Overview | Solve Area and Circumference Problems Involving Circles

STANDARDS FOR MATHEMATICAL PRACTICE (SMP)

SMP 1, 2, 3, 4, 5, and 6 are integrated into the Try-Discuss-Connect routine.*

This lesson provides additional support for:

4 Model with mathematics.

7 Look for and make use of structure.

* See page 1q to learn how every lesson includes these SMP.

Objectives

Content Objectives

- Derive formulas for circumference and area of a circle.
- Understand that a circle's diameter and its circumference have a proportional relationship; the constant of proportionality is called pi (π).
- Use proportional reasoning and the formula for circumference of a circle to solve problems involving radius, circumference, and diameter.
- Use the formula for area of a circle to solve area problems.

Language Objectives

- Explain the relationship between *circumference, diameter, radius,* and *area* using the lesson vocabulary.
- Connect strategies for determining the proportional relationship between diameter and circumference by analyzing models and describing how the strategies are alike and different.
- Interpret and solve word problems about circles using circumference and area formulas.

Prior Knowledge

- Know that the perimeter is the distance around a two-dimensional figure.
- Know that the area of a two-dimensional figure is the amount of space enclosed inside it and that area is measured in square units.
- Know that a proportional relationship is one in which ratios of quantities all share the same constant of proportionality, or unit rate.

Vocabulary

Math Vocabulary

center (of a circle) a point that is the same distance from every point on the circle.

circle a two-dimensional shape in which every point is the same distance from the center.

circumference the distance around the outside of a circle. It can be thought of as the perimeter of the circle.

diameter a line segment that goes through the center of a circle and has endpoints on the circle. Also, the distance across a circle through the center.

pi (π) in a circle, the quotient $\frac{\text{circumference}}{\text{diameter}}$. Common approximations are 3.14 and $\frac{22}{7}$.

radius (of a circle) a line segment from the center of a circle to any point on the circle. Also, the distance from the center to any point on a circle.

Review the following key terms.

area the amount of space inside a closed two-dimensional figure. Area is measured in square units such as square centimeters.

unit rate the numerical part of a rate. For example, the rate 3 miles per hour has a unit rate of 3. For the ratio $a : b$, the unit rate is the quotient $\frac{a}{b}$.

Academic Vocabulary

approximate almost exact.

Learning Progression

Earlier in Grade 7, students learned about ratios, rates, scale models, and proportional relationships. They learned that the measurements of a geometric figure can be proportional to one another and what makes one figure a scale copy of another.

In this lesson, students apply their knowledge to recognize that the circumference of a circle is proportional to its diameter and that pi is the constant of proportionality. They identify the area of a circle as a property that is related to its circumference and radius but that is not proportional to either of them.

In Grade 8, students will apply their understanding of proportional relationships to identify the properties of linear functions. They will extend their knowledge of circles to find the volumes of spheres, cylinders, and cones.

Pacing Guide

Items marked with are available on the **Teacher Toolbox**.

	MATERIALS	DIFFERENTIATION
SESSION 1 **Explore** Circumference of a Circle (35–50 min)		
• **Start** (5 min) • **Try It** (5–10 min) • **Discuss It** (10–15 min) • **Connect It** (10–15 min) • **Close: Exit Ticket** (5 min) **Additional Practice** (pages 101–102)	**Math Toolkit** grid paper, string Presentation Slides	**PREPARE** Interactive Tutorial **RETEACH or REINFORCE** Visual Model **Materials** For display: meter stick, sticky notes
SESSION 2 **Develop** Using the Relationship Between a Circle's Circumference and Diameter (45–60 min)		
• **Start** (5 min) • **Try It** (10–15 min) • **Discuss It** (10–15 min) • **Connect It** (15–20 min) • **Close: Exit Ticket** (5 min) **Additional Practice** (pages 107–108)	**Math Toolkit** compasses, flexible tape measures, rulers, string Presentation Slides	**RETEACH or REINFORCE** Hands-On Activity **Materials** For each pair: at least 3 different-sized circular objects, ruler, string **REINFORCE** Fluency & Skills Practice **EXTEND** Deepen Understanding
SESSION 3 **Develop** Finding the Area of a Circle (45–60 min)		
• **Start** (5 min) • **Try It** (10–15 min) • **Discuss It** (10–15 min) • **Connect It** (15–20 min) • **Close: Exit Ticket** (5 min) **Additional Practice** (pages 113–114)	**Math Toolkit** grid paper, tracing paper Presentation Slides	**RETEACH or REINFORCE** Hands-On Activity **Materials** For each pair: ruler, scissors, tape Activity Sheet *Grid Paper* **REINFORCE** Fluency & Skills Practice **EXTEND** Deepen Understanding
SESSION 4 **Refine** Solving Circumference and Area Problems Involving Circles (45–60 min)		
• **Start** (5 min) • **Monitor & Guide** (15–20 min) • **Group & Differentiate** (20–30 min) • **Close: Exit Ticket** (5 min)	**Math Toolkit** Have items from previous sessions available for students. Presentation Slides	**RETEACH** Hands-On Activity **Materials** For each small group: compass, Activity Sheet *Grid Paper* **REINFORCE** Problems 4–10 **EXTEND** Challenge **PERSONALIZE** i-Ready
Lesson 6 Quiz or **Digital Comprehension Check**		**RETEACH** Tools for Instruction **REINFORCE** Math Center Activity **EXTEND** Enrichment Activity

Connect to Culture

➤ **Use these activities to connect with and leverage the diverse backgrounds and experiences of all students. Engage students in sharing what they know about contexts before you add the information given here.**

SESSION 1 ■ □ □ □

Try It When you fly over the Great Plains in an airplane, you can see green sections of irrigated crops surrounded by dry grasslands. Soon after its invention in the 1940s, the center pivot irrigation system became very popular because it delivered water easily across a wide area. In 1976, *Scientific American* published the opinion that the invention was the most significant mechanical improvement to agriculture since the tractor. Ask students to speculate about how farms were watered before this system was invented.

SESSION 2 ■ ■ □ □

Apply It **Problem 9** The word *gong* comes from the language of Java, an island in southeast Asia. Gongs date back thousands of years in regions across Asia, and they remain in use today in many cultures. A gong is generally circular and flat, like a plate, and made of metal. Like bells and chimes, a gong is a type of percussion instrument, meaning it is played by being struck. The sound of a gong is often used as an effect in television and radio programs, often to signify the end of an event. Ask students to name and discuss other examples of circular shapes in music, such as drum faces, cymbals, tambourines, and the rim of the bell of a horn.

SESSION 3 ■ ■ ■ □

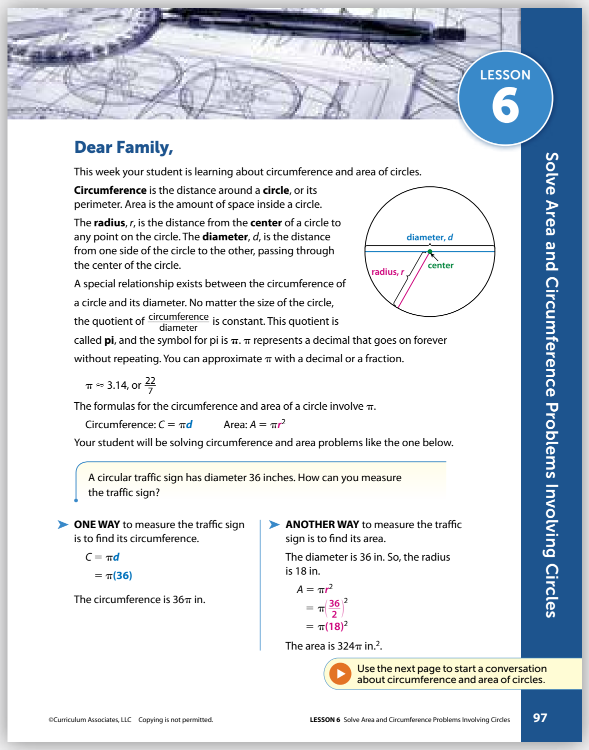
Apply It **Problem 8** Ask students to describe their experiences playing darts or similar games, such as lawn darts or ring toss. Games of darts are played in basements, club halls, recreation centers, and many places where people gather to relax. However, darts is also played as a serious sport. Professional darts is played according to a long list of rules and regulations. For example, the dartboard must be centered at a height of 5 feet, 8 inches above the ground, and the horizontal distance from the board to the toe line must be 7 feet, 9.25 inches.

SESSION 4 ■ ■ ■ ■

Apply It **Problem 3** Paving stones, or pavers, can be made of natural stone, brick, or concrete. Each has different properties, including cost, strength, durability, and resistance to stains. All provide a hard, relatively flat surface for outdoor use. Patios can take on a wide variety of colors and designs based on the pavers used to make them. Ask students to discuss the patios or outdoor walkways with designs made from pavers or bricks that they have seen.

Connect to Family and Community

➤ **After the Explore session, have students use the Family Letter to let their families know what they are learning and to encourage family involvement.**



LESSON 6

Solve Area and Circumference Problems Involving Circles

Dear Family,

This week your student is learning about circumference and area of circles.

Circumference is the distance around a **circle**, or its perimeter. Area is the amount of space inside a circle.

The **radius**, r, is the distance from the **center** of a circle to any point on the circle. The **diameter**, d, is the distance from one side of the circle to the other, passing through the center of the circle.

A special relationship exists between the circumference of a circle and its diameter. No matter the size of the circle, the quotient of $\frac{\text{circumference}}{\text{diameter}}$ is constant. This quotient is called **pi**, and the symbol for pi is π. π represents a decimal that goes on forever without repeating. You can approximate π with a decimal or a fraction.

$\pi \approx 3.14$, or $\frac{22}{7}$

The formulas for the circumference and area of a circle involve π.

Circumference: $C = \pi d$ Area: $A = \pi r^2$

Your student will be solving circumference and area problems like the one below.

A circular traffic sign has diameter 36 inches. How can you measure the traffic sign?

➤ **ONE WAY** to measure the traffic sign is to find its circumference.

$$C = \pi d$$
$$= \pi(36)$$

The circumference is 36π in.

➤ **ANOTHER WAY** to measure the traffic sign is to find its area.

The diameter is 36 in. So, the radius is 18 in.

$$A = \pi r^2$$
$$= \pi\left(\frac{36}{2}\right)^2$$
$$= \pi(18)^2$$

The area is 324π in.^{2}.

Use the next page to start a conversation about circumference and area of circles.

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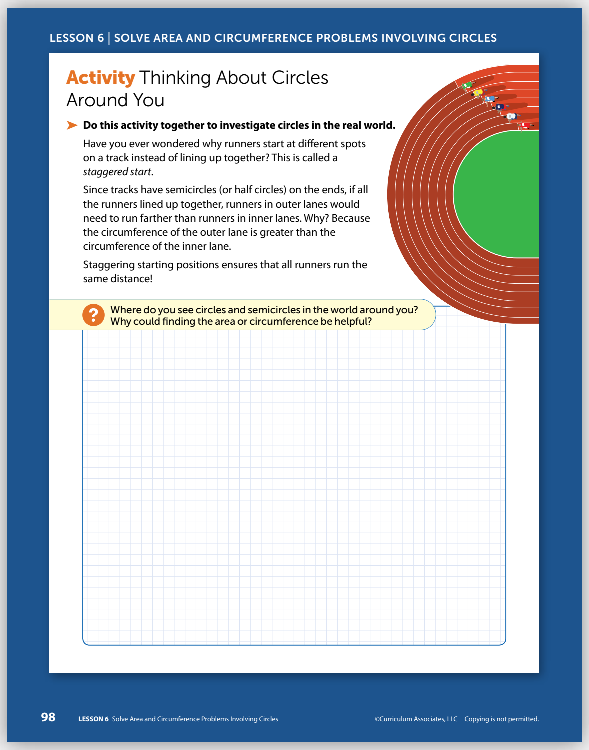

LESSON 6 | SOLVE AREA AND CIRCUMFERENCE PROBLEMS INVOLVING CIRCLES

Activity Thinking About Circles Around You

➤ **Do this activity together to investigate circles in the real world.**

Have you ever wondered why runners start at different spots on a track instead of lining up together? This is called a *staggered start*.

Since tracks have semicircles (or half circles) on the ends, if all the runners lined up together, runners in outer lanes would need to run farther than runners in inner lanes. Why? Because the circumference of the outer lane is greater than the circumference of the inner lane.

Staggering starting positions ensures that all runners run the same distance!

Where do you see circles and semicircles in the world around you? Why could finding the area or circumference be helpful?

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Connect to Language

➤ **For English language learners, use the Differentiation chart to scaffold the language in each session. Use the Academic Vocabulary routine for academic terms before Session 1.**

DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS Use with **Session 1** **Try It**

Levels 1–3: Listening/Speaking

Help students make sense of Try It by explaining the photograph of the center pivot irrigation system. Then display the photograph from ***Connect to Culture*** and help students list words to describe it. Tell students that irrigation systems, like the one shown, create these green circles.

Read aloud Try It and use **Act It Out** to support understanding of the phrase *turn around a center point*. Adapt **Say It Another Way** by asking questions and guiding students to respond. (e.g., *What does Hai order, or buy?*) Have students respond with words, phrases, gestures, and/or drawings. If students' ideas are unclear, reword so others can understand.

Levels 2–4: Listening/Speaking

Help students make sense of Try It by discussing the photograph of the center pivot irrigation system and reading the caption. Then display the photograph from ***Connect to Culture*** and have partners turn and talk about how the green circles were created.

Read aloud Try It and invite volunteers to use **Act It Out** to support the second sentence. Adapt **Say It Another Way** by reading each sentence of the problem and providing students with think time to consider how to paraphrase that sentence. Ask for volunteers to paraphrase the text. Reword any unclear statements, or ask another student to do so, so that others understand. Confirm with the speaker that the rewording is correct.

Levels 3–5: Listening/Speaking

Help students make sense of Try It. Have students turn and talk about how the word problem connects to their discussion from ***Connect to Culture*** and the photograph of the center pivot system. Have students ask for clarification of terms. Invite them to offer additional prior knowledge related to farming or irrigation by asking: *How might a farmer or gardener bring water when there is not enough rain?*

Use **Say It Another Way** to have students paraphrase each sentence of the problem. Remind students to be respectful if they disagree with another's ideas. Select students who showed thumbs down and have them explain what is inaccurate or missing in the paraphrase.

Explore Circumference of a Circle

Purpose

- **Explore** the idea that a circle is defined by points that are equidistant from a fixed center point.
- **Understand** the linear measurements that describe circles, including diameter, radius, and circumference.

START CONNECT TO PRIOR KNOWLEDGE

Same and Different

A	B
circle	square
C	D
equilateral triangle	right triangle

Possible Solutions

All are shapes that are two-dimensional, or flat.

The circle is the only shape without straight sides.

The square and equilateral triangle both have equal side lengths.

The square and right triangle both have right angles, although the right triangle has other angles as well.

WHY? Support students' understanding of the properties of different shapes.

TRY IT SMP 1, 2, 3, 4, 5, 6

Make Sense of the Problem

See ***Connect to Culture*** to support student engagement. Before students work on Try It, use **Say It Another Way** to help them make sense of the problem. Invite volunteers to describe the problem in their own words, including a description of the irrigation system. Ask: *How do you think the irrigation system works? Where is the center point of the system?*

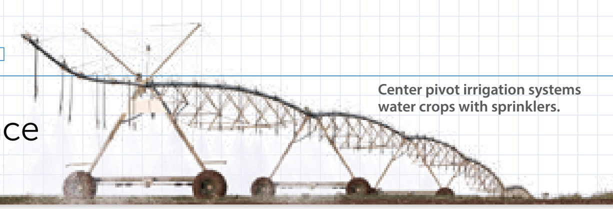
DISCUSS IT SMP 2, 3, 6

Support Partner Discussion

After students work on Try It, have them respond to Discuss It with a partner. Listen for understanding that:

- the distance between the center point and the other end of the system is constant.
- the system rotates around the center, so the distance across the watered space is twice the length of the system.

Explore Circumference of a Circle



Previously, you learned about area and perimeter of polygons.
In this lesson, you will learn about the area and circumference of circles.

➤ **Use what you know to try to solve the problem below.**

Hai orders a center pivot irrigation system to water his fields. The system is made up of a line of connected pipes that turn around a center point. Hai's system will be 1,320 feet in length. What is the shape of the space Hai's system will water? What is the longest distance across the watered space?

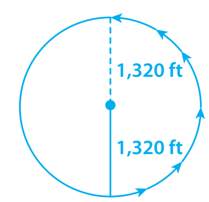
TRY IT **Math Toolkit** grid paper, string

Possible work:

SAMPLE A

The shape I made is a circle.

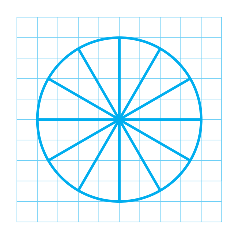
The system extends 1,320 ft in one direction. When it turns so it is directly opposite its starting position, it extends 1,320 ft in the opposite direction.
So, the distance across the circle is 1,320 ft + 1,320 ft = 2,640 ft.



SAMPLE B

I used 4-unit lines to represent the length of the 1,320-foot system. When I connected the ends of all the lines, I could see that I was making a circle. A line from one side of the circle to the other is 8 units long.

Since 4 units represents 1,320 ft, 8 units represents 2,640 ft.



DISCUSS IT

Ask: How is your strategy similar to mine? How is it different?

Share: My strategy is similar to yours . . . It is different . . .

Learning Target SMP 1, SMP 2, SMP 3, SMP 4, SMP 5, SMP 6, SMP 7
Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.

Common Misconception Listen for students who confuse the concept of the length of the system, which is 1,320 feet, with the width of the circle, which is twice the system's length. As students share their strategies, ask them to describe how the system moves around a central point. Ask students to use a diagram to show the widest distance across the circle, which is 2 times 1,320 feet, or 2,640 feet.

Select and Sequence Student Strategies

Select 2–3 samples that represent the range of student thinking in your classroom. Here is one possible order for class discussion:

- sketch of the circular shape of the watered area
- (misconception) diagram that shows the pipe's length as the full distance across the circular area rather than as half that distance
- labeled diagram of watered area on grid paper

Facilitate Whole Class Discussion

Call on students to share selected strategies. As students share, remind the class to look at each speaker and try to understand the ideas being expressed.

Guide students to **Compare and Connect** the representations. Allow students individual think time to consider all the strategies before beginning the discussion.

ASK *How do the solutions of [student name] and [student name] both show that the shape of the watered area is a circle?*

LISTEN FOR Both solutions show how the 1,320-foot length of the system rotates about a center point and that its far end traces a circle.

CONNECT IT

SMP 2, 4, 5

1 Look Back Look for understanding that the distance of 1,320 feet represents the distance from the center of the circle to the edge of the circle and that the greatest distance across the circle is twice this value.

DIFFERENTIATION | RETEACH or REINFORCE

Visual Model

Rotate a radius to form a circle.

If students are unsure about the function of the center pivot irrigation system, then use this activity to help them visualize it.

Materials For display: meter stick, sticky notes

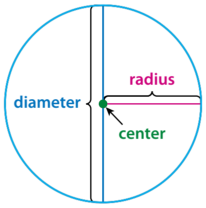
- Have one student model the center point of the system by standing in the middle of a clear space and holding a meter stick at one end so it is parallel to the floor. The meter stick represents the irrigation system.
- Ask the student modeling the center to slowly rotate while holding the meter stick. After every step or two of the rotation, have another student place a sticky note on the floor under the far end of the meter stick. After several notes have been placed, ask students to describe the shape of the path the notes are forming. The students should continue rotating and placing sticky notes for one full rotation.
- Ask: *How does this activity show the shape of the irrigated region?* [The shape is the region covered by the meter stick, which is a circle.]
- Ask: *What is the distance from the center to the edge of the circle? What is the distance across the circle?* [1 meter; 2 meters]

CONNECT IT

1 Look Back What is the shape of the space Hai's system will water? What is the longest distance across the space? How do you know?

A circle; 2,640 ft; Possible explanation: Half the distance across the circle is 1,320 ft, so the distance across the whole circle must be 2,640 ft.

2 Look Ahead Hai's irrigation system waters in the shape of a **circle**. Every point on the edge of a circle is the same distance from the **center**. The **radius** of Hai's system is 1,320 feet, so the **diameter** is 2,640 feet.



a. The radius of a circle is the distance from the edge to the center. The diameter is the distance across a circle through the center. What is the relationship between the radius and the diameter of any circle?

Possible answers: The radius of a circle is half the diameter, or the diameter is twice the radius.

b. You can draw more than one diameter on a circle. Why?

There are many lines that you can draw from edge to edge on a circle that go through the center.

c. Suppose two different diameters are drawn on a circle. Explain how you can use these diameters to find the center of the circle.

Every diameter goes through the center of a circle. So, if you draw two different diameters, the place where they cross must be the center of the circle.

d. The distance around a circle is called the **circumference**. Trace the circumference of the circle. How is the circumference of a circle like the perimeter of a square?

See diagram; Both the circumference of a circle and the perimeter of a square are the distance around the shape.

3 Reflect You can draw more than one radius on a circle. What must be true about all of the radii? (*Radii* is the plural of *radius*.)

Possible answer: All the radii start at the center of the circle and end at the circumference. They are all straight lines and they all have the same length.

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2 Look Ahead Point out that any circle can be described by its center and its radius or diameter. Students should recognize that the radius is the distance from the center of the circle to any point on the edge and the diameter is the distance across the circle through the center, which is twice the radius.

Ask a volunteer to restate the definitions *of center, radius, diameter,* and *circumference* in the volunteer's own words. Support student understanding by using the diagram.

CLOSE EXIT TICKET

3 Reflect Look for understanding that all the radii of a circle have the same length.

Common Misconception If students confuse the radius with other line segments across a circle, such as a diameter or a chord, then draw a diagram of a circle with an unlabeled radius, diameter, and chord (segment across the circle that is not through the center). Have students compare and contrast the three segments and identify the radius.

Prepare for Solving Circumference and Area Problems Involving Circles

Support Vocabulary Development

Assign **Prepare for Solving Circumference and Area Problems Involving Circles** as extra practice in class or as homework.

If you have students complete this in class, then use the guidance below.

Ask students to consider the term *area*. Have a volunteer explain what students should write or draw in each section of the organizer. Challenge students to write three or more things they know about area and to show examples using different figures.

Have students work in pairs to complete the graphic organizer. If students have trouble getting started, you may want to ask volunteers to draw a figure on the board and point out the difference between its area and its perimeter or circumference. Provide support as needed. Invite pairs to share their completed organizers and prompt a whole-class comparative discussion of the examples that students provided.

Have students look at the rectangle in problem 2 and discuss how some of the examples students included in their graphic organizers could help them decide which of the given measures is the area of the rectangle.

Problem Notes

1. Students should understand that area is a property of an enclosed two-dimensional figure and is a measurement of the amount of space inside the figure. One way to measure or estimate area is to fill the space with unit squares and then count the number of squares.

2. Students should understand the difference between the units in this problem: feet and square feet. Students should recognize that a foot is a linear unit that measures distance, such as a side length or the distance around or across a figure. A square foot is a two-dimensional unit used to measure the area of a figure.

LESSON 6 | SESSION 1 Name:

Prepare for Solving Circumference and Area Problems Involving Circles

1. Think about what you know about the area of two-dimensional figures. Fill in each box. Use words, numbers, and pictures. Show as many ideas as you can. Possible answers:

What Is It?
Area is the amount of space inside a shape. It is measured in square units.

What I Know About It
Some shapes have formulas you can use to find the area.

Square = side • side

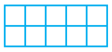
Triangle = $\frac{1}{2}$ • base • height

Parallelogram = base • height

area

Examples

← area

Examples

A = 10 units^2

2. Is the area of the rectangle 40 feet or 96 square feet? Explain.
96 ft^2; Possible explanation: Area is measured in square units. Multiply the base and height to find the area of the rectangle.

12 ft
8 ft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REAL-WORLD CONNECTION

The odometer on a car keeps track of how far the car has traveled. Most odometers work by counting the number of times the wheels of the car rotate. For each rotation, the car travels a distance equal to the circumference of the car's tire. So, the total distance a car has traveled is the number of times its wheels have turned times the circumference of a tire. Ask students to think of other situations when the circumference or area of a circle is important.

Additional Practice

3 Problem 3 provides another look at a real-life construction of a circular region. This problem is similar to the Try It problem about Hai and the irrigation system. The main difference is that the length of the system in problem 3 is slightly shorter. As students solve problem 3, have them refer to the solution to the Try It.

Students may want to draw a diagram to help them solve.

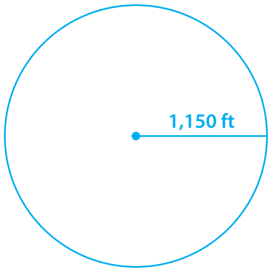
Suggest that students use **Three Reads**, asking themselves one of the following questions each time they read the problem:

- *What is this problem about?*
- *What is the question I am trying to answer?*
- *What information is important?*

3 Brian orders a center pivot irrigation system to water his fields. The system is made up of a line of connected pipes that turn around a center point. Brian's system will be 1,150 ft long.

a. What is the longest distance across the space the system will water? Show your work.

Possible work:



The radius of the circle is 1,150 ft. The longest distance across the space is the diameter of the circle.

The diameter is twice the radius, so $2 \cdot 1{,}150 = 2{,}300$.

SOLUTION The diameter of the space is 2,300 ft.

b. Check your answer to problem 3a. Show your work.

The diameter of a circle is the same as the radius plus the radius. So, the diameter of the area Brian's system can water is $1{,}150 \text{ ft} + 1{,}150 \text{ ft} = 2{,}300 \text{ ft}$.

center pivot irrigation system

DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Use with **Session 2** **Connect It**

ACADEMIC VOCABULARY

Exact means accurate and precise. An exact amount is not an estimate.

Levels 1–3: Reading/Speaking

Guide students as they interpret and discuss Connect It problem 5. Display and discuss the academic terms *approximate* and *exact*. Support understanding by asking students to estimate and count things in the classroom.

Provide these frames to help students talk about problem:

- *The formula for the circumference of a circle is _____.*
- *An approximate value of π is _____.*
- *The exact circumference is _____.*
- *The approximate circumference is _____.*

Levels 2–4: Reading/Speaking

Adapt **Three Reads** to support students as they interpret and discuss Connect It problem 5. First review the definitions for *exact* and *approximate*. After the first read, have students turn and talk about the difference between an exact circumference and an approximate circumference.

Have partners work together to complete the second and third read and answer the corresponding questions. Have them sketch and label the circle described and use the model to restate the problem.

Levels 3–5: Reading/Speaking

Use **Three Reads** to support students as they make sense of Connect It problem 5. Adapt the routine by asking students to preview the problem to identify unfamiliar vocabulary. Clarify the terms as needed.

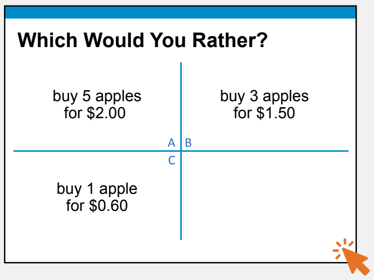
Before each reading, display the question students will answer. After the read, provide students with individual think time to consider their responses. Have students compare their responses with partners. Call on several students to share their ideas. To prompt students to use precise language, call on volunteers to reword vague or unclear statements.

Develop Using the Relationship Between a Circle's Circumference and Diameter

Purpose

- **Develop** strategies to determine the circumference of a circle.
- **Recognize** that the circumference and diameter of a circle are proportional, with a constant of proportionality of pi (π).

START CONNECT TO PRIOR KNOWLEDGE



Possible Solutions

A offers the lowest unit rate of $0.40 per apple.

B might be the best choice if you only want 3 apples.

C is the best choice if you only want 1 apple or to spend just $0.60.

WHY? Support students' ability to identify and compare ratios.

DEVELOP ACADEMIC LANGUAGE

WHY? Develop awareness of precision in academic language.

HOW? Explain that words and phrases that add or qualify details make definitions more precise. Display: *Diameter is the distance from one side of a circle to the other, passing through the center of the circle.* Ask students to discuss how the final phrase makes the definition more precise. Then have them discuss the use of *approximate circumference* and *exact circumference* in Apply It.

TRY IT SMP 1, 2, 3, 4, 5, 6, 7

Make Sense of the Problem

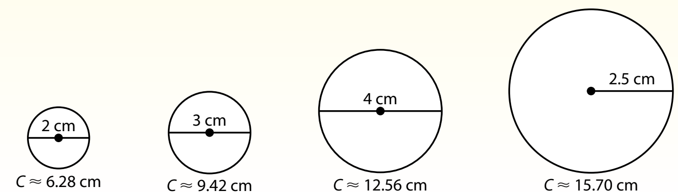
Before students work on Try It, use **Notice and Wonder** to help them make sense of the problem. If students have trouble getting started, ask what they notice about the diameters or the circumferences. Record students' suggestions. If time allows, revisit the lists after the lesson to see what was addressed.

LESSON 6 | SESSION 2 ■ ■ □ □

Develop Using the Relationship Between a Circle's Circumference and Diameter

➤ **Read and try to solve the problem below.**

Look at the circumference of each of the circles below. What do you think would be the circumference of a circle with diameter 1 cm?



TRY IT Math Toolkit compasses, flexible tape measures, rulers, string

Possible work:

SAMPLE A

Diameter (cm)	2	3	4	2.5(2) = 5	1
Circumference (cm)	6.28	9.42	12.56	15.70	3.14

$9.42 - 6.28 = 3.14$ $12.56 - 9.42 = 3.14$ $15.70 - 12.56 = 3.14$

$6.28 - 3.14 = 3.14$

The circumference will be about 3.14 cm when the diameter is 1 cm.

SAMPLE B

$\frac{6.28}{2} = 3.14$ $\frac{9.24}{3} = 3.14$ $\frac{12.56}{4} = 3.14$ $\frac{15.70}{2(2.5)} = 3.14$

$\frac{C}{1} = 3.14$

$C = 3.14$

DISCUSS IT

Ask: Why did you choose that strategy to find the circumference of a circle with diameter 1 cm?

Share: I chose to use . . . because . . .

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DISCUSS IT SMP 2, 3, 6

Support Partner Discussion

After students work on Try It, have them respond to Discuss It with a partner. If students need support in getting started, prompt them to ask each other questions that help them identify what they agree on and help them build on each other's ideas, such as:

- *What patterns or relationships did you see in the measurements that helped you come up with a strategy?*
- *How is the measurement of 2.5 cm for the fourth circle different from the other measurements?*

Error Alert If students misinterpret 2.5 cm as the diameter of the circle, ask students to look closely at the circles and identify whether the segment labeled is the radius or the diameter. Then, for the last circle, ask students to calculate the diameter.

Select and Sequence Student Strategies

Select 2–3 samples that represent the range of student thinking in your classroom. Here is one possible order for class discussion:

- compass and string used to draw and measure the circumference of a circle of diameter 1 unit
- pattern of 3.14-cm change in circumference for every 1-cm change in diameter
- constant ratio of circumference to diameter

Facilitate Whole Class Discussion

Call on students to share selected strategies. Afterward, ask students to take individual think time and then turn and talk to discuss the relationship between diameter and circumference.

Guide students to **Compare and Connect** the representations. Remind students to be respectful if they disagree and to disagree with the idea, not the person.

> **ASK** *How does each representation show that the relationship between circumference and diameter is proportional?*
>
> **LISTEN FOR** The quotient of circumference and diameter is constant. Each increase of 1 unit in the diameter increases the circumference by the same amount.

Model It

If students presented these models, have students connect these models to those presented in class.

If no student presented at least one of these models, have students first analyze key features of the models and then connect them to the models presented in class.

> **ASK** *How is the constant of proportionality shown in the graph?*
>
> **LISTEN FOR** In the graph, the constant of proportionality is $y \div x$ for any point (x, y) on the line. Here, it is approximately equal to 3.14.

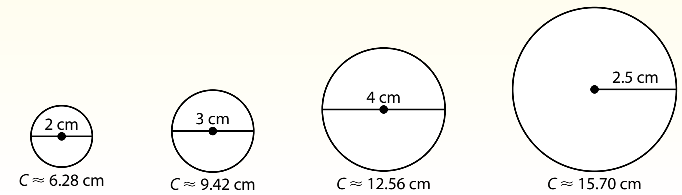
For the graph, prompt students to recognize that the line describes a proportional relationship. Ask: *For each point on the graph, are the ratios of y to x equivalent? What is the unit rate?*

For the model that divides circumferences by diameters, prompt students to recognize the usefulness of the constant of proportionality. Ask: *Could you use the constant of proportionality of approximately 3.14 to find the circumference of any circle? How?*

LESSON 6 | SESSION 2

➤ **Explore different ways to investigate the relationship between the circumference and diameter of a circle.**

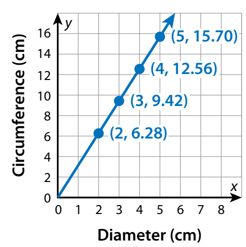
Look at the circumference of each of the circles below. What do you think would be the circumference of a circle with diameter 1 cm?



Model It

You can make a graph to look at the relationship between circumference and diameter for each circle.

The x-axis shows the diameter. The y-axis shows the approximate circumference.



Model It

You can look for a constant of proportionality in the relationship between the approximate circumference of each circle and the diameter.

Divide the circumference of each circle by its diameter.

$\frac{6.28}{2} = 3.14$ $\quad \frac{9.42}{3} = 3.14$ $\quad \frac{12.56}{4} = 3.14$ $\quad \frac{15.70}{5} = 3.14$

$r = 2.5$ cm, so $d = 5$ cm.

The constant of proportionality is 3.14.

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DIFFERENTIATION | EXTEND

SMP 7

Deepen Understanding

Interpreting a Graph Relating Diameter and Circumference

Prompt students to use the graph to explain the relationship between diameter and circumference.

> **ASK** *How can the graph be used to approximate the circumference of a circle of diameter 6 cm?*
>
> **LISTEN FOR** Extend the graph up and find the y-coordinate of the line when $x = 6$ cm, which is slightly less than 19 cm.

> **ASK** *Suppose the graph had radius instead of diameter on the x-axis. What are some points that would be on the graph?*
>
> **LISTEN FOR** (1, 6.28), (2, 12.56), and other points along the line $y = 2\pi x$.

> **ASK** *How would the graph with radius on the x-axis be similar to the graph with diameter on the x-axis? How would it be different?*
>
> **LISTEN FOR** It would have some of the same y-coordinates; It would go up more quickly; The y-coordinates would be twice as high on the radius graphs as on the diameter graph.

Develop Using the Relationship Between a Circle's Circumference and Diameter

CONNECT IT

SMP 2, 4, 5, 6

Remind students that the quantities and the relationships between them are the same in each representation. Explain that they will now use those relationships to reason about how to find the circumference of a circle.

Before students begin to record and expand on their work in Model It, tell them that problem 3 will prepare them to write the formula asked for in problem 4.

Monitor and Confirm Understanding 1 – 2

- The wavy symbol ≈ means that a value is approximate, not exact. The value 3.14 is approximately the quotient of circumference and diameter.
- In the diagram of the largest circle, 2.5 cm is the radius and 15.70 cm is the circumference. The graph shows the relationship between the diameter and the circumference. The diameter of this circle is 5 cm, so the point is (5, 15.70).

Facilitate Whole Class Discussion

3 Look for understanding of the idea of a constant of proportionality.

> **ASK** *How can you tell that a value close to 3.14 is a constant of proportionality?*
>
> **LISTEN FOR** The unit rate for the ratio of circumference to diameter is approximately this value.

4 Look for understanding of the constant pi.

> **ASK** *How is pi similar to other constants of proportionality you have studied? How is it different?*
>
> **LISTEN FOR** Pi is similar because describes a unit rate for a relationship between two quantities. It is different because it is a decimal that goes on forever and can only be approximated in calculations.

5 Look for understanding of the different ways of expressing circumference.

> **ASK** *Why is stating the circumference as "6 pi centimeters" more precise than a measurement such as 18.84 centimeters?*
>
> **LISTEN FOR** Pi is a never-ending decimal, so approximating pi with any value, such as 3.14, introduces a slight inaccuracy.

6 **Reflect** Have all students focus on the strategies used to solve the Try It. If time allows, have students discuss their ideas with a partner.

CONNECT IT

➤ **Use the problem from the previous page to help you understand how to find the circumference of a circle.**

1 Each circumference is given using the ≈ symbol. What does that mean about the circumferences? What is the approximate circumference of a circle with diameter 1 cm?

The circumferences given are approximate, not exact; 3.14 cm

2 Look at the first **Model It**. Why is the point (5, 15.70) on the graph?

You are looking for the relationship between diameter and circumference. When the diameter is 15.70 cm, the radius is 2.5 cm, so the diameter is 2(2.5), or 5, cm.

3 Look at the second **Model It**. What does the constant of proportionality tell you about the relationship between the circumference and diameter of a circle?

Possible answer: The circumference of a circle is always about 3.14 times the diameter of the circle.

4 The quotient $\frac{\text{circumference}}{\text{diameter}}$ is called **pi (π)**. π represents a decimal that goes on forever without repeating. Use π to write a formula for the circumference of a circle, C, when you know the diameter, d. $C = d \cdot \pi$

5 An exact circumference uses π. To find an approximate circumference you can use 3.14 or $\frac{22}{7}$ as a value for π. What is the exact circumference for a circle with diameter 6 cm? What is the approximate circumference?

6π cm; 18.84 or $\frac{132}{7}$ cm

6 **Reflect** Think about all the models and strategies you have discussed today. Describe how one of them helped you better understand how to find the circumference of a circle.

Responses will vary. Check student responses.

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DIFFERENTIATION | RETEACH or REINFORCE

Hands-On Activity

Use examples of circles to calculate pi.

If students are unsure about the meaning of pi, then use this activity to show how pi is the constant of proportionality for the relationship between circumference and diameter.

Materials For each pair: at least 3 different-sized circular objects, ruler, string

- Have students make a three-column table with the heads *Circumference, Diameter,* and *C ÷ d.*
- For each object, have students use the ruler and a string to carefully measure the circumference and diameter. Then have them calculate circumference ÷ diameter. They should record their results in the table.
- Ask: *What do you notice about all the quotients you calculated?* [All the quotients are close to 3.14.]
- Ask: *How do your results indicate that the relationship between circumference and diameter is proportional?* [The quotients, which are unit rates, are approximately equal.]

Remind students that the constant of proportionality is called pi, and write $\pi \approx 3.14$ on the board.

Apply It

For all problems, encourage students to use a model to support their thinking. Allow some leeway in precision; students' approximations may differ for problem 9. Remind students that pi is approximately 3.14 and does not exactly equal this value.

7 Students should recognize that the diameter of 12 feet is an exact value, not an approximation. This is because the diameter is calculated using the exact circumference 12π feet. An approximation of π is not used.

8 Because the diameter d is always 2 times the radius r, students could also write and apply the equation $C = \pi \cdot 2r$, or $C = 2\pi r$. Substituting $r = 4$ into this equation shows that $C = 8 \cdot \pi$.

Apply It

➤ **Use what you learned to solve these problems.**

7 The circumference of a circle is 12π feet. What is the diameter? Show your work.

Possible work:

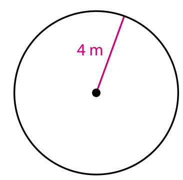
$C = d \cdot \pi$

$12\pi = d \cdot \pi$

$12 = d$

SOLUTION The diameter is 12 ft.

8 What is the circumference of the circle? Write your answer using π. Show your work.



Possible work:

The diameter is $2 \cdot 4$ m or 8 m.

$C = d \cdot \pi$

$= 8 \cdot \pi$

SOLUTION The circumference of the circle is 8π m.

9 The diameter of a gong is 20 inches. Find the approximate circumference of the gong, using 3.14 for π. Then find the exact circumference of the gong. Show your work.

Possible work:

$C = d \cdot \pi$

$\approx 20 \cdot 3.14$ $\quad$ $C = 20 \cdot \pi$

$= 62.8$ $\quad$ $= 20\pi$

SOLUTION The approximate circumference of the gong is 62.8 in. The exact circumference of the gong is 20π in.

CLOSE EXIT TICKET

9 See ***Connect to Culture*** to support student engagement. Students' solutions should show an understanding of:

- the application of the equation $C = d \cdot \pi$ to calculate circumference.
- the substitution of 3.14 for π to find an approximate measurement of the circumference in inches.

Error Alert If students state that 62.8 inches is the exact circumference of the gong, then remind them that 3.14 is only a close approximation of pi, not its exact value. Ask interested students to research the value of pi to several decimal places and to report their findings to the class.

Practice Using the Relationship Between a Circle's Circumference and Diameter

Problem Notes

Assign **Practice: Using the Relationship Between a Circle's Circumference and Diameter** as extra practice in class or as homework.

1. **a.** *Basic*

   b. Students may also use the equation $C = \pi \cdot 2r$. ***Medium***

2. Students may point out that the coin must also be thin enough to fit through the slot. Pennies, nickels, dimes, and quarters each have different thicknesses. ***Medium***

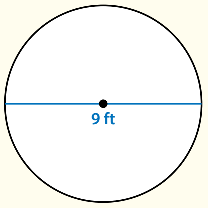
LESSON 6 | SESSION 2 Name:

Practice Using the Relationship Between a Circle's Circumference and Diameter

➤ **Study the Example showing how to find the circumference of a circle. Then solve problems 1–5.**

Example

A model of a circular pool is shown. What is the exact circumference of the pool?



You can use a formula to find the circumference of a circle.

The diagram shows that the diameter of the pool is 9 ft.

$C = \pi d$

$= \pi \cdot 9$

$= 9\pi$

Using π gives an exact circumference. The circumference of the pool is 9π ft.

1. **a.** A circular dining room table top has a radius of 22 inches. What is the diameter of the table top? 44 in.

   b. What is the circumference of the table top? Write your answer using π. Show your work.

   Possible work:

   $C = \pi d$

   $= \pi \cdot 44$

   SOLUTION The circumference of the table top is 44π in.

2. A circular coin has circumference 32π millimeters. Will the coin fit through a slot that is 35 millimeters long? Explain.

   Yes; Possible explanation: The circumference of the coin is 32π mm, so the diameter of the coin is 32 mm. Since 32 mm $<$ 35 mm, the coin will fit through the slot.

Vocabulary

circumference
the distance around the outside of a circle.

diameter
the distance across a circle through the center.

pi (π)
in a circle, the quotient $\frac{\text{circumference}}{\text{diameter}}$.
Common approximations are 3.14 and $\frac{22}{7}$.

radius
the distance from the center of a circle to any point on the circle.

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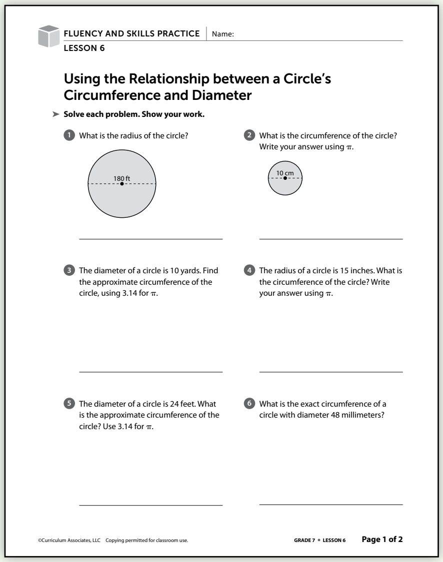
Fluency & Skills Practice

Using the Relationship Between a Circle's Circumference and Diameter

In this activity, students use the formula for the circumference of a circle to calculate various circumferences given either a diameter or a radius. They also find the radius of a circle given its circumference.


FLUENCY AND SKILLS PRACTICE | Name:
LESSON 6

Using the Relationship between a Circle's Circumference and Diameter

➤ Solve each problem. Show your work.

1. What is the radius of the circle? (180 ft)
2. What is the circumference of the circle? Write your answer using π. (10 cm)
3. The diameter of a circle is 10 yards. Find the approximate circumference of the circle, using 3.14 for π.
4. The radius of a circle is 15 inches. What is the circumference of the circle? Write your answer using π.
5. The diameter of a circle is 24 feet. What is the approximate circumference of the circle? Use 3.14 for π.
6. What is the exact circumference of a circle with diameter 48 millimeters?

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Additional Practice

3 **a.** Students may also substitute $r = 14$ directly into a formula that relates circumference and radius: $C = \pi \cdot 2r$. ***Medium***

b. Students may also use the fraction $\frac{22}{7}$ as an estimate for pi. This gives an approximate circumference of $\frac{22}{7} \cdot 28$ cm, or 88 cm, which is close to the estimate of 85 cm. ***Challenge***

4 Students may also express the circumference as the product $\pi \cdot 18$ in., although stating π as the second factor is conventional. ***Medium***

5 Students should recognize that if they use the equation $C = \pi \cdot d$, they will be solving for d, the diameter. The problem asks for the radius, so the value of d must be divided by 2. ***Challenge***

LESSON 6 | SESSION 2

3 Destiny draws a circle with radius 14 centimeters.

a. What is the circumference of Destiny's circle? Write your answer using π. Show your work.

Possible work:

$r = 14$ cm and $d = 2r$, so $d = 28$ cm

$C = \pi \cdot 28$

SOLUTION The circumference is 28π cm.

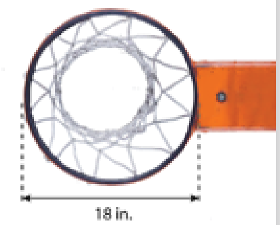
b. Is 85 centimeters a reasonable estimate for the circumference of Destiny's circle? Explain.

Yes; Possible explanation: The exact circumference is 28π cm. Since $\pi \approx 3.14$, use 3 for the value of π to estimate 28π.

$28 \cdot 3 = 84$

Since 84 is close to 85, 85 cm is a reasonable estimate for the circumference of Destiny's circle.

4 The diameter of a basketball rim is 18 inches. What is the circumference of the rim? Write your answer using π. Show your work.



Possible work:

$C = \pi d$

$C = \pi \cdot 18$

SOLUTION The circumference is 18π in.

5 A circular mirror has a circumference of 50π inches. What is the radius of the mirror? Show your work.

Possible work:

$C = \pi d$

$50\pi = \pi d$

$50 = d$

The diameter is 50 in. Since the diameter is twice the radius, the radius is $(50 \div 2)$ in., or 25 in.

SOLUTION The radius of the mirror is 25 in.

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DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Use with **Session 3** **Start**

MATH TERM

Decompose means to break into parts.

ACADEMIC VOCABULARY

Visualize means to form a mental picture of something.

Levels 1–3: Speaking/Writing

Guide students as they make connections between Model It and Picture It to deepen their understanding of the formula for the area of a circle.

Read Model It aloud. Begin a **Co-Constructed Word Bank** with the terms *decompose* and *parallelogram*. Encourage students to add terms to the word bank that might be used to discuss the model, such as *divide, rearrange, formula, base,* and *height*. Read Picture It aloud. Have students add new words to the bank and put a check mark next to existing terms that apply to Picture It.

Levels 2–4: Speaking/Writing

Guide students as they make connections between Model It and Picture It to deepen their understanding of the formula for the area of a circle.

Read and discuss Model It and Picture It. With students, create a **Co-Constructed Word Bank** that includes terms that might be used to discuss the models. Display a graphic organizer for comparison, such as a two-column chart. Have students compare the models with a partner.

Call on volunteers to share ideas. Record responses on the graphic organizer.

Levels 3–5: Speaking/Writing

Guide students as they make connections between Model It and Picture It to deepen their understanding of the formula for the area of a circle.

Use **Notice and Wonder** to stimulate conversation about the diagrams in Model It and Picture It. Have students cover the text so that only the diagrams are showing. Ask:

- *What do you notice?*
- *What do you wonder?*

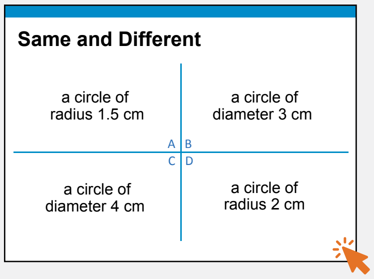
Record responses. Refer to relevant responses as you read and discuss Model It and Picture It with students.

Develop Finding the Area of a Circle

Purpose

- **Develop** strategies to determine the area of a circle.
- **Recognize** that the area of a circle is π times the square of the radius.

START CONNECT TO PRIOR KNOWLEDGE



Possible Solutions

All are circles.

A and B are the same size and have the smaller circumference.

C and D are the same size and have the greater circumference.

WHY? Support students' ability to compare and analyze the dimensions of circles.

DEVELOP ACADEMIC LANGUAGE

WHY? Support students as they make connections among ideas in discussion.

HOW? Explain that trying to make connections between what people say during a discussion can help everyone understand the ideas better or come up with new ideas. Prompt students to connect their strategies to other strategies by showing how they are alike and how they are different. Have students use this sentence frame:

- *My ideas are related to ____'s ideas because ____.*

TRY IT SMP 1, 2, 4, 5, 6

Make Sense of the Problem

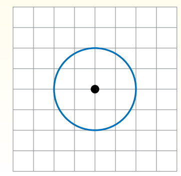
Before students work on Try It, use **Notice and Wonder** to help them make sense of the problem. Have students turn and talk to generate ideas before beginning the class discussion. If no one comments on the grid, ask: *What do you notice about the circle and the grid squares? What does the grid reveal about the circle?*

LESSON 6 | SESSION 3

Develop Finding the Area of a Circle

Read and try to solve the problem below.

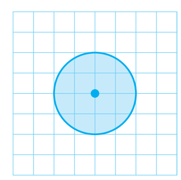
What is the approximate area of a circle with radius 2 units?



TRY IT Math Toolkit grid paper, tracing paper

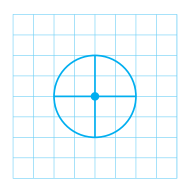
Possible work:

SAMPLE A



Counting the complete or nearly complete squares inside the circle, the area is about 12 square units.

SAMPLE B



I can divide the circle into quarters. Each quarter circle has about 3 complete or nearly complete squares. Since $3 \cdot 4 = 12$, the area is about 12 square units, maybe a little more.

DISCUSS IT

Ask: What did you do first to find the area of the circle?

Share: First, I . . .

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DISCUSS IT SMP 2, 3, 6

Support Partner Discussion

After students work on Try It, encourage them respond to Discuss It with a partner. Remind students that they should pause and check to see if partners have questions about the ideas being discussed. If students need support in extending their discussion, prompt them to ask each other questions such as:

- *Why does counting squares on grid paper only give an estimate of the area of a circle?*
- *How does the location of the circle on the grid paper affect your answer?*

Common Misconception Listen for students who think that counting the squares the circle covers can show the exact area of a circle. As students share their strategies, have them describe the regions where the edge of the circle crosses the grid lines. Ask students to explain how they counted squares that were only partially inside the circle.

Select and Sequence Student Strategies

Select 2–3 samples that represent the range of student thinking in your classroom. Here is one possible order for class discussion:

- counting grid squares inside the whole circle to estimate its area
- (misconception) counting grid squares and stating that the result is the exact area
- counting grid squares to estimate the area of a fraction of a circle and then multiplying to estimate the area of the whole circle

Facilitate Whole Class Discussion

Call on students to share selected strategies. Allow individual think time for students to process each strategy after it is presented.

Guide students to **Compare and Connect** the representations. Encourage students to listen to the different strategies and connect them by thinking about how they are the same and how they are different.

ASK *Do any of these strategies give the exact area of the circle?*

LISTEN FOR Because the circle is curved, it is not possible to count the exact number of squares inside it.

Model It & Picture It

If students presented these models, have students connect these models to those presented in class.

If no student presented at least one of these models, have students first analyze key features of the models and then connect them to the models presented in class.

ASK *What do the strategies shown in Model It and Picture It have in common?*

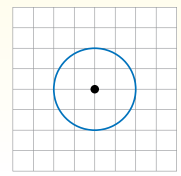
LISTEN FOR Both strategies divide a circle into parts and then rearrange the parts into a new shape. The new shapes are similar to familiar polygons, but they are not exactly the same because their sides are not straight segments.

For the parallelogram model, prompt students to explain how the model works. Ask: *How do you know the base and height of the parallelogram? How are these dimensions useful?*

For the triangle model, prompt students to explain how the model is constructed. *Say: Describe the "unrolling" of the circle in your own words.*

➤ **Explore different ways to find the area of a circle.**

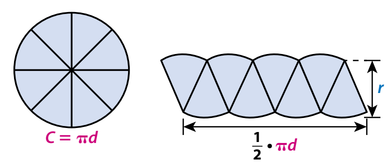
What is the approximate area of a circle with radius 2 units?



Model It

You can decompose a circle.

Divide a circle into equal parts. Then compose the parts into a figure that looks like a parallelogram.

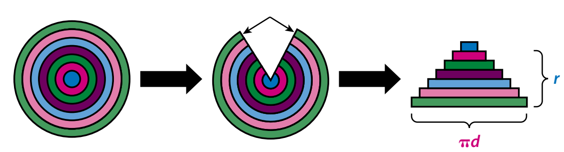


A formula for the area of the circle is $A = \frac{1}{2}C \cdot r$ or $\frac{1}{2} \cdot \pi d \cdot r$.

Picture It

You can visualize "unrolling" a circle into a triangle.

Think of a circle as made of rings. Slice the rings along a radius. Then unroll the rings to make a triangular shape.



The formula for the area of a triangle is $A = \frac{1}{2}bh$. The length of the base of the triangle is the circumference of the circle, πd. The height of the triangle is r.

So, a formula for the area of a circle is $A = \frac{1}{2} \cdot \pi d \cdot r$.

DIFFERENTIATION | EXTEND

SMP 4

Deepen Understanding

Using a Model to Derive the Area Formula for a Circle

Prompt students to explain the parallelogram model for the area of a circle.

ASK *Look at the circle divided into parts in Model It. How are the parts rearranged?*

LISTEN FOR The parts are arranged in a row, with their straight edges touching and their curved edges along the top and bottom. The result is a shape similar to a parallelogram.

ASK *If the circle were divided into 16 equal parts instead of 8, what would the figure made from the parts look like?*

LISTEN FOR Each part would be thinner. The top and bottom of the parallelogram-like figure would have less-pronounced curves. The base would still be $\frac{1}{2}\pi d$ and the height would still be r.

ASK *What would happen if you kept dividing the circle into smaller and smaller equal parts before rearranging the parts?*

LISTEN FOR The shape made from the parts would become closer and closer to a true parallelogram.

Develop Finding the Area of a Circle

CONNECT IT

SMP 2, 4, 5, 6

Remind students that the quantities and the relationships between them are the same in each representation. Explain that they will now use those relationships to reason about how to find the area of a circle.

Before students begin to record and expand on their work in Model It, tell them that problems 1 and 2 will prepare them to develop the formula in problem 3.

Monitor and Confirm Understanding 1 – 2

- The base of the parallelogram is made up of 4 of the 8 equal curved sections that make up the whole circle, so the length of each base is half of the circumference.
- The base of the triangle is made from the outer ring of the circle, so its length is equal to the circumference of the circle.

Facilitate Whole Class Discussion

3 Look for understanding that the diameter, *d*, can be replaced with twice the radius, $2r$.

> **ASK** *How can you simplify the formula after you substitute $2r$ for d?*
>
> **LISTEN FOR** The product of $\frac{1}{2}$ and $2r$ is equal to r. Multiplying r by πr gives the formula.

4 Look for understanding of how to apply and interpret the formula for the area of a circle.

> **ASK** *Why is it incorrect to say that the area of the circle is exactly 12.56 cm^2?*
>
> **LISTEN FOR** The value 3.14 is an approximation of π, so using it to calculate the area results in an approximate area.

5 Look for understanding of the relationship among the dimensions of the circle.

6 **Reflect** Have all students focus on the strategies used to solve the Try It. If time allows, have students discuss their ideas with a partner.

CONNECT IT

➤ **Use the problem from the previous page to help you understand how to find the area of a circle.**

1 Look at **Model It**. Why is the base of the parallelogram $\frac{1}{2}$ the circumference of the circle?
The circle is divided into triangles, and the triangles are arranged so that the base of the parallelogram is made up of $\frac{1}{2}$ of the triangles.

2 Look at **Picture It**. Why is the base of the triangle the same as the circumference of the circle?
The outer ring of the circle makes up the base of the triangle.

3 **Model It** and **Picture It** both show the area of a circle using the formula $A = \frac{1}{2} \cdot \pi \cdot d \cdot r$. What is a formula for the area of a circle that only uses r?
$A = r \cdot \pi \cdot r$ or $A = \pi r^2$.

4 The formula for the area of a circle contains π. That means you can find both an exact and an approximate area of a circle. What is the approximate area of a circle with radius 2 cm? Why might you want to find an approximate area of a circle?
12.56 cm^2; Possible answer: If you need to find how much space something will take up, knowing the approximate area lets you better picture the area the object takes up.

5 The formula for circumference uses diameter. The formula for area uses radius. Explain how you can find the area of a circle if you know the circumference.
Possible explanation: Use the circumference to find the diameter of the circle. Then divide by 2 to find the radius. Then square the radius and multiply by π.

6 **Reflect** Think about all the models and strategies you have discussed today. Describe how one of them helped you better understand how to solve the **Try It** problem.
Responses will vary. Check student responses.

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DIFFERENTIATION | RETEACH or REINFORCE

Hands-On Activity

Demonstrate the derivation of the area formula for a circle.

If students are unsure about the derivation of the area formula for a circle, then have them follow the patterns shown in Model It to demonstrate the procedure.

Materials For each pair: ruler, scissors, tape, Activity Sheet *Grid Paper*

- Have pairs draw a circle on grid paper where the radius is equal to a whole number of grid units. Have them calculate and record the area of the circle using the formula $A = \pi r^2$, using 3.14 as an approximation for π.
- Have students use the ruler as a straight edge to divide the circle into 8 pieces, as shown in Model It. Then have them cut and reassemble the pieces, as shown.
- Have students use the grid lines to measure the approximate base and height of the parallelogram and then calculate the area as the product of base and height.
- Ask: *How does the area you calculated for the parallelogram compare to the area of the circle? How do you explain this comparison?* [The areas are similar, but they are not exactly the same. If the shape were a true parallelogram, then the base would be $\frac{1}{2} \cdot \pi d$, and the height would be the radius of the circle, *r*.]

Apply It

For all problems, encourage students to use a model to support their thinking. Remind students that when pi (π) is replaced with an estimate, such as 3.14, the answer becomes an approximation and not an exact value. Students can use $\approx$ to show that a step in their work is approximately equal to the previous step.

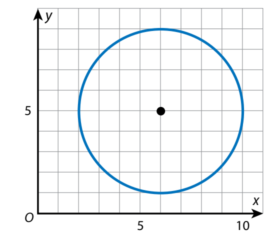
7. Students may find the radius by counting the grid units between the center (6, 5) and a point on the circle, such as (10, 5). Then they can apply the formula for area.

8. See ***Connect to Culture*** to support student engagement. Students should recognize that the problem states the diameter of the dartboard, while the formula for area uses radius.

Apply It

➤ **Use what you learned to solve these problems.**

7. Gaspar draws a circle in the coordinate plane with the center at (6, 5). What is the area of Gaspar's circle? Write your answer using π. Show your work.



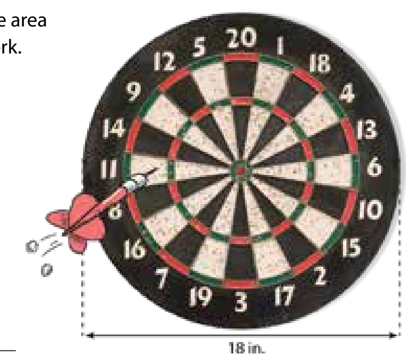
Possible work:

The radius of the circle is 4 units.

$A = \pi r^2$
$= \pi \cdot 4 \cdot 4$
$= 16\pi$

SOLUTION The area of Gaspar's circle is 16π square units.

8. The diameter of a circular dartboard is 18 inches. What is the area of the dartboard? Write your answer using π. Show your work.



Possible work:

$r = \frac{1}{2} \cdot d$
$= \frac{1}{2} \cdot 18$
$= 9$
$A = \pi \cdot 9 \cdot 9$
$A = 81\pi$

SOLUTION The area of the dartboard is 81π in.^{2}.

9. The radius of a circle is 10 cm. What is the approximate area of the circle? What is the exact area of the circle? Show your work.

Possible work:

Approximate	Exact
$A \approx 3.14 \cdot 10 \cdot 10$	$A = \pi \cdot 10 \cdot 10$
$= 314$	$= 100\pi$

SOLUTION The approximate area of the circle is 314 cm^2. The exact area of the circle is 100π cm^2.

CLOSE **EXIT TICKET**

9. Students' solutions should show an understanding that:
   - the equation $A = \pi r^2$ can be used to calculate the area of a circle.
   - an exact area will include the number π.
   - using 3.14 as an approximation of π gives an approximate area.
   - area is expressed in square units: in this case, square centimeters.

Error Alert If students report the area in centimeters instead of square centimeters, then remind them that area is measured in square units.

Practice Finding the Area of a Circle

Problem Notes

Assign **Practice: Finding the Area of a Circle** as extra practice in class or as homework.

1 **a.** Students should include the measurement label in., not in.^{2}. ***Basic***

b. Students should recognize that the unit for the area is square inches. ***Medium***

2 Students may apply the formula $A = \pi r^2$. They should recognize that the answer is in square meters. ***Medium***

LESSON 6 | SESSION 3 Name:

Practice Finding the Area of a Circle

➤ **Study the Example showing how to find the area of a circle. Then solve problems 1–5.**

Example

What is the exact area of the circle?

To find the radius, count the number of units from the center of the circle to the edge. The radius is 6 units.

Then find the area of the circle.

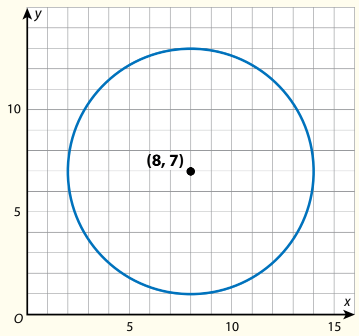
$A = \pi r^2$

$= \pi(6)^2$

$= 36\pi$

To find the exact area, use π.

The area of the circle is 36π units^2.



1 A round tablecloth has a diameter of 30 inches.

a. What is the radius of the tablecloth? 15 in.

b. What is the area of the tablecloth? Write your answer using π. Show your work. Possible work:

$r = 15$ in.

$A = \pi \cdot 15^2$

$= 225\pi$

SOLUTION The area of the tablecloth is 225π in.^{2}.

2 A circular garden has a radius of 3 meters. What is the area of the garden? Write your answer using π. Show your work. Possible work:

$r = 3$ m

$A = \pi \cdot 3^2$

$= \pi \cdot 9$

SOLUTION The area of the garden is 9π m^2.

Vocabulary

area
the amount of space inside a closed two-dimensional figure. Area is measured in square units.

diameter
the distance across a circle through the center.

radius
the distance from the center of a circle to any point on the circle.

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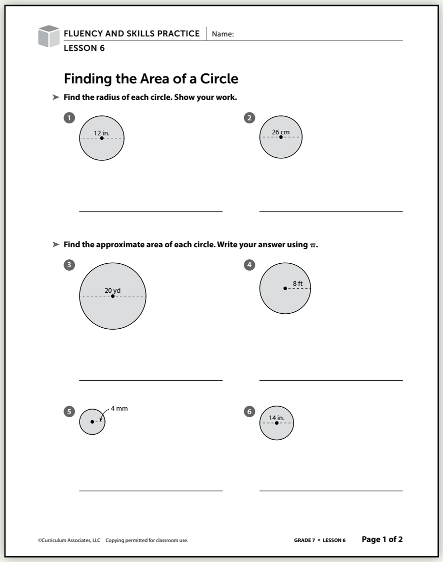
Fluency & Skills Practice

Finding the Area of a Circle

In this activity, students find the area of circles given a diameter or radius using the formula for the area of a circle. They also calculate the radius and area of a circle given its diameter using an approximation for π.

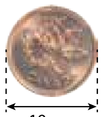


3. Students may use an approximation for π and find an approximation of the difference, rather than the exact difference. ***Challenge***

4. Students should state the area in units of square millimeters, abbreviated mm^2. ***Medium***

5. Students should recognize that the answer should include π and should be expressed in square centimeters, abbreviated cm^2. ***Medium***

3. The diameter of a Canadian penny is 19 millimeters. The diameter of a Canadian nickel is 21 millimeters. How much greater is the area of a Canadian nickel than the area of a Canadian penny? Show your work.



19 mm

21 mm

Possible work:

Penny:

The diameter is 19 mm. The radius is half the diameter, or 9.5 mm.

$A = \pi \cdot 9.5 \cdot 9.5$

$= 90.25\pi$

Nickel:

The diameter is 21 mm. The radius is half the diameter, or 10.5 mm.

$A = \pi \cdot 10.5 \cdot 10.5$

$= 110.25\pi$

$110.25\pi - 90.25\pi = 20\pi$

The exact difference is 20π mm^2, which is about $20(3.14)$ mm^2.

SOLUTION **The area of a Canadian nickel is 20π mm^2 greater.**

4. A circle has radius 12 millimeters. What is its area? Use 3.14 for π. Show your work.

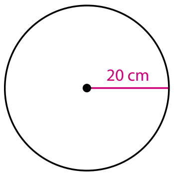
Possible work:

$A \approx 3.14 \cdot 12 \cdot 12$

$= 452.16$

SOLUTION **The area of the circle is about 452.16 mm^2.**

5. What is the exact area of the circle below? Show your work.



Possible work: $r = 20$ cm

$A = \pi \cdot 20 \cdot 20$

$= 400\pi$

SOLUTION **The area is 400π cm^2.**

DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Use with **Session 4** **Apply It**

Levels 1–3: Reading/Writing

Read Apply It problem 11 as students follow along. Review the lesson's math vocabulary and definitions with students. Have partners turn and talk about what they will draw, what they will measure, and what they will calculate.

Display the terms *radius, diameter, circumference,* and *area*. Have students tell what they will measure first. Then have them tell the order in which they will find the other information. Record the steps.

Provide sentence frames to help students express ideas in complete sentences:

- *The ____ of the circle is ____.*
- *This means that the ____ is ____.*

Levels 2–4: Reading/Writing

Have students read Apply It problem 11 with partners. Have them discuss what they need to include in their drawings and in their written responses. Have students tell the order in which they will complete the steps, using *first, next, then,* and *finally*. Ask students to explain their ideas clearly. Remind them to pause to ask for questions and comments from their partners.

Encourage students to use the steps they discussed to write their responses. Provide the following sentence frames to support writing:

- *Since I know ____, I can ____.*
- *I can use the ____ to find the ____.*

Levels 3–5: Reading/Writing

Have students read Apply It problem 11. Ask them to draw the circle and label the radius or diameter. Encourage students to consider how they plan to find the circumference and the area. Ask students to explain their ideas to a partner. Then have them solve individually and explain in writing.

Next, have them exchange written responses with partners. Have students read their partners' responses and confirm that an exact answer was provided using pi (π). Ask students to review the lesson vocabulary and confirm that terms were used accurately in the responses. Have students circle any words or phrases that need attention and write an explanation of what needs to be revised.

Refine Solving Circumference and Area Problems Involving Circles

Purpose

- **Refine** strategies for solving problems involving the dimensions, circumference, or area of a circle.
- **Refine** understanding of proportional relationships related to circles.

START CHECK FOR UNDERSTANDING

A circle has circumference 6π inches. What is the area of the circle?

Solution

9π in.^{2}

WHY? Confirm students' understanding of the relationship between the circumference, diameter, radius, and area of a circle.

MONITOR & GUIDE

Before students begin to work, use their responses to the **Start** to determine those who will benefit from additional support. Use the **Error Analysis** table below to guide remediation.

Have all students complete the Example and problems 1–3, using Consider This and Pair/Share as appropriate. Observe and monitor their reasoning and guide or redirect students as needed.

LESSON 6 | SESSION 4

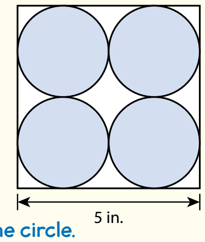
Refine Solving Circumference and Area Problems Involving Circles

➤ **Complete the Example below. Then solve problems 1–11.**

Example

Four identical circles are drawn within a square. What is the total area of the circles?

Look at how you could use the formula for the area of a circle.



The diameter of each circle is $\frac{5}{2}$ in.

The radius of each circle is $\frac{5}{2} \div 2 = \frac{5}{4}$ in.

Total shaded area is 4 times the area of one circle.

Area of one circle $= \pi r^2$

$= \pi \cdot \left(\frac{5}{4}\right)^2$

$= \pi \cdot \frac{25}{16}$

SOLUTION The area of the shaded part is $\frac{25}{4}\pi$ in.^{2}.

CONSIDER THIS . . . The distance across two of the circles is 5 inches.

PAIR/SHARE How would your answer change if the side length of the square was 6 inches?

Apply It

1. A circle with radius 3 centimeters is cut from a square piece of felt. The sides of the felt square are 8 centimeters long. How much felt is left over? Use 3.14 for π. Show your work. Possible work:

Area of square is 8^2 or 64 cm^2.

Area of circle $\approx 3.14 \cdot 3^2$

$= 3.14 \cdot 9$

$= 28.26$

Area left over: $64 - 28.26 = 35.74$

SOLUTION Approximately 35.74 cm^2 of felt is left over.

CONSIDER THIS . . . The formula for the area of a square is $A = s^2$.

PAIR/SHARE Is your answer exact or approximate? Why?

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START ERROR ANALYSIS

If the error is . . .	Students may . . .	To support understanding . . .
36π in.^{2}	have misapplied the formula for the circumference of a circle, or confused the diameter with the radius.	Ask students to write the equations for the circumference and area of a circle: $C = \pi d$ and $A = \pi r^2$. Then have them explain how the diameter d in the first question relates to the radius r in the second equation.
144π in.^{2}	have confused the meanings of *radius* and *diameter*.	Ask students to draw a circle and label its radius, diameter, and circumference. Then have students write two equations for the circumference: $C = \pi d$ and $C = 2\pi r$.
9π in.	not understand or recognize the different units for linear measurements and area.	Ask students to explain the difference between inches and square inches, and the different uses for these two units.

Refine

Example

Guide students in understanding the Example. Ask:

- *How can you use the side length of the square to find the diameter of each circle?*
- *Why is it useful to find the radius of each circle, instead of only its diameter?*
- *Do you think you could find the total area of the unshaded regions of the square? How?*

Help all students focus on the Example and responses to the questions by asking them to agree, disagree, or add on to classmates' responses.

Look for understanding of the key steps of the problem: finding the radius of one circle, applying the formula for the area of one circle, and multiplying the area of the circle by 4, which is the number of circles in the diagram.

Apply It

1 Students may make a sketch to help them visualize the problem. They should recognize that solving this problem requires three steps: finding the area of the square, finding the area of the circle, and subtracting to find the difference. Students should report the answer as an estimate or an approximate value because the calculation used an approximation for π. *DOK 2*

2 Students may add labels to the diagram to help visualize the problem. Students should recognize that the distance around two semicircles of the same diameter is equal to the circumference of one full circle with that diameter. *DOK 2*

3 See ***Connect to Culture*** to support student engagement.

C is correct. The area of the patio is equal to $\pi(6\text{ ft})^2$, or about 113 ft^2. Multiply by 10.50 to find the total of about \$1,187.

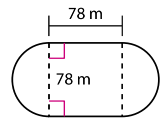
A is not correct. This answer may be the result of multiplying 12 feet by the unit cost, which does not account for the area of each stone.

B is not correct. This answer may be the result of calculating the circumference of the patio instead of its area and using 3 to approximate pi.

D is not correct. This answer may be the result of using the diameter instead of the radius in the formula for the area of a circle.

DOK 3

2 A middle school has an oval track with the dimensions shown. What is the distance around the track? Use 3.14 for π. Show your work.



Possible work:

The two half circles form a circle with diameter 78 m.

$C = \pi d$

$\approx 3.14 \cdot 78$

$= 244.92$

Each straight side is 78 m.

$78 + 78 + 244.92 = 400.92$

SOLUTION The distance around the track is about 400.92 m.

CONSIDER THIS . . .
You can think of the track as a square with half circles on the right and left sides.

PAIR/SHARE
Why might you want to find an approximate distance?

3 Mr. Aba builds a circular patio with a diameter of 12 feet. He covers the patio with paving stones. The cost of the paving stones is \$10.50 per square foot. To the nearest dollar, how much do the paving stones cost?

A \$126

B \$378

C \$1,187

D \$4,748

Jacob chose D as the correct answer. How might he have gotten that answer?

Possible answer: He might have used the diameter instead of the radius when he found the area of the circle. If he found $3.14 \cdot 12^2$, he would have gotten 452.16 ft^2 for the area and about \$4,748 for the cost.

CONSIDER THIS . . .
Multiply the number of square feet by the cost per square foot to find the total cost.

PAIR/SHARE
How could you estimate the area to check that your answer makes sense?

GROUP & DIFFERENTIATE

Identify groupings for differentiation based on the **Start** and problems 1–3. A recommended sequence of activities for each group is suggested below. Use the resources on the next page to differentiate and close the lesson.

Approaching Proficiency

- **RETEACH** Hands-On Activity
- **REINFORCE** Problems 4, 5, 8

Meeting Proficiency

- **REINFORCE** Problems 4–10

Extending Beyond Proficiency

- **REINFORCE** Problems 4–10
- **EXTEND** Challenge

Have all students complete the **Close: Exit Ticket.**

Resources for Differentiation are found on the next page.

Refine Solving Circumference and Area Problems Involving Circles

Apply It

4 Students may reason that the formula for the perimeter of a semicircle is $P = \frac{1}{2}\pi d + d$. The first term shows the length of the arc of the semicircle, and the second term shows the length of the diameter that forms the straight side of the figure. *DOK 2*

5 **A is correct.** The diameter of the circle is twice the radius, or $2 \cdot 7$ cm. The circumference is the product of the diameter and pi.

D is correct. The diameter of the circle is twice the radius, making it 14 cm. The product of the diameter and pi is the circumference.

B is not correct. This answer may be the result of misapplying the formula $C = \pi d$ or using the diameter as the radius.

C is not correct. This answer may be the result of finding the diameter by dividing the radius by 2 instead of multiplying by 2.

E is not correct. This answer may be the result of confusing the formula for the area of a circle with the formula for the circumference.

DOK 2

6 Students may explain that the area is proportional to the square of the radius, not to the radius by itself. *DOK 3*

7 Students should recognize that the area formula can be used to find the radius. Then, they can find the diameter and the circumference. *DOK 3*

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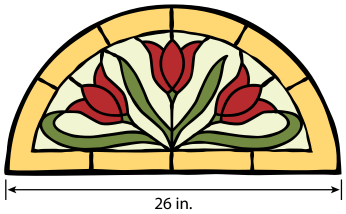
4 The stained glass window shown is a half circle. What is the perimeter of the window? Use 3.14 for π. Show your work.



Possible work:

$C = \pi d$

$\approx 3.14 \cdot 26$

$= 81.64$

The circumference of the curved part of the half circle is half of 81.64 in., or 40.82 in.

Total perimeter $= 40.82 + 26$

$= 66.82$

SOLUTION The perimeter is about 66.82 in.

5 Erin wants to find the circumference of a circle with radius 7 cm. Which of the following can she use to find the circumference of the circle? Select all that apply.

A $2 \cdot 7 \cdot \pi$

B $2 \cdot 14 \cdot \pi$

C $\frac{7}{2} \cdot \pi$

D $14 \cdot \pi$

E $49 \cdot \pi$

6 Is there a proportional relationship between the area and radius of a circle? Explain.

No; Possible explanation: When the radius is 1 unit, the ratio of area to radius is 3.14 : 1. When the radius is 2 units, the ratio of area to radius is 12.56 : 2. The ratios 3.14 : 1 and 12.56 : 2 are not equivalent ratios, so there is not a proportional relationship between the area and radius of a circle.

7 The area of a circle is 25π ft^2. What is the circumference of the circle? Explain.

10π ft; Possible explanation: The formula for the area of a circle is $A = \pi r^2$, so the radius of the circle is 5 ft. The diameter is twice the radius, so the radius is 10 ft. The formula for the circumference of a circle is $C = d\pi$, so the circumference of the circle is 10π ft.

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DIFFERENTIATION

RETEACH

Hands-On Activity

Construct circles on grid paper and calculate their measurements.

Students approaching proficiency with solving problems involving circles will benefit from matching drawings of circles to their dimensions and area.

Materials For each small group: compass, Activity Sheet *Grid Paper*

- Assign each group a number from 4 to 10. Assign each number to at most one group.
- Have students use the grid paper to construct a circle with their assigned number of units as a radius. Then have them label the circle with the radius, diameter, circumference, and area. Tell students to express the radius and diameter in units of grid lines on the grid paper, and to express the circumference and area exactly, as multiples of pi.
- Display the diagrams in order of increasing radius. Then discuss and evaluate the students' work.
- Ask: *What do you think is the most useful dimension of a circle to identify? Explain*. [Possible answer: The radius; when you know the radius, you can easily calculate the other dimensions and the area.]
- Ask: *How is pi useful in this activity?* [Pi is the constant used for finding both the circumference and area.]

8 Students should recognize that the line shows that the radius of each circle is $\frac{15}{6}$ in. *DOK 3*

9 Students may also identify the quotient as approximately 3.14 or $\frac{22}{7}$. *DOK 1*

10 Students may apply the formula $C = \pi d$ to find that the circumference of the wheel is 28π, or about 87.92 inches. Then they can multiply the circumference by 10 to find the total distance. *DOK 3*

CLOSE EXIT TICKET

11 **Math Journal** Look for understanding of the formulas for the circumference and area of a circle and the units in which circumference and area are stated.

Error Alert If students confuse the formulas for the circumference and area of a circle, then have them compare the two formulas. Have them relate the square units of area to the square of the radius in the area formula.

End of Lesson Checklist

INTERACTIVE GLOSSARY Support students by suggesting that they include a diagram of a circle with labels for the circumference and diameter.

SELF CHECK Have students review and check off any new skills on the Unit 1 Opener.

LESSON 6 | SESSION 4

8 A box holds three circular flower pots, each with the same diameter. What is the exact area of the base of one pot? Show your work.

Possible work:

The diameter of each pot is $\frac{15}{3}$, or 5, in. The radius is $\frac{5}{2}$ in.

$A = \pi r^2$

$= \pi \cdot \left(\frac{5}{2}\right)^2$

$= \frac{25}{4}\pi$

SOLUTION The area of the base of one pot is $\frac{25}{4}\pi$ in.^{2}.

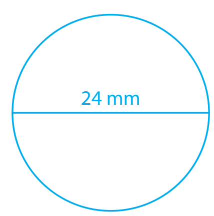
9 The quotient of a circle's circumference divided by its diameter is π.

10 The radius of the wheel of a unicycle is 14 inches. What is the distance, in inches, that the unicycle covers after 10 full rotations of the wheel? Use 3.14 for π.

879.2

11 **Math Journal** Draw a circle and give its radius or diameter. Then find the circumference and the area of the circle. Write your answers using π.

Possible answer:



The diameter of the circle is 24 mm, so its radius is 12 mm.

The circumference is 24π mm.

$A = \pi \cdot 12^2$

$= 144\pi$

The area is 144π mm^2.

End of Lesson Checklist

- [ ] **INTERACTIVE GLOSSARY** Find the entries for *circumference* and *pi* (π). Rewrite the definitions in your own words.
- [ ] **SELF CHECK** Go back to the Unit 1 Opener and see what you can check off.

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REINFORCE

Problems 4–10

Solve problems involving the dimensions and area of circles.

Students meeting proficiency will benefit from additional work with solving multi-step problems involving circumference and area.

- Have students work on their own or with a partner to solve the problems.
- Encourage students to show their work.

EXTEND

Challenge

Show how doubling the radius of a circle affects its circumference and area.

Students extending beyond proficiency will benefit from multi-step problems involving circumference and area.

- Have students work with a partner to solve. Say: *A circle has a radius of 2 cm. When the radius is doubled, by what factor does the circumference increase? By what factor does the area increase?*
- Students should find that the circumference increases by a factor of 2, while the area increases by a factor of 4.
- Repeat with other initial values of radius, and generalize to show a pattern for all circles.

PERSONALIZE

i-Ready

Provide students with opportunities to work on their personalized instruction path with *i-Ready* Online Instruction to:

- fill prerequisite gaps.
- build up grade-level skills.

Overview | Solve Problems Involving Area and Surface Area

STANDARDS FOR MATHEMATICAL PRACTICE (SMP)

SMP 1, 2, 3, 4, 5, and 6 are integrated into the Try-Discuss-Connect routine.*

This lesson provides additional support for:

2 Reason abstractly and quantitatively.

3 Construct viable arguments and critique the reasoning of others.

7 Look for and make use of structure.

* See page 1q to learn how every lesson includes these SMP.

Objectives

Content Objectives

- Use given areas and given lengths to solve problems involving unknown lengths of two-dimensional composite figures.
- Use given surface areas and given lengths to solve problems involving unknown lengths of right prisms.
- Apply knowledge of surface area of right prisms to solve real-world and mathematical problems involving surface areas of composite figures.

Language Objectives

- Justify in writing, claims about the unknown dimensions of composite figures by describing known information about the figures that compose them.
- Understand and use lesson vocabulary to describe prisms and composite figures.
- Make connections among strategies to find surface areas of composite figures during partner and class discussions.

Prior Knowledge

- Find the areas of triangles and quadrilaterals.
- Apply strategies to find the areas of composite figures.
- Use nets or other visual models to represent the faces of three-dimensional figures and find surface area.

Vocabulary

Math Vocabulary

There is no new vocabulary. Review the following key terms.

decompose to break into parts. You can break apart numbers and shapes.

dimension length in one direction. A figure may have one, two, or three dimensions.

face a flat surface of a solid shape.

right rectangular prism a right prism where the bases and other faces are all rectangles.

right triangular prism a right prism where the bases are triangles and the other faces are rectangles.

surface area the sum of the areas of all the faces of a three-dimensional figure.

Academic Vocabulary

claim to state that something is true or false.

composite made of different parts.

in terms of in relationship to or in units named by.

Learning Progression

In Grade 6, students determined the areas of triangles and quadrilaterals. They also composed and decomposed composite figures to find their areas. They used nets to find the surface area of three-dimensional figures with rectangular and triangular faces.

Earlier in Grade 7, students learned the formula for area of a circle and found the areas of circles.

In this lesson, students extend their understanding of area and surface area to composite three-dimensional shapes. They will write and solve equations to find an unknown linear dimension of a composite figure. They will solve real-world problems that involve area.

Later in Grade 7, students will extend their skills to find the volumes of prisms and other three-dimensional figures. including composite figures. They will identify plane sections of three-dimensional figures. They will explore the relationship between the surface area and volume of three-dimensional figures.

Pacing Guide

Items marked with are available on the **Teacher Toolbox**.

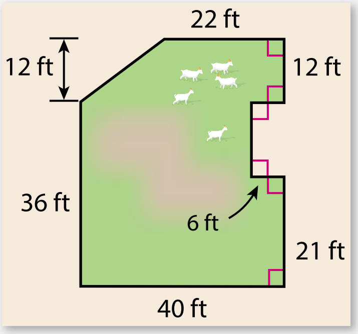
	MATERIALS	DIFFERENTIATION
SESSION 1 **Explore** Finding Composite Areas (35–50 min)		
• **Start** (5 min) • **Try It** (5–10 min) • **Discuss It** (10–15 min) • **Connect It** (10–15 min) • **Close: Exit Ticket** (5 min) **Additional Practice** (pages 545–546)	**Math Toolkit** grid paper, tracing paper Presentation Slides	**PREPARE** Interactive Tutorial **RETEACH or REINFORCE** Hands-On Activity **Materials** For each student: scissors, tape, 2 copies of Activity Sheet *1-Centimeter Grid Paper*
SESSION 2 **Develop** Solving Problems Involving Area (45–60 min)		
• **Start** (5 min) • **Try It** (10–15 min) • **Discuss It** (10–15 min) • **Connect It** (15–20 min) • **Close: Exit Ticket** (5 min) **Additional Practice** (pages 551–552)	**Math Toolkit** grid paper, tracing paper Presentation Slides	**RETEACH or REINFORCE** Visual Model **REINFORCE** Fluency & Skills Practice **EXTEND** Deepen Understanding
SESSION 3 **Develop** Solving Problems Involving Surface Area (45–60 min)		
• **Start** (5 min) • **Try It** (10–15 min) • **Discuss It** (10–15 min) • **Connect It** (15–20 min) • **Close: Exit Ticket** (5 min) **Additional Practice** (pages 557–558)	**Math Toolkit** geometric solids, grid paper, isometric dot paper Presentation Slides	**RETEACH or REINFORCE** Hands-On Activity **Materials** For each pair: scissors, 2 copies of Activity Sheet *1-Centimeter Grid Paper* **REINFORCE** Fluency & Skills Practice **EXTEND** Deepen Understanding
SESSION 4 **Develop** Solving Problems Involving Surface Area of Composite Figures (45–60 min)		
• **Start** (5 min) • **Try It** (10–15 min) • **Discuss It** (10–15 min) • **Connect It** (15–20 min) • **Close: Exit Ticket** (5 min) **Additional Practice** (pages 563–564)	**Math Toolkit** dot paper, geometric solids, grid paper, isometric dot paper Presentation Slides	**RETEACH or REINFORCE** Hands-On Activity **Materials** For each pair: 25 unit cubes (9 of one color, 16 of another color) **REINFORCE** Fluency & Skills Practice **EXTEND** Deepen Understanding
SESSION 5 **Refine** Solving Problems Involving Area and Surface Area (45–60 min)		
• **Start** (5 min) • **Monitor & Guide** (15–20 min) • **Group & Differentiate** (20–30 min) • **Close: Exit Ticket** (5 min)	**Math Toolkit** Have items from previous sessions available for students. Presentation Slides	**RETEACH** Visual Model **Materials** For display: 10 unit cubes **REINFORCE** Problems 4–8 **EXTEND** Challenge **PERSONALIZE** i-Ready
Lesson 25 Quiz or **Digital Comprehension Check**		**RETEACH** Tools for Instruction **REINFORCE** Math Center Activity **EXTEND** Enrichment Activity

Connect to Culture

➤ **Use these activities to connect with and leverage the diverse backgrounds and experiences of all students. Engage students in sharing what they know about contexts before you add the information given here.**

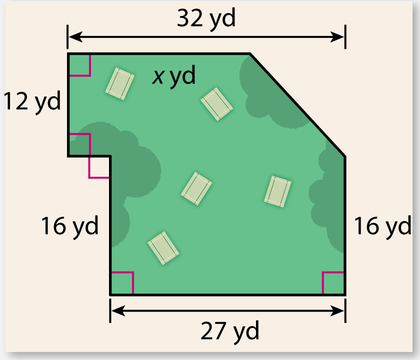
SESSION 1 ■ □ □ □ □

Try It Survey your students' interest in or experience with keeping pets or livestock at home. Animal science experts who have studied the containment of livestock argue that the minimum area that a penned animal needs depends on the type of animal, time span of containment, and the shape of the pen. Ask students to share their experiences with keeping a pet or livestock in an enclosed space. Prompt for details about the size and shape of the pen and the animal's behavior.



SESSION 2 ■ ■ □ □ □

Try It Ask students to describe possible advantages and disadvantages to having an outdoor eating space at a school. At a conference of the School Nutrition Association, members shared ideas such as inviting food trucks to the school, installing self-service entrée bars, and conducting cooking demonstrations in the classroom. Schools are also using new technology to predict the amount of food that students eat, which can help reduce wasted food. Have students share innovations they have heard of or would like to see pertaining to school lunches and food.



SESSION 4 ■ ■ ■ ■ □

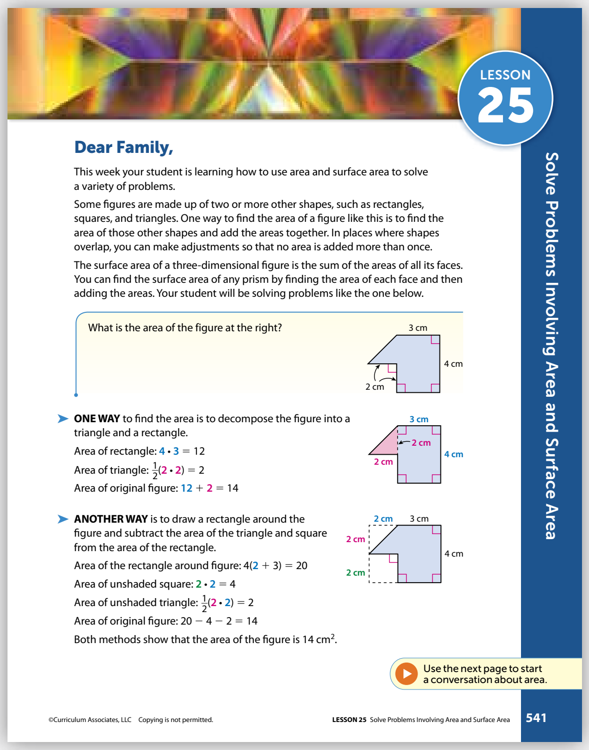
Try It Ask for a show of hands from students who have or want to have a pet cat. One challenge to keeping a pet cat is that it needs to scratch. Cats scratch for many reasons, including removing dead skin, marking their territory, and exercising their feet and claws. Fortunately, many cats can be trained to practice their scratching behavior on a single place, such as a scratching post or block. Cats are attracted to the nubby, coarse surfaces that are used to cover the posts. Have students share any strategies they know for training cats not to scratch.

SESSION 5 ■ ■ ■ ■ ■

Apply It **Problem 4** A courtyard is generally defined as a roofless outdoor space that is surrounded on all sides by the walls of a house or by several buildings. Courtyards were common in homes and public spaces of ancient Rome. Later they were included in monasteries, allowing monks to privately enjoy an outdoor space. Ask students to describe how they would design a courtyard for their home or community.

Connect to Family and Community

➤ **After the Explore session, have students use the Family Letter to let their families know what they are learning and to encourage family involvement.**



LESSON 25

Solve Problems Involving Area and Surface Area

Dear Family,

This week your student is learning how to use area and surface area to solve a variety of problems.

Some figures are made up of two or more other shapes, such as rectangles, squares, and triangles. One way to find the area of a figure like this is to find the area of those other shapes and add the areas together. In places where shapes overlap, you can make adjustments so that no area is added more than once.

The surface area of a three-dimensional figure is the sum of the areas of all its faces. You can find the surface area of any prism by finding the area of each face and then adding the areas. Your student will be solving problems like the one below.

What is the area of the figure at the right?

3 cm
4 cm
2 cm

➤ **ONE WAY** to find the area is to decompose the figure into a triangle and a rectangle.

Area of rectangle: $4 \cdot 3 = 12$

Area of triangle: $\frac{1}{2}(2 \cdot 2) = 2$

Area of original figure: $12 + 2 = 14$

3 cm
2 cm
4 cm
2 cm

➤ **ANOTHER WAY** is to draw a rectangle around the figure and subtract the area of the triangle and square from the area of the rectangle.

Area of the rectangle around figure: $4(2 + 3) = 20$

Area of unshaded square: $2 \cdot 2 = 4$

Area of unshaded triangle: $\frac{1}{2}(2 \cdot 2) = 2$

Area of original figure: $20 - 4 - 2 = 14$

Both methods show that the area of the figure is 14 cm^2.

2 cm
3 cm
2 cm
4 cm
2 cm

Use the next page to start a conversation about area.

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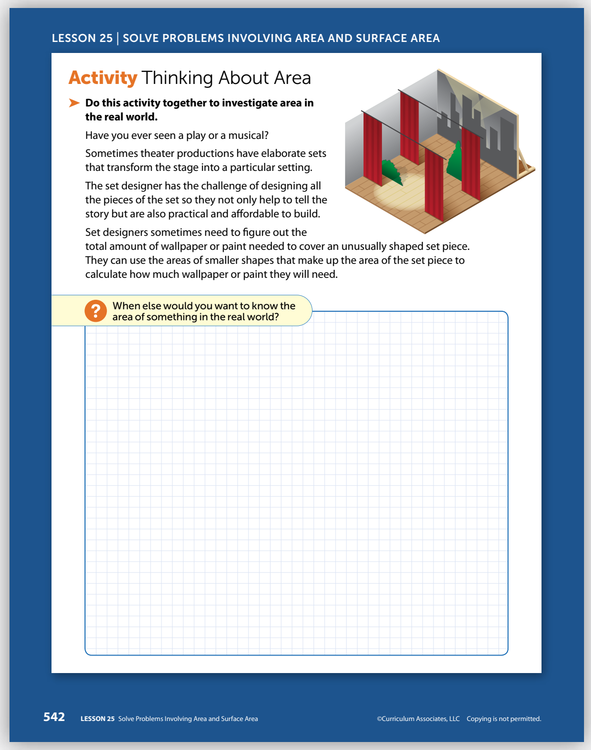


LESSON 25 | SOLVE PROBLEMS INVOLVING AREA AND SURFACE AREA

Activity Thinking About Area

➤ **Do this activity together to investigate area in the real world.**

Have you ever seen a play or a musical?

Sometimes theater productions have elaborate sets that transform the stage into a particular setting.

The set designer has the challenge of designing all the pieces of the set so they not only help to tell the story but are also practical and affordable to build.

Set designers sometimes need to figure out the total amount of wallpaper or paint needed to cover an unusually shaped set piece. They can use the areas of smaller shapes that make up the area of the set piece to calculate how much wallpaper or paint they will need.

When else would you want to know the area of something in the real world?

542 LESSON 25 Solve Problems Involving Area and Surface Area ©Curriculum Associates, LLC Copying is not permitted.

Connect to Language

➤ **For English language learners, use the Differentiation chart to scaffold the language in each session. Use the Academic Language routine for academic terms before Session 1.**

DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS Use with **Session 1** **Connect It**

Levels 1–3: Reading/Listening

Prepare students for problem 2 by reviewing *quantity, variable, expression, known,* and *unknown.* Ask for cognates, such as the Spanish words *cantidad, variable,* and *expresión*.

Display the figure and a T-chart labeled *Sides* and *Side Lengths.* Ask students to list the sides, identify whether the lengths are quantities or variables, and tell if the lengths are known or unknown:

- *I see that side ____ has the length ____. The label is a ____. The side length is ____.*
- *I see that side ____ does not have a label. The side length is ____. We can write an ____.*

Levels 2–4: Reading/Listening

Help students interpret Connect It problem 2 by discussing sides and side lengths. Read the problem with students and connect to the figure by having them identify side lengths labeled with a quantity or variable. Ask: *Which sides do not show a side length?* Have students circle the notation for $\overline{DE}$ and $\overline{EF}$ in the problem. Then ask: *What can we write to figure out the length of these sides? What expression is in problem 2a? What do the terms of the expression represent?*

If needed, ask students to use the Interactive Glossary to define *term*. Then help with other meanings. To clarify the meaning of *in terms of* in the problem, ask: *What variable is the expression 24 – x in terms of?*

Levels 3–5: Reading/Listening

Have students read and interpret Connect It problem 2 by discussing sides and side lengths. Ask: *What does the problem use to represent the sides? The side lengths?* Ensure that students connect the notation for $\overline{DE}$ and $\overline{EF}$ to the figure. Ask students to list sides, tell if lengths are known, and tell which are represented with a quantity, variable, or expression.

Next, ask partners to find the phrase *in terms of* and use context clues to discuss the meaning. Confirm understanding using **Say It Another Way**. Then have students discuss the lesson's academic vocabulary definition. Ask: *What expression is in problem 2a? Which variable is the expression in terms of?*

Explore Finding Composite Areas

Purpose

- **Explore** the idea that unknown side lengths can be expressed in terms of variables.
- **Understand** that areas of composite figures can be represented by algebraic expressions when one or more side lengths are known.

START CONNECT TO PRIOR KNOWLEDGE

Same and Different

A	B
A square with sides 3 in.	A rectangle with sides 12 in. and 2 in.
C	**D**
A right triangle with legs 6 in. and 8 in.	A square with sides 5 in.

Possible Solutions

All have at least one right angle.

A, B, and D are quadrilaterals and C is a triangle.

A has the least area, B and C have the same area, and D has the greatest area.

WHY? Support students' ability to compare polygons based on their areas and other properties.

TRY IT

SMP 1, 2, 4, 5, 6

Make Sense of the Problem

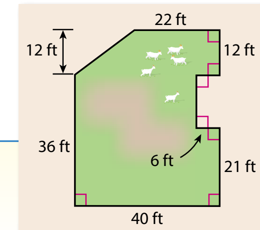
See ***Connect to Culture*** to support student engagement. Before students work on Try It, use **Notice and Wonder** to help them make sense of the problem. Ask students to study the diagram of the plan for the goat pen and encourage them to wonder about the composite figure it depicts. If time allows at the end of the session, return to the list of things that students noticed and wondered about to discuss what was relevant to the problem and what could lead to further questions that could be answered with mathematics.

Explore Finding Composite Areas

Previously, you learned about area and surface area. In this lesson, you will learn about solving problems that involve area and surface area of composite figures.

➤ **Use what you know to try to solve the problem below.**

Tyler's family builds a new pen on their dairy goat farm. The diagram represents the new pen. Each goat needs at least 50 ft^2 of space. What is the greatest number of goats the new pen can hold?

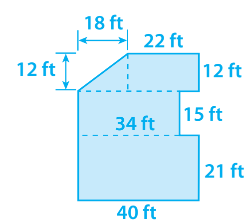


TRY IT

Math Toolkit grid paper, tracing paper

Possible work:

SAMPLE A



Total area: triangle plus 3 rectangles

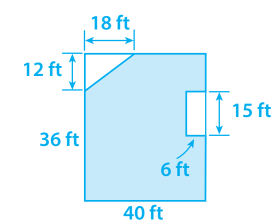
$\frac{1}{2}(18 \cdot 12) + (22 \cdot 12) +$

$(34 \cdot 15) + (40 \cdot 21) = 1{,}722$

$1{,}722 \div 50 = 34.44$

The pen can hold up to 34 goats.

SAMPLE B



Total area: enclosing rectangle minus triangle and rectangle

$40(12 + 36) - \frac{1}{2}(18 \cdot 12) -$

$(6 \cdot 15) = 1{,}722$

$1{,}722 \div 50 = 34.44$

The pen can hold up to 34 goats.

DISCUSS IT

Ask: How is the way you found the greatest number of goats similar to the way I did? How is it different?

Share: My method is similar to yours because . . . It is different because . . .

Learning Target SMP 1, SMP 2, SMP 3, SMP 4, SMP 5, SMP 6, SMP 7
Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

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DISCUSS IT

SMP 2, 3, 6

Support Partner Discussion

After students work on Try It, have them respond to Discuss It with a partner. Listen for understanding of:

- decomposing a composite figure into known shapes.
- applying formulas to find the area of a composite figure.
- using information about area to determine capacity per square foot.

Common Misconception Listen for students who decompose the figure into shapes with only the given lengths, such as a 40 ft-by-21 ft rectangle, a 36 ft-by-6 ft rectangle, and a 12 ft-by-22 ft triangle. As students share their strategies, have them draw a model of the composite figure they are finding the area of and compare it to the pen.

Select and Sequence Student Strategies

Select 2–3 samples that represent the range of student thinking in your classroom. Here is one possible order for class discussion:

- making a scale copy of the figure on grid paper and counting squares to find the area
- (common misconception) finding the areas of shapes that do not actually form the figure
- decomposing the figure into known shapes, finding the area of each, and finding the sum

Facilitate Whole Class Discussion

Call on students to share selected strategies. As they listen to their classmates, have students evaluate the strategies and agree and build on them. Remind students that one way to agree and build on ideas is to give another example.

Guide students to **Compare and Connect** the representations. Before they discuss the question below, provide students with individual think time to prepare and refine their responses.

ASK *How are the strategies of [student name] and [student name] similar to each other?*

LISTEN FOR Both strategies decompose the composite figure into smaller shapes to find the area. Both divide the area by 50 to find out the number of goats.

CONNECT IT

SMP 2, 4, 5

1 Look Back Look for understanding that the quotient must be rounded down to 34 to give the goats enough area.

DIFFERENTIATION | RETEACH or REINFORCE

Hands-On Activity

Find the area of a composite figure.

If students are unsure about how to find the area of a composite figure, then have them find and compare the areas of related composite figures.

Materials For each student: scissors, tape, 2 copies of Activity Sheet *1-Centimeter Grid Paper*

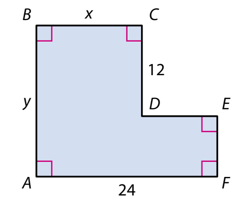
- Have each student cut out a 4-by-6 rectangle, a 3-by-6 rectangle, and a right triangle with base 3 and height 4, use the shapes to build a composite figure without overlap, and find its area.
- Have partners compare their work. Ask: *How does arranging the shapes differently affect the area of the composite figure?* [The area remains the same.]
- Then have partners use their shapes to form a new composite figure, find its area, and discuss patterns they see in the areas of composite figures.

CONNECT IT

1 Look Back What is the greatest number of goats the new pen can hold? Explain how you know.

34; Possible explanation: The area of the pen divided by 50 ft^2 per goat is 34.44. You can not have part of a goat, so the new pen can hold 34 goats.

2 Look Ahead To find the number of goats that can fit in the pen, you may have used the side lengths you knew to find an unknown side length. Sometimes a side length in a figure is labeled with a variable. You may be able to write expressions for other side lengths in terms of the variable.



a. How do you know that $24 - x$ represents the length of $\overline{DE}$?

The length of $\overline{BC}$ plus the length of $\overline{DE}$ is equal to 24. So, the length of $\overline{DE}$ is equal to 24 minus the length of $\overline{BC}$, or x.

b. Write an expression for the length of $\overline{EF}$ in terms of y. $y - 12$

c. Explain how the expression $12x + 24(y - 12)$ represents the area of the figure.

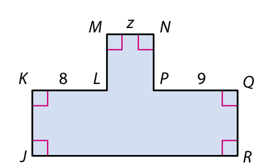
You can divide the figure into two rectangles horizontally by extending $\overline{DE}$ left to meet $\overline{AB}$. Then you can find the sum of the areas of the two rectangles. The area of the top rectangle is $12x$. The area of the bottom rectangle is $24(y - 12)$. So, the area of the rectangle is $12x + 24(y - 12)$.

d. Explain how $24y - 12(24 - x)$ also represents the area of the figure.

You can think of the figure as a large rectangle with base $\overline{AF}$ and height $\overline{AB}$ minus a smaller rectangle with base $\overline{DE}$ and height $\overline{DC}$. The area of the larger rectangle is $24y$. The area of the smaller rectangle is $12(24 - x)$. So, the area of the figure is $24y - 12(24 - x)$.

3 Reflect How do you know that the expression $z + 17$ represents the length of $\overline{JR}$?

The length of $\overline{JR}$ is equal to the sum of the lengths of $\overline{KL}$, $\overline{MN}$, and $\overline{PQ}$. So, the length of $\overline{JR}$ is equal to $8 + z + 9$, or $z + 17$.



2 Look Ahead Point out that a relationship involving subtraction can also be described using addition. In the example, x plus the length of $\overline{DE}$ is equal to 24 and y is equal to 12 plus the length of $\overline{EF}$.

CLOSE EXIT TICKET

3 Reflect Look for understanding that together, the three short horizontal sides of the figure span the same horizontal distance as the length of $\overline{JR}$, the long horizontal side at the bottom. The sum of the lengths of the three short sides can be expressed algebraically as $8 + z + 9$, or $z + 17$.

Error Alert If students recognize that $z + 17$ is the same as $z + 8 + 9$ but do not recognize that JR is the same as $KL + MN + PQ$, then model drawing vertical lines to decompose the figure into 3 rectangles. Ask students to explain why the shapes are rectangles and what that tells them about the side lengths.

Prepare for Solving Problems Involving Area and Surface Area

Support Vocabulary Development

Assign **Prepare for Solving Problems Involving Area and Surface Area** as extra practice in class or as homework.

If you have students complete this in class, then use the guidance below.

Ask students to consider the term *surface area.* Remind students that they just found the area of a composite figure. Encourage students to think about the relationship between the terms *area* and *surface area*.

Have students work in pairs to complete the graphic organizer. Invite pairs to share their completed organizers, and prompt a whole-class comparative discussion of the definitions, examples, and non-examples given.

Have students review the expression that Muna proposes and discuss with a partner whether the expression accurately identifies the surface area of the rectangular prism. Encourage students to refer to their graphic organizer and to their knowledge of rectangular prisms to help them construct their response.

Problem Notes

1. Students should understand that surface area is measured in square units, such as square inches or square centimeters, and that it is the total area of the surface, or set of surfaces, of a three-dimensional figure. Students may point out that surface area can be found from analyzing a net that corresponds to a three-dimensional figure.
2. Students should recognize that a rectangular prism has 6 faces. Muna's expression only includes the area of 3 faces. The actual surface area is twice Muna's expression.

LESSON 25 | SESSION 1 Name:

Prepare for Solving Problems Involving Area and Surface Area

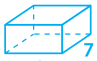
1. Think about what you know about three-dimensional figures and surface area. Fill in each box. Use words, numbers, and pictures. Show as many ideas as you can.
Possible answers:

What Is It?
the total area of the surfaces of a three-dimensional figure

What I Know About It
You need to include the areas of all the faces, including faces on the back or bottom.
Surface area is measured in square units, just like area.

surface area

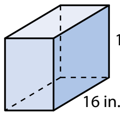
Example



Surface Area = 162 ft^2

Non-Example
To find the amount of space inside a three-dimensional figure, you should find the volume, not the surface area.

2. Muna claims that the expression (8)(16) + (8)(12) + (16)(12) represents the surface area, in square inches, of the right rectangular prism shown. Is Muna correct? Explain.



No; Possible explanation: Muna added the areas of only 3 faces of the prism. She needs to add the areas of all 6 faces to find the surface area.

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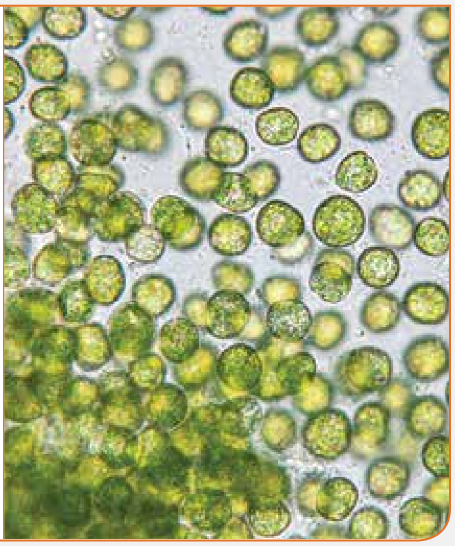
REAL-WORLD CONNECTION

Biologists use the concept of surface area to explain the sizes of cells. The width of a typical plant or animal cell is between 10 and 100 micrometers. A micrometer (μm) is one millionth of a meter, or one thousandth of a millimeter, too small for the unaided eye to see. As the cell grows larger, its volume increases at a greater rate than its surface area. If the cell were to grow beyond its size, the cell membrane would not be large enough to transport the oxygen and other materials that the cell needs to survive, while waste products, such as carbon dioxide, would build up inside the cell. Ask students to think of other real-world examples involving surface area.

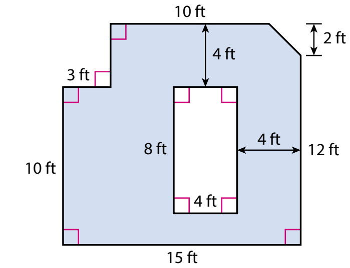


3 Problem 3 provides another look at finding the area of a composite figure and then applying a rate to solve a problem. This problem is similar to the problem about the maximum number of goats allowed in a pen. In both problems, students are asked to find and use the area of a composite figure to find a quantity in proportion to it.

Students may also think of the area of the composite figure as the area of a surrounding rectangle minus the areas of two rectangles and a triangle.

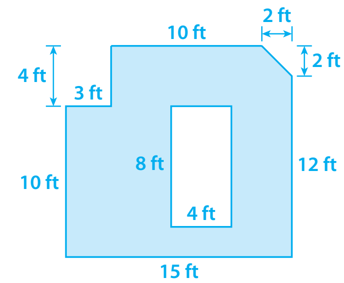
Suggest that students use **Three Reads** to help them make sense of the problem.

3 A kitchen floor needs new tile. The shaded region in the diagram represents the floor that needs tile. The new tile costs \$5.40 per square foot.



a. What is the total cost of the new tile for the kitchen? Show your work.

Possible work:



Think of the tiled area as a 15 ft-by-14 ft rectangle with a 4 ft-by-8 ft rectangle, a 3 ft-by-4 ft rectangle, and a 2 ft-by-2 ft triangle removed.

$$15(12 + 2) - (4 \cdot 8) - (3 \cdot 4) - \frac{1}{2}(2 \cdot 2) = 210 - 32 - 12 - 2$$
$$= 164$$

$164(5.40) = 885.60$

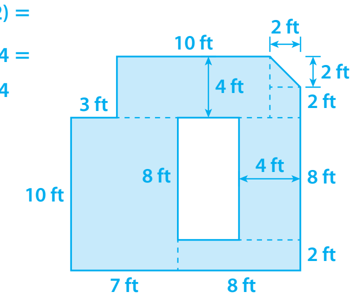
SOLUTION The total cost is \$885.60.

b. Check your answer to problem 3a. Show your work.

Possible work:

$$(7 \cdot 10) + (10 \cdot 4) + (4 \cdot 8) + (8 \cdot 2) + \frac{1}{2}(2 \cdot 2) + (2 \cdot 2) =$$
$$70 + 40 + 32 + 16 + 2 + 4 =$$
$$164$$

$164(5.40) = 885.60$



DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Use with **Session 2** **Connect It**

Levels 1–3: Speaking/Writing

Help students interpret Connect It problem 4 and write responses. Review the lesson vocabulary term *decompose* and connect it to the Spanish cognate *descomponer,* if appropriate. Provide these frames to help students connect to previous strategies and describe one way to decompose the figure to find the value of *x*. Record for reference.

- *Picture It shows how to ____ the figure into smaller ____. The smaller figures are ____.*
- *Model It uses an equation to ____. The equation can help me find ____.*

Give students time to solve for *x*. Have partners show how they used decomposing and their strategies. Help partners change the sentence frames for written responses.

Levels 2–4: Speaking/Writing

Help students interpret Connect It problem 4 and then write responses. Guide students to make sense of *decompose* by connecting different meanings and discussing word parts. Have students tell how *decompose* relates to science and then connect to the definition of the lesson vocabulary term. Next, ask students to discuss how *decompose* relates to *compose.* Then use **Say It Another Way** to confirm understanding of the problem.

To help students write, suggest they use sequence words like *first, next, then,* and *finally.* Remind students to specify in their explanations how they decomposed the figure and why it was helpful.

Levels 3–5: Speaking/Writing

Have students read and interpret Connect It problem 4. To make sense of lesson vocabulary term *decompose,* ask students to discuss its multiple meanings and tell how the prefix *de-* changes the meaning of the base *compose*.

Next, have partners prepare to write by discussing how to use *decompose* in their explanations. Remind them that effective writing uses transitions to show the flow of ideas. Present this example: *The first step in my strategy ____.* Have partners discuss other transitions they might use.

Ask students to write a clear explanation of how decomposing relates to their overall strategy for finding the value of *x.*

Develop Solving Problems Involving Area

Purpose

- **Develop** strategies for solving real-world and mathematical problems involving the area of composite figures.
- **Recognize** that you can decompose complicated shapes to solve area problems, including problems that involve using area to find an unknown side length.

START CONNECT TO PRIOR KNOWLEDGE

Always, Sometimes, Never

A A hexagon can be decomposed into triangles.
B A hexagon can be decomposed into circles.
C A hexagon can be decomposed into quadrilaterals.
D A hexagon can be decomposed into rectangles.

Possible Solutions

A and C are always true, even for irregular hexagons.

B is never true, since hexagons do not have curves.

D is sometimes true. L-shaped hexagons can be decomposed into 2 rectangles.

WHY? Support students' ability to compose and decompose composite shapes.

DEVELOP ACADEMIC LANGUAGE

WHY? Support understanding of *composite* through related words.

HOW? Draw a composite figure on the board and use *composite* to identify it. Have students identify smaller figures that *compose* or form the figure. Tell students that sometimes we *decompose* complex shapes in order to analyze them. Then have them use *decompose* to describe the process. Invite students to list and define other words related to *composite*.

TRY IT

SMP 1, 2, 4, 5, 6

Make Sense of the Problem

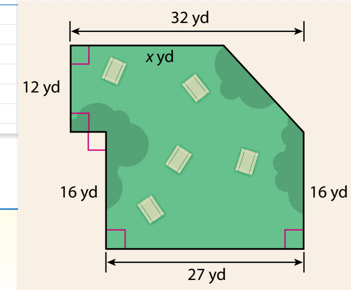
See ***Connect to Culture*** to support student engagement. Before students work on Try It, use **Say It Another Way** to help them make sense of the problem. Have students rephrase the problem statement and describe the composite figure in their own words. Use a show of thumbs from the class to evaluate the description. Allow students to revise their descriptions as necessary.

LESSON 25 | SESSION 2

Develop Solving Problems Involving Area

➤ **Read and try to solve the problem below.**

The diagram represents an outdoor eating space at a school. The area of the outdoor eating space is 750 yd^2. What is the value of x?



TRY IT

Math Toolkit grid paper, tracing paper

Possible work:

SAMPLE A

The area of the outdoor eating space is equal to the area of two rectangles and a triangle.

Area of one rectangle: $16 \cdot 27 = 432$

Area of the other rectangle: $12x$

Area of the triangle: $\frac{1}{2}(32 - x)12 = 192 - 6x$

$750 = 432 + 12x + 192 - 6x$

$750 = 624 + 6x$

$126 = 6x$

$21 = x$

The value of x is 21.

SAMPLE B

The area of the eating space is equal to the area of the outer rectangle minus the areas of the unshaded rectangle and triangle.

$750 = (32 \cdot 28) - (5 \cdot 16) - \frac{1}{2}(32 - x)(12)$

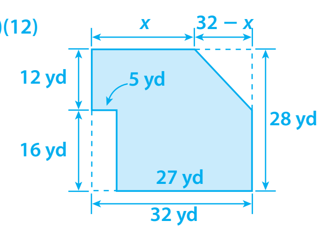
$750 = 896 - 80 - 192 + 6x$

$750 = 624 + 6x$

$126 = 6x$

$21 = x$

The value of x is 21.



DISCUSS IT

Ask: What did you do first to find the value of x?

Share: First, I . . .

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DISCUSS IT

SMP 2, 3, 6

Support Partner Discussion

After students work on Try It, encourage them to respond to Discuss It with a partner. If students need support in getting started, prompt them to ask each other questions such as:

- *How did you decompose the composite figure that represents the eating space?*
- *How did you decide whether to represent the area as a sum or a difference?*
- *How did you represent the variable x in an equation for area?*

Error Alert If students conclude that the length of the top base of the trapezoid is the same as the length of the rectangle, 27 yd, or that the length of the rectangle is the same as the length of the bottom base of the trapezoid, 32 yd, then have them explain how they identified the unlabeled side lengths. Encourage students to trace lines on the diagram to confirm that they are applying the dimensions accurately.

Select and Sequence Student Strategies

Select 2–3 samples that represent the range of student thinking in your classroom. Here is one possible order:

- decomposing the figure into smaller shapes, such as two rectangles and a triangle
- finding the difference between the area of an enclosing rectangle and the unshaded areas
- writing and solving an equation that uses a variable to represent the unknown measure

Facilitate Whole Class Discussion

Call on students to share selected strategies. After students listen to others' strategies, prompt them to check their understanding by trying to paraphrase the speaker's ideas. Let them ask questions about ideas that are unclear.

Guide students to **Compare and Connect** the representations. Have students turn and talk to share their thoughts about the question below with one another.

ASK *What do all of the strategies have in common?*

LISTEN FOR They used expressions to represent the unidentified side lengths and represented the area of the composite figure as the sum or difference of the areas of the smaller shapes.

Picture It & Model It

If students presented these models, have students connect these models to those presented in class.

If no student presented at least of these models, have students first analyze key features of the models, and then connect them to the models presented in class.

ASK *How is the diagram in Picture It related to the equation in Model It?*

LISTEN FOR The picture shows how the figure can be decomposed into smaller shapes. The equation shows that the combined area of the smaller shapes equals 750 yd^2.

For the diagram, prompt students to explain how the composite figure was decomposed.

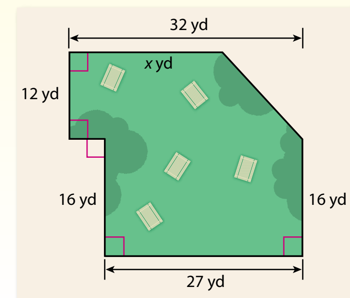
- *Why was it useful to add dotted lines to this picture?*

For the equation, prompt students to identify the meaning of each term.

- *What does each term in the equation represent?*

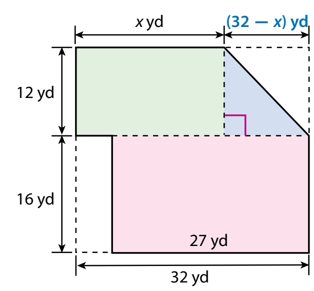
➤ **Explore different ways to solve problems involving area.**

The diagram represents an outdoor eating space at a school. The area of the outdoor eating space is 750 yd^2. What is the value of x?



Picture It

You can decompose a composite figure into smaller shapes.



Model It

You can write and solve an equation to find an unknown measurement.

The sum of the areas of the two rectangles and the triangle is equal to the area of the eating space.

$$(x \cdot 12) + (27 \cdot 16) + \frac{1}{2}(32 - x)(12) = 750$$
$$12x + 432 + 6(32 - x) = 750$$
$$12x + 432 + 192 - 6x = 750$$
$$6x + 624 = 750$$
$$6x = 126$$

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DIFFERENTIATION | EXTEND

SMP 3

Deepen Understanding

Constructing Arguments to Support Strategies for Solving Problems Involving Composite Figures

Prompt students to explain a strategy for solving a problem involving a composite figure and to construct an argument in support of the strategy.

ASK *Look at Picture It. Why are the dashed lines perpendicular?*

LISTEN FOR The perpendicular lines form rectangles. If they were not perpendicular, you would not have enough information to find the areas.

ASK *What if I drew a different set of dotted lines to cut the composite figure in another way? Would the strategy still produce the same answer? Explain.*

LISTEN FOR Yes. Composite figures can often be decomposed into different groups of smaller shapes, and different smaller shapes can be used to find the area.

ASK *Why is finding the areas helpful in solving for x in this problem?*

LISTEN FOR The relationship between the known area and the known and unknown side lengths can be stated as an equation.

Develop Solving Problems Involving Area

CONNECT IT

SMP 2, 4, 5, 6

Remind students that the quantities and the relationships between them are the same in each representation. Explain that they will now use those relationships to reason about strategies for finding unknown side lengths in composite figures.

Before students begin to record and expand on their work in Picture It & Model It, tell them that problems 3 and 4 will prepare them to provide the explanation asked for in problem 5.

Monitor and Confirm Understanding 1 – 2

- The base, b, of the triangle satisfies the equation $x + b = 32$. Solving for b shows that $b = 32 - x$.
- The area of a triangle is $\frac{1}{2}bh$. For a right triangle, the base and height are also the lengths of the two legs of the triangle.

Facilitate Whole Class Discussion

3 Students should recognize that the area of a composite figure is equal to the sum of the areas of the shapes it is composed of.

4 Look for the idea that an expression for the area of the composite figure leads to an equation that can be solved for the unknown variable, x.

> **ASK** *Could you find x using just the side lengths in the diagram? Explain.*
>
> **LISTEN FOR** The side lengths in the diagram do not give enough information to find x. You need the area to write and solve an equation for x.

5 Look for the idea that the strategies applied to solve Try It can be adapted for a wide variety of problems involving a known area and an unknown side length.

> **ASK** *Suppose you were given a similar problem with a figure composed of different shapes. How would your strategy be the same and different?*
>
> **LISTEN FOR** I would still need to decompose the figure into smaller shapes and write an equation using area to solve for an unknown, but the expressions I use for the smaller areas would be different.

6 **Reflect** Have all students focus on the strategies used to solve the Try It. If time allows, have students discuss their ideas with a partner.

CONNECT IT

➤ **Use the problem from the previous page to help you understand how to solve problems involving area.**

1 Look at the shaded triangle in **Picture It**. Why can you use the expression $32 - x$ to represent the base of the triangle in yards?

The base of the outer rectangle is equal to 32 yd and the base of the triangle is equal to the base of the outer rectangle minus x.

2 Look at the first equation in **Model It**. Explain why the expression $\frac{1}{2}(32 - x)(12)$ represents the area of the shaded triangle.

The area of a triangle is equal to $\frac{1}{2}$ its base times its height. The triangle has base $(32 - x)$ yd and height 12 yd.

3 Why does the sum of the areas of the two rectangles and the triangle equal the area of the composite figure?

When you decompose a figure into smaller shapes, the total area does not change.

4 What is the value of x? How does decomposing the figure help you find the value of x?

21; Possible answer: You can decompose the figure into smaller shapes that you know how to find the area of to write an equation that you can solve for x.

5 How can you use the area of a figure to find an unknown side length of the figure?

Possible answer: Write an expression for the area that uses a variable for the unknown length. Set the expression equal to the given area. Then solve the equation for the variable.

6 **Reflect** Think about all the models and strategies you have discussed today. Describe how one of them helped you better understand how to solve the **Try It** problem.

Responses will vary. Check student responses.

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DIFFERENTIATION | RETEACH or REINFORCE

Visual Model

Compare strategies for finding an unknown side length.

If students are unsure about using area to find unknown side lengths, then have them compare different solutions.

- Display an irregular hexagon formed by removing an x unit-by-4 unit rectangle from the corner of an 8 unit-by-15 unit rectangle.
- Say: *The area of the figure is 100 units²*. Write the equation: $60 + 4(15 - x) = 100$.
- Ask: *Where does the equation come from? How do you know you can use the equation to find x?* [You can decompose the figure into a rectangle with area 60 units^2 and a rectangle with area $4(15 - x)$ units^2. Together, they have the area of the composite figure, 100 units^2.]
- Repeat with the equations $4x + 8(15 - x) = 100$ and $120 - 4x = 100$.
- Have students generalize about using area to find a side length. [You use expressions that represent the area of the figure to write an equation and solve for x.]
- Have a volunteer solve each equation. Guide students to understand that the value of x will be the same, since x represents the same value in each equation.

Apply It

For all problems, encourage students to use models of the composite figures and various ways to decompose them to support their thinking. Students may see the value of using different approaches to solving different problems.

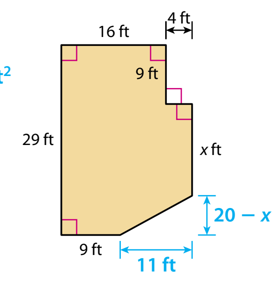
7 Students may also decompose the composite figure into two rectangles and a trapezoid, and write an equation for the total area as the sum of the three smaller areas.

8 Students may prefer to find the area of the shaded portions instead. One strategy is to decompose the figure into 2 triangles, one with base and height 8 and the other with base 5 and height 18.

Apply It

➤ **Use what you learned to solve these problems.**

7 The diagram shows a plan for a deck. The area of the deck is 511 ft^2. What is the value of x? Show your work. Possible work:



Enclosing rectangle area − areas of rectangle and triangle = 511 ft^2

$$(16 + 4)(29) - (4 \cdot 9) - \frac{1}{2}(11)(20 - x) = 511$$

$$580 - 36 - 110 + 5.5x = 511$$

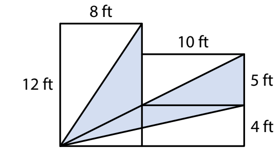
$$434 + 5.5x = 511$$

$$5.5x = 77$$

$$x = 14$$

SOLUTION The value of x is 14.

8 Portions of three rectangles are shaded as shown. What is the area of the shaded region? Show your work. Possible work:



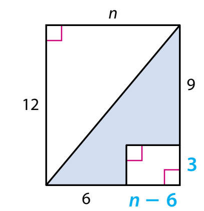
Total area: $(8 \cdot 12) + (10 \cdot 5) + (10 \cdot 4) = 186$

Unshaded area: $\frac{1}{2}(8 \cdot 12) + \frac{1}{2}(10 \cdot 5) + \frac{1}{2}(8 + 10)(4) = 109$

Shaded area: $186 - 109 = 77$

SOLUTION The area of the shaded region is 77 ft^2.

9 The area of the shaded region of the figure is 48 $units^2$. What is n? Show your work. Possible work:



The area of half the figure is $\frac{1}{2}(n \cdot 12)$.

$$\frac{1}{2}(n \cdot 12) - (n - 6)(3) = 48$$

$$6n - 3n + 18 = 48$$

$$3n + 18 = 48$$

$$3n = 30$$

$$n = 10$$

SOLUTION n is 10 units.

CLOSE EXIT TICKET

9 Students' solutions should show an understanding of:

- decomposing a composite figure as the difference between two smaller shapes.
- writing and solving an equation that relates the areas of smaller shapes to the known area of the composite figure.

Error Alert If students try to solve the problem by decomposing the shaded region into shapes that they do not have enough information to find the area of, then suggest they begin by considering what other sides they can write an expression for. Guide them to find the side lengths $n - 6$ and 3. Then prompt them to consider which shapes they can find the area for.

Practice Solving Problems Involving Area

Problem Notes

Assign **Practice Solving Problems Involving Area** as extra practice in class or as homework.

1. Students may also multiply the number of unshaded squares, 4, by the area of each square, 4.41. ***Basic***

2. Students may reason that since the area of the shaded region is 78 in.^{2} and the area of the rectangle is 96 in.^{2}, the area of the unshaded region is 16 in.^{2}. They can use this to write and solve the equation $16 = \frac{1}{2}(12)(x - 8)$. ***Medium***

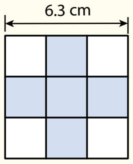
LESSON 25 | SESSION 2 Name:

Practice Solving Problems Involving Area

➤ **Study the Example showing how to solve a problem involving area. Then solve problems 1–5.**

Example

Each side of a square is divided into three equal sections. The square is then shaded as shown. What is the area of the shaded part?



You can think of the shaded region as 5 small squares of equal size.

Each side of the larger square is divided into thirds. So, each small square has side length $6.3 \div 3$, or 2.1 cm.

So, each small square has area (2.1)(2.1), or 4.41 cm^2.

So, the area of the shaded region is 5(4.41), or 22.05 cm^2.

1. What is the area of the unshaded part of the square in the Example? Show your work.

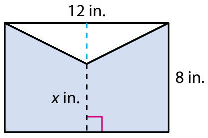
   Possible work:

   One fewer square is unshaded.

   $22.05 - 4.41 = 17.64$

   SOLUTION The area of the unshaded region is 17.64 cm^2.

2. A portion of a rectangle is shaded as shown. The area of the shaded region is 78 in.^{2}. What is the value of x? Show your work.



   Possible work:

   Height of unshaded region: $(8 - x)$ in.

   Rectangle area = shaded region area + unshaded region area

   $12 \cdot 8 = 78 + \frac{1}{2}(12)(8 - x)$

   $96 = 78 + 48 - 6x$

   $96 = 126 - 6x$

   $-30 = -6x$

   $5 = x$

   SOLUTION The value of x is 5.

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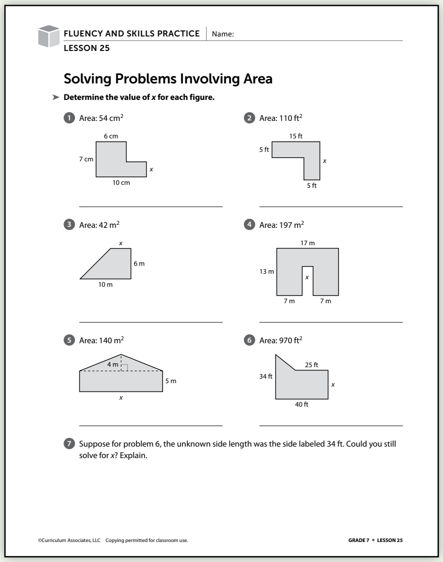
Fluency & Skills Practice

Solving Problems Involving Area

In this activity, students determine the area of composite figures.


FLUENCY AND SKILLS PRACTICE | Name:
LESSON 25

Solving Problems Involving Area

➤ Determine the value of x for each figure.

1. Area: 54 cm²
2. Area: 110 ft²
3. Area: 42 m²
4. Area: 197 m²
5. Area: 140 m²
6. Area: 970 ft²
7. Suppose for problem 6, the unknown side length was the side labeled 34 ft. Could you still solve for x? Explain.

3 Students may also apply the formula for the area of a trapezoid, which is $A = \frac{1}{2}(b_1 + b_2)h$. ***Challenge***

4 **A is correct.** Students should recognize that a vertical line can decompose the shape into a 16 cm-by-n cm rectangle and a triangle of base n and height 11 cm.

B is not correct. This answer uses 16 cm as the base of the triangle.

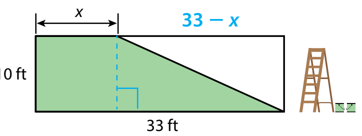
C is not correct. This answer incorrectly finds the difference between the area of an enclosing rectangle and unshaded regions. The area of the unshaded region can be represented by $\frac{1}{2}(n)(11)$.

D is not correct. This answer incorrectly finds the difference between the area of an enclosing rectangle and unshaded regions.

Medium

5 Students may reason that since 3 bags would cover a rectangular region that is 90 yd-by-30 yd, the landscaper needs no more than 3 bags. They can also reason that part of the figure can be decomposed into a rectangular region that is 1,620 yd^2 and two rectangular regions that are each 140 yd^2, the sum of which is more than 1,800 yd^2, so the landscaper will need more than 2 bags. ***Medium***

3 An art class plans to paint part of a rectangular wall in the cafeteria and leave the rest of the wall white, as shown. The painted section will take up $\frac{2}{3}$ of the area of the wall. What is x? Show your work.



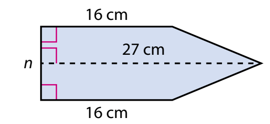
Possible work:

The painted section is composed of a square and a triangle.

$$(x \cdot 10) + \frac{1}{2}(33 - x)(10) = \frac{2}{3}(33 \cdot 10)$$
$$10x + 165 - 5x = 220$$
$$5x + 165 = 220$$
$$5x = 55$$
$$x = 11$$

SOLUTION x is 11 ft.

4 The figure has area 193.5 cm^2. Which equation can be used to find the value of n, in centimeters?



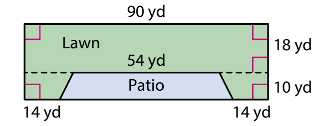
(A) $16n + \frac{1}{2}(11n) = 193.5$

B $16n + \frac{1}{2}(16)(11) = 193.5$

C $27n - \frac{1}{2}(16n) = 193.5$

D $27n - \frac{1}{2}(16)(11) = 193.5$

5 The diagram shows the plan for a lawn. A landscaper needs to buy grass seed to cover the lawn. One bag of grass seed covers an area of 900 yd^2. How many bags of seed does the landscaper need to buy to cover the lawn? Show your work.



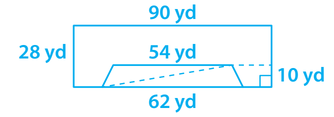
Possible work:

Outer rectangle area: $90 \cdot 28 = 2{,}520$

Patio area: $\frac{1}{2}(54 \cdot 10) + \frac{1}{2}(62 \cdot 10) = 580$

Lawn area: $2{,}520 - 580 = 1{,}940$

Bags: $1{,}940 \div 900 = 2.1\overline{5}$



SOLUTION The landscaper needs to buy 3 bags.

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DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Use with **Session 3** **Connect It**

Levels 1–3: Speaking/Writing

Read Connect It problem 3 as students follow along. Ask: *What is the area of one base? What other area is in the problem? Which face of the prism has the area 15 cm^2?* Review the lesson vocabulary term *surface area* and connect to the Spanish cognate *superficie*, if appropriate. Ask: *How can we find the surface area?*

Ask students to use a net, diagram, or equation to find the surface area. Help them write using these sentence frames:

- *I know ____. I also know ____.*
- *I can find the surface area by ____.*

Levels 2–4: Speaking/Writing

Read Connect It problem 3 with students. Then ask partners to discuss the lesson vocabulary terms *surface area, face,* and *right rectangular prism*. Next, have partners make sense of the information in the problem and share possible strategies to find the surface area. Remind students to support ideas by connecting to the solid or to other problems they have already solved. Ask: *How is this problem the same as others we have solved? How is it different? What do we know about the prism? Can we find the surface area?*

Encourage students to write explanations by telling what they know about the prism and how they might use that information.

Levels 3–5: Speaking/Writing

Have students read Connect It problem 3. Ask students to consider the details in the problem that might help them find the surface area. Have students turn to partners and discuss the information in the problem and strategies to reason about the surface area.

Call on students to share their ideas. Ask students to reword responses using math terms, like the lesson vocabulary *surface area, face,* and *right rectangular prism*.

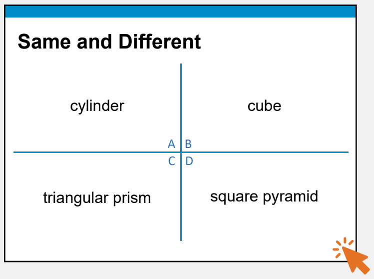
Next, have students write explanations. Encourage students to present the information they know about the prism and tell how they might use it to find the surface area.

Develop Solving Problems Involving Surface Area

Purpose

- **Develop** strategies for solving real-world and mathematical problems involving surface areas of prisms.
- **Recognize** that the surface area of any prism can be found by adding the areas of its faces.

START CONNECT TO PRIOR KNOWLEDGE



Possible Solutions

All are three-dimensional figures.

A is the only figure with a curved surface.

B, C, and D all have at least one rectangular face, but only B has only rectangular faces.

C and D both have 5 faces, each of which is either a triangle or a rectangle.

WHY? Support students' ability to identify and compare three-dimensional figures.

DEVELOP ACADEMIC LANGUAGE

WHY? Support students as they justify solutions with definitions, properties, and prior knowledge.

HOW? Review that one way to justify a solution is to share definitions, properties, and what you already know to explain and defend your strategy and thinking. Recognize when students defend their ideas during the class discussion about surface area and encourage them to do so in other academic discussions.

TRY IT SMP 1, 2, 4, 5, 6

Make Sense of the Problem

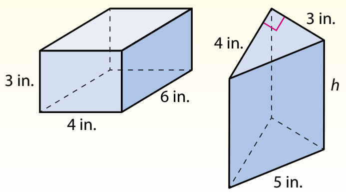
Before students work on Try It, use **Three Reads** to help them make sense of the problem. After the first read, ask students what the problem is about and what it means for two prisms to have the same surface area. After the second read, have them identify the problem. After the third read, have students describe the important quantities and relationships needed to solve the problem.

LESSON 25 | SESSION 3

Develop Solving Problems Involving Surface Area

➤ **Read and try to solve the problem below.**

The right rectangular prism and the right triangular prism have the same surface area. What is the height in inches, h, of the triangular prism?



TRY IT **Math Toolkit** geometric solids, grid paper, isometric dot paper

Possible work:

SAMPLE A

S.A. of rectangular prism: $2(4 \cdot 6 + 4 \cdot 3 + 6 \cdot 3) = 108$

Triangular prism:

Area of rectangular faces: $3h + 4h + 5h = 12h$

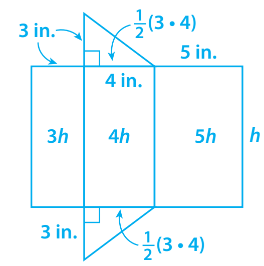
Area of triangular faces: $2\left[\frac{1}{2}(3 \cdot 4)\right] = 12$

S.A. of triangular prism = S.A. of rectangular prism

$12h + 12 = 108$

$12h = 96$

$h = 8$ The height of the triangular prism is 8 in.



SAMPLE B

S.A. of rectangular prism = S.A. of triangular prism

$2(4 \cdot 6 + 4 \cdot 3 + 6 \cdot 3) = 3h + 4h + 5h + 2\left[\frac{1}{2}(3 \cdot 4)\right]$

$2(24 + 12 + 18) = 12h + 12$

$12(4 + 2 + 3) = 12(h + 1)$

$9 = h + 1$

$8 = h$

The height of the triangular prism is 8 in.

DISCUSS IT

Ask: How do you know your answer is reasonable?

Share: My answer is reasonable because . . .

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DISCUSS IT SMP 2, 3, 6

Support Partner Discussion

After students work on Try It, have them respond to Discuss It with a partner. If students need support in getting started, prompt them to ask each other questions such as:

- *How do you know which parts of the surface area involve the variable, h, and which do not?*
- *How did you know your expression for the surface area was correct?*

Common Misconception Listen for students who neglect to find the area for one of the hidden faces of a prism, such as the face that forms the bottom of the figure. As students share their strategies, ask them to identify each of the 6 faces of the rectangular prism or the 5 faces of the triangular prism. Encourage students to recognize any faces that a classmate counted that they might have omitted.

Select and Sequence Student Strategies

Select 2–3 samples that represent the range of student thinking in your classroom. Here is one possible order for class discussion:

- (common misconception) using the diagram to find the area of all of the visible faces
- drawing nets to represent the surface area
- finding the area of the rectangular faces as the sum of side lengths multiplied by the height
- writing an equation that relates the surface area of each figure

Facilitate Whole Class Discussion

Call on students to share selected strategies. As they listen, if students disagree with the accuracy or usefulness of a strategy, have them refer to the diagrams to help explain why they disagree.

Guide students to **Compare and Connect** the representations. Encourage students to turn and talk to compare the different ideas and strategies that were presented.

ASK *What are some different ways to represent the surface area of the triangular prism?*

LISTEN FOR The areas of the three rectangular faces of the prisms can be found individually, or as a large 12 in.-by-h in. rectangle.

Picture It & Model It

If students presented these models, have students connect these models to those presented in class.

If no student presented at least one of these models, have students first analyze key features of the models, and then connect them to the models presented in class.

ASK *How do the pictures of the faces shown in Picture It relate to the equations in Model It?*

LISTEN FOR The pictures show the side lengths of each face. The equations use the same side lengths to find the surface area.

For the drawing of the faces, prompt students to relate the diagrams of the two triangular faces to the diagram of the triangular prism.

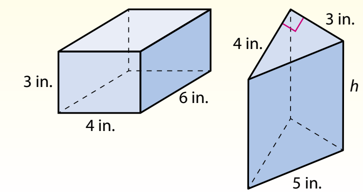
- *Are these the only triangular faces in the prism? How do you know?*

For the equations, prompt students to relate the terms of the equations to the faces of the prisms.

- *In the equation for the rectangular prism, why does each term have a factor of 2?*
- *In the equation for the triangular prism, why does only the first term have a factor of 2?*

➤ **Explore different ways to solve problems involving surface area.**

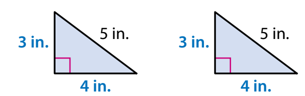
The right rectangular prism and the right triangular prism have the same surface area. What is the height in inches, h, of the triangular prism?



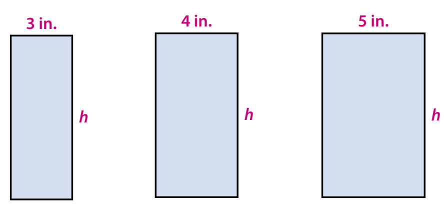
Picture It

You can draw the faces of a prism with their dimensions.

The bases of the triangular prism are the same shape and size.



The other three faces of the triangular prism are rectangles.



Model It

You can write and solve an equation to find an unknown measurement.

Surface area of rectangular prism: $2(4 \cdot 6) + 2(4 \cdot 3) + 2(6 \cdot 3) = 108$

Surface area of triangular prism: $2\left(\frac{1}{2}\right)(4 \cdot 3) + 3h + 4h + 5h = 12 + 12h$

Surface area of rectangular prism = Surface area of triangular prism

$108 = 12 + 12h$

$96 = 12h$

DIFFERENTIATION | EXTEND

SMP 2

Deepen Understanding

Using Abstract Reasoning to Solve Problems Involving Surface Area

Prompt students to reason about dimensions of three-dimensional figures.

ASK *Why is knowing three dimensions enough to find the surface area of the right rectangular prism but not the right triangular prism?*

LISTEN FOR The rectangular faces of the rectangular prism all have the same width but 2 different lengths. The rectangular faces of the triangular prism have the same width but 3 different lengths.

ASK *Two right rectangular prisms have the same height and bases with the same area. Does this mean they must have the same surface area? How do you know?*

LISTEN FOR No. A different right rectangular prism with base area 12 in.^{2} and height 6 in. could have a 2 in.-by-6 in. base and surface area 120 in.^{2}.

ASK *Given all the side lengths of a right triangular prism, can you always find the surface area? Why or why not?*

LISTEN FOR No, because to find the area of the triangular faces, I need the base and the height, which are not always the same as the side lengths.

Develop Solving Problems Involving Surface Area

CONNECT IT

SMP 2, 4, 5, 6

Remind students that the quantities and the relationships between them are the same in each representation. Explain that they will now use those relationships to reason about problems involving surface area.

Before students begin to record and expand on their work in Picture It & Model It, tell them that problem 2 will prepare them to provide the explanation asked for in problem 3.

Monitor and Confirm Understanding 1

- A triangular prism has 2 identical triangular bases and 3 rectangular faces that can all be different sizes.

Facilitate Whole Class Discussion

2 Look for the idea that the problem states that the two figures have equal surface area, and that this relationship helps solve for the value of *h*.

> **ASK** *How can you determine the value for h?*
>
> **LISTEN FOR** Represent the surface area of the triangular prism with an expression in terms of *h* and set it equal to the surface area of the rectangular prism to solve for *h*.

3 Students should recognize that the surface area of a right rectangular prism is equal to the sum of the areas of its faces.

> **ASK** *Why is it important to know that the figure is a rectangular prism to find its surface area?*
>
> **LISTEN FOR** You need to know that the figure is a rectangular prism so you know how many faces it has. A rectangular prism has 4 non-base faces.

4 **Reflect** Have all students focus on the strategies used to solve the Try It. If time allows, have students discuss their ideas with a partner.

CONNECT IT

➤ **Use the problem from the previous page to help you understand how to solve problems involving surface area.**

1 Look at the expression for the surface area of the triangular prism in **Model It**.

Why is $\left(\frac{1}{2}\right)(4 \cdot 3)$ the only part of the expression that is multiplied by 2?

The prism has two bases with area $\left(\frac{1}{2}\right)(4 \cdot 3)$. None of the rectangular faces are the same shape and size, so none of the other areas are multiplied by 2.

2 What is the height of the triangular prism? How does knowing the surface area of the prism help you find the height?

8 in.; You can use the surface area to write an equation and solve for the height.

3 Another rectangular prism has bases with area 14 cm^2 each. Each of the other faces has area 15 cm^2. Why is this enough information to find the surface area of the prism?

Possible explanation: Since rectangular prisms have 2 bases with the same area and 4 other faces with the same area, you can find the surface area by adding 2 times 14 and 4 times 15.

4 **Reflect** Think about all the models and strategies you have discussed today. Describe how one of them helped you better understand how to solve problems involving surface area.

Responses will vary. Check student responses.

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DIFFERENTIATION | RETEACH or REINFORCE

Hands-On Activity

Make and analyze nets to find the surface area of prisms.

If students are unsure about strategies for finding surface area, then use this activity to provide them with a concrete model.

Materials: For each pair: scissors, 2 copies of Activity Sheet *1-Centimeter Grid Paper*

- Have partners work together to draw a net for a 3 cm-by-3 cm-by-8 cm right rectangular prism. Remind students that not all arrangements will fold into a rectangular prism. Encourage them to fold their nets to ensure they are accurate.
- Have pairs find the surface area for the rectangular prism.
- Ask: *How is a net useful for finding the surface area of a prism?* [Since the net shows each face, I can use it to find the area of each face then add them to find the surface area.]
- Ask: *Some of the faces are the same. How does this help you find the surface area?* [Instead of adding the areas of each face individually, I can multiply the number of identical faces by the area of each face.]
- Extend the activity by repeating with a right triangular prism. Use a 3 cm-by-4 cm-by-5 cm base and 8 cm height. Have students discuss how their nets are similar and different.

Apply It

For all problems, encourage students to draw nets or other visual models to support their thinking. Allow some leeway in precision; for example, the side lengths of the rectangles, triangles, or other two-dimensional figures may be hand-drawn and not to scale but should be labeled with the correct dimensions.

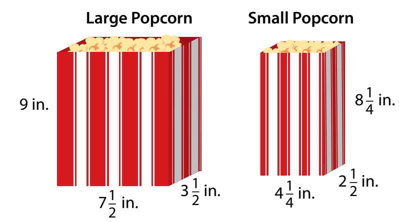
5 Students should recognize that a bottom face is included in each bag, but the matching top face is not included. Therefore, the area of the bottom face is not multiplied by 2 in the expression for the surface area.

6 Students should recognize that this problem can be solved in two steps: first, finding the surface area of the box, and second, finding a 10% increase, or 110%, of the surface area.

Apply It

➤ **Use what you learned to solve these problems.**

5 A company makes popcorn bags in two sizes. Each bag is shaped like a right rectangular prism, but it is open at the top. How many more square inches of paper are needed to make a large bag than a small bag? Show your work. Possible work:



Large: $\left(7\frac{1}{2} \cdot 3\frac{1}{2}\right) + 2\left(7\frac{1}{2} \cdot 9\right) + 2\left(3\frac{1}{2} \cdot 9\right) = 224\frac{1}{4}$

Small: $\left(4\frac{1}{4} \cdot 2\frac{1}{2}\right) + 2\left(4\frac{1}{4} \cdot 8\frac{1}{4}\right) + 2\left(2\frac{1}{2} \cdot 8\frac{1}{4}\right) = 122$

$224\frac{1}{4} - 122 = 102\frac{1}{4}$

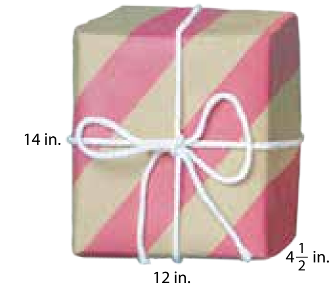
SOLUTION A large bag needs $102\frac{1}{4}$ in.^{2} more paper.

6 Grace needs to cover a box shaped like a right rectangular prism with wrapping paper. Grace needs 10% more paper than the surface area of the box. How many square inches of wrapping paper does she need? Show your work.

Possible work:

$2(14 \cdot 12) + 2\left(14 \cdot 4\frac{1}{2}\right) + 2\left(12 \cdot 4\frac{1}{2}\right) = 570$

$1.1(570) = 627$

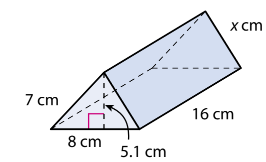


SOLUTION Grace needs 627 in.^{2} of wrapping paper.

7 The surface area of the right triangular prism is 376.8 cm^2. What is the value of x? Show your work.

Possible work:

$$2\left(\frac{1}{2}\right)(8)(5.1) + (8)(16) + (7)(16) + (x)(16) = 376.8$$
$$280.8 + 16x = 376.8$$
$$16x = 96$$
$$x = 6$$



SOLUTION The value of x is 6.

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CLOSE EXIT TICKET

7 Students' solutions should show an understanding of:

- identifying the faces of a triangular prism.
- writing an expression for the sum of the areas of the faces.
- solving an equation to find the unknown dimension of the prism.

Error Alert If students account for only one triangular face when finding the surface area and find that $x = 7.275$, then have them identify the faces of the triangular prism as shown in the diagram. Ask students to relate each term in their expression for surface area to the individual faces.

Practice Solving Problems Involving Surface Area

Problem Notes

Assign **Practice Solving Problems Involving Surface Area** as extra practice in class or as homework.

1 **a.** Students should recognize that the triangular bases are isosceles triangles, so they have two sides that are the same length, meaning the rectangular faces have the same length and area. ***Basic***

b. Students should recognize that the number 2 signifies two identical faces. ***Medium***

2 **D is correct.** Students may solve the problem by writing an expression that represents the surface area of the prism as the sum of the areas of its 6 faces and then simplifying the expression.

A is not correct. This answer applies the formula for the volume of a rectangular prism instead of the surface area.

B is not correct. This answer accounts for only one face of each pair of faces that have length *h*.

C is not correct. This answer may be the result of accounting for only one face of each of the three pairs of faces.

Medium

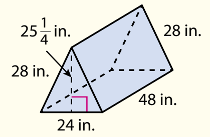
 Name:

Practice Solving Problems Involving Surface Area

➤ **Study the Example showing how to solve a problem involving surface area. Then solve problems 1–5.**

Example

A small greenhouse for a vegetable garden is shaped like a right triangular prism. Clear plastic covers all faces of the greenhouse except the bottom. How many square inches of clear plastic cover the greenhouse?



You can find the amount of plastic by adding the areas of the faces of the greenhouse.

Area of the triangular faces: $\frac{1}{2}\left(24 \cdot 25\frac{1}{4}\right) = 303$

Area of the rectangular faces: $48 \cdot 28 = 1{,}344$

Total surface area: $2(303) + 2(1{,}344) = 3{,}294$

There are 3,294 in.^{2} of clear plastic covering the greenhouse.

1 **a.** The expression $2(303) + 2(1{,}344)$ can be used to find the area of the plastic in the Example. Why is each term multiplied by 2?

Each term represents the combined area of 2 identical faces.

b. Teresa writes the expression $2(303 + 1{,}344 + 1{,}152)$ to find the total surface area of the greenhouse, including the bottom. Explain why her expression is incorrect.

Teresa's expression includes the area of the bottom face multiplied by 2, but there is only one face with that area.

2 A right rectangular prism has length 10 in. and width 8 in. The surface area of the prism is 376 in.^{2}. Which equation can be used to find the height in inches, *h*, of the prism?

A $80h = 376$

B $160 + 18h = 376$

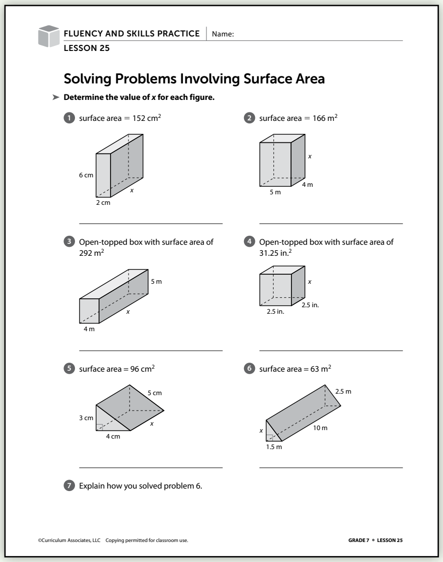
C $80 + 18h = 376$

D $160 + 36h = 376$

Fluency & Skills Practice

Solving Problems Involving Surface Area

In this activity, students determine the missing dimensions of three-dimensional figures when given the surface areas.


FLUENCY AND SKILLS PRACTICE | Name:
LESSON 25

Solving Problems Involving Surface Area

➤ Determine the value of x for each figure.

1 surface area = 152 cm^2

2 surface area = 166 m^2

3 Open-topped box with surface area of 292 m^2

4 Open-topped box with surface area of 31.25 in.^{2}

5 surface area = 96 cm^2

6 surface area = 63 m^2

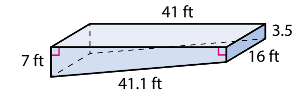
7 Explain how you solved problem 6.

3. Students should recognize that the surface area to be painted consists of the areas of five faces: the rectangular bottom of the pool, the trapezoidal front and back walls, and the two rectangular left and right ends. Students should also know to round up the number of gallons, since the customer may only purchase whole gallons of paint and needs to have enough paint to cover all of the surfaces of the pool. ***Challenge***

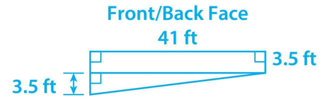
4. Students may also solve the problem by reasoning that when the length and width of a rectangle are both doubled, its area increases by a factor of 4. ***Medium***

5. Students should recognize that the total area of the awning is equal to the sum of three separate areas: the two triangular faces and the slanted rectangular face. ***Medium***

3. The swimming pool at an apartment complex is shaped like a right prism. The bottom and sides of the pool need to be repainted. One gallon of paint covers up to 125 ft^2. Paint can only be purchased in whole gallons. How many gallons of paint will the painters need to purchase? Show your work.



Possible work:



Area of front/back face:

$(41)(3.5) + \frac{1}{2}(41)(3.5) = 215.25$

Area to be painted:

$2(215.25) + (16)(7) + (16)(3.5) + (41.1)(16) = 1{,}256.1$

Gallons of paint: $1{,}256.1 \div 125 = 10.0488$

SOLUTION The painters will need to purchase 11 gallons of paint.

4. A right rectangular prism has length 15 cm, width 10 cm, and height 5 cm. Savanna claims that doubling the length, width, and height of the prism will double its surface area. Is Savanna correct? Explain.

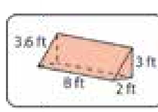
Savanna is not correct; Possible explanation: The surface area of the larger prism is 4 times the surface area of the smaller prism.

Smaller prism: $2(15 \cdot 10) + 2(15 \cdot 5) + 2(10 \cdot 5) = 550$

Larger prism: $2(30 \cdot 20) + 2(30 \cdot 10) + 2(20 \cdot 10) = 2{,}200$

$2{,}200 \div 550 = 4$

5. The awning for a window is shaped like a right triangular prism. The fabric of the awning covers one rectangular and two triangular faces of the prism, as shown. The fabric for the awning costs \$0.55 per square foot. What is the cost of fabric for the awning? Show your work.



Possible work:

$2\left(\frac{1}{2}\right)(2)(3) + (8)(3.6) = 34.8$

$34.8(0.55) = 19.14$

SOLUTION The fabric costs \$19.14.

DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Use with **Session 4** **Picture It/Model It**

Levels 1–3: Reading/Speaking

To support Picture It and Model It, help students make sense of the adjectives *front, back, left, right, top, bottom, inner,* and *outer.* Display the adjectives and ask students to tell which terms have opposite meanings. Guide students to write each term on a sticky note. Invite volunteers to label classroom objects with the terms, such as the top and bottom of a desk or the inner left wall of a closet.

Next, help students make sense of the terms in Picture It by connecting to the labeled objects. Help students describe each face using these sentence frames and 1–2 adjectives:

- *This is the ___ face of the figure.*
- *It is also the ___ face of the figure.*

Levels 2–4: Reading/Speaking

Help students make sense of descriptive language in Picture It and Model It. Ask students to identify pairs of adjectives with opposite meanings, like *front* and *back, inner* and *outer.* Have students look at Picture It. Ask: *What helps us know each figure represents more than one face?* [Possible answer: the *-s* in *faces*] Guide students to identify faces described by a series of adjectives, and help students rephrase the descriptions using complete sentences. Give an example:

- *The figure represents both the left inner face and the right inner face.*

Next, help students connect Picture It and Model It by matching the descriptions of the figures with those of the expressions.

Levels 3–5: Reading/Speaking

To prepare for Picture It and Model It, review how students' work showed all the faces of the block. Ask students to show their work and describe the faces using complete sentences. Record any adjectives or other descriptive phrases in students' responses. Then have students connect the words they used to the descriptions in Picture It and Model It.

Have students read Model It sentence 1 and identify the adjectives, like *outside* and *inside.* Ask students to list other words to describe the surfaces. Next, ask partners to discuss how the expressions in Model It correspond to figures in Picture It. Ask: *How many faces does each figure represent? How do you know?*

Develop Solving Problems Involving Surface Area of Composite Figures

Purpose

- **Develop** strategies for solving real-world and mathematical problems involving surface area of composite figures.
- **Recognize** when you decompose a composite figure to find its surface area, you need to account for only the areas that are part of the surface of the original figure.

START CONNECT TO PRIOR KNOWLEDGE

Same and Different

Finding the surface area of . . .

A	B
a cube	a rectangular prism with two square faces
C	**D**
a rectangular prism with no square faces	a triangular prism

Possible Solutions

All are the sums of the areas of faces.

A is the sum of the areas of 6 identical faces.

A, B, and C are the sums of rectangular faces.

D is the sum of triangular and rectangular faces.

WHY? Support students' ability to evaluate the surface areas of prisms.

DEVELOP ACADEMIC LANGUAGE

WHY? Use precise language to talk about composite figures and surface area.

HOW? Ask students to list words and phrases that describe composite figures and surface areas. Discuss whether the words are more precise or more generic (e.g., precise: *right rectangular prism*, *height*; generic: *block, side*). Have partners sort precise and generic terms. Encourage them to use precise language in order to explain their ideas clearly during discussion.

TRY IT

SMP 1, 2, 4, 5, 6

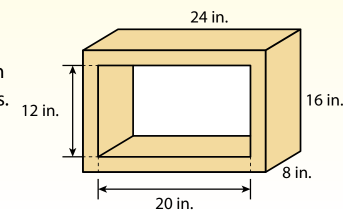
Make Sense of the Problem

See ***Connect to Culture*** to support student engagement. Before students work on Try It, use **Notice and Wonder** to help them make sense of the problem. Ask students to describe the shape of the scratching block in their own words.

Develop Solving Problems Involving Surface Area of Composite Figures

➤ **Read and try to solve the problem below.**

A company sells cardboard scratching blocks for cats. The block is shaped like a right rectangular prism with a rectangular hole cut through its center. A cat can scratch on any face of the block, including the bottom and inside faces. What is the total area of the block's scratching surface?



TRY IT Math Toolkit dot paper, geometric solids, grid paper, isometric dot paper

Possible work:

SAMPLE A

Outside Face	Area
Top/Bottom	$24 \cdot 8 = 192$
Right/Left	$8 \cdot 16 = 128$
Front/Back	$(24 \cdot 16) - (20 \cdot 12) = 144$

Inside Face	Area
Top/Bottom	$20 \cdot 8 = 160$
Right/Left	$8 \cdot 12 = 96$

$2(192 + 128 + 144 + 160 + 96) = 1{,}440$

The total area is 1,440 in.^{2}.

SAMPLE B

The front face is 2 in. thick.

Area of front/back face: $2(24 \cdot 2) + 2(2 \cdot 12) = 144$

Area of top/bottom face: $24 \cdot 8 = 192$

Area of right/left face: $8 \cdot 16 = 128$

Area of inside top/bottom face: $20 \cdot 8 = 160$

Area of inside right/left face: $8 \cdot 12 = 96$

Surface area: $2(144) + 2(192) + 2(128) + 2(160) + 2(96) = 1{,}440$

The scratching surface has a total surface area of 1,440 in.^{2}.

DISCUSS IT

Ask: How does your work show all the faces of the block?

Share: My work shows all the faces of the block by . . .

DISCUSS IT

SMP 2, 3, 6

Support Partner Discussion

After students work on Try It, have them respond to Discuss It with a partner. If students need support in getting started, prompt them to ask each other questions such as:

- *How did you count or identify all of the faces of the scratching block?*
- *How did you find the area of the rectangular face with the hole in it?*
- *How did you identify faces that have equal area?*

Common Misconception Listen for students who subtract the surface area of one rectangular prism from another rectangular prism. As students share their strategies, encourage them to explain how they accounted for the missing faces and sections from the prism.

Select and Sequence Student Strategies

Select 2–3 samples that represent the range of student thinking in your classroom. Here is one possible order for class discussion:

- drawing and finding the area of each face
- (common misconception) finding the area of the total rectangular prism and subtracting the area of the smaller rectangular prism
- applying symmetry to identify identical faces and multiply by 2 when appropriate

Facilitate Whole Class Discussion

Call on students to share selected strategies. Before they present, remind students that one way to justify a solution is to tell why it is reasonable, given the problem context.

Guide students to **Compare and Connect** the representations. If time is short, have students turn and talk to critique or evaluate their strategies.

ASK *How do the strategies of [student name] and [student name] both make sure that all faces of the block are included in the surface area?*

LISTEN FOR The strategies classify the faces according to their position, including outside and inside the block.

Picture It & Model It

If students presented these models, have students connect these models to those presented in class.

If no student presented at least one of these models, have students first analyze key features of the models and then connect them to the models presented in class.

ASK *How are the pictures related to the expressions?*

LISTEN FOR The pictures show the dimensions of each face and help classify them to write an expression for the area of each.

For the pictures, prompt students to notice the labels that locate each face on the block.

- *Why is it useful to identify the location of each face in the block?*

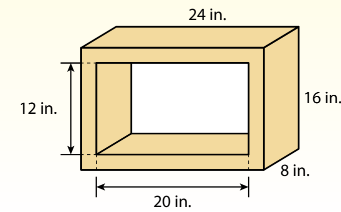
For the expressions, prompt students to relate each expression to the face it represents.

- *Why do all of the expressions include a factor of 2?*
- *Why are the areas of the front and back faces expressed as a difference?*

LESSON 25 | SESSION 4

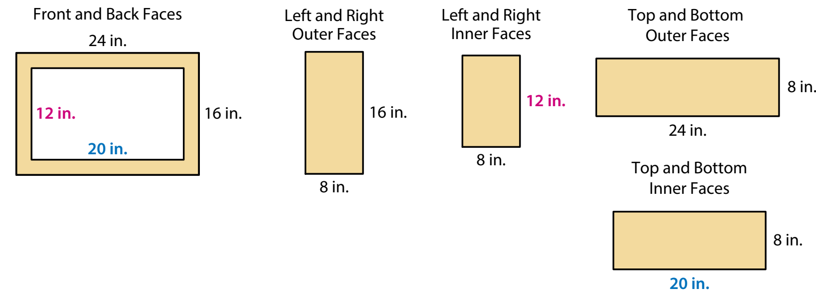
➤ **Explore different ways to solve problems involving surface area of composite figures.**

A company sells cardboard scratching blocks for cats. The block is shaped like a right rectangular prism with a rectangular hole cut through its center. A cat can scratch on any face of the block, including the bottom and inside faces. What is the total area of the block's scratching surface?



Picture It

You can draw the faces of a figure with their dimensions.

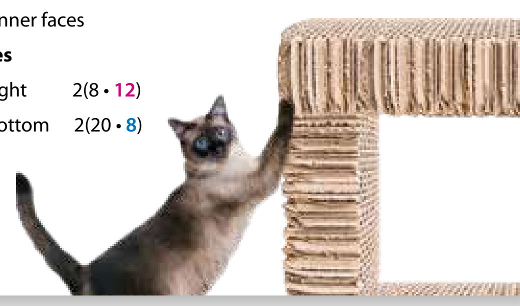


Model It

You can think of the total surface area as the area of the outside surfaces plus the area of the inside surfaces.

Total surface area = area of outer faces + area of inner faces

Outer Faces		**Inner Faces**	
Left and right	$2(8 \cdot 16)$	Left and right	$2(8 \cdot 12)$
Top and bottom	$2(24 \cdot 8)$	Top and bottom	$2(20 \cdot 8)$
Front and back	$2[(24 \cdot 16) - (20 \cdot 12)]$		



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DIFFERENTIATION | EXTEND

SMP 7

Deepen Understanding

Making Use of Structure to Find the Surface Area of Composite Figures

Prompt students to think about how the structure of a composite figure impacts its surface area by visualizing how changes in the figure impact its surface area.

ASK *Suppose the scratching block did not have a rectangular prism cut out. How would that change the area of the scratching surface?*

LISTEN FOR The surface area would decrease. Even though the area of the front and back faces increase, you lose all 4 inner faces, which have a greater total area.

ASK *Suppose that you attached a block the size of the cut-out on top of the block instead of cutting it out. How would that change the total area of the scratching surface?*

LISTEN FOR There are 5 additional faces instead of 4, and less area is lost to the overlapping surfaces than the cut-out surfaces, so the total surface area would increase.

ASK *How can you use the structure of a composite figure to help you find its surface area?*

LISTEN FOR You use the structure of a figure when deciding whether to add or subtract areas and when multiplying the area of a face by the number of identical faces.

Develop Solving Problems Involving Surface Area of Composite Figures

CONNECT IT

SMP 2, 4, 5, 6

Remind students that the quantities and the relationships between them are the same in each representation. Explain that they will now use those relationships to reason about the surface area of composite figures.

Before students begin to record and expand on their work in Picture It & Model It, tell them that problem 3 will prepare them to provide the explanation asked for in problem 4.

Monitor and Confirm Understanding 1 – 2

- The area of the front and back faces is the difference between the areas of two rectangles.
- Without the hole, the block becomes a right rectangular prism of length 24 in., width 8 in., and height 16 in. Its surface area would be 1,408 in.^{2}.

Facilitate Whole Class Discussion

3 Look for understanding that the cut-out rectangular prism fills the hole that extends between the front and back of the block.

> **ASK** *How can Felipe's strategy be modified to make it accurate?*
>
> **LISTEN FOR** After you add the two areas, subtract the area of the front and back of the smaller prism twice, once for each prism.

4 Look for the idea that the general strategy for finding surface area can always be applied, but that figures with holes may require more attention to precision and details.

> **ASK** *When a hole is cut out of a rectangular prism, how does the surface area change?*
>
> **LISTEN FOR** The hole adds surface area inside the prism and removes area from two outer faces.

5 **Reflect** Have all students focus on the strategies used to solve the Try It. If time allows, have students discuss their ideas with a partner.

CONNECT IT

➤ **Use the problem from the previous page to help you understand how to solve problems involving surface area of composite figures.**

1 Look at the drawing of the front and back faces in **Picture It**. How does **Model It** show how to find their area?

Model It shows that you can find the area of the outside rectangle and subtract the area of the inside rectangle. Then you have just the area for the border.

2 What is the total area of the block's scratching surface? Would the surface area of the block be greater or less if it did not have a hole? Explain.

1,440 in.^{2}; less; Without the hole, the area of the front and back faces would be greater, but there would be no inner faces, so the total area would be less.

3 You can think of the block as a rectangular prism that had a smaller rectangular prism cut out of it. Felipe claims that you can add the surface areas of the two prisms to find the total surface area of the block. Why is Felipe incorrect?

Felipe is incorrect because you would count the parts that got cut out of the front and the back twice. You do not want to include those parts.

4 How is finding the surface area of a three-dimensional figure with a hole in it like finding the surface area of a solid three-dimensional figure? How is it different?

For both types of figures, you find the surface area by adding the areas of all faces on the surface of the figure. When a figure has a hole, you need to include the areas of the faces inside the hole.

5 **Reflect** Think about all the models and strategies you have discussed today. Describe how one of them helped you better understand how to solve the **Try It** problem.

Responses will vary. Check student responses.

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DIFFERENTIATION | RETEACH or REINFORCE

Hands-On Activity

Use a concrete model to represent a composite figure.

If students are unsure how to find the surface areas of composite figures, then have them work with a concrete model.

Materials For each pair: 25 unit cubes (9 of one color, 16 of another color)

- Have pairs arrange the cubes into a 1-by-5-by-5 right square prism, with the 9 cubes of one color in the center. Then have students remove the 9-cube inner prism.
- Ask: *How are the surface areas of the prism you removed and the resulting figure related?* [The area of the side faces equal the areas of the inner faces, and the area of the top and bottom faces equal the cut-out areas.]
- Have students discuss two methods to find the surface area of the figure, then use one to find the surface area. [Possible methods: Find the sum of the areas of each of the 10 faces or find the surface area of the original rectangular prism then add and subtract areas of faces of the removed prism as appropriate; 64 units^2]
- If time allows, repeat the activity with a 1-by-2-by-4 prism removed from center of a 1-by-4-by-6 prism. [64 units^2]

Apply It

For all problems, encourage students to use models to support their thinking. Allow some leeway in precision; for example, the dimensions stated on a model need not be exactly proportional in the student's sketch.

6 Students may recognize that each of the two side walls of the shed have the shape of an irregular pentagon and that the shape can be decomposed into a rectangle and a right triangle. Students should also include the areas of the front and back walls and recognize that only the front wall has a door and windows.

7 **C is correct.** Students may solve the problem by finding the sum of the surface areas of both rectangular prisms and then subtracting 2 times the area of one face of the cube.

A is not correct. This answer may be the result of subtracting the overlapping area once instead of twice.

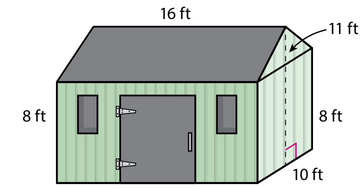
B is not correct. This answer may be the result of counting an extra 36 in.^{2}, or the area of one face of the cube, in the surface area.

D is not correct. This answer is the sum of the surface areas of the two prisms.

Apply It

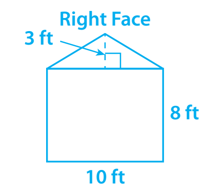
➤ **Use what you learned to solve these problems.**

6 A garden shed is shaped like a right prism. The shed has one 7 ft-by-6 ft rectangular door and two 3 ft-by-1$\frac{1}{2}$ ft rectangular windows. The outer walls will be painted, but not the roof, door, or windows. What is the total surface area to be painted? Show your work.



Possible work:

Area of right face: $(10 \cdot 8) + \frac{1}{2}(10 \cdot 3) = 95$

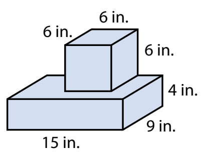


Area to be painted:

$2(95) + 2(16 \cdot 8) - (7 \cdot 6) - 2\left(3 \cdot 1\frac{1}{2}\right) = 395$

SOLUTION The area to be painted is 395 ft^2.

7 The three-dimensional figure shown is composed of right prisms. What is the total surface area of the figure?



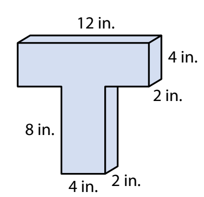
A 510 in.^{2}

B 642 in.^{2}

C 606 in.^{2}

D 678 in.^{2}

8 Tyrone makes a wooden letter T to hang on his bedroom wall. The T is a right prism. Tyrone plans to cover all the faces except the back with fabric. How many square inches of fabric will he need? Show your work.



Possible work:

Area from top prism:

$(12 \cdot 4) + 2(2 \cdot 4) + 12(2 \cdot 2) - (4 \cdot 2) = 104$

Area from bottom prism:

$(4 \cdot 2) + (4 \cdot 8) + 2(2 \cdot 8) = 72$

Total area: $104 + 72 = 176$

SOLUTION Tyrone needs 176 in.^{2} of fabric.

CLOSE **EXIT TICKET**

8 Students' solutions should show an understanding of:

- identifying the faces of a composite figure.
- finding the surface area as the sum of the areas of its faces, subtracting for overlapping areas.

Error Alert If students decompose the composite figure into two rectangular prisms but do not account for the area of the overlap, then form a T-shape with two geometric solids shaped like right rectangular prisms. Ask students to describe how the surface area changes when the T-shape is formed.

Practice Solving Problems Involving Surface Area of Composite Figures

Problem Notes

Assign **Practice Solving Problems Involving Surface Area of Composite Figures** as extra practice in class or as homework.

1 **a.** Students should understand that the area where the two shapes meet is not included in the surface area of the composite figure, so it must be subtracted from the surface area once for each face. ***Basic***

b. Students may draw a model or show calculations to explain. ***Challenge***

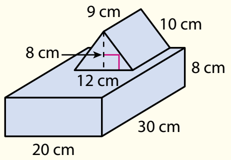
LESSON 25 | SESSION 4 Name:

Practice Solving Problems Involving Surface Area of Composite Figures

➤ **Study the Example showing how to solve problems involving surface area of composite figures. Then solve problems 1–4.**

Example

The three-dimensional figure shown is composed of right prisms. What is the total surface area of the figure?



You can think of the figure as a right triangular prism on top of a right rectangular prism.

Triangular prism: $2\left(\frac{1}{2}\right)(12 \cdot 8) + 2(10 \cdot 9) + (12 \cdot 9) = 384$

Rectangular prism: $2(30 \cdot 20) + 2(20 \cdot 8) + 2(30 \cdot 8) = 2{,}000$

The bottom face of the triangular prism and part of the top face of the rectangular prism are not part of the surface of the composite figure. Subtract the areas of these parts.

$384 + 2{,}000 - (12 \cdot 9) - (12 \cdot 9) = 2{,}168$

The total surface area of the figure is 2,168 cm^2.

1 **a.** Why is $(12 \cdot 9)$ subtracted twice in the expression for the surface area of the figure from the Example?

The product represents the area of the bottom face of the triangular prism, which covers part of the top face of the rectangular prism. Since the area was included in the total surface areas of both prisms, it must be subtracted twice.

b. Consider the figure in the Example. Suppose the triangular prism was moved so that a triangular base touches the rectangular prism instead. Would the total surface area of the figure increase or decrease? Explain your reasoning.

increase; Possible explanation: Since the triangular base has a smaller area than the 12 cm-by-9 cm face, a smaller area is subtracted from the total surface area of each prism.

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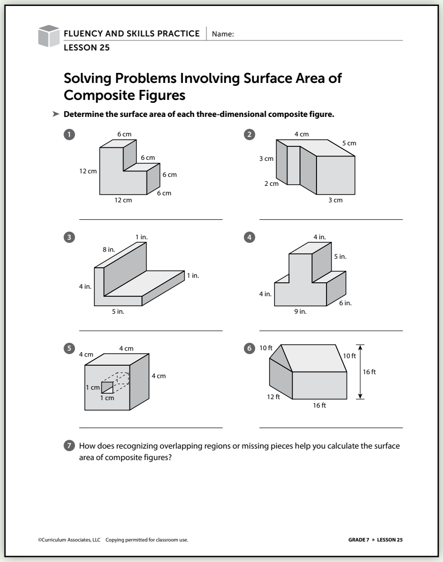
Fluency & Skills Practice

Solving Problems Involving Surface Area of Composite Figures

In this activity, students determine the surface area of three-dimensional composite figures.



Additional Practice

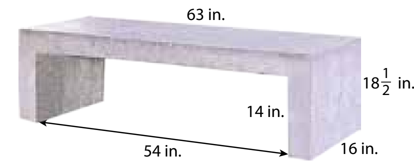
2 Students may find the sum or difference of the areas of the faces of two rectangular prisms. ***Medium***

3 Students may decompose the composite figure into three rectangular prisms, find the surface area as the sum of the surface areas of the smaller prisms, and subtract the overlapping areas. ***Medium***

4 Students may model the box as a large rectangular prism from which a smaller prism is removed. They may find the surface area as the sum of the surface areas of the two prisms and subtract the area of the top face of the smaller prism twice. ***Challenge***

LESSON 25 | SESSION 4

2 All faces of a bench except the two faces that rest on the ground will be coated with a water-resistant paint. The bench is a right prism. What is the total area to be coated with the paint? Show your work.



Possible work:

Top area: $63 \cdot 16 = 1{,}008$

Left and right areas: $16 \cdot 18\frac{1}{2} = 296$

Top inside area: $54 \cdot 16 = 864$

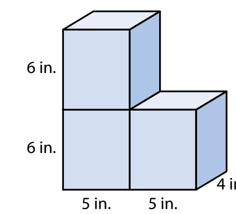
Left and right inside areas: $16 \cdot 14 = 224$

Front and back areas: $\left(63 \cdot 18\frac{1}{2}\right) - (54 \cdot 14) = 409\frac{1}{2}$

$1{,}008 + 2(296) + 2\left(409\frac{1}{2}\right) + 864 + 2(224) = 3{,}731$

SOLUTION The total area to be coated is 3,731 in.^{2}.

3 All sides of the three-dimensional figure shown meet at right angles. What is the surface area of the three-dimensional figure? Show your work.



Possible work:

Front and back areas: $3(5 \cdot 6) = 90$

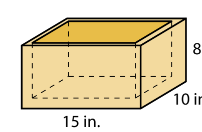
Right and left areas: $2(4 \cdot 6) = 48$

Top and bottom areas: $2(5 \cdot 4) = 40$

$2(90) + 2(48) + 2(40) = 356$

SOLUTION The surface area is 356 in.^{2}.

4 Indira makes a wooden box without a lid. All the faces of the box meet at right angles. Indira plans to paint all surfaces of the box, including the inside and outside. The interior of the box has length 13 in., width 8 in., and depth 7 in. Indira can cover up to 1,350 in.^{2} with $\frac{1}{2}$ cup of paint. Will she need more than $\frac{1}{2}$ cup of paint to cover the box? Explain how you know.



No; Possible explanation: The surface area of the outside of the box is $2(15 \cdot 10) + 2(15 \cdot 8) + 2(10 \cdot 8) - (13 \cdot 8)$, or 596 in.^{2}. The surface area of the inside is $(13 \cdot 8) + 2(13 \cdot 7) + 2(8 \cdot 7)$, or 398 in.^{2}. The total surface area is $596 + 398$, or 994 in.^{2}, which is less than 1,350 in.^{2}.

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DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Use with **Session 5** **Apply It**

Levels 1–3: Reading/Writing

Support Apply It problem 4 by summarizing key descriptions and quantities. Guide students to add the details to the figure.

Next, help students share their reasoning about Ju-long's estimate and then write explanations. Ask students to estimate the weight of the gravel using sketches or models. Then help students explain if their estimate is more or less than 2,000 lb. Suggest phrases to support written explanations such as *I know that . . . , One reason is . . . , This means that . . . .*

Next, adapt **Stronger and Clearer Each Time** to have students read responses to partners and then work together to revise written sentences.

Levels 2–4: Reading/Writing

Help students make sense of Apply It problem 4 using a **Co-Constructed Word Bank**. Review the terms with students and then ask them to tell how the problem relates to the figure. Guide students to add in more details that can help them estimate the weight of the gravel.

Next, have students evaluate Ju-long's estimate and write an explanation of their reasoning. Use **Stronger and Clearer Each Time** to help students write their reasoning clearly. Suggest that students tell what they notice or assumed about the problem, what they decided to do as a result, and why.

After each round of partner discussion, have students review and refine their writing.

Levels 3–5: Reading/Writing

Have students make sense of Apply It problem 4 through partner discussion. Ask partners to identify the key descriptions and quantities in the problem and to discuss how the figure represents the information. Suggest that students add any details to the figure that may help them estimate the weight of the gravel.

Next, ask students to evaluate Ju-long's estimate and write an explanation of their reasoning. Remind students to justify their evaluation by writing one or more reasons it makes sense. Then have students use **Stronger and Clearer Each Time** to discuss their ideas with parters and revise their writing.

Refine Solving Problems Involving Area and Surface Area

Purpose

- **Refine** strategies for solving problems involving area and surface area.
- **Refine** understanding of decomposing and composing figures to find linear dimensions, area, and surface area.

START CHECK FOR UNDERSTANDING

A figure is made by gluing one face of cube with edge length 7 cm to a face of cube with edge length 8 cm. What is the surface area of the figure?

Solution

580 cm^2

WHY? Confirm students' ability to identify a composite figure from a verbal description and find its surface area.

MONITOR & GUIDE

Before students begin to work, use their responses to the **Start** to determine those who will benefit from additional support. Use the **Error Analysis** table below to guide remediation.

Have all students complete the Example and problems 1–3, using Consider This and Pair/Share as appropriate. Observe and monitor their reasoning and guide or redirect students as needed.

LESSON 25 | SESSION 5

Refine Solving Problems Involving Area and Surface Area

➤ **Complete the Example below. Then solve problems 1–9.**

Example

The four walls in a classroom need to be painted. The room is 27 ft long, 32 ft wide, and 10 ft high. The room has two windows, each with width 3 ft 9 in. and height 2 ft 1 in. The door has width 3 ft and height 6 ft 10 in. One gallon can of paint covers about 400 ft^2. Estimate the number of gallon cans of paint needed for the walls.

Look at how you could show your work using rounding.

Round dimensions to the nearest foot and find each area.

Area of the walls: $2(27 \cdot 10) + 2(32 \cdot 10) = 1{,}180$

Area of the door: $2(4 \cdot 2) = 16$

Area of the windows: $(3 \cdot 7) = 21$

Area of walls minus area of doors and windows:

$1{,}180 - 16 - 21 = 1{,}143$

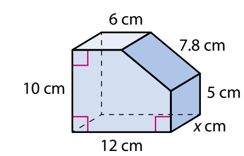
SOLUTION About 3 gallon cans of paint are needed.

CONSIDER THIS . . .
There are 12 inches in 1 foot.

PAIR/SHARE
How would your estimate change if the ceiling of the classroom also needed to be painted? Why?

Apply It

1 The right prism has surface area 536.4 cm^2. What is the value of x? Show your work.



Possible work:

Front and back areas:

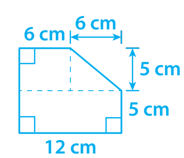
$(12 \cdot 5) + (6 \cdot 5) + \frac{1}{2}(6 \cdot 5) = 105$

$2(105) + 6x + 7.8x + 5x + 12x + 10x = 536.4$

$210 + 40.8x = 536.4$

$40.8x = 326.4$

$x = 8$



SOLUTION The value of x is 8.

CONSIDER THIS . . .
The front face can be decomposed into two rectangles and a triangle.

PAIR/SHARE
How can you check that you accounted for all the faces of the prism?

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START ERROR ANALYSIS

If the error is . . .	Students may . . .	To support understanding . . .
678 cm^2	think that the surface area of a composite figure is equal to the sum of the surface areas of its component figures.	Have students model the figure described with geometric solids, then relate the surface areas of the separated cubes to the surface area of the composite figure.
550 cm^2	have found the sum of the surface areas of the two cubes, and then subtracted twice the area of a face of the large cube instead of a face of the smaller cube.	Ask students to draw a picture to visualize the composite figure and then to identify the area that overlaps between the two cubes. Have them relate the overlap area to the strategy they used for finding surface area.
629 cm^2	have added the surface areas of the two cubes, and then subtracted area of a face of the small cube only one time, instead of subtracting the area of the face two times.	Ask students to explain their strategy and to relate each step of their calculations to one of the faces of the composite figure. Ask students to explain how they accounted for the overlapping area where the two cubes are glued together.

Example

Guide students in understanding the Example. Ask:

- *Why is it useful and reasonable to round the dimensions to the nearest foot?*
- *How is finding the area to be painted similar to finding the surface area of a rectangular prism? How is it different?*
- *How could you check the solution?*

Help all students focus on the Example and responses to the questions by asking them to agree, disagree, or add on to classmates' answers.

Look for understanding that the area of the classroom to be painted can be modeled as a composite shape, formed by large rectangles (representing the walls) with smaller rectangles removed (representing the windows and doors).

Apply It

1 Students should recognize that the front and back faces are pentagons, while the other 5 faces are each rectangles. They may decompose the pentagon as the sum of two rectangles and a triangle or the difference between the surrounding rectangle and a triangle. ***DOK 2***

2 Students may model the display shelf as a large rectangular prism from which a smaller prism has been removed. They may find the area to be stained as the difference between the sum of the surface area of each prism and the removed front and back faces. ***DOK 2***

3 **C is correct.** Students should recognize that the area of the composite shape is the difference between the areas of the surrounding rectangle and a smaller rectangle removed from one corner.

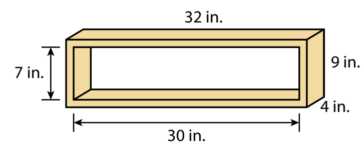
A is not correct. This answer subtracts just the height, 5, instead of the area of the small rectangle, $5x$, from the area of the surrounding rectangle.

B is not correct. This answer adds the area of the small rectangle to the area of the surrounding rectangle.

D is not correct. This answer ignores the rectangle removed from the surrounding rectangle.

DOK 3

2 Ignacio makes a display shelf from 4 wooden boards. All angles formed by the boards are right angles. Ignacio plans to stain all faces of the shelf, except the back face, which will be against the wall. What is the total area Ignacio will stain? Show your work.



Possible work:

Front area: $(32 \cdot 9) - (30 \cdot 7) = 78$

Left and right areas: $4 \cdot 9 = 36$

Top and bottom areas: $32 \cdot 4 = 128$

Left and right inside areas: $7 \cdot 4 = 28$

Top and bottom inside areas: $30 \cdot 4 = 120$

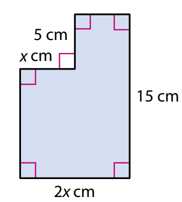
$78 + 2(36) + 2(128) + 2(28) + 2(120) = 702$

SOLUTION The total area Ignacio will stain is 702 in.^{2}.

CONSIDER THIS . . .
The shelf is shaped like a right rectangular prism with a rectangular hole through its center.

PAIR/SHARE
What is a different way you could find the total area to be stained?

3 The figure at the right has area 125 cm^2. Which equation can be used to find the value of x?



A $125 = 30x - 5$

B $35x = 125$

C $25x = 125$

D $125 \div 15 = 2x$

Josephine chose B as the correct answer. How might she have gotten that answer?

Possible answer: She might have drawn an enclosing rectangle with area $15(2x)$, and then added the $5x$ rectangle in the upper left corner instead of subtracting it.

CONSIDER THIS . . .
You do not need to know the value of x to solve the problem.

PAIR/SHARE
Did you find the lengths of any unknown sides? Which ones?

GROUP & DIFFERENTIATE

Identify groupings for differentiation based on the **Start** and problems 1–3. A recommended sequence of activities for each group is suggested below. Use the resources on the next page to differentiate and close the lesson.

Approaching Proficiency

- **RETEACH** Visual Model
- **REINFORCE** Problems 5, 7, 8

Meeting Proficiency

- **REINFORCE** Problems 4–8

Extending Beyond Proficiency

- **REINFORCE** Problems 4–8
- **EXTEND** Challenge

Have all students complete the **Close: Exit Ticket**.

Resources for Differentiation are found on the next page.

Refine Solving Problems Involving Area and Surface Area

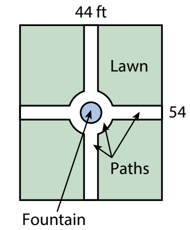
Apply It

4 See ***Connect to Culture*** to support student engagement. Students should recognize that they do not need to find the total area of the paths precisely. They may stop when they can conclude that the weight of the gravel will exceed 2,000 pounds. ***DOK 3***

5 Students should recognize that the exact position of the smaller cube over the larger cube is not specified in the diagram, but that does not affect the surface area. ***DOK 2***

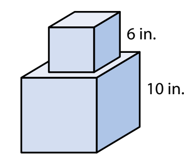
6 Students may also represent the area as the sum of the areas of three rectangles and one triangle. ***DOK 3***

4 The center of a rectangular courtyard has a circular fountain with radius 3 ft. All paths in the courtyard are 4 ft wide. Each path will be covered with gravel. The gravel needed to cover 1 ft^2 weighs about 15 lb. Ju-long estimates that the gravel needed to cover the paths will weigh less than 2,000 lb. Is Ju-long's estimate reasonable? Explain.



No; Possible explanation: The gravel for the circular path weighs about $15(3.14 \times 7^2 - 3.14 \times 3^2) = 1{,}884$ lb, which is close to 2,000 lb. Once the weights for the other paths are added in, the total weight will be much greater than 2,000 lb.

5 The prism shown is made of two cubes. What is its total surface area? Show your work.



Possible work:

Total S. A.: large cube S. A. + small cube S. A. − 2(overlap)

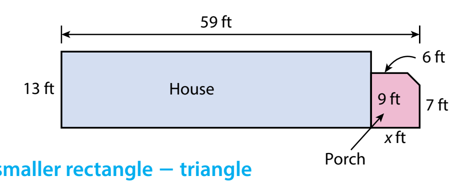
$6(10 \cdot 10) + 6(6 \cdot 6) - 2(6 \cdot 6)$

$600 + 4(6 \cdot 6)$

$600 + 144 = 744$

SOLUTION The total surface area of the prism is 744 $in.^2$.

6 The diagram shows a plan for a rectangular house with an attached porch. The combined area of the house and the porch is 733 ft^2. What is the value of x? Show your work.



Possible work:

Combined area = enclosing rectangle − smaller rectangle − triangle

$733 = 59(13) - x(13 - 9) - 0.5(9 - 7)(x - 6)$

$733 = 767 - 4x - 0.5(2)(x - 6)$

$733 = 767 - 4x - x + 6$

$733 = 773 - 5x$

$-40 = -5x$

$x = 8$

SOLUTION The value of x is 8.

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DIFFERENTIATION

RETEACH

Visual Model

Use concrete models to find surface area.

Students approaching proficiency with finding the surface area of composite figures will benefit from constructing and analyzing concrete models.

Materials For display: 10 unit cubes

- Display a cube. Ask: *Each face has area 1 cm^2. What is the cube's surface area?* [6 cm^2]
- Display a second cube. Ask: *What is the combined surface area of the two cubes?* [12 cm^2]
- Stack the cubes. Ask: *Is the total surface area still 12 cm^2? Explain.* [No, it is only 10 cm^2 because the two touching faces are no longer on the surface.]
- Stack a third cube. Ask: *What is the surface area?* [14 cm^2] Rearrange the cubes into an L. Ask: *Why does changing the shape of the figure not change the total surface area?* [There are still 14 faces on the surface and 4 faces touching.]
- Display a fourth cube. Compare the surface areas of a 1-by-1-by-4 and a 1-by-2-by-2 rectangular prism. [18 cm^2; 16 cm^2] Ask: *Why does the surface area change even though the number of cubes stays the same?* [The number of touching faces changes.]
- Guide students to consider strategies they can use to find the surface area of cubes that touch, such as counting the number of faces on the surface or subtracting the number of touching faces from the total number of faces, which is 6 times the number of cubes.
- Extend the activity by having students apply their strategies to different configurations of 5 or more cubes. Ask students to predict whether formations will increase, decrease, or maintain the surface area and justify their claims.

7 Students may reason that the length and width do not change, while the height increases by 2.4 in. as each prism is added. *DOK 2*

8 **B is correct.** Students may decompose the surface of the figure into triangles and rectangles to solve for *x*.

A is not correct. This answer includes the area of only one base.

C is not correct. This answer confuses volume and surface area.

D is not correct. This answer may come from treating both the front and back faces as squares with side length 5 cm.

DOK 3

CLOSE EXIT TICKET

9 **Math Journal** Look for understanding that the sum of the surface areas includes 2 faces that are no longer on the surface.

Error Alert If students think Naomi is correct, then ask them to relate the surface area to each exterior face of the composite figure.

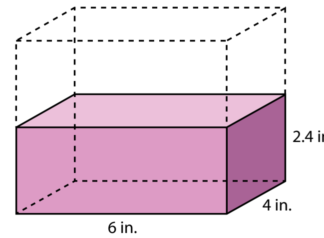
End of Lesson Checklist

INTERACTIVE GLOSSARY Support students by suggesting that they look back at Session 4 Connect It problem 3 to see how the word *claim* is used.

SELF CHECK Have students review and check off any new skills on the Unit 6 Opener.

LESSON 25 | SESSION 5

7 Kadeem stacks right rectangular prisms like the one shown. He aligns each prism on top of the previous prism to make a larger prism. The larger prism has surface area 288 in.^{2}. How many prisms does Kadeem stack? Show your work.



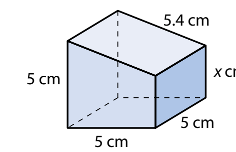
Possible work:

n = number of prisms in stack

$$2(6)(4) + 2(6)(2.4n) + 2(4)(2.4n) = 288$$
$$48 + 28.8n + 19.2n = 288$$
$$48 + 48n = 288$$
$$48n = 240$$
$$n = 5$$

SOLUTION Kadeem stacks 5 prisms.

8 The right prism has surface area 132 cm^2. What is *x*?



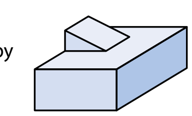
A $5\frac{2}{3}$

B 3

C $1.\overline{296}$

D 1

9 **Math Journal** The figure shown is a right triangular prism on top of a right rectangular prism. Naomi claims that she can find the surface area of the figure by adding the surface areas of the two prisms and then subtracting the area of the bottom face of the triangular prism. Is Naomi correct? Explain.



No; Possible explanation: Naomi also needs to subtract the area of the top face of the rectangular prism that is covered by the triangular prism.

End of Lesson Checklist

- [ ] **INTERACTIVE GLOSSARY** Write a new entry for *claim*. Tell what you do when you *claim* something about a three-dimensional figure.
- [ ] **SELF CHECK** Go back to the Unit 6 Opener and see what you can check off.

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REINFORCE

Problems 4–8

Solve area and surface area problems.

Students meeting proficiency will benefit from additional work with writing, solving, and interpreting inequalities by solving problems in a variety of formats.

- Have students work on their own or with a partner to solve the problems.
- Encourage students to show their work.

EXTEND

Challenge

Describe composite figures with a given surface area.

Students extending beyond proficiency will benefit from solving challenging problems with partners.

- Have partners work together to describe a composite figure that has a surface area of 100 cm^2.
- As time allows, challenge partners to come up with several different examples, such as two rectangular prisms joined together or a large prism with a hole in the shape of a small prism.

PERSONALIZE

i-Ready

Provide students with opportunities to work on their personalized instruction path with *i-Ready* Online Instruction to:

- fill prerequisite gaps.
- build up grade-level skills.