
> **LISTEN FOR** Ramona's hourly wage is the same, $10 per hour.

> **ASK** *Suppose Ramona's new job paid $15 per hour instead of $10 per hour. How would this change the graph and the equation of line q?*
>
> **LISTEN FOR** The slope of line *q* would be 15. It would no longer be parallel to line *p*. It would be steeper. If the bonus was still $100, the equation would be $y = 15x + 100$.

> **ASK** *Suppose Ramona's new job had the same pay rate as her old job but offered a $200 sign-on bonus, instead of a $100 bonus. How would this change the graph and the equation of line q?*
>
> **LISTEN FOR** The line would still be parallel to line *p*, but it would be higher up, with a *y*-intercept of 200, instead of 100. The equation would be $y = 10x + 200$.

Develop Deriving $y = mx + b$

CONNECT IT

SMP 2, 4, 5, 6

Remind students that the slopes and *y*-intercepts are the same in each representation. Explain that they will now use those representations to reason about deriving the equation $y = mx + b$.

Before students begin to record and expand on their work in Model It & Analyze It, tell them that their explanation in problem 2 will prepare them to provide the equation asked for in problem 3.

Monitor and Confirm Understanding ① – ②

- Line *p* can be mapped onto line *q* by translating it up 100 units.
- The *y*-intercept of line *q*, which is 100, represents Ramona's sign-on bonus.
- The equations for the old and new jobs have the same slope because Ramona's pay rate was the same at both jobs. The equations have different *y*-intercepts because she got a sign-on bonus only at the new job.

Facilitate Whole Class Discussion

③ Look for understanding that when a linear equation is written in slope-intercept form, $y = mx + b$, the value of *m* is the slope and the value of *b* is the *y*-intercept.

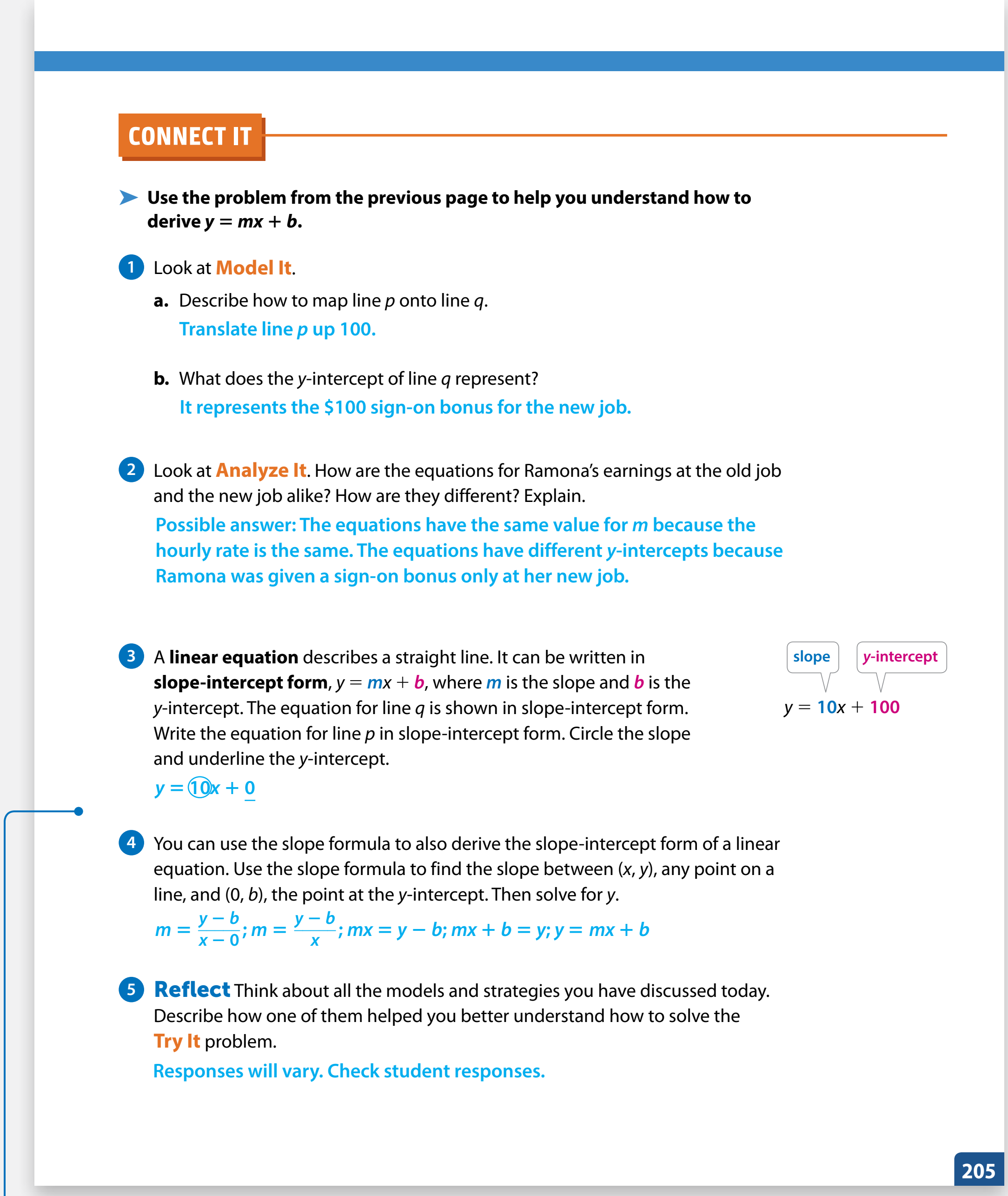
> **ASK** *If two equations have different values of m and the same value of b, how will the graphs of the equations compare?*
>
> **LISTEN FOR** The graphs will be lines with different slopes, or steepness, but they will cross the *y*-axis at the same point.

④ Look for understanding that the two points being substituted into the slope formula are (*x*, *y*) and (0, *b*).

> **ASK** *Why might you want to label (x, y) as (x_1, y_1), and (0, b) as (x_2, y_2)?*
>
> **LISTEN FOR** It will help you substitute the coordinates in the correct order in the slope formula.

> **ASK** *Once you find m, what steps do you have to take to solve for y?*
>
> **LISTEN FOR** Multiply each side of the equation by *x* and then add *b* to each side of the equation.

⑤ **Reflect** Have all students focus on the strategies used to solve the Try It. If time allows, have students discuss their ideas with a partner.



CONNECT IT

➤ **Use the problem from the previous page to help you understand how to derive $y = mx + b$.**

① Look at **Model It**.

a. Describe how to map line *p* onto line *q*.

Translate line *p* up 100.

b. What does the *y*-intercept of line *q* represent?

It represents the $100 sign-on bonus for the new job.

② Look at **Analyze It**. How are the equations for Ramona's earnings at the old job and the new job alike? How are they different? Explain.

Possible answer: The equations have the same value for *m* because the hourly rate is the same. The equations have different *y*-intercepts because Ramona was given a sign-on bonus only at her new job.

③ A **linear equation** describes a straight line. It can be written in **slope-intercept form**, $y = mx + b$, where *m* is the slope and *b* is the *y*-intercept. The equation for line *q* is shown in slope-intercept form. Write the equation for line *p* in slope-intercept form. Circle the slope and underline the *y*-intercept.

$$y = \overset{\text{slope}}{10}x + \overset{y\text{-intercept}}{100}$$

$y = \enclose{circle}{10}x + \underline{0}$

④ You can use the slope formula to also derive the slope-intercept form of a linear equation. Use the slope formula to find the slope between (*x*, *y*), any point on a line, and (0, *b*), the point at the *y*-intercept. Then solve for *y*.

$m = \frac{y - b}{x - 0}; m = \frac{y - b}{x}; mx = y - b; mx + b = y; y = mx + b$

⑤ **Reflect** Think about all the models and strategies you have discussed today. Describe how one of them helped you better understand how to solve the **Try It** problem.

Responses will vary. Check student responses.

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DIFFERENTIATION | RETEACH or REINFORCE

Hands-On Activity

Analyze slopes and *y*-intercepts.

If students are unsure about the impact of the slope and y-intercept of a line on the equation $y = mx + b$, then use this activity to solidify their understanding.

Materials For each pair: 3 chenille stems, tape, Activity Sheet *Graph Paper*

- Have pairs draw axes on the graph paper and tape a chenille stem to show $y = 3x$.
- Instruct pairs to place another chenille stem on the first one, and then move it up 5 units, keeping it parallel to the first line. They should then tape it in place.
- Ask: *What is the slope of this line?* [3] *What is the y-intercept?* [5]
- Ask: *What is the equation of this line?* [$y = 3x + 5$]
- Instruct pairs to place another chenille stem on the first one and then move it down 2 units, parallel to the first line. They should then tape it in place.
- Ask: *What is the equation of this line?* [$y = 3x - 2$]
- Ask: *What part of the equation $y = mx + b$ shows the slope of the line?* [*m*] *What part shows the y-intercept?* [*b*]

Apply It

For all problems, encourage students to use the axes, scales, and labels of the graphs to support their thinking.

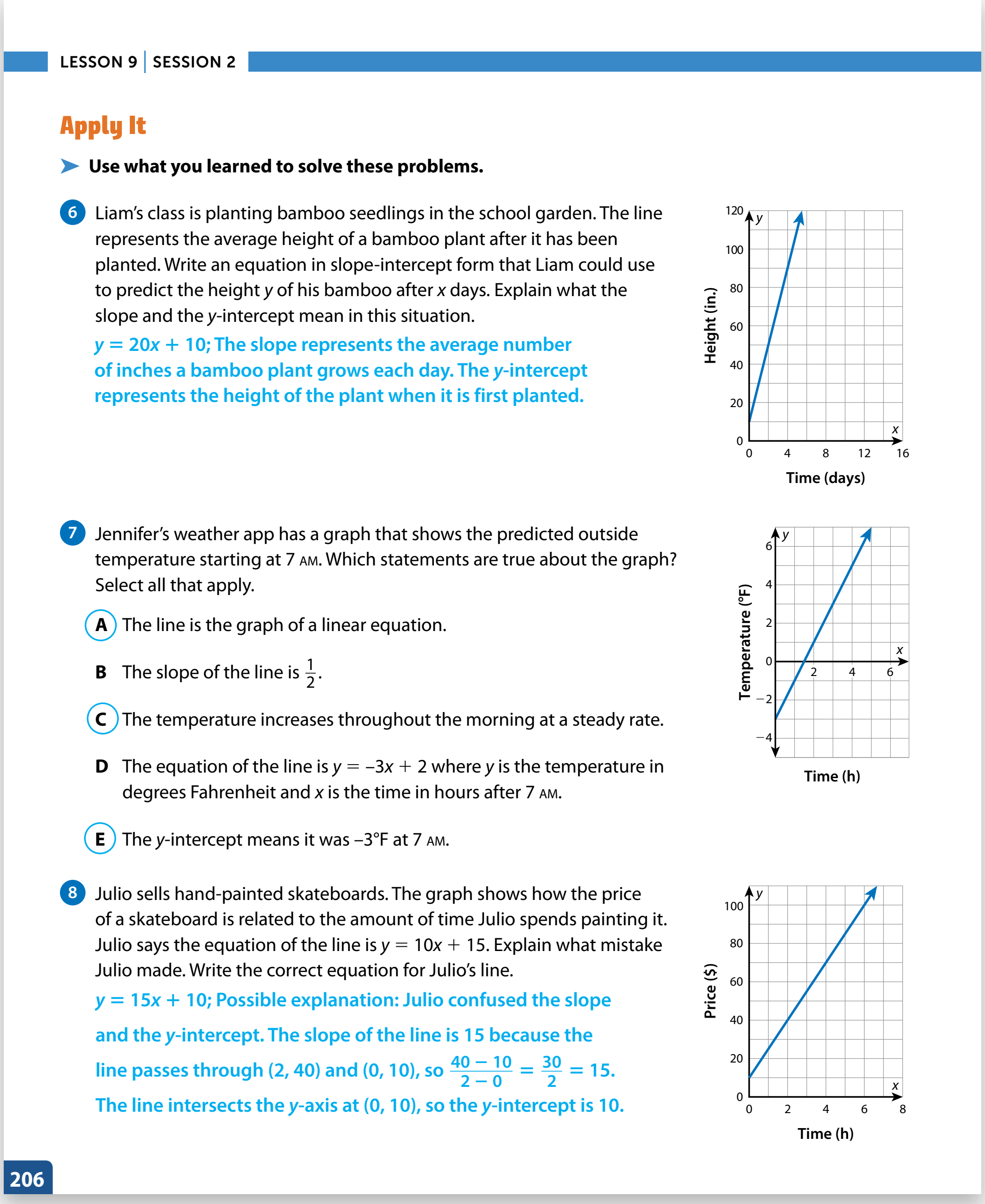
⑥ See ***Connect to Culture*** to support student engagement. Students should be able to read the *y*-intercept from the graph. To find the slope, they will have to look for another point on the line that they can easily read from the graph. The line passes through both (0, 10) and (2, 50), so the slope is $\frac{50-10}{2-0} = \frac{40}{2}$, or 20.

Encourage students to utilize the axis labels when writing their explanations.

⑦ **A, C, and E are correct.** The graph represents a linear equation because it is a straight line. The fact that the graph is a straight line indicates that the temperature increases throughout the morning at a steady rate. The line crosses the *y*-axis at (0, −3). The 0 represents the starting time, which the problem states is 7 AM. So, −3 represents the predicted temperature at 7 AM.

B is not correct. The line passes through the points (0, −3) and (1, −1), so the slope is 2.

D is not correct. This equation has the slope and *y*-intercept exchanged or confused. The equation of the line is $y = 2x - 3$.



Apply It

➤ **Use what you learned to solve these problems.**

⑥ Liam's class is planting bamboo seedlings in the school garden. The line represents the average height of a bamboo plant after it has been planted. Write an equation in slope-intercept form that Liam could use to predict the height *y* of his bamboo after *x* days. Explain what the slope and the *y*-intercept mean in this situation.

$y = 20x + 10$; The slope represents the average number of inches a bamboo plant grows each day. The *y*-intercept represents the height of the plant when it is first planted.

⑦ Jennifer's weather app has a graph that shows the predicted outside temperature starting at 7 AM. Which statements are true about the graph? Select all that apply.

Ⓐ The line is the graph of a linear equation.

B The slope of the line is $\frac{1}{2}$.

Ⓒ The temperature increases throughout the morning at a steady rate.

D The equation of the line is $y = -3x + 2$ where *y* is the temperature in degrees Fahrenheit and *x* is the time in hours after 7 AM.

Ⓔ The *y*-intercept means it was −3°F at 7 AM.

⑧ Julio sells hand-painted skateboards. The graph shows how the price of a skateboard is related to the amount of time Julio spends painting it. Julio says the equation of the line is $y = 10x + 15$. Explain what mistake Julio made. Write the correct equation for Julio's line.

$y = 15x + 10$; Possible explanation: Julio confused the slope and the *y*-intercept. The slope of the line is 15 because the line passes through (2, 40) and (0, 10), so $\frac{40-10}{2-0} = \frac{30}{2} = 15$. The line intersects the *y*-axis at (0, 10), so the *y*-intercept is 10.

CLOSE **EXIT TICKET**

⑧ Students' solutions should show an understanding that:

- an equation of the form $y = mx + b$ is a linear equation where *m* is the slope and *b* is the *y*-intercept.
- the slope is found by applying the formula to two points through which the line passes.
- the *y*-intercept is the *y*-coordinate of the point where the line intersects the *y*-axis. It is the *y*-coordinate of the point with *x*-coordinate zero.

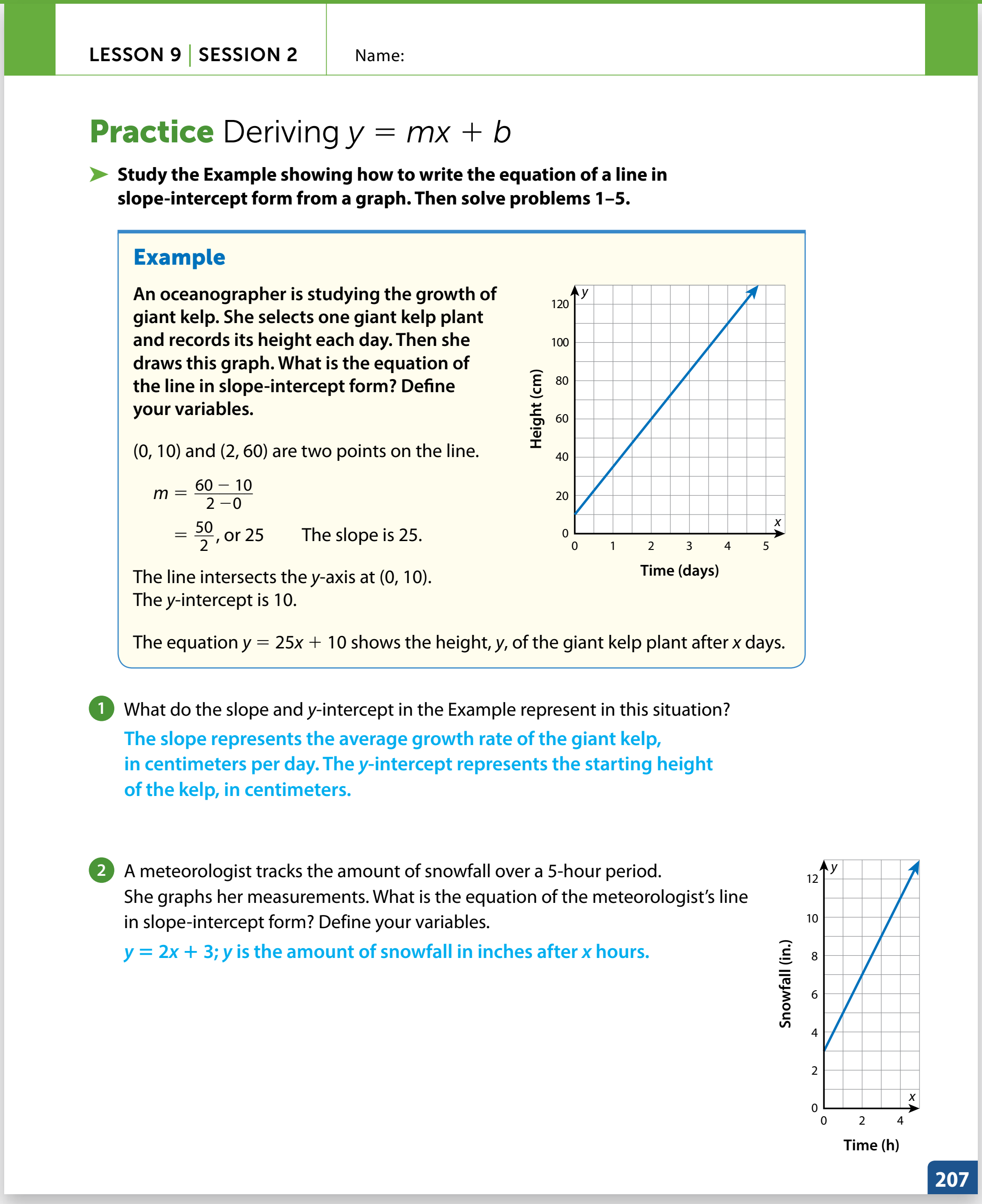
Error Alert If students incorrectly apply the slope formula and write $m = -15$, then demonstrate how the coordinates of the points must be substituted into the formula in the same order. Also point out that a negative slope does not make sense in this situation because the price of the skateboard is increasing with time rather than decreasing.

Prepare Deriving $y = mx + b$

Problem Notes

Assign **Practice Deriving $y = mx + b$** as extra practice in class or as homework.

1. Students should use the information provided in the Example to interpret the meaning of the slope and y-intercept in the context of the situation. ***Medium***
2. Students can examine the graph to identify the y-intercept as 3. To find the slope, they can identify two points and use the formula or use the graph to find the rise over run between two points. ***Medium***



LESSON 9 | SESSION 2 Name:

Practice Deriving $y = mx + b$

➤ **Study the Example showing how to write the equation of a line in slope-intercept form from a graph. Then solve problems 1–5.**

Example

An oceanographer is studying the growth of giant kelp. She selects one giant kelp plant and records its height each day. Then she draws this graph. What is the equation of the line in slope-intercept form? Define your variables.

(0, 10) and (2, 60) are two points on the line.

$$m = \frac{60 - 10}{2 - 0}$$

$$= \frac{50}{2}, \text{ or } 25 \qquad \text{The slope is 25.}$$

The line intersects the y-axis at (0, 10).
The y-intercept is 10.

The equation $y = 25x + 10$ shows the height, y, of the giant kelp plant after x days.

1. What do the slope and y-intercept in the Example represent in this situation?
   The slope represents the average growth rate of the giant kelp, in centimeters per day. The y-intercept represents the starting height of the kelp, in centimeters.

2. A meteorologist tracks the amount of snowfall over a 5-hour period. She graphs her measurements. What is the equation of the meteorologist's line in slope-intercept form? Define your variables.
   $y = 2x + 3$; y is the amount of snowfall in inches after x hours.

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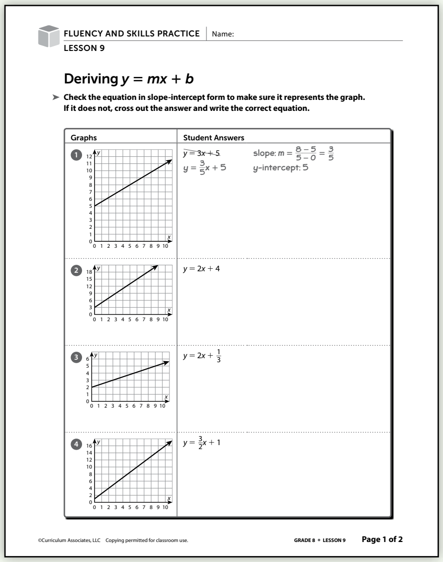
Fluency & Skills Practice

Deriving $y = mx + b$

In this activity, students are given graphs of lines and possible equations. They check whether the equation is correct for the graph shown. If the equation is not correct, they write the correct equation.



FLUENCY AND SKILLS PRACTICE | Name:
LESSON 9

Deriving $y = mx + b$

➤ **Check the equation in slope-intercept form to make sure it represents the graph. If it does not, cross out the answer and write the correct equation.**

Graphs	Student Answers
①	~~$y = 3x + 5$~~ $y = \frac{3}{5}x + 5$ slope: $m = \frac{8-5}{5-0} = \frac{3}{5}$ y-intercept: 5
②	$y = 2x + 4$
③	$y = 2x + \frac{1}{3}$
④	$y = \frac{3}{2}x + 1$

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Additional Practice

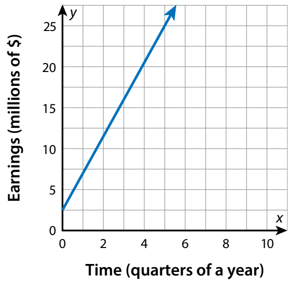
③ Students can identify the rise and the run between (0, 2.5) and (5, 25) using the grid instead of using the formula, but they need to note that each vertical grid unit is 2.5 rather than 1. ***Challenge***

④ **a.** *Basic*

b. *Basic*

⑤ Students may solve this using the process of elimination. For example: the only equation and graph with a negative *y*-intercept is the first equation and the last graph. The first graph has a *y*-intercept of 2, so that must correspond with the second equation. Similar thinking could be applied to the third equation and second graph. When it comes to matching the last combination, they will be the only ones remaining. ***Medium***

③ The growth in earnings for a digital music service is shown in the graph. What is the equation of the line? Show your work. Define your variables.



Possible work:

$b = 2.5$, so one point is (0, 2.5). One other point on the line is (5, 25).

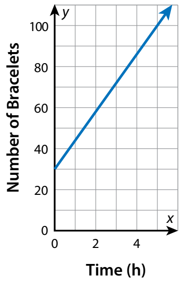
$$m = \frac{25 - 2.5}{5 - 0}$$

$$= \frac{22.5}{5}$$

$$= \frac{4.5}{1}\text{, or } 4.5$$

SOLUTION The equation $y = 4.5x + 2.5$ can be used to find *y*, the earnings in millions of dollars, after *x* quarters of a year.

④ Daria and her brother want to make 100 bracelets to sell at a craft fair. They have made some already. Daria made this graph to show how they can reach their goal. The equation of Daria's line is $y = 14x + 30$ where *y* is the number of bracelets and *x* is the time in hours.

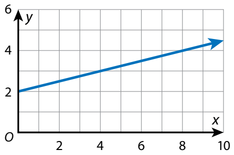


a. What is the slope of the line? 14

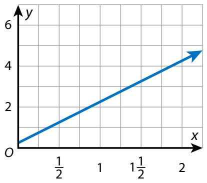
b. What is the *y*-intercept? 30

⑤ Write each linear equation under the graph of its line.

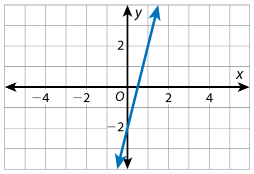
$y = 4x - 2$ $\quad$ $y = \frac{1}{4}x + 2$ $\quad$ $y = 2x + \frac{1}{4}$



$y = \frac{1}{4}x + 2$



$y = 2x + \frac{1}{4}$



$y = 4x - 2$

DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Use with **Session 3** **Graph It**

Levels 1–3: Reading/Speaking

Read the problem in Graph It. Review *capacity*. Then have students tell how many gallons the barrel can hold. Use **Act It Out** with drawings or gestures to explore the meanings of *decrease* and *increase*. Then read each statement and help students restate or explain:

- *A slope of* $-\frac{3}{1}$ *means that every time the water goes down by ____, the time goes up by ____.*
- *The equivalent quotient* $-\frac{30}{10}$ *means that every time the water ____, the time ____.*

Help partners explain (10, 30) and (20, 0):

- *After ____ minutes, there are ____ gallons left.*

Levels 2–4: Reading/Speaking

Read the problem in Graph It with students. Have students tell how many gallons the barrel can hold. Use **Act It Out** to have students demonstrate the meanings of *decrease* and *increase*. Then help them read and explain each statement. Ask: *What happens every time the water goes down by 3?*

Why is $-\frac{30}{10}$ *equivalent to* $-\frac{3}{1}$*? What happens for every decrease in y?*

Have students find the *y*-intercept. Then have them turn to partners and take turns reading and explaining (10, 30) and (20, 0). Provide a sentence frame:

- *The point (____,____) means that after ____ minutes, there are ____.*

Levels 3–5: Reading/Speaking

Have students read Graph It and think about how the steps connect to the graph. Then have them turn to a partner to share ideas. Partners take turns stating ideas and providing feedback. Provide sentence starters:

- *This sentence/phrase means ____ and the graph shows ____.*
- *You said the graph shows ____.*
- *I think you are right because ____.*
- *I have a different idea because ____.*

Then have partners use complete sentences to explain what (10, 30) and (20, 0) represent.

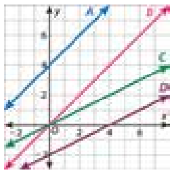
Develop Graphing a Linear Equation of the Form $y = mx + b$

Purpose

- **Develop** strategies for graphing a linear equation of the form $y = mx + b$.
- **Recognize** that when an equation is in slope-intercept form, the slope and y-intercept can be read directly from the equation and used to graph the equation.

START CONNECT TO PRIOR KNOWLEDGE

Same and Different



Possible Solutions

All represent linear equations with positive slopes.

Lines A and B are parallel and have the same slope.

Lines C and D are parallel and have the same slope.

Lines B and C have y-intercepts of 0.

Line A has a positive y-intercept.

Line D has a negative y-intercept.

WHY? Support students' facility with identifying and comparing slopes and y-intercepts of lines.

DEVELOP ACADEMIC LANGUAGE

WHY? Support understanding of sentences with passive voice and clauses.

HOW? Help students interpret the third sentence of Try It. Explain that *can be used* means that someone can use it. Read the first part of the sentence and have students tell what they can use to find y. Then point to the comma. Explain that the comma indicates that the sentence will provide more information. Read the second part and have students tell what y represents.

TRY IT

SMP 1, 2, 4, 5, 6

Make Sense of the Problem

See ***Connect to Culture*** to support student engagement. Before students work on Try It, use **Three Reads** to help them make sense of the problem. Draw attention to the key words and emphasize the components of the equation to ensure students interpret the information correctly.

LESSON 9 | SESSION 3 ■ ■ ■ □ □

Develop Graphing a Linear Equation of the Form $y = mx + b$

➤ **Read and try to solve the problem below.**

A 60-gallon rain barrel is filled to capacity. Elena opens the stopper to let water drain out to water her garden.

The equation $y = -3x + 60$ can be used to find y, the number of gallons of water left after the barrel drains for x minutes. Graph the equation.

TRY IT

Math Toolkit graph paper, straightedges

Possible work:

SAMPLE A

$x = 5$: $y = -3(5) + 60$; $y = -15 + 60$; $y = 45$

(5, 45) is a point on the line.

$x = 10$: $y = -3(10) + 60$; $y = -30 + 60$; $y = 30$

(10, 30) is a point on the line.

Draw a line through (5, 45) and (10, 30). See graph.

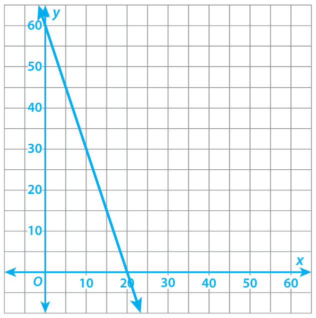
SAMPLE B

The y-intercept is 60, so the line intersects the y-axis at (0, 60).

Substitute 0 for y.

$$0 = -3x + 60$$
$$0 - 60 = -3x + 60 - 60$$
$$-60 = -3x$$
$$\frac{-60}{-3} = \frac{-3x}{-3}$$
$$20 = x$$

The line goes through (20, 0) and (0, 60). See graph.



DISCUSS IT

Ask: How did you start to graph the equation?

Share: I started graphing by . . .

DISCUSS IT

SMP 2, 3, 4, 5, 6, 7

Support Partner Discussion

After students work on Try It, encourage them to respond to Discuss It with a partner. If students need support in getting started, prompt them to ask each other questions such as:

- *What information in the equation helped you graph it?*
- *How many points did you plot before you drew a line?*

Error Alert If students graph the wrong line, then they may have confused the x- and y-axes or made a mistake in their calculations. Review the coordinate plane with students and remind them to check their work. You may want to suggest that students plot more than two points to see whether all will fall along the same line as a way to check their work.

Select and Sequence Student Strategies

Select 2–3 samples that represent the range of student thinking in your classroom. Here is one possible order for class discussion:

- substitution of two or more nonzero values for *x* to find points on the line
- point at the *y*-intercept and one or more additional points used to graph line
- point at the *y*-intercept and $\frac{\text{rise}}{\text{run}}$ used to find one or more additional points

Facilitate Whole Class Discussion

Call on students to share selected strategies. Review the idea that one way to connect strategies is to describe how they are alike and how they are different.

Guide students to **Compare and Connect** the strategies by allowing them some individual think time to process the ideas.

> **ASK** *Did all of these strategies involve plotting points? Did all of the strategies use the slope?*
>
> **LISTEN FOR** Some strategies plotted two or more points to draw the line. These strategies did not use the slope. Some strategies used the *y*-intercept to find one point and then used the slope to find one or more other points.

Analyze It & Graph It

If students presented these models, have students connect these models to those presented in class.

If no student presented at least one of these models, have students first analyze key features of the models, and then connect them to the models presented in class.

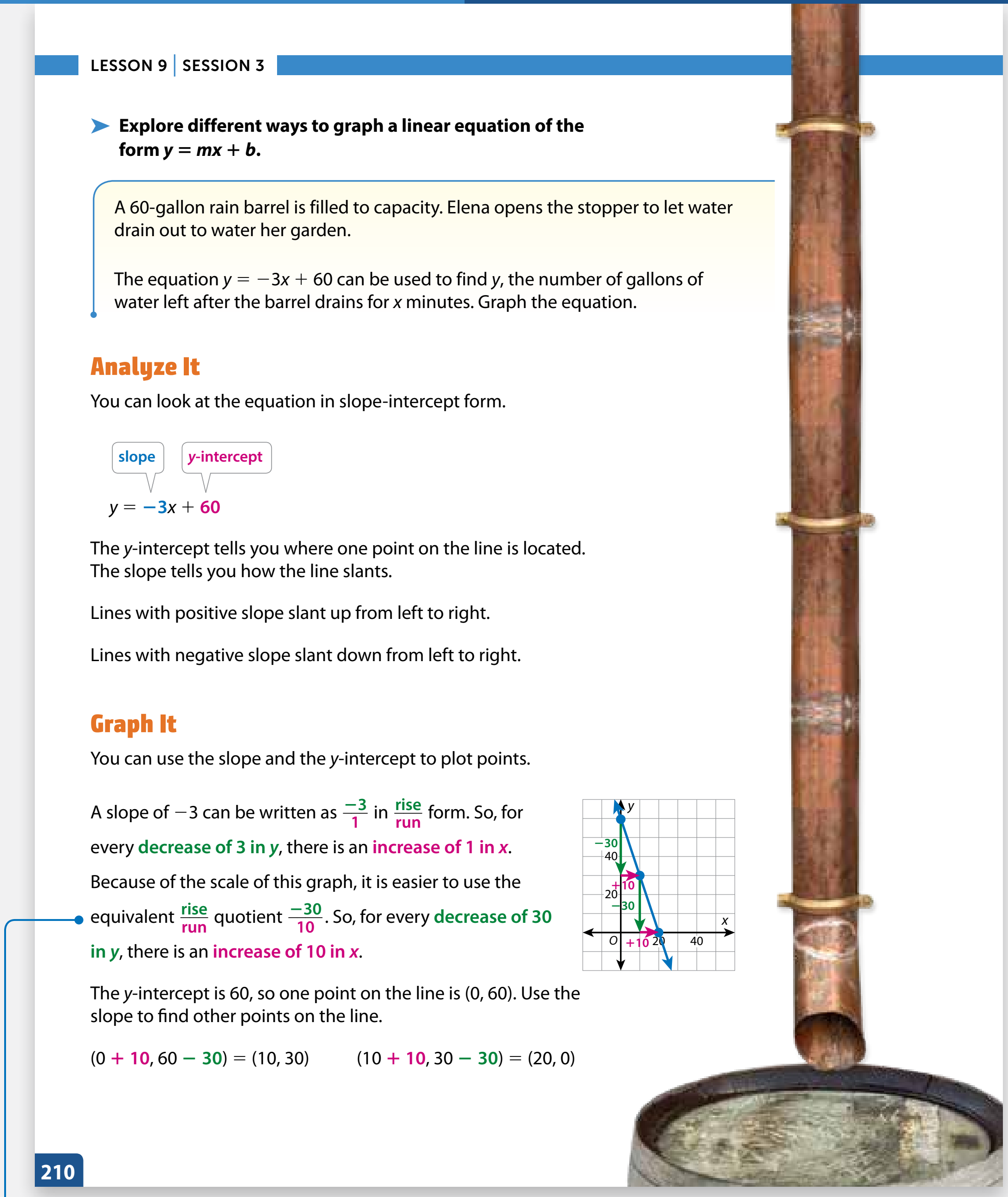
> **ASK** *What is different about the slope and appearance of this line, compared to the lines from the previous session?*
>
> **LISTEN FOR** The slope is negative. The line slants downward from left to right rather than upward.

For the equation in slope-intercept form, prompt students to identify the slope and *y*-intercept from the equation.

- *What is the slope? What is the y-intercept?*
- *How do you think the negative slope will affect the graph of the line?*

For the graph, prompt students to note how the slope and *y*-intercept are used to graph the line.

- *How is the slope written, and why do you think this is the case?*
- *How can you use the y-intercept and the slope to find another point on the line?*



LESSON 9 | SESSION 3

Explore different ways to graph a linear equation of the form $y = mx + b$.

A 60-gallon rain barrel is filled to capacity. Elena opens the stopper to let water drain out to water her garden.

The equation $y = -3x + 60$ can be used to find *y*, the number of gallons of water left after the barrel drains for *x* minutes. Graph the equation.

Analyze It

You can look at the equation in slope-intercept form.

slope
y-intercept

$y = -3x + 60$

The *y*-intercept tells you where one point on the line is located. The slope tells you how the line slants.

Lines with positive slope slant up from left to right.

Lines with negative slope slant down from left to right.

Graph It

You can use the slope and the *y*-intercept to plot points.

A slope of −3 can be written as $\frac{-3}{1}$ in $\frac{\text{rise}}{\text{run}}$ form. So, for every decrease of 3 in *y*, there is an increase of 1 in *x*.

Because of the scale of this graph, it is easier to use the equivalent $\frac{\text{rise}}{\text{run}}$ quotient $\frac{-30}{10}$. So, for every decrease of 30 in *y*, there is an increase of 10 in *x*.

The *y*-intercept is 60, so one point on the line is (0, 60). Use the slope to find other points on the line.

$(0 + 10, 60 - 30) = (10, 30)$ $(10 + 10, 30 - 30) = (20, 0)$

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DIFFERENTIATION | EXTEND

SMP 7

Deepen Understanding

Using Structure to Apply Slope in Equivalent Forms

Prompt students to consider how different equivalent forms of the slope relate to the graph of a line.

> **ASK** *In the quotient that represents the slope, the numerator is negative and the denominator is positive. Could you also use $\frac{3}{-1}$ or $\frac{30}{-10}$ to represent the slope? Why or why not?*
>
> **LISTEN FOR** Yes, regardless of whether the negative sign is written in the numerator or denominator, the value of the slope remains the same.

> **ASK** *How might using $\frac{30}{-10}$ impact how you calculate the coordinates of your points?*
>
> **LISTEN FOR** To find points, subtract 10 from the *x*-coordinate and add 30 to the *y*-coordinate of the point at the *y*-intercept. For example, $(0 - 10, 60 + 30) = (-10, 90)$, then $(-10 - 10, 90 + 30) = (-20, 120)$.

> **ASK** *Could you use these points to graph the line? Explain.*
>
> **LISTEN FOR** The points could be used to graph the line, but the graph's axes would have to be extended up and to the left so the points could be plotted.

Develop Graphing a Linear Equation of the Form $y = mx + b$

CONNECT IT

SMP 2, 4, 5, 6

Remind students that the quantities and the relationships between them are the same in each representation. Explain that they will now use those quantities and relationships to reason about how to graph a linear equation of the form $y = mx + b$.

Before students begin to record and expand on their work in Analyze It & Graph It, tell them that problems 1–3 will prepare them to provide the description asked for in problem 4.

Monitor and Confirm Understanding ① – ③

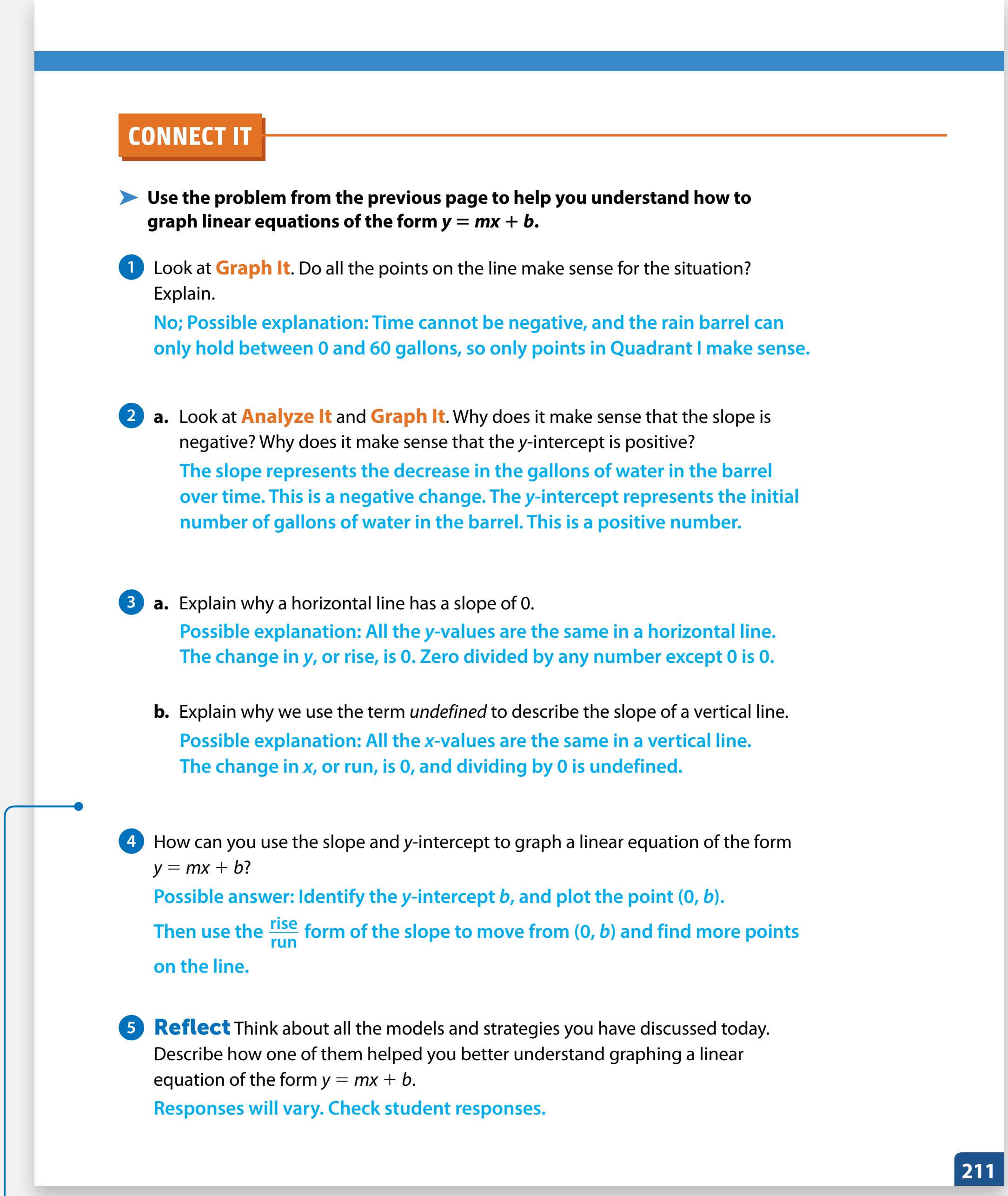
- Because time and the volume of water in the rain barrel cannot be negative, only the points in Quadrant I make sense for the situation.
- The slope is negative because the number of gallons of water in the rain barrel decreases as time goes by. The y-intercept is positive because there is water in the barrel to start.
- A horizontal line has a slope of 0 because there is no change in y as x increases. A vertical line has an undefined slope because the change in x (the run) is 0, and dividing by 0 is undefined. It is unknown whether the line is going straight up and has a positive slope, or is going straight down and has a negative slope.

Facilitate Whole Class Discussion

④ Look for the idea that if you are given the equation of a line in slope-intercept form, you can use the slope and y-intercept to graph the line in the coordinate plane.

> **ASK** *If you start by plotting a point at the y-intercept, how does the* $\frac{rise}{run}$ *form of the slope help you find more points on the line?*
>
> **LISTEN FOR** From the point at the y-intercept, use the rise to determine how far to move up or down and the run to determine how far to move right or left to plot the next point.

⑤ **Reflect** Have all students focus on the strategies used to solve the Try It. If time allows, have students discuss their ideas with a partner.



CONNECT IT

➤ **Use the problem from the previous page to help you understand how to graph linear equations of the form $y = mx + b$.**

① Look at **Graph It**. Do all the points on the line make sense for the situation? Explain.

No; Possible explanation: Time cannot be negative, and the rain barrel can only hold between 0 and 60 gallons, so only points in Quadrant I make sense.

② **a.** Look at **Analyze It** and **Graph It**. Why does it make sense that the slope is negative? Why does it make sense that the y-intercept is positive?

The slope represents the decrease in the gallons of water in the barrel over time. This is a negative change. The y-intercept represents the initial number of gallons of water in the barrel. This is a positive number.

③ **a.** Explain why a horizontal line has a slope of 0.

Possible explanation: All the y-values are the same in a horizontal line. The change in y, or rise, is 0. Zero divided by any number except 0 is 0.

b. Explain why we use the term *undefined* to describe the slope of a vertical line.

Possible explanation: All the x-values are the same in a vertical line. The change in x, or run, is 0, and dividing by 0 is undefined.

④ How can you use the slope and y-intercept to graph a linear equation of the form $y = mx + b$?

Possible answer: Identify the y-intercept b, and plot the point $(0, b)$. Then use the $\frac{rise}{run}$ form of the slope to move from $(0, b)$ and find more points on the line.

⑤ **Reflect** Think about all the models and strategies you have discussed today. Describe how one of them helped you better understand graphing a linear equation of the form $y = mx + b$.

Responses will vary. Check student responses.

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DIFFERENTIATION | RETEACH or REINFORCE

Visual Model

Use m and b to graph linear equations of the form $y = mx + b$.

If students are unsure about graphing equations of the form $y = mx + b$, then use this activity to help them understand the process.

Materials For display: large coordinate plane

- Display the equation $y = 2x + 4$. Tell students that they will graph this line.
- Ask: *What is the y-intercept of this equation?* [4] Ask: *How is the y-intercept shown on a graph?* [It is the y-coordinate of the point where the line crosses the y-axis.]
- Ask: *Where does the line cross the y-axis?* [(0, 4)] Invite a student to plot a point at (0, 4).
- Ask: *What is the slope?* [2] Ask: *How can you write this as* $\frac{rise}{run}$*?* $\left[\frac{2}{1}\right]$
- Show how to start at the point at the y-intercept and go up 2 units and right 1 unit. Plot (1, 6).
- Invite a student to show how to go from (1, 6) up 2 units and right 1 unit. Invite another to show how to go from (2, 8) up 2 units and right 1 unit.
- Have a student draw the line through the points.

Apply It

For problems that require making a graph, encourage students to take care in choosing a scale and labeling the axes.

6 Some students may need to sketch the graph in order to help them write their description. Remind students to describe the slope as change in units since there is no real-world context to which this equation applies.

7 This question allows students to demonstrate that they understand what the equations of horizontal and vertical lines look like, as well as how to graph them. Be sure students understand that the graph of $y = -1$ includes all points with a y-coordinate of -1, so it is a horizontal line that passes through -1 on the y-axis. The graph of $x = -1$ includes all points with an x-coordinate of -1, so it is a vertical line that passes through -1 on the x-axis.

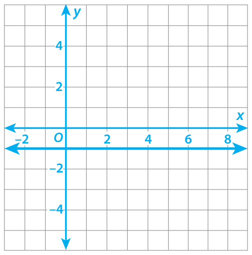
Apply It

➤ **Use what you learned to solve these problems.**

6 Describe what the graph of the equation $y = 50x + 125$ will look like.

The graph will be a straight line. The slope is positive, so the line will slant up from left to right, increasing 50 vertical units for every 1 horizontal unit. The *y*-intercept is 125, so the line will cross the *y*-axis at (0, 125).

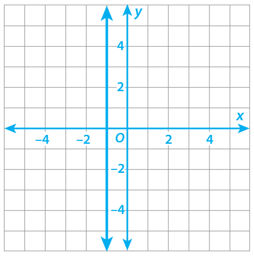
7 Graph the equations $y = -1$ and $x = -1$. What is the slope of each line? What is the y-intercept of each line?



$y = -1$

slope: 0

y-intercept: −1

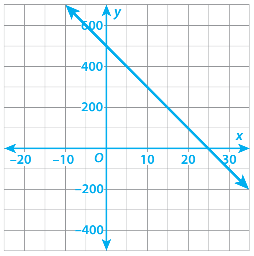


$x = -1$

slope: undefined

y-intercept: none

8 Graph the equation $y = -20x + 500$.



CLOSE **EXIT TICKET**

8 **Reflect** Students' solutions should show an understanding that:

- the y-intercept of the line is the value of b in $y = mx + b$, so in this case it is 500.
- the slope of the line is the value of m in $y = mx + b$, so in this case it is -20.
- a line with negative slope slants down from left to right.
- a line can be graphed by starting at the point at the y-intercept and using the slope to plot at least one additional point.

Error Alert If students graph a line that slants upward, ask them to identify the slope from the equation. Ask them what a negative slope tells them about the rise and the run. If necessary, point out that in the quotient $\frac{\text{rise}}{\text{run}}$, either the rise or the run must be negative, so the line will slant downward.

Practice Graphing a Linear Equation of the Form $y = mx + b$

Problem Notes

Assign **Practice Graphing a Linear Equation of the Form $y = mx + b$** as extra practice in class or as homework.

① Students should understand that the word *draining* indicates a negative value for slope. Students should recognize that since the slope is negative, the graph will slant down from left to right.

Students may use axis scales that are different from the sample answer. ***Medium***

Practice Graphing a Linear Equation of the Form $y = mx + b$

➤ **Study the Example showing how to graph a linear equation of the form $y = mx + b$. Then solve problems 1–4.**

Example

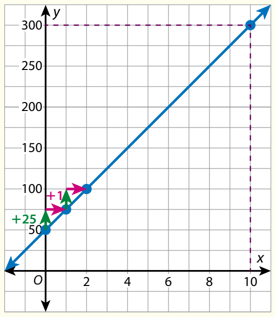
Mr. Díaz uses a hose to fill a kiddie pool with water. When full, the pool holds 300 gallons of water. The equation $y = 25x + 50$ can be used to find the number of gallons of water, y, in the pool x minutes after he turns on the hose. Graph the equation. How long does it take to fill the pool?

The y-intercept is 50, so the line intersects the y-axis at (0, 50). The slope is 25, or $\frac{25}{1}$. There is a vertical change of 25 for every horizontal change of 1.

$(0 + 1, 50 + 25) = (1, 75)$

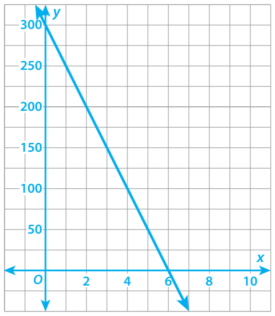
$(1 + 1, 75 + 25) = (2, 100)$

Plot the points and draw a line through them. The pool is filled when the number of gallons, y, is 300. This corresponds to an x-value of 10, so it takes 10 minutes to fill the pool.



① At the end of the day, Mr. Díaz drains the pool. The equation $y = -50x + 300$ can be used to find y, the number of gallons of water left after draining the pool for x minutes. Graph the equation. How long does it take to drain the pool? Explain.

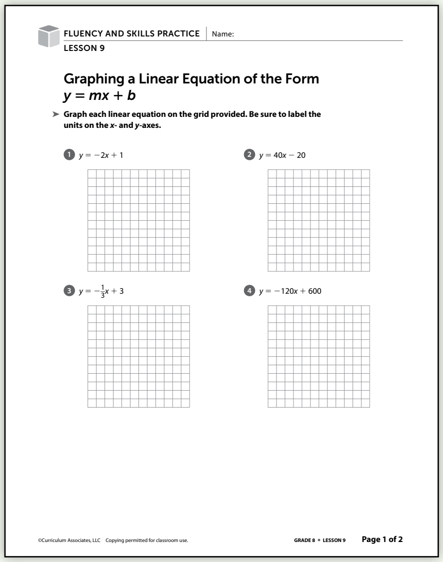
6 minutes; The pool is empty when the number of gallons, y, is 0. This happens when $x = 6$.



Fluency & Skills Practice

Graphing a Linear Equation of the Form $y = mx + b$

In this activity, students are given linear equations of the form $y = mx + b$ and a blank grid, and then they draw the x- and y-axes and graph the equation.



FLUENCY AND SKILLS PRACTICE | Name:
LESSON 9

Graphing a Linear Equation of the Form
$y = mx + b$

➤ Graph each linear equation on the grid provided. Be sure to label the units on the x- and y-axes.

① $y = -2x + 1$

② $y = 40x - 20$

③ $y = -\frac{1}{3}x + 3$

④ $y = -120x + 600$

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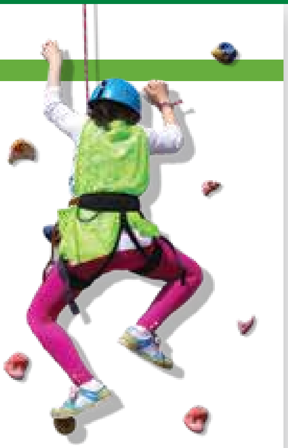
GRADE 8 • LESSON 9 Page 1 of 2

Additional Practice

② Students may find the slope and y-intercept directly from the equation because it is in $y = mx + b$ form. ***Basic***

③ Students may also write the slope as $\frac{1}{-2}$, with the negative sign in the denominator. The graph of the line will be the same no matter what form of the slope they use. ***Challenge***

④ Students may realize that the equation $y = 3$ represents all points with a y-coordinate of 3 and must be a horizontal line through 3 on the y-axis. Similarly, $x = 3$ represents all points with an x-coordinate of 3 and must be a vertical line through 3 on the x-axis. ***Medium***

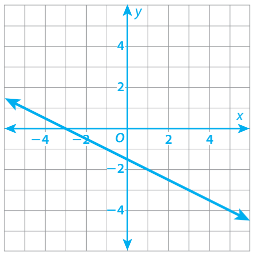


② Tameka signs up for membership at a rock climbing gym. She pays a one-time $100 membership fee. Then she will pay a $25 monthly fee. The equation $y = 25x + 100$ can be used to find y, the total cost of a gym membership for x months. What is the slope of the line? What is the y-intercept?

Slope = 25

y-intercept = 100

③ Graph the linear equation $y = -\frac{1}{2}x - 1.5$. Show your work.

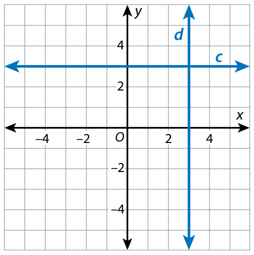


Possible work:

The y-intercept is −1.5, so the line intersects the y-axis at (0, −1.5).

$m = -\frac{1}{2}$, or $\frac{-1}{2}$, so move down 1 unit and right 2 units from the y-intercept to plot the next point.

④ Which line has the equation $y = 3$? Which has equation $x = 3$? Explain how you know.



Line c has equation $y = 3$. Line d has equation $x = 3$. Possible explanation: Line c has slope 0 and y-intercept 3, so its equation is $y = 0x + 3$, or just $y = 3$. Line d is made up of every point with an x-coordinate of 3. It has no y-intercept, and its slope is not defined, so it cannot be written in slope-intercept form. Its equation is $x = 3$.

DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Use with **Session 4** **Apply It**

Levels 1–3: Listening/Speaking

Read Apply It problem 8. Help students understand the scenario. Use pictures to illustrate *marine biologist, underwater, drone,* and *coral reef*. Ask: *What does the x-axis represent?* Repeat for the *y*-axis. Help students graph the equation. Provide sentence frames for students to share solutions with partners and tell whether they agree or disagree:

- *The slope of the line is _____.*
- *The y-intercept is _____.*

Then have partners describe the situation based on the graph:

- *The drone is _____ over time.*
- *The point (_____, _____) means _____.*
- *When x is 40, the drone is _____.*

Levels 2–4: Listening/Speaking

Have students read Apply It problem 8 with a partner. Ask them to circle unknown words or words they can use as they work on the problem. Help students create a **Co-Constructed Word Bank** with the math terms they circled. You may also need to discuss *underwater, drone,* and *coral reef.* Have partners graph the equation. Allow think time and then have them meet with other partners. Have students describe the situation based on the graph. Ask partners to say whether they agree or disagree with the descriptions:

- *I agree that the _____ means _____.*
- *I disagree because _____ does not mean _____. It means _____.*

Levels 3–5: Reading/Writing

Have students read Apply It problem 8. Allow them time to work independently to graph the line and answer the questions. Encourage them to use math terminology such as *slope, y-intercept,* and *quadrant.* Then have students turn to a partner to discuss the problem and compare answers.

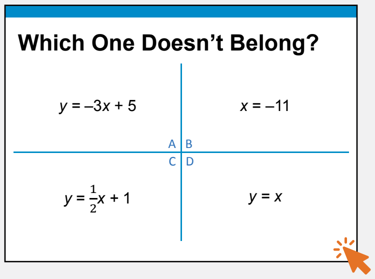
Encourage students to use *agree* and *because* to elaborate on their partner's responses or *disagree* and *because* to explain why they think the responses are unclear or incorrect.

Develop Graphing a Linear Equation Given in Any Form

Purpose

- **Develop** strategies for graphing a linear equation given in any form.
- **Recognize** that any linear equation can be graphed using two points that fall on the line.

START CONNECT TO PRIOR KNOWLEDGE



Which One Doesn't Belong?

A $y = -3x + 5$

B $x = -11$

C $y = \frac{1}{2}x + 1$

D $y = x$

Possible Solutions

A represents the only line with a negative slope.

B represents the only line with an undefined slope.

C represents the only line with a fractional slope.

D represents the only line that passes through the origin.

WHY? Support students' facility with analyzing linear equations.

DEVELOP ACADEMIC LANGUAGE

WHY? Guide students to be specific when they disagree with an idea.

HOW? Prompt students to show the parts of an idea or strategy that they disagree with and tell why they think it is incorrect. Encourage students to suggest corrections or improvement to the idea. It may be helpful to practice this interaction in a role-play using Apply It problem 7.

TRY IT

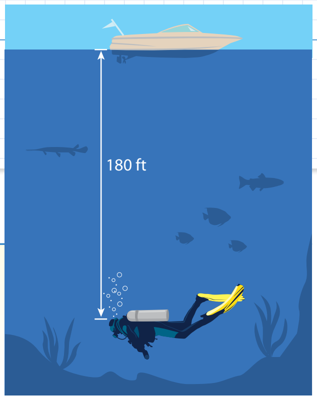
SMP 1, 2, 4, 5, 6

Make Sense of the Problem

See ***Connect to Culture*** to support student engagement. Before students work on Try It, use **Three Reads** to help students understand the problem. After the first read, ask what the problem is about. After the second read, ask a student what the problem is asking them to do. Finally, have students pair up. Have one partner read the problem to the other. Ask: *What are the important quantities and relationships in the problem?*

LESSON 9 | SESSION 4 ■ ■ ■ ■ □

Develop Graphing a Linear Equation Given in Any Form



➤ **Read and try to solve the problem below.**

A scuba diver dives to 180 feet below sea level. The linear equation $-60x + 2y = -360$ represents his trip back to the surface. The variable y is his elevation in feet relative to sea level after x minutes. Graph the equation.

TRY IT

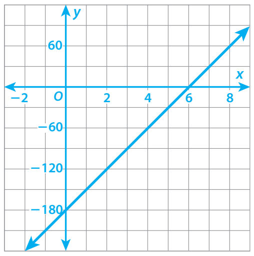
Math Toolkit graph paper, straightedges

Possible work:

SAMPLE A **See graph.**

Points on the line:

x	0	1	2	3
y	−180	−150	−120	−90



SAMPLE B **See graph.**

The graph will be a line, so I need two points.

$x = 0$:

$-60(0) + 2y = -360$

$2y = -360$

$y = -180$

$(0, -180)$

$x = 4$:

$-60(4) + 2y = -360$

$-240 + 2y = -360$

$2y = -120$

$y = -60$

$(4, -60)$

DISCUSS IT

Ask: How is your strategy similar to mine? How is it different?

Share: My strategy is similar to yours . . . It is different . . .

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DISCUSS IT

SMP 2, 3, 6

Support Partner Discussion

After students work on Try It, encourage them to respond to Discuss It with a partner. If students need support in getting started, prompt them to ask each other questions such as:

- *What prompted you to use your strategy?*
- *What did you do first to start graphing the equation?*

Common Misconception Listen for students who think that the coefficient of x is always the slope even if the equation is not in slope-intercept form. As students share their strategies, remind them that if they want to find the slope without writing the equation in $y = mx + b$ form, they could find two points that make the equation true and then use the slope formula.

Select and Sequence Student Strategies

Select 2–3 samples that represent the range of student thinking in your classroom. Here is one possible order for class discussion:

- table of values to find points on the graph
- (misconception) a point and a slope of –60 used to graph the line
- *x*- and *y*-intercepts calculated and their points plotted
- equation rewritten in slope-intercept form and graphed

Facilitate Whole Class Discussion

Call on students to share selected strategies. Remind students to be specific when explaining why they disagree with another student's idea.

Guide students to **Compare and Connect** the representations. Allow time for students to turn and talk with a partner.

> **ASK** *What quadrant of the graph is most important to show for this situation?*
>
> **LISTEN FOR** Only points in Quadrant IV make sense because time is never negative and the diver is always below or at the surface.

Model It & Solve It

If students presented these models, have students connect these models to those presented in class.

If no student presented at least one of these models, have students first analyze key features of the models, and then connect them to the models presented in class.

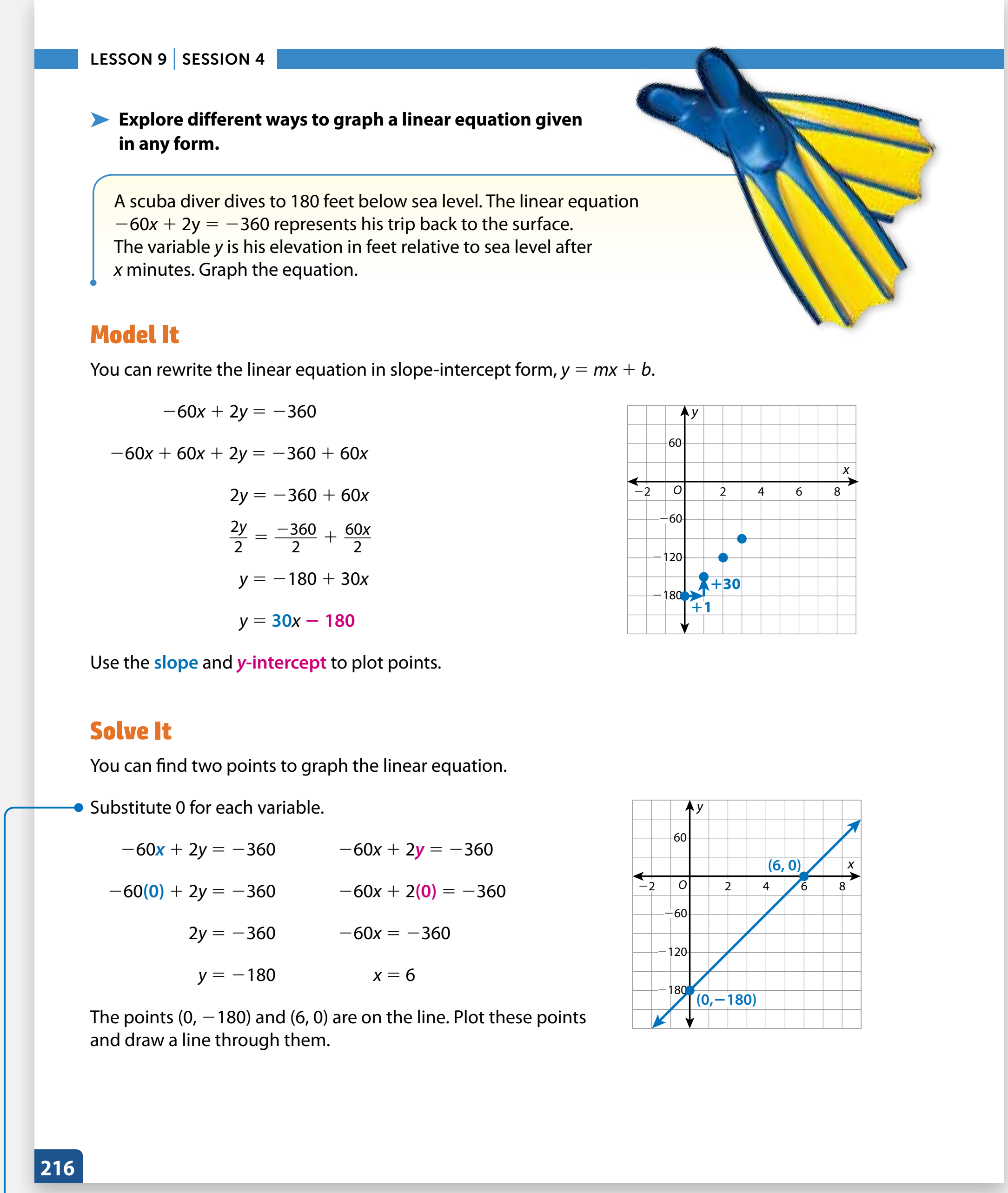
> **ASK** *The diver is below sea level. So why is the slope of the graph positive?*
>
> **LISTEN FOR** The slope represents the *change* in elevation. The diver's elevation is increasing as he gets closer to the surface, so the slope is positive.

For the graph generated from slope-intercept form, prompt students to examine the algebraic steps taken to rewrite the equation.

- *What are the y-intercept and slope?*
- *What do they represent in the context of the problem?*

For the graph generated from two points, prompt students to consider why this strategy is efficient.

- *Why is it efficient to use points with one coordinate equal to 0 when graphing the equation?*
- *What do each of these special points represent in the context of the problem?*



LESSON 9 | SESSION 4

➤ **Explore different ways to graph a linear equation given in any form.**

A scuba diver dives to 180 feet below sea level. The linear equation $-60x + 2y = -360$ represents his trip back to the surface. The variable *y* is his elevation in feet relative to sea level after *x* minutes. Graph the equation.

Model It

You can rewrite the linear equation in slope-intercept form, $y = mx + b$.

$$-60x + 2y = -360$$
$$-60x + 60x + 2y = -360 + 60x$$
$$2y = -360 + 60x$$
$$\frac{2y}{2} = \frac{-360}{2} + \frac{60x}{2}$$
$$y = -180 + 30x$$
$$y = 30x - 180$$

Use the slope and *y*-intercept to plot points.

Solve It

You can find two points to graph the linear equation.

Substitute 0 for each variable.

$$-60x + 2y = -360 \qquad -60x + 2y = -360$$
$$-60(0) + 2y = -360 \qquad -60x + 2(0) = -360$$
$$2y = -360 \qquad -60x = -360$$
$$y = -180 \qquad x = 6$$

The points (0, −180) and (6, 0) are on the line. Plot these points and draw a line through them.

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DIFFERENTIATION | EXTEND

SMP 5

Deepen Understanding

Using Appropriate Tools to Graph Equations

Prompt students to think about why substituting 0 for each variable is an efficient way to graph the line.

> **ASK** *Why does substituting 0 for one variable make it easy to find the value of the other variable?*
>
> **LISTEN FOR** Because 0 times any number is 0, and 0 added to any term is that term.

> **ASK** *How does using a point with 0 as one of its coordinates make the point easy to find on the graph?*
>
> **LISTEN FOR** The point is located on an axis.

> **ASK** *Suppose a line intersects both axes at the points (p, 0) and (0, q). How can you use these two points to write an equation of the line?*
>
> **LISTEN FOR** The slope, *m*, of the line will be $\frac{q-0}{0-p}$, or $-\frac{q}{p}$. The *y*-intercept, *b*, for the line will be *q*. So, an equation for the line will be $y = -\frac{q}{p}x + q$.

Develop Graphing a Linear Equation Given in Any Form

CONNECT IT

SMP 2, 4, 5, 6

Remind students that the quantities and the relationships between them are the same in each representation. Explain that they will now use those relationships to reason about how to graph a linear equation in any form.

Before students begin to record and expand on their work in Model It & Solve It, tell them that problem 3 will prepare them to provide the description asked for in problem 4.

Monitor and Confirm Understanding ① – ②

- Writing an equation in slope-intercept form makes it easy to identify the slope and *y*-intercept and use them to graph the line.
- When you substitute 0 for either *x* or *y*, you eliminate one of the terms, which makes it easier to solve for the other variable.

Facilitate Whole Class Discussion

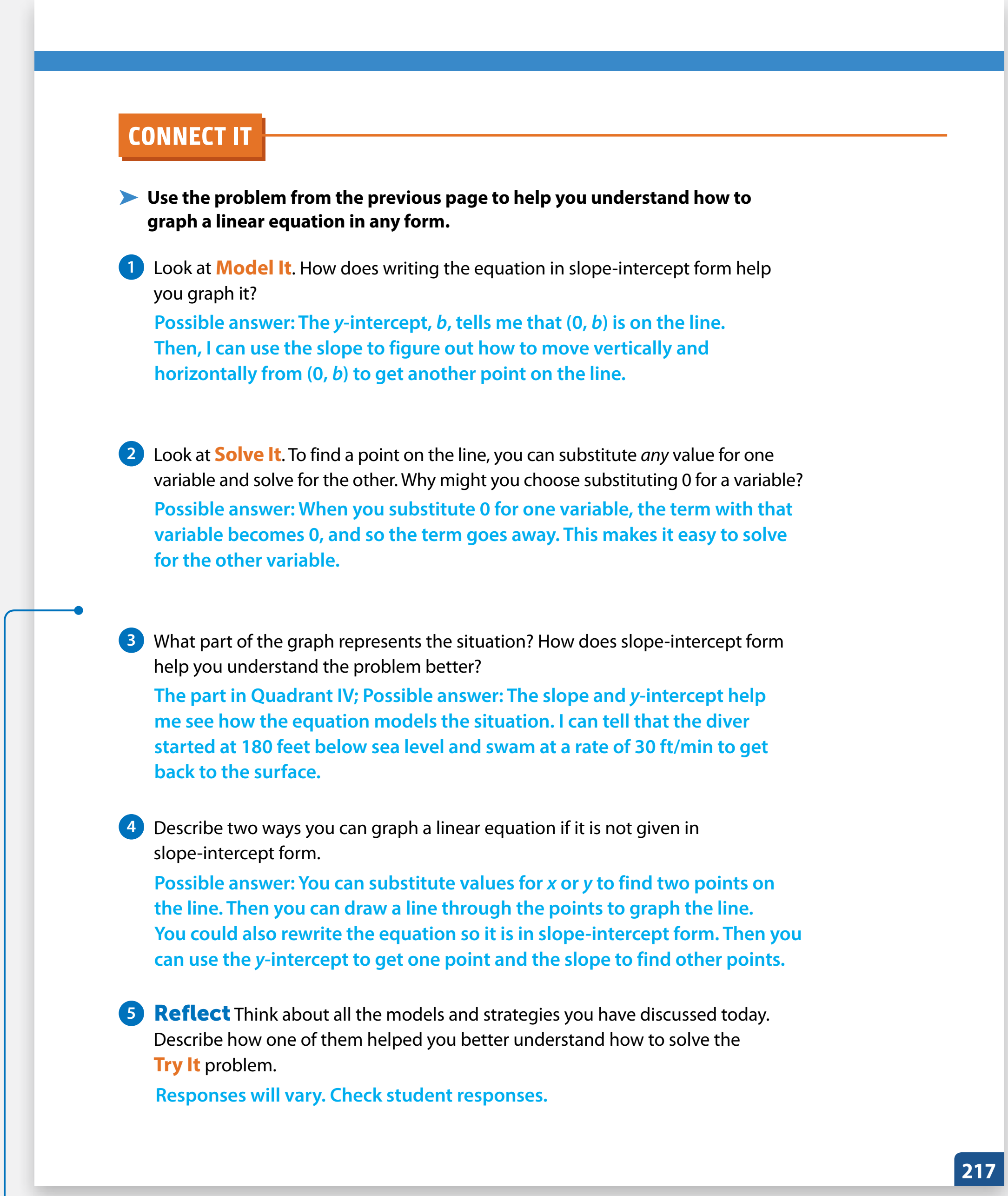
③ Students should recognize that negative values for time do not make sense in this context. Also, positive values for *y* do not make sense for the diver.

> **ASK** *The point (7, 30) is on the graph. What would this mean in terms of the context of the problem? Does this make sense?*
>
> **LISTEN FOR** At 7 minutes, the diver is 30 feet above the surface. This does not make sense because the diver can only be at or below the surface.

④ Look for the idea that a linear equation given in any form can be graphed in the coordinate plane using any two points that fall on the line.

> **ASK** *Which method for graphing a linear equation do you feel is most efficient and/or makes the most sense?*
>
> **LISTEN FOR** Students will probably gravitate to the method that is the easiest for them to understand. Any correct method is a correct response.

⑤ **Reflect** Have all students focus on the strategies used to solve the Try It. If time allows, have students discuss their ideas with a partner.



CONNECT IT

➤ **Use the problem from the previous page to help you understand how to graph a linear equation in any form.**

① Look at **Model It**. How does writing the equation in slope-intercept form help you graph it?

Possible answer: The *y*-intercept, *b*, tells me that (0, *b*) is on the line. Then, I can use the slope to figure out how to move vertically and horizontally from (0, *b*) to get another point on the line.

② Look at **Solve It**. To find a point on the line, you can substitute *any* value for one variable and solve for the other. Why might you choose substituting 0 for a variable?

Possible answer: When you substitute 0 for one variable, the term with that variable becomes 0, and so the term goes away. This makes it easy to solve for the other variable.

③ What part of the graph represents the situation? How does slope-intercept form help you understand the problem better?

The part in Quadrant IV; Possible answer: The slope and *y*-intercept help me see how the equation models the situation. I can tell that the diver started at 180 feet below sea level and swam at a rate of 30 ft/min to get back to the surface.

④ Describe two ways you can graph a linear equation if it is not given in slope-intercept form.

Possible answer: You can substitute values for *x* or *y* to find two points on the line. Then you can draw a line through the points to graph the line. You could also rewrite the equation so it is in slope-intercept form. Then you can use the *y*-intercept to get one point and the slope to find other points.

⑤ **Reflect** Think about all the models and strategies you have discussed today. Describe how one of them helped you better understand how to solve the **Try It** problem.

Responses will vary. Check student responses.

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DIFFERENTIATION | RETEACH or REINFORCE

Visual Model

Model equations in $y = mx + b$ form.

If students are unsure about graphing equations in slope-intercept form, then use this activity to provide them an opportunity to gain a better understanding.

Materials For display: large coordinate plane

- Display the equation $3y + 2x = 12$.
- Ask: *Why can't you immediately identify and use the slope and y-intercept to graph the equation?* [The equation is not in the form $y = mx + b$.]
- Ask: *When $x = 0$, what one-step equation results?* [$3y = 12$]
- Ask: *What is the value of y?* [4] *What ordered pair does that represent?* [(0, 4)]
- Invite a student to plot a point at (0, 4).
- Ask: *When $y = 0$, what one-step equation results?* [$2x = 12$]
- Ask: *What is the value of x?* [6] *What ordered pair does that represent?* [(6, 0)]
- Invite a student to plot a point at (6, 0).
- Invite a student to draw a line through both points.

Apply It

For all problems, encourage students to use a model to support their thinking.

⑥ Students may also graph the line by substituting to find the coordinates of two points.

⑦ Students should recognize that the equation should be written in slope-intercept form in order to be able to see the slope from the equation.

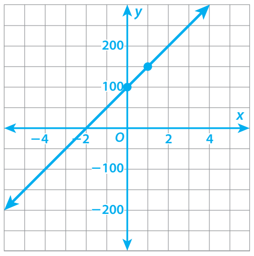
Apply It

➤ **Use what you learned to solve these problems.**

⑥ Graph the linear equation $-150x + 3y - 300 = 0$. Show your work.

Possible work: I can rewrite the equation in slope-intercept form and then use the *y*-intercept and slope to graph the line.

$$-150x + 3y - 300 = 0$$
$$3y = 150x + 300$$
$$y = 50x + 100$$



⑦ Kiara said the line with equation $28x - \frac{1}{2}y = -20$ has a slope of 28. What mistake did Kiara make?

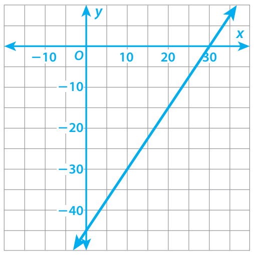
Possible answer: Kiara gave the coefficient of *x* as the slope, without first writing the equation in slope-intercept form. In the slope-intercept form, the equation is $y = 56x + 40$, so the slope is 56.

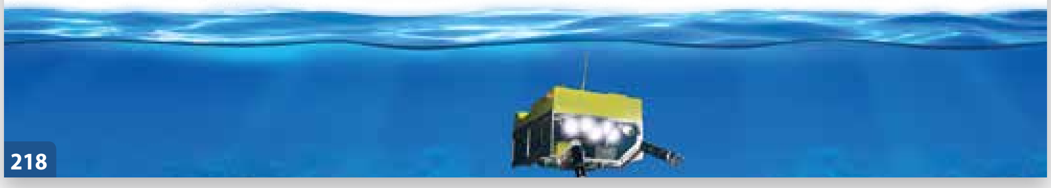
⑧ A marine biologist is using an underwater drone to study a delicate coral reef. The linear equation $20y - 30x = -900$ gives the drone's elevation, *y*, in meters from the surface of the water after *x* seconds. Graph the equation. What are the slope and *y*-intercept of the line? What part of the graph represents this situation?

The slope is $\frac{3}{2}$, or 1.5.

The *y*-intercept is −45.

The situation is represented in Quadrant IV of the graph.





CLOSE **EXIT TICKET**

⑧ Students' solutions should show an understanding of:

- graphing an equation that is not in slope-intercept form.
- identifying the slope and *y*-intercept of an equation that is not in slope-intercept form.
- choosing an appropriate scale to graph an equation.

Error Alert If students make an algebraic mistake when manipulating equations into $y = mx + b$ form, encourage them to look over their work and check the reasonableness of their solution. Selecting another point that makes the original equation true and making sure it is on the graph of the line is one way of doing this.

Practice Graphing a Linear Equation Given in Any Form

Problem Notes

Assign **Practice Graphing a Linear Equation Given in Any Form** as extra practice in class or as homework.

1. **a.** *Basic*

   b. Students identify the *y*-intercept and slope from their equation in part a or by using the graph in the example. *Basic*

2. Students may substitute both the *x*- and *y*-values for each point into the equation to see whether the resulting statements are true or false. *Challenge*

LESSON 9 | SESSION 4 Name:

Practice Graphing a Linear Equation Given in Any Form

➤ **Study the Example showing how to graph a linear equation given in any form. Then solve problems 1–4.**

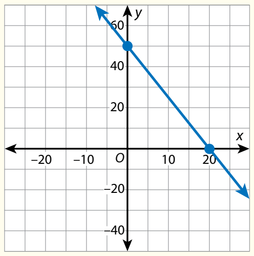
Example

Conan has some money to spend on gas for his car. The linear equation $5x + 2y = 100$ represents y, the amount of money he has left after buying x gallons of gas. Graph the equation. What part of the graph represents this situation?

Find two points on the line by substituting 0 for x and y.

$$\begin{aligned} 5(0) + 2y &= 100 \\ 2y &= 100 \\ y &= 50 \end{aligned} \qquad \begin{aligned} 5x + 2(0) &= 100 \\ 5x &= 100 \\ x &= 20 \end{aligned}$$

Plot (0, 50) and (20, 0) and draw a line through them. The situation is represented in Quadrant I of the graph.



① **a.** Write the equation from the Example in slope-intercept form.

$y = -\frac{5}{2}x + 50$

b. What is the slope of the line? What is the *y*-intercept? $-\frac{5}{2}$; 50

② Madison is reeling in her kite string at a steady rate. The linear equation $3y - 9x = -81$ can be used to find y, the number of feet of kite string she still needs to reel in after x seconds. Are the points $(0, -27)$ and $(-9, 0)$ on the line? Show your work.

Possible work:

$$\begin{aligned} 3(-27) - 9(0) &\overset{?}{=} -81 \\ -81 - 0 &\overset{?}{=} -81 \\ -81 &= -81 \end{aligned} \qquad \begin{aligned} 3(0) - 9(-9) &\overset{?}{=} -81 \\ 0 + 81 &\overset{?}{=} -81 \\ 81 &\neq -81 \end{aligned}$$

SOLUTION (0, −27) is on the line, but (−9, 0) is not.

Vocabulary

slope

for any two points on a line, the $\frac{\text{rise}}{\text{run}}$ or $\frac{\text{change in } y}{\text{change in } x}$.

slope-intercept form

a linear equation in the form $y = mx + b$, where m is the slope and b is the y-intercept.

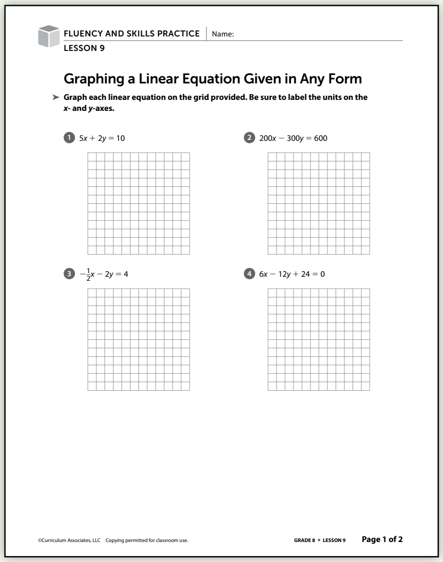
***y*-intercept**

the *y*-coordinate of the point where a line intersects the *y*-axis.

Fluency & Skills Practice

Graphing a Linear Equation Given in Any Form

In this activity, students are given linear equations in forms other than $y = mx + b$ and a blank grid, and then they draw the *x*- and *y*-axes and graph the equation.



FLUENCY AND SKILLS PRACTICE | Name:
LESSON 9

Graphing a Linear Equation Given in Any Form

➤ **Graph each linear equation on the grid provided. Be sure to label the units on the *x*- and *y*-axes.**

① $5x + 2y = 10$

② $200x - 300y = 600$

③ $-\frac{1}{2}x - 2y = 4$

④ $6x - 12y + 24 = 0$

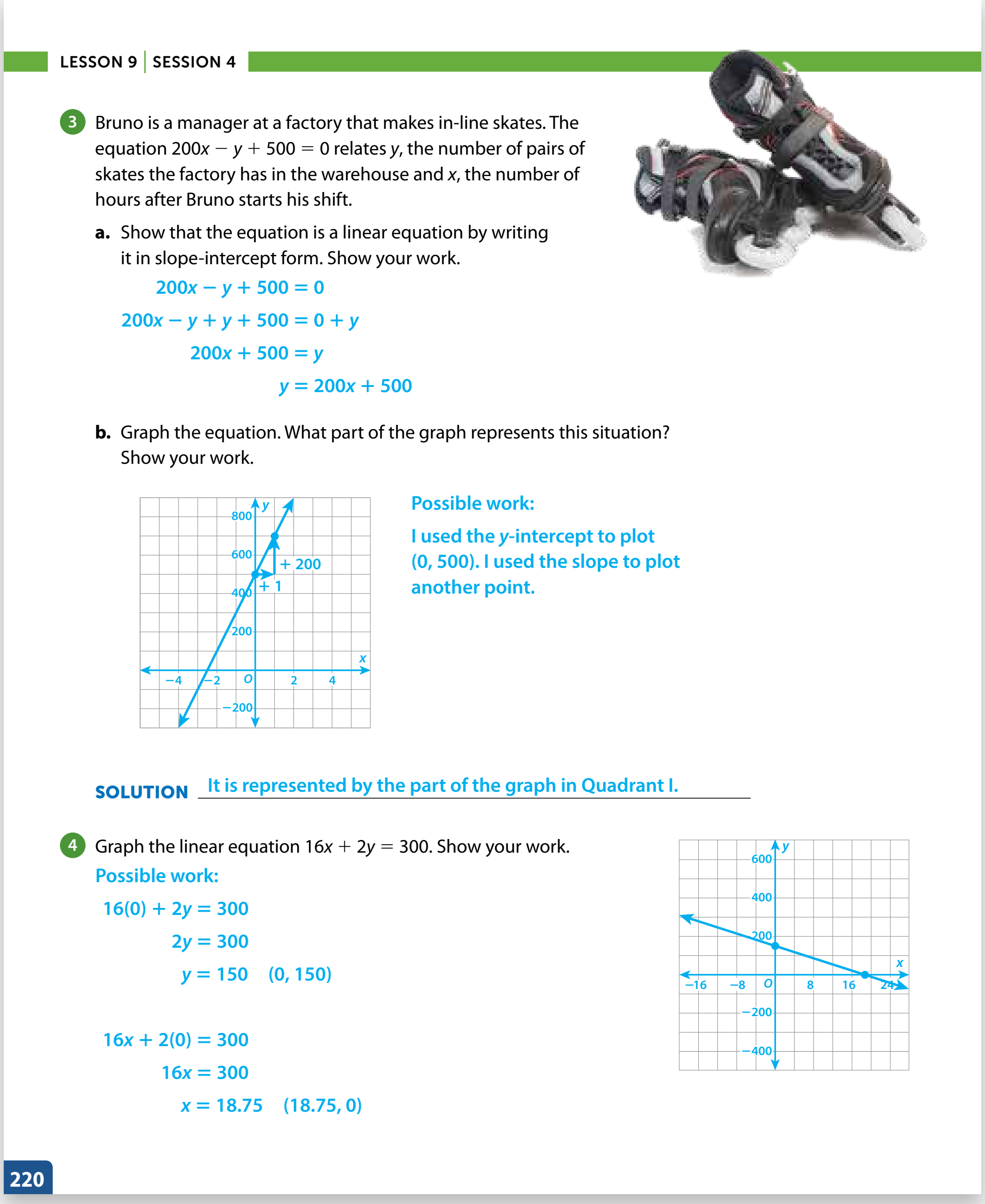
GRADE 8 • LESSON 9 Page 1 of 2

LESSON 9 | SESSION 4
Additional Practice

③ **a.** *Basic*

b. Students may instead plot the *y*-intercept at (0, 500) and the *x*-intercept at (−2.5, 0). ***Medium***

④ Students may rewrite the equation in slope-intercept form as $y = -8x + 150$ and use the slope and *y*-intercept to graph the equation. ***Medium***



③ Bruno is a manager at a factory that makes in-line skates. The equation $200x - y + 500 = 0$ relates *y*, the number of pairs of skates the factory has in the warehouse and *x*, the number of hours after Bruno starts his shift.

a. Show that the equation is a linear equation by writing it in slope-intercept form. Show your work.

$$200x - y + 500 = 0$$
$$200x - y + y + 500 = 0 + y$$
$$200x + 500 = y$$
$$y = 200x + 500$$

b. Graph the equation. What part of the graph represents this situation? Show your work.

Possible work:

I used the *y*-intercept to plot (0, 500). I used the slope to plot another point.

SOLUTION It is represented by the part of the graph in Quadrant I.

④ Graph the linear equation $16x + 2y = 300$. Show your work.

Possible work:

$$16(0) + 2y = 300$$
$$2y = 300$$
$$y = 150 \quad (0, 150)$$

$$16x + 2(0) = 300$$
$$16x = 300$$
$$x = 18.75 \quad (18.75, 0)$$

DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Use with **Session 5** **Apply It**

Levels 1–3: Speaking/Writing

Read Apply It problem 2 aloud as students follow along. Write the word *botanist* on the board. Point out that the Spanish word for *botanist* is a cognate: *botánica* or *botanico*. Invite Spanish-speaking students to pronounce the words and explain the endings -*a* (feminine) and -*o* (masculine).

Have students work with a partner to choose variables and explain what they represent. Provide the following sentence frames to help them write and discuss:

- *The first variable I chose is* ____. *It represents* ____. *The second variable I chose is* ____. *It represents* ____.
- *So, the equation is* ____.

Levels 2–4: Speaking/Writing

Read Apply It problem 2 with students. Write *botanist* on the board. Invite volunteers to tell its meaning and share cognates they know. Encourage students to tell what Spanish cognate they would use to talk about the botanist in the problem. (*botánico;* masculine) Ask: *How do you know?* (the pronoun *he* in the problem)

Have partners explain to each other what it means to define a variable. Then have them define their variables independently and write the equation of the line in slope-intercept form. Have partners read the equation and explain their variables. Encourage them to use complete sentences.

Levels 3–5: Speaking/Writing

Have students read Apply It problem 2. Invite students to discuss the meaning of *botanist* and share cognates they know. Create a chart with the headings: *English* and *Cognate*. Begin the chart with the word *botanist* and its cognates. Then have students suggest other words with cognates from the problem. When the chart is complete, invite volunteers to compare and pronounce the words.

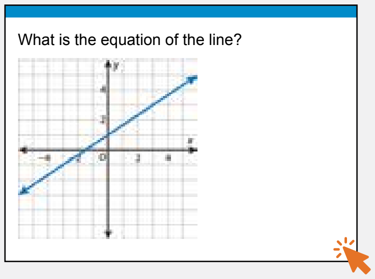
Have students turn to partners to read the problem and **Say It Another Way.** Encourage them to say what it means to define a variable. Then have them define their variables and share with other partners.

Refine Deriving and Graphing Linear Equations of the Form $y = mx + b$

Purpose

- **Refine** strategies for analyzing and graphing linear equations.
- **Refine** understanding of how to write an equation for a graphed line in slope-intercept form, how to graph linear equations in various forms, and how to interpret slope and y-intercept in context.

START CHECK FOR UNDERSTANDING


What is the equation of the line?

Solution

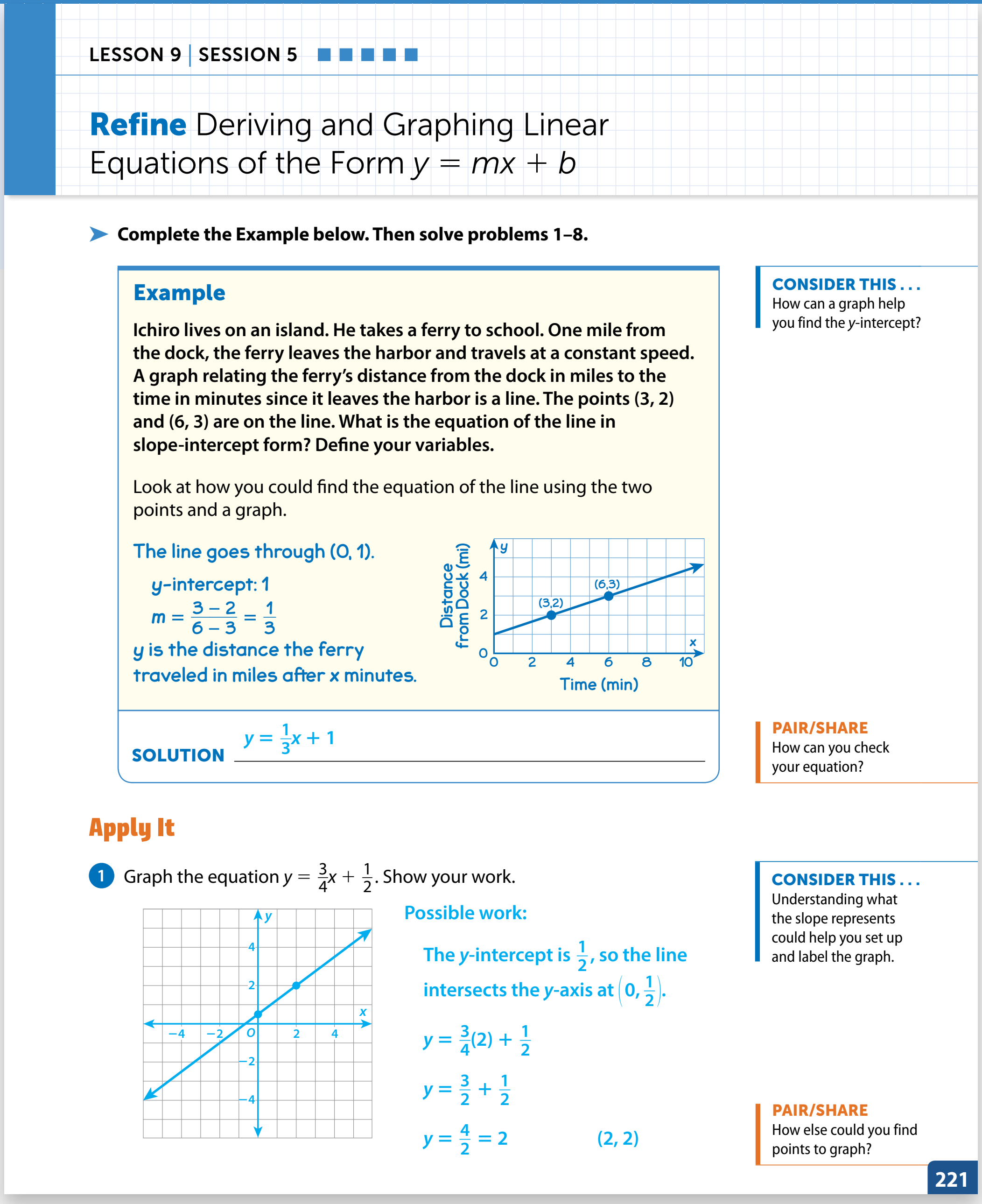
$y = \frac{2}{3}x + 1$

WHY? Confirm students' understanding of writing an equation for a graphed line in slope-intercept form, identifying common errors to address as needed.

MONITOR & GUIDE

Before students begin to work, use their responses to the **Start** to determine those who will benefit from additional support. Use the **Error Analysis** table below to guide remediation.

Have all students complete the Example and problems 1–3, using Consider This and Pair/Share as appropriate. Observe and monitor their reasoning and guide or redirect students as needed.



LESSON 9 | SESSION 5 ■■■■■

Refine Deriving and Graphing Linear Equations of the Form $y = mx + b$

➤ **Complete the Example below. Then solve problems 1–8.**

Example

Ichiro lives on an island. He takes a ferry to school. One mile from the dock, the ferry leaves the harbor and travels at a constant speed. A graph relating the ferry's distance from the dock in miles to the time in minutes since it leaves the harbor is a line. The points (3, 2) and (6, 3) are on the line. What is the equation of the line in slope-intercept form? Define your variables.

Look at how you could find the equation of the line using the two points and a graph.

The line goes through (0, 1).

y-intercept: 1

$m = \frac{3-2}{6-3} = \frac{1}{3}$

y is the distance the ferry traveled in miles after x minutes.

SOLUTION $y = \frac{1}{3}x + 1$

CONSIDER THIS . . . How can a graph help you find the y-intercept?

PAIR/SHARE How can you check your equation?

Apply It

1 Graph the equation $y = \frac{3}{4}x + \frac{1}{2}$. Show your work.

Possible work:

The y-intercept is $\frac{1}{2}$, so the line intersects the y-axis at $\left(0, \frac{1}{2}\right)$.

$y = \frac{3}{4}(2) + \frac{1}{2}$

$y = \frac{3}{2} + \frac{1}{2}$

$y = \frac{4}{2} = 2$ (2, 2)

CONSIDER THIS . . . Understanding what the slope represents could help you set up and label the graph.

PAIR/SHARE How else could you find points to graph?

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START ERROR ANALYSIS

If the error is . . .	Students may . . .	To support understanding . . .
$y = \frac{3}{2}x + 1$	have written the slope as $\frac{\text{run}}{\text{rise}}$ instead of $\frac{\text{rise}}{\text{run}}$.	Ask students to review the meaning of *slope*. Elicit from students that the slope is the vertical change divided by the horizontal change, or the rise divided by the run.
$y = x + \frac{2}{3}$	have confused the slope and y-intercept.	Ask students to review the equation in the form $y = mx + b$ by having them explain the meaning of the values of m and b.
$y = \frac{2}{3x} + 1$	have written x in the denominator of the fraction.	Ask students to consider $2x$, $2 \cdot x$, and $\frac{2}{x}$. Point out that the first two expressions are equivalent, but they are not equivalent to the last one. Multiplying by x is very different from dividing by x. Encourage students to write the x after the fraction, to show that the whole fraction is being multiplied by x.

Example

Guide students in understanding the Example. Ask:

- *How can extending a line through the points (3, 2) and (6, 3) help you identify the y-intercept?*
- *What are two different ways to identify the slope of the line?*
- *What is the unit of measurement for speed in this scenario? How can you use this to help you define your variables?*

Help all students focus on the Example and responses to the questions by asking them to critique classmates' responses.

Look for understanding that the slope formula can be used to find the slope of the line and that the *y*-intercept is the *y*-coordinate of the point where a line intersects the *y*-axis.

Apply It

1. Students may find the slope and the *y*-intercept from the equation and use them to plot points, or they may use substitution to find coordinates of two points on the line. *DOK 2*

2. Students may check their equations by choosing another point on the line, such as (10, 45). When they substitute 10 for *x*, they can confirm $y = 45$. *DOK 1*

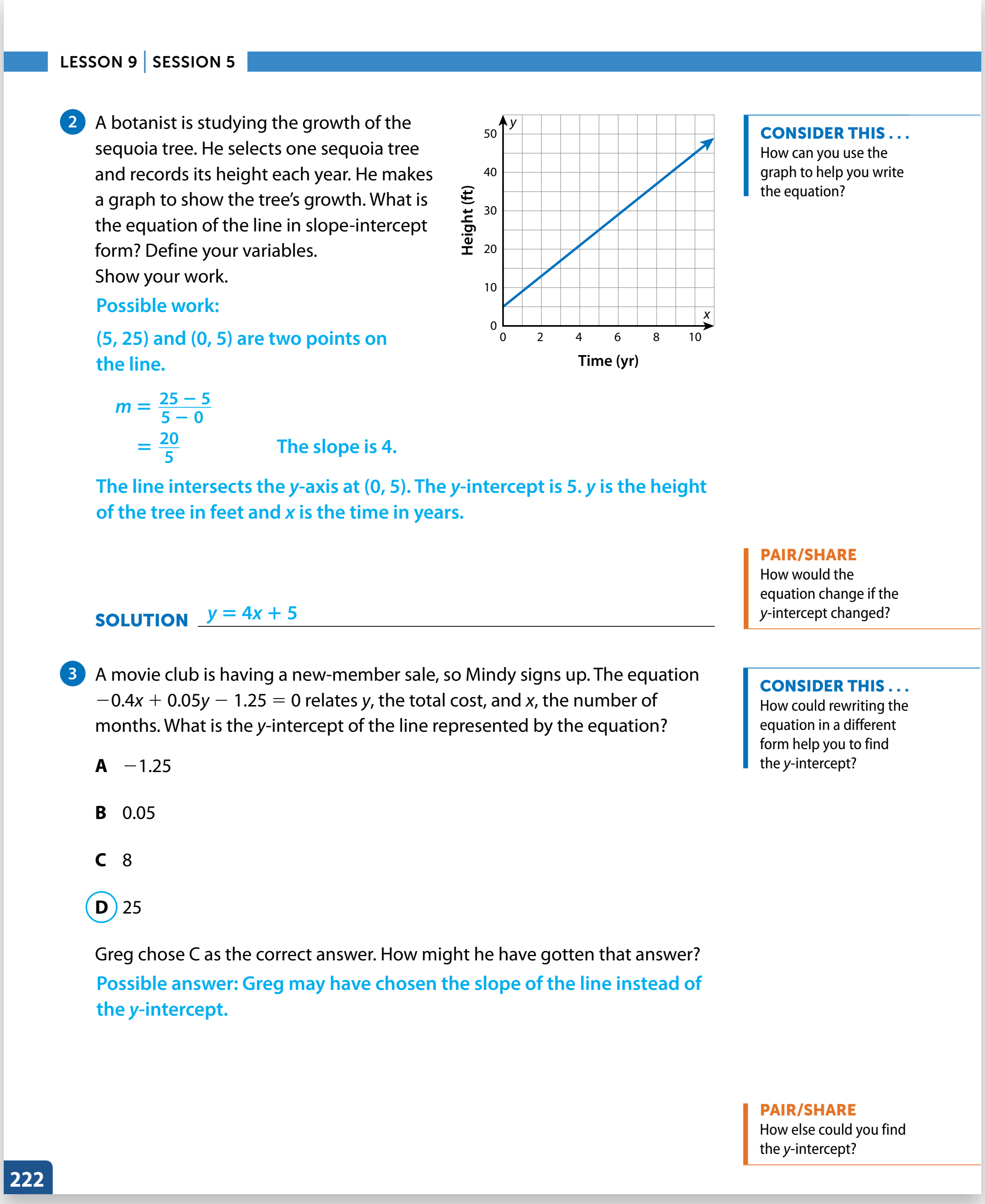
3. **D is correct.** Students may solve this problem by rewriting the equation in slope-intercept form, $y = mx + b$.

   A is not correct. This answer is the constant in the original equation.

   B is not correct. This answer is the coefficient of the variable *y* in the original equation.

   C is not correct. This answer is the slope.

   DOK 3



② A botanist is studying the growth of the sequoia tree. He selects one sequoia tree and records its height each year. He makes a graph to show the tree's growth. What is the equation of the line in slope-intercept form? Define your variables.
Show your work.

Possible work:

(5, 25) and (0, 5) are two points on the line.

$$m = \frac{25 - 5}{5 - 0} = \frac{20}{5}$$

The slope is 4.

The line intersects the *y*-axis at (0, 5). The *y*-intercept is 5. *y* is the height of the tree in feet and *x* is the time in years.

SOLUTION $y = 4x + 5$

CONSIDER THIS . . .
How can you use the graph to help you write the equation?

PAIR/SHARE
How would the equation change if the *y*-intercept changed?

③ A movie club is having a new-member sale, so Mindy signs up. The equation $-0.4x + 0.05y - 1.25 = 0$ relates *y*, the total cost, and *x*, the number of months. What is the *y*-intercept of the line represented by the equation?

A −1.25

B 0.05

C 8

[✓ **D**] 25

Greg chose C as the correct answer. How might he have gotten that answer?

Possible answer: Greg may have chosen the slope of the line instead of the *y*-intercept.

CONSIDER THIS . . .
How could rewriting the equation in a different form help you to find the *y*-intercept?

PAIR/SHARE
How else could you find the *y*-intercept?

GROUP & DIFFERENTIATE

Identify groupings for differentiation based on the **Start** and problems 1–3. A recommended sequence of activities for each group is suggested below. Use the resources on the next page to differentiate and close the lesson.

Approaching Proficiency
- **RETEACH** Visual Model
- **REINFORCE** Problems 4, 5, 6

Meeting Proficiency
- **REINFORCE** Problems 4–7

Extending Beyond Proficiency
- **REINFORCE** Problems 4–7
- **EXTEND** Challenge

Have all students complete the **Close: Exit Ticket.**

Resources for Differentiation are found on the next page.

Refine Deriving and Graphing Linear Equations of the Form $y = mx + b$

Apply It

④ Students may solve the problem by identifying two points on the line. Then they may find the quotient of the vertical change and the horizontal change to identify the slope. Finally, they may identify the *y*-intercept by examining the graph to see where the line intersects the *y*-axis. ***DOK 1***

⑤ The slope-intercept form of the equation is $y = 20x + 50$.

a. When the equation is in slope-intercept form, the coefficient of *x* is 20. So, the slope is 20.

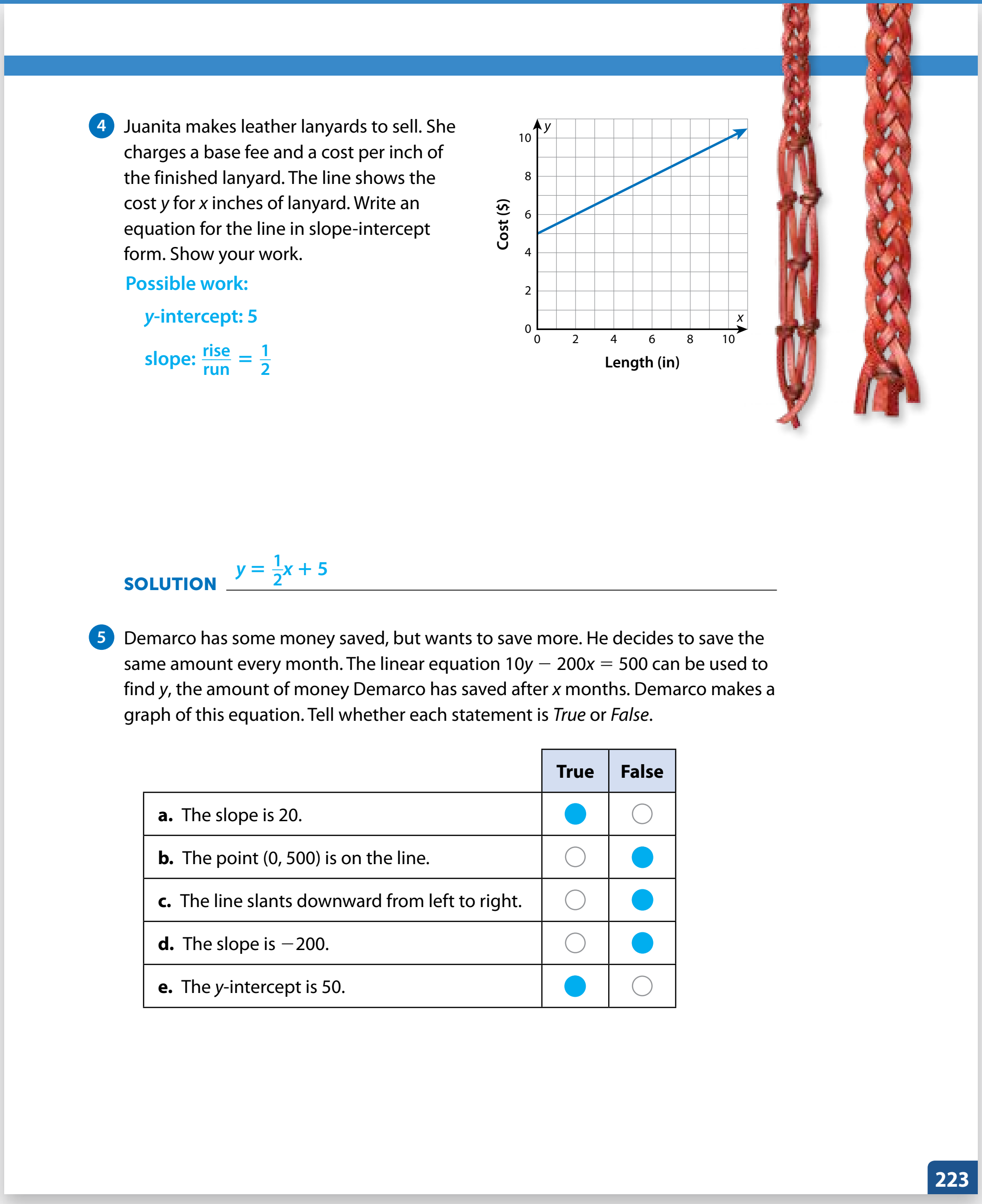
b. The line passes through (0, 50), not (0, 500).

c. The slope is positive, which means the line slants upward.

d. The coefficient of *x* in the initial equation is −200, but this equation is not in slope-intercept form.

e. When the equation is in slope-intercept form, the value of *b* is 50. So, the *y*-intercept is 50.

DOK 2



④ Juanita makes leather lanyards to sell. She charges a base fee and a cost per inch of the finished lanyard. The line shows the cost *y* for *x* inches of lanyard. Write an equation for the line in slope-intercept form. Show your work.

Possible work:

y-intercept: 5

slope: $\frac{\text{rise}}{\text{run}} = \frac{1}{2}$

SOLUTION $y = \frac{1}{2}x + 5$

⑤ Demarco has some money saved, but wants to save more. He decides to save the same amount every month. The linear equation $10y - 200x = 500$ can be used to find *y*, the amount of money Demarco has saved after *x* months. Demarco makes a graph of this equation. Tell whether each statement is *True* or *False*.

	True	False
a. The slope is 20.	●	○
b. The point (0, 500) is on the line.	○	●
c. The line slants downward from left to right.	○	●
d. The slope is −200.	○	●
e. The *y*-intercept is 50.	●	○

DIFFERENTIATION

RETEACH

Visual Model

Comparing linear equations and their graphs.

Students approaching proficiency with different forms of linear equations will benefit from manipulating and comparing their slopes and y-intercepts.

Materials For display: large coordinate plane

- Write the linear equation $y = 2x - 1$.
- Demonstrate how to graph the equation.
- Have a volunteer provide an equation that has the same slope but a different *y*-intercept. Then ask them to graph it for the class. (Repeat this process a couple more times using a variety of student volunteers.)
- Ask: *What do you notice about the graphs of these equations?* [The lines have the same slope and different *y*-intercepts. The lines are parallel.]
- Now start with a clean coordinate plane and write $y = -\frac{1}{2}x + 3$.
- Demonstrate how to graph the equation.
- Have a volunteer provide an equation that has a different slope but the same *y*-intercept. Then ask them to graph it for the class. (Repeat this process a couple more times using a variety of student volunteers.)
- Ask: *What do you notice about the graphs of these equations?* [The lines are not parallel, and they all cross the *y*-axis at the same point, (0, 3).]

LESSON 9 | SESSION 5

Refine

⑥ Students may identify the slope by looking at the equation. An equation of the form $y = mx$ has a slope of m. Horizontal lines have a slope of 0. Vertical lines have a slope that is undefined. *DOK 2*

⑦ Students may solve the problem by plotting the points and drawing a line through them. Then they could extend the line to find the point where the line intersects the y-axis. *DOK 3*

CLOSE EXIT TICKET

⑧ **Math Journal** Look for understanding that either two points or the slope and y-intercept (or another point) are needed to graph a linear equation. *DOK 3*

Error Alert If students can describe only one way to graph a linear equation, then have them review a problem in the Student Worktext, such as Refine problem 1, as a guide for graphing a linear equation.

☑ End of Lesson Checklist

INTERACTIVE GLOSSARY Support students by suggesting they write $y = 3x + 5$, where 3 and the label *slope* are written with a blue pencil, and 5 and the label *y-intercept* are written with a purple pencil. The colors will help them quickly identify the slope and y-intercept.

SELF CHECK Have students review and check off any new skills in the Unit 3 Opener.

⑥ The slope of the line represented by $y = 5x$ is 5.

The slope of the line represented by $y = 3$ is 0.

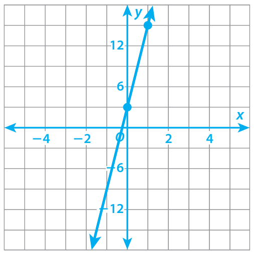
The slope of the line represented by $x = 4$ is undefined.

⑦ What is the y-intercept of a line that passes through the points (2, 7) and (6, 1)?

−	1	.	5		

⑧ **Math Journal** Write a linear equation. Describe two ways you can graph the equation. Then graph the equation.

Possible answer: $y = 12x + 3$; I can substitute values into the equation to find and plot two points on the line and then draw a line through them. Or, since the equation is in slope-intercept form, I can use the slope 12 and y-intercept 3. I can plot the point (0, 3) and then use the slope to move up 12 and to the right 1 from (0, 3) to plot another point on the line. Then I can connect these points with a line.



☑ End of Lesson Checklist

☐ **INTERACTIVE GLOSSARY** Find the entry for *slope-intercept form.* Sketch a graph of an equation in slope-intercept form.

☐ **SELF CHECK** Go back to the Unit 3 Opener and see what you can check off.

REINFORCE

Problems 4–7

Solve problems using slope-intercept form.

Students meeting proficiency will benefit from additional work with writing an equation for a graphed line in slope-intercept form and graphing linear equations in various forms by solving problems in a variety of formats.

- Have students work on their own or with a partner to solve the problems.
- Encourage students to show their work.

EXTEND

Challenge

Solve problems given a point and the slope.

Students extending beyond proficiency will benefit from writing an equation of a line given a point and the slope.

- Have students work with a partner to solve this problem: *What is the y-intercept of a line that goes through the point (−2, 4) and has a slope of −3?*
- Encourage students to plot the given point and then use the slope to plot other points on the line, where one of the points is the y-intercept.
- Repeat, this time having partners solve the following problem: *What is the y-intercept of a line that goes through the point (10, −2) and has a slope of* $-\frac{2}{3}$*?*

PERSONALIZE

i-Ready®

Provide students with opportunities to work on their personalized instruction path with *i-Ready* Online Instruction to:

- fill prerequisite gaps.
- build up grade-level skills.

Overview | Determine the Number of Solutions to One-Variable Equations

STANDARDS FOR MATHEMATICAL PRACTICE (SMP)

SMP 1, 2, 3, 4, 5, and 6 are integrated into the Try-Discuss-Connect routine.*

This lesson provides additional support for:

7 Look for and make use of structure.

8 Look for and express regularity in repeated reasoning.

* See page 1o to learn how every lesson includes these SMP.

Objectives

Content Objectives

- Identify equations with infinitely many solutions or no solution.
- Write equations that have exactly one solution, infinitely many solutions, or no solution.
- Determine what constant term or variable term to use to complete an equation for a given number of solutions.

Language Objectives

- Use math vocabulary and the phrases *true statement, false statement* to talk and write about equations with infinitely many solutions or no solution.
- Write or complete equations for a given number of solutions by comparing and interpreting terms and selecting the terms that will make a statement true or false.
- Explain why the completion of an equation for a given number of solutions is correct, using lesson vocabulary terms including *constant terms, coefficient, variable, expression, and solution*.
- Use precise language including math vocabulary and sequence words to describe and explain a strategy in response to questions about equations.

Prior Knowledge

- Use substitution to check whether a value is a solution to an equation.
- Solve one-variable equations with variables on both sides.

Vocabulary

Math Vocabulary

There is no new vocabulary. Review the following key terms.

distributive property multiplying each term in a sum or difference by a common factor does not change the value of the expression. For any numbers a, b, and c, $a(b + c) = ab + ac$.

expression a group of numbers, variables, and/or operation symbols that represents a mathematical relationship. An expression without variables, such as $3 + 4$, is called a *numerical expression*. An expression with variables, such as $5b^2$, is called an *algebraic expression*.

like terms two or more terms that have the same variable factors.

linear equation an equation whose graph is a straight line.

term a number, a variable, or a product of numbers, variables, and/or expressions. A term may include an exponent.

variable a letter that represents an unknown number. In some cases, a variable may represent more than one number.

Academic Vocabulary

in terms of in relationship to or in units named by.

infinitely many no end to the number of. When an equation has infinitely many solutions, any number is a solution.

Learning Progression

Earlier in Grade 8, students used the distributive property to simplify sides of equations as a step in solving one-variable equations. They also combined like terms and performed the same operation on both sides of an equation to solve these equations. Students used substitution to verify that a solution they found makes the original equation true.

➤

In this lesson, students learn that not all linear equations have one and only one solution. Some linear equations have infinitely many solutions, and some have no solution.

➤

Later in Grade 8, students will solve systems of linear equations, which can have no, one, or infinitely many solutions.

LESSON 11

Overview

Pacing Guide

Items marked with ↖ are available on the **Teacher Toolbox**.

	MATERIALS	DIFFERENTIATION
SESSION 1 **Explore** The Number of Solutions to One-Variable Linear Equations (35–50 min)		
• **Start** (5 min) • **Try It** (5–10 min) • **Discuss It** (10–15 min) • **Connect It** (10–15 min) • **Close: Exit Ticket** (5 min) **Additional Practice** (pages 251–252)	**Math Toolkit** algebra tiles, grid paper Presentation Slides ↖	**PREPARE** Interactive Tutorial ↖ **RETEACH or REINFORCE** Hands-On Activity **Materials** For each pair: algebra tiles (at least 15 *x*-tiles and 25 1-tiles)
SESSION 2 **Develop** Determining the Number of Solutions to One-Variable Equations (45–60 min)		
• **Start** (5 min) • **Try It** (10–15 min) • **Discuss It** (10–15 min) • **Connect It** (15–20 min) • **Close: Exit Ticket** (5 min) **Additional Practice** (pages 257–258)	**Math Toolkit** algebra tiles Presentation Slides ↖	**RETEACH or REINFORCE** Hands-On Activity **Materials** For each student: algebra tiles (at least 10 *x*-tiles and 10 1-tiles) **REINFORCE** Fluency & Skills Practice ↖ **EXTEND** Deepen Understanding
SESSION 3 **Develop** Writing an Equation with No, One, or Infinitely Many Solutions (45–60 min)		
• **Start** (5 min) • **Try It** (10–15 min) • **Discuss It** (10–15 min) • **Connect It** (15–20 min) • **Close: Exit Ticket** (5 min) **Additional Practice** (pages 263–264)	**Math Toolkit** algebra tiles Presentation Slides ↖	**RETEACH or REINFORCE** Hands-On Activity **Materials** For each student: algebra tiles (at least 10 *x*-tiles and 10 1-tiles), index card **REINFORCE** Fluency & Skills Practice ↖ **EXTEND** Deepen Understanding
SESSION 4 **Refine** Determining the Number of Solutions to One-Variable Equations (45–60 min)		
• **Start** (5 min) • **Monitor & Guide** (15–20 min) • **Group & Differentiate** (20–30 min) • **Close: Exit Ticket** (5 min)	**Math Toolkit** Have items from previous sessions available for students. Presentation Slides ↖	**RETEACH** Visual Model **REINFORCE** Problems 4–8 **EXTEND** Challenge **PERSONALIZE** i-Ready
Lesson 11 Quiz ↖ or **Digital Comprehension Check**		**RETEACH** Tools for Instruction ↖ **REINFORCE** Math Center Activity ↖ **EXTEND** Enrichment Activity ↖

LESSON 11

Overview | Determine the Number of Solutions to One-Variable Equations

Connect to Culture

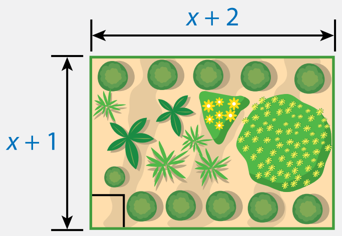
➤ **Use these activities to connect with and leverage the diverse backgrounds and experiences of all students. Engage students in sharing what they know about contexts before you add the information given here.**

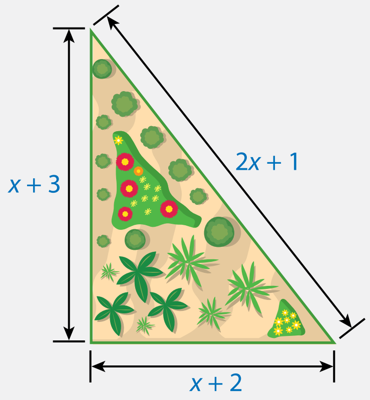
SESSION 2 ■ ■ □ □

Try It Ask students whether they have ever seen an animal known as a sloth. Have students familiar with this animal share what the animal looks like and how it behaves. Sloths have long claws, which makes it hard for them to walk, resulting in their slow movement. These animals are usually found hanging from branches of trees in tropical forests of South America and Central America, but can also be seen in a controlled environment such as an animal park. There are actually two varieties of sloths. They can be two-toed or three-toed. Sloths live between 20 and 30 years but can live longer in a controlled environment.

SESSION 3 ■ ■ ■ □

Practice **Problem 3** Gardening is a popular activity on Earth, but there is a garden in space as well! The "space garden" on the International Space Station is part of an experiment to develop efficient methods for growing vegetables that astronauts can safely eat in space. Being able to grow food in space will mean that less food needs to be carried from Earth on long missions. The experiment is also considering the psychological benefits of gardening for crew members, who may find it comforting and relaxing during long periods away from home. Ask students what vegetables they think should be grown in the "space garden."





SESSION 4 ■ ■ ■ ■

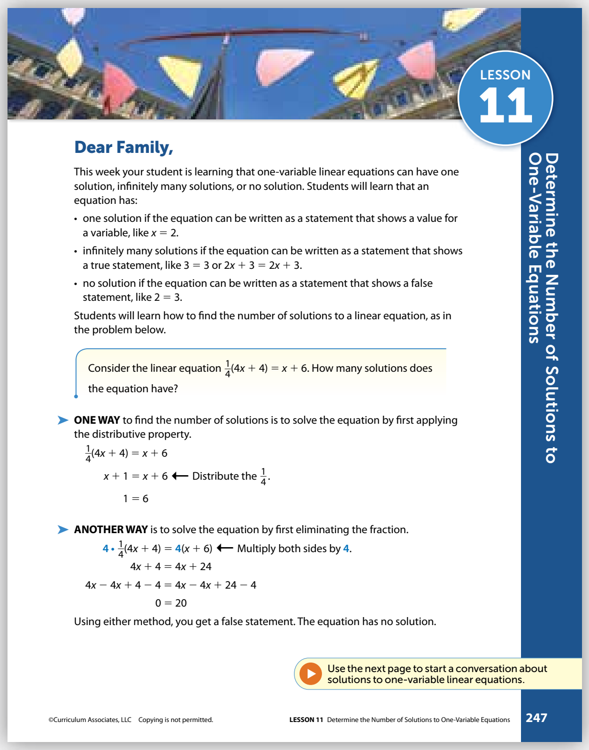
Apply It **Problem 7** The use of ribbon in arts and crafts is popular because of its different colors, textures, and widths. Ribbons are also an important part of a sport called rhythmic gymnastics. Some rhythmic gymnastics routines involve creating S and spiral shapes with a 6-meter-long satin ribbon. The most difficult moves involving tossing the ribbon in the air, performing a series of graceful, acrobatic movements, and then catching the ribbon before it falls to the ground. Ask students who have seen or participated in a rhythmic gymnastics event to share their experiences.

LESSON 11

Overview

Connect to Family and Community

➤ **After the Explore session, have students use the Family Letter to let their families know what they are learning and to encourage family involvement.**



LESSON 11

Determine the Number of Solutions to One-Variable Equations

Dear Family,

This week your student is learning that one-variable linear equations can have one solution, infinitely many solutions, or no solution. Students will learn that an equation has:

- one solution if the equation can be written as a statement that shows a value for a variable, like $x = 2$.
- infinitely many solutions if the equation can be written as a statement that shows a true statement, like $3 = 3$ or $2x + 3 = 2x + 3$.
- no solution if the equation can be written as a statement that shows a false statement, like $2 = 3$.

Students will learn how to find the number of solutions to a linear equation, as in the problem below.

> Consider the linear equation $\frac{1}{4}(4x + 4) = x + 6$. How many solutions does the equation have?

➤ **ONE WAY** to find the number of solutions is to solve the equation by first applying the distributive property.

$$\frac{1}{4}(4x + 4) = x + 6$$

$$x + 1 = x + 6 \longleftarrow \text{Distribute the } \tfrac{1}{4}.$$

$$1 = 6$$

➤ **ANOTHER WAY** is to solve the equation by first eliminating the fraction.

$$4 \cdot \frac{1}{4}(4x + 4) = 4(x + 6) \longleftarrow \text{Multiply both sides by } 4.$$

$$4x + 4 = 4x + 24$$

$$4x - 4x + 4 - 4 = 4x - 4x + 24 - 4$$

$$0 = 20$$

Using either method, you get a false statement. The equation has no solution.

Use the next page to start a conversation about solutions to one-variable linear equations.

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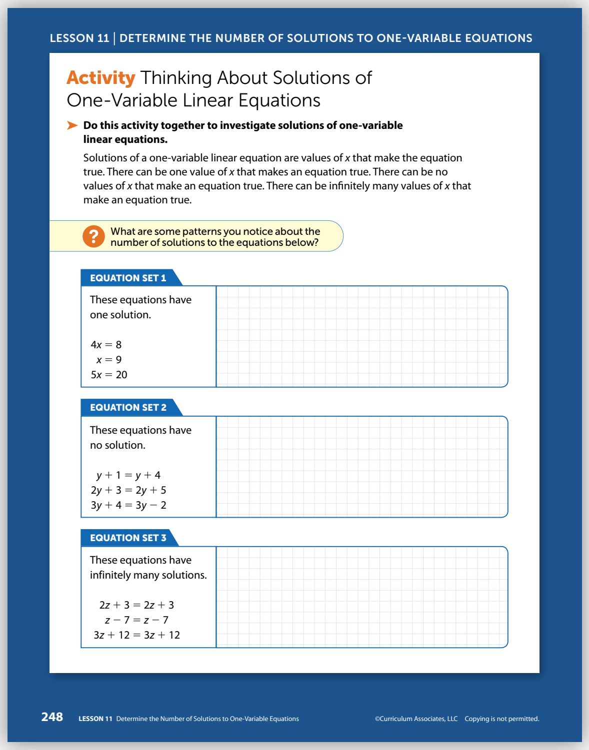

LESSON 11 | DETERMINE THE NUMBER OF SOLUTIONS TO ONE-VARIABLE EQUATIONS

Activity Thinking About Solutions of One-Variable Linear Equations

➤ **Do this activity together to investigate solutions of one-variable linear equations.**

Solutions of a one-variable linear equation are values of *x* that make the equation true. There can be one value of *x* that makes an equation true. There can be no values of *x* that make an equation true. There can be infinitely many values of *x* that make an equation true.

? What are some patterns you notice about the number of solutions to the equations below?

EQUATION SET 1

These equations have one solution.

$4x = 8$

$x = 9$

$5x = 20$

EQUATION SET 2

These equations have no solution.

$y + 1 = y + 4$

$2y + 3 = 2y + 5$

$3y + 4 = 3y - 2$

EQUATION SET 3

These equations have infinitely many solutions.

$2z + 3 = 2z + 3$

$z - 7 = z - 7$

$3z + 12 = 3z + 12$

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Connect to Language

➤ **For English language learners, use the Differentiation chart to scaffold the language in each session. Use the Academic Vocabulary routine for academic terms before Session 1.**

DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Use with **Session 1** **Connect It**

MATH TERMS

A *value* is a quantity.

A *variable* is a letter that represents one or more unknown numbers.

Levels 1–3: Reading/Speaking

To help students with the language in Connect It problem 2, read it aloud. Reread the first sentence. Ask what the statements $x = 5$ and $t = 17.8$ represent. Read the second sentence. Say it in a different way: *When you solve an equation and get one number for the variable, that means the value of that variable makes the equation true.* Have students give an example of an equation and the value of the variable that makes it true using that language. Before reading the next sentence explain that the word *however* usually means that what comes next is going to be different. Have students read to find out and discuss the definition of *infinitely many*.

Levels 2–4: Reading/Speaking

Read Connect It problem 2 with students. Display the lesson vocabulary *infinitely many*. Have students compare and discuss the words. Then display the Math Terms. Invite students to provide examples of variables and values. Other students can share cognates for the words.

Have students turn to a partner to work on problems 2a and 2b. Then ask partners to talk about the equations. Ask: *How many solutions does* $\frac{5x}{5} = \frac{2}{5}$ *have? How many solutions does* $9x - 5 = 9x - 5$ *have? When is each equation true?* Encourage partners to use terms *value, solution, true,* and *infinitely many* in their answers.

Levels 3–5: Reading/Speaking

Have students read Connect It problem 2. Have students read the definition of *infinitely many* and discuss. Have them read the problem again and explain the text of the problem in their own words, using the phrases *value of the variable, solution, infinitely many,* and *true*.

Have students answer problems 2a and 2b independently. Then have them meet with a partner again to compare and discuss answers. Encourage them to ask each other the following questions:

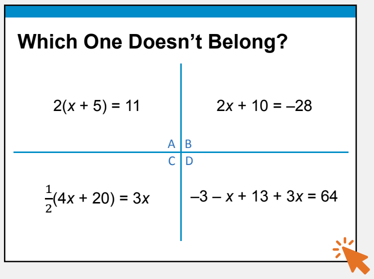
- *How many solutions does (equation) have? What does this mean?*

Explore The Number of Solutions to One-Variable Linear Equations

Purpose

- **Explore** the idea that one-variable equations can have infinitely many solutions.
- **Understand** that some linear equations in one variable have infinitely many values that make the equation true.

START CONNECT TO PRIOR KNOWLEDGE



Which One Doesn't Belong?

A $2(x + 5) = 11$

B $2x + 10 = -28$

C $\frac{1}{2}(4x + 20) = 3x$

D $-3 - x + 13 + 3x = 64$

Possible Solutions

A is the only equation with a noninteger solution.

B is the only equation with a negative solution.

C is the only equation with variables on both sides.

D is the only equation that involves combining like terms to simplify the left side.

WHY? Support students' facility with simplifying in order to solve linear equations.

TRY IT

SMP 1, 2, 4, 5, 6

Make Sense of the Problem

Before students work on Try It, use **Notice and Wonder** to help them make sense of the problem. Have them think about what might make this equation different from other equations they have solved.

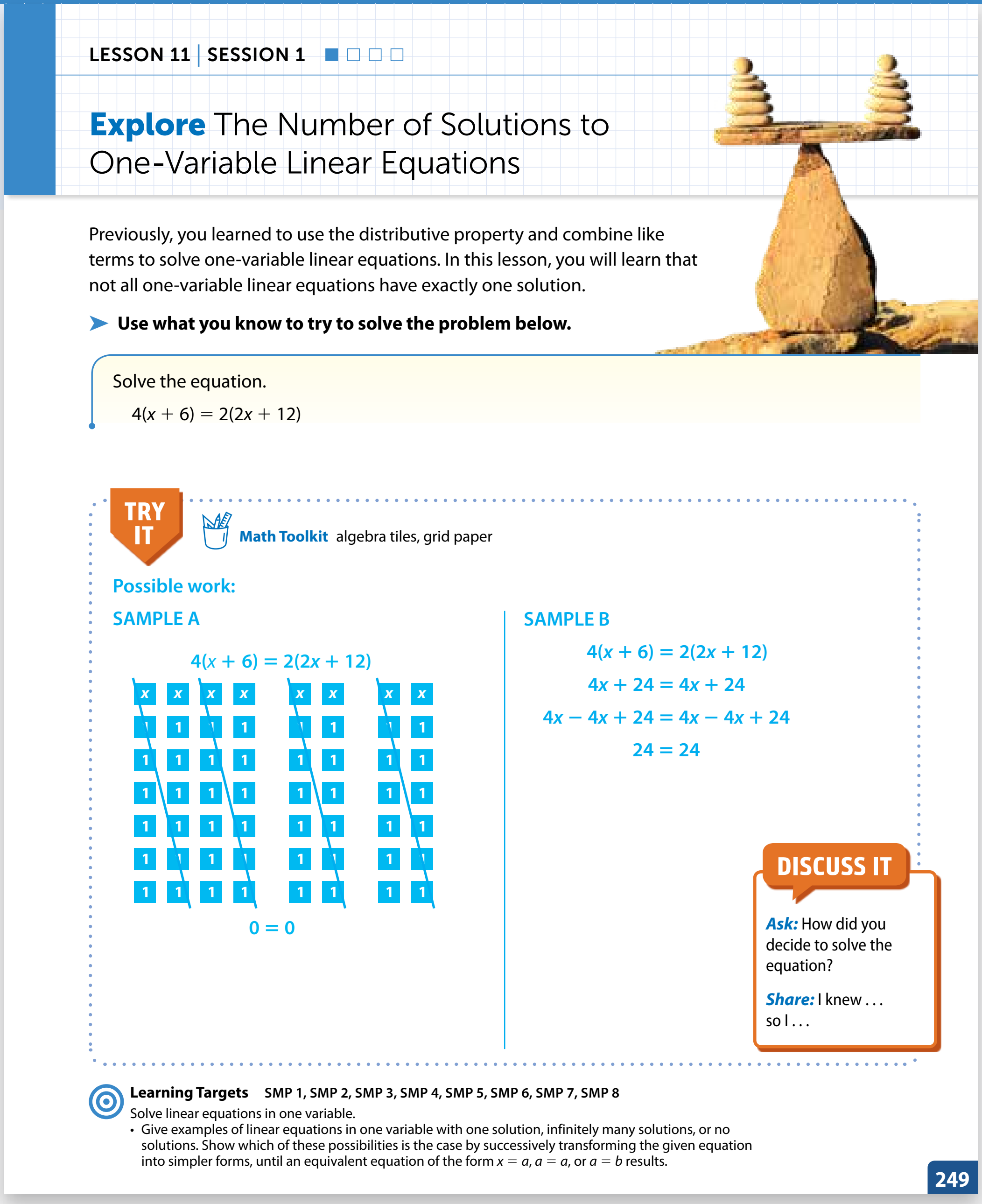
DISCUSS IT

SMP 2, 3, 6

Support Partner Discussion

After students work on Try It, have them respond to Discuss It with a partner. Listen for understanding of:

- using algebraic properties.
- performing operations on both sides of the equation.



LESSON 11 | SESSION 1 ■ □ □ □

Explore The Number of Solutions to One-Variable Linear Equations

Previously, you learned to use the distributive property and combine like terms to solve one-variable linear equations. In this lesson, you will learn that not all one-variable linear equations have exactly one solution.

➤ **Use what you know to try to solve the problem below.**

Solve the equation.

$4(x + 6) = 2(2x + 12)$

TRY IT Math Toolkit algebra tiles, grid paper

Possible work:

SAMPLE A

$4(x + 6) = 2(2x + 12)$

$0 = 0$

SAMPLE B

$4(x + 6) = 2(2x + 12)$

$4x + 24 = 4x + 24$

$4x - 4x + 24 = 4x - 4x + 24$

$24 = 24$

DISCUSS IT

Ask: How did you decide to solve the equation?

Share: I knew . . . so I . . .

Learning Targets SMP 1, SMP 2, SMP 3, SMP 4, SMP 5, SMP 6, SMP 7, SMP 8

Solve linear equations in one variable.

- Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results.

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Common Misconception Listen for students who think that because all of the terms can be eliminated, the equation has no solution. As students share their strategies, elicit that because $0 = 0$ is always true, the original equation must always be true. Encourage them to substitute several values of x and confirm that the equation is true for those values.

Select and Sequence Student Strategies

Select 2–3 samples that represent the range of student thinking in your classroom. Here is one possible order for class discussion:

- algebra tiles
- (misconception) conclusion that the equation has no solution
- hanger diagram
- algebraic manipulation

Facilitate Whole Class Discussion

Call on students to share selected strategies. Remind students to project their voices while they explain how they solved the problem.

Guide students to **Compare and Connect** the representations. Use turn and talk to help students think through their responses before sharing with the group.

> **ASK** *How is solving this equation different from solving equations you have worked with previously?*
>
> **LISTEN FOR** When this equation is solved, the variable term is eliminated.

CONNECT IT

SMP 2, 4, 5

① **Look Back** Look for understanding that after the x-terms are eliminated, the equation shows a number equal to itself. Any number substituted for x makes the equation true.

DIFFERENTIATION | RETEACH or REINFORCE

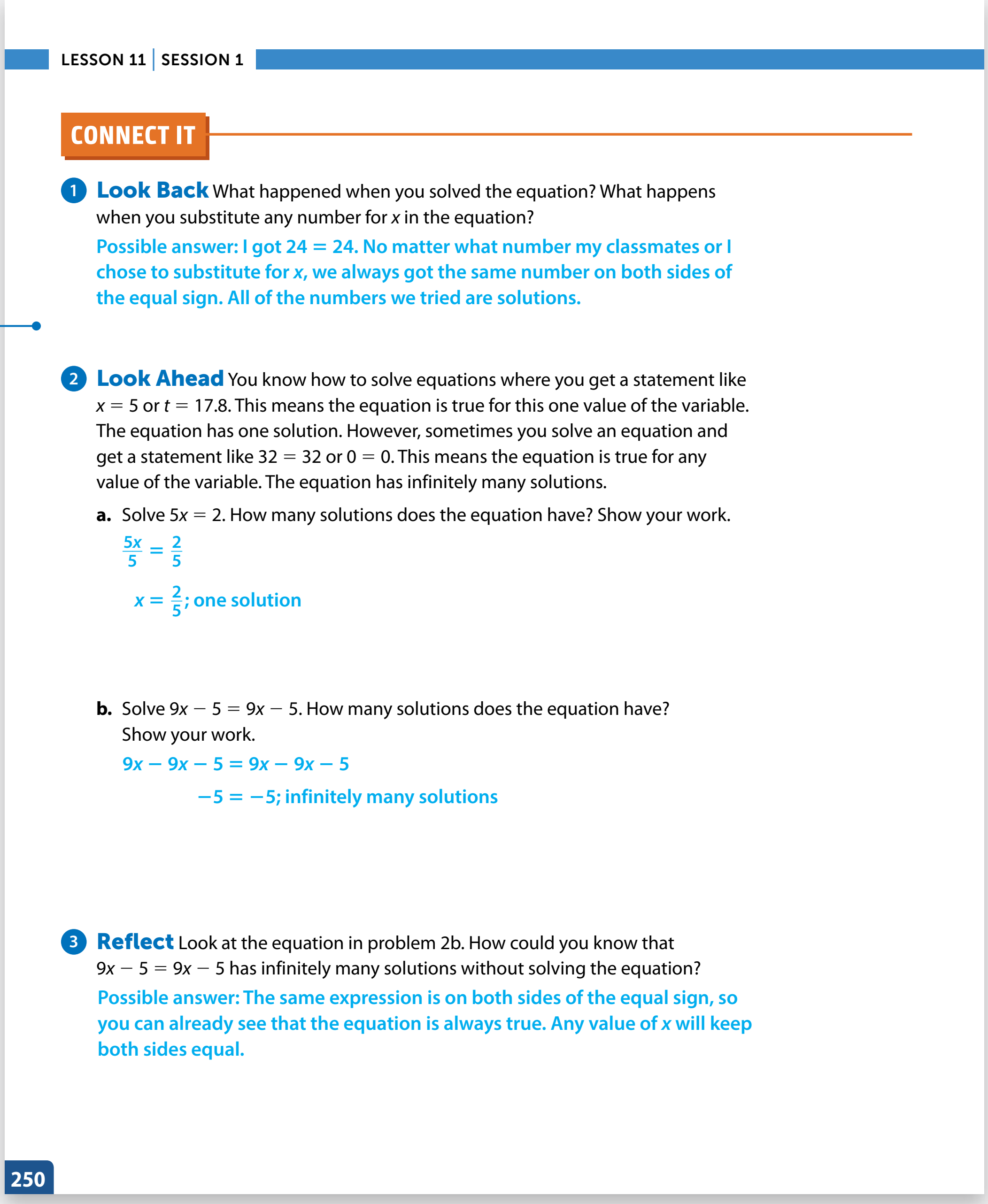
Hands-On Activity

Use algebra tiles to solve an equation in which the variable is eliminated.

If students are unsure about how to solve equations in which the variable gets eliminated, then use this activity to help them gain confidence.

Materials For each pair: algebra tiles (at least 15 x-tiles and 25 1-tiles)

- Display the equation $3(2x + 4) = 6(x + 2)$.
- Ask: *How can you model this equation with algebra tiles?* [Show 3 groups of $2x + 4$ on the left side of the mat and 6 groups of $x + 2$ on the right side.]
- Have pairs make the model, one student per side.
- Ask: *How many x-tiles are on each side?* [6]
- Instruct each student to remove 6 x-tiles from their side of the equation.
- Ask: *What equation represents the algebra tiles that remain?* [$12 = 12$]
- Reinforce that this means there are infinitely many solutions for x.
- Have students consider the original equation, $3(2x + 4) = 6(x + 2)$ and verify that any number chosen for x will make the equation true.
- Instruct one student to choose any number. Have one partner substitute this number into the left side of the equation. Have the other partner substitute this number into the right side. Partners should simplify their side and confirm that their simplified sides match.
- Repeat, having the other partner choose an arbitrary number for x.



LESSON 11 | SESSION 1

CONNECT IT

① **Look Back** What happened when you solved the equation? What happens when you substitute any number for x in the equation?

Possible answer: I got $24 = 24$. No matter what number my classmates or I chose to substitute for x, we always got the same number on both sides of the equal sign. All of the numbers we tried are solutions.

② **Look Ahead** You know how to solve equations where you get a statement like $x = 5$ or $t = 17.8$. This means the equation is true for this one value of the variable. The equation has one solution. However, sometimes you solve an equation and get a statement like $32 = 32$ or $0 = 0$. This means the equation is true for any value of the variable. The equation has infinitely many solutions.

a. Solve $5x = 2$. How many solutions does the equation have? Show your work.

$$\frac{5x}{5} = \frac{2}{5}$$

$x = \frac{2}{5}$; one solution

b. Solve $9x - 5 = 9x - 5$. How many solutions does the equation have? Show your work.

$$9x - 9x - 5 = 9x - 9x - 5$$

$-5 = -5$; infinitely many solutions

③ **Reflect** Look at the equation in problem 2b. How could you know that $9x - 5 = 9x - 5$ has infinitely many solutions without solving the equation?

Possible answer: The same expression is on both sides of the equal sign, so you can already see that the equation is always true. Any value of x will keep both sides equal.

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② **Look Ahead** Point out that when you solve an equation and the variables are eliminated and the constants are the same, the equation has infinitely many solutions. Students should recognize that having infinitely many solutions means that any value of x makes the equation true.

CLOSE **EXIT TICKET**

③ **Reflect** Look for understanding that when two equivalent expressions form an equation, there are infinitely many solutions to the equation.

Common Misconception If students think that a resulting statement such as $-5 = -5$ indicates that the solution is $x = -5$, then have them substitute other values for x into the equation. Students will see that a true statement is formed each time. Explain the difference between resulting statements of $x = -5$ and $-5 = -5$. Stress that if the statement $-5 = -5$ is true for any value of x, then the equation has infinitely many solutions.

Prepare for Determining the Number of Solutions to an Equation

Support Vocabulary Development

Assign **Prepare for Determining the Number of Solutions to an Equation** as extra practice in class or as homework.

If you have students complete this in class, then use the guidance below.

Ask students to consider the term *expression*. They should be able to distinguish an expression from an equation and to identify the characteristics of an expression.

Have students work in pairs to complete the graphic organizer. Invite pairs to share their completed organizers and prompt a whole-class comparative discussion of the definitions and examples given.

Have students look at the expressions in problem 2 and discuss with a partner the difference between an expression and an equation.

Problem Notes

① Students should understand that an expression has numbers and/or variables that are added, subtracted, multiplied, or divided and that represent a mathematical relationship. Student responses might include that an expression can have one term or more than one term.

② Students should recognize that the difference between an expression and an equation is the inclusion of an equal sign.

Prepare for Determining the Number of Solutions to an Equation

① Think about what you know about expressions in mathematical statements.
Fill in each box. Use words, numbers, and pictures. Show as many ideas as you can.
Possible answers:

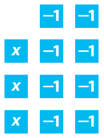
What Is It?
a group of numbers and/or variables that are added, subtracted, multiplied, or divided and that represent a mathematical relationship

What I Know About It
There are numerical expressions and algebraic expressions.
Numerical expressions do not have variables.
Algebraic expressions can have variables.
Expressions do not have =.

expression

Examples
$4x - 3$
$\frac{1}{3}m$

Examples
$3x - 8$



Examples
$4 - 6$
$5 + a$

② Which of the following are expressions? Circle your answers.

[✓ $4x + 7$] $6b + 1 = 13$

$192 \div 8 = 24$ [✓ $y - 17$]

REAL-WORLD CONNECTION

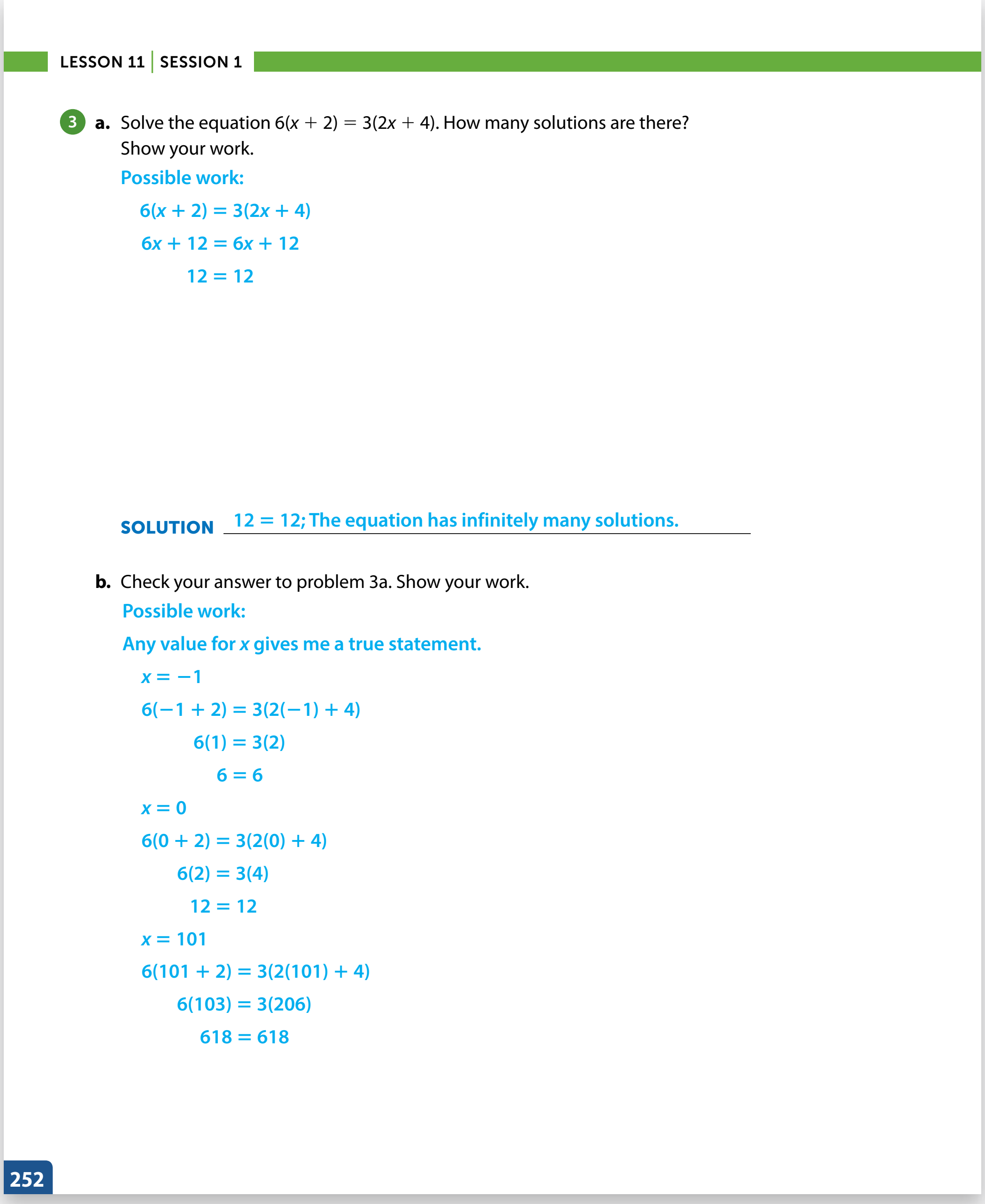
Conducting a cost analysis is a way for companies to know the financial impact of starting a new incentive for their sales team. An equation with the cost of the incentive program on one side and the expected revenue from sales on the other can be made. This equation may have no solution, one solution, or infinitely many solutions. Each of these reveals the potential a company can expect to gain or lose as a result of trying to reward its staff. For example, if there are infinitely many solutions to such an equation, the company may rationalize that any cost would be offset by an equal amount of sales. Therefore, the company could spend any amount on the incentives. Ask students to think of other real-world examples when knowing the number of solutions to an equation might be useful.

Additional Practice

③ Problem 3 provides another look at an equation with variables on both sides. This problem is similar to the problem in the Try It. In both problems, the distributive property is used on both sides and the variables are eliminated, resulting in a true statement. Like the problem in the Try It, this equation has infinitely many solutions.

Students may use algebra tiles or algebraic manipulation to solve.

Suggest that students use **Notice and Wonder** to help them understand the problem and think about what may happen after the distributive property is used.



LESSON 11 | SESSION 1

③ **a.** Solve the equation $6(x + 2) = 3(2x + 4)$. How many solutions are there? Show your work.

Possible work:

$6(x + 2) = 3(2x + 4)$

$6x + 12 = 6x + 12$

$12 = 12$

SOLUTION $12 = 12$; The equation has infinitely many solutions.

b. Check your answer to problem 3a. Show your work.

Possible work:

Any value for x gives me a true statement.

$x = -1$

$6(-1 + 2) = 3(2(-1) + 4)$

$6(1) = 3(2)$

$6 = 6$

$x = 0$

$6(0 + 2) = 3(2(0) + 4)$

$6(2) = 3(4)$

$12 = 12$

$x = 101$

$6(101 + 2) = 3(2(101) + 4)$

$6(103) = 3(206)$

$618 = 618$

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DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Use with **Session 2** **Apply It**

Levels 1–3: Speaking/Writing

To help students explain a conclusion, read Apply It problem 6 as students follow along. Write the words *no solution* on the board. Ask students to turn to a partner and share a way to rephrase this. Point out the academic vocabulary word *conclude.* Then read each sentence and ask questions to help students paraphrase: *What does Gabriel do? What does he get? What does he decide or conclude?* Allow think time for partners to write their answers. Then restate Gabriel's conclusion for students: *Gabriel decides that there is no number that will make the equation true. Think about Gabriel's conclusion and decide if he is correct. What do you conclude?*

- *I conclude that Gabriel is* _____.

Levels 2–4: Speaking/Writing

To help students explain a conclusion, read Apply It problem 6 with students. Write the words *no solution* on the board. Ask students to turn to a partner and think of as many ways to rephrase this as possible. Give them 1 minute to discuss and write, then call on volunteers to share their answers. Read the first two sentences and ask questions to help students paraphrase: *What does Gabriel do? What does he get?* Point out the academic vocabulary word *conclude.* Have students read the third sentence and paraphrase. Then have students discuss and share their own conclusions:

- *I* _____ *that Gabriel is* _____ *because* _____.

Levels 3–5: Speaking/Writing

To help students explain a conclusion, have students read Apply It problem 6 and turn to a partner to **Say It Another Way.** Point out the academic vocabulary word *conclude.* Encourage students to discuss how they can replace the word.

Next, have students meet with other partners and discuss how they paraphrased. Ask: *Did you replace the same words or phrases? What word or words did you use instead of* conclude*? Did you say* no solution *another way? How?*

After students have discussed, have them write a response to the problem. Ask: *How did you conclude that Gabriel was right/wrong?*

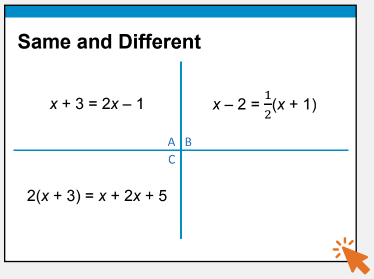
Ask students to share and explain their answers to the class.

Develop Determining the Number of Solutions to One-Variable Equations

Purpose

- **Develop** strategies for transforming one-variable equations to determine the number of solutions.
- **Recognize** that as soon as an equation is known to be false, it can be concluded that the equation has no solution.

START CONNECT TO PRIOR KNOWLEDGE



Same and Different

A $x + 3 = 2x - 1$

B $x - 2 = \frac{1}{2}(x + 1)$

C $2(x + 3) = x + 2x + 5$

Possible Solutions

All are one-variable equations.

B and C involve applying the distributive property to solve.

Only B can be solved by eliminating a fraction.

Only C has like terms to combine.

WHY? Support students' facility with recognizing ways to solve equations.

DEVELOP ACADEMIC LANGUAGE

WHY? Support understanding of the word *eliminate*.

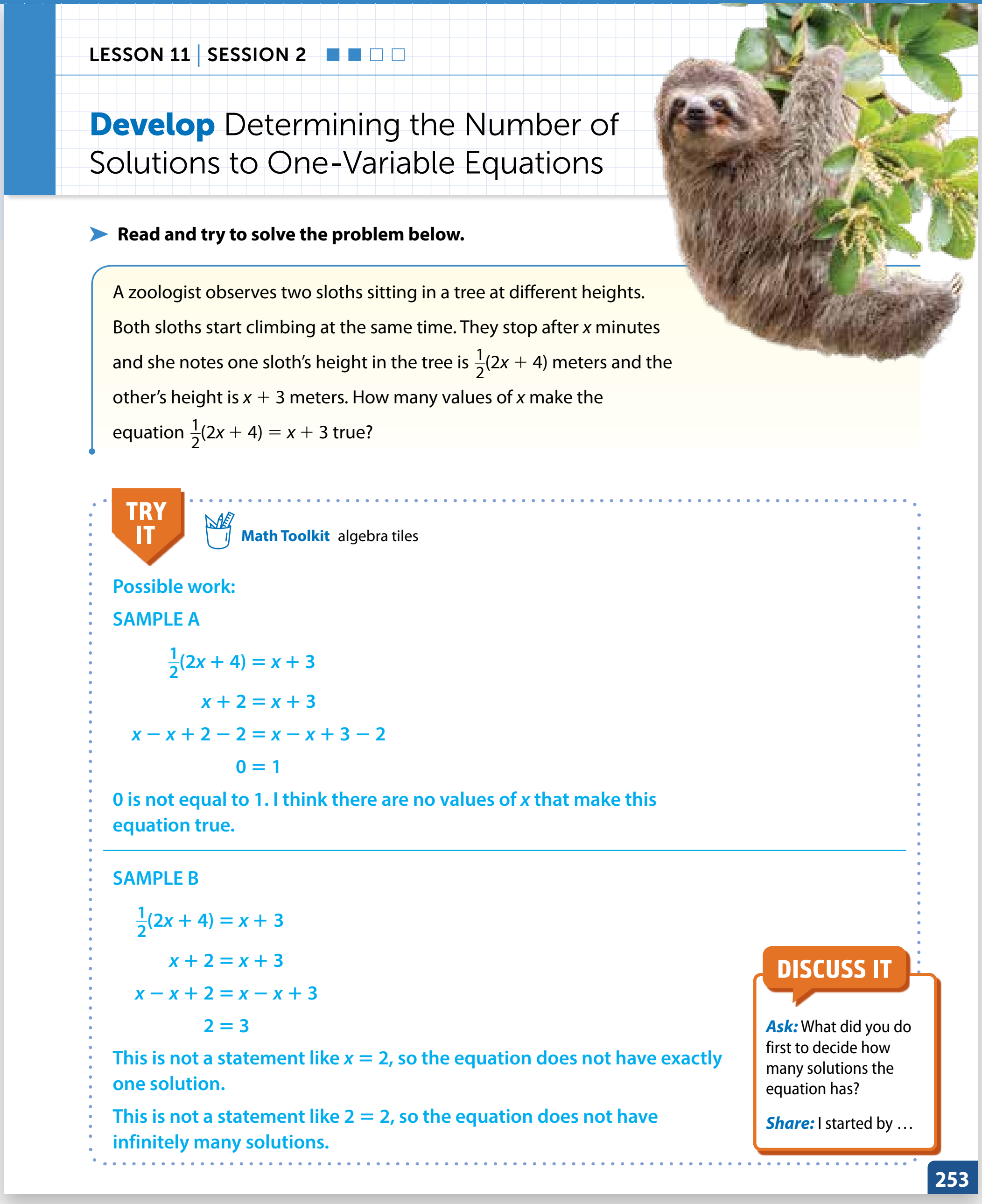
HOW? Read the second Model It and have students think about what it means to eliminate something. Have students share their ideas and give examples. Explain that in an equation, you need to keep both sides balanced, so you cannot just cross the fraction off. Show how you can multiply by a number that will make the fraction equal to 1. Have students explain how the fraction was eliminated.

TRY IT

SMP 1, 2, 4, 5, 6

Make Sense of the Problem

See ***Connect to Culture*** to support student engagement. Before students work on Try It, use **Say It Another Way** to help them make sense of the problem. Have students read the problem with a partner and use Say It Another Way to confirm understanding before they begin work.



LESSON 11 | SESSION 2 ■ ■ □ □

Develop Determining the Number of Solutions to One-Variable Equations

➤ **Read and try to solve the problem below.**

A zoologist observes two sloths sitting in a tree at different heights. Both sloths start climbing at the same time. They stop after *x* minutes and she notes one sloth's height in the tree is $\frac{1}{2}(2x + 4)$ meters and the other's height is $x + 3$ meters. How many values of *x* make the equation $\frac{1}{2}(2x + 4) = x + 3$ true?

TRY IT

Math Toolkit algebra tiles

Possible work:

SAMPLE A

$$\frac{1}{2}(2x + 4) = x + 3$$
$$x + 2 = x + 3$$
$$x - x + 2 - 2 = x - x + 3 - 2$$
$$0 = 1$$

0 is not equal to 1. I think there are no values of *x* that make this equation true.

SAMPLE B

$$\frac{1}{2}(2x + 4) = x + 3$$
$$x + 2 = x + 3$$
$$x - x + 2 = x - x + 3$$
$$2 = 3$$

This is not a statement like $x = 2$, so the equation does not have exactly one solution.

This is not a statement like $2 = 2$, so the equation does not have infinitely many solutions.

DISCUSS IT

Ask: What did you do first to decide how many solutions the equation has?

Share: I started by …

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DISCUSS IT

SMP 2, 3, 6

Support Partner Discussion

After students work on Try It, have them explain their work and then respond to Discuss It with a partner. If students need support in getting started, prompt them to ask each other questions such as:

- *What is the meaning of the solution to a one-variable equation?*
- *If you are solving and get a true statement like 2 = 2, what does that mean?*
- *If you are solving and get a false statement like 2 = 4, what do you think that means?*

Common Misconception Listen for students who say that the equation has infinitely many solutions because the variable terms are eliminated. As students share their strategies, discuss the fact that the resulting statement is *never* true. Help students see that this means the original equation is also never true and therefore has no solution.

Select and Sequence Student Strategies

Select 2–3 samples that represent the range of student thinking in your classroom. Here is one possible order for class discussion:

- hanger diagram
- (misconception) conclusion that the equation has an infinite number of solutions
- algebraic manipulation using the distributive property first
- algebraic manipulation by multiplying both sides by 2 first

Facilitate Whole Class Discussion

Call on students to share selected strategies. Prompt students to build on ideas they agree with by showing why the strategy is reasonable. Have students use the sentence starter: *I know this strategy makes sense because _____.*

Guide students to **Compare and Connect** the representations. Ask students to take individual think time and then turn and talk to answer the question below.

> **ASK** *How does each strategy show that the end statement when solving the equation is false?*
>
> **LISTEN FOR** After the variable term is eliminated, there is a number on one side of the equal sign and a different number on the other side.

Model It

If students presented these models, have students connect these models to those presented in class.

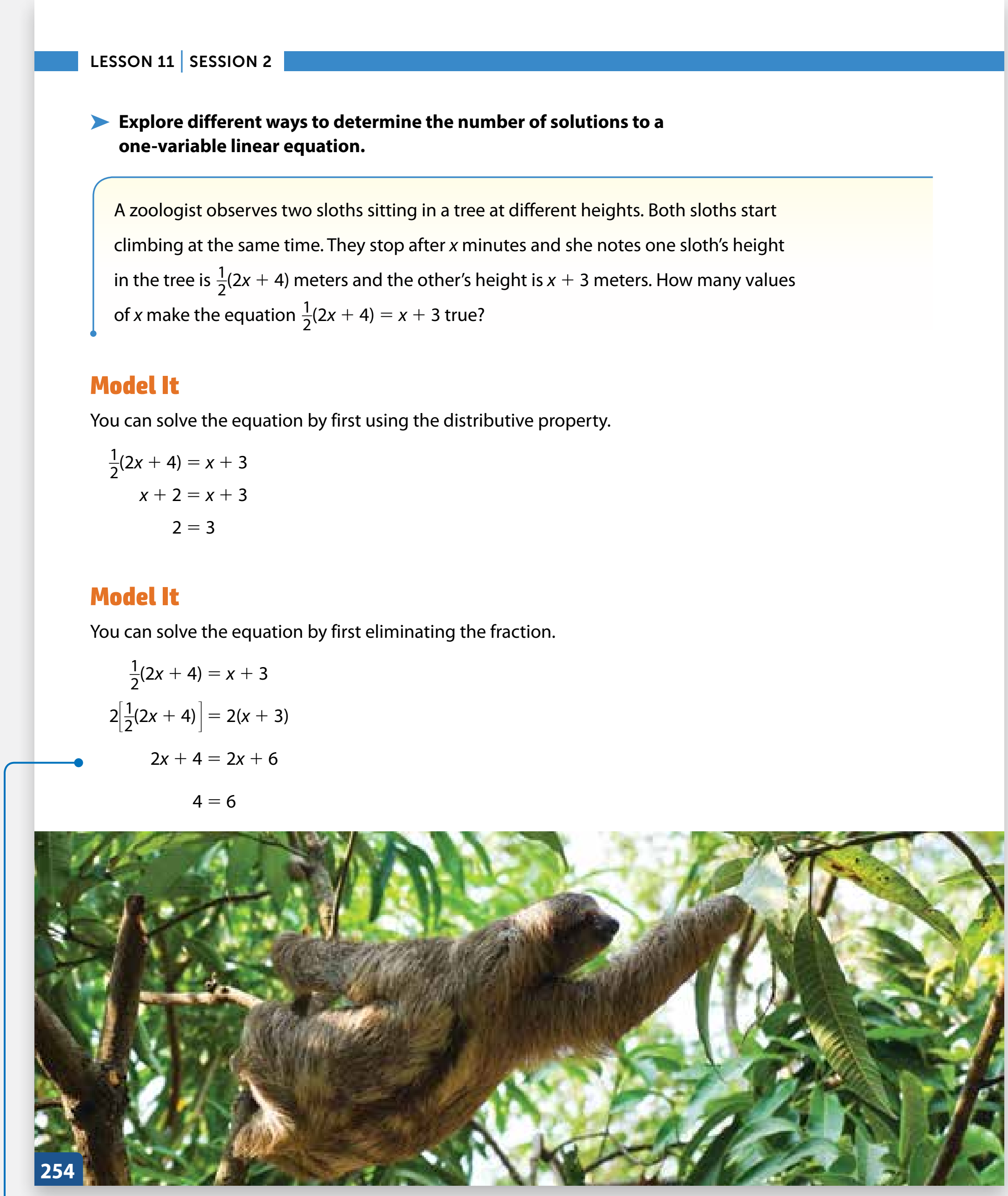
If no student presented at least one of these models, have students first analyze key features of the models, and then connect them to the models presented in class.

> **ASK** *What is different about the final statements in each model? What is the same?*
>
> **LISTEN FOR** The numbers in each final mathematical statement are different, but they are both false statements.

For the model that solved the equation by first using the distributive property, prompt students to think about why the variable is eliminated. *Why is there no variable in the final mathematical statement?*

For the model that eliminates the fraction, prompt students to consider how this model differs from the first model.

- *Why is the final math statement different in this model?*
- *How is this difference significant?*



LESSON 11 | SESSION 2

➤ **Explore different ways to determine the number of solutions to a one-variable linear equation.**

A zoologist observes two sloths sitting in a tree at different heights. Both sloths start climbing at the same time. They stop after x minutes and she notes one sloth's height in the tree is $\frac{1}{2}(2x + 4)$ meters and the other's height is $x + 3$ meters. How many values of x make the equation $\frac{1}{2}(2x + 4) = x + 3$ true?

Model It

You can solve the equation by first using the distributive property.

$$\frac{1}{2}(2x + 4) = x + 3$$
$$x + 2 = x + 3$$
$$2 = 3$$

Model It

You can solve the equation by first eliminating the fraction.

$$\frac{1}{2}(2x + 4) = x + 3$$
$$2\left[\frac{1}{2}(2x + 4)\right] = 2(x + 3)$$
$$2x + 4 = 2x + 6$$
$$4 = 6$$

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DIFFERENTIATION | EXTEND

SMP 8

Deepen Understanding

Using Repeated Reasoning to Analyze Equations with No Solution

Have students look at the equation $2x + 4 = 2x + 6$ in the second Model It. Have a volunteer explain what each side of the equation means. [The left side means take a number and double it and then add 4. The right side means take that same number and double it and then add 6.]

> **ASK** *What does x represent? Does it represent the same number on both sides of the equation, or can it be a different number?*
>
> **LISTEN FOR** The variable x represents an unknown value. It must have the same value wherever it is used in the equation.

> **ASK** *Is there any positive number that can be substituted for x so that the left side and the right side of the equation have the same value? What about substituting 0 for x? What about substituting a negative number?*
>
> **LISTEN FOR** The number on the right side will always be 2 more than the number on the left side, so there is no number that is a solution for x.

Generalize Invite students to suggest other equations that have no solution and to explain how they know no values of the variable will make the equations true.

Develop Determining the Number of Solutions to One-Variable Equations

CONNECT IT

SMP 2, 4, 5, 6

Remind students that even though the final mathematical statement in each model is different, the initial equation and the interpretation of the result is the same in each representation. Explain that they will now use those representations to reason about when it is possible to conclude an equation has no solution.

Before students begin to record and expand on their work in Model It, tell them that problems 1–3 will prepare them to provide the explanation asked for in problem 4.

Monitor and Confirm Understanding ① – ②

- The equations $2 = 3$ and $4 = 6$ are false statements. They do not indicate that 2, 3, 4, or 6 is a solution.
- If solving an equation does not result in a statement that is always true, such as $0 = 0$, then the equation does not have infinitely many solutions.
- If solving an equation does not result in a statement of the form $x = a$, where a is a number, then the equation does not have exactly one solution.

Facilitate Whole Class Discussion

③ Students should understand that if the solution process results in a statement that is clearly false, then the original equation is also false. Therefore, the equation has no solution.

> **ASK** *What do the statements $2 = 3$ and $4 = 6$ indicate about what happens when x-values are substituted into the original equation?*
>
> **LISTEN FOR** No matter what value is substituted for x, the left and right sides of the equation will not be equal.

④ Look for the idea that the moment students recognize an equation is a false statement, they can conclude the equation has no solution.

> **ASK** *How can you tell just by looking that $x + 2 = x + 3$ has no solution?*
>
> **LISTEN FOR** Adding 2 to a number will never give the same result as adding 3 to that same number.

⑤ **Reflect** Have all students focus on the strategies used to solve the Try It. If time allows, have students discuss their ideas with a partner.

CONNECT IT

➤ **Use the problem from the previous page to help you understand how to determine the number of solutions to a one-variable linear equation.**

① The final statements for the **Model Its** are $2 = 3$ and $4 = 6$. Is either a true statement? Is 2, 3, 4, or 6 a solution of the equation? Explain.

No; No; $2 = 3$ and $4 = 6$ are false statements. When you substitute 2, 3, 4, or 6 into the original equation for x, you also get false statements.

② Does the equation $\frac{1}{2}(2x + 4) = x + 3$ have infinitely many solutions? Does it have exactly one solution? Explain.

No; No; Possible explanation: A final statement like $0 = 0$ means the equation has infinitely many solutions. A final statement like $x = 0$ means the equation has exactly one solution.

③ The statements $2 = 3$ and $4 = 6$ are simplified versions of the original equation. Because they are false statements, the original equation is also a false statement. Why does it make sense to conclude that the equation has no solution? What does this mean in terms of the situation?

Possible answer: No matter what number you substitute for x, you get a false statement. The equation is not true for any value of x. This means that the two sloths were never at the same height in the tree at the same time during this climb.

④ Look at the equation $x + 2 = x + 3$ in the first **Model It**. How can you tell that this equation has no solution without solving further?

Possible answer: No value for x can make both sides equal, so you can already tell that this is a false statement.

⑤ **Reflect** Think about all the models and strategies you discussed today. Describe how one of them helped you better understand how to solve the **Try It** problem.

Responses will vary. Check student responses.

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DIFFERENTIATION | RETEACH or REINFORCE

Hands-On Activity

Use algebra tiles to visualize equations with no solution and infinitely many solutions.

If students are unsure about how end statements indicate that an equation has no solution or infinitely many solutions, then use this activity to help them understand.

Materials For each student: algebra tiles (at least 10 x-tiles and 10 1-tiles)

- Have students model the equation $2(x + 1) = 2x + 3$ using algebra tiles.
- Next have them solve the equation, which should result in $2 = 3$.
- Ask: *What tiles are left?* [2 unit tiles on one side and 3 unit tiles on the other]
- Ask: *What can you conclude?* [The result is not a balanced equation, so there is no solution. When substituting values for x, no values will make the equation true.]
- Have students model the equation $4(x + 1) = 2(2x + 2)$ using algebra tiles.
- Next have them solve the equation, which should result in $4 = 4$ or $0 = 0$.
- Ask: *What tiles are left?* [4 unit tiles on each side or no tiles on each side]
- Ask: *Why doesn't this mean there is no solution?* [The result is a balanced equation, so there must be a solution. When substituting values for x, any value will make the equation true.]

Apply It

For all problems, encourage students to use a model to support their thinking.

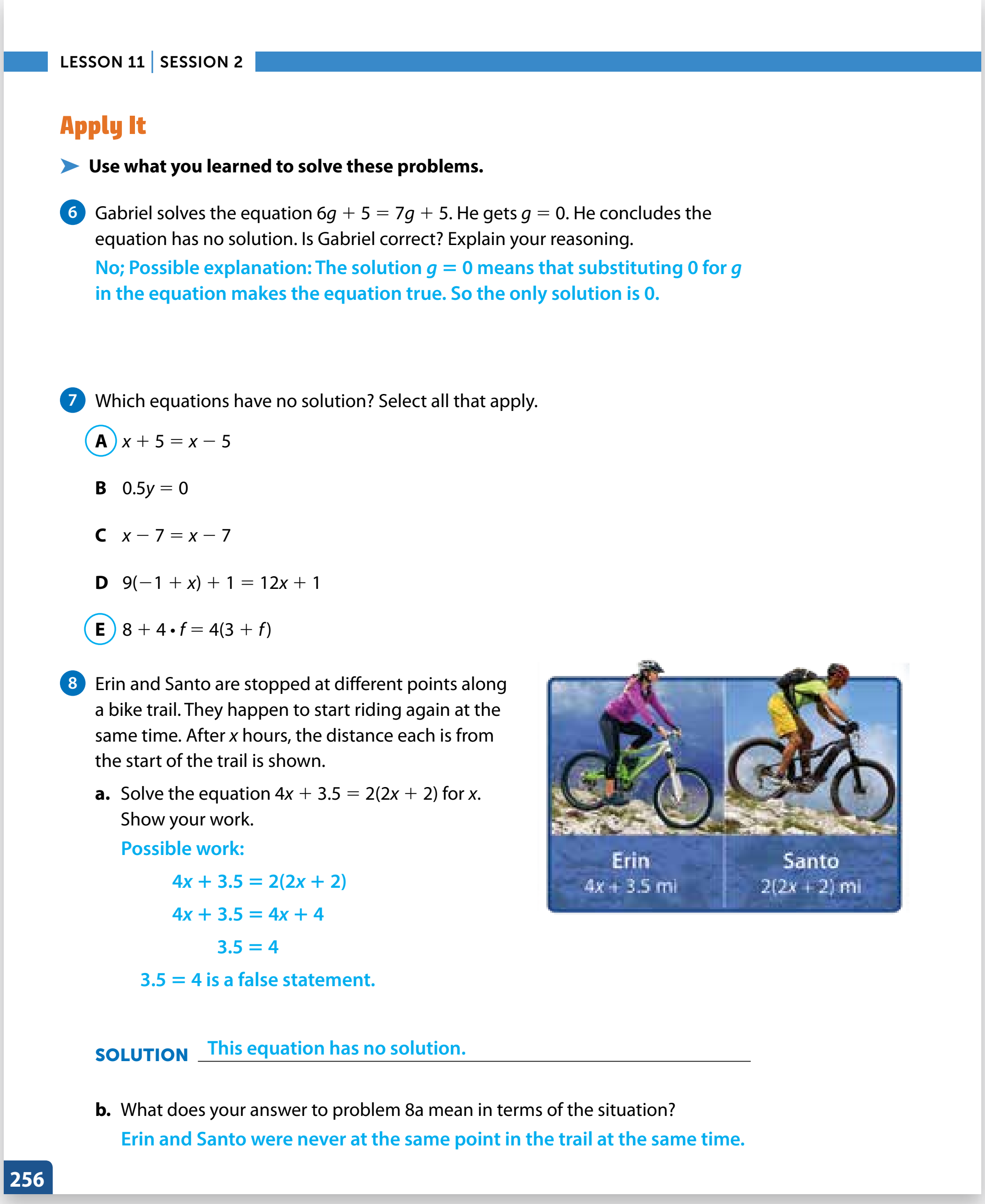
⑥ Students should understand that $g = 0$ is a true statement when (and only when) g is equal to 0. So, 0 is the solution. Students can substitute 0 in the original equation to verify that this is true.

⑦ **A and E are correct.** Students may solve A by subtracting the x from each side in the equation to leave a false statement. They may solve E by first distributing and then eliminating the x-term from each side to leave a false statement.

B is not correct. The solution of this equation is 0.

C is not correct. This equation has infinitely many solutions.

D is not correct. The solution of this equation is -3.



LESSON 11 | SESSION 2

Apply It

➤ **Use what you learned to solve these problems.**

⑥ Gabriel solves the equation $6g + 5 = 7g + 5$. He gets $g = 0$. He concludes the equation has no solution. Is Gabriel correct? Explain your reasoning.

No; Possible explanation: The solution $g = 0$ means that substituting 0 for g in the equation makes the equation true. So the only solution is 0.

⑦ Which equations have no solution? Select all that apply.

[✓ **A**] $x + 5 = x - 5$

B $0.5y = 0$

C $x - 7 = x - 7$

D $9(-1 + x) + 1 = 12x + 1$

[✓ **E**] $8 + 4 \cdot f = 4(3 + f)$

⑧ Erin and Santo are stopped at different points along a bike trail. They happen to start riding again at the same time. After x hours, the distance each is from the start of the trail is shown.

a. Solve the equation $4x + 3.5 = 2(2x + 2)$ for x. Show your work.

Possible work:

$$4x + 3.5 = 2(2x + 2)$$
$$4x + 3.5 = 4x + 4$$
$$3.5 = 4$$

$3.5 = 4$ is a false statement.

SOLUTION This equation has no solution.

b. What does your answer to problem 8a mean in terms of the situation?

Erin and Santo were never at the same point in the trail at the same time.

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CLOSE EXIT TICKET

⑧ Students' solutions should show an understanding of:

- solving a multi-step one-variable equation.
- interpreting the false statement to mean that no value of x makes the equation true.
- interpreting the solution in terms of the context.

Error Alert If students find a solution or say that there are infinitely many solutions, have them go back and check their work. Point out that, if they perform the steps correctly, the resulting statement will always be false, indicating that the original equation has no solution.

Practice Determining the Number of Solutions to One-Variable Equations

Problem Notes

Assign **Practice Determining the Number of Solutions to One-Variable Equations** as extra practice in class or as homework.

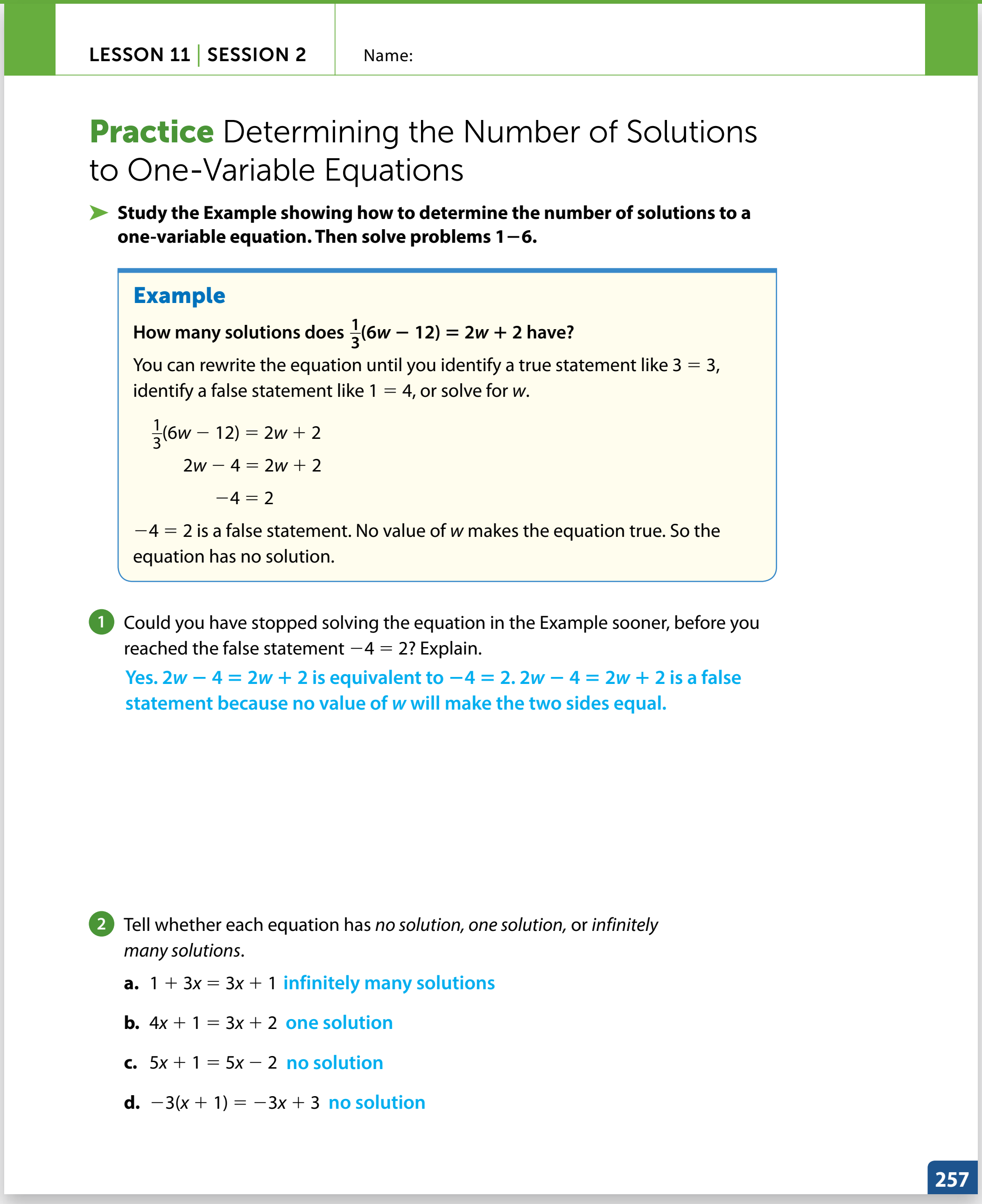
① From the step $2w - 4 = 2w + 2$, students can see that since the coefficient of w is the same on both sides, the w term will be eliminated, leaving the false statement $-4 = 2$. ***Medium***

② **a.** ***Basic***

b. Students may solve by subtracting $3x$ from each side and then subtracting 1 from each side to get $x = 1$. ***Basic***

c. Students may solve by recognizing the coefficient of x is the same on both sides and has the same sign, so it will drop out and result in a false statement. ***Basic***

d. Students may solve by distributing on the left side, then recognizing the x-terms will drop out and result in a false statement. ***Basic***



LESSON 11 | SESSION 2 Name:

Practice Determining the Number of Solutions to One-Variable Equations

➤ **Study the Example showing how to determine the number of solutions to a one-variable equation. Then solve problems 1–6.**

Example

How many solutions does $\frac{1}{3}(6w - 12) = 2w + 2$ have?

You can rewrite the equation until you identify a true statement like $3 = 3$, identify a false statement like $1 = 4$, or solve for w.

$$\frac{1}{3}(6w - 12) = 2w + 2$$
$$2w - 4 = 2w + 2$$
$$-4 = 2$$

$-4 = 2$ is a false statement. No value of w makes the equation true. So the equation has no solution.

① Could you have stopped solving the equation in the Example sooner, before you reached the false statement $-4 = 2$? Explain.

Yes. $2w - 4 = 2w + 2$ is equivalent to $-4 = 2$. $2w - 4 = 2w + 2$ is a false statement because no value of w will make the two sides equal.

② Tell whether each equation has *no solution*, *one solution*, or *infinitely many solutions*.

a. $1 + 3x = 3x + 1$ **infinitely many solutions**

b. $4x + 1 = 3x + 2$ **one solution**

c. $5x + 1 = 5x - 2$ **no solution**

d. $-3(x + 1) = -3x + 3$ **no solution**

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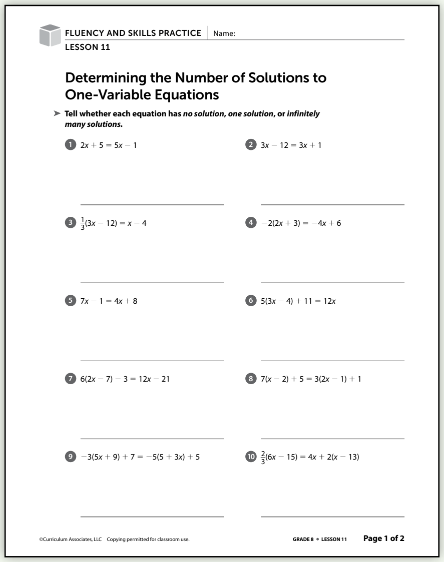
Fluency & Skills Practice

Determining the Number of Solutions to One-Variable Equations

In this activity, students practice solving one-variable linear equations and they determine whether each equation has no solution, one solution, or infinitely many solutions.



FLUENCY AND SKILLS PRACTICE | Name:
LESSON 11

Determining the Number of Solutions to One-Variable Equations

➤ **Tell whether each equation has *no solution*, *one solution*, or *infinitely many solutions*.**

① $2x + 5 = 5x - 1$

② $3x - 12 = 3x + 1$

③ $\frac{1}{3}(3x - 12) = x - 4$

④ $-2(2x + 3) = -4x + 6$

⑤ $7x - 1 = 4x + 8$

⑥ $5(3x - 4) + 11 = 12x$

⑦ $6(2x - 7) - 3 = 12x - 21$

⑧ $7(x - 2) + 5 = 3(2x - 1) + 1$

⑨ $-3(5x + 9) + 7 = -5(5 + 3x) + 5$

⑩ $\frac{2}{3}(6x - 15) = 4x + 2(x - 13)$

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GRADE 8 • LESSON 11 Page 1 of 2

Additional Practice

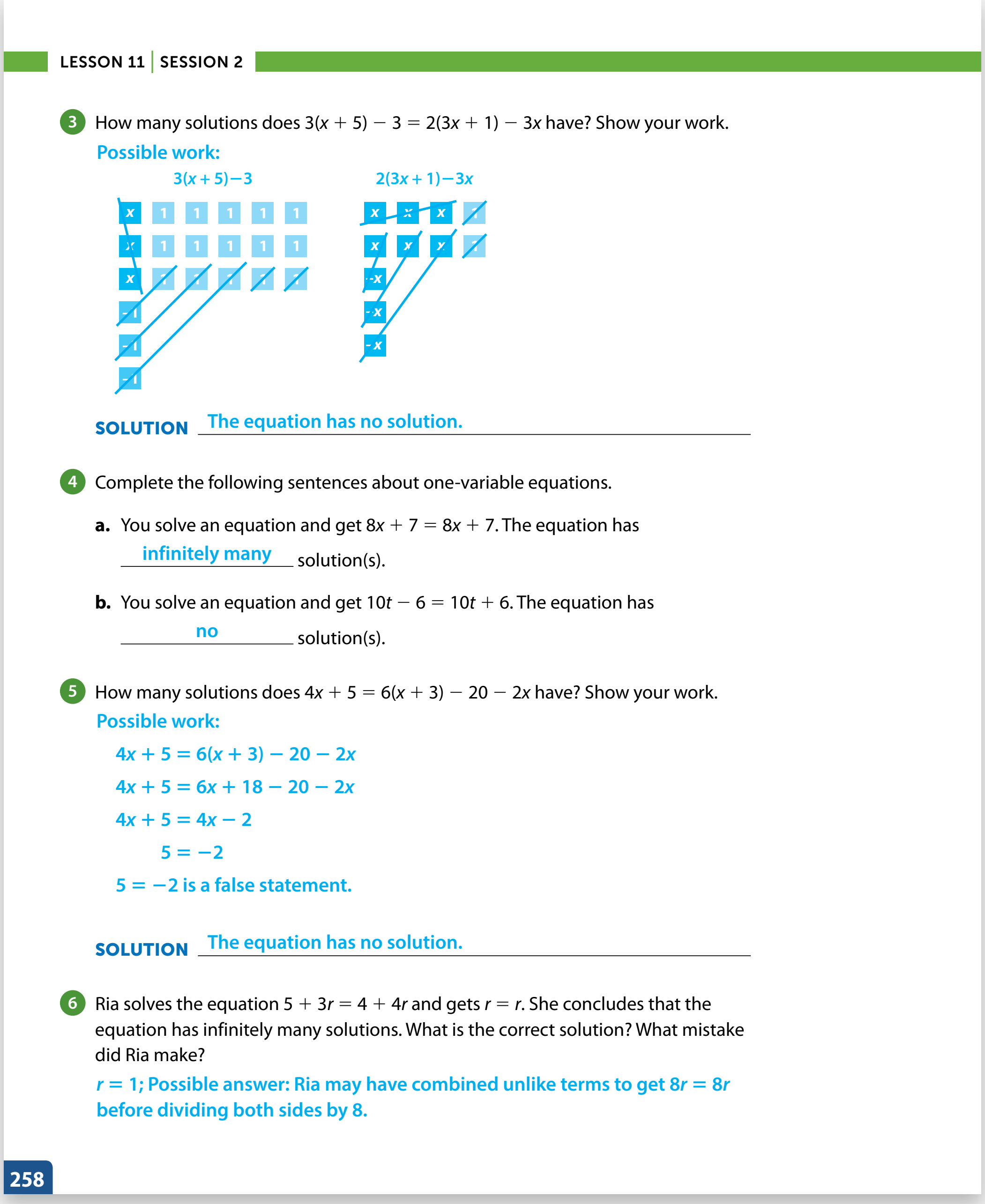
③ Students may also solve the equation algebraically to get 12 = 2. The false statement means the equation has no solution. ***Medium***

④ **a.** Students may immediately recognize that the expressions on the left and right sides of the equation are the same. ***Basic***

b. Students may immediately recognize that the *x*-terms will drop out and result in a false statement. ***Basic***

⑤ Students may stop solving at $4x + 5 = 4x - 2$ and recognize the *x*-terms will drop out and leave a false statement. ***Medium***

⑥ Students may also think Ria's error is that when she subtracted 4 from both sides instead of writing 1, she wrote *r*. ***Challenge***



LESSON 11 | SESSION 2

③ How many solutions does $3(x + 5) - 3 = 2(3x + 1) - 3x$ have? Show your work.

Possible work:

SOLUTION The equation has no solution.

④ Complete the following sentences about one-variable equations.

a. You solve an equation and get $8x + 7 = 8x + 7$. The equation has infinitely many solution(s).

b. You solve an equation and get $10t - 6 = 10t + 6$. The equation has no solution(s).

⑤ How many solutions does $4x + 5 = 6(x + 3) - 20 - 2x$ have? Show your work.

Possible work:

$4x + 5 = 6(x + 3) - 20 - 2x$

$4x + 5 = 6x + 18 - 20 - 2x$

$4x + 5 = 4x - 2$

$5 = -2$

$5 = -2$ is a false statement.

SOLUTION The equation has no solution.

⑥ Ria solves the equation $5 + 3r = 4 + 4r$ and gets $r = r$. She concludes that the equation has infinitely many solutions. What is the correct solution? What mistake did Ria make?

$r = 1$; Possible answer: Ria may have combined unlike terms to get $8r = 8r$ before dividing both sides by 8.

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DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Use with **Session 3** **Apply It**

Levels 1–3: Listening/Speaking

To help students write terms, read Apply It problem 7 aloud as students follow along. Review the phrases *constant term* and *variable term*. Use sentences to clarify meaning: *The noise does not stop. I cannot concentrate because of the constant noise. The word* variable *has the same root as* vary. *If something varies, it changes.* Have students turn to a partner to come up with other examples. Then have students define *constant term* and *variable term* in their own words and provide examples.

- *A constant term* _____. *An example is* _____.
- *A variable term* _____. *An example is* _____.

Have students work on 7a–d and use *constant term* and *variable term* to state their answers.

Levels 2–4: Listening/Speaking

To help students fill in terms, read Apply It problem 7 with students. Review the phrases *constant term* and *variable term*. Ask the following questions to confirm understanding: *The noise does not stop. What word can you use to describe the noise? How are the words* vary *and* variable *alike? What does* variable *mean?* Have students turn to a partner to come up with other examples. Then have students define *constant term* and *variable term* in their own words. Have them write an expression that contains both a variable term and a constant term and have a partner identify each kind of term.

Have students work on 7a–d and use *constant term* and *variable term* to state their answers.

Levels 3–5: Listening/Speaking

Have students read Apply It problem 7. Call on volunteers to explain the meaning of *constant term* and *variable term*. Then ask other volunteers to provide examples in an expression that contains both variable and constant terms. Have them share their expressions and point out the variable and constant terms in them.

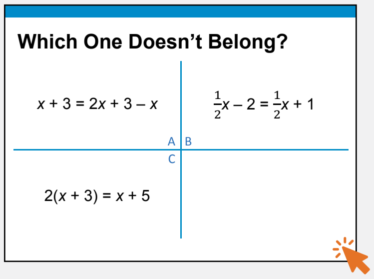
Have students work on 7a–d independently and then meet with partners to compare and discuss their answers. Have partners take turns identifying each other's answers as *constant term* or *variable term*. Have them explain why their answers may be different but correct.

Develop Writing an Equation with No, One, or Infinitely Many Solutions

Purpose

- **Develop** strategies for writing an equation with no, one, or an infinite number of solutions.
- **Recognize** that if both sides of an equation have the same variable terms and the same constants, the equation has an infinite number of solutions, and if both sides of an equation have the same variable terms and different constants, the equation has no solution.

START CONNECT TO PRIOR KNOWLEDGE



Which One Doesn't Belong?

A: $x + 3 = 2x + 3 - x$

B: $\frac{1}{2}x - 2 = \frac{1}{2}x + 1$

C: $2(x + 3) = x + 5$

Possible Solutions

A is the only equation with infinitely many solutions and where combining like terms is needed to simplify one side.

B is the only equation that has no solution and fractional coefficients.

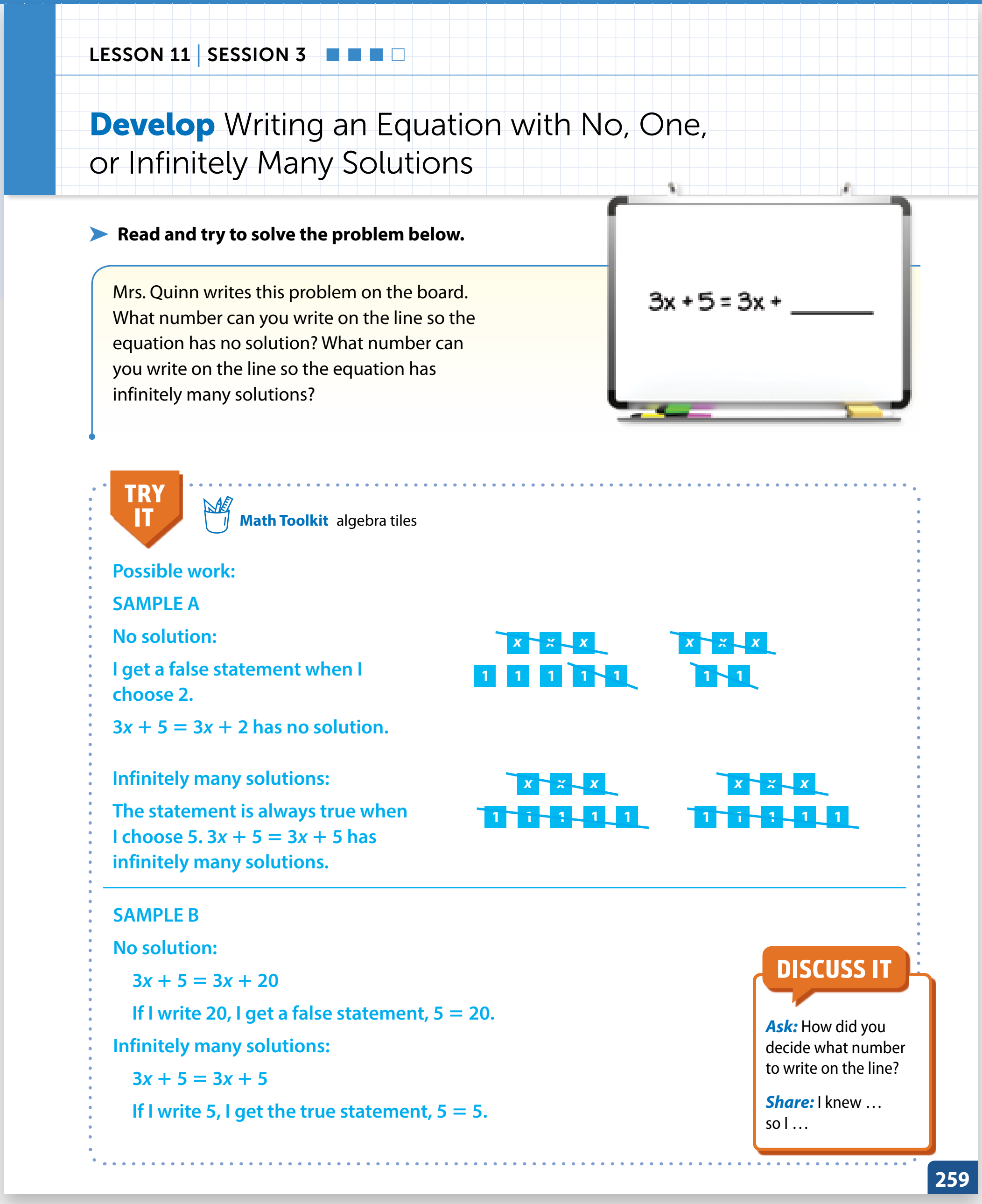
C is the only equation with exactly one solution and involves the distributive property.

WHY? Support students' facility with finding the number of solutions to a one-variable equation, as well as identifying the similarities and differences among the solution processes.

DEVELOP ACADEMIC LANGUAGE

WHY? Support students as they craft clear explanations using precise language.

HOW? Remind students that using precise mathematical language and complete sentences in their explanations makes them clearer and easier to understand. Before answering the question in Discuss It, review how students could use *infinitely many* and *equation* in their responses. Have students use precise language when crafting explanations and share them with a partner. If the partner has questions, it may mean that the explanation was not detailed enough. Have students refine their explanations, focusing on using precise language, and read them to other students.



LESSON 11 | SESSION 3 ■ ■ ■ □

Develop Writing an Equation with No, One, or Infinitely Many Solutions

➤ **Read and try to solve the problem below.**

Mrs. Quinn writes this problem on the board. What number can you write on the line so the equation has no solution? What number can you write on the line so the equation has infinitely many solutions?

$3x + 5 = 3x +$ ____

TRY IT Math Toolkit algebra tiles

Possible work:

SAMPLE A

No solution:

I get a false statement when I choose 2.

$3x + 5 = 3x + 2$ has no solution.

Infinitely many solutions:

The statement is always true when I choose 5. $3x + 5 = 3x + 5$ has infinitely many solutions.

SAMPLE B

No solution:

$3x + 5 = 3x + 20$

If I write 20, I get a false statement, $5 = 20$.

Infinitely many solutions:

$3x + 5 = 3x + 5$

If I write 5, I get the true statement, $5 = 5$.

DISCUSS IT

Ask: How did you decide what number to write on the line?

Share: I knew … so I …

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TRY IT

SMP 1, 2, 4, 5, 6

Make Sense of the Problem

Before students work on Try It, use **Three Reads** to help them make sense of the problem. Have partners use the routine, alternating the reading between.

DISCUSS IT

SMP 2, 3, 6

Support Partner Discussion

After students work on Try It, encourage them to respond to Discuss It with a partner. Listen for understanding that:

- a statement that is never true means the equation has no solution.
- a statement that is always true means the equation has an infinite number of solutions.

Error Alert If students are confusing equations with infinitely many solutions with equations with no solution, then remind them that a resulting true statement means an equation has infinitely many solutions and a resulting false statement means an equation has no solution. Students should realize there is only one number that makes the equation have infinitely many solutions but many numbers that make the equation have no solution.

Select and Sequence Student Strategies

Select 2–3 samples that represent the range of student thinking in your classroom. Here is one possible order for class discussion:

- algebra tiles
- hanger diagram
- guess and check
- algebraic manipulation

Facilitate Whole Class Discussion

Call on students to share selected strategies. Reinforce with students that clear explanations use complete sentences and precise vocabulary.

Guide students to **Compare and Connect** the representations. After a student shares an idea, ask another student to reword any unclear statements. Confirm with the speaker that the rewording is accurate.

> **ASK** *What is consistent about the strategies shared?*
>
> **LISTEN FOR** For the equations with infinitely many solutions, all the strategies resulted in equations with a 5 in the blank. For the equations with no solution, all the resulting equations had a number other than 5 in the blank.

Model It & Analyze It

If students presented these models, have students connect these models to those presented in class.

If no student presented at least one of these models, have students first analyze key features of the models, and then connect them to the models presented in class.

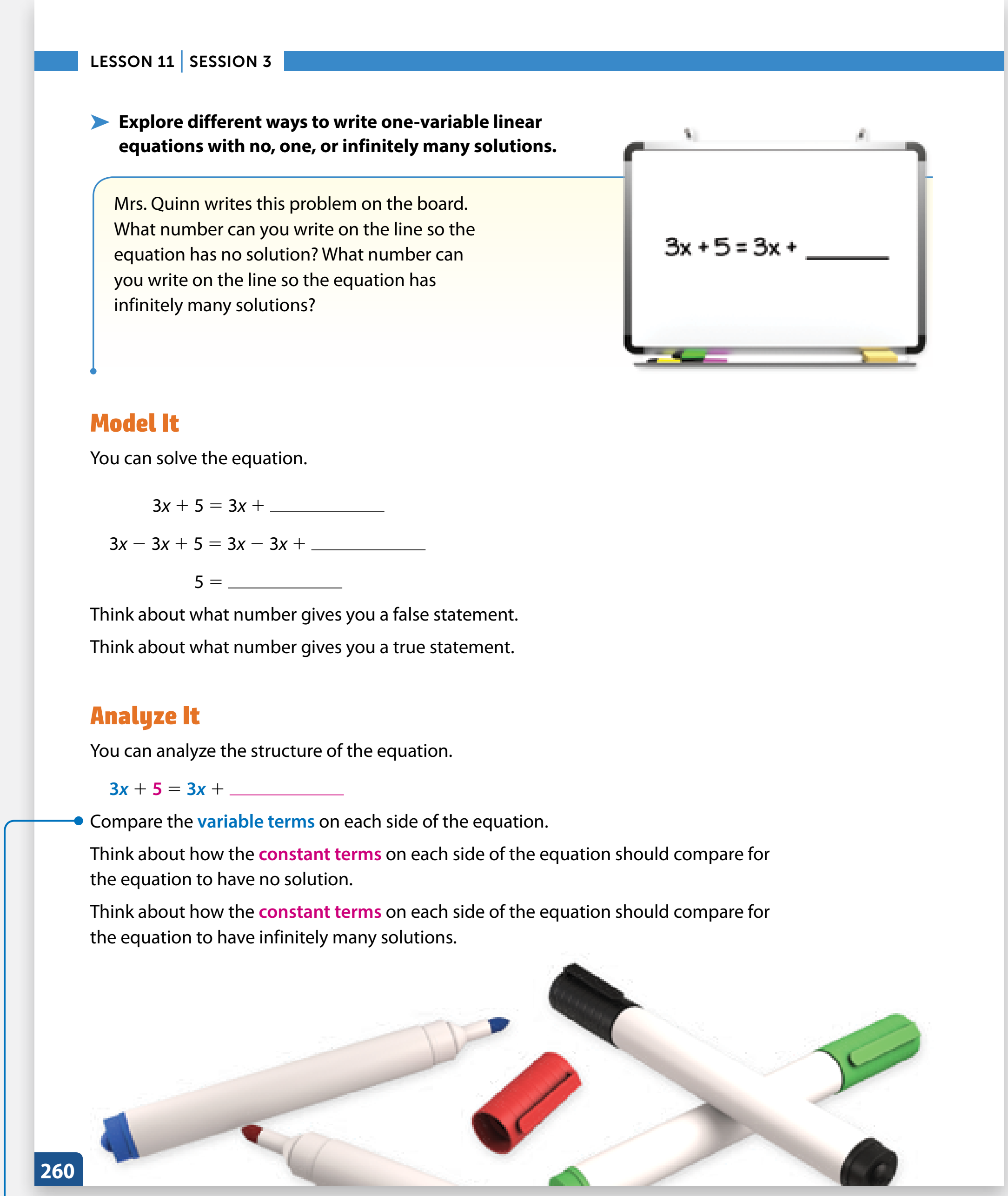
> **ASK** *How do the models differ?*
>
> **LISTEN FOR** They are different because one eliminates the variable and one analyzes the equation with the variable still included.

For the model in which the equation was solved, prompt students to consider the structure of the expressions in the equation and make observations.

- *What happens to the x-terms when an equation has either no solution or infinitely many?*
- *What is true about the constant terms when an equation has no solution?*
- *What is true about the constant terms when an equation has infinitely many solutions?*

For the model in which structure was analyzed, prompt students to consider the characteristics of the x-terms and constant terms.

- *What do you notice about the x-terms?*
- *What constant must be on the right side of the equation for it to have infinitely many solutions?*



LESSON 11 | SESSION 3

➤ Explore different ways to write one-variable linear equations with no, one, or infinitely many solutions.

Mrs. Quinn writes this problem on the board. What number can you write on the line so the equation has no solution? What number can you write on the line so the equation has infinitely many solutions?

$3x + 5 = 3x +$ ______

Model It

You can solve the equation.

$$3x + 5 = 3x + \underline{\qquad\qquad}$$

$$3x - 3x + 5 = 3x - 3x + \underline{\qquad\qquad}$$

$$5 = \underline{\qquad\qquad}$$

Think about what number gives you a false statement.

Think about what number gives you a true statement.

Analyze It

You can analyze the structure of the equation.

$3x + 5 = 3x +$ ______

Compare the variable terms on each side of the equation.

Think about how the constant terms on each side of the equation should compare for the equation to have no solution.

Think about how the constant terms on each side of the equation should compare for the equation to have infinitely many solutions.

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DIFFERENTIATION | EXTEND

SMP 7

Deepen Understanding

Looking for Structure in Equations with No, One, or Infinitely Many Solutions

Prompt students to think about the terms in a more complex equation that generate no solution or infinitely many solutions. Display the equation $10x - 2 = 5(\_\_\_\_ + 2x)$.

> **ASK** *How can you just look at this equation to be able to tell whether it has no solution, one solution, or infinitely many solutions prior to doing any operations on both sides?*
>
> **LISTEN FOR** Determine the coefficient of the variable on the right side. If the variable terms have the same coefficient, the equation has infinitely many solutions or no solution.

> **ASK** *What number could be written on the line so that there are infinitely many solutions? How do you know?*
>
> **LISTEN FOR** $-\frac{2}{5}$; The variable terms are the same, so the constant terms would have to be equal for there to be infinitely many solutions. $5 \bullet \left(-\frac{2}{5}\right) = -2$

> **ASK** *What number could be written on the line so that there is no solution?*
>
> **LISTEN FOR** Any number except for $-\frac{2}{5}$

Develop Writing an Equation with No, One, or Infinitely Many Solutions

CONNECT IT

SMP 2, 4, 5, 6

Remind students that the equation is the same in both representations. Explain that they will now use those representations to reason about how to write one-variable linear equations with different numbers of solutions.

Before students begin to record and expand on their work in Model It & Analyze It, tell them that problem 1 will prepare them to provide the explanation asked for in problem 2.

Monitor and Confirm Understanding ①

- The constant terms must be the same for an equation to have infinitely many solutions.
- The constant terms must be different for an equation to have no solution.

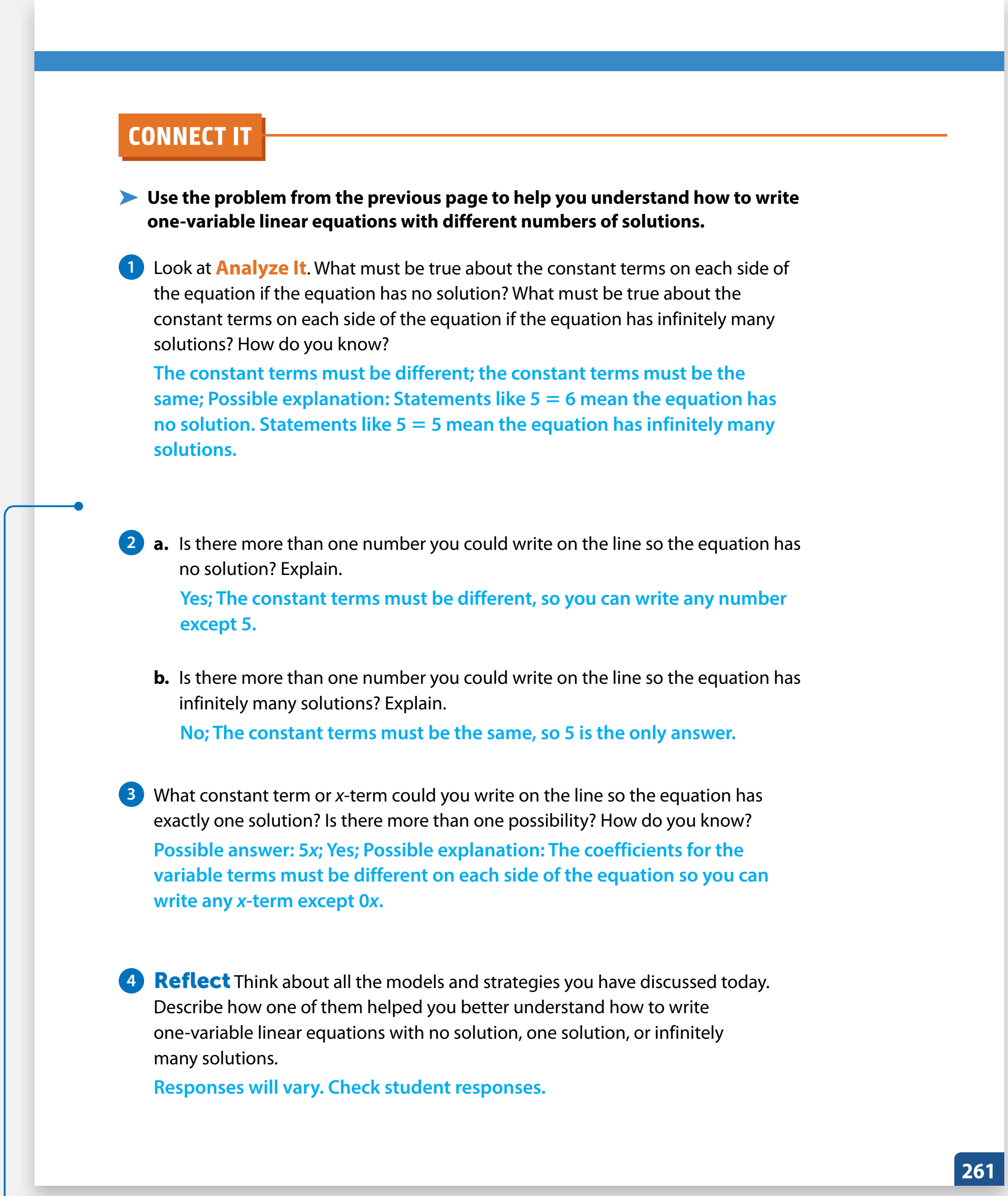
Facilitate Whole Class Discussion

② Look for the idea that to write an equation with no solution, any number except 5 could be written on the line. For infinitely many solutions, only the number 5 can be written on the line.

> **ASK** *How can you recognize the number of solutions for an equation without solving?*
>
> **LISTEN FOR** If both sides of an equation have the same variable terms and different constants, the equation has no solution. If both sides of an equation have the same variable terms and the same constants, the equation has an infinite number of solutions.

③ This question takes the Try It problem one step further by having students think about what variable term they could write on the line to get an equation with exactly one solution. Students should recognize that writing any x-term except $0x$ and combining it with $3x$ will make the x-terms on the two sides different. This means the x-terms will not drop out when the equation is solved, giving a final statement of $x = a$ for a number a.

④ **Reflect** Have all students focus on the strategies used to solve the Try It. If time allows, have students discuss their ideas with a partner.



CONNECT IT

➤ **Use the problem from the previous page to help you understand how to write one-variable linear equations with different numbers of solutions.**

① Look at **Analyze It**. What must be true about the constant terms on each side of the equation if the equation has no solution? What must be true about the constant terms on each side of the equation if the equation has infinitely many solutions? How do you know?

The constant terms must be different; the constant terms must be the same; Possible explanation: Statements like $5 = 6$ mean the equation has no solution. Statements like $5 = 5$ mean the equation has infinitely many solutions.

② **a.** Is there more than one number you could write on the line so the equation has no solution? Explain.

Yes; The constant terms must be different, so you can write any number except 5.

b. Is there more than one number you could write on the line so the equation has infinitely many solutions? Explain.

No; The constant terms must be the same, so 5 is the only answer.

③ What constant term or x-term could you write on the line so the equation has exactly one solution? Is there more than one possibility? How do you know?

Possible answer: $5x$; Yes; Possible explanation: The coefficients for the variable terms must be different on each side of the equation so you can write any x-term except $0x$.

④ **Reflect** Think about all the models and strategies you have discussed today. Describe how one of them helped you better understand how to write one-variable linear equations with no solution, one solution, or infinitely many solutions.

Responses will vary. Check student responses.

DIFFERENTIATION | RETEACH or REINFORCE

Hands-On Activity

Use algebra tiles to see the structure of equations with no solution and infinitely many solutions.

If students are unsure about how to recognize the number of solutions of an equation without solving, then use this activity to provide a concrete model to help them make the connection.

Materials For each student: algebra tiles (at least 10 x-tiles and 10 1-tiles), index card

- Have students model the open-ended problem from Try It using algebra tiles. Have them use an index card for the blank space on the right side.
- Ask: *What do you notice about the x-terms on each side of the equation?* [There are the same number of x-tiles on each side.]
- Have students remove the same number of x-tiles from each side. Ask: *What results after the x-terms are removed?* [There are 5 unit tiles on the left side of the equation and none on the right.]
- Ask: *What would need to be on the right side for the equation to have infinitely many solutions?* [5 unit tiles]
- Ask: *What would need to be on the right side for the equation to have no solution?* [any number of unit tiles other than 5]

Apply It

For all problems, encourage students to use a model to support their thinking.

⑤ Students may also solve this problem by simplifying to $-4 = 6$.

⑥ **A, B, D, and E are correct.** Students may rewrite the equation as $16x + 4c = 16x + 2$. The variable terms on both sides are the same. Any value of c except $\frac{1}{2}$ will make the constant terms different, resulting in a false statement. So, the equation will have no solution.

C is not correct. If the value of c is $\frac{1}{2}$, then the resulting equation can be written as $16x + 2 = 16x + 2$, which has an infinite number of solutions.

Apply It

➤ **Use what you learned to solve these problems.**

⑤ Hai's teacher writes the equation $3x - 4 = 2(x + 3) + x$. Hai concludes that the equation has infinitely many solutions. Is Hai correct? Explain.

No; The equation can be rewritten as $3x - 4 = 3x + 6$. The variable terms on each side of the equation are the same and the constant terms are different, so the equation has no solution.

⑥ Which numbers could you substitute for c so the equation $4(4x + c) = 2(8x + 1)$ has no solution? Select all that apply.

[✓ A] 0

[✓ B] $\frac{1}{4}$

C $\frac{1}{2}$

[✓ D] 1

[✓ E] 2

⑦ Write a constant term or variable term on the line so that each equation has the number of solutions shown.

a. No solution:

$\frac{2}{7}m + 1 = \frac{2}{7}m +$ Possible answer: any number except 1

b. One solution:

$m + 1 = m +$ Possible answer: any non-zero m-term

c. Infinitely many solutions:

$3p + 3 = 3p +$ 3

d. Infinitely many solutions:

$2x + 4 = 2x -$ -4

CLOSE EXIT TICKET

⑦ Students' solutions should show an understanding that:

- if both sides of an equation have the same variable terms and different constants, the equation has no solution.
- if both sides of an equation have the same variable terms and the same constants, the equation has an infinite number of solutions.
- if variable terms are different on each side, then the equation has exactly one solution.

Error Alert If students answer 1 to problem 7a and any number except 3 for problem 7c and -4 for problem 7d, then they are confusing the structure of equations for no solution and infinitely many solutions. Have them revisit the Hands-On Activity.

Practice Writing an Equation with No, One, or Infinitely Many Solutions

Problem Notes

Assign **Practice Writing an Equation with No, One, or Infinitely Many Solutions** as extra practice in class or as homework.

① **a.** Students should recognize that the constant terms need to be different. They can insert any constant other than 7. ***Medium***

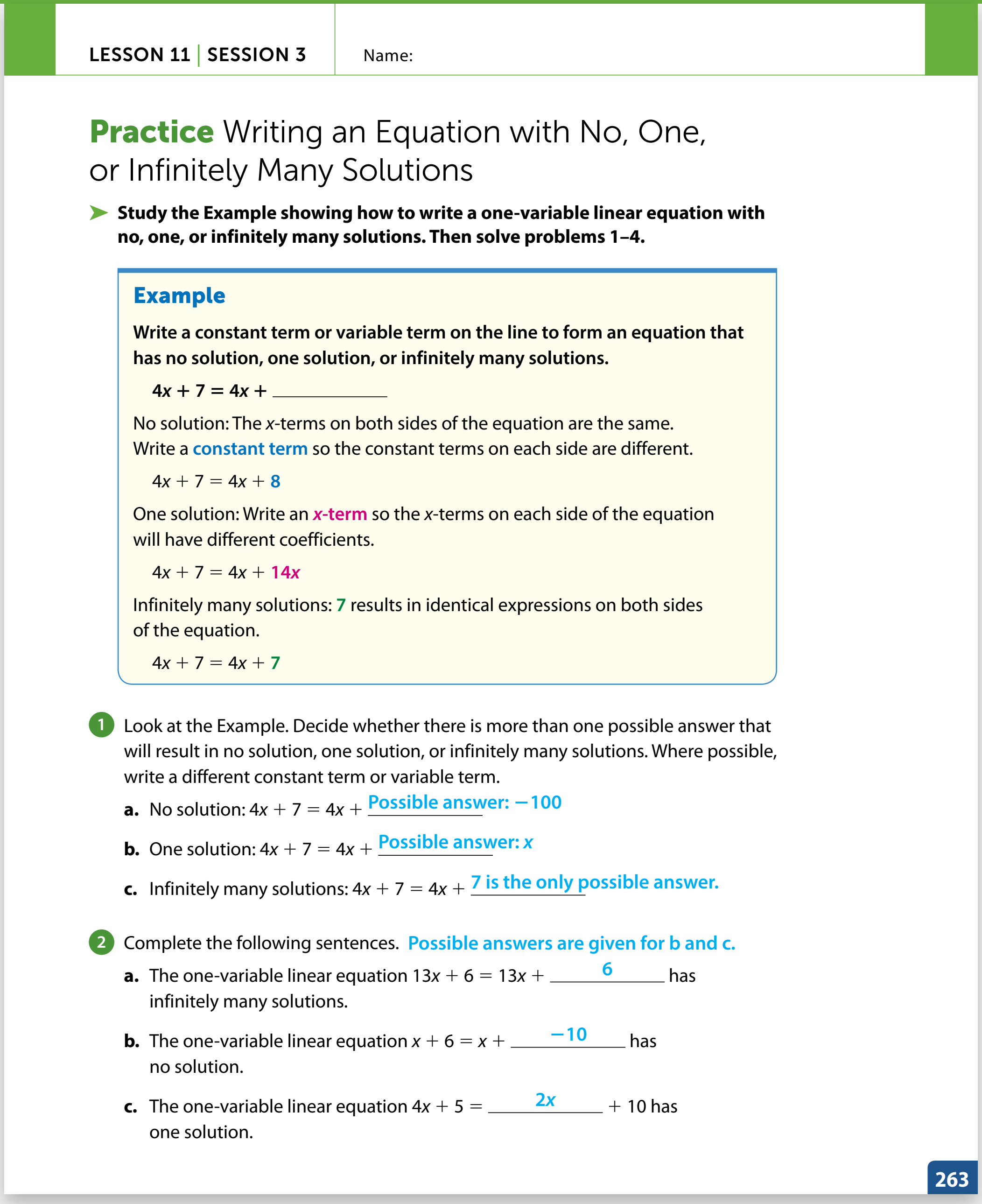
b. Students should recognize that the x-terms need to be different. They can insert any x-term except $0x$. ***Medium***

c. Students recognize that the constant terms need to be the same. ***Basic***

② **a.** Students should recognize that the constant terms need to be the same. ***Basic***

b. Students should recognize that the constant terms need to be different. They can insert any constant other than 6. ***Basic***

c. Students should recognize that the x-terms need to be different. They can insert any x-term other than $4x$. ***Basic***



LESSON 11 | SESSION 3 Name:

Practice Writing an Equation with No, One, or Infinitely Many Solutions

➤ **Study the Example showing how to write a one-variable linear equation with no, one, or infinitely many solutions. Then solve problems 1–4.**

Example

Write a constant term or variable term on the line to form an equation that has no solution, one solution, or infinitely many solutions.

$4x + 7 = 4x +$ ______

No solution: The x-terms on both sides of the equation are the same. Write a constant term so the constant terms on each side are different.

$4x + 7 = 4x + 8$

One solution: Write an x-term so the x-terms on each side of the equation will have different coefficients.

$4x + 7 = 4x + 14x$

Infinitely many solutions: 7 results in identical expressions on both sides of the equation.

$4x + 7 = 4x + 7$

① Look at the Example. Decide whether there is more than one possible answer that will result in no solution, one solution, or infinitely many solutions. Where possible, write a different constant term or variable term.

a. No solution: $4x + 7 = 4x +$ Possible answer: -100

b. One solution: $4x + 7 = 4x +$ Possible answer: x

c. Infinitely many solutions: $4x + 7 = 4x +$ 7 is the only possible answer.

② Complete the following sentences. Possible answers are given for b and c.

a. The one-variable linear equation $13x + 6 = 13x +$ ___6___ has infinitely many solutions.

b. The one-variable linear equation $x + 6 = x +$ ___-10___ has no solution.

c. The one-variable linear equation $4x + 5 =$ ___$2x$___ $+ 10$ has one solution.

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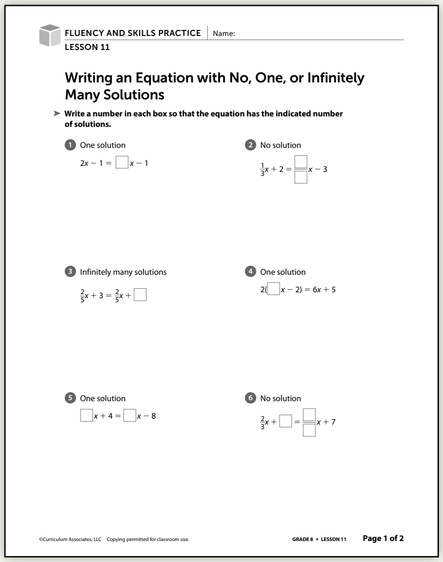
Fluency & Skills Practice

Writing an Equation with No, One, or Infinitely Many Solutions

In this activity, students fill in missing numbers in equations so that each equation has either no solution, one solution, or infinitely many solutions.



FLUENCY AND SKILLS PRACTICE | Name:
LESSON 11

Writing an Equation with No, One, or Infinitely Many Solutions

➤ **Write a number in each box so that the equation has the indicated number of solutions.**

① One solution
$2x - 1 = \square x - 1$

② No solution
$\frac{1}{3}x + 2 = \frac{\square}{\square}x - 3$

③ Infinitely many solutions
$\frac{2}{5}x + 3 = \frac{2}{5}x + \square$

④ One solution
$2(\square x - 2) = 6x + 5$

⑤ One solution
$\square x + 4 = \square x - 8$

⑥ No solution
$\frac{2}{3}x + \square = \frac{\square}{\square}x + 7$

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Additional Practice

③ See ***Connect to Culture*** to support student engagement.

a. Students may also solve to get $0 = 0$. This is not the most efficient method, but it does result in the correct answer. ***Medium***

b. Students may solve by reasoning that each side length of the garden can be set equal to 0 to find the value of x that makes that length equal to 0. The value of x must be greater than the least of those values. ***Challenge***

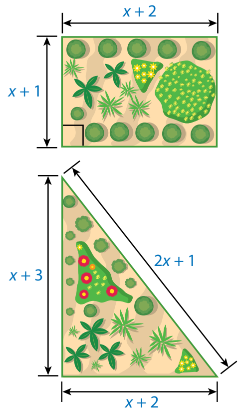
④ **a.** Students may solve this problem by recognizing that the constant terms need to be different. They can write an expression with a variable term of $2h$ and a constant term of any number other than 6. ***Medium***

b. Students may solve this problem by recognizing that the h-terms need to be different and the constant terms can be the same or different. They can create an expression with an h-term other than $2h$. ***Medium***

c. Students may solve this problem by recognizing that the h-terms and constant terms need to be the same. ***Medium***

LESSON 11 | SESSION 3

③ Two garden beds are shown. The perimeters of the two gardens are equal.



a. Write an equation that sets the perimeters equal. Then solve the equation.

$2(x + 2) + 2(x + 1) = (2x + 1) + (x + 3) + (x + 2)$

$2x + 4 + 2x + 2 = 2x + 1 + x + 3 + x + 2$

$4x + 6 = 4x + 6$

infinitely many solutions

b. The side length of a garden cannot be a negative number or zero. What value(s) of x make the equation you wrote in problem 3a true in the context of this problem?

x can be any number that keeps all side lengths positive; x can be any number greater than -0.5.

④ Write an expression on the line to form an equation that has no solution, one solution, or infinitely many solutions.

a. No solution

$2(h + 3) =$ Possible answer: $2h + 7$

b. One solution

$2h + 5 =$ Possible answer: $6h + 10$

c. Infinitely many solutions

$2h - 12 =$ Possible answer: $2h - 12$

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DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Use with **Session 4** **Apply It**

Levels 1–3: Reading/Writing

To help students explain whether Vivian is correct, read Apply It problem 7 aloud. Ask students to think for a minute about the two lengths of ribbon. Record the sentences and have partners read and complete the sentence with *always*, *sometimes*, or *never*:

- *If the solid ribbon costs 6p and the striped ribbon costs 6p+9, then the striped ribbon _____ costs more than the solid ribbon.*

Have students turn to a partner to discuss what is incorrect about Vivian's thinking:

- *Vivian thinks the costs are always/sometimes/never equal.*
- *This is incorrect because _____.*

Levels 2–4: Reading/Writing

To help students explain whether Vivian is correct, read Apply It problem 7 with students. Ask students to think for a minute about the two costs of ribbon. Then ask them to work with a partner to read and complete the following sentence with *always*, *sometimes*, or *never*:

- *If the solid ribbon costs 6p and the striped ribbon costs 6p+9, then the striped ribbon _____ costs more than the solid ribbon.*

Have students turn to a partner to discuss Vivian's thinking. Provide these sentence starters:

- *Vivian thinks the cost of both kinds of ribbons is _____. This is not correct because _____.*

Levels 3–5: Reading/Writing

To help students explain whether Vivian is correct, have students read Apply It problem 7. Ask them to construct a written response to the question. Encourage them to use precise math language to describe why Vivian's reasoning is incorrect.

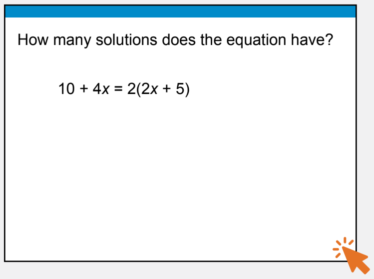
Have students share answers with a partner. Encourage partners to give each other feedback and revise their responses when appropriate. Have students read the answers and build on by adding math terminology and describing the lengths of the ribbons.

Refine Determining the Number of Solutions to One-Variable Equations

Purpose

- **Refine** strategies for determining the number of solutions to one-variable linear equations.
- **Refine** understanding of how to write one-variable linear equations with no, one, or infinitely many solutions.

START CHECK FOR UNDERSTANDING



How many solutions does the equation have?

$10 + 4x = 2(2x + 5)$

Solution

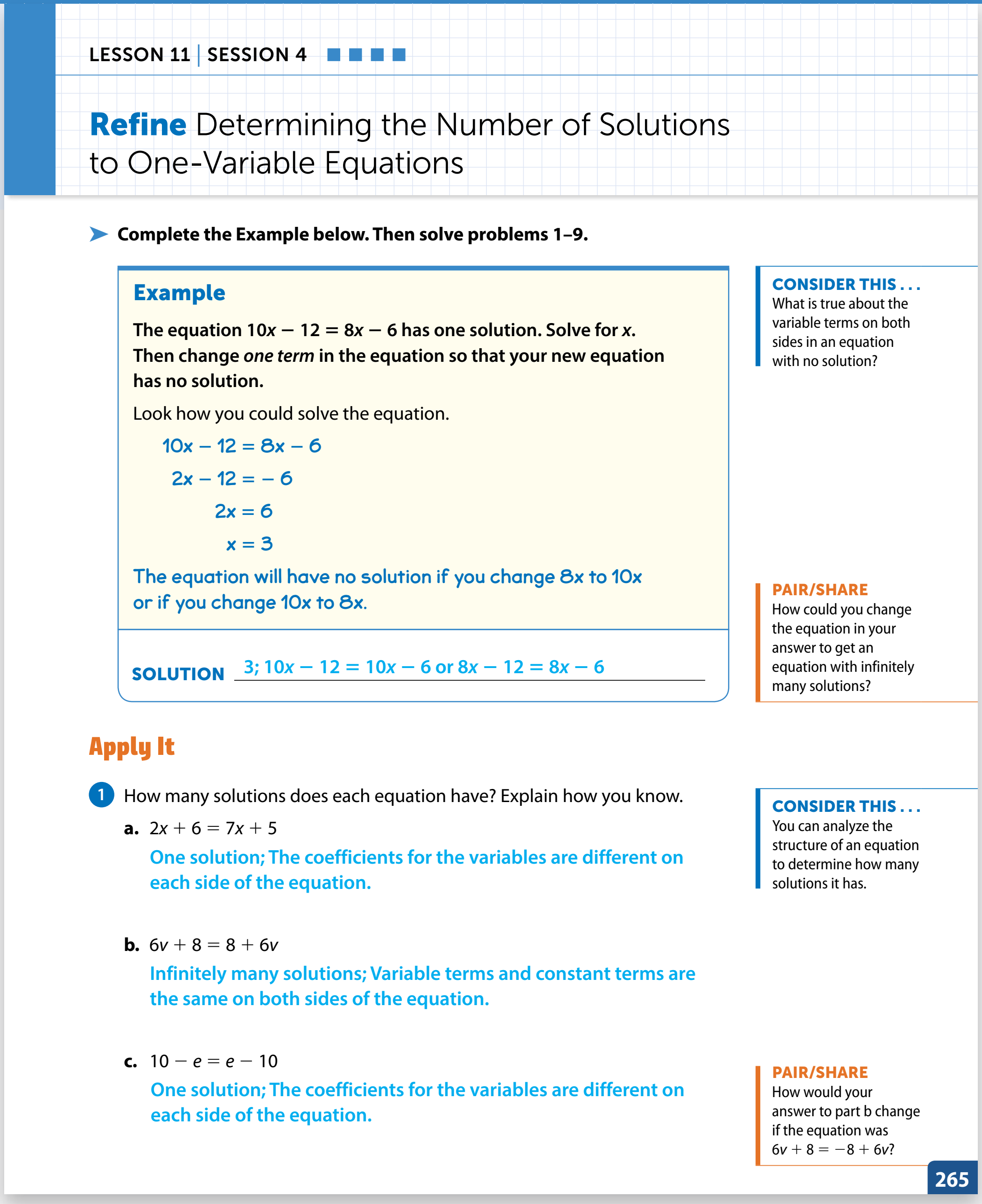
infinitely many solutions

WHY? Confirm students' understanding of determining the number of solutions to a one-variable equation, identifying common errors to address as needed.

MONITOR & GUIDE

Before students begin to work, use their responses to the **Start** to determine those who will benefit from additional support. Use the **Error Analysis** table below to guide remediation.

Have all students complete the Example and problems 1–3, using Consider This and Pair/Share as appropriate. Observe and monitor their reasoning and guide or redirect students as needed.



LESSON 11 | SESSION 4 ■■■■

Refine Determining the Number of Solutions to One-Variable Equations

➤ **Complete the Example below. Then solve problems 1–9.**

Example

The equation $10x - 12 = 8x - 6$ has one solution. Solve for x. Then change *one term* in the equation so that your new equation has no solution.

Look how you could solve the equation.

$$10x - 12 = 8x - 6$$
$$2x - 12 = -6$$
$$2x = 6$$
$$x = 3$$

The equation will have no solution if you change $8x$ to $10x$ or if you change $10x$ to $8x$.

SOLUTION 3; $10x - 12 = 10x - 6$ or $8x - 12 = 8x - 6$

CONSIDER THIS . . . What is true about the variable terms on both sides in an equation with no solution?

PAIR/SHARE How could you change the equation in your answer to get an equation with infinitely many solutions?

Apply It

① How many solutions does each equation have? Explain how you know.

a. $2x + 6 = 7x + 5$

One solution; The coefficients for the variables are different on each side of the equation.

b. $6v + 8 = 8 + 6v$

Infinitely many solutions; Variable terms and constant terms are the same on both sides of the equation.

c. $10 - e = e - 10$

One solution; The coefficients for the variables are different on each side of the equation.

CONSIDER THIS . . . You can analyze the structure of an equation to determine how many solutions it has.

PAIR/SHARE How would your answer to part b change if the equation was $6v + 8 = -8 + 6v$?

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START ERROR ANALYSIS

If the error is . . .	Students may . . .	To support understanding . . .
no solution	not have distributed 2 to all terms in the parentheses.	Ask students to model the original equation using algebra tiles to help them determine the solution.
one solution	have combined the constant and variable terms on the left side of the equation thinking them like terms.	Review the meaning of like terms with students. Have them perform the distribution. Then have them identify the variable terms and the constant terms and ask which terms can be combined.

Example

Guide students in understanding the Example. Ask:

- *What is true about the variable terms on both sides of an equation that has no solution?*
- *What is true about the constants on both sides of an equation that has no solution?*

Help all students focus on the Example and responses to the questions by reminding students that good listeners use engaged body language, such as looking at the speaker and nodding to show understanding.

Look for understanding that if both sides of an equation have the same variable terms and different constants, the equation has no solution.

Apply It

① **a.** *DOK 1*

b. *DOK 1*

c. Students should realize that the variable terms are different. The variable term on the left is $-e$, while the variable term on the right is e. So, the equation has one solution. *DOK 1*

② **a.** Students may write any x term other than $0x$ on the line to create an equation with different variable terms on the two sides. *DOK 1*

b. Students may write any constant except -3 on the line. *DOK 1*

c. Students must write -3 on the line to create an equation with an infinite number of solutions. *DOK 1*

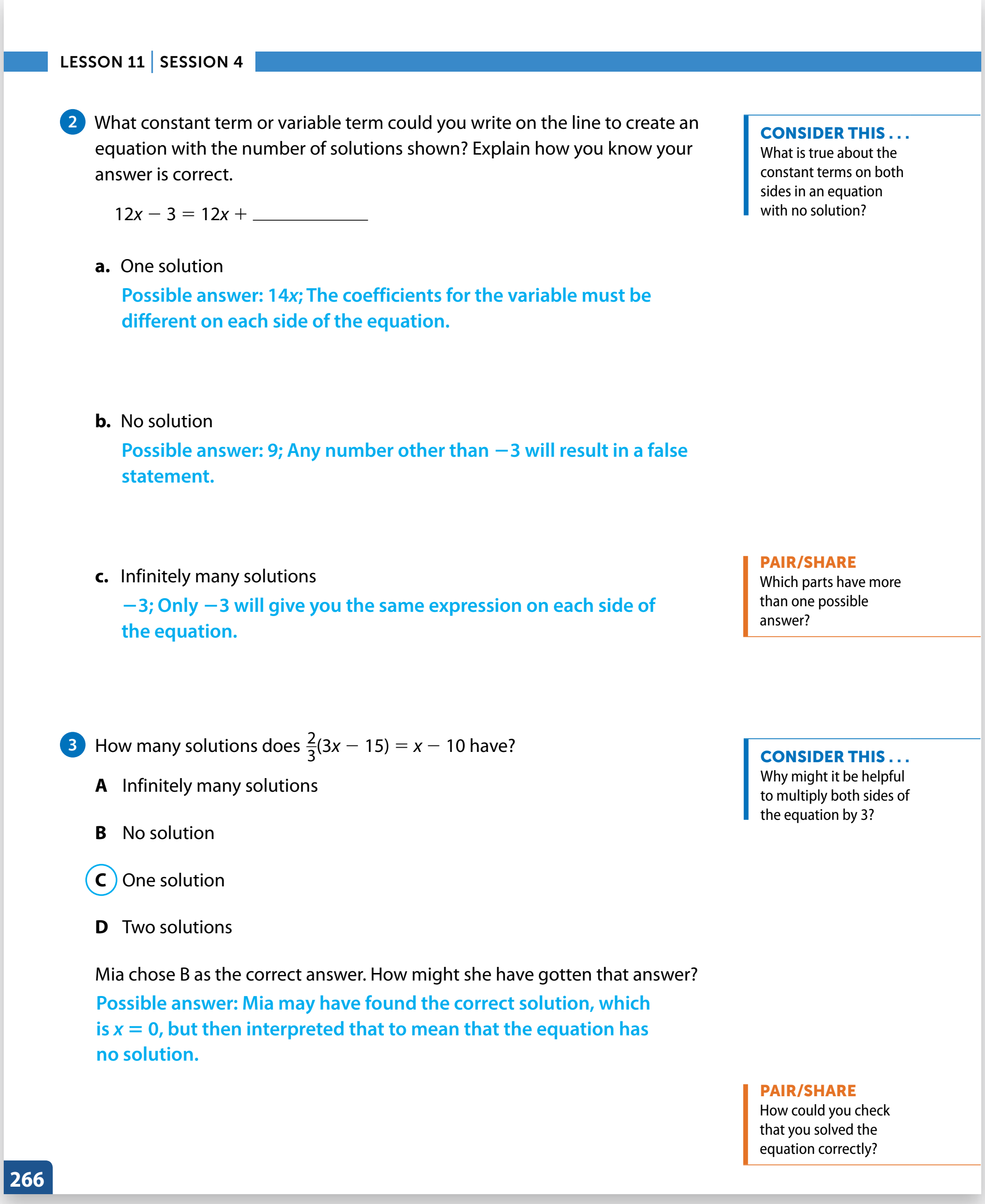
③ **C** **is correct**. Students may solve the problem by first using the distributive property to eliminate the fraction and then solving the equation for x.

A is not correct. The variable terms and constants on each side of the equation are not the same.

B is not correct. The solution is $x = 0$.

D is not correct. This answer is not possible; a linear equation cannot have only 2 solutions.

DOK 3



LESSON 11 | SESSION 4

② What constant term or variable term could you write on the line to create an equation with the number of solutions shown? Explain how you know your answer is correct.

$12x - 3 = 12x +$ ____________

a. One solution

Possible answer: 14x; The coefficients for the variable must be different on each side of the equation.

b. No solution

Possible answer: 9; Any number other than −3 will result in a false statement.

c. Infinitely many solutions

−3; Only −3 will give you the same expression on each side of the equation.

CONSIDER THIS . . .
What is true about the constant terms on both sides in an equation with no solution?

PAIR/SHARE
Which parts have more than one possible answer?

③ How many solutions does $\frac{2}{3}(3x - 15) = x - 10$ have?

A Infinitely many solutions

B No solution

[✓ **C**] One solution

D Two solutions

Mia chose B as the correct answer. How might she have gotten that answer?

Possible answer: Mia may have found the correct solution, which is $x = 0$, but then interpreted that to mean that the equation has no solution.

CONSIDER THIS . . .
Why might it be helpful to multiply both sides of the equation by 3?

PAIR/SHARE
How could you check that you solved the equation correctly?

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GROUP & DIFFERENTIATE

Identify groupings for differentiation based on the **Start** and problems 1–3. A recommended sequence of activities for each group is suggested below. Use the resources on the next page to differentiate and close the lesson.

Approaching Proficiency

- **RETEACH** Visual Model
- **REINFORCE** Problems 5, 6, 8

Meeting Proficiency

- **REINFORCE** Problems 4–8

Extending Beyond Proficiency

- **REINFORCE** Problems 4–8
- **EXTEND** Challenge

Have all students complete the **Close: Exit Ticket.**

Resources for Differentiation are found on the next page.

Refine Determining the Number of Solutions to One-Variable Equations

Apply It

④ **a.** For an equation to have one solution, the variable terms cannot be the same. *DOK 2*

b. For an equation to have infinitely many solutions, the variable terms and constants must be the same. *DOK 2*

c. For an equation to have no solution, the variable terms must be the same and the constants different. *DOK 2*

⑤ **C is correct.** The expressions have the same variable terms and different constant terms.

A is not correct. This expression has the same terms. The equation has infinitely many solutions.

B is not correct. This expression has a different variable term. The equation has one solution.

D is not correct. This expression has a different variable term. The equation has one solution.

DOK 1

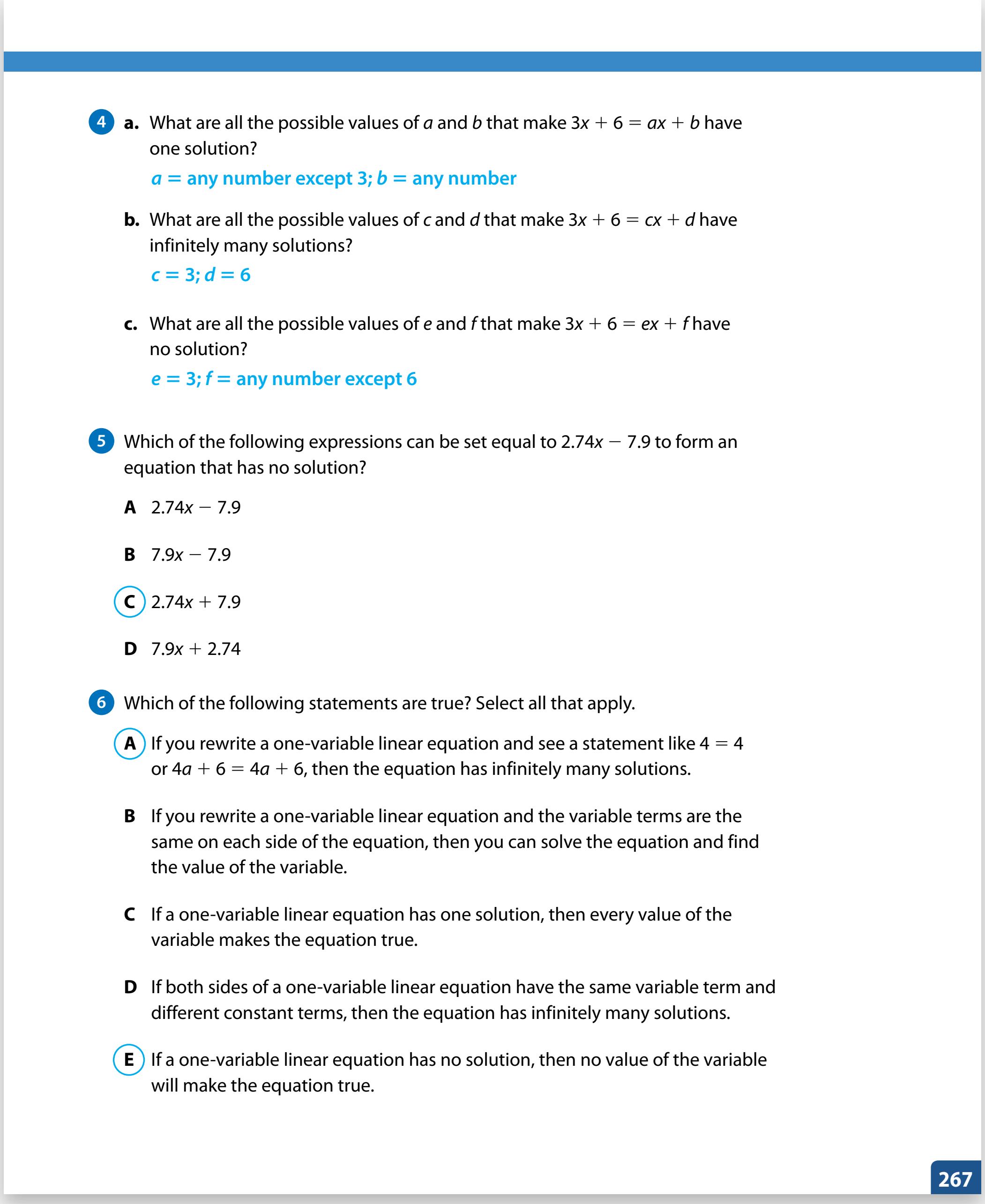
⑥ **A and E are correct.** An equation resulting in a true statement has infinitely many solutions. An equation resulting in a false statement has no solution.

B is not correct. This will either have no solution or infinitely many solutions.

C is not correct. Only one value makes it true.

D is not correct. It will have no solution.

DOK 2



④ **a.** What are all the possible values of a and b that make $3x + 6 = ax + b$ have one solution?

a = any number except 3; b = any number

b. What are all the possible values of c and d that make $3x + 6 = cx + d$ have infinitely many solutions?

$c = 3$; $d = 6$

c. What are all the possible values of e and f that make $3x + 6 = ex + f$ have no solution?

$e = 3$; f = any number except 6

⑤ Which of the following expressions can be set equal to $2.74x - 7.9$ to form an equation that has no solution?

A $2.74x - 7.9$

B $7.9x - 7.9$

[✓ **C**] $2.74x + 7.9$

D $7.9x + 2.74$

⑥ Which of the following statements are true? Select all that apply.

[✓ **A**] If you rewrite a one-variable linear equation and see a statement like $4 = 4$ or $4a + 6 = 4a + 6$, then the equation has infinitely many solutions.

B If you rewrite a one-variable linear equation and the variable terms are the same on each side of the equation, then you can solve the equation and find the value of the variable.

C If a one-variable linear equation has one solution, then every value of the variable makes the equation true.

D If both sides of a one-variable linear equation have the same variable term and different constant terms, then the equation has infinitely many solutions.

[✓ **E**] If a one-variable linear equation has no solution, then no value of the variable will make the equation true.

DIFFERENTIATION

RETEACH

Visual Model

Model equations with infinitely many solutions or no solution.

Students approaching proficiency with finding the number of solutions to a one-variable linear equation will benefit from solving equations by using a model.

- Display the equation $4x + 8 = 2(4 + 2x)$.
- Invite a student to simplify the right side of the equation.
- Ask: *How does the right side of the equation compare to the left side once the right side is simplified using the distributive property?* [The right side will have the same terms as the left side.]
- Ask: *What does this relationship tell you?* [The equation $4x + 8 = 2(4 + 2x)$ has infinitely many solutions.]
- Display the equation $3(x + 2) = x + 2(x + 1)$.
- Invite a student to simplify the left side of the equation. Then invite a student to simplify the right side of the equation.
- Ask: *How does the left side of the equation compare to the right side once the sides are simplified?* [The sides have the same variable terms but different constant terms.]
- Ask: *What does this relationship tell you?* [The equation $3(x + 2) = x + 2(x + 1)$ has no solution.]

7 See ***Connect to Culture*** to support student engagement. Students may analyze the structure of the equation and compare the variable terms and the constants on each side. *DOK 3*

8 **a.** Students may write an equation that has the same variable terms but different constants on each side. *DOK 2*

b. Students may write an equation that has the same variable terms and constants on each side of the equation. *DOK 2*

CLOSE **EXIT TICKET**

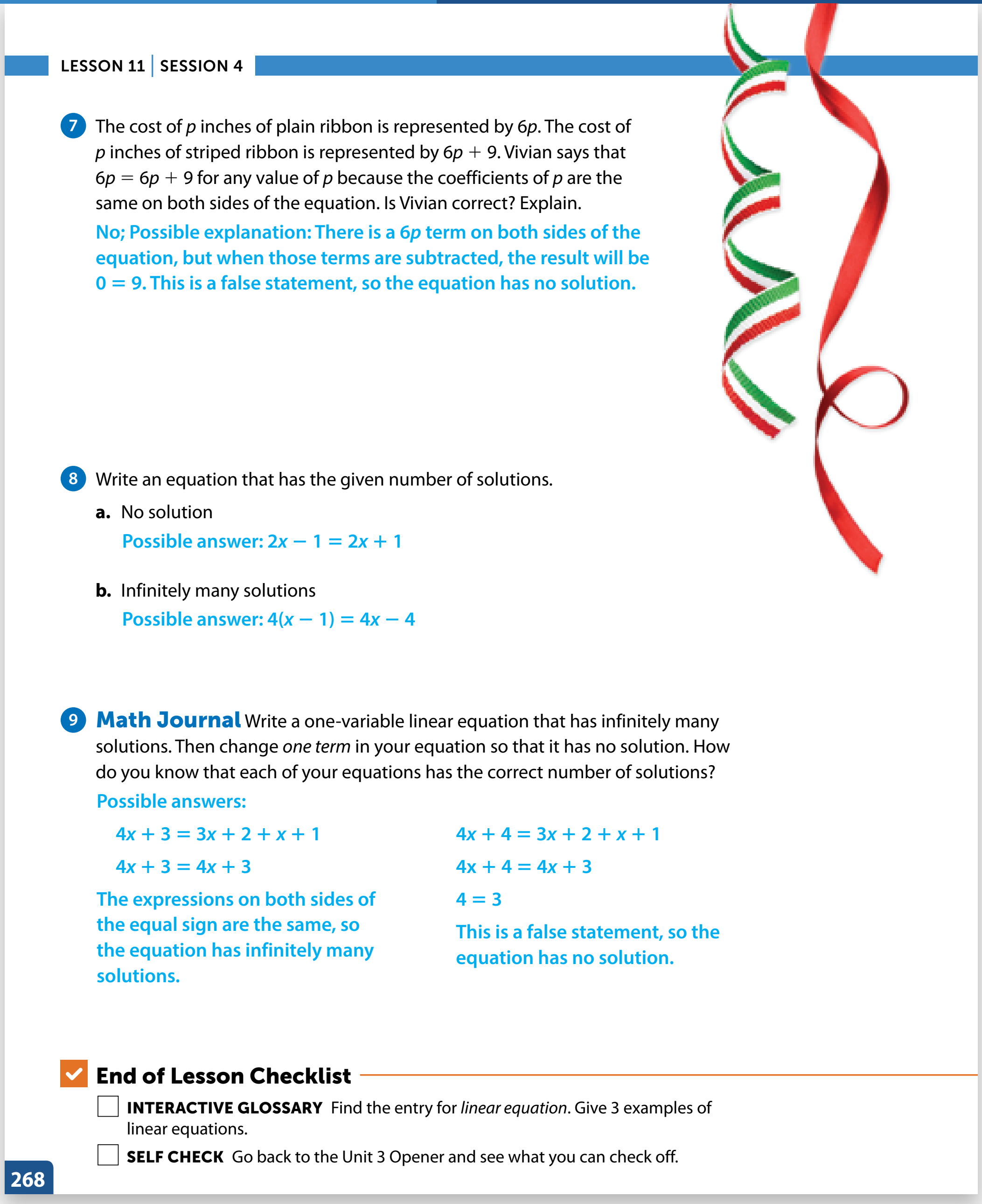
9 **Math Journal** Look for understanding of how the variable terms and constant terms on each side of a one-variable equation compare to get the desired number of solutions. *DOK 3*

Error Alert If students write an equation that simplifies to $x = 0$, then remind them that $x = 0$ means the solution is 0, not that the equation has no solution. Students can substitute 0 for x to verify this.

End of Lesson Checklist

INTERACTIVE GLOSSARY Support students by suggesting they make a list of similarities of the three linear equations they gave.

SELF CHECK Have students review and check off any new skills on the Unit 3 Opener.



LESSON 11 | SESSION 4

7 The cost of p inches of plain ribbon is represented by $6p$. The cost of p inches of striped ribbon is represented by $6p + 9$. Vivian says that $6p = 6p + 9$ for any value of p because the coefficients of p are the same on both sides of the equation. Is Vivian correct? Explain.

No; Possible explanation: There is a $6p$ term on both sides of the equation, but when those terms are subtracted, the result will be $0 = 9$. This is a false statement, so the equation has no solution.

8 Write an equation that has the given number of solutions.

a. No solution

Possible answer: $2x - 1 = 2x + 1$

b. Infinitely many solutions

Possible answer: $4(x - 1) = 4x - 4$

9 **Math Journal** Write a one-variable linear equation that has infinitely many solutions. Then change *one term* in your equation so that it has no solution. How do you know that each of your equations has the correct number of solutions?

Possible answers:

$4x + 3 = 3x + 2 + x + 1$
$4x + 3 = 4x + 3$

The expressions on both sides of the equal sign are the same, so the equation has infinitely many solutions.

$4x + 4 = 3x + 2 + x + 1$
$4x + 4 = 4x + 3$
$4 = 3$

This is a false statement, so the equation has no solution.

End of Lesson Checklist

☐ **INTERACTIVE GLOSSARY** Find the entry for *linear equation*. Give 3 examples of linear equations.

☐ **SELF CHECK** Go back to the Unit 3 Opener and see what you can check off.

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REINFORCE

Problems 4–8

Solve one-variable linear equations.

Students meeting proficiency will benefit from additional work with determining the number of solutions to one-variable linear equations by solving problems in a variety of formats.

- Have students work on their own or with a partner to solve the problems.
- Encourage students to show their work.

EXTEND

Challenge

Solve critical thinking questions about one-variable linear equations.

Students extending beyond proficiency will benefit from writing true statements about a one-variable equation.

- Have students work with a partner to solve this problem: *Given the equation $a(2x + b) = -3 + 12x$, find values of a and b that result in an equation with infinitely many solutions and an equation with no solution.*
- Some students may first use the distributive property to simplify the left side of the equation. Explain that they can set the variable terms equal to each other and can set the constant terms equal to each other.

PERSONALIZE

i-Ready

Provide students with opportunities to work on their personalized instruction path with *i-Ready* Online Instruction to:

- fill prerequisite gaps.
- build up grade-level skills.

LESSON 14

Overview | Represent and Solve Problems with Systems of Linear Equations

STANDARDS FOR MATHEMATICAL PRACTICE (SMP)

SMP 1, 2, 3, 4, 5, and 6 are integrated into the Try-Discuss-Connect routine.*

This lesson provides additional support for:

2 Reason abstractly and quantitatively.

3 Construct viable arguments and critique the reasoning of others.

* See page 1o to learn how every lesson includes these SMP.

Objectives

Content Objectives

- Represent mathematical and real-world problems with two related linear equations in two variables.
- Graph systems of linear equations to estimate solutions.
- Solve systems of linear equations algebraically.
- Understand that a system of linear equations may have one solution, no solution, or infinitely many solutions.

Language Objectives

- Read and interpret problems that include phrases in the form of *Let* <variable> *be* to write systems of linear equations.
- Describe how the graph of a system of linear equations represents the solution.
- Explain how to solve a system of equations using lesson vocabulary and the terms *substitution* and *elimination*.
- Explain how graphs and slopes help determine whether or where two lines intersect.
- Participate in partner conversations by using passive and active voice phrases such as *represented by* and *represents* to connect variables and equations with quantities or situations.

Prior Knowledge

- Graph a system of linear equations to determine its solution.
- Solve a system of linear equations algebraically.

Vocabulary

Math Vocabulary

There is no new vocabulary. Review the following key terms.

expression a group of numbers, variables, and/or operation symbols that represents a mathematical relationship. An expression without variables, such as $3 + 4$, is called a *numerical expression*. An expression with variables, such as $5b^2$, is called an *algebraic expression*.

parallel lines lines that are always the same distance apart and never intersect.

system of linear equations a group of related linear equations in which a solution makes all the equations true at the same time. A system of equations can have zero, one, or infinitely many solutions.

***y*-intercept** the *y*-coordinate of the point where a line, or graph of a function, intersects the *y*-axis.

Academic Vocabulary

determine to decide something based on evidence or facts.

Learning Progression

Earlier in Grade 8, students wrote linear equations to represent real-world situations. They estimated solutions to systems of linear equations using graphs and solved systems of equations algebraically using substitution and elimination. Students solved systems of equations that represent real-world problems and interpreted the meanings of the equations and solutions in the context of the problem.

➤

In this lesson, students write and solve systems of two linear equations that model real-world situations and mathematical problems.

➤

Later in Grade 8, students will learn how to model real-world situations with linear functions.

LESSON 14

Overview

Pacing Guide

Items marked with 🖱 are available on the **Teacher Toolbox**.

	MATERIALS	DIFFERENTIATION
SESSION 1 **Explore** Representing and Solving Problems with Systems of Linear Equations (35–50 min)		
- **Start** (5 min) - **Try It** (5–10 min) - **Discuss It** (10–15 min) - **Connect It** (10–15 min) - **Close: Exit Ticket** (5 min) **Additional Practice** (pages 313–314)	**Math Toolkit** counters, graph paper, straightedges Presentation Slides 🖱	**PREPARE** Interactive Tutorial 🖱 **RETEACH or REINFORCE** Hands-On Activity **Materials** For each group: 30 two-color counters
SESSION 2 **Develop** Solving Real-World Problems with Systems of Linear Equations (45–60 min)		
- **Start** (5 min) - **Try It** (10–15 min) - **Discuss It** (10–15 min) - **Connect It** (15–20 min) - **Close: Exit Ticket** (5 min) **Additional Practice** (pages 319–320)	**Math Toolkit** counters, graph paper, straightedges Presentation Slides 🖱	**RETEACH or REINFORCE** Visual Model **Materials** For display: 6 nickels and 4 dimes **REINFORCE** Fluency & Skills Practice 🖱 **EXTEND** Deepen Understanding
SESSION 3 **Develop** Solving Mathematical Problems Involving Systems of Linear Equations (45–60 min)		
- **Start** (5 min) - **Try It** (10–15 min) - **Discuss It** (10–15 min) - **Connect It** (15–20 min) - **Close: Exit Ticket** (5 min) **Additional Practice** (pages 325–326)	**Math Toolkit** graph paper, straightedges Presentation Slides 🖱	**RETEACH or REINFORCE** Hands-On Activity **Materials** For each student: 1 geoboard, 4 rubber bands **REINFORCE** Fluency & Skills Practice 🖱 **EXTEND** Deepen Understanding
SESSION 4 **Refine** Representing and Solving Problems with Systems of Linear Equations (45–60 min)		
- **Start** (5 min) - **Monitor & Guide** (15–20 min) - **Group & Differentiate** (20–30 min) - **Close: Exit Ticket** (5 min)	**Math Toolkit** Have items from previous sessions available for students. Presentation Slides 🖱	**RETEACH** Visual Model **REINFORCE** Problems 4–8 **EXTEND** Challenge **PERSONALIZE** i-Ready
Lesson 14 Quiz 🖱 or **Digital Comprehension Check**		**RETEACH** Tools for Instruction 🖱 **REINFORCE** Math Center Activity 🖱 **EXTEND** Enrichment Activity 🖱

LESSON 14

Overview | Represent and Solve Problems with Systems of Linear Equations

Connect to Culture

Use these activities to connect with and leverage the diverse backgrounds and experiences of all students. Engage students in sharing what they know about contexts before you add the information given here.

SESSION 1 ■ □ □ □

Try It Ask students to share a goal for which they had to save money and have them describe how they saved the money for that goal. There are many personal preferences on the best way to save and keep track of money. There are mobile phone applications available that can help users work toward a savings goal. They may indicate how much is saved and how many more weeks it will be until a savings goal is reached. Some savers opt to open an additional bank account and transfer money each week or month into that account until a goal is met. Others may withdraw or keep cash on hand and store it in a safe place until enough money is saved. Whichever method is used to save money toward a goal, there is a certain amount of self-discipline required to be successful.

SESSION 2 ■ ■ □ □

Try It Restaurants put a lot of thought into the seating arrangements they use. Tables that seat two and four people are fairly common, and pushing several smaller tables together can be done to accommodate a large group of people. Restaurants can rearrange tables and chairs depending on the diners they are serving. Flexible seating can make it easier to accommodate diners in wheelchairs. Ask students about the seating in the school cafeteria and how it compares to seating in a restaurant. Why do they think the school chose the arrangement of seats that they have?

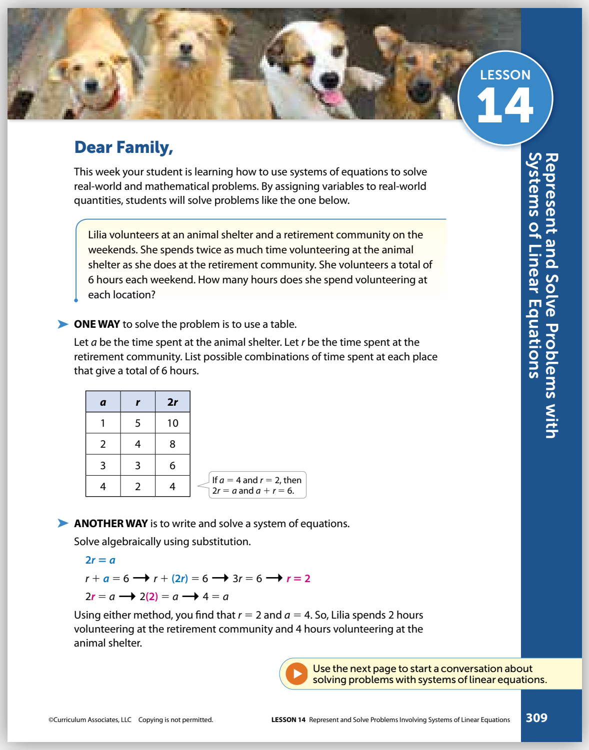
Apply It **Problem 5** Ask students whether they have been to a rowing event such as a regatta. Invite them to share their experiences. The Head of the Charles Regatta is a rowing race that takes place at the banks of the Charles River in Massachusetts. The race began in 1965, and since then, it has grown into a two-day event in which more than 11,000 people participate in rowing competitions. It is considered the world's largest rowing event and draws over 300,000 spectators.

SESSION 4 ■ ■ ■ ■

Apply It **Problem 4** Ask students to describe things they may have done to raise money or collect donations for charity. Many knitters make more knitted items than they can use or give to family and friends, so they may try knitting for charity. Although hats and scarves are certainly accepted by many charities, there are a lot more opportunities available to creative crafters. Wildlife rescue organizations may use knitted bird nests, which are softer and less likely to injure a baby bird than a wooden or plastic bowl. In Australia, knitters even contribute knitted pouches to help raise orphaned baby kangaroos.

Connect to Family and Community

➤ **After the Explore session, have students use the Family Letter to let their families know what they are learning and to encourage family involvement.**



LESSON 14

Represent and Solve Problems with Systems of Linear Equations

Dear Family,

This week your student is learning how to use systems of equations to solve real-world and mathematical problems. By assigning variables to real-world quantities, students will solve problems like the one below.

> Lilia volunteers at an animal shelter and a retirement community on the weekends. She spends twice as much time volunteering at the animal shelter as she does at the retirement community. She volunteers a total of 6 hours each weekend. How many hours does she spend volunteering at each location?

➤ **ONE WAY** to solve the problem is to use a table.

Let *a* be the time spent at the animal shelter. Let *r* be the time spent at the retirement community. List possible combinations of time spent at each place that give a total of 6 hours.

a	r	$2r$
1	5	10
2	4	8
3	3	6
4	2	4

If $a = 4$ and $r = 2$, then $2r = a$ and $a + r = 6$.

➤ **ANOTHER WAY** is to write and solve a system of equations.

Solve algebraically using substitution.

$2r = a$

$r + a = 6 \longrightarrow r + (2r) = 6 \longrightarrow 3r = 6 \longrightarrow r = 2$

$2r = a \longrightarrow 2(2) = a \longrightarrow 4 = a$

Using either method, you find that $r = 2$ and $a = 4$. So, Lilia spends 2 hours volunteering at the retirement community and 4 hours volunteering at the animal shelter.

Use the next page to start a conversation about solving problems with systems of linear equations.

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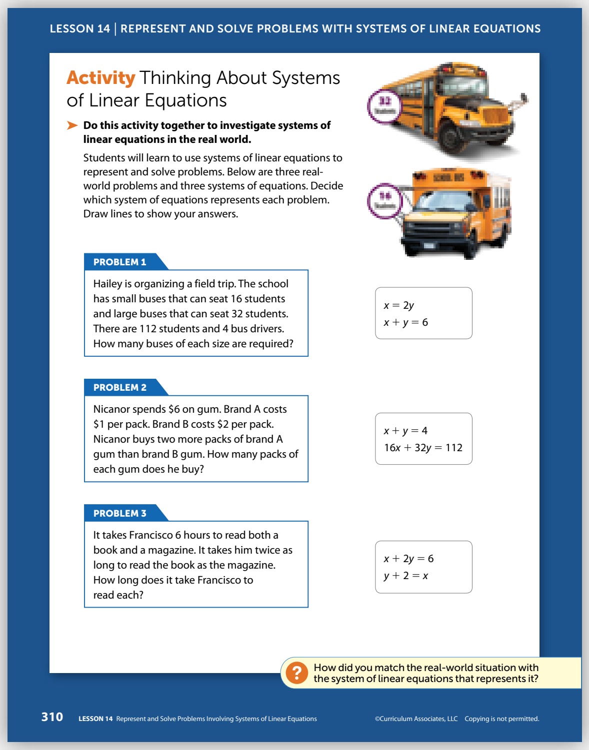

LESSON 14 | REPRESENT AND SOLVE PROBLEMS WITH SYSTEMS OF LINEAR EQUATIONS

Activity Thinking About Systems of Linear Equations

➤ **Do this activity together to investigate systems of linear equations in the real world.**

Students will learn to use systems of linear equations to represent and solve problems. Below are three real-world problems and three systems of equations. Decide which system of equations represents each problem. Draw lines to show your answers.

PROBLEM 1

Hailey is organizing a field trip. The school has small buses that can seat 16 students and large buses that can seat 32 students. There are 112 students and 4 bus drivers. How many buses of each size are required?

PROBLEM 2

Nicanor spends $6 on gum. Brand A costs $1 per pack. Brand B costs $2 per pack. Nicanor buys two more packs of brand A gum than brand B gum. How many packs of each gum does he buy?

PROBLEM 3

It takes Francisco 6 hours to read both a book and a magazine. It takes him twice as long to read the book as the magazine. How long does it take Francisco to read each?

$x = 2y$
$x + y = 6$

$x + y = 4$
$16x + 32y = 112$

$x + 2y = 6$
$y + 2 = x$

How did you match the real-world situation with the system of linear equations that represents it?

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Connect to Language

➤ **For English language learners, use the Differentiation chart to scaffold the language in each session. Use the Academic Vocabulary routine for academic terms before Session 1.**

DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS Use with **Session 1** **Connect It**

MATH TERM

An *equation* is a mathematical statement that uses an equal sign (=) to show that two expressions have the same value.

ACADEMIC VOCABULARY

A *quantity* is an amount or number of something that can be counted or measured.

Levels 1–3: Reading/Writing

Display the Math Term and Academic Vocabulary. Have students offer cognates for the words and add them to a **Co-Constructed Word Bank.** Then read Connect It problem 2. Have students circle words to add to the Word Bank, for example *unknown, different, system,* and *variable*. Have students refer to the bank as they answer problems 2a–b:

- *One quantity is _____ and the other is _____.*
- *There are two _____, so I need two _____.*

Ask students to identify the amounts given for each person and week and then write an equation for each person.

Levels 2–4: Reading/Writing

Display the Math Term and Academic Vocabulary. Read Connect It problem 2. Have students circle and share words to add to a **Co-Constructed Word Bank.** Encourage students to add cognates for the words. Then have them refer to the bank as they respond to problems 2a–b. Ask: *What are the quantities? How many unknowns are there? How many equations will you need?*

Then have them work with a partner to find and circle information from Try It that can help them write the expressions and equation in problems 2c–e. Ask: *What does the variable x represent? What does y represent?*

Levels 3–5: Reading/Writing

Display the Math Term and Academic Vocabulary. Have students read Connect It problem 2. Have students turn to partners to list and discuss the values they know based on Try It. Then have them read each part of problem 2 and tell how they can use the information from their lists to answer.

Allow think time for students to respond to problems 2a–d. Then ask partners to take turns explaining how each piece of information in the problem connects to an expression or equation they wrote.

Explore Representing and Solving Problems with Systems of Linear Equations

Purpose

- **Explore** the idea that a real-world problem with two unknown values can be solved mathematically.
- **Understand** that when there are two unknowns, a system of two equations is needed to find the solutions algebraically.

START CONNECT TO PRIOR KNOWLEDGE

Same and Different

A
$y = 2x + 1$
$y = 2x - 5$

B
$-2x + 2y = -6$
$x - y = 3$

C
$y = -3x + 8$
$y = \frac{1}{3}x - 2$

D
$y = x$
$8y = -7x + 15$

Possible Solutions

All show a system of linear equations.

Both equations in A and C are written in slope-intercept form.

A has no solution.

B has an infinite number of solutions.

C and D each have exactly one solution.

WHY? Support students' facility with recognizing the number of solutions for a system of equations.

TRY IT SMP 1, 2, 4, 5, 6

Make Sense of the Problem

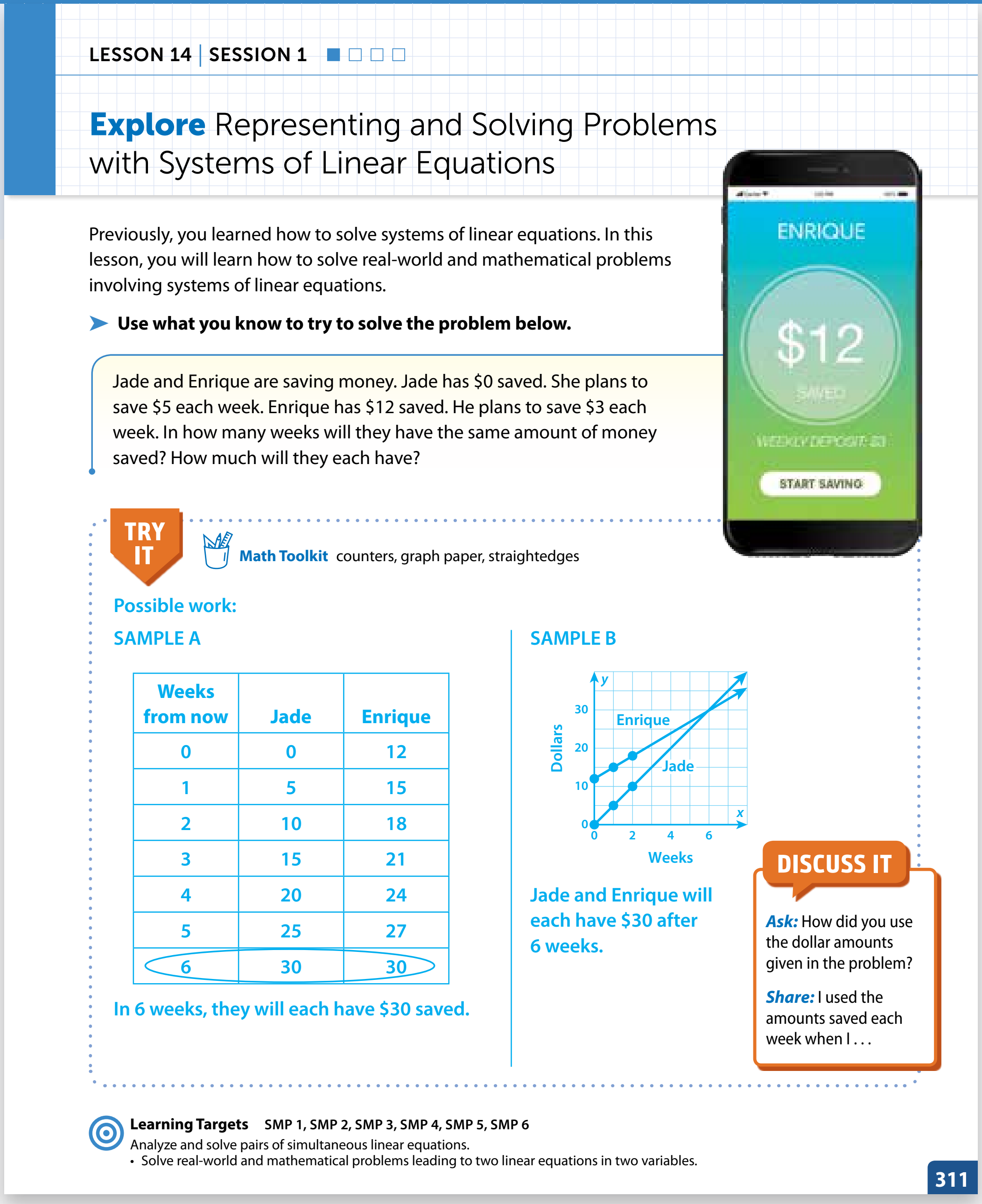
See ***Connect to Culture*** to support student engagement. Before students work on Try It, use **Say It Another Way** to help them make sense of the problem. Have students paraphrase the questions in the problem and explain in their own words what information has been provided.

DISCUSS IT SMP 2, 3, 6

Support Partner Discussion

After students work on Try It, have them respond to Discuss It with a partner. Listen for understanding that:

- \$0 and \$12 are the starting amounts Jade and Enrique have, respectively.
- \$5 is the amount added to Jade's total each week; 5 is the unit rate.
- \$3 is the amount added to Enrique's total each week; 3 is the unit rate.



LESSON 14 | SESSION 1 ■□□□

Explore Representing and Solving Problems with Systems of Linear Equations

Previously, you learned how to solve systems of linear equations. In this lesson, you will learn how to solve real-world and mathematical problems involving systems of linear equations.

➤ **Use what you know to try to solve the problem below.**

Jade and Enrique are saving money. Jade has \$0 saved. She plans to save \$5 each week. Enrique has \$12 saved. He plans to save \$3 each week. In how many weeks will they have the same amount of money saved? How much will they each have?

TRY IT **Math Toolkit** counters, graph paper, straightedges

Possible work:

SAMPLE A

Weeks from now	Jade	Enrique
0	0	12
1	5	15
2	10	18
3	15	21
4	20	24
5	25	27
6	30	30

In 6 weeks, they will each have \$30 saved.

SAMPLE B

Jade and Enrique will each have \$30 after 6 weeks.

DISCUSS IT

Ask: How did you use the dollar amounts given in the problem?

Share: I used the amounts saved each week when I . . .

Learning Targets SMP 1, SMP 2, SMP 3, SMP 4, SMP 5, SMP 6
Analyze and solve pairs of simultaneous linear equations.
- Solve real-world and mathematical problems leading to two linear equations in two variables.

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Common Misconception If students answer that Jade and Enrique have the same amount of money after 7 weeks, then they are likely treating the starting amounts as $x = 1$, not $x = 0$. Explain the difference between the money already saved before weekly savings begin and the money saved after weekly savings begin on Week 1.

Select and Sequence Student Strategies

Select 2–3 samples that represent the range of student thinking in your classroom. Here is one possible order for class discussion:

- table used to compare weekly savings
- (misconception) any model assigning the starting amounts to Week 1
- graphs showing weeks versus savings for Jade and Enrique
- single equation $5x = 12 + 3x$ used to find number of weeks
- system of equations written and solved algebraically

Facilitate Whole Class Discussion

Call on students to share selected strategies. Remind students that a good explanation describes what you did and why you decided to do it.

Guide students to **Compare and Connect** the representations. Have them explain what each part of their representation represents. Call on students to repeat key points in others' explanations.

> **ASK** *How did you show the amount of money Jade had at the end of each week? How did you show the amount of money Enrique had at the end of each week?*
>
> **LISTEN FOR** When using tables, lists of ordered pairs, or graphs, the x-value is the week number and the corresponding y-value shows the amount each person had at the end of that week.

CONNECT IT

SMP 2, 4, 5

① **Look Back** Look for understanding that the total amount of money saved by each person changes every week and that after a certain number of weeks, the amounts Jade and Enrique have saved will be the same.

DIFFERENTIATION | RETEACH OR REINFORCE

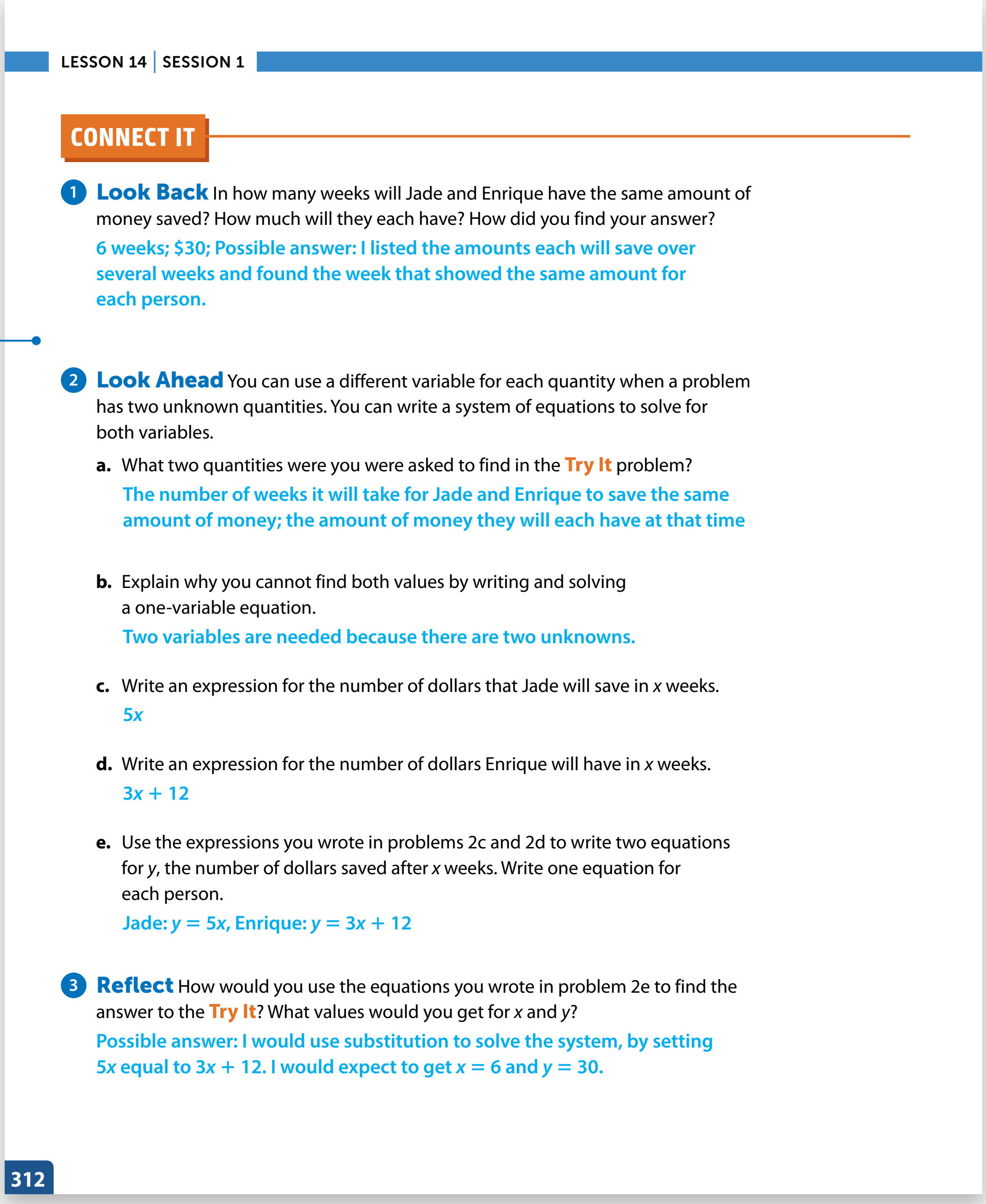
Hands-On Activity

Use counters to model a problem.

If students are unsure about how to find the number of weeks when the amounts are the same, then use this activity to model the situation.

Materials For each group: 30 two-color counters

- Instruct students that they will be modeling the problem in the Try It with counters.
- They should let each red counter represent \$5 and each yellow counter represent \$1.
- Assign one student to be Jade and one student to be Enrique in each group. Others in the group can hand them their weekly savings.
- Have a student give Enrique \$12. Have Jade and Enrique each report their savings for Week 0.
- Have students give Jade \$5 and Enrique \$3 to represent their savings in Week 1.
- Ask: *Do Jade and Enrique have the same amount of money?* [no]
- Continue handing Jade \$5 and Enrique \$3. After each repetition, have groups record the week number and current amounts. Then ask whether the amounts are the same.
- Once the amounts are the same, explain that the problem has been solved.



LESSON 14 | SESSION 1

CONNECT IT

① **Look Back** In how many weeks will Jade and Enrique have the same amount of money saved? How much will they each have? How did you find your answer?

6 weeks; \$30; Possible answer: I listed the amounts each will save over several weeks and found the week that showed the same amount for each person.

② **Look Ahead** You can use a different variable for each quantity when a problem has two unknown quantities. You can write a system of equations to solve for both variables.

a. What two quantities were you were asked to find in the **Try It** problem?

The number of weeks it will take for Jade and Enrique to save the same amount of money; the amount of money they will each have at that time

b. Explain why you cannot find both values by writing and solving a one-variable equation.

Two variables are needed because there are two unknowns.

c. Write an expression for the number of dollars that Jade will save in x weeks.

$5x$

d. Write an expression for the number of dollars Enrique will have in x weeks.

$3x + 12$

e. Use the expressions you wrote in problems 2c and 2d to write two equations for y, the number of dollars saved after x weeks. Write one equation for each person.

Jade: $y = 5x$, Enrique: $y = 3x + 12$

③ **Reflect** How would you use the equations you wrote in problem 2e to find the answer to the **Try It**? What values would you get for x and y?

Possible answer: I would use substitution to solve the system, by setting $5x$ equal to $3x + 12$. I would expect to get $x = 6$ and $y = 30$.

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② **Look Ahead** Encourage students to reread the Try It problem statement to see that two variables are needed, one to represent each person's savings because both the number of weeks and the amount saved are unknown. Students should also recognize that because there are two people with different starting amounts and different rates of saving, two equations are needed.

CLOSE **EXIT TICKET**

③ **Reflect** Look for understanding of an efficient method to solve the system of equations generated. Students are likely to write two equations that are each already solved for y, which makes substitution an efficient choice.

Error Alert If students get a solution of $x = 1.5$ and $y = 7.5$, then they most likely added $3x$ to both sides after substituting $5x$ for y. Have students think about whether their solution makes sense in context and encourage them to go back and check their work.

Prepare for Representing and Solving Problems with Systems of Linear Equations

Support Vocabulary Development

Assign **Prepare for Representing and Solving Problems with Systems of Linear Equations** as extra practice in class or as homework.

If you have students complete this in class, then use the guidance below.

Ask students to consider the term *y-intercept*. Have students think about where they may have heard the word *intercept* outside of math class and how the real-world meaning relates to the meaning of *y*-intercept.

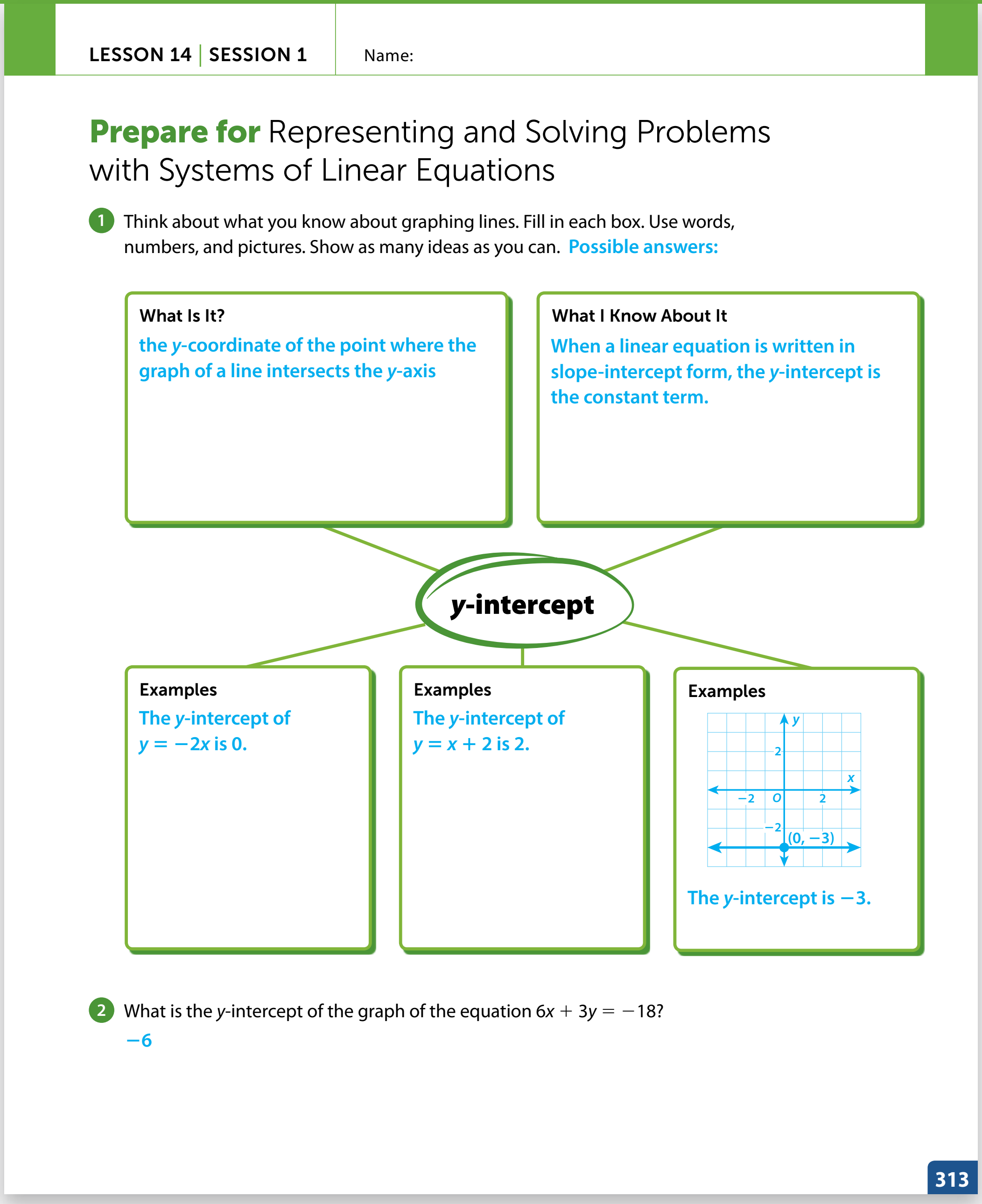
Have students work in pairs to complete the graphic organizer. Invite pairs to share their completed organizers and prompt a whole-class comparative discussion of the definitions, known information, and examples presented.

Have students look at the equation in problem 2 and discuss with a partner how to find the *y*-intercept. Encourage them to brainstorm one algebraic way and one graphical way to identify the *y*-intercept.

Problem Notes

① Students should understand that the *y*-intercept is the *y*-coordinate of the point where a graph intersects the *y*-axis. Student responses might include that the *y*-intercept is the value of *b* when an equation is rewritten in slope-intercept form. Students should recognize that the *x*-coordinate of the point where the *y*-intercept occurs is 0.

② Students may manipulate the equation so that it is in slope-intercept form, $y = -2x - 6$, and then identify the value of *b* as -6. They may also find the value of *y* when $x = 0$. Or, they may graph the equation using two points such as $(-3, 0)$ and $(1, -8)$ and then identify where the line crosses the *y*-axis.



LESSON 14 | SESSION 1 Name:

Prepare for Representing and Solving Problems with Systems of Linear Equations

① Think about what you know about graphing lines. Fill in each box. Use words, numbers, and pictures. Show as many ideas as you can. **Possible answers:**

What Is It?
the *y*-coordinate of the point where the graph of a line intersects the *y*-axis

What I Know About It
When a linear equation is written in slope-intercept form, the *y*-intercept is the constant term.

***y*-intercept**

Examples
The *y*-intercept of $y = -2x$ is 0.

Examples
The *y*-intercept of $y = x + 2$ is 2.

Examples

The *y*-intercept is −3.

② What is the *y*-intercept of the graph of the equation $6x + 3y = -18$?
−6

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REAL-WORLD CONNECTION

Mixture problems in a chemistry lab are often solved with a system of equations. Consider a scenario where a chemist is mixing two acid solutions to make a third acid solution. One equation relates the number of liters of solution A mixed with the number of liters of solution B to make the total number of liters of solution C. The other equation relates the percent of acid in solution A mixed with the percent of acid in solution B to find the total percent of acid in solution C. When the equations are solved as a system, the ordered pair represents the number of liters of solutions A and B that were mixed to make solution C. Ask students to think of other real-world examples where knowing how to solve a system of equations might be useful.

Additional Practice

③ Problem 3 provides another look at writing and solving a system of equations in context. This problem is similar to the problem about Jade and Enrique saving money. In both problems, there are two unknowns and equations involving two variables that can be generated from given information. This problem asks for the number of weeks after which two people will have volunteered the same number of hours.

Students may want to use a table, a graph, or algebraic methods to solve.

Suggest that students use **Say It Another Way** to help them solve the problem. Encourage them to write out a statement that paraphrases the questions being asked.

③ Adrian and Cyrus volunteer for a community service organization the number of hours shown. Cyrus has already volunteered 8 hours when Adrian begins to volunteer.

Community Service Hours

Adrian	Cyrus
4 h per week	2 h per week

a. After how many weeks will they both have volunteered the same number of hours? How many hours will each of them have volunteered at that time? Show your work.

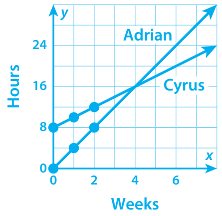
Possible work:

Weeks from now	Adrian	Cyrus
0	0	8
1	4	10
2	8	12
3	12	14
4	16	16

SOLUTION 4 weeks; 16 hours

b. Check your answer to problem 3a. Show your work.

Possible work:



After 4 weeks, Adrian and Cyrus will each have volunteered 16 hours.

DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Use with **Session 2** Apply It

Levels 1–3: Reading/Speaking

Modify **Three Reads** to help students interpret Apply It problem 5. After Read 1, have students look at the illustration as you explain the boating terms *crew*, *regatta*, and *row*. After Read 2, ask partners to find and discuss the sentence with *Let x be* and *Let y be*. Explain that the phrase *Let <variable> be* defines a variable. Ask: *In this problem, what will x be? What will y be?*

After Read 3, have partners find and highlight the important quantities and relationships, including the unit labels.

Allow think time for partners to work on identifying the system that represents the distance for each crew.

Levels 2–4: Reading/Speaking

Modify **Three Reads** to help students with Apply It problem 5. After Read 1, explain the boating terms as needed. After Read 2, have students turn to a partner to find and discuss sentences with *Let x be* or *Let y be*. Ask: *What do "Let x be" and "Let y be" define in this problem?*

After Read 3, have partners identify the important quantities and relationships in the problem. Remind them to pay attention to the unit labels.

Allow think time for students to work on the problem individually. Have them meet with partners again to discuss which system represents the distance for each crew.

Levels 3–5: Reading/Speaking

Use **Three Reads** to have pairs read Apply It problem 5 together. After Read 1, have partners explain what the problem is about. After Read 2, ask students to identify values that are important in the problem. Ask students to find and explain the sentences that define the variables. After Read 3, have partners discuss how they will use each variable to write the system of equations.

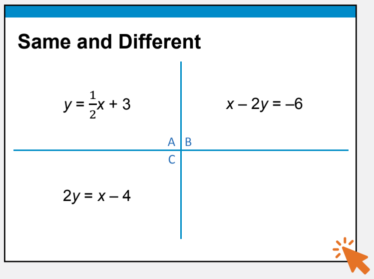
Allow think time for students to work on the problem individually. Then have them meet with partners again and discuss how the equations connect to the information in the problem.

Develop Solving Real-World Problems with Systems of Linear Equations

Purpose

- **Develop** strategies for modeling real-world problems with systems of two linear equations.
- **Recognize** that each variable in a system of linear equations must represent the same quantity in both of the equations.

START CONNECT TO PRIOR KNOWLEDGE



Same and Different

A $y = \frac{1}{2}x + 3$

B $x - 2y = -6$

C $2y = x - 4$

Possible Solutions

All are linear equations and represent lines with the same slope.

A and B represent lines with the same *y*-intercept.

C represents the only line with a negatve *y*-intercept.

WHY? Support students' facility with transforming equations to identify characteristics of their graphs.

DEVELOP ACADEMIC LANGUAGE

WHY? Support students in making connections between tables and equations.

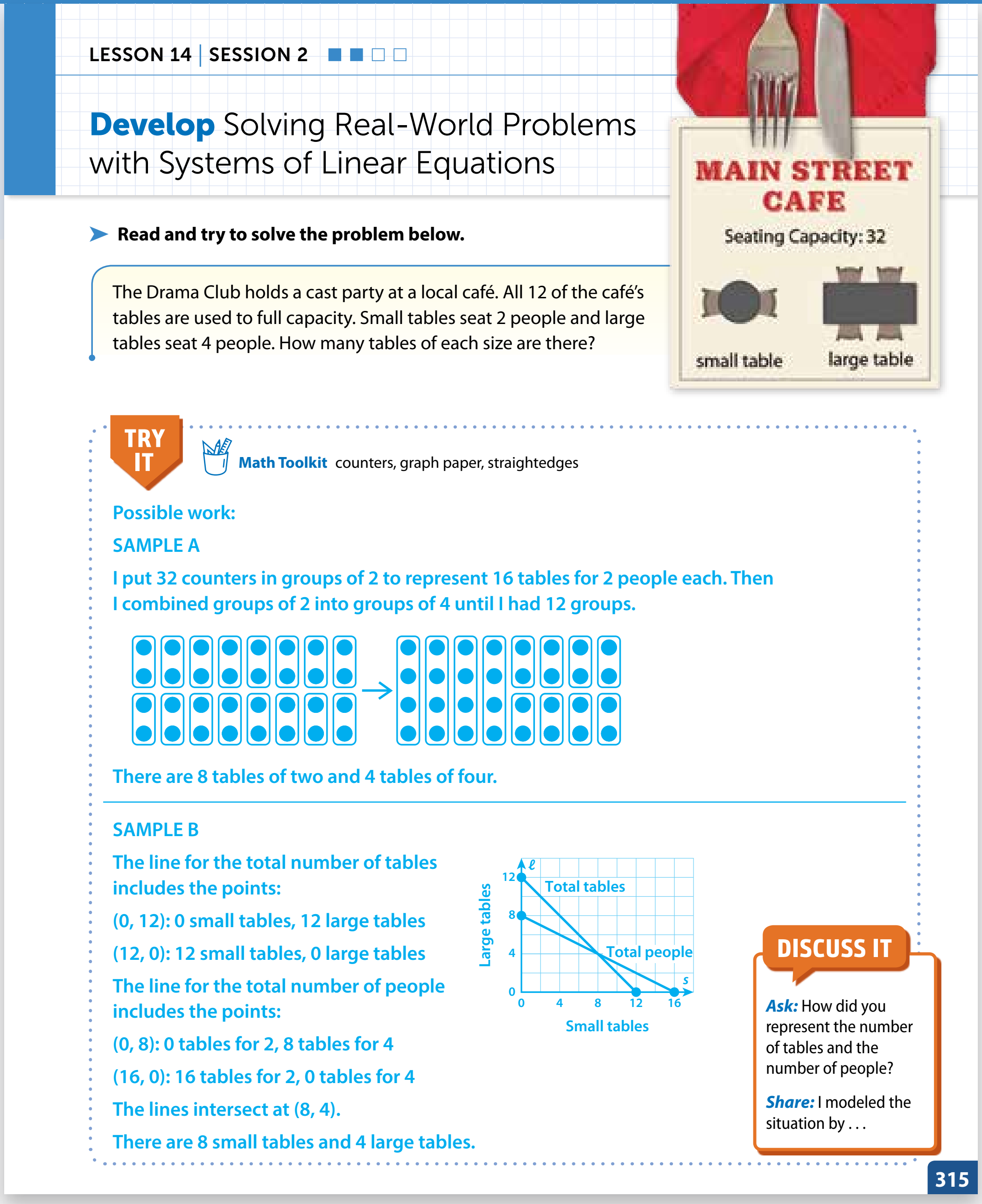
HOW? Explain that one way to connect different methods is to explain how each method represents the situation in the problem. Have students analyze the three-column table in the first Model It. Have students explain what each column represents. Then have pairs discuss how the equations in the second Model It connect to the table. Ask: *How does the first equation connect to the table?*

TRY IT

SMP 1, 2, 4, 5, 6

Make Sense of the Problem

See ***Connect to Culture*** to support student engagement. Before students work on Try It, have them read the problem with a partner and use **Notice and Wonder** to help them make sense of the problem. Ask students to identify the key information given in the graphic on the page.



LESSON 14 | SESSION 2 ■ ■ □ □

Develop Solving Real-World Problems with Systems of Linear Equations

➤ **Read and try to solve the problem below.**

The Drama Club holds a cast party at a local café. All 12 of the café's tables are used to full capacity. Small tables seat 2 people and large tables seat 4 people. How many tables of each size are there?

TRY IT

Math Toolkit counters, graph paper, straightedges

Possible work:

SAMPLE A

I put 32 counters in groups of 2 to represent 16 tables for 2 people each. Then I combined groups of 2 into groups of 4 until I had 12 groups.

There are 8 tables of two and 4 tables of four.

SAMPLE B

The line for the total number of tables includes the points:

(0, 12): 0 small tables, 12 large tables

(12, 0): 12 small tables, 0 large tables

The line for the total number of people includes the points:

(0, 8): 0 tables for 2, 8 tables for 4

(16, 0): 16 tables for 2, 0 tables for 4

The lines intersect at (8, 4).

There are 8 small tables and 4 large tables.

DISCUSS IT

Ask: How did you represent the number of tables and the number of people?

Share: I modeled the situation by . . .

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DISCUSS IT

SMP 2, 3, 6

Support Partner Discussion

After students work on Try It, encourage them to respond to Discuss It with a partner. Listen for understanding that:

- there are two unknowns: the number of large tables and the number of small tables.
- the total number of tables is 12, and the total number of people is 32.
- the total number of people is 4 times the number of large tables plus 2 times the number of small tables.
- 12 small tables are not enough for 32 people and 12 large tables are too many, so there must be some tables of each size.

Common Misconception Listen for students who are trying to write one equation relating the total number of people to the number of small tables and one equation relating the total number of people to the number of large tables. As students share their strategies, highlight the connection between table size and the number of people who can be seated so students can see the two cannot be separated.

Select and Sequence Student Strategies

Select 2–3 samples that represent the range of student thinking in your classroom. Here is one possible order for class discussion:

- counters or a diagram
- (misconception) equations relating the total number of people to the number of tables of each size
- table showing relationships and combinations
- graph with lines for the total number of people and total number of tables
- system of equations

Facilitate Whole Class Discussion

Call on students to share selected strategies. Remind them to project their voices and to pause to ask for questions and comments from classmates.

Guide students to **Compare and Connect** the representations. Prompt students to connect strategies by showing how each represents the important relationships in the problem.

> **ASK** *How does each representation show the key relationships in the problem?*
>
> **LISTEN FOR** Each shows that the total number of tables is 12 and the total number of people is 32.

Model It

If students presented these models, have students connect these models to those presented in class.

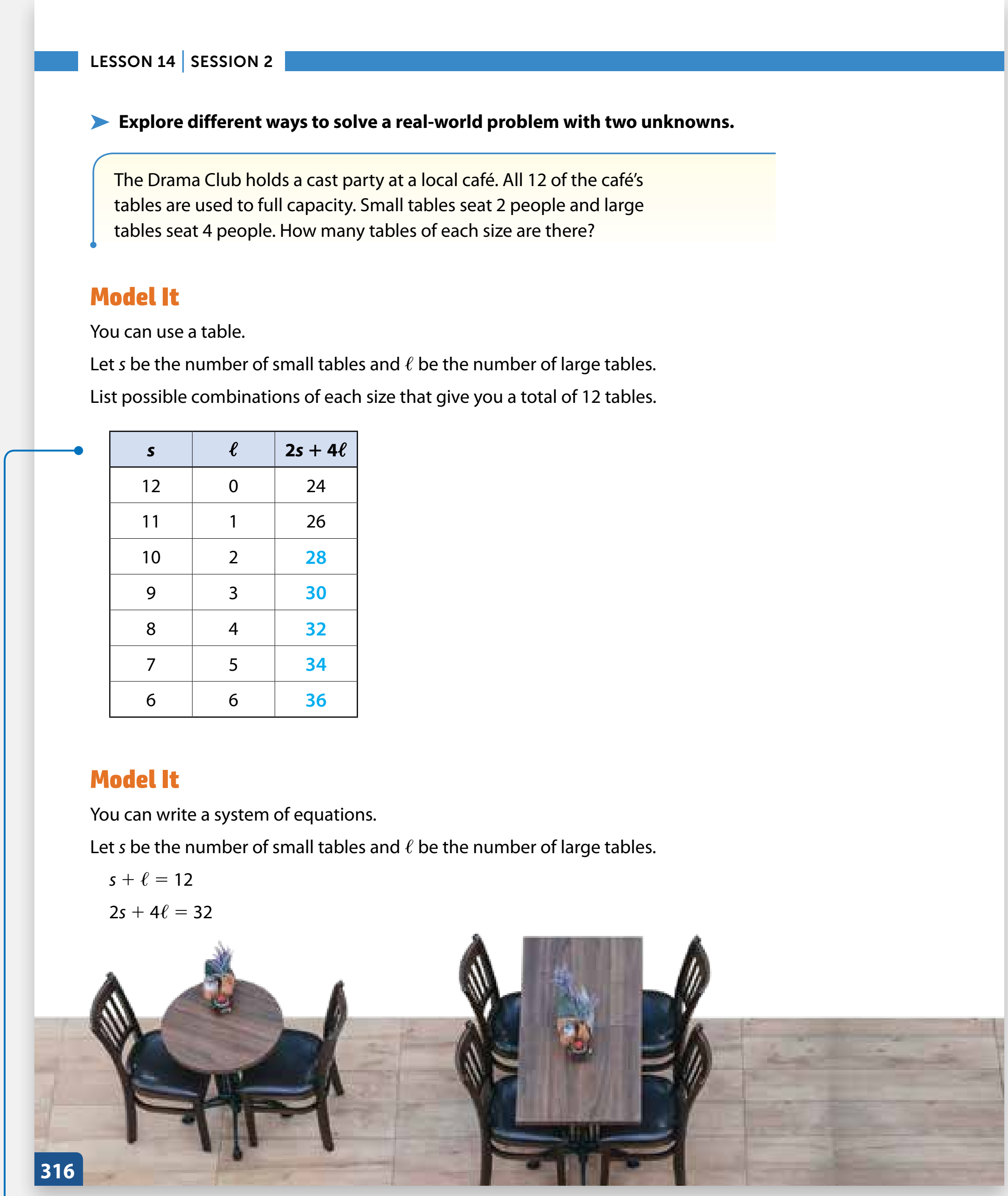
If no student presented at least one of these models, have students first analyze key features of the models, and then connect them to the models presented in class.

> **ASK** *How are the two Model Its the same? How are they different?*
>
> **LISTEN FOR** Both use variables to represent the unknown values and show the relationship between tables and people. The first is an organized way of showing all the possibilities while the second uses algebra to solve.

For the table, prompt students to consider the connections between number of people that can be seated and the number of tables.

- *How are the numbers of each size table related?*
- *What happens to the number of people who can be seated as the number of large tables goes up?*

For the system of equations, prompt students to think about how the variables and numbers relate to the word problem. Ask: *What connection from the problem is made in the first equation? The second equation?*



LESSON 14 | SESSION 2

➤ **Explore different ways to solve a real-world problem with two unknowns.**

The Drama Club holds a cast party at a local café. All 12 of the café's tables are used to full capacity. Small tables seat 2 people and large tables seat 4 people. How many tables of each size are there?

Model It

You can use a table.

Let s be the number of small tables and ℓ be the number of large tables.

List possible combinations of each size that give you a total of 12 tables.

s	ℓ	$2s + 4\ell$
12	0	24
11	1	26
10	2	28
9	3	30
8	4	32
7	5	34
6	6	36

Model It

You can write a system of equations.

Let s be the number of small tables and ℓ be the number of large tables.

$s + \ell = 12$

$2s + 4\ell = 32$

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DIFFERENTIATION | EXTEND

SMP 2

Deepen Understanding

Making Sense of Quantities and the Relationships Between Them

Prompt students to consider how the last column in the table would change if the number of people that could be seated at each large table changed.

> **ASK** *Suppose a large table seats only 3 people. How would the expression at the top of the last column change? Why?*
>
> **LISTEN FOR** It would be $2s + 3\ell$ because the total number of people would be 2 times the number of small tables plus 3 times the number of large tables.

> **ASK** *How would the values in the last column change? Why?*
>
> **LISTEN FOR** They would go up by 1 each time instead of 2. Removing 1 small table takes away 2 people and adding 1 large one adds 3 people. Only 1 person is added in all.

> **ASK** *How would the last column change if the large tables sat 6 people?*
>
> **LISTEN FOR** The expression at the top would be $2s + 6\ell$. The values would go up by 4 each time because removing 1 small table takes away 2 people and adding one large table adds 6 people. So, 4 people would be added in all.

Practice Solving Real-World Problems with Systems of Linear Equations

CONNECT IT

SMP 2, 4, 5, 6

Remind students that quantities and the relationships between them are the same in each representation. Explain that they will now use those relationships to reason about how to solve problems with two unknowns.

Before students begin to record and expand on their work in Model It, tell them that problems 1 and 2 will prepare them to provide the explanation asked for in problem 3.

Monitor and Confirm Understanding ① – ②

- $2s + 4\ell$ is the number of people who can sit at s small tables and ℓ large tables.
- The value in the $2s + 4\ell$ column is 32 when $s = 8$ and $\ell = 4$, so 8 small tables and 4 large tables are needed to seat the 32 people.
- In the system, $s + \ell = 12$ shows that the total number of tables is 12, and $2s + 4\ell = 32$ shows the total number of people is 32.
- The solution to the system is the same as the answer found by using the table.

Facilitate Whole Class Discussion

③ Look for understanding that a system of equations is a more efficient model than a table, particularly when problems involve large numbers.

> **ASK** *Although this problem is similar to the Model It problem, what is different about it?*
>
> **LISTEN FOR** There are large numbers of people and tables in this problem, which can make the table strategy take longer.

> **ASK** *Are there situations where certain strategies are more efficient? When?*
>
> **LISTEN FOR** When the numbers involved are larger, it can make sense to use a system of equations.

④ **Reflect** Have all students focus on the strategies used to solve the Try It. If time allows, have students discuss their ideas with a partner.

CONNECT IT

➤ **Use the problem from the previous page to help you understand how to solve problems with two unknowns.**

① **a.** Look at the first **Model It**. What does the expression $2s + 4\ell$ represent?

$2s + 4\ell$ represents the total number of people who can sit at s small tables and ℓ large tables.

b. Complete the table. What combination of tables will seat 32 people? How do you know?

See table; 8 small tables and 4 large tables; The row where $s = 8$ and $\ell = 4$ shows a value of 32 for $2s + 4\ell$.

② **a.** Look at the second **Model It**. What does each equation in the system represent?

$s + \ell = 12$ represents the total number of tables at the café. $2s + 4\ell = 32$ represents the total number of people who can sit at the tables.

b. Solve the system. Do you get the same answer as you did in problem 1b?

$s = 8$, $\ell = 4$; Yes.

③ Look at this problem: A banquet hall has seating for 200 people. Some tables seat 6 people and some tables seat 10 people. There are 26 tables in all. How many tables are there of each size?

a. Write a system of equations to represent the problem.

$s + \ell = 26$, $6s + 10\ell = 200$

b. Why might someone choose to use a system of equations to solve this problem instead of making a table?

Possible answer: Because there are so many tables in the banquet hall, it would take a long time to list all the possible combinations.

④ **Reflect** Think about all the models and strategies you have discussed today. Describe how one of them helped you better understand how to solve the **Try It** problem.

Responses will vary. Check student responses.

DIFFERENTIATION | RETEACH or REINFORCE

Visual Model

Model to connect verbal descriptions and mathematical statements.

If students are unsure about writing systems of two equations where each variable represents the same quantity in both equations, then use this activity to help make connections between verbal descriptions and mathematical statements.

Materials For display: 6 nickels and 4 dimes

- Display the nickels and dimes for students. Ask: *What relationships do these coins have?* [There are 10 total coins. The coins are worth \$0.70; there are 6 nickels worth \$0.30 and 4 dimes worth \$0.40.] Write the responses on the board.
- Cover the coins. Ask: *Imagine the number of each type of coin is not known. Which of your descriptions could be used to determine the number of each type of coin?* [There are 10 total coins, and the coins are worth \$0.70.]
- Have students write 2 equations for the relationships. [$n + d = 10$; $0.05n + 0.10d = 0.70$]
- Instruct students to write a word problem for the coins. [Sample answer: Brenna has \$0.70 in nickels and dimes. She has a total of 10 coins. How many of each type of coin does Brenna have?]

Additional Practice

Apply It

For all problems, encourage students to use a system of equations. Students may use other strategies and models, including tables or graphs, to help support their thinking.

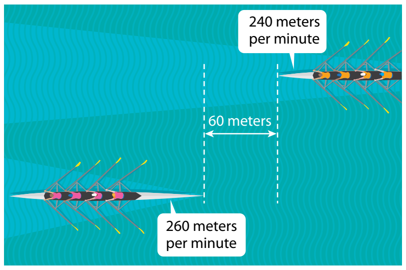
5 See ***Connect to Culture*** to support student engagement. Students should note that x is the number of minutes after the *second* crew starts. The first crew travels $240x$ meters in x minutes, but since they have already rowed 60 meters when the second crew starts, their distance from the start after x minutes is $240x + 60$ meters.

6 Students may also solve the equation $x + y = 10$ for x or y and use the substitution method.

Apply It

➤ **Use what you learned to solve these problems.**

5 Crew teams are racing in a regatta. Every 15 seconds a new crew starts the race. Today, the first crew rows at a speed of 240 meters per minute. They are 60 m ahead when the next crew starts, rowing 260 m per minute. Let x be the number of minutes after the second crew starts and y be the distance rowed. Write a system of equations that can be solved to find out when the two crews are the same distance from the start.



$y = 260x$, $y = 240x + 60$

6 Lian buys 10 packs of batteries. C batteries are sold in packs of 6. AAA batteries are sold in packs of 8. Lian buys 72 batteries in all. Let x be the number of packs of C batteries. Let y be the number of packs of AAA batteries. Write and solve a system of equations to find how many packs of each type of battery Lian buys. Show your work. Possible work:

$x + y = 10 \quad \times(-6) \longrightarrow \quad -6x - 6y = -60 \qquad x + y = 10$

$6x + 8y = 72 \quad \longrightarrow \quad \underline{+ \quad 6x + 8y = 72} \qquad x + 6 = 10$

$2y = 12 \qquad x = 4$

$y = 6$

SOLUTION Lian buys 4 packs of C batteries and 6 packs of AAA batteries.

7 You have $3.10 in dimes and quarters. You have 3 more dimes than quarters. Write an equation that relates the number of coins and an equation for the value of the coins. How many of each kind of coin do you have? Show your work.

Possible work: Let d = the number of dimes, q = the number of quarters

Number of coins: $d = q + 3$ Value of coins: $0.10d + 0.25q = 3.10$

$0.10(q + 3) + 0.25q = 3.10$ $\qquad d = q + 3$

$0.10q + 0.30 + 0.25q = 3.10$ $\qquad d = 8 + 3$

$0.35q = 2.80$ $\qquad d = 11$

$q = 8$

SOLUTION I have 11 dimes and 8 quarters.

CLOSE EXIT TICKET

7 Students' solutions should show an understanding of:

- creating a system of equations from a verbal description.
- solving a system of equations using substitution or elimination.

Error Alert If students try to write an equation for the total number of coins, then have them read the second sentence again. Help them see that the sentence does not say anything about the total number of coins, but it does indicate the relationship between the dimes and quarters.

Practice Solving Real-World Problems with Systems of Linear Equations

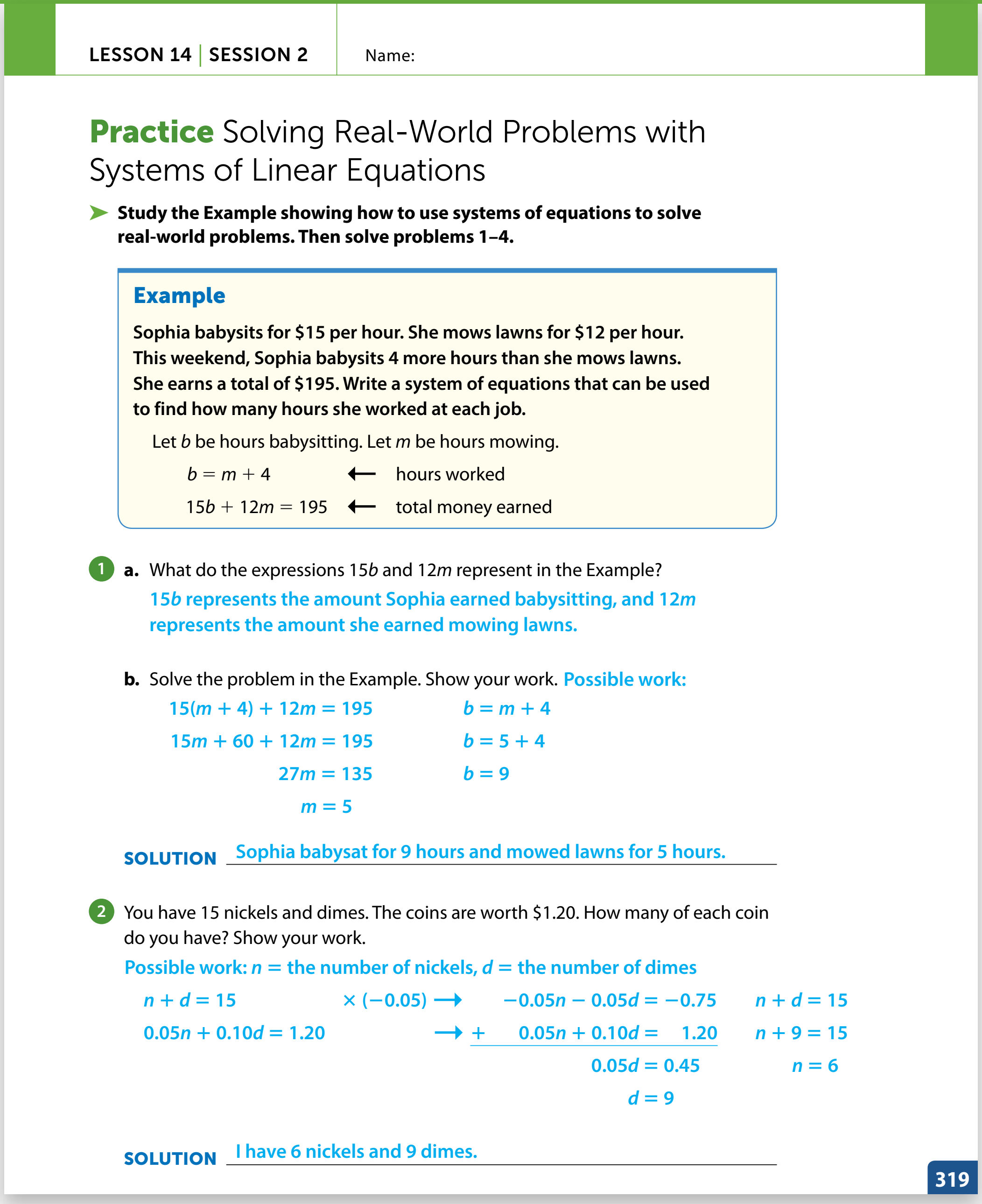
Problem Notes

Assign **Practice Solving Real-World Problems with Systems of Linear Equations** as extra practice in class or as homework.

① **a.** Students should recognize that b represents the number of hours Sophia babysits, and \$15 is the amount she earns per hour for babysitting, so those values are multiplied. Students should recognize that m represents the number of hours Sophia mows lawns and \$12 is the amount she earns per hour for mowing, so those values are multiplied. ***Medium***

b. Although elimination can also be used, substitution is the most efficient solution method since the first equation is already solved for b. ***Basic***

② Students may also solve $n + d = 15$ for n or d and use the substitution method. ***Medium***



LESSON 14 | SESSION 2 Name:

Practice Solving Real-World Problems with Systems of Linear Equations

➤ **Study the Example showing how to use systems of equations to solve real-world problems. Then solve problems 1–4.**

Example

Sophia babysits for \$15 per hour. She mows lawns for \$12 per hour. This weekend, Sophia babysits 4 more hours than she mows lawns. She earns a total of \$195. Write a system of equations that can be used to find how many hours she worked at each job.

Let b be hours babysitting. Let m be hours mowing.

$b = m + 4$ ⟵ hours worked

$15b + 12m = 195$ ⟵ total money earned

① **a.** What do the expressions $15b$ and $12m$ represent in the Example?

15b represents the amount Sophia earned babysitting, and 12m represents the amount she earned mowing lawns.

b. Solve the problem in the Example. Show your work. Possible work:

$$\begin{array}{rl} 15(m + 4) + 12m = 195 & \quad b = m + 4 \\ 15m + 60 + 12m = 195 & \quad b = 5 + 4 \\ 27m = 135 & \quad b = 9 \\ m = 5 & \end{array}$$

SOLUTION Sophia babysat for 9 hours and mowed lawns for 5 hours.

② You have 15 nickels and dimes. The coins are worth \$1.20. How many of each coin do you have? Show your work.

Possible work: n = the number of nickels, d = the number of dimes

$$\begin{array}{lcrl} n + d = 15 & \times(-0.05) \longrightarrow & -0.05n - 0.05d = -0.75 & \quad n + d = 15 \\ 0.05n + 0.10d = 1.20 & \longrightarrow + & \underline{0.05n + 0.10d = 1.20} & \quad n + 9 = 15 \\ & & 0.05d = 0.45 & \quad n = 6 \\ & & d = 9 & \end{array}$$

SOLUTION I have 6 nickels and 9 dimes.

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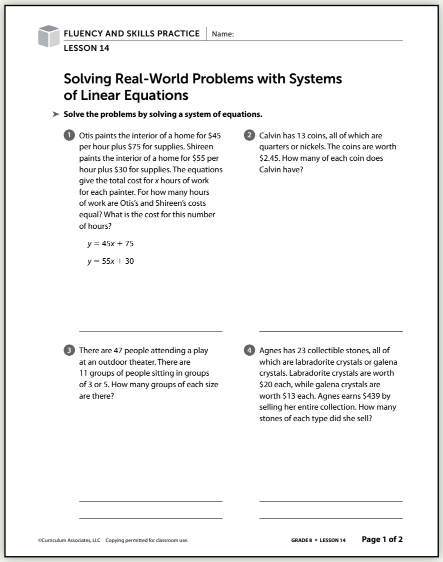
Fluency & Skills Practice

Solving Real-World Problems with Systems of Linear Equations

In this activity, students solve real-world probelms by writing and solving systems of linear equations.



FLUENCY AND SKILLS PRACTICE | Name:
LESSON 14

Solving Real-World Problems with Systems of Linear Equations

➤ **Solve the problems by solving a system of equations.**

① Otis paints the interior of a home for \$45 per hour plus \$75 for supplies. Shireen paints the interior of a home for \$55 per hour plus \$30 for supplies. The equations give the total cost for x hours of work for each painter. For how many hours of work are Otis's and Shireen's costs equal? What is the cost for this number of hours?

$y = 45x + 75$

$y = 55x + 30$

② Calvin has 13 coins, all of which are quarters or nickels. The coins are worth \$2.45. How many of each coin does Calvin have?

③ There are 47 people attending a play at an outdoor theater. There are 11 groups of people sitting in groups of 3 or 5. How many groups of each size are there?

④ Agnes has 23 collectible stones, all of which are labradorite crystals or galena crystals. Labradorite crystals are worth \$20 each, while galena crystals are worth \$13 each. Agnes earns \$439 by selling her entire collection. How many stones of each type did she sell?

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Additional Practice

③ **a.** ***Basic***

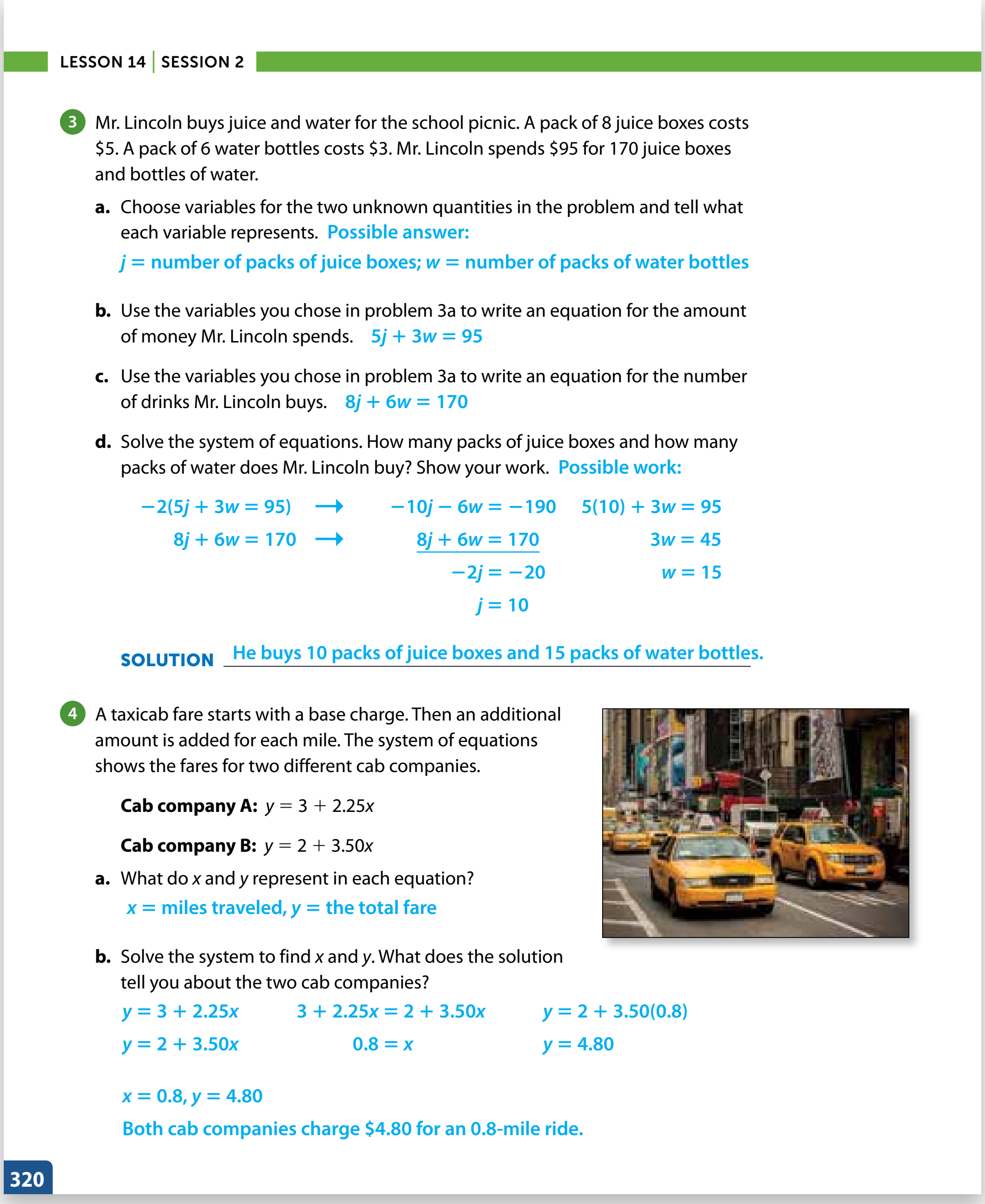
b. Students may write $3w + 5j = 95$, writing the variable terms in a different order than in the answer shown. ***Challenge***

c. Students may write the variable terms in a different order. Since elimination is likely the most efficient solution method, students should write the variable terms in the same order as in the previous equation. ***Challenge***

d. Students may recognize that 6 is double 3 and use elimination to eliminate the w-term. However, some students may multiply both equations by a number to eliminate the j-term. ***Medium***

④ **a.** ***Basic***

b. Students may use elimination to solve the system. Students may interpret the solution by connecting the amounts to the variable descriptions. They should recognize that the result is the distance for which the cab companies charge the same fee and the amount of that fee. ***Medium***



③ Mr. Lincoln buys juice and water for the school picnic. A pack of 8 juice boxes costs \$5. A pack of 6 water bottles costs \$3. Mr. Lincoln spends \$95 for 170 juice boxes and bottles of water.

a. Choose variables for the two unknown quantities in the problem and tell what each variable represents. **Possible answer:**
j = number of packs of juice boxes; w = number of packs of water bottles

b. Use the variables you chose in problem 3a to write an equation for the amount of money Mr. Lincoln spends. $5j + 3w = 95$

c. Use the variables you chose in problem 3a to write an equation for the number of drinks Mr. Lincoln buys. $8j + 6w = 170$

d. Solve the system of equations. How many packs of juice boxes and how many packs of water does Mr. Lincoln buy? Show your work. **Possible work:**

$$\begin{array}{rcr} -2(5j + 3w = 95) & \longrightarrow & -10j - 6w = -190 \\ 8j + 6w = 170 & \longrightarrow & \underline{8j + 6w = 170} \\ & & -2j = -20 \\ & & j = 10 \end{array} \qquad \begin{array}{r} 5(10) + 3w = 95 \\ 3w = 45 \\ w = 15 \end{array}$$

SOLUTION He buys 10 packs of juice boxes and 15 packs of water bottles.

④ A taxicab fare starts with a base charge. Then an additional amount is added for each mile. The system of equations shows the fares for two different cab companies.

Cab company A: $y = 3 + 2.25x$

Cab company B: $y = 2 + 3.50x$

a. What do x and y represent in each equation?
x = miles traveled, y = the total fare

b. Solve the system to find x and y. What does the solution tell you about the two cab companies?

$$\begin{array}{l} y = 3 + 2.25x \\ y = 2 + 3.50x \end{array} \qquad \begin{array}{c} 3 + 2.25x = 2 + 3.50x \\ 0.8 = x \end{array} \qquad \begin{array}{l} y = 2 + 3.50(0.8) \\ y = 4.80 \end{array}$$

$x = 0.8$, $y = 4.80$
Both cab companies charge \$4.80 for an 0.8-mile ride.

DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Use with **Session 3** **Apply It**

ACADEMIC VOCABULARY

Represent means to use as a sign, symbol, or example for something.

Parentheses ( ) are symbols used in pairs to group. In equations, parentheses can be used to show multiplication or mark a change to the order of operations.

Levels 1–3: Reading/Listening

After students read Apply It problem 6a, help them identify the numbers, symbols, and relationships described in the first two sentences.

Review *represent* and ask: *What can you use to represent the two numbers?* [variables] Clarify that the numbers are called *one of the numbers* and *the other number* in the second sentence. Next, ask students to tell what word an equal sign can represent [is] and to identify operations indicated by *three times, sum of, more than, one-third of.* Suggest students write parentheses around *sum of two numbers* to preserve the order of operations.

Levels 2–4: Reading/Listening

After students read Apply It problem 6a, have them identify the numbers, symbols, and relationships described in the first two sentences.

Ask students to identify words that indicate operations. Ask: *Do you find the sum or multiply first?* Point out that they should use parentheses. Review *represent* and provide the frame:

- *What can you use to represent ____?*

Ask for ways to complete the frame, such as the word *is* and the phrases *the sum of two numbers, one of the numbers, the other number, more than,* and *one-third*. Have partners take turns asking and answering questions.

Levels 3–5: Reading/Listening

After students read Apply It problem 6a, have them use **Say It Another Way** to confirm their understanding and work with partners to identify the numbers, symbols, and relationships described in the first two sentences. Display these frames:

- *What can you use to represent ____?*
- *What is represented by a ____?*

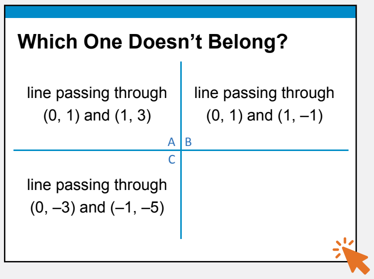
Have students discuss the difference between *represent* and *represented by.* Then have partners use the frames to discuss how to represent the words and phrases in the sentence, including those used for operations. Have them discuss if they should use parentheses.

Develop Solving Mathematical Problems Involving Systems of Linear Equations

Purpose

- **Develop** strategies for solving mathematical problems with systems of two linear equations.
- **Recognize** that given a pair of points on each of two lines, it is possible to determine whether the lines intersect and to write and solve a system of equations to find the intersection point(s).

START CONNECT TO PRIOR KNOWLEDGE



Which One Doesn't Belong?

A	B
line passing through (0, 1) and (1, 3)	line passing through (0, 1) and (1, –1)
C	
line passing through (0, –3) and (–1, –5)	

Possible Solutions

A is the only line that has both given points in the first quadrant.

B has a different slope than A and C.

C has a different *y*-intercept than A and B.

WHY? Support students' facility with using points on a line to identify slope and *y*-intercept.

DEVELOP ACADEMIC LANGUAGE

WHY? Understand how *this* can be used to refer back to previous parts of a text.

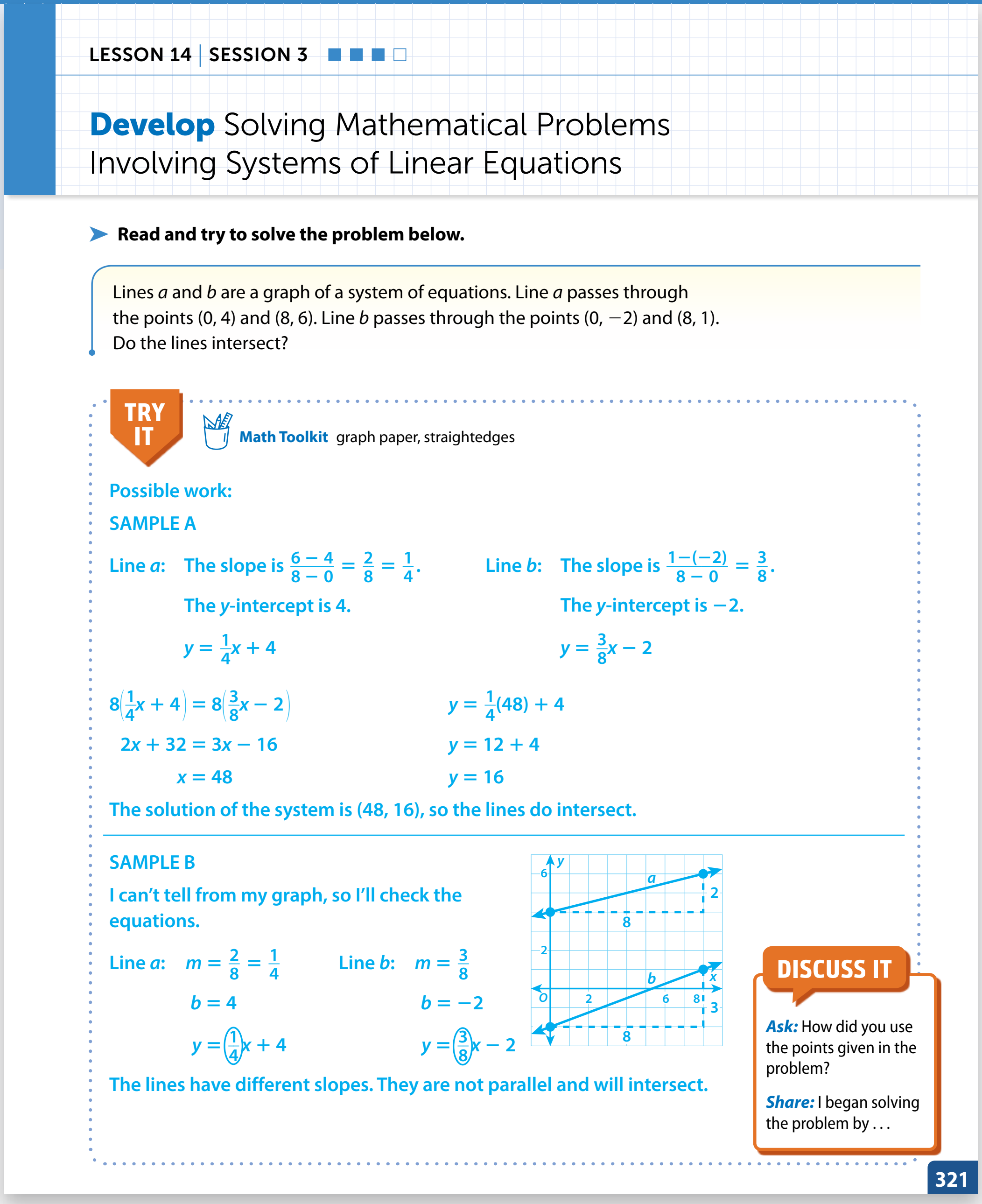
HOW? Read Connect It problem 1 with students. Have them circle *this* in the third question. Ask students to define *this*, referring back to previous sentences or questions as needed. Discuss how words like *this*, *it*, and *that* can be used to refer back to previous parts in a text. Explain that it is important to be clear on what these words refer to in different contexts.

TRY IT

SMP 1, 2, 4, 5, 6

Make Sense of the Problem

Before students work on Try It, use **Notice and Wonder** to help them make sense of the problem. When all ideas have been shared, read the problem statement aloud and have students indicate which items they noticed and wondered might be most relevant to the problem.



LESSON 14 | SESSION 3 ■■■□

Develop Solving Mathematical Problems Involving Systems of Linear Equations

➤ **Read and try to solve the problem below.**

Lines *a* and *b* are a graph of a system of equations. Line *a* passes through the points (0, 4) and (8, 6). Line *b* passes through the points (0, −2) and (8, 1). Do the lines intersect?

TRY IT **Math Toolkit** graph paper, straightedges

Possible work:

SAMPLE A

Line *a*: The slope is $\frac{6-4}{8-0} = \frac{2}{8} = \frac{1}{4}$.

The *y*-intercept is 4.

$y = \frac{1}{4}x + 4$

Line *b*: The slope is $\frac{1-(-2)}{8-0} = \frac{3}{8}$.

The *y*-intercept is −2.

$y = \frac{3}{8}x - 2$

$8\left(\frac{1}{4}x + 4\right) = 8\left(\frac{3}{8}x - 2\right)$

$2x + 32 = 3x - 16$

$x = 48$

$y = \frac{1}{4}(48) + 4$

$y = 12 + 4$

$y = 16$

The solution of the system is (48, 16), so the lines do intersect.

SAMPLE B

I can't tell from my graph, so I'll check the equations.

Line *a*: $m = \frac{2}{8} = \frac{1}{4}$

$b = 4$

$y = \boxed{\frac{1}{4}}x + 4$

Line *b*: $m = \frac{3}{8}$

$b = -2$

$y = \boxed{\frac{3}{8}}x - 2$

The lines have different slopes. They are not parallel and will intersect.

DISCUSS IT

Ask: How did you use the points given in the problem?

Share: I began solving the problem by . . .

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DISCUSS IT

SMP 2, 3, 6

Support Partner Discussion

After students work on Try It, encourage them to respond to Discuss It with a partner. Listen for understanding that:

- points with an *x*-coordinate of 0 reveal the *y*-intercept of the line.
- the two points given for each line can be used to find slope.
- lines with different slopes intersect and the point of intersection represents the solution to a system of equations.

Error Alert If students calculate slope by dividing the difference in *x*-coordinates by the difference in *y*-coordinates or by subtracting the *x*-coordinates in a different order than they subtract the *y*-coordinates, then have them review the slope formula, $m = \frac{y_2 - y_1}{x_2 - x_1}$. You might suggest that they write (x_1, y_1) and (x_2, y_2) above or next to the coordinates of the points for each line to ensure they substitute correctly into the formula.

Select and Sequence Student Strategies

Select 2–3 samples that represent the range of student thinking in your classroom. Here is one possible order for class discussion:

- graphs of equations drawn and compared
- slopes of lines calculated and compared
- equations of lines compared
- system of equations written and solved

Facilitate Whole Class Discussion

Call on students to share selected strategies. Suggest students who present use sentence starters such as *I know this approach works because* ____. or *I know this answer is correct because* ____.

Guide students to **Compare and Connect** the representations. Before having a class discussion, ask students to turn and talk to a partner about the connections they observe to practice sharing their ideas with clarity.

> **ASK** *How did the strategies presented use the idea of slope?*
>
> **LISTEN FOR** One strategy compared the slopes of the lines to conclude the lines intersected. Another used slope along with the *y*-intercept to write the equations that could be solved as a system to determine if the lines intersected.

Picture It & Model It

If students presented these models, have students connect these models to those presented in class.

If no student presented at least one of these models, have students first analyze key features of the models, and then connect them to the models presented in class.

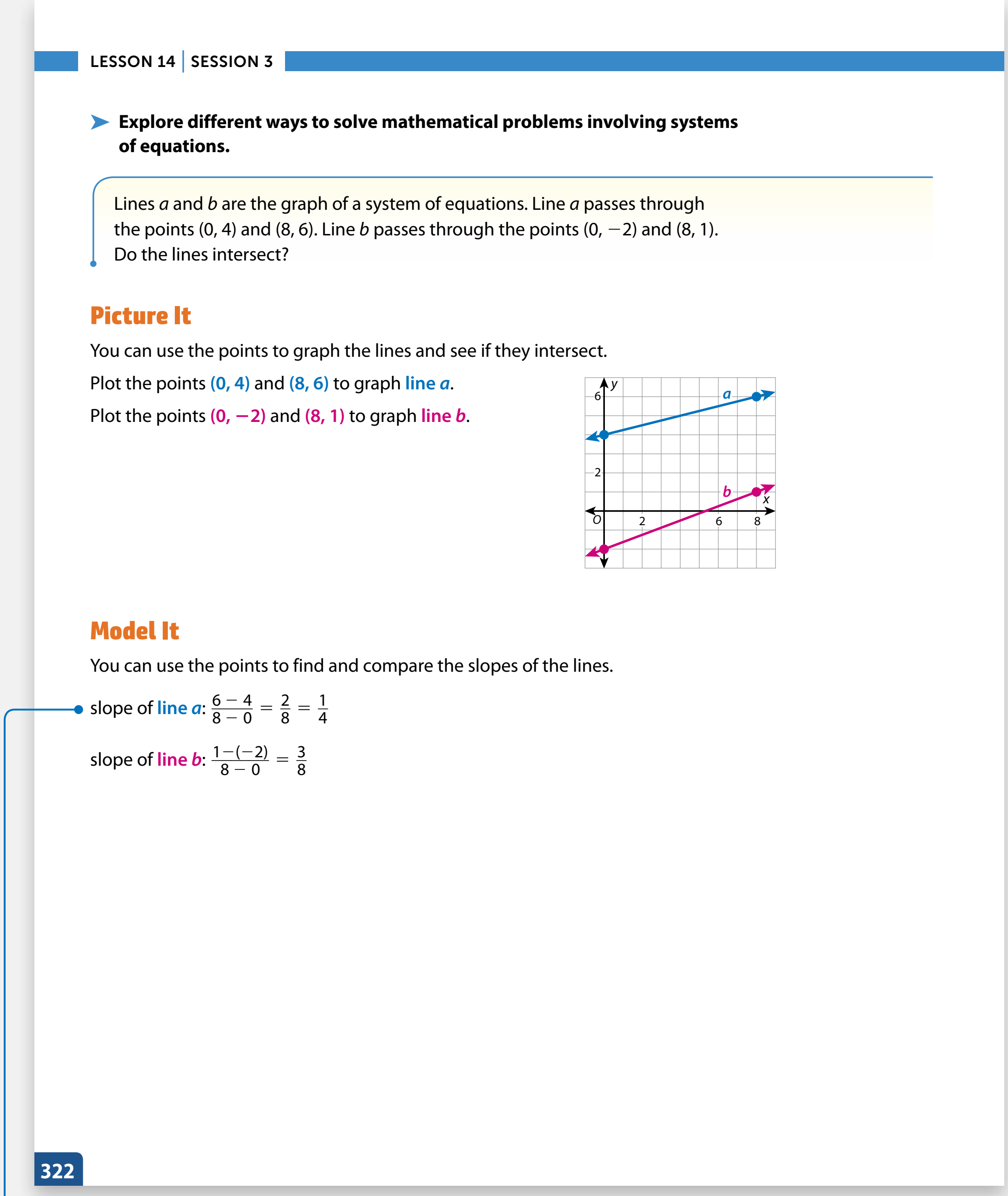
> **ASK** *How are the slopes calculated in Model It reflected in Picture It?*
>
> **LISTEN FOR** The slope of line *a* in Model It is less than the slope of line *b*, which means line *a* is less steep than line *b*. This can be seen in Picture It.

For the graph, prompt students to consider whether the lines intersect.

- *Do the lines intersect in the part of the coordinate plane shown?*
- *Will the lines intersect if they are extended?*

For the comparison of slopes, prompt students to think about what the slopes reveal about how the lines are related.

- *How do the slopes compare?*
- *What does this tell you about whether the lines intersect?*



LESSON 14 | SESSION 3

➤ **Explore different ways to solve mathematical problems involving systems of equations.**

Lines *a* and *b* are the graph of a system of equations. Line *a* passes through the points (0, 4) and (8, 6). Line *b* passes through the points (0, −2) and (8, 1). Do the lines intersect?

Picture It

You can use the points to graph the lines and see if they intersect.

Plot the points (0, 4) and (8, 6) to graph line *a*.

Plot the points (0, −2) and (8, 1) to graph line *b*.

Model It

You can use the points to find and compare the slopes of the lines.

slope of line *a*: $\frac{6-4}{8-0} = \frac{2}{8} = \frac{1}{4}$

slope of line *b*: $\frac{1-(-2)}{8-0} = \frac{3}{8}$

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DIFFERENTIATION | EXTEND

SMP 3

Deepen Understanding

Using Slope to Show Whether Points Are on a Line

Prompt students to consider how they can use slope to determine whether a given point is on line *a*.

> **ASK** *What is the slope between any two points on line a? How do you know?*
>
> **LISTEN FOR** It is $\frac{1}{4}$. The slope between any two points on a line is the same.
>
> **ASK** *You know that (0, 4) and (8, 6) are on line a. How can you use slope to determine whether (16, 8) is also on line a?*
>
> **LISTEN FOR** Find the slope between (16, 8) and either of the other two points. If it is $\frac{1}{4}$, then (16, 8) is also on line *a*.
>
> **ASK** *If you are given three points, how can you use slope to tell whether they are all on the same line?*
>
> **LISTEN FOR** Find the slope between the first and second point. Then find the slope between the third point and either of the other points. If both slopes are the same, then all three points are on the same line.

Develop Solving Mathematical Problems Involving Systems of Linear Equations

CONNECT IT

SMP 2, 4, 5, 6

Remind students that the points are the same in each representation. Explain that they will now use those representations to reason about how to determine whether lines through two given pairs of points intersect.

Before students begin to record and expand on their work in Picture It & Model It, tell them that problems 1 and 2 will prepare them to provide the explanation asked for in problem 3.

Monitor and Confirm Understanding ① – ②

- The graph shows that the distance between the lines is decreasing as x increases. This indicates that the lines will eventually intersect.
- Because the slopes of the lines are different, the lines will intersect at one point.

Facilitate Whole Class Discussion

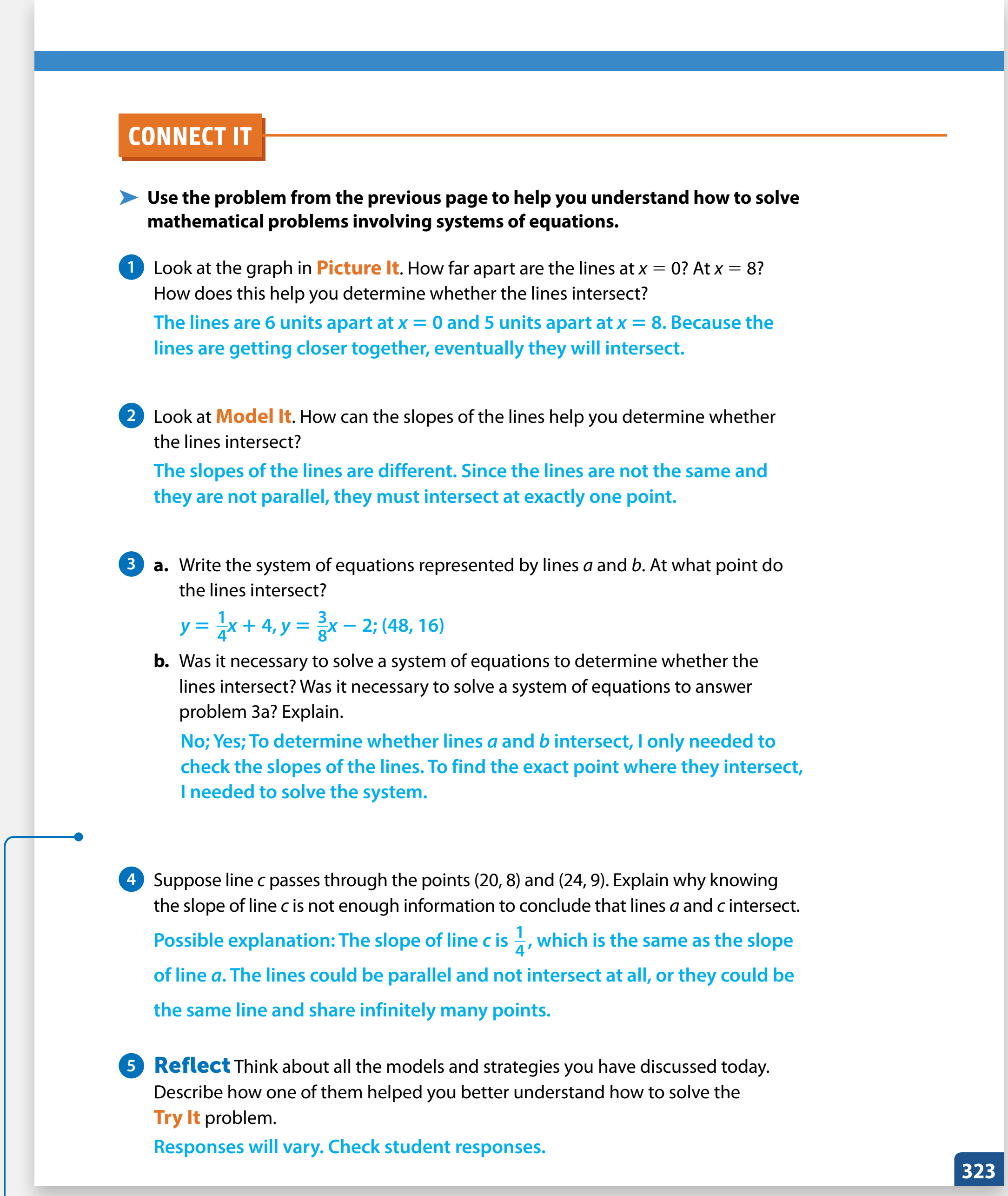
③ Look for understanding that it is not necessary to solve the system in order to determine whether two lines intersect.

> **ASK** *Why do you not need to solve the system to tell if lines a and b intersect?*
>
> **LISTEN FOR** I know that any lines with different slopes intersect, so I just need to compare the slopes and see that they are different.

> **ASK** *Why do you need to solve the system to solve problem 3a?*
>
> **LISTEN FOR** The problem asks for the point where the lines intersect. The intersection point is the solution to the system.

④ Look for understanding that two lines with the same slope may be parallel or may be the same line. You might challenge students by asking them how they can determine whether the lines are the same. One way is to find the slope between one of the given points on line c and one of the given points on line a. If the slope is $\frac{1}{4}$, then the lines are the same.

⑤ **Reflect** Have all students focus on the strategies used to solve the Try It. If time allows, have students discuss their ideas with a partner.



CONNECT IT

➤ **Use the problem from the previous page to help you understand how to solve mathematical problems involving systems of equations.**

① Look at the graph in **Picture It**. How far apart are the lines at $x = 0$? At $x = 8$? How does this help you determine whether the lines intersect?

The lines are 6 units apart at $x = 0$ and 5 units apart at $x = 8$. Because the lines are getting closer together, eventually they will intersect.

② Look at **Model It**. How can the slopes of the lines help you determine whether the lines intersect?

The slopes of the lines are different. Since the lines are not the same and they are not parallel, they must intersect at exactly one point.

③ **a.** Write the system of equations represented by lines a and b. At what point do the lines intersect?

$y = \frac{1}{4}x + 4$, $y = \frac{3}{8}x - 2$; (48, 16)

b. Was it necessary to solve a system of equations to determine whether the lines intersect? Was it necessary to solve a system of equations to answer problem 3a? Explain.

No; Yes; To determine whether lines a and b intersect, I only needed to check the slopes of the lines. To find the exact point where they intersect, I needed to solve the system.

④ Suppose line c passes through the points (20, 8) and (24, 9). Explain why knowing the slope of line c is not enough information to conclude that lines a and c intersect.

Possible explanation: The slope of line c is $\frac{1}{4}$, which is the same as the slope of line a. The lines could be parallel and not intersect at all, or they could be the same line and share infinitely many points.

⑤ **Reflect** Think about all the models and strategies you have discussed today. Describe how one of them helped you better understand how to solve the **Try It** problem.

Responses will vary. Check student responses.

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DIFFERENTIATION | RETEACH or REINFORCE

Hands-On Activity

Use a geoboard to explore slope relationships in systems of equations.

If students are unsure what the equations of lines in a system reveal about its solution, then use this activity to clarify relationships between lines and solutions in a system.

Materials For each student: 1 geoboard, 4 rubber bands

- Have students use 2 rubber bands for axes and 2 more to make a line with slope $\frac{1}{2}$ and another with slope $\frac{1}{4}$. Have students write the equations of their system.
- Ask: *Do/Will the lines intersect?* [yes] *What do you know about x and y at the point of intersection?* [x and y make each equation true.] *What does this mean in terms of the number of solutions to the system of equations?* [There is one solution to the system.]
- Now have students make two different lines with the same slope.
- Ask: *Do the lines intersect?* [no] *What does this mean in terms of the solution to the system of equations?* [No value of x and y make both equations true. There is no solution.]
- Ask: *Can lines have the same slope and share any points?* [Yes, if they are the same line.] *How does this change the solution?* [All values of x and y make both equations true. There are infinitely many solutions.]

Apply It

For all problems, encourage students to use a model to support their thinking. Allow some leeway in precision; for example, students may use a coordinate plane to draw rough sketches of lines to support their thinking.

⑥ **a.** Students should recognize they must use the relationships in the verbal descriptions to create a system of equations.

b. Students may switch x and y in the second equation of the system, but the result is the same.

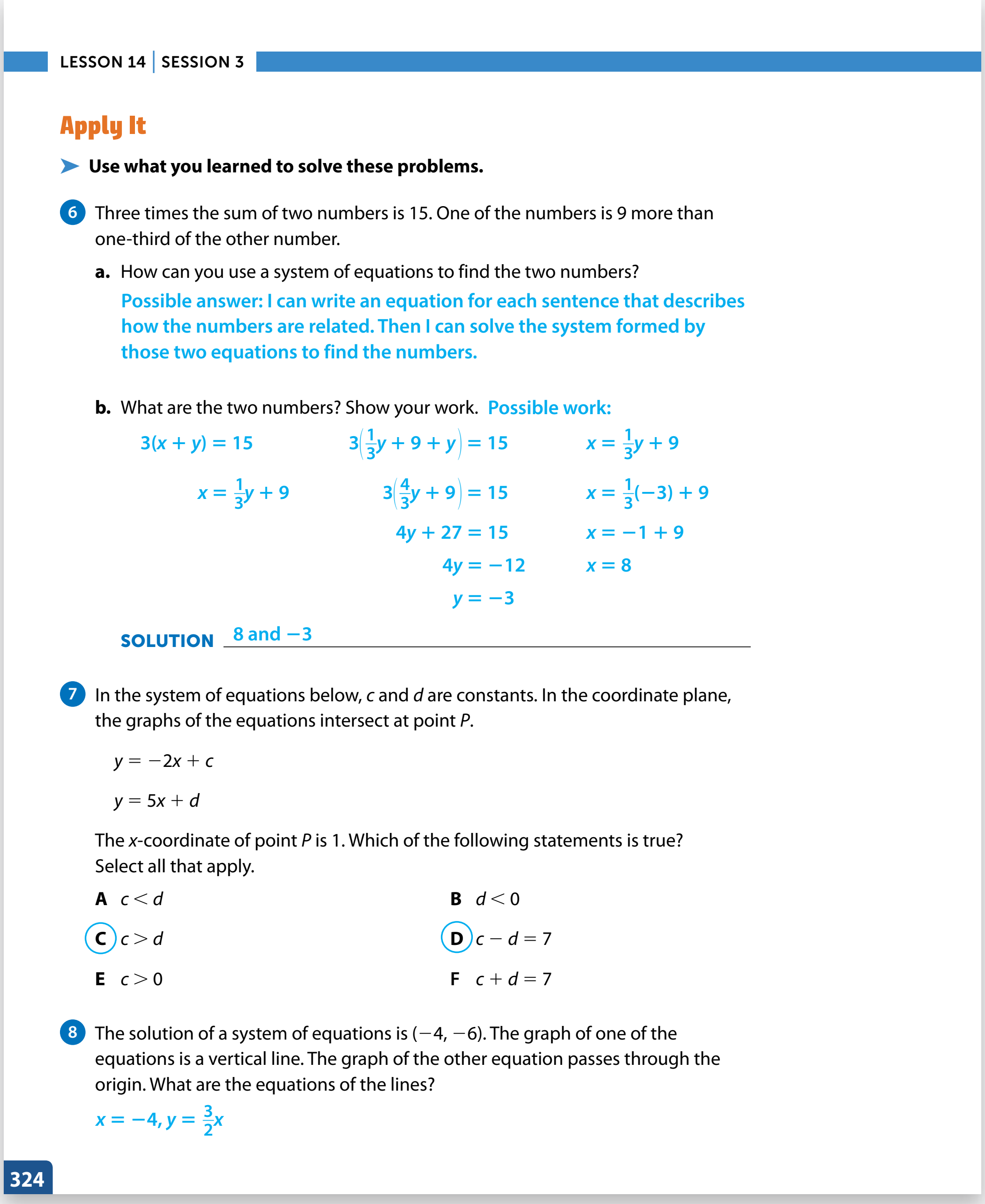
⑦ **C and D are correct**. Substituting 1 for x and setting the expressions for y equal gives $-2 + c = 5 + d$, or $c = 7 + d$. This indicates that $c > d$ and that $c - d = 7$.

A is not correct. This answer gives an incorrect relationship between c and d.

B is not correct. There is not enough information to make this conclusion.

E is not correct. There is not enough information to make this conclusion.

F is not correct. This answer is the result of incorrectly adding d to both sides to simplify after substituting 1 for the x-coordinate.



Apply It

➤ **Use what you learned to solve these problems.**

⑥ Three times the sum of two numbers is 15. One of the numbers is 9 more than one-third of the other number.

a. How can you use a system of equations to find the two numbers?

Possible answer: I can write an equation for each sentence that describes how the numbers are related. Then I can solve the system formed by those two equations to find the numbers.

b. What are the two numbers? Show your work. Possible work:

$3(x + y) = 15$

$x = \frac{1}{3}y + 9$

$3\left(\frac{1}{3}y + 9 + y\right) = 15$

$3\left(\frac{4}{3}y + 9\right) = 15$

$4y + 27 = 15$

$4y = -12$

$y = -3$

$x = \frac{1}{3}y + 9$

$x = \frac{1}{3}(-3) + 9$

$x = -1 + 9$

$x = 8$

SOLUTION 8 and -3

⑦ In the system of equations below, c and d are constants. In the coordinate plane, the graphs of the equations intersect at point P.

$y = -2x + c$

$y = 5x + d$

The x-coordinate of point P is 1. Which of the following statements is true? Select all that apply.

A $c < d$

B $d < 0$

[✓ **C**] $c > d$

[✓ **D**] $c - d = 7$

E $c > 0$

F $c + d = 7$

⑧ The solution of a system of equations is $(-4, -6)$. The graph of one of the equations is a vertical line. The graph of the other equation passes through the origin. What are the equations of the lines?

$x = -4, y = \frac{3}{2}x$

CLOSE **EXIT TICKET**

⑧ Students' solutions should show an understanding that:

- a vertical line has the same x-value for every point, so the equation of the vertical line through $(-4, -6)$ is $x = -4$.
- the second line has a y-intercept of 0 because it passes through the origin.
- the origin and solution point can be used to find the slope of the second line.

Error Alert If students write the first equation as $y = -4$, then review the meaning of *vertical* and have them consider which coordinates are the same and which are different on the graph of a vertical line.

Practice Solving Mathematical Problems Involving Systems of Linear Equations

Problem Notes

Assign **Practice Solving Mathematical Problems Involving Systems of Linear Equations** as extra practice in class or as homework.

① **a.** *Basic*

b. Students may use the intersection point and the given points on the y-axis to find the slopes of the lines. They may then write the equations using the slope and corresponding y-intercept. ***Medium***

c. Students may also check by solving the system of equations. ***Medium***

② Students may also use substitution to solve the system. ***Medium***

Practice Solving Mathematical Problems Involving Systems of Linear Equations

➤ **Study the Example showing how to solve a mathematical problem involving systems of linear equations. Then solve problems 1–6.**

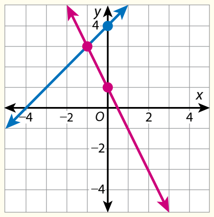
Example

A line with slope −2 and a line with slope 1 intersect at the point (−1, 3). Graph the system. What are the equations of the lines?

Plot the point (−1, 3). Use the slopes given to plot another point for each line.

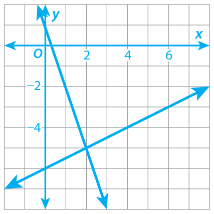
The line with slope −2 crosses the y-axis at $y = 1$. The equation of the line is $y = -2x + 1$.

The line with slope 1 crosses the y-axis at $y = 4$. The equation of the line is $y = x + 4$.



① Two lines intersect at the point (2, −5). The lines cross the y-axis at (0, 1) and (0, −6).

a. Graph the system.



b. What are the equations of the lines? $y = -3x + 1, y = \frac{1}{2}x - 6$

c. Check that (2, −5) is the solution of the system of equations you wrote in problem 2b.

$y = -3x + 1$ $\qquad$ $y = \frac{1}{2}x - 6$

$-5 \overset{?}{=} -3(2) + 1$ $\qquad$ $-5 \overset{?}{=} \frac{1}{2}(2) - 6$

$-5 = -5$ TRUE $\qquad$ $-5 = -5$ TRUE

② The sum of two numbers is 147. The difference of the two numbers is 25. What are the two numbers? Show your work. Possible work:

$x + y = 147$ $\qquad$ $x + y = 147$

$\underline{x - y = 25}$ $\qquad$ $86 + y = 147$

$2x = 172$ $\qquad$ $y = 61$

$x = 86$

SOLUTION 86 and 61

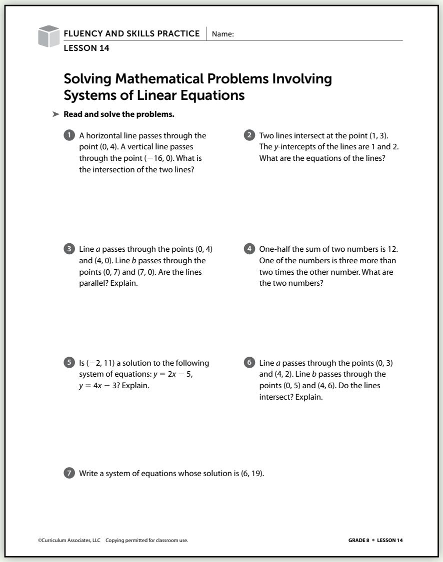
Vocabulary

system of linear equations
a group of related linear equations in which a solution makes all the equations true at the same time.

Fluency & Skills Practice

Solving Mathematical Problems Involving Systems of Linear Equations

In this activity, students solve mathematical problems by writing and solving systems of linear equations.



FLUENCY AND SKILLS PRACTICE | Name:
LESSON 14

Solving Mathematical Problems Involving Systems of Linear Equations

➤ **Read and solve the problems.**

❶ A horizontal line passes through the point (0, 4). A vertical line passes through the point (−16, 0). What is the intersection of the two lines?

❷ Two lines intersect at the point (1, 3). The y-intercepts of the lines are 1 and 2. What are the equations of the lines?

❸ Line a passes through the points (0, 4) and (4, 0). Line b passes through the points (0, 7) and (7, 0). Are the lines parallel? Explain.

❹ One-half the sum of two numbers is 12. One of the numbers is three more than two times the other number. What are the two numbers?

❺ Is (−2, 11) a solution to the following system of equations: $y = 2x - 5$, $y = 4x - 3$? Explain.

❻ Line a passes through the points (0, 3) and (4, 2). Line b passes through the points (0, 5) and (4, 6). Do the lines intersect? Explain.

❼ Write a system of equations whose solution is (6, 19).

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LESSON 14 | SESSION 3

Additional Practice

③ Students may also use elimination to solve the system. ***Challenge***

④ **a.** The lines do not intersect; they have the same slope but different y-intercepts. ***Medium***

b. The lines have the same slope, 2. ***Medium***

c. The y-coordinates of the given points with 0 as the x-coordinate are the y-intercepts, and they are not the same. ***Medium***

d. The lines have the same slope, so they are parallel. ***Medium***

⑤ **A and F are correct.** Students may solve the problem by substituting 2 for x in each equation and finding the corresponding value of y.

B is not correct. This answer divides 8 by 2 in the first equation and adds it to j.

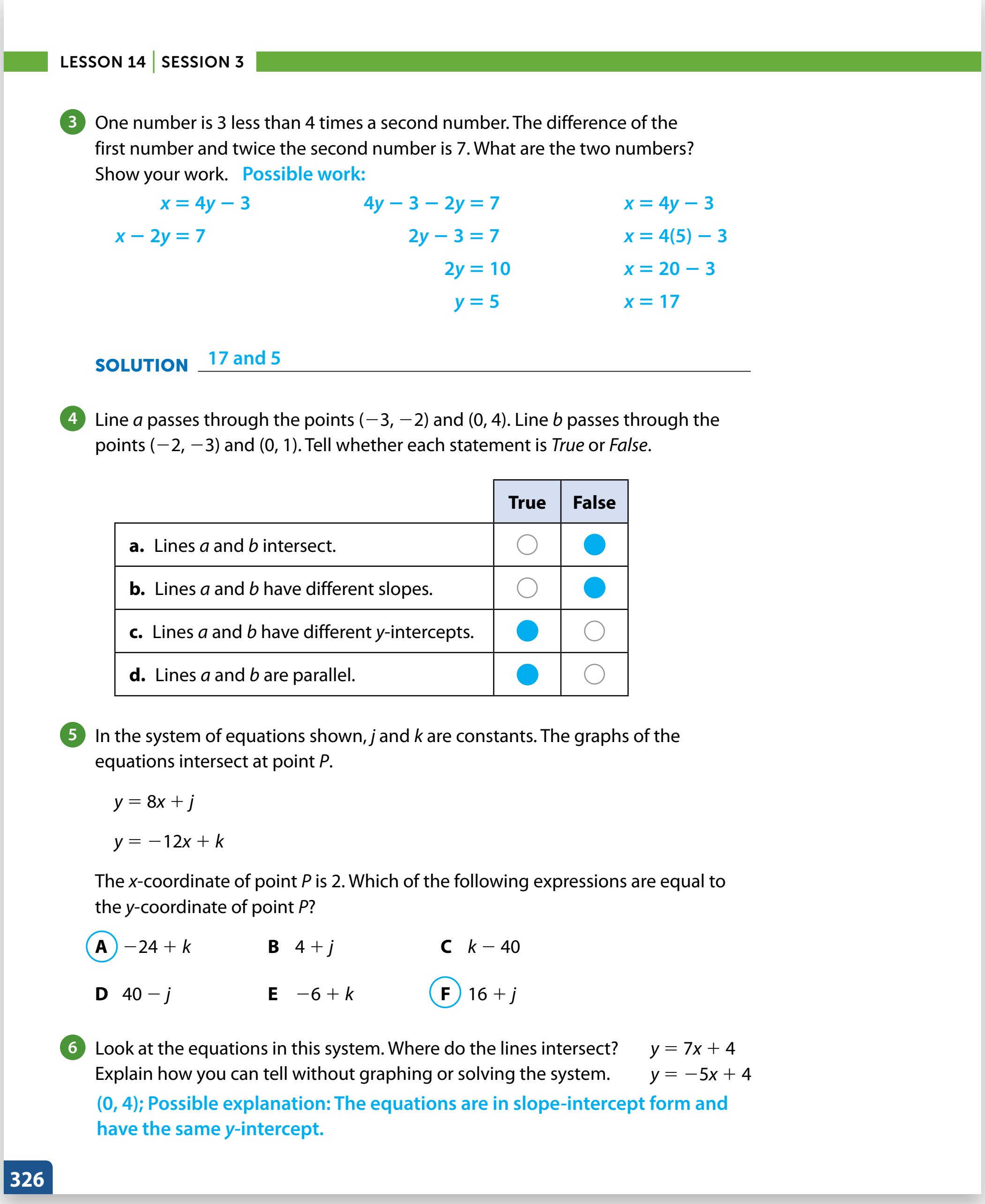
C is not correct. This solves the system by substitution, then substitutes the x-value. It shows the relationship between j and k.

D is not correct. This solves the system by substitution, then substitutes the x-value. It shows the relationship between j and k.

E is not correct. This answer divides -12 by 2 in the first equation and adds it to k.

Medium

⑥ The slopes are different, so the lines intersect. The y-intercept of both lines is 4. Because the lines can intersect at only one point, that point must be (0, 4). ***Medium***



③ One number is 3 less than 4 times a second number. The difference of the first number and twice the second number is 7. What are the two numbers? Show your work. **Possible work:**

$x = 4y - 3$
$x - 2y = 7$

$4y - 3 - 2y = 7$
$2y - 3 = 7$
$2y = 10$
$y = 5$

$x = 4y - 3$
$x = 4(5) - 3$
$x = 20 - 3$
$x = 17$

SOLUTION 17 and 5

④ Line a passes through the points $(-3, -2)$ and (0, 4). Line b passes through the points $(-2, -3)$ and (0, 1). Tell whether each statement is *True* or *False*.

	True	False
a. Lines a and b intersect.	○	●
b. Lines a and b have different slopes.	○	●
c. Lines a and b have different y-intercepts.	●	○
d. Lines a and b are parallel.	●	○

⑤ In the system of equations shown, j and k are constants. The graphs of the equations intersect at point P.

$y = 8x + j$

$y = -12x + k$

The x-coordinate of point P is 2. Which of the following expressions are equal to the y-coordinate of point P?

[✓ **A**] $-24 + k$
B $4 + j$
C $k - 40$
D $40 - j$
E $-6 + k$
[✓ **F**] $16 + j$

⑥ Look at the equations in this system. Where do the lines intersect? Explain how you can tell without graphing or solving the system.

$y = 7x + 4$
$y = -5x + 4$

(0, 4); Possible explanation: The equations are in slope-intercept form and have the same y-intercept.

DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Use with **Session 4** **Apply It**

Levels 1–3: Reading/Writing

Modify **Three Reads** to have students read Apply It problem 9 with partners. After the first read, have students tell what Cameron and Olivia are doing. After the next read, help students identify important values. Then chunk the third sentence to help students read and understand the task: *Write and solve a system of equations/ to find/ the price of each notebook/ and /the price of each package of pens.* Ask: *What do you need to write? What are the two things you will find?* Help students work on the problem and write what the variables and equation represent:

- *In this problem, the _____ represents _____.*

Levels 2–4: Reading/Writing

Modify **Three Reads** to have students read Apply It problem 9. After the first read, have students describe the situation to a partner. After the second read, have students underline and describe important values in the problem, for example, *$16 is the amount Cameron pays for _____.* Then chunk the third sentence to help students read and understand the task. Monitor as partners discuss the chunks. Then ask students to write the equations and explain what they represent. Provide sentence starters to help them connect:

- *In this problem, the variable/equation _____.*

Levels 3–5: Speaking/Writing

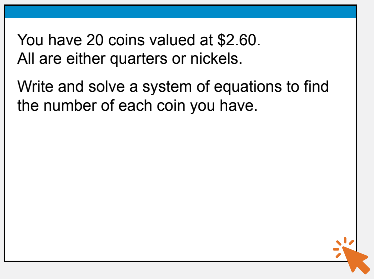
Use **Three Reads** to have students work on Apply It problem 9. After the first read, have students describe the situation to a partner. After the second read, have students identify and describe important values in the problem. Then have students read the third sentence. Ask them to underline the words or phrases that indicate what they need to do and find. Then ask students to write the equations and explain what they represent. Have students meet with other partners to tell how their equation connects with the situation in the problem.

Refine Representing and Solving Problems with Systems of Linear Equations

Purpose

- **Refine** strategies for writing and solving systems of equations that model real-world and mathematical problems.
- **Refine** understanding of how to identify and interpret the information in a problem in order to write the system of equations that models the problem.

START CHECK FOR UNDERSTANDING



You have 20 coins valued at $2.60. All are either quarters or nickels.

Write and solve a system of equations to find the number of each coin you have.

Solution

$q + n = 20$

$0.25q + 0.5n = 2.60$;

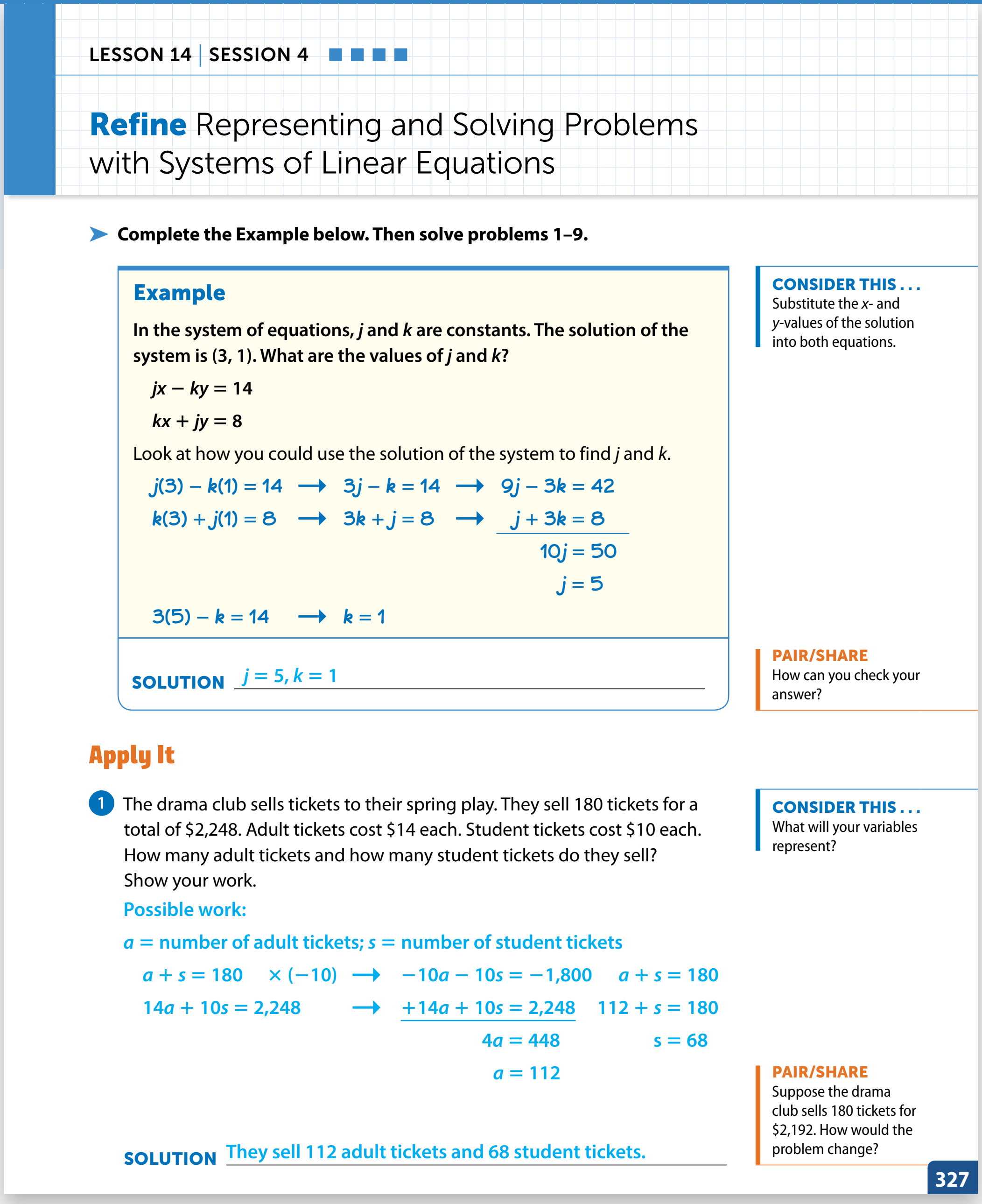
8 quarters, 12 nickels

WHY? Confirm students' understanding of writing and solving a system of equations to model a real-world problem, identifying common errors to address as needed.

MONITOR & GUIDE

Before students begin to work, use their responses to the **Start** to determine those who will benefit from additional support. Use the **Error Analysis** table below to guide remediation.

Have all students complete the Example and problems 1–3, using Consider This and Pair/Share as appropriate. Observe and monitor their reasoning and guide or redirect students as needed.



LESSON 14 | SESSION 4 ■■■■

Refine Representing and Solving Problems with Systems of Linear Equations

➤ **Complete the Example below. Then solve problems 1–9.**

Example

In the system of equations, j and k are constants. The solution of the system is (3, 1). What are the values of j and k?

$jx - ky = 14$

$kx + jy = 8$

Look at how you could use the solution of the system to find j and k.

$$j(3) - k(1) = 14 \longrightarrow 3j - k = 14 \longrightarrow 9j - 3k = 42$$
$$k(3) + j(1) = 8 \longrightarrow 3k + j = 8 \longrightarrow \underline{j + 3k = 8}$$
$$10j = 50$$
$$j = 5$$
$$3(5) - k = 14 \longrightarrow k = 1$$

SOLUTION $j = 5, k = 1$

CONSIDER THIS . . . Substitute the x- and y-values of the solution into both equations.

PAIR/SHARE How can you check your answer?

Apply It

① The drama club sells tickets to their spring play. They sell 180 tickets for a total of $2,248. Adult tickets cost $14 each. Student tickets cost $10 each. How many adult tickets and how many student tickets do they sell? Show your work.

Possible work:

a = number of adult tickets; s = number of student tickets

$$a + s = 180 \quad \times(-10) \longrightarrow -10a - 10s = -1{,}800 \qquad a + s = 180$$
$$14a + 10s = 2{,}248 \longrightarrow \underline{+14a + 10s = 2{,}248} \qquad 112 + s = 180$$
$$4a = 448 \qquad s = 68$$
$$a = 112$$

SOLUTION They sell 112 adult tickets and 68 student tickets.

CONSIDER THIS . . . What will your variables represent?

PAIR/SHARE Suppose the drama club sells 180 tickets for $2,192. How would the problem change?

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START ERROR ANALYSIS

If the error is . . .	Students may . . .	To support understanding . . .
12 quarters, 8 nickels	have switched the variables.	Ask students to define variables before writing and solving a system of equations. Encourage them to use variables that connect to the item, like q for quarters and d for dimes.
10 quarters, 2 nickels	have found the maximum number of quarters you could have, 10, which means you need 2 nickels to make $2.60.	Remind students that although the value of the coins needs to be $2.60, the number of coins also needs to be 20. Have them write an equation for each piece of information and help them see that these two equations make up the system of equations they need to solve.
any combination of 20 coins except 12 quarters and 8 nickels	have found a combination of quarters and nickels that have a sum of 20, but do not have a value of $2.60.	Prompt students to consider the value of a quarter and a nickel in decimal form. Ask students how they can use the value of a quarter and a nickel to write a system of equations.

Example

Guide students in understanding the Example. Ask:

- *What does the first number in the ordered pair (3, 1) represent?*
- *What does the second number in the ordered pair (3, 1) represent?*
- *After substituting (3, 1) into the system, which method is easier to use to solve the system?*

Help all students focus on the Example and responses to the questions by asking them to critique their classmates' responses.

Look for understanding that the ordered pair (3, 1) should be substituted into the system for x and y. Then the resulting system can be solved for j and k using elimination.

Apply It

1. Students may also solve the system by multiplying the equation that represents the number of tickets by -14 and eliminating the variable a. *DOK 2*

2. Line b intersects line a at point (–2, 2). If b is the y-intercept of line b, then the slope between the intersection point and (0, b) is $\frac{b-2}{2}$.

   Because the slope is greater than 0, b must be greater than 2. Because the slope is less than 1, b must be less than 4. So, line b passes through $(-2, 2)$ and has a y-intercept between 2 and 4. *DOK 2*

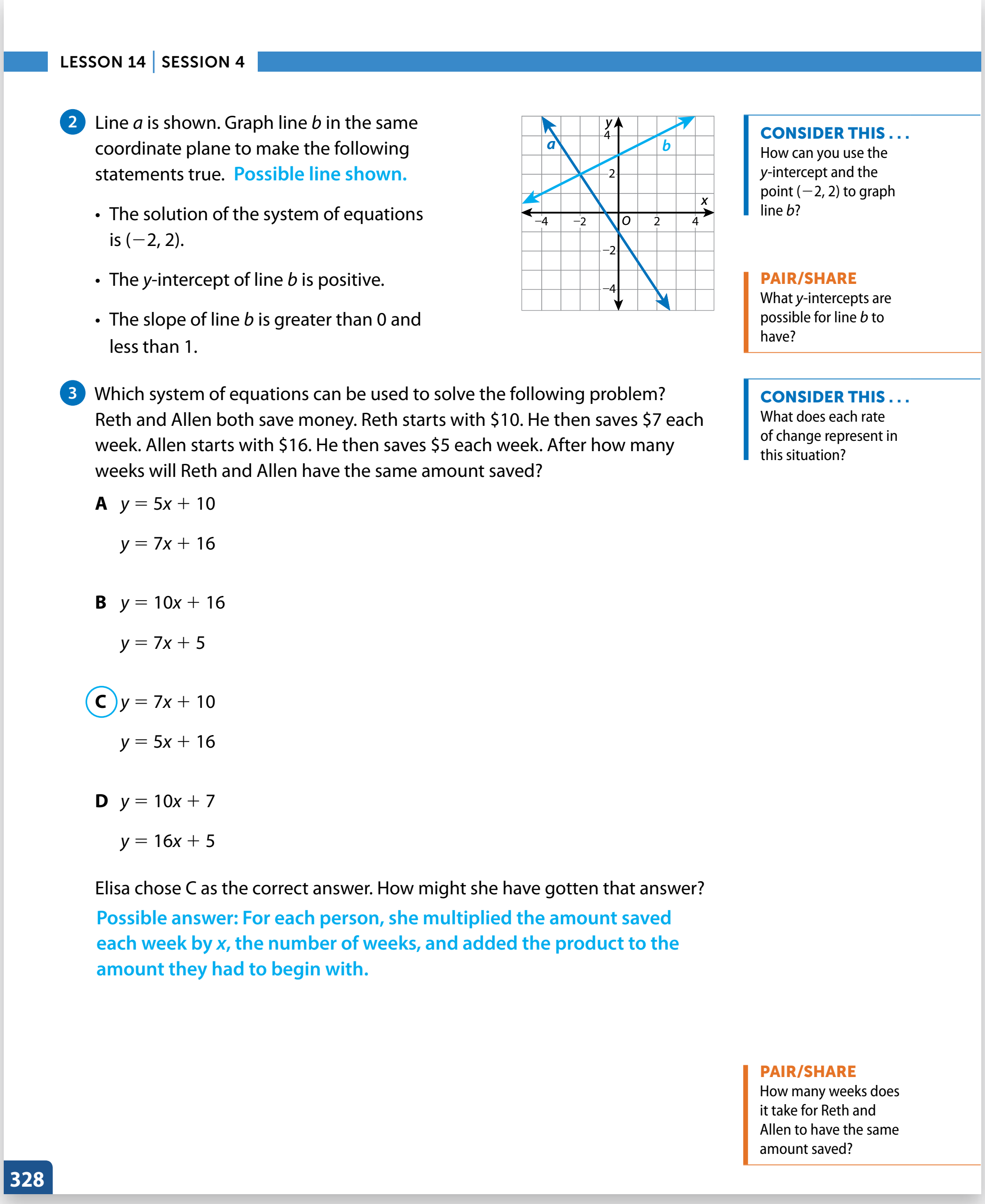
3. **C is correct.** Reth's equation is $y = 7x + 10$ because he starts with \$10 and saves \$7 each week. Allen's equation is $y = 5x + 16$ because he starts with \$16 and saves \$5 each week.

   A is not correct. This answer switches the amount saved each week for Reth and Allen.

   B is not correct. In this answer, the initial amounts saved were used to write the first equation, and the amounts saved each week were used to write the second equation.

   D is not correct. In this answer, the initial amount saved was multiplied by the number of weeks and added to the amount saved each week.

   DOK 3



LESSON 14 | SESSION 4

2 Line a is shown. Graph line b in the same coordinate plane to make the following statements true. Possible line shown.

- The solution of the system of equations is $(-2, 2)$.
- The y-intercept of line b is positive.
- The slope of line b is greater than 0 and less than 1.

CONSIDER THIS . . .
How can you use the y-intercept and the point $(-2, 2)$ to graph line b?

PAIR/SHARE
What y-intercepts are possible for line b to have?

3 Which system of equations can be used to solve the following problem?
Reth and Allen both save money. Reth starts with \$10. He then saves \$7 each week. Allen starts with \$16. He then saves \$5 each week. After how many weeks will Reth and Allen have the same amount saved?

A $y = 5x + 10$
$y = 7x + 16$

B $y = 10x + 16$
$y = 7x + 5$

[✓ **C**] $y = 7x + 10$
$y = 5x + 16$

D $y = 10x + 7$
$y = 16x + 5$

Elisa chose C as the correct answer. How might she have gotten that answer?

Possible answer: For each person, she multiplied the amount saved each week by x, the number of weeks, and added the product to the amount they had to begin with.

CONSIDER THIS . . .
What does each rate of change represent in this situation?

PAIR/SHARE
How many weeks does it take for Reth and Allen to have the same amount saved?

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GROUP & DIFFERENTIATE

Identify groupings for differentiation based on the **Start** and problems 1–3. A recommended sequence of activities for each group is suggested below. Use the resources on the next page to differentiate and close the lesson.

Approaching Proficiency

- **RETEACH** Visual Model
- **REINFORCE** Problems 4, 6, 8

Meeting Proficiency

- **REINFORCE** Problems 4–8

Extending Beyond Proficiency

- **REINFORCE** Problems 4–8
- **EXTEND** Challenge

Have all students complete the **Close: Exit Ticket**.

Resources for Differentiation are found on the next page.

Refine Representing and Solving Problems with Systems of Linear Equations

Apply It

④ See ***Connect to Culture*** to support student engagement.

a. Students may choose related variables. *DOK 1*

b. One equation should relate the yards of yarn for a hat and the yards of yarn for a scarf to the total number of yards of yarn. The other should relate the hours it takes to knit a hat and the hours it takes to knit a scarf to the number of hours knitted. *DOK 2*

c. Students may multiply the second equation by −22 and then add to eliminate *h*. *DOK 2*

⑤ Students may compare slopes and *y*-intercepts.

C is correct. The slope of the line containing (−6, 4) and (−2, 1) is the same as the slope of line *a*, but the *y*-intercepts are different. The lines are parallel. So, there is no solution.

A is not correct. The line containing (2, 1) and (6, 4) has a different slope than line *a*.

B is not correct. The slope and *y*-intercept of the line containing (1, −1) and (9, −7) are the same as line *a*. The lines are the same.

D is not correct. The line containing (3, 1) and (5, −4) has a different slope than line *a*.

DOK 2

⑥ Students may solve the system they write by solving one equation for *x* or *y* and then using the substitution method. *DOK 1*

④ Evelyn knits hats and scarves for charity. She records the time it takes and the amount of yarn needed to make one of each item. Last winter Evelyn knitted for 180 hours. She used 2,520 yards of yarn. How many hats and scarves did Evelyn knit?

Hat	Scarf
5 hours	15 hours
110 yards	150 yards

a. Choose variables for the two unknown quantities in the problem and tell what each variable represents.

Possible answer:
h = number of hats; s = number of scarves

b. Write a system of two equations to represent the situation.

$110h + 150s = 2{,}520$, $5h + 15s = 180$

c. How many hats and scarves did Evelyn knit? Show your work.

Possible work:

$110h + 150s = 2{,}520 \longrightarrow 110h + 150s = 2{,}520$

$-10(5h + 15s = 180) \longrightarrow \underline{-50h - 150s = -1{,}800}$

$60h = 720$

$h = 12$

$5h + 15s = 180$

$5(12) + 15s = 180$

$15s = 120$

$s = 8$

SOLUTION Evelyn knit 12 hats and 8 scarves.

⑤ Lines *a* and *b* form a system with no solution. The points (−3, 2) and (5, −4) lie on line *a*. Which two points could lie on line *b*?

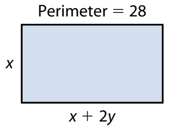
A (2, 1) and (6, 4)

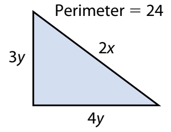
B (1, −1) and (9, −7)

[✓ **C**] (−6, 4) and (−2, 1)

D (3, 1) and (5, −4)

⑥ What are the values of *x* and *y* in the figures shown? Show your work.





Possible work:

$4x + 4y = 28 \longrightarrow 4x + 4y = 28$

$2x + 7y = 24 \quad \times(-2) \longrightarrow \underline{+\ -4x - 14y = -48}$

$-10y = -20$

$y = 2$

$4x + 4(2) = 28 \longrightarrow 4x = 20 \longrightarrow x = 5$

SOLUTION $x = 5, y = 2$

DIFFERENTIATION

RETEACH

Visual Model

Use a trivia game to understand a system of linear equations.

Students approaching proficiency with writing and solving systems of equations to solve real-world problems will benefit from extra practice writing and solving problems.

- Present students with the scenario: *Suppose you are playing a trivia game. You answer 7 questions correctly. Some questions are worth 3 points and others are worth 5 points. Your total score is 27 points.*
- Ask: *What variables should be defined for this problem*? [*x* can represent the number of 3-point questions answered correctly and *y* can represent the number of 5-point questions answered correctly.]
- Ask: *What equation represents the number of questions answered correctly?* [$x + y = 7$]
- Ask: *What equation represents the number of points earned?* [$3x + 5y = 27$]
- Divide students into 3 groups. Assign one group to solve the system of equations by graphing, one group to solve the system using substitution, and one group to solve the system using elimination.
- Allow time for students to solve the system and have each group present their solution to the class. Ensure each group has an answer of $x = 4$ and $y = 3$.
- Discuss the meaning of the solution to the system. Ask: *How many points did you earn from 3-point questions?* [12] Ask: *How many points did you earn from 5-point questions?* [15]

Refine

7 The first equation in the system represents the fact that the sum of the number of liters of Solution A and the number of liters of Solution B is equal to 10 liters of mixture. *DOK 2*

8 **a.** Students may reason the representations from the given system. *DOK 2*

b. Students may solve the system by substitution. *DOK 2*

CLOSE EXIT TICKET

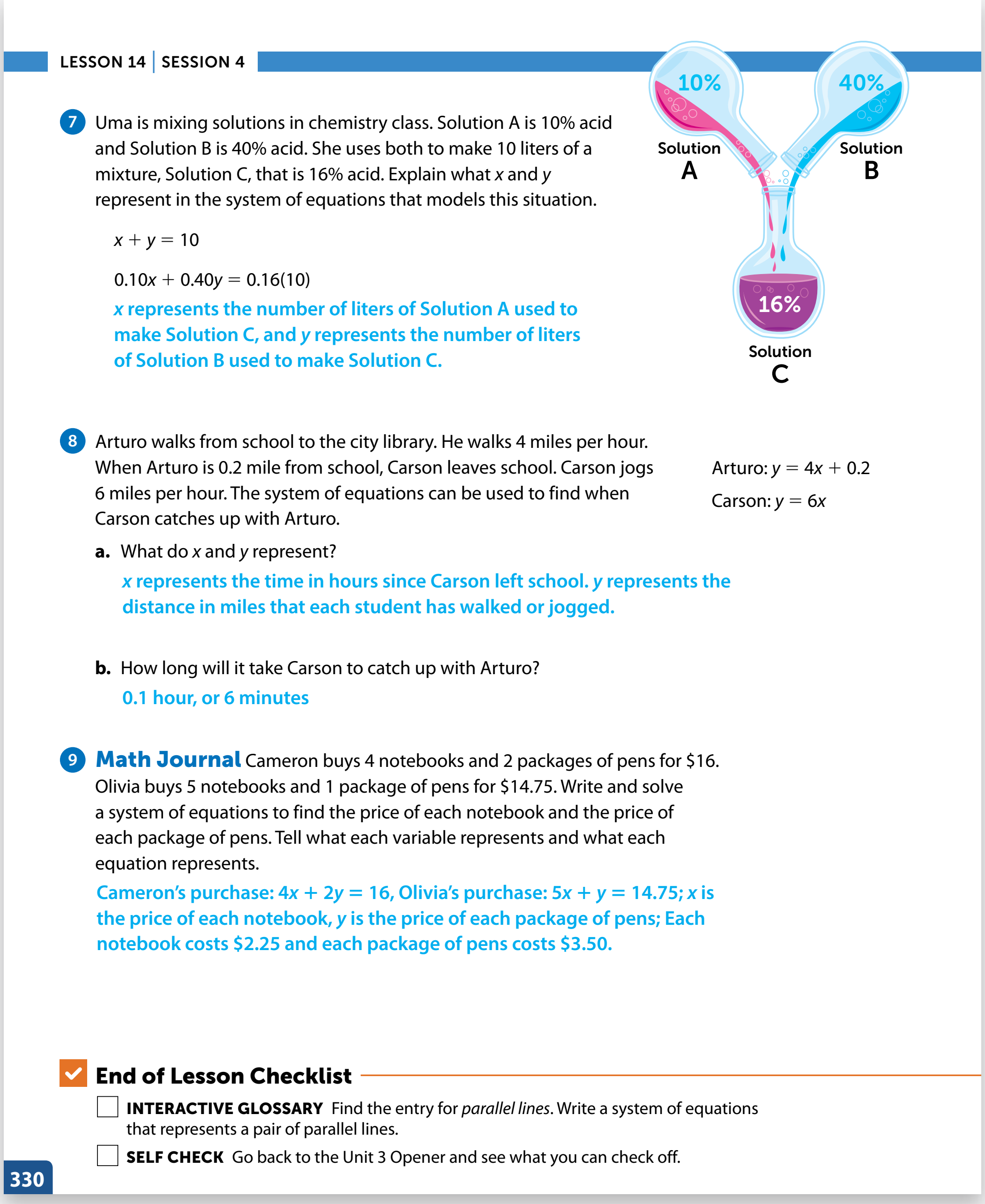
9 **Math Journal** Look for understanding of how to write and solve a system of equations to solve a problem with two unknowns.

Error Alert If students write the system as $4x + 5y = 16$ and $2x + y = 14.75$, then have them connect the problem to each part of their equations. Cameron's cost for notebooks plus pens is equal to his total cost. Olivia's cost for notebooks plus pens is equal to her total cost.

✓ End of Lesson Checklist

INTERACTIVE GLOSSARY Support students by having them discuss with a partner what must be true about the slopes and *y*-intercepts of two lines that are parallel. Then, have students write the system, with both equations in slope-intercept form.

SELF CHECK Have students review and check off any new skills on the Unit 3 Opener.



LESSON 14 | SESSION 4

7 Uma is mixing solutions in chemistry class. Solution A is 10% acid and Solution B is 40% acid. She uses both to make 10 liters of a mixture, Solution C, that is 16% acid. Explain what *x* and *y* represent in the system of equations that models this situation.

$x + y = 10$

$0.10x + 0.40y = 0.16(10)$

x represents the number of liters of Solution A used to make Solution C, and *y* represents the number of liters of Solution B used to make Solution C.

8 Arturo walks from school to the city library. He walks 4 miles per hour. When Arturo is 0.2 mile from school, Carson leaves school. Carson jogs 6 miles per hour. The system of equations can be used to find when Carson catches up with Arturo.

Arturo: $y = 4x + 0.2$

Carson: $y = 6x$

a. What do *x* and *y* represent?

x represents the time in hours since Carson left school. *y* represents the distance in miles that each student has walked or jogged.

b. How long will it take Carson to catch up with Arturo?

0.1 hour, or 6 minutes

9 **Math Journal** Cameron buys 4 notebooks and 2 packages of pens for $16. Olivia buys 5 notebooks and 1 package of pens for $14.75. Write and solve a system of equations to find the price of each notebook and the price of each package of pens. Tell what each variable represents and what each equation represents.

Cameron's purchase: $4x + 2y = 16$, Olivia's purchase: $5x + y = 14.75$; *x* is the price of each notebook, *y* is the price of each package of pens; Each notebook costs $2.25 and each package of pens costs $3.50.

✓ **End of Lesson Checklist**

☐ **INTERACTIVE GLOSSARY** Find the entry for *parallel lines*. Write a system of equations that represents a pair of parallel lines.

☐ **SELF CHECK** Go back to the Unit 3 Opener and see what you can check off.

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REINFORCE

Problems 4–8

Write and solve real-world systems.

Students meeting proficiency will benefit from additional work with writing and solving systems of linear equations that model real-world and mathematical problems by solving problems in a variety of formats.

- Have students work on their own or with a partner to solve the problems.
- Encourage students to show their work.

EXTEND

Challenge

Solve a system to identify a mystery number.

Students extending beyond proficiency will benefit from writing and solving a system of equations to solve a problem.

- Have students work with a partner to solve:

  The sum of the digits of a two-digit whole number is $\frac{1}{4}$ the value of the number. The tens digit is half of the ones digit. What is the number?
- Students can use a variable for the tens digit and a variable for the ones digit and then use the information given to write a system of equations.

PERSONALIZE

i-Ready

Provide students with opportunities to work on their personalized instruction path with *i-Ready* Online Instruction to:

- fill prerequisite gaps.
- build up grade-level skills.