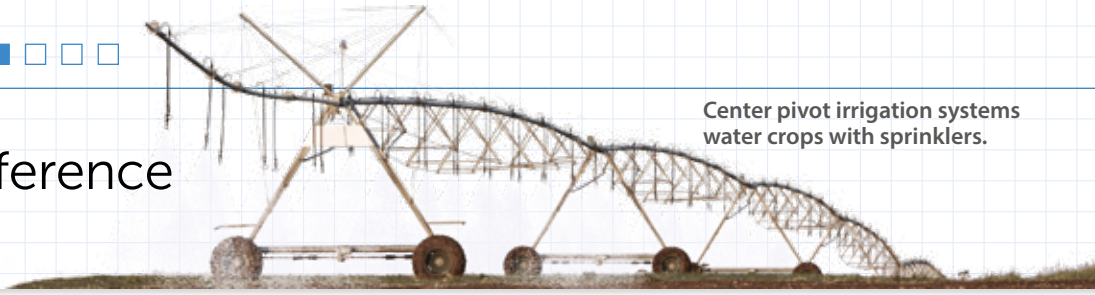


Center pivot irrigation systems water crops with sprinklers.

Explore Circumference of a Circle

Previously, you learned about area and perimeter of polygons.
In this lesson, you will learn about the area and circumference of circles.

➤ **Use what you know to try to solve the problem below.**

Hai orders a center pivot irrigation system to water his fields. The system is made up of a line of connected pipes that turn around a center point. Hai's system will be 1,320 feet in length. What is the shape of the space Hai's system will water? What is the longest distance across the watered space?

TRY IT

Math Toolkit grid paper, string

DISCUSS IT

Ask: How is your strategy similar to mine? How is it different?

Share: My strategy is similar to yours . . . It is different . . .

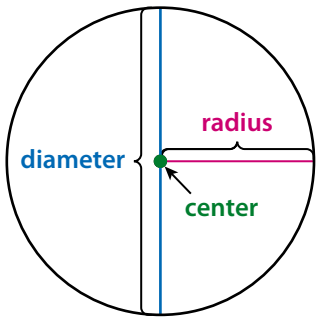
Learning Target **SMP 1, SMP 2, SMP 3, SMP 4, SMP 5, SMP 6, SMP 7**

Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.

CONNECT IT

1 Look Back What is the shape of the space Hai's system will water? What is the longest distance across the space? How do you know?

2 Look Ahead Hai's irrigation system waters in the shape of a **circle**. Every point on the edge of a circle is the same distance from the **center**. The **radius** of Hai's system is 1,320 feet, so the **diameter** is 2,640 feet.



a. The radius of a circle is the distance from the edge to the center. The diameter is the distance across a circle through the center. What is the relationship between the radius and the diameter of any circle?

b. You can draw more than one diameter on a circle. Why?

c. Suppose two different diameters are drawn on a circle. Explain how you can use these diameters to find the center of the circle.

d. The distance around a circle is called the **circumference**. Trace the circumference of the circle. How is the circumference of a circle like the perimeter of a square?

3 Reflect You can draw more than one radius on a circle. What must be true about all of the radii? (*Radii* is the plural of *radius*.)

 Name:

Prepare for Solving Circumference and Area Problems Involving Circles

1. Think about what you know about the area of two-dimensional figures. Fill in each box. Use words, numbers, and pictures. Show as many ideas as you can.

What Is It?

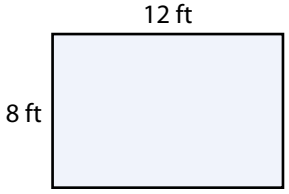
What I Know About It

area

Examples

Examples

2. Is the area of the rectangle 40 feet or 96 square feet? Explain.



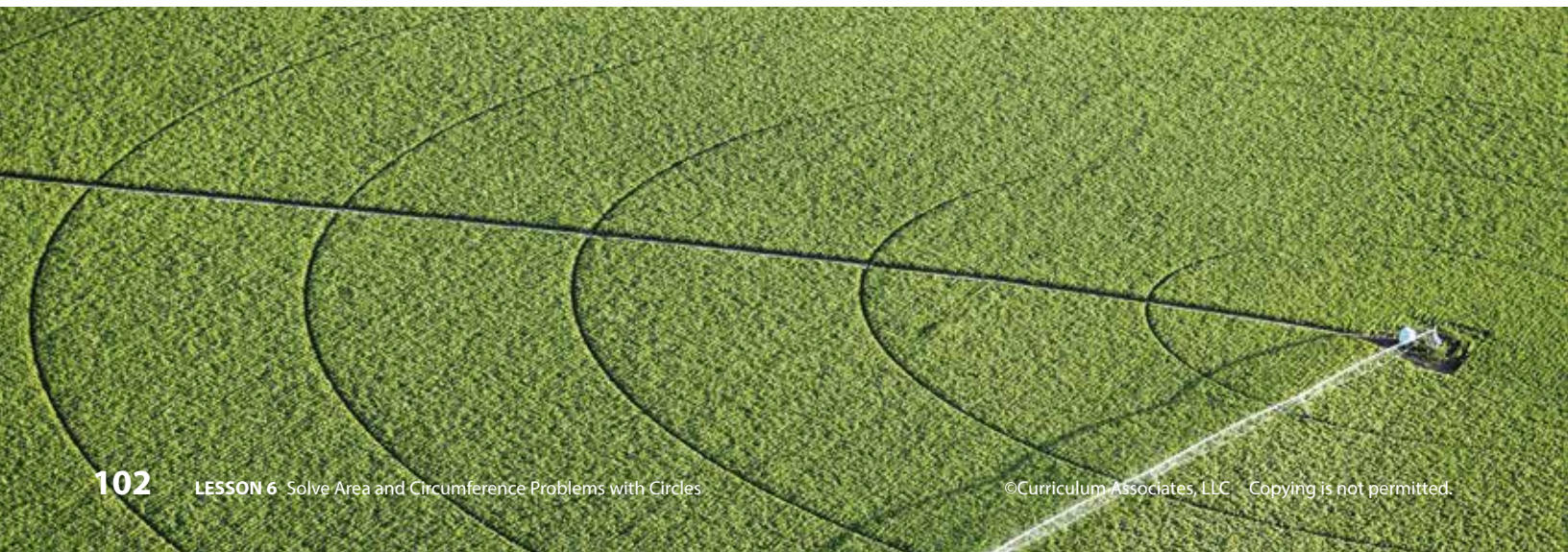
3 Brian orders a center pivot irrigation system to water his fields. The system is made up of a line of connected pipes that turn around a center point. Brian's system will be 1,150 ft long.

a. What is the longest distance across the space the system will water? Show your work.

SOLUTION _____

b. Check your answer to problem 3a. Show your work.

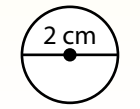
center pivot irrigation system



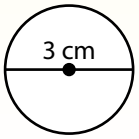
Develop Using the Relationship Between a Circle's Circumference and Diameter

➤ **Read and try to solve the problem below.**

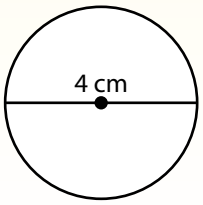
Look at the circumference of each of the circles below. What do you think would be the circumference of a circle with diameter 1 cm?



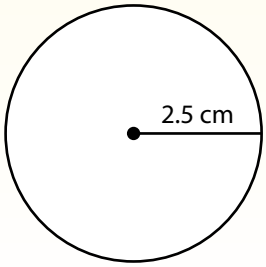
$C \approx 6.28$ cm



$C \approx 9.42$ cm



$C \approx 12.56$ cm



$C \approx 15.70$ cm

TRY IT

Math Toolkit compasses, flexible tape measures, rulers, string

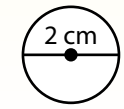
DISCUSS IT

Ask: Why did you choose that strategy to find the circumference of a circle with diameter 1 cm?

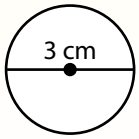
Share: I chose to use . . . because . . .

Explore different ways to investigate the relationship between the circumference and diameter of a circle.

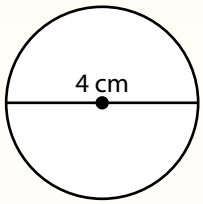
Look at the circumference of each of the circles below. What do you think would be the circumference of a circle with diameter 1 cm?



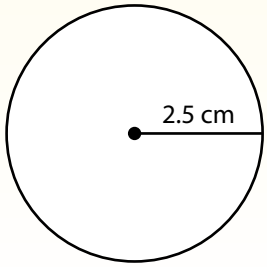
$C \approx 6.28$ cm



$C \approx 9.42$ cm



$C \approx 12.56$ cm

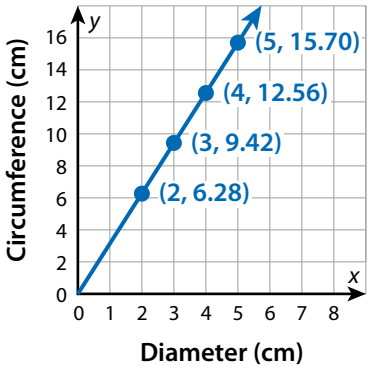


$C \approx 15.70$ cm

Model It

You can make a graph to look at the relationship between circumference and diameter for each circle.

The *x*-axis shows the diameter. The *y*-axis shows the approximate circumference.



Model It

You can look for a constant of proportionality in the relationship between the approximate circumference of each circle and the diameter.

Divide the circumference of each circle by its diameter.

$\frac{6.28}{2} = 3.14$ $\frac{9.42}{3} = 3.14$ $\frac{12.56}{4} = 3.14$ $\frac{15.70}{5} = 3.14$

$r = 2.5$ cm, so $d = 5$ cm.

The constant of proportionality is 3.14.

CONNECT IT

➤ **Use the problem from the previous page to help you understand how to find the circumference of a circle.**

1. Each circumference is given using the ≈ symbol. What does that mean about the circumferences? What is the approximate circumference of a circle with diameter 1 cm?

2. Look at the first **Model It**. Why is the point (5, 15.70) on the graph?

3. Look at the second **Model It**. What does the constant of proportionality tell you about the relationship between the circumference and diameter of a circle?

4. The quotient $\frac{\text{circumference}}{\text{diameter}}$ is called **pi (π)**. π represents a decimal that goes on forever without repeating. Use π to write a formula for the circumference of a circle, C, when you know the diameter, d.

5. An exact circumference uses π. To find an approximate circumference you can use 3.14 or $\frac{22}{7}$ as a value for π. What is the exact circumference for a circle with diameter 6 cm? What is the approximate circumference?

6. **Reflect** Think about all the models and strategies you have discussed today. Describe how one of them helped you better understand how to find the circumference of a circle.

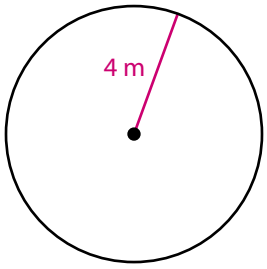
Apply It

Use what you learned to solve these problems.

7 The circumference of a circle is 12π feet. What is the diameter? Show your work.

SOLUTION _____

8 What is the circumference of the circle? Write your answer using π. Show your work.



SOLUTION _____

9 The diameter of a gong is 20 inches. Find the approximate circumference of the gong, using 3.14 for π. Then find the exact circumference of the gong. Show your work.

SOLUTION _____

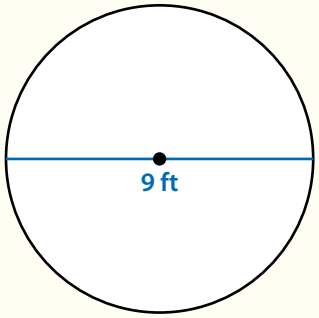
_____

Practice Using the Relationship Between a Circle's Circumference and Diameter

➤ **Study the Example showing how to find the circumference of a circle. Then solve problems 1–5.**

Example

A model of a circular pool is shown. What is the exact circumference of the pool?



You can use a formula to find the circumference of a circle.

The diagram shows that the diameter of the pool is 9 ft.

$C = \pi d$

$= \pi \cdot 9$

$= 9\pi$

Using π gives an exact circumference. The circumference of the pool is 9π ft.

1 **a.** A circular dining room table top has a radius of 22 inches. What is the diameter of the table top?

b. What is the circumference of the table top? Write your answer using π. Show your work.

SOLUTION _____

2 A circular coin has circumference 32π millimeters. Will the coin fit through a slot that is 35 millimeters long? Explain.

Vocabulary

circumference
the distance around the outside of a circle.

diameter
the distance across a circle through the center.

pi (π)
in a circle, the quotient $\frac{\text{circumference}}{\text{diameter}}$.
Common approximations are 3.14 and $\frac{22}{7}$.

radius
the distance from the center of a circle to any point on the circle.

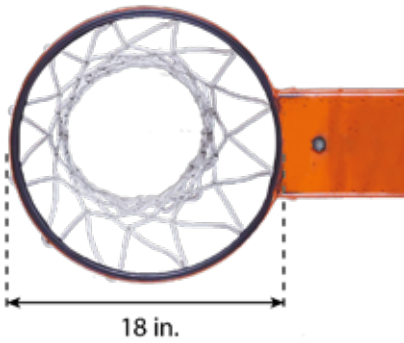
3 Destiny draws a circle with radius 14 centimeters.

a. What is the circumference of Destiny's circle? Write your answer using π. Show your work.

SOLUTION _____

b. Is 85 centimeters a reasonable estimate for the circumference of Destiny's circle? Explain.

4 The diameter of a basketball rim is 18 inches. What is the circumference of the rim? Write your answer using π. Show your work.



SOLUTION _____

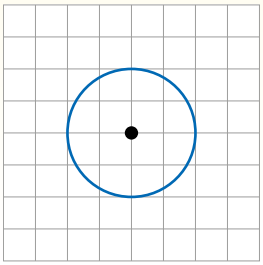
5 A circular mirror has a circumference of 50π inches. What is the radius of the mirror? Show your work.

SOLUTION _____

Develop Finding the Area of a Circle

Read and try to solve the problem below.

What is the approximate area of a circle with radius 2 units?



TRY IT

Math Toolkit grid paper, tracing paper

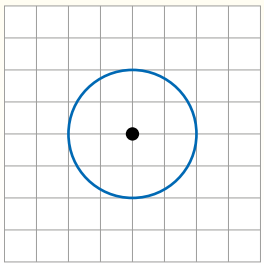
DISCUSS IT

Ask: What did you do first to find the area of the circle?

Share: First, I . . .

➤ **Explore different ways to find the area of a circle.**

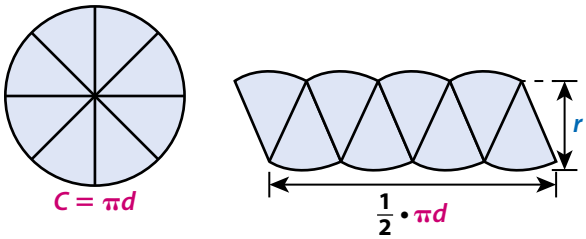
What is the approximate area of a circle with radius 2 units?



Model It

You can decompose a circle.

Divide a circle into equal parts. Then compose the parts into a figure that looks like a parallelogram.

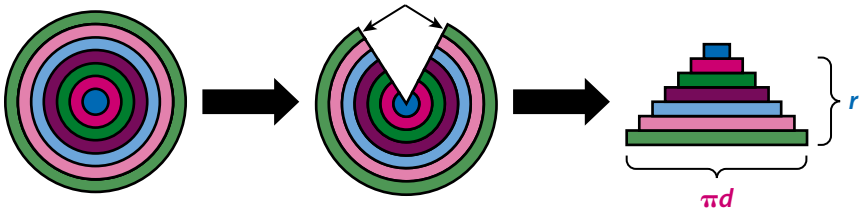


A formula for the area of the circle is $A = \frac{1}{2}C \cdot r$ or $\frac{1}{2} \cdot \pi d \cdot r$.

Picture It

You can visualize "unrolling" a circle into a triangle.

Think of a circle as made of rings. Slice the rings along a radius. Then unroll the rings to make a triangular shape.



The formula for the area of a triangle is $A = \frac{1}{2}bh$. The length of the base of the triangle is the circumference of the circle, πd. The height of the triangle is r.

So, a formula for the area of a circle is $A = \frac{1}{2} \cdot \pi d \cdot r$.

CONNECT IT

Use the problem from the previous page to help you understand how to find the area of a circle.

1. Look at **Model It**. Why is the base of the parallelogram $\frac{1}{2}$ the circumference of the circle?

2. Look at **Picture It**. Why is the base of the triangle the same as the circumference of the circle?

3. **Model It** and **Picture It** both show the area of a circle using the formula $A = \frac{1}{2} \cdot \pi \cdot d \cdot r$. What is a formula for the area of a circle that only uses r?

4. The formula for the area of a circle contains π. That means you can find both an exact and an approximate area of a circle. What is the approximate area of a circle with radius 2 cm? Why might you want to find an approximate area of a circle?

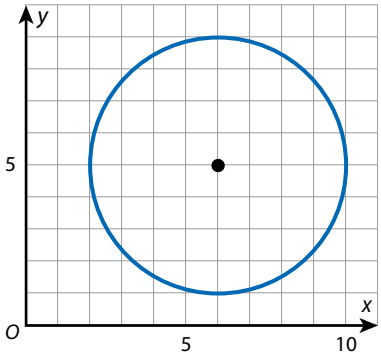
5. The formula for circumference uses diameter. The formula for area uses radius. Explain how you can find the area of a circle if you know the circumference.

6. **Reflect** Think about all the models and strategies you have discussed today. Describe how one of them helped you better understand how to solve the **Try It** problem.

Apply It

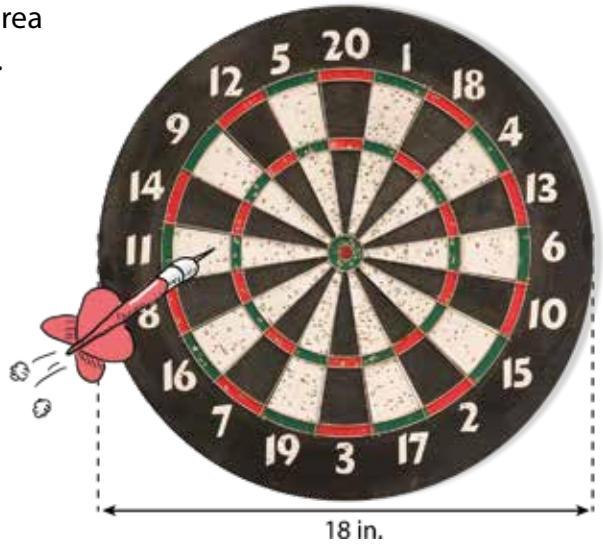
Use what you learned to solve these problems.

7 Gaspar draws a circle in the coordinate plane with the center at (6, 5). What is the area of Gaspar's circle? Write your answer using π. Show your work.



SOLUTION _____

8 The diameter of a circular dartboard is 18 inches. What is the area of the dartboard? Write your answer using π. Show your work.



SOLUTION _____

9 The radius of a circle is 10 cm. What is the approximate area of the circle? What is the exact area of the circle? Show your work.

SOLUTION _____

_____

Practice Finding the Area of a Circle

➤ **Study the Example showing how to find the area of a circle. Then solve problems 1–5.**

Example

What is the exact area of the circle?

To find the radius, count the number of units from the center of the circle to the edge. The radius is 6 units.

Then find the area of the circle.

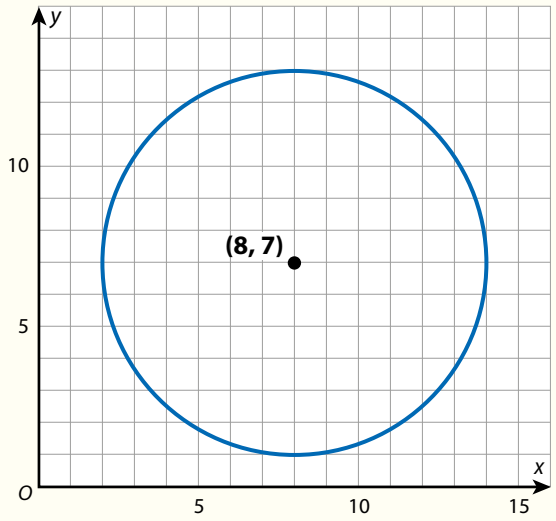
$A = \pi r^2$

$= \pi(6)^2$

$= 36\pi$

To find the exact area, use π.

The area of the circle is 36π units^2.



1. A round tablecloth has a diameter of 30 inches.

   a. What is the radius of the tablecloth?

   b. What is the area of the tablecloth? Write your answer using π. Show your work.

SOLUTION _____

2. A circular garden has a radius of 3 meters. What is the area of the garden? Write your answer using π. Show your work.

SOLUTION _____

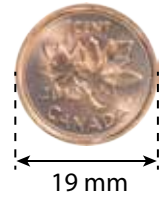
Vocabulary

area
the amount of space inside a closed two-dimensional figure. Area is measured in square units.

diameter
the distance across a circle through the center.

radius
the distance from the center of a circle to any point on the circle.

3 The diameter of a Canadian penny is 19 millimeters. The diameter of a Canadian nickel is 21 millimeters. How much greater is the area of a Canadian nickel than the area of a Canadian penny? Show your work.

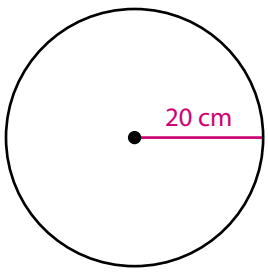


SOLUTION _____

4 A circle has radius 12 millimeters. What is its area? Use 3.14 for π. Show your work.

SOLUTION _____

5 What is the exact area of the circle below? Show your work.



SOLUTION _____

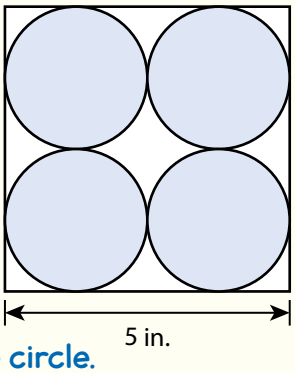
Refine Solving Circumference and Area Problems Involving Circles

➤ **Complete the Example below. Then solve problems 1–11.**

Example

Four identical circles are drawn within a square. What is the total area of the circles?

Look at how you could use the formula for the area of a circle.



The diameter of each circle is $\frac{5}{2}$ in.

The radius of each circle is $\frac{5}{2} \div 2 = \frac{5}{4}$ in.

Total shaded area is 4 times the area of one circle.

$$\text{Area of one circle} = \pi r^2$$

$$= \pi \cdot \left(\frac{5}{4}\right)^2$$

$$= \pi \cdot \frac{25}{16}$$

SOLUTION _____

CONSIDER THIS . . .
The distance across two of the circles is 5 inches.

PAIR/SHARE
How would your answer change if the side length of the square was 6 inches?

Apply It

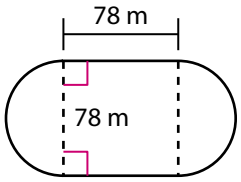
1 A circle with radius 3 centimeters is cut from a square piece of felt. The sides of the felt square are 8 centimeters long. How much felt is left over? Use 3.14 for π. Show your work.

CONSIDER THIS . . .
The formula for the area of a square is $A = s^2$.

SOLUTION _____

PAIR/SHARE
Is your answer exact or approximate? Why?

2 A middle school has an oval track with the dimensions shown. What is the distance around the track? Use 3.14 for π. Show your work.



CONSIDER THIS . . .
You can think of the track as a square with half circles on the right and left sides.

PAIR/SHARE
Why might you want to find an approximate distance?

SOLUTION

3 Mr. Aba builds a circular patio with a diameter of 12 feet. He covers the patio with paving stones. The cost of the paving stones is $10.50 per square foot. To the nearest dollar, how much do the paving stones cost?

A $126

B $378

C $1,187

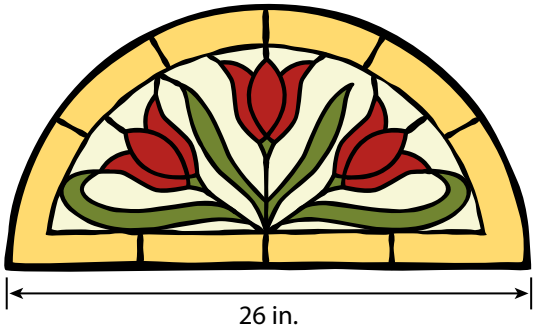
D $4,748

Jacob chose D as the correct answer. How might he have gotten that answer?

CONSIDER THIS . . .
Multiply the number of square feet by the cost per square foot to find the total cost.

PAIR/SHARE
How could you estimate the area to check that your answer makes sense?

4 The stained glass window shown is a half circle. What is the perimeter of the window? Use 3.14 for π. Show your work.



SOLUTION

5 Erin wants to find the circumference of a circle with radius 7 cm. Which of the following can she use to find the circumference of the circle? Select all that apply.

A $2 \cdot 7 \cdot \pi$

B $2 \cdot 14 \cdot \pi$

C $\frac{7}{2} \cdot \pi$

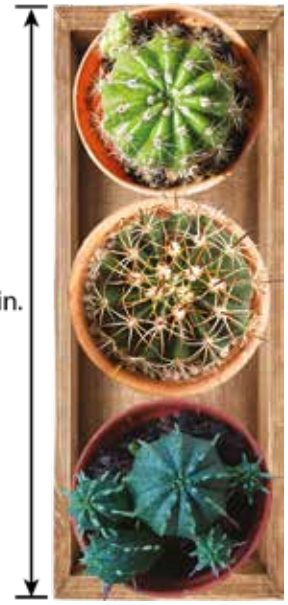
D $14 \cdot \pi$

E $49 \cdot \pi$

6 Is there a proportional relationship between the area and radius of a circle? Explain.

7 The area of a circle is 25π ft^2. What is the circumference of the circle? Explain.

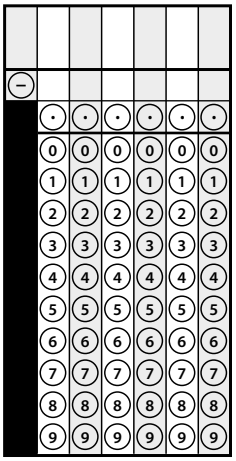
8 A box holds three circular flower pots, each with the same diameter. What is the exact area of the base of one pot? Show your work.



SOLUTION _____

9 The quotient of a circle's circumference divided by its diameter is _____ .

10 The radius of the wheel of a unicycle is 14 inches. What is the distance, in inches, that the unicycle covers after 10 full rotations of the wheel? Use 3.14 for π.



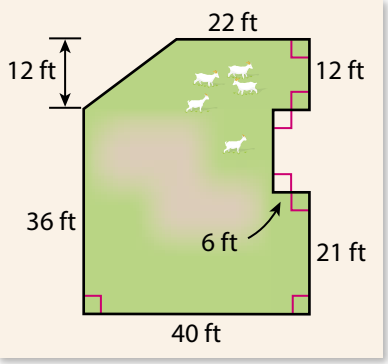
11 **Math Journal** Draw a circle and give its radius or diameter. Then find the circumference and the area of the circle. Write your answers using π.

End of Lesson Checklist

- ☐ **INTERACTIVE GLOSSARY** Find the entries for *circumference* and *pi* (π). Rewrite the definitions in your own words.
- ☐ **SELF CHECK** Go back to the Unit 1 Opener and see what you can check off.

Explore Finding Composite Areas

Previously, you learned about area and surface area. In this lesson, you will learn about solving problems that involve area and surface area of composite figures.



➤ **Use what you know to try to solve the problem below.**

Tyler's family builds a new pen on their dairy goat farm. The diagram represents the new pen. Each goat needs at least 50 ft^2 of space. What is the greatest number of goats the new pen can hold?

TRY IT

Math Toolkit grid paper, tracing paper

DISCUSS IT

Ask: How is the way you found the greatest number of goats similar to the way I did? How is it different?

Share: My method is similar to yours because . . . It is different because . . .

Learning Target SMP 1, SMP 2, SMP 3, SMP 4, SMP 5, SMP 6, SMP 7

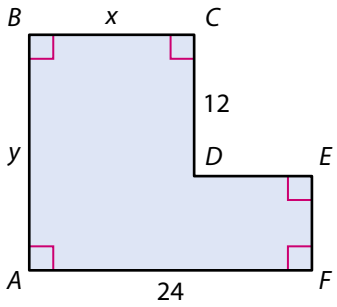
Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

CONNECT IT

1 Look Back What is the greatest number of goats the new pen can hold? Explain how you know.

2 Look Ahead To find the number of goats that can fit in the pen, you may have used the side lengths you knew to find an unknown side length. Sometimes a side length in a figure is labeled with a variable. You may be able to write expressions for other side lengths in terms of the variable.

a. How do you know that $24 - x$ represents the length of $\overline{DE}$?

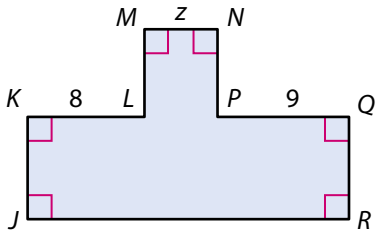


b. Write an expression for the length of $\overline{EF}$ in terms of y.

c. Explain how the expression $12x + 24(y - 12)$ represents the area of the figure.

d. Explain how $24y - 12(24 - x)$ also represents the area of the figure.

3 Reflect How do you know that the expression $z + 17$ represents the length of $\overline{JR}$?



 Name:

Prepare for Solving Problems Involving Area and Surface Area

1 Think about what you know about three-dimensional figures and surface area. Fill in each box. Use words, numbers, and pictures. Show as many ideas as you can.

What Is It?

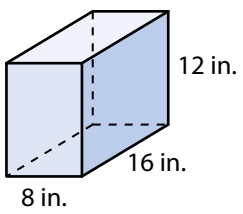
What I Know About It

surface area

Example

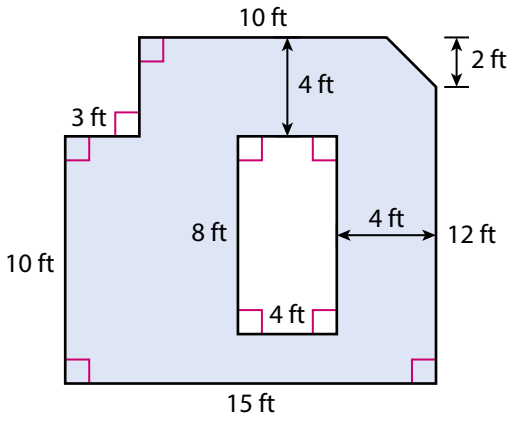
Non-Example

2 Muna claims that the expression (8)(16) + (8)(12) + (16)(12) represents the surface area, in square inches, of the right rectangular prism shown. Is Muna correct? Explain.



3 A kitchen floor needs new tile. The shaded region in the diagram represents the floor that needs tile. The new tile costs $5.40 per square foot.

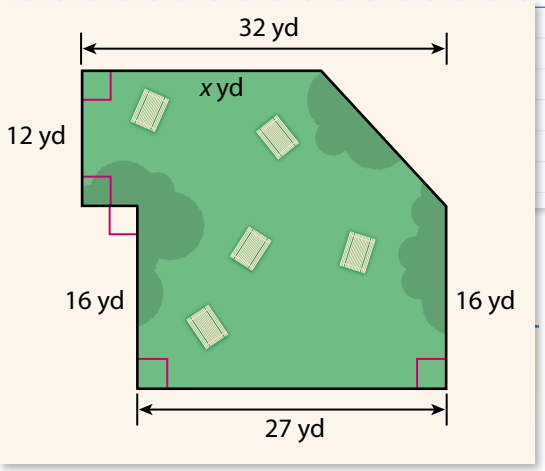
a. What is the total cost of the new tile for the kitchen? Show your work.



SOLUTION _____

b. Check your answer to problem 3a. Show your work.

Develop Solving Problems Involving Area



➤ **Read and try to solve the problem below.**

The diagram represents an outdoor eating space at a school. The area of the outdoor eating space is 750 yd^2. What is the value of x?

TRY IT

Math Toolkit grid paper, tracing paper

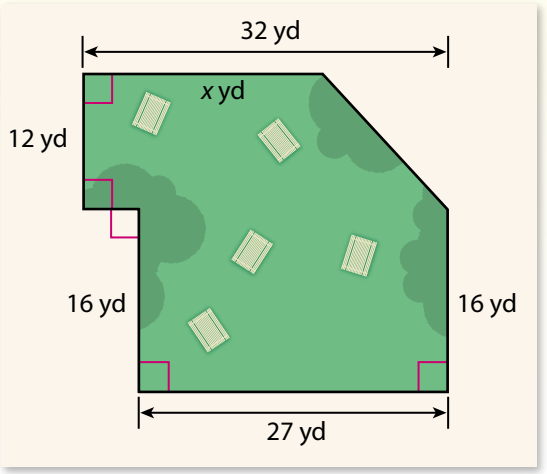
DISCUSS IT

Ask: What did you do first to find the value of x?

Share: First, I . . .

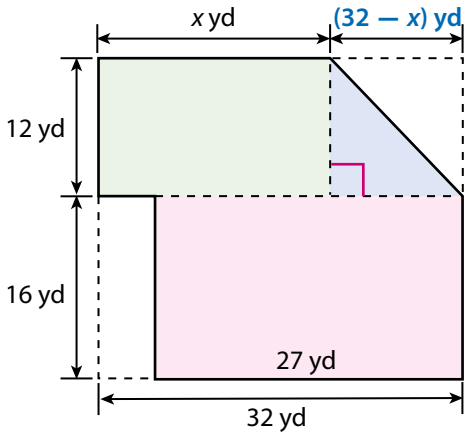
➤ **Explore different ways to solve problems involving area.**

The diagram represents an outdoor eating space at a school. The area of the outdoor eating space is 750 yd^2. What is the value of x?



Picture It

You can decompose a composite figure into smaller shapes.



Model It

You can write and solve an equation to find an unknown measurement.

The sum of the areas of the two rectangles and the triangle is equal to the area of the eating space.

$$(x \cdot 12) + (27 \cdot 16) + \frac{1}{2}(32 - x)(12) = 750$$

$$12x + 432 + 6(32 - x) = 750$$

$$12x + 432 + 192 - 6x = 750$$

$$6x + 624 = 750$$

$$6x = 126$$

CONNECT IT

➤ **Use the problem from the previous page to help you understand how to solve problems involving area.**

1. Look at the shaded triangle in **Picture It**. Why can you use the expression $32 - x$ to represent the base of the triangle in yards?

2. Look at the first equation in **Model It**. Explain why the expression $\frac{1}{2}(32 - x)(12)$ represents the area of the shaded triangle.

3. Why does the sum of the areas of the two rectangles and the triangle equal the area of the composite figure?

4. What is the value of x? How does decomposing the figure help you find the value of x?

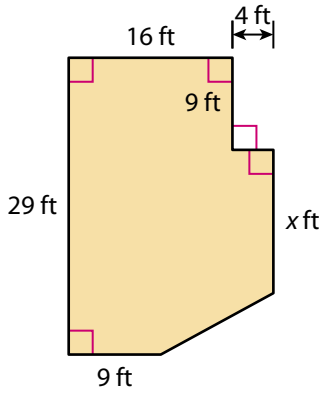
5. How can you use the area of a figure to find an unknown side length of the figure?

6. **Reflect** Think about all the models and strategies you have discussed today. Describe how one of them helped you better understand how to solve the **Try It** problem.

Apply It

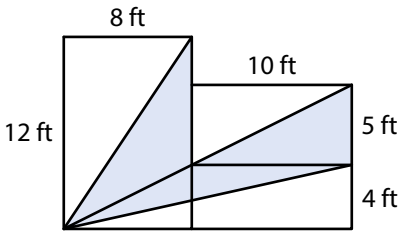
Use what you learned to solve these problems.

7 The diagram shows a plan for a deck. The area of the deck is 511 ft^2. What is the value of x? Show your work.



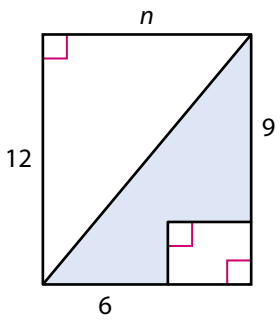
SOLUTION _____

8 Portions of three rectangles are shaded as shown. What is the area of the shaded region? Show your work.



SOLUTION _____

9 The area of the shaded region of the figure is 48 $units^2$. What is n? Show your work.



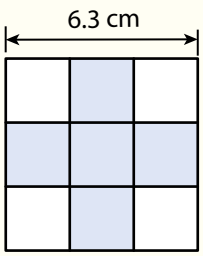
SOLUTION _____

Practice Solving Problems Involving Area

➤ **Study the Example showing how to solve a problem involving area. Then solve problems 1–5.**

Example

Each side of a square is divided into three equal sections. The square is then shaded as shown. What is the area of the shaded part?



You can think of the shaded region as 5 small squares of equal size.

Each side of the larger square is divided into thirds. So, each small square has side length 6.3 ÷ 3, or 2.1 cm.

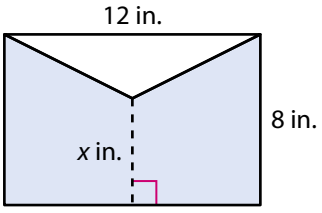
So, each small square has area (2.1)(2.1), or 4.41 cm^2.

So, the area of the shaded region is 5(4.41), or 22.05 cm^2.

1 What is the area of the unshaded part of the square in the Example? Show your work.

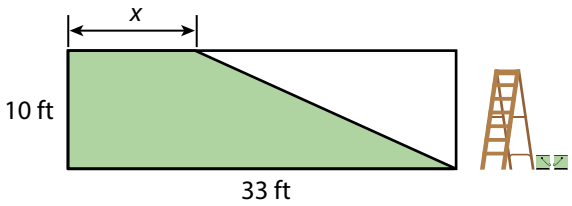
SOLUTION _____

2 A portion of a rectangle is shaded as shown. The area of the shaded region is 78 $in.^2$. What is the value of x? Show your work.



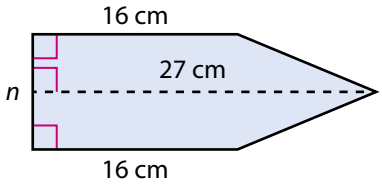
SOLUTION _____

3 An art class plans to paint part of a rectangular wall in the cafeteria and leave the rest of the wall white, as shown. The painted section will take up $\frac{2}{3}$ of the area of the wall. What is x? Show your work.



SOLUTION

4 The figure has area 193.5 cm^2. Which equation can be used to find the value of n, in centimeters?



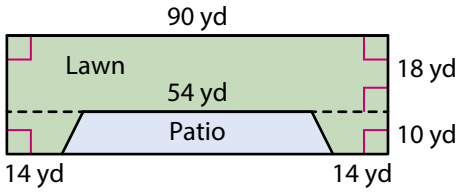
A $16n + \frac{1}{2}(11n) = 193.5$

B $16n + \frac{1}{2}(16)(11) = 193.5$

C $27n - \frac{1}{2}(16n) = 193.5$

D $27n - \frac{1}{2}(16)(11) = 193.5$

5 The diagram shows the plan for a lawn. A landscaper needs to buy grass seed to cover the lawn. One bag of grass seed covers an area of 900 yd^2. How many bags of seed does the landscaper need to buy to cover the lawn? Show your work.

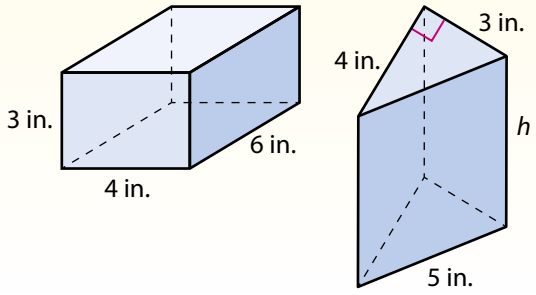


SOLUTION

Develop Solving Problems Involving Surface Area

➤ **Read and try to solve the problem below.**

The right rectangular prism and the right triangular prism have the same surface area. What is the height in inches, h, of the triangular prism?



TRY IT

Math Toolkit geometric solids, grid paper, isometric dot paper

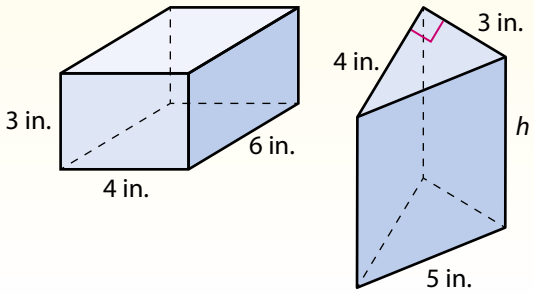
DISCUSS IT

Ask: How do you know your answer is reasonable?

Share: My answer is reasonable because . . .

➤ **Explore different ways to solve problems involving surface area.**

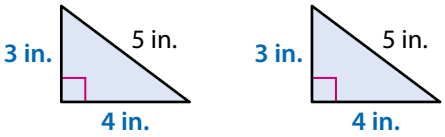
The right rectangular prism and the right triangular prism have the same surface area. What is the height in inches, h, of the triangular prism?



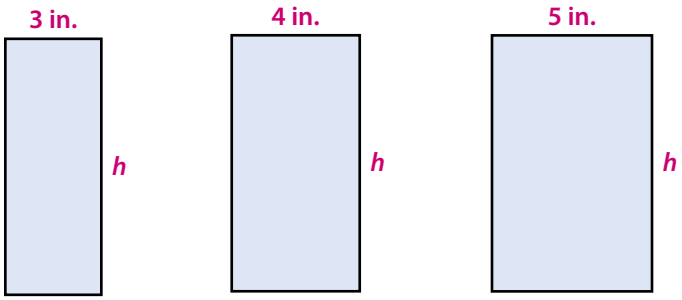
Picture It

You can draw the faces of a prism with their dimensions.

The bases of the triangular prism are the same shape and size.



The other three faces of the triangular prism are rectangles.



Model It

You can write and solve an equation to find an unknown measurement.

Surface area of rectangular prism: $2(4 \cdot 6) + 2(4 \cdot 3) + 2(6 \cdot 3) = 108$

Surface area of triangular prism: $2\left(\frac{1}{2}\right)(4 \cdot 3) + 3h + 4h + 5h = 12 + 12h$

Surface area of rectangular prism = Surface area of triangular prism

$$108 = 12 + 12h$$

$$96 = 12h$$

CONNECT IT

➤ **Use the problem from the previous page to help you understand how to solve problems involving surface area.**

1. Look at the expression for the surface area of the triangular prism in **Model It**.
   Why is $\left(\frac{1}{2}\right)(4 \cdot 3)$ the only part of the expression that is multiplied by 2?

2. What is the height of the triangular prism? How does knowing the surface area of the prism help you find the height?

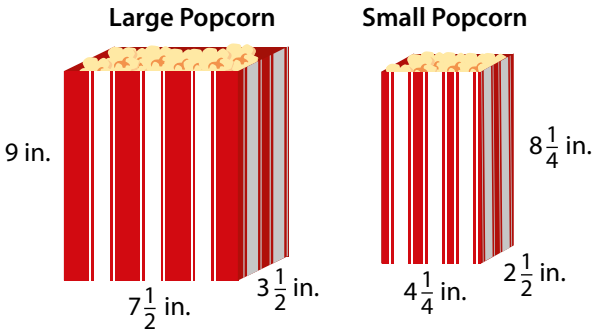
3. Another rectangular prism has bases with area 14 cm^2 each. Each of the other faces has area 15 cm^2. Why is this enough information to find the surface area of the prism?

4. **Reflect** Think about all the models and strategies you have discussed today. Describe how one of them helped you better understand how to solve problems involving surface area.

Apply It

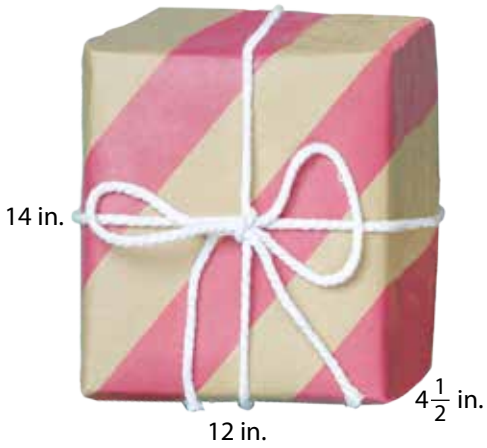
Use what you learned to solve these problems.

5 A company makes popcorn bags in two sizes. Each bag is shaped like a right rectangular prism, but it is open at the top. How many more square inches of paper are needed to make a large bag than a small bag? Show your work.



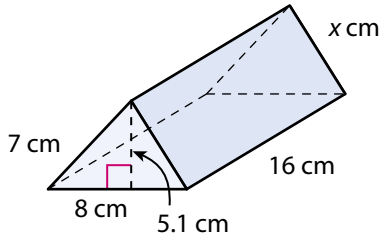
SOLUTION

6 Grace needs to cover a box shaped like a right rectangular prism with wrapping paper. Grace needs 10% more paper than the surface area of the box. How many square inches of wrapping paper does she need? Show your work.



SOLUTION

7 The surface area of the right triangular prism is 376.8 cm^2. What is the value of x? Show your work.



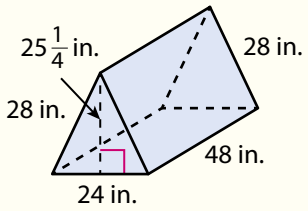
SOLUTION

Practice Solving Problems Involving Surface Area

➤ **Study the Example showing how to solve a problem involving surface area. Then solve problems 1–5.**

Example

A small greenhouse for a vegetable garden is shaped like a right triangular prism. Clear plastic covers all faces of the greenhouse except the bottom. How many square inches of clear plastic cover the greenhouse?



You can find the amount of plastic by adding the areas of the faces of the greenhouse.

Area of the triangular faces: $\frac{1}{2}\left(24 \cdot 25\frac{1}{4}\right) = 303$

Area of the rectangular faces: $48 \cdot 28 = 1{,}344$

Total surface area: $2(303) + 2(1{,}344) = 3{,}294$

There are 3,294 in.^{2} of clear plastic covering the greenhouse.

1 **a.** The expression $2(303) + 2(1{,}344)$ can be used to find the area of the plastic in the Example. Why is each term multiplied by 2?

b. Teresa writes the expression $2(303 + 1{,}344 + 1{,}152)$ to find the total surface area of the greenhouse, including the bottom. Explain why her expression is incorrect.

2 A right rectangular prism has length 10 in. and width 8 in. The surface area of the prism is 376 in.^{2}. Which equation can be used to find the height in inches, h, of the prism?

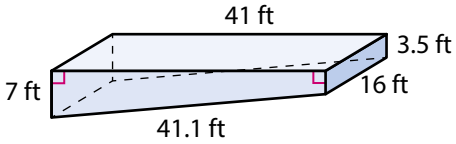
A $80h = 376$

B $160 + 18h = 376$

C $80 + 18h = 376$

D $160 + 36h = 376$

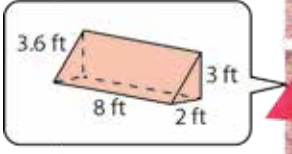
3 The swimming pool at an apartment complex is shaped like a right prism. The bottom and sides of the pool need to be repainted. One gallon of paint covers up to 125 ft^2. Paint can only be purchased in whole gallons. How many gallons of paint will the painters need to purchase? Show your work.

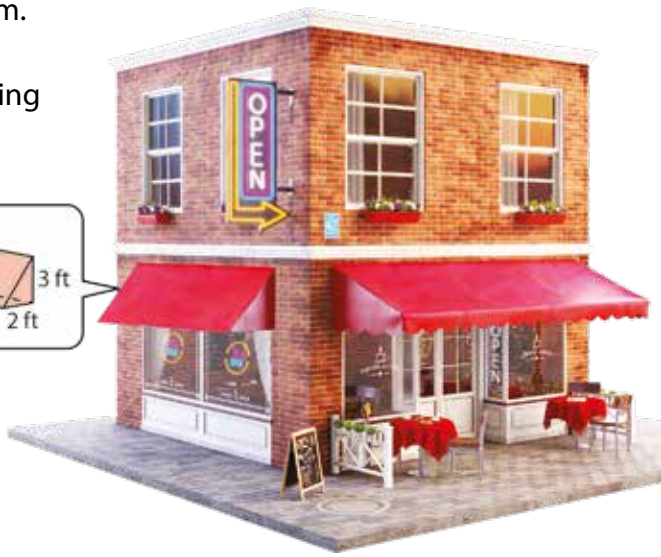


SOLUTION

4 A right rectangular prism has length 15 cm, width 10 cm, and height 5 cm. Savanna claims that doubling the length, width, and height of the prism will double its surface area. Is Savanna correct? Explain.

5 The awning for a window is shaped like a right triangular prism. The fabric of the awning covers one rectangular and two triangular faces of the prism, as shown. The fabric for the awning costs $0.55 per square foot. What is the cost of fabric for the awning? Show your work.



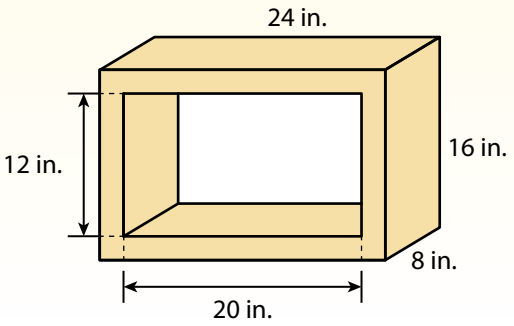


SOLUTION

Develop Solving Problems Involving Surface Area of Composite Figures

➤ **Read and try to solve the problem below.**

A company sells cardboard scratching blocks for cats. The block is shaped like a right rectangular prism with a rectangular hole cut through its center. A cat can scratch on any face of the block, including the bottom and inside faces. What is the total area of the block's scratching surface?



TRY IT

Math Toolkit dot paper, geometric solids, grid paper, isometric dot paper

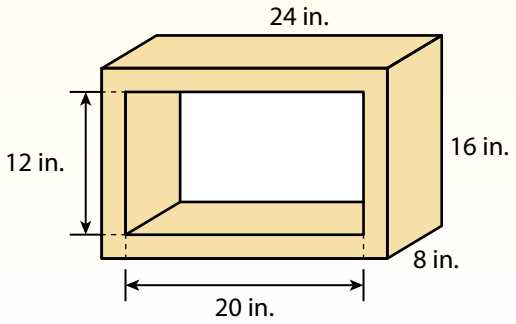
DISCUSS IT

Ask: How does your work show all the faces of the block?

Share: My work shows all the faces of the block by . . .

➤ **Explore different ways to solve problems involving surface area of composite figures.**

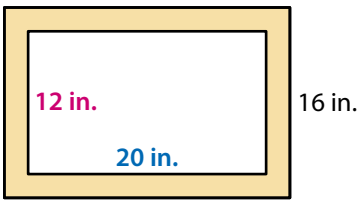
A company sells cardboard scratching blocks for cats. The block is shaped like a right rectangular prism with a rectangular hole cut through its center. A cat can scratch on any face of the block, including the bottom and inside faces. What is the total area of the block's scratching surface?



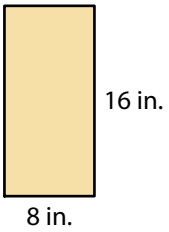
Picture It

You can draw the faces of a figure with their dimensions.

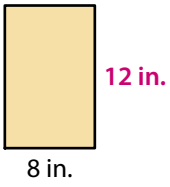
Front and Back Faces



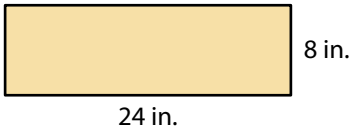
Left and Right Outer Faces



Left and Right Inner Faces



Top and Bottom Outer Faces



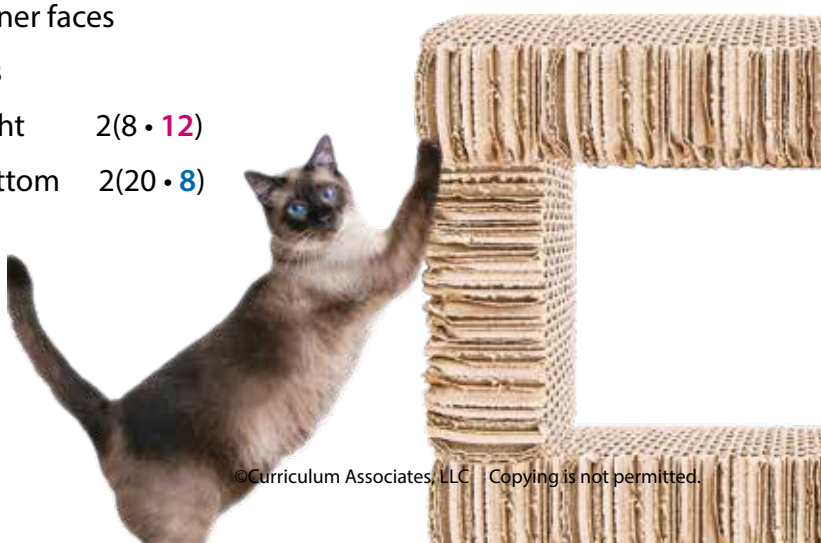
Top and Bottom Inner Faces

Model It

You can think of the total surface area as the area of the outside surfaces plus the area of the inside surfaces.

Total surface area = area of outer faces + area of inner faces

Outer Faces		**Inner Faces**	
Left and right	$2(8 \cdot 16)$	Left and right	$2(8 \cdot 12)$
Top and bottom	$2(24 \cdot 8)$	Top and bottom	$2(20 \cdot 8)$
Front and back	$2[(24 \cdot 16) - (20 \cdot 12)]$		



CONNECT IT

Use the problem from the previous page to help you understand how to solve problems involving surface area of composite figures.

1. Look at the drawing of the front and back faces in **Picture It**. How does **Model It** show how to find their area?

2. What is the total area of the block's scratching surface? Would the surface area of the block be greater or less if it did not have a hole? Explain.

3. You can think of the block as a rectangular prism that had a smaller rectangular prism cut out of it. Felipe claims that you can add the surface areas of the two prisms to find the total surface area of the block. Why is Felipe incorrect?

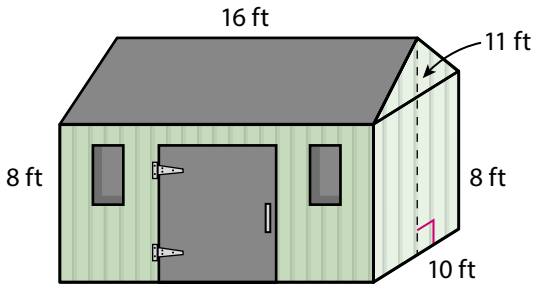
4. How is finding the surface area of a three-dimensional figure with a hole in it like finding the surface area of a solid three-dimensional figure? How is it different?

5. **Reflect** Think about all the models and strategies you have discussed today. Describe how one of them helped you better understand how to solve the **Try It** problem.

Apply It

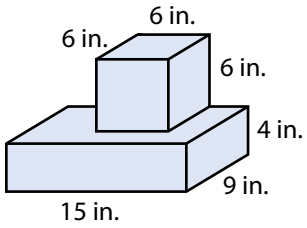
Use what you learned to solve these problems.

6 A garden shed is shaped like a right prism. The shed has one 7 ft-by-6 ft rectangular door and two 3 ft-by-$1\frac{1}{2}$ ft rectangular windows. The outer walls will be painted, but not the roof, door, or windows. What is the total surface area to be painted? Show your work.



SOLUTION

7 The three-dimensional figure shown is composed of right prisms. What is the total surface area of the figure?



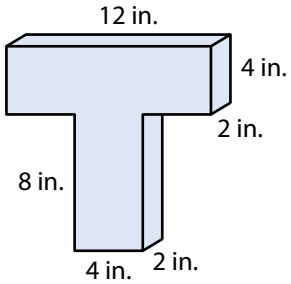
A 510 in.^{2}

B 642 in.^{2}

C 606 in.^{2}

D 678 in.^{2}

8 Tyrone makes a wooden letter T to hang on his bedroom wall. The T is a right prism. Tyrone plans to cover all the faces except the back with fabric. How many square inches of fabric will he need? Show your work.



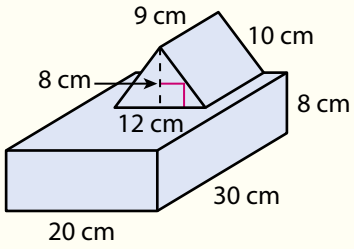
SOLUTION

Practice Solving Problems Involving Surface Area of Composite Figures

➤ **Study the Example showing how to solve problems involving surface area of composite figures. Then solve problems 1–4.**

Example

The three-dimensional figure shown is composed of right prisms. What is the total surface area of the figure?



You can think of the figure as a right triangular prism on top of a right rectangular prism.

Triangular prism: $2\left(\frac{1}{2}\right)(12 \cdot 8) + 2(10 \cdot 9) + (12 \cdot 9) = 384$

Rectangular prism: $2(30 \cdot 20) + 2(20 \cdot 8) + 2(30 \cdot 8) = 2{,}000$

The **bottom face** of the triangular prism and **part of the top face** of the rectangular prism are not part of the surface of the composite figure. Subtract the areas of these parts.

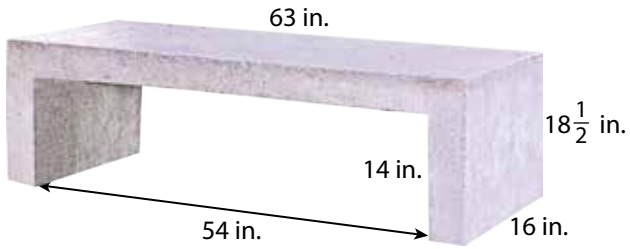
$384 + 2{,}000 - (12 \cdot 9) - (12 \cdot 9) = 2{,}168$

The total surface area of the figure is 2,168 cm^2.

1 **a.** Why is $(12 \cdot 9)$ subtracted twice in the expression for the surface area of the figure from the Example?

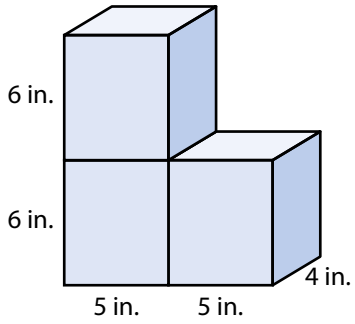
b. Consider the figure in the Example. Suppose the triangular prism was moved so that a triangular base touches the rectangular prism instead. Would the total surface area of the figure increase or decrease? Explain your reasoning.

2 All faces of a bench except the two faces that rest on the ground will be coated with a water-resistant paint. The bench is a right prism. What is the total area to be coated with the paint? Show your work.



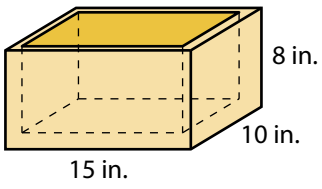
SOLUTION

3 All sides of the three-dimensional figure shown meet at right angles. What is the surface area of the three-dimensional figure? Show your work.



SOLUTION

4 Indira makes a wooden box without a lid. All the faces of the box meet at right angles. Indira plans to paint all surfaces of the box, including the inside and outside. The interior of the box has length 13 in., width 8 in., and depth 7 in. Indira can cover up to 1,350 in.^{2} with $\frac{1}{2}$ cup of paint. Will she need more than $\frac{1}{2}$ cup of paint to cover the box? Explain how you know.



Refine Solving Problems Involving Area and Surface Area

➤ **Complete the Example below. Then solve problems 1–9.**

Example

The four walls in a classroom need to be painted. The room is 27 ft long, 32 ft wide, and 10 ft high. The room has two windows, each with width 3 ft 9 in. and height 2 ft 1 in. The door has width 3 ft and height 6 ft 10 in. One gallon can of paint covers about 400 ft^2. Estimate the number of gallon cans of paint needed for the walls.

Look at how you could show your work using rounding.

Round dimensions to the nearest foot and find each area.

Area of the walls: $2(27 \cdot 10) + 2(32 \cdot 10) = 1{,}180$

Area of the door: $2(4 \cdot 2) = 16$

Area of the windows: $(3 \cdot 7) = 21$

Area of walls minus area of doors and windows:

$1{,}180 - 16 - 21 = 1{,}143$

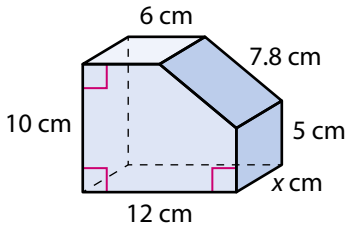
SOLUTION _____

CONSIDER THIS . . .
There are 12 inches in 1 foot.

PAIR/SHARE
How would your estimate change if the ceiling of the classroom also needed to be painted? Why?

Apply It

1 The right prism has surface area 536.4 cm^2. What is the value of x? Show your work.

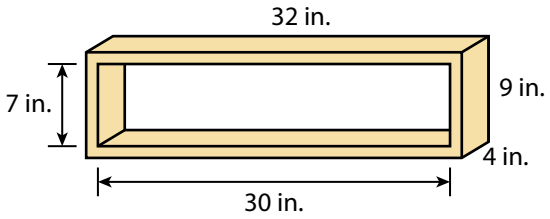


SOLUTION _____

CONSIDER THIS . . .
The front face can be decomposed into two rectangles and a triangle.

PAIR/SHARE
How can you check that you accounted for all the faces of the prism?

2 Ignacio makes a display shelf from 4 wooden boards. All angles formed by the boards are right angles. Ignacio plans to stain all faces of the shelf, except the back face, which will be against the wall. What is the total area Ignacio will stain? Show your work.

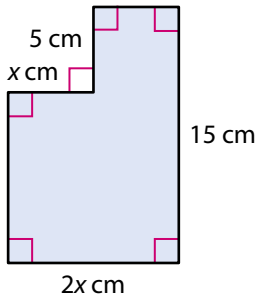


CONSIDER THIS . . .
The shelf is shaped like a right rectangular prism with a rectangular hole through its center.

PAIR/SHARE
What is a different way you could find the total area to be stained?

SOLUTION

3 The figure at the right has area 125 cm^2. Which equation can be used to find the value of x?



A $125 = 30x - 5$

B $35x = 125$

C $25x = 125$

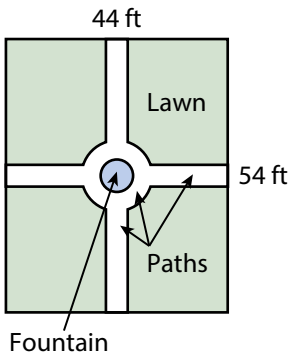
D $125 \div 15 = 2x$

Josephine chose B as the correct answer. How might she have gotten that answer?

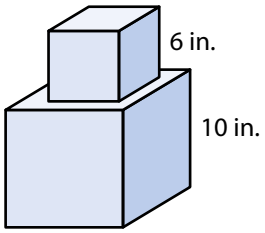
CONSIDER THIS . . .
You do not need to know the value of x to solve the problem.

PAIR/SHARE
Did you find the lengths of any unknown sides? Which ones?

4 The center of a rectangular courtyard has a circular fountain with radius 3 ft. All paths in the courtyard are 4 ft wide. Each path will be covered with gravel. The gravel needed to cover 1 ft^2 weighs about 15 lb. Ju-long estimates that the gravel needed to cover the paths will weigh less than 2,000 lb. Is Ju-long's estimate reasonable? Explain.

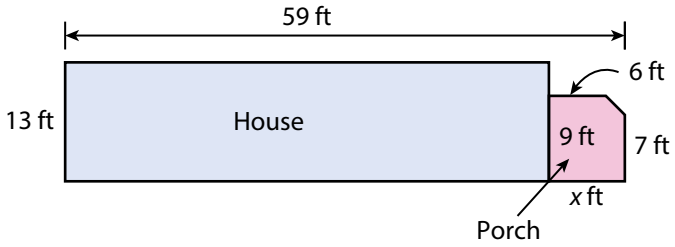


5 The prism shown is made of two cubes. What is its total surface area? Show your work.



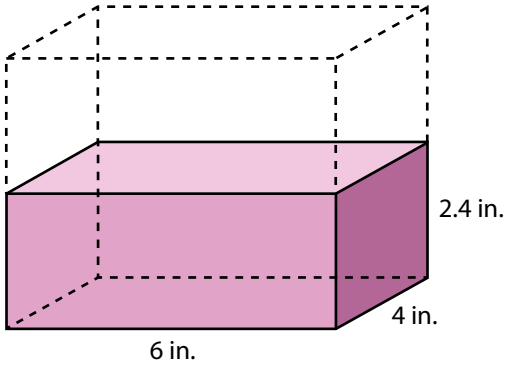
SOLUTION

6 The diagram shows a plan for a rectangular house with an attached porch. The combined area of the house and the porch is 733 ft^2. What is the value of x? Show your work.



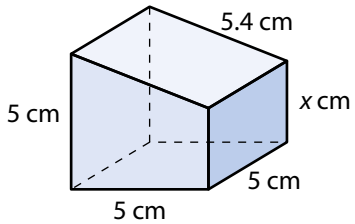
SOLUTION

7 Kadeem stacks right rectangular prisms like the one shown. He aligns each prism on top of the previous prism to make a larger prism. The larger prism has surface area 288 in.^{2}. How many prisms does Kadeem stack? Show your work.



SOLUTION

8 The right prism has surface area 132 cm^2. What is x?



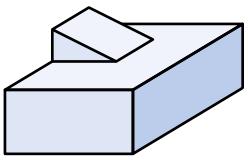
A $5\frac{2}{3}$

B 3

C $1.\overline{296}$

D 1

9 **Math Journal** The figure shown is a right triangular prism on top of a right rectangular prism. Naomi claims that she can find the surface area of the figure by adding the surface areas of the two prisms and then subtracting the area of the bottom face of the triangular prism. Is Naomi correct? Explain.



End of Lesson Checklist

- ☐ **INTERACTIVE GLOSSARY** Write a new entry for *claim*. Tell what you do when you *claim* something about a three-dimensional figure.
- ☐ **SELF CHECK** Go back to the Unit 6 Opener and see what you can check off.