

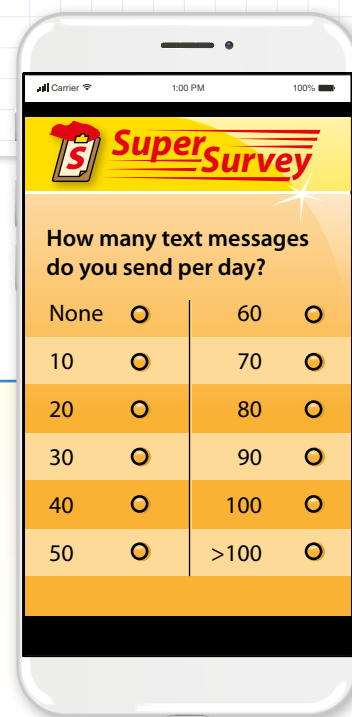
Explore Random Samples

Previously, you learned about random samples. In this lesson, you will learn about using random samples to make estimates.

► **Use what you know to try to solve the problem below.**

Vivian surveys a random sample of Grade 7 students in the school band. She asks the students how many text messages they send each day. The median of her sample data is 50.

Edward surveys a random sample of all Grade 7 students. He asks the students how many text messages they send each day. The median of his sample data is 60. Whose result is more likely to be representative of all the Grade 7 students? Why?



**TRY
IT**



Math Toolkit bags, bowls, buttons, cups, index cards, number cubes

DISCUSS IT

Ask: How did you reach that conclusion?

Share: First, I thought . . .



Learning Target SMP 1, SMP 2, SMP 3, SMP 4, SMP 5, SMP 6

Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions.

CONNECT IT

- 1 Look Back** Is Vivian's or Edward's result more likely to be representative of the whole grade? Why?

- 2 Look Ahead** While Vivian and Edward surveyed different populations, they both surveyed random samples. You can use data from a random sample to make an estimate or an inference about a population.
 - a.** Sofia surveys a random sample of students in her school. She finds that 20 people in her random sample have pierced ears. Is it reasonable for Sofia to estimate that 20 people in the population of students at her school have pierced ears? Explain.

 - b.** Sofia also finds that 50% of the students in her random sample wear glasses. Why is it a more reasonable inference that about 50% of the students in her school wear glasses than exactly 50% of the students in her school wear glasses?

- 3 Reflect** How is making an inference about a population different from knowing something for certain about a population?

Prepare for Reasoning About Random Samples

- 1 Think about what you know about random samples. Fill in each box. Use words, numbers, and pictures. Show as many ideas as you can.

The diagram is a concept map centered on the term "random sample". It consists of a central oval containing the text "random sample". Four lines radiate from this central oval to four rectangular boxes arranged in a square pattern around it. The boxes are labeled as follows:

- Top-left box: **In Your Own Words**
- Top-right box: **What I Know About It**
- Bottom-left box: **Examples**
- Bottom-right box: **Non-Examples**

- 2 José is interested in the favorite activities of the students in his school. He surveys all the members of the school's book club. Did José survey a random sample of all the students in his school? Explain.

- 3 On the last day of school, all of the students in Jordan and Dara's middle school compete in a long jump contest. Jordan surveys a random sample from all students in the school about how far they can long jump. The mean of her sample data is 11 ft 6 in.

Dara surveys a random sample of Grade 8 students in the school. The mean of his sample data is 12 ft 2 in.

- a. Whose answer do you think is more likely to be representative of all the students in the school? Explain.

- b. Justify your answer in a different way.



Develop Making Inferences from Samples About Populations

► Read and try to solve the problem below.

There are 406 students at Destiny's school. Destiny surveys 20 randomly chosen students from her school about how they get to school.

Based on the data from the sample, about how many students in the school should Destiny expect to take the subway?

Transportation Method	Frequency
Subway	8
Walk	4
Bus	4
Bike	2
Car	2

TRY
IT



Math Toolkit double number lines, grid paper

DISCUSS IT

Ask: How do you know your answer is reasonable?

Share: I know my answer is reasonable because . . .

➤ **Explore different ways to use a random sample to make inferences about a population.**

There are 406 students at Destiny's school. Destiny surveys 20 randomly chosen students from her school about how they get to school.

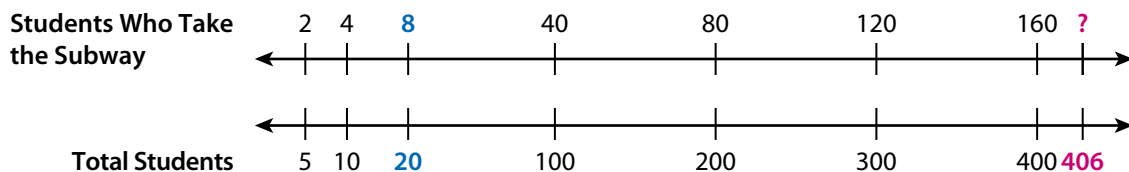
Based on the data from the sample, about how many students in the school should Destiny expect to take the subway?

Transportation Method	Frequency
Subway	8
Walk	4
Bus	4
Bike	2
Car	2

Model It

You can use a double number line to make an inference about the population.

The data come from a random sample, so you can expect them to be representative of the population.



Model It

You can use percents to make an inference about a population.

Find the percent of students in the sample that take the subway.

$$\frac{8}{20} = 0.4, \text{ or } 40\%$$

So, you can infer that 40% of all the students in the population take the subway.

$$0.4(406)$$



CONNECT IT

► **Use the problem from the previous page to help you understand how to use a random sample to make inferences about a population.**

- 1 Look at the first **Model It**. What two quantities does it show a proportional relationship between?
- 2 Look at $0.4(406)$ in the second **Model It**. How is the random sample represented in the expression?
- 3 Based on Destiny's random sample, about how many students can she expect to take the subway? Is this the same as the result of finding 40% of 406? Explain.
- 4 Destiny's friends Xavier and Querida also each survey 20 students selected at random from the school. Based on their samples, Xavier infers that about 142 students take the subway to school and Querida infers that about 183 students do. Explain why both of their inferences are reasonable.
- 5 How can you use proportional reasoning to make an inference about a population from a random sample of that population?
- 6 **Reflect** Think about all the models and strategies you have discussed today. Describe how one of them helped you better understand how to use a random sample to make inferences about a population.

Apply It

► Use what you learned to solve these problems.

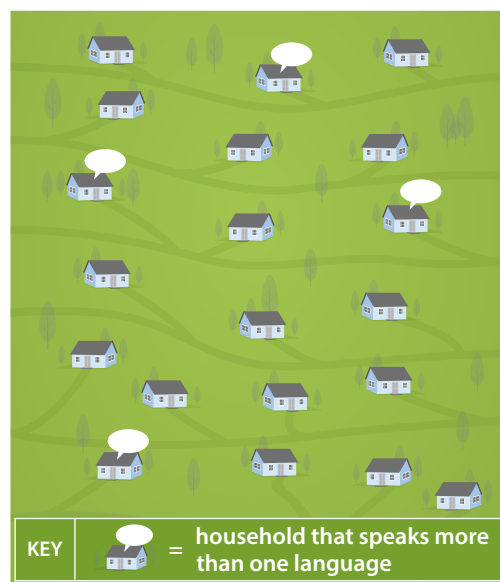
- 7 A random sample of Grade 8 students at a school are asked whether they plan to take computer science in high school. Of those asked, 15 students plan to take computer science, 5 do not, and 7 are unsure. There are 326 Grade 8 students in the school. Based on the sample, about how many Grade 8 students in the school plan to take computer science in high school? Show your work.

SOLUTION

- 8 Yolanda asks a random sample of 50 students at her school what they do after school. Of the 50 students, 30 say they play a sport. Based on the sample, about what percent of students at Yolanda's school play a sport after school?

- A 15% B 30%
C 50% D 60%

- 9 There are 5,119 households in Salvador's town. Salvador surveys a random sample of 20 households from his town. He finds that more than one language is spoken in 4 of the households. Based on the sample, in about how many households in the town should Salvador expect more than one language to be spoken? Show your work.



SOLUTION

Practice Making Inferences from Samples About Populations

- **Study the Example showing how to use a random sample to make an inference about a population. Then solve problems 1–4.**

Example

Garrett is running for class president. He wants to know if he is likely to win, so his friend Jacob surveys a random sample of 10 students in his school. Of the 10 students, 7 say they will vote for Garrett. Suppose all 233 students in the school vote in the election for class president. About how many students should Garrett expect to vote for him?

Since $\frac{7}{10}$ of students in the sample say they will vote for Garrett, he should expect about $\frac{7}{10}$ of the population to vote for him.

$$\frac{7}{10}(233) = 163\frac{1}{10}$$

Garrett should expect about 163 students to vote for him.

- 1 Jacob conducts another survey of students in the school in the Example. This time, he surveys a random sample of 30 students.
- a. In Jacob's sample, 24 students say they will vote for Garrett. Based on this sample, about how many students in the school should Garrett expect to vote for him? Show your work.

SOLUTION

- b. Using Jacob's surveys, can Garrett know for certain how many students plan to vote for him? Explain why or why not.

Vocabulary

random sample

a sample in which every element in the population has an equal chance of being selected.

- 2 Mindy works at a movie theater. One Friday, she collects a random sample of the type of tickets sold in the afternoon and the evening. She estimates that when 400 tickets are sold on a Friday evening, about 100 of them will be senior tickets. Is Mindy's estimate reasonable? Explain.

Time of Day	Adult	Senior
Afternoon	12	48
Evening	45	15

- 3 Students at a certain high school have to take an arts or technology class. A random sample of 60 students from the high school are surveyed. Each student is asked which class they take. Based on the survey results, which of the following statements are true? Select all that apply.

Class	Number of Students
Dance	19
Electronics	8
Music	7
Painting	15
Photography	11

- A** There are many excellent dancers at the high school.
- B** About 25% of the students at the high school take painting.
- C** Of every 30 students in the high school, about 11 of them take photography.
- D** Next year, 7 out of every 60 students at the high school will take music.
- E** In a group of 120 students from the high school, about 16 of the students likely take electronics.
- 4 Moses writes a paper on fruit for health class. He surveys a random sample of students at his school about their favorite fruit. In his sample 46 students say strawberries. The other 34 students in his sample say a different fruit. There are 506 students in Moses's school. What inference can Moses make about the number of students in his school who would say strawberries are their favorite fruit? Show your work.

SOLUTION _____

Develop Making Inferences from Multiple Samples

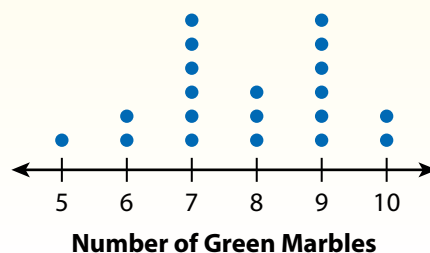


► Read and try to solve the problem below.

Mr. Seda has a bag with 500 marbles. The marbles are either green or yellow. He has 20 students in his class take turns selecting 10 marbles from the bag without looking. Each student records the number of green marbles and then returns the marbles to the bag.

What is a reasonable estimate for the number of green marbles in the bag?

Samples from Mr. Seda's Class



TRY
IT



Math Toolkit double number lines, grid paper

DISCUSS IT

Ask: Why did you choose that strategy to find a reasonable estimate?

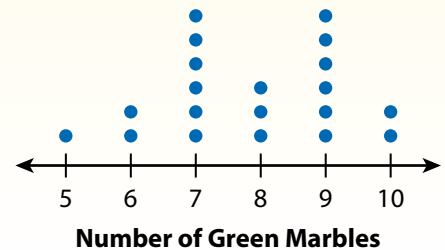
Share: I chose that strategy because ...

➤ **Explore different ways to make inferences about a population from multiple random samples.**

Mr. Seda has a bag with 500 marbles. Some marbles are green, and the rest are yellow. He has 20 students in his class each select 10 marbles from the bag without looking. Each student records the number of green marbles and then returns the marbles to the bag.

What is a reasonable estimate for the number of green marbles in the bag?

Samples from Mr. Seda's Class



Analyze It

You can use the mean of the samples to make an inference.

Each sample has 10 marbles. The mean of the number of green marbles in each sample is 7.85.

You can expect the fraction $\frac{\text{green marbles}}{\text{total marbles}}$ to be about the same for the population and the average of the samples.

Let g represent the number of green marbles in the population.

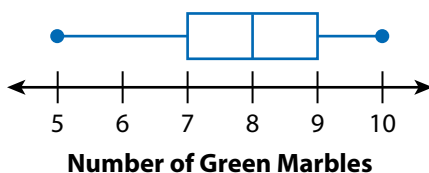
$$\frac{7.85}{10} = \frac{g}{500}$$

Analyze It

You can use the median of the samples to make an inference.

Use a box plot to organize the samples.

Samples from Mr. Seda's Class



Each sample has 10 marbles. The median number of green marbles in each sample is 8.

Let g represent the number of green marbles in the population.

$$\frac{8}{10} = \frac{g}{500}$$

CONNECT IT

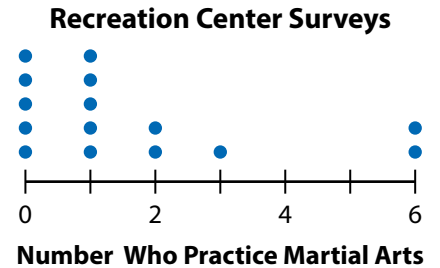
- Use the problem from the previous page to help you understand how to use multiple random samples to make an inference about a population.
- 1 Look at the **Analyze Its**. How many green marbles do you estimate are in the bag when you use the mean? What about when you use the median?
 - 2 Why are the estimates different even though both use a measure of center?
 - 3 Are both estimates reasonable? Why?
 - 4 The first student who selected a sample got 6 green marbles in his sample. Why might making an inference from just this one sample be misleading?
 - 5 How can using more than one sample help you make a better inference about a population than using only one sample?
 - 6 **Reflect** Think about all the models and strategies you have discussed today. Describe how one of them helped you better understand how to solve the **Try It** problem.

Apply It

► Use what you learned to solve these problems.

- 7 Staff at a recreation center are deciding whether to offer martial arts classes. Each of 15 staff members surveys a random sample of 8 recreation center members about whether they practice martial arts.

Riley uses the median of the results and infers that $\frac{1}{8}$ of the recreation center members practice martial arts. Brian uses the mean and infers that $\frac{1}{5}$ do. Explain why Riley's inference is more reasonable than Brian's.



- 8 Caleb, Aimee, and Ramón all survey random samples of 40 students in their school about whether they babysit their younger siblings. They use the results to make an inference about what percent of students in the school babysit their younger siblings. Which range of inferences is most reasonable?

- A 27.5% to 30%
- B 20% to 30%
- C 8% to 12%
- D 10.3% to 11%

Student Who Conducted Survey	Number of Students Who Babysit Siblings
Caleb	<div> </div> <div> </div> <div> </div>
Aimee	<div> </div> <div> </div> <div> </div>
Ramón	<div> </div> <div> </div> <div> </div>

- 9 Mason has a bag with 171 beads. Some beads are red, some are orange, and the rest are yellow. He selects 8 beads without looking, counts how many are orange, and then puts the beads back in the bag. He does this 12 times. What is a reasonable estimate for the number of orange beads in the bag? Show your work.

Number of Orange Beads	2	3	4	5	6
Frequency	2	5	3	1	1

SOLUTION _____

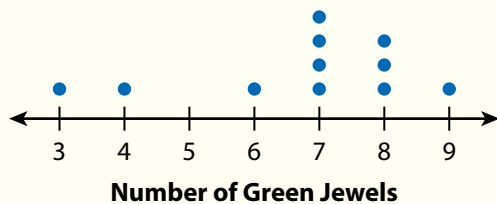
Practice Making Inferences from Multiple Samples

- Study the Example showing how to use multiple random samples to make an inference about a population. Then solve problems 1–4.

Example

A box in Mrs. McClary's class has 200 plastic jewels. She has 11 students each select a random sample of 10 jewels, count the number of green jewels, and then return all 10 jewels to the box. Estimate the number of green jewels in the box.

Samples from Mrs. McClary's Class



The median number of green jewels in each sample is 7. There are 10 jewels in a sample. There are 200 jewels in the population.

$$\frac{7}{10} \times 200 = 140$$

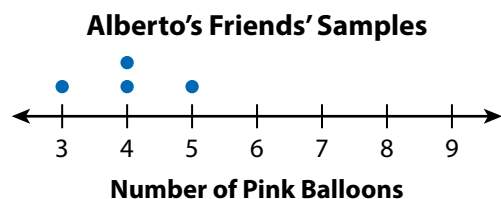
A reasonable estimate for the number of green jewels in the box is 140.

- 1 Use the mean to find an estimate for the number of green jewels in the box in the Example. Show your work.

SOLUTION

- 2 Describe a situation when an inference from the mean and an inference from the median might be very different.

- 3 Alberto buys a bag of 600 balloons. Some of the balloons are pink, and the rest are green. He wants to estimate the number of pink balloons in the bag. He asks 4 friends to each select a random sample of 10 balloons. Each friend returns the sample before the next friend selects a sample.



Based on the data, what is a reasonable estimate for the number of pink balloons in the bag? Show your work.

SOLUTION

- 4 A jar holds 3,000 marbles of four different colors. Neena tries to guess the number of marbles of each color. She is allowed to take 4 random samples of 200 marbles each.

Based on Neena's data, which of the following statements are true? Select all that apply.

	Blue	Green	Red	White
Sample 1	82	42	36	40
Sample 2	70	51	49	30
Sample 3	58	65	45	32
Sample 4	65	62	38	35

- A** The total number of red marbles in the jar is approximately 168.
- B** There are likely more blue marbles in the jar than any other color.
- C** There are probably more white marbles than green marbles in the jar.
- D** Approximately 21% to 33% of the marbles in the jar are green.
- E** There are probably about twice as many blue marbles as green marbles in the jar.

Refine Reasoning About Random Samples

► Complete the Example below. Then solve problems 1–8.

Example

There are 150 cards in a bag. Each card is either black or red. Carter selects 5 cards at random, records the number of red cards, and returns the cards. He does this until he has 12 samples of 5 cards.

Carter's Results: 0, 1, 1, 1, 2, 2, 2, 2, 2, 3, 3, 5

Kwame does the same, but he uses 10 cards in his samples.

Kwame's Results: 1, 2, 3, 3, 3, 4, 4, 4, 4, 5, 5, 7

What is a reasonable estimate for the number of red cards in the bag?

Look at how you could show your work using percents.

The median number of red cards in Carter's samples is 2.

$$\frac{2}{5} = 40\%$$

The median number of red cards in Kwame's samples is 4.

$$\frac{4}{10} = 40\%$$

SOLUTION _____

CONSIDER THIS . . .

To find the percent of cards that are red, divide the number that are red by the number of cards in the sample.

PAIR/SHARE

Why would it be inaccurate to make an estimate using the median of Carter and Kwame's combined samples?

Apply It

- 1 A school has 800 students. The staff plans to order a bag for each student. They ask 120 randomly selected students which color bag they prefer. Based on this sample, how many bags of each color should the staff order? Show your work.

Color	Quantity
Blue	48
Black	54
Gray	18

CONSIDER THIS . . .

The sum of the quantities of each color bag ordered should be 800.

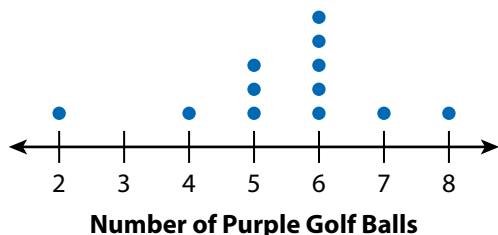
PAIR/SHARE

How could you solve this problem with a different strategy?

SOLUTION _____

- 2 A box contains a mix of 800 golf balls. Some are white and the rest are purple. Each student in Mrs. Ramírez's class selects a random sample of 20 golf balls from the box, counts the purple golf balls, and returns the sample to the box.

Samples from Mrs. Ramírez's Class



What is a reasonable estimate for the number of purple golf balls in the box?
Show your work.

SOLUTION _____

CONSIDER THIS ...

You could use the mean or the median to make an estimate.

PAIR/SHARE

How would your confidence in your estimate change if there were 25 students in Ms. Ramírez's class?

- 3 Mr. Shen selects 20 of his 140 students at random to survey about their pets. His survey indicates that 8 students have a dog, 9 students have a cat, 3 students have another kind of pet, and 5 students have no pet. How many of his students should Mr. Shen expect to have a cat?

- A 9
- B 50
- C 56
- D 63

Nadia chose B as the correct answer. How might she have gotten that answer?

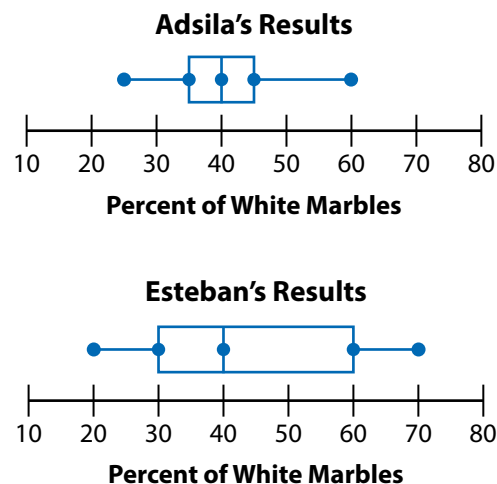
CONSIDER THIS ...

Some students have more than one pet.

PAIR/SHARE

How can you check your answer?

- 4 Adsila and Esteban have a bag containing 250 marbles. Each marble is red, black, or white. Adsila and Esteban each select a sample of marbles from the bag, record the percent that are white, and return all the marbles to the bag. They each collect 15 samples. Their results are shown in the box plots. If you could only use one sample to make an estimate of the number of white marbles in the bag, would you rather use one of Adsila's samples or one of Esteban's? Why?



- 5 Emma has 3 random samples of the same size from a population of 650 people. In one sample, 78% of people have brown eyes. In another, 85% have brown eyes. In the third, 83% have brown eyes. About how many people with brown eyes should she expect are in the population? Show your work.

SOLUTION

- 6 A jar contains 1,800 marbles. James enters a contest to guess the number of any one color of marbles in the jar. He is allowed to take 4 random samples of 200 marbles each.

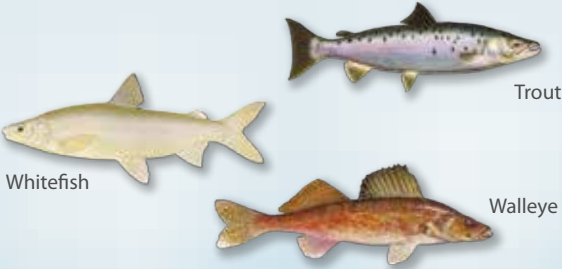
- a. In James's samples, the color _____ has the most variation and the color _____ has the least variation.

	Blue	Green	Red	White
Sample 1	41	37	63	49
Sample 2	58	34	54	54
Sample 3	52	29	59	60
Sample 4	47	30	72	51

- b. James can choose which color of marble to make his guess about. Which color do you think he should choose? Why?

- 7 Aniyah studies the fish populations in a lake. She catches fish, tags them, identifies the type, and returns them to the lake. She takes two random samples in the winter and two in the summer. She organizes her data in the table at the right. Which inferences about the fish populations in the lake are reasonable? Select all that apply.

	Trout	Whitefish	Walleye	Sample Size
Winter	42	44	14	100
	46	42	12	100
Summer	91	84	25	200
	85	89	26	200



Whitefish

Trout

Walleye

- A The total number of fish in the lake is 600.
- B The walleye population comprises anywhere from 12% to 14% of the total population in both the winter and summer.
- C The number of whitefish in the lake is greater than the number of trout.
- D The ratios of the populations of trout, whitefish, and walleye are relatively stable from the winter to the summer.
- E The populations of trout, whitefish, and walleye are approximately twice as large in the summer as in the winter.
- 8 **Math Journal** Write a word problem about taking a random sample and making an inference about the population. Then explain how to solve your problem.

✓ End of Lesson Checklist

- ☐ **INTERACTIVE GLOSSARY** Write a new entry for *inference*. Tell what you do when you make an *inference* about a population from a random sample.
- ☐ **SELF CHECK** Go back to the Unit 5 Opener and see what you can check off.