Overview | Write Equivalent Expressions Involving Rational Numbers

STANDARDS FOR MATHEMATICAL PRACTICE (SMP)

SMP 1, 2, 3, 4, 5, and 6 are integrated into the Try-Discuss-Connect routine.*

This lesson provides additional support for:

2 Reason abstractly and quantitatively.

7 Look for and make use of structure.

* See page 1q to learn how every lesson includes these SMP.

Objectives

Content Objectives

- Use addition and subtraction to generate equivalent expressions with rational coefficients and constants.
- Use factoring and expanding to generate equivalent expressions with rational coefficients and constants.
- Evaluate expressions with rational terms.
- Apply the properties of operations and order of operations to manipulate expressions with negative constants and coefficients, particularly when using the distributive property.

Language Objectives

- Demonstrate understanding of the lesson vocabulary by responding appropriately to oral and written questions about strategies for generating equivalent expressions.
- Read and interpret word problems by identifying important quantities and relationships in order to evaluate expressions.
- Use precise language to explain strategies for using properties of operations to factor expressions with rational terms.

Prior Knowledge

- Identify terms, variables, and coefficients in an expression.
- Use the four operations with rational numbers.
- Use the distributive property to factor and expand expressions with positive rational coefficients.

Vocabulary

Math Vocabulary

factor (verb) to rewrite an expression as a product of factors.

Review the following key terms.

equivalent expressions two or more expressions in different forms that always name the same value.

factor (noun) a number, or expression with parentheses, that is multiplied.

like terms two or more terms that have the same variable factors.

rational number a number that can be expressed as the fraction $\frac{a}{b}$ where a and b are integers and $b \neq 0$. Rational numbers include integers, fractions, repeating decimals, and terminating decimals.

term a number, a variable, or a product of numbers, variables, and/or expressions. A term may include an exponent.

Academic Vocabulary

determine to decide something based on evidence or facts.

Learning Progression

In Grade 6, students learned to identify terms, variables, and coefficients in an expression. They also identified equivalent expressions and applied the distributive property to factor and expand expressions with positive rational coefficients.

Earlier in Grade 7, students applied the four operations to evaluate expressions with positive and negative rational numbers.

In this lesson, students extend their abilities to generate, identify, and compare equivalent expressions by working with expressions with negative rational coefficients. They apply the distributive property to show whether two expressions are equivalent.

Later in Grade 7, students will develop an understanding of how equivalent forms of linear expressions can highlight different aspects of a problem. They will also apply their ability to manipulate linear expressions involving rational numbers to write and solve multi-step linear equations and inequalities.

Pacing Guide

Items marked with are available on the **Teacher Toolbox**.

	MATERIALS	DIFFERENTIATION
SESSION 1 **Explore** Equivalent Expressions (35–50 min)		
• **Start** (5 min) • **Try It** (5–10 min) • **Discuss It** (10–15 min) • **Connect It** (10–15 min) • **Close: Exit Ticket** (5 min) **Additional Practice** (pages 311–312)	**Math Toolkit** grid paper, sticky notes Presentation Slides	**PREPARE** Interactive Tutorial **RETEACH or REINFORCE** Hands-On Activity **Materials** For each pair: 7 counters, 7 unit cubes
SESSION 2 **Develop** Expanding Expressions (45–60 min)		
• **Start** (5 min) • **Try It** (10–15 min) • **Discuss It** (10–15 min) • **Connect It** (15–20 min) • **Close: Exit Ticket** (5 min) **Additional Practice** (pages 317–318)	**Math Toolkit** grid paper, sticky notes Presentation Slides	**RETEACH or REINFORCE** Visual Model **REINFORCE** Fluency & Skills Practice **EXTEND** Deepen Understanding
SESSION 3 **Develop** Factoring Expressions (45–60 min)		
• **Start** (5 min) • **Try It** (10–15 min) • **Discuss It** (10–15 min) • **Connect It** (15–20 min) • **Close: Exit Ticket** (5 min) **Additional Practice** (pages 323–324)	**Math Toolkit** algebra tiles, grid paper Presentation Slides	**RETEACH or REINFORCE** Visual Model **REINFORCE** Fluency & Skills Practice **EXTEND** Deepen Understanding
SESSION 4 **Refine** Writing Equivalent Expressions Involving Rational Numbers (45–60 min)		
• **Start** (5 min) • **Monitor & Guide** (15–20 min) • **Group & Differentiate** (20–30 min) • **Close: Exit Ticket** (5 min)	**Math Toolkit** Have items from previous sessions available for students. Presentation Slides	**RETEACH** Hands-On Activity **Materials** For each pair: 10 counters, 10 unit cubes **REINFORCE** Problems 4–8 **EXTEND** Challenge **PERSONALIZE** i-Ready
Lesson 15 Quiz or **Digital Comprehension Check**		**RETEACH** Tools for Instruction **REINFORCE** Math Center Activity **EXTEND** Enrichment Activity

Connect to Culture

➤ **Use these activities to connect with and leverage the diverse backgrounds and experiences of all students. Engage students in sharing what they know about contexts before you add the information given here.**

SESSION 1 ■ □ □ □

Try It One of the first popular and successful video games was available in arcades beginning in 1972. The game was an electronic version of ping-pong, in which players moved a cursor to bounce a moving square across the screen. The game used a very simple scoring system, in which 1 point was awarded after each play. Modern video games employ a variety of complicated scoring systems, in which players gain or lose multiple points as a result of various actions. Ask students to share their experiences with accumulating points playing video and computer games.

SESSION 3 ■ ■ ■ □

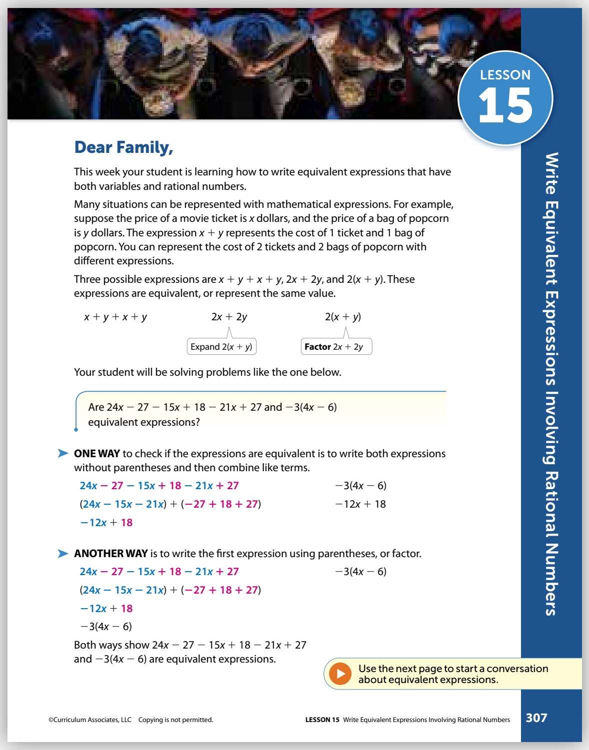
Apply It **Problem 6** Ask students when they think cell phones first became available for sale. Record a few guesses to compare to the actual year. The first commercial cell phone was put on the market in 1983. It was large and bulky, ran on batteries that lasted for 30 minutes, and was priced at about $4,000. Today, cell phones are much more useful and economical. The phones both transmit and receive radio waves, which are translated into electronic signals that are carried through a network of wires and cables. Several communications companies sell cell phone service to consumers for a variety of fee structures. Consumers may pay a flat fee per month or a fee based on minutes of use.

SESSION 4 ■ ■ ■ ■

Apply It **Problem 4** Survey students to see if they think there are too many, too few, or just enough parking spaces in your community. Have a few volunteers justify their answers. One study says the United States has eight parking spots for every car in the country. One researcher has calculated that the total parking space available for each car is greater than the housing space for each person. Parking lots take up space that could be used for other purposes, and they affect the environment because ground that has been paved over for parking lots cannot absorb water when it rains or snows. Many cities have begun studying ways to reduce the size of parking lots and to use them more effectively.

Connect to Family and Community

➤ **After the Explore session, have students use the Family Letter to let their families know what they are learning and to encourage family involvement.**


LESSON 15

Write Equivalent Expressions Involving Rational Numbers

Dear Family,

This week your student is learning how to write equivalent expressions that have both variables and rational numbers.

Many situations can be represented with mathematical expressions. For example, suppose the price of a movie ticket is x dollars, and the price of a bag of popcorn is y dollars. The expression $x + y$ represents the cost of 1 ticket and 1 bag of popcorn. You can represent the cost of 2 tickets and 2 bags of popcorn with different expressions.

Three possible expressions are $x + y + x + y$, $2x + 2y$, and $2(x + y)$. These expressions are equivalent, or represent the same value.

$x + y + x + y$ | $2x + 2y$ (Expand $2(x + y)$) | $2(x + y)$ (**Factor** $2x + 2y$)

Your student will be solving problems like the one below.

Are $24x - 27 - 15x + 18 - 21x + 27$ and $-3(4x - 6)$ equivalent expressions?

➤ **ONE WAY** to check if the expressions are equivalent is to write both expressions without parentheses and then combine like terms.

$24x - 27 - 15x + 18 - 21x + 27$ | $-3(4x - 6)$

$(24x - 15x - 21x) + (-27 + 18 + 27)$ | $-12x + 18$

$-12x + 18$

➤ **ANOTHER WAY** is to write the first expression using parentheses, or factor.

$24x - 27 - 15x + 18 - 21x + 27$ | $-3(4x - 6)$

$(24x - 15x - 21x) + (-27 + 18 + 27)$

$-12x + 18$

$-3(4x - 6)$

Both ways show $24x - 27 - 15x + 18 - 21x + 27$ and $-3(4x - 6)$ are equivalent expressions.

Use the next page to start a conversation about equivalent expressions.

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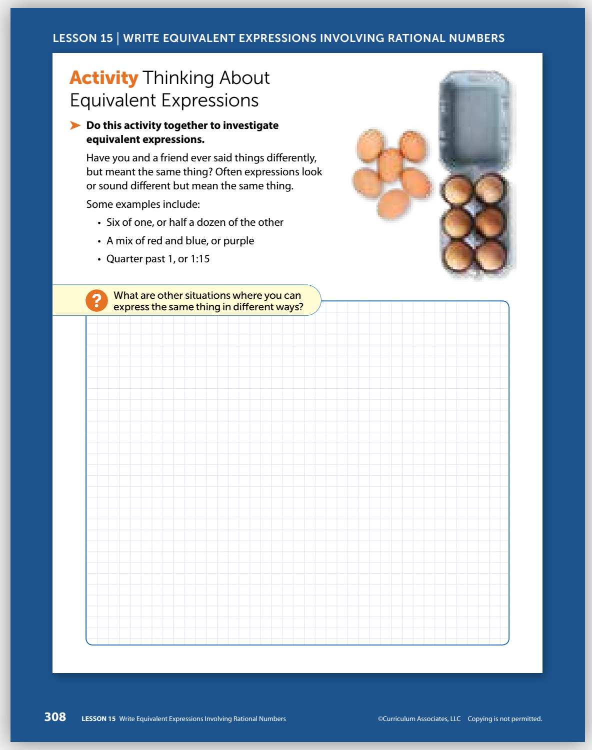
LESSON 15 | WRITE EQUIVALENT EXPRESSIONS INVOLVING RATIONAL NUMBERS

Activity Thinking About Equivalent Expressions

➤ **Do this activity together to investigate equivalent expressions.**

Have you and a friend ever said things differently, but meant the same thing? Often expressions look or sound different but mean the same thing.

Some examples include:

- Six of one, or half a dozen of the other
- A mix of red and blue, or purple
- Quarter past 1, or 1:15

What are other situations where you can express the same thing in different ways?

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Connect to Language

➤ **For English language learners, use the Differentiation chart to scaffold the language in each session. Use the Academic Vocabulary routine for academic terms before Session 1.**

DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS Use with **Session 1** Connect It

MATH TERM

To *evaluate* is to find the value of an expression.

ACADEMIC VOCABULARY

An *original expression* is the first expression, or an expression before it was changed.

Levels 1–3: Reading/Speaking

Read Connect It problem 2 with students and help them talk about finding equivalent expressions. Use a **Co-Constructed Word Bank** to help clarify terms such as *original expression, equivalent expression, expand,* and *factor*. Use a rubber band to model *expanding* and connect to *expand an expression*.

Ask students to identify the operations used to expand and factor using:

- *We can expand/factor the expression ____.*
- *We can multiply/divide ____ to get ____.*
- *The equivalent expression is ____.*

Levels 2–4: Reading/Speaking

Read Connect It problem 2 with students and have them talk about finding equivalent expressions. Use a **Co-Constructed Word Bank** to identify important terms that might be used to discuss the problem. Ask students to use gestures to represent *expand*, and then connect to *expand an expression*.

Provide individual think time, and then have students turn and talk with a partner about the following questions:

- *How can you find an equivalent expression?*
- *How do you know the expressions are equivalent?*

Levels 3–5: Reading/Speaking

Have students read Connect It problem 2 and talk about finding equivalent expressions. Ask students to use a **Co-Constructed Word Bank** to identify mathematical language and clarify meanings. Have students discuss multiple-meaning words, like *expand* and *factor*.

Provide time for students to compare the expressions and determine the operations used to expand or factor the original expression. Then have partners discuss their ideas. Encourage them to use precise language, including the terms in the word bank.

Explore Equivalent Expressions

Purpose

- **Explore** the idea that there are different ways to write equivalent expressions.
- **Understand** that the strategies used to add and subtract rational numbers can be used to add and subtract rational terms in an expression.

START CONNECT TO PRIOR KNOWLEDGE

Same and Different

A: $9b$	B: $6b + 3b$
C: $2b + 5b + b$	D: $3b + 4b + 2b$

Possible Solutions

All the expressions include the variable b.

A is the only expression with one term.

B is the only expression with two terms.

A, B, and D are equivalent.

C and D both have three terms.

WHY? Support students' ability to compare expressions with positive coefficients and to identify equivalent expressions.

TRY IT SMP 1, 2, 4, 5, 6

Make Sense of the Problem

See ***Connect to Culture*** to support student engagement. Before students work on Try It, use **Three Reads** to help them make sense of the problem. After the first read, ask: *What is the problem about?* After the second read, ask: *What are you trying to find out?* After the third read, ask: *What are the important quantities and relationships in the problem? In particular, what is the purpose of the long expression?*

DISCUSS IT SMP 2, 3, 6

Support Partner Discussion

After students work on Try It, have them respond to Discuss It with a partner. Listen for understanding of:

- evaluating the expression by substituting 201 for the variable t.
- strategies for combining terms in the expression.

LESSON 15 | SESSION 1

Explore Equivalent Expressions

Previously, you worked with equivalent expressions that have positive coefficients. In this lesson, you will work with equivalent expressions that have negative coefficients.

➤ **Use what you know to try to solve the problem below.**

In a certain video game, players earn bonus points after finishing a level. The number of bonus points is based on how many seconds, t, it takes the player to finish the level. The game uses the expression $-5.5 + 500 + 0.9t + \left(\frac{11}{2} - 1.9t\right)$ to determine the number of bonus points. Zahara finishes a level in 201 seconds. How many bonus points does she earn?

TRY IT **Math Toolkit** grid paper, sticky notes

Possible work:

SAMPLE A

$-5.5 + 500 + 0.9t + \left(\frac{11}{2} - 1.9t\right)$

$-5.5 + 500 + 0.9t + \frac{11}{2} - 1.9t$

$-5.5 + 500 + 0.9(201) + \frac{11}{2} - 1.9(201)$

$-5.5 + 500 + 180.9 + 5.5 - 381.9$

$500 + 180.9 - 381.9$

$680.9 - 381.9$

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SAMPLE B

$-5.5 + 500 + 0.9t + \left(\frac{11}{2} - 1.9t\right)$

$-5.5 + 500 + 0.9t + \frac{11}{2} - 1.9t$

$-5.5 + \frac{11}{2} + 500 + 0.9t - 1.9t$

$\left(-5.5 + \frac{11}{2}\right) + 500 + (0.9t - 1.9t)$

$500 - t$

$500 - 201 = 299$

DISCUSS IT

Ask: What did you do first to determine how many bonus points Zahara earns?

Share: First, I . . . because . . .

Learning Target SMP 1, SMP 2, SMP 3, SMP 4, SMP 5, SMP 6, SMP 7
Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.

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Common Misconception Listen for students who do not combine the decimal -5.5 and fraction $\frac{11}{2}$ in the expression. As students share their strategies, ask them to explain how they could combine -5.5 and $\frac{11}{2}$ or how they could combine any decimal and fraction.

Select and Sequence Student Strategies

Select 2–3 samples that represent the range of student thinking in your classroom. Here is one possible order for class discussion:

- substituting 201 for t and evaluating the expression to find the number of bonus points
- **(misconception)** evaluating the expression incompletely by not combining the decimal and fraction
- combining like terms, substituting 201 for t, and evaluating

Facilitate Whole Class Discussion

Call on students to share selected strategies. To ensure that everyone hears presented strategies, ask students to repeat key parts.

Guide students to **Compare and Connect** the representations. After each strategy is presented, ask students to justify why the strategy is reasonable and leads to the correct answer.

ASK *Why do the strategies of [student name] and [student name] both lead to the same solution?*

LISTEN FOR Both combine like terms and substitute the given value for the variable. The order of substituting the value of *t* and combining like terms is different.

CONNECT IT

SMP 2, 4, 5

1 Look Back Look for understanding that combining like terms will generate an equivalent expression.

DIFFERENTIATION | RETEACH or REINFORCE

Hands-On Activity

Model equivalent expressions.

If students are unsure why an original expression is equivalent to a simplified expression, then use this activity with counters to model the expression.

Materials For each pair: 7 counters, 7 unit cubes

- Display the expression shown below.
  $3a + 2b - 2a - b + 4a + 3b$
- Tell students that each variable can be shown by a different type of object. Have students model the expression by adding or taking away corresponding objects. Students begin by adding 3 counters and 2 cubes, then remove 2 counters, and so on.
- Ask: *How many objects do you have in the end? What is another way to write this result?* [5 counters and 4 cubes; $5a + 4b$]
- Write the new expression. Ask: *Why are these two expressions equivalent?* [Both expressions represent the same number of counters and cubes.]
- Repeat with another expression, such as $4a + 2b - a - b + 2a + 4b$. Extend the activity by having students compare and contrast all 4 expressions.
- Continue with other examples. Ask students to write the equivalent expression. Guide students to see that combining like terms shows how many in all of each type of object or variable.

LESSON 15 | SESSION 1

CONNECT IT

1 Look Back How many bonus points does Zahara earn? What is another expression the game could use to determine the number of bonus points she earns?

299; Possible answer: $500 - t$

2 Look Ahead In the **Try It**, you evaluated an expression. Sometimes it can be helpful to find an equivalent expression to evaluate instead. One way to find an equivalent expression is to use the distributive property.

a. Sometimes you can use the distributive property to expand an expression. You can expand $-3(5 - x)$ into the equivalent expression $-15 + 3x$. Show how you can use the distributive property to expand $-4(-a + 2)$ to get an equivalent expression.

Possible work:

$-4(-a + 2)$

$-4(-a) + (-4)(2)$

$4a - 8$

b. Sometimes you can use the distributive property to **factor** an expression. You can factor $3z - 6$ to get the equivalent expression $3(z - 2)$. Show how you can use the distributive property to factor $2g - 10$ to get an equivalent expression.

Possible work:

$2g - 10$

$2(g - 5)$

c. Can you factor $-2x + 6$ to get $-2(x - 3)$? Explain.

Yes; Possible explanation: When you factor out -2 from $-2x$, you get x. When you factor out -2 from 6, you get 6 divided by -2, which is -3. So, you get $-2[x + (-3)]$, which is the same as $-2(x - 3)$.

3 Reflect Explain why the expressions $-6(5 - k)$ and $6k - 30$ are equivalent.

Possible explanation: When you expand $-6(5 - k)$, you get $-30 - (-6k)$. You can rewrite $-(-6k)$ as $+6k$ and reorder the terms to get $6k - 30$. Since you can rewrite the first expression as the second, they are equivalent.

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2 Look Ahead Point out that for some expressions the distributive property can be used to generate equivalent expressions. Students should recognize that in some cases the distributive property can be applied either to expand or factor an expression.

Ask a volunteer to explain the meaning of *expand* and *factor*. Support student understanding by applying one specific example for both terms. For example, the expression $2(x - 5)$ can be expanded to form the equivalent expression $2x - 10$, while $2x - 10$ can be factored to form $2(x - 5)$.

CLOSE EXIT TICKET

3 Reflect Look for understanding that the distributive property and the commutative property of addition were used to generate equivalent expressions.

Common Misconception If students expand $-6(5 - k)$ to form $-30 - 6k$, then have them expand a related expression that includes a plus sign: $-6(5 + k)$. Then have them compare the expanded versions of the two expressions.

Prepare for Writing Equivalent Expressions Involving Rational Numbers

Support Vocabulary Development

Assign **Prepare for Writing Equivalent Expressions Involving Rational Numbers** as extra practice in class or as homework.

If you have students complete this in class, then use the guidance below.

Ask students to consider the term *term*. Remind them to focus on the mathematical use of the word. Encourage students to use the specific examples of terms to clarify their definitions or improve their explanations.

Have students work in pairs to complete the graphic organizer. Invite pairs to share their completed organizers and prompt a whole-class comparative discussion of the definitions, illustrations, examples, and non-examples given.

Have students review Aiden's statement in problem 2 and discuss with a partner whether equivalent expressions always, sometimes, or never have the same number of terms. Encourage students to refer to the examples they listed in the graphic organizer to help them construct their response.

Problem Notes

1. Students should understand that a term is a part of a mathematical expression and may include a number, a variable, or the product or quotient of a number and a variable. Student responses might include a wide variety of specific examples of terms, including a number only (5), a variable only (x), or the product of a number and a variable ($5x$). Students should recognize that the terms of an expression are separated by addition or subtraction signs.

2. Students should recognize that it is possible to have equivalent expressions with the same number of terms but also possible that the expressions have different numbers of terms. The expression $5x$ could have equivalent expressions of many terms, such as $x + x + x + x + x$.

LESSON 15 | SESSION 1 Name:

Prepare for Writing Equivalent Expressions Involving Rational Numbers

1. Think about what you know about the terms of an expression. Fill in each box. Use words, numbers, and pictures. Show as many ideas as you can. **Possible answers:**

In My Own Words

A term is a number, a variable, or the product of numbers, variables, and/or expressions.

My Illustrations

$32 + 10a + 6a$

32, 10*a*, and 6*a* are all terms of the expression.

term

Examples

10

y

$5a$

c^2

Non-Examples

$3 + 2b - a$

$y = 23x$

2. Aiden says that equivalent expressions always have the same number of terms. Is Aiden correct? If he is, explain why. If he is not correct, give a counterexample.

No; Possible explanation: A counterexample is the expressions $3x + 2x$ and $5x$. They are equivalent expressions, but they do not have the same number of terms.

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REAL-WORLD CONNECTION

Fences are used to mark or protect the perimeters of gardens, yards, parks, and livestock pens. If a fence encloses a rectangle of length ℓ and width w, then the perimeter of the fence is equal to $\ell + w + \ell + w$, which is equivalent to either $2\ell + 2w$, or $2(\ell + w)$. Suppose a fencing contractor needs to give an estimate of the cost to fence a rectangular yard of length 60 feet and width 40 feet using fencing that costs \$8.00 per foot. The contractor will multiply the cost per foot by the total amount of the fencing in feet. The estimate could read $8[2(40 + 60)] = 1{,}600$ or $8(60 + 40 + 60 + 40) = 1{,}600$, for a total cost of \$1,600.00. Ask students to think of other real-world examples when equivalent expressions might be useful.

Additional Practice

3 Problem 3 provides another look at writing and evaluating equivalent expressions. This problem is similar to the problem about Zahara's bonus points in the video game. In both problems, an expression with several terms is evaluated at a specific value of a variable. This problem asks for the perimeter of a star given an expression for the perimeter in terms of its side length, ℓ.

Suggest that students use **Say It Another Way** to explain the question being asked in the problem, what the expression represents, and the meaning of the variable ℓ.

Students may want to substitute the given value for ℓ and then apply the order of operations to evaluate.

3 Jake cuts out five-pointed stars. They are different sizes, but they all have the same shape. The side lengths within each star are the same. To find the perimeter of each star, Jake uses the expression $12\ell + \frac{1}{2} - 4\ell - 8.5 + 2\ell + 8$, where ℓ is the side length of the star.

a. Find the perimeter of a star with side length 6 inches. Show your work.

Possible work:

$12\ell + \frac{1}{2} - 4\ell - 8.5 + 2\ell + 8$

$12\ell - 4\ell + 2\ell + \frac{1}{2} - 8.5 + 8$

10ℓ

$10(6)$

60

SOLUTION 60 inches

b. Check your answer to problem 3a. Show your work.

Possible work:

$12(6) + \frac{1}{2} - 4(6) - 8.5 + 2(6) + 8$

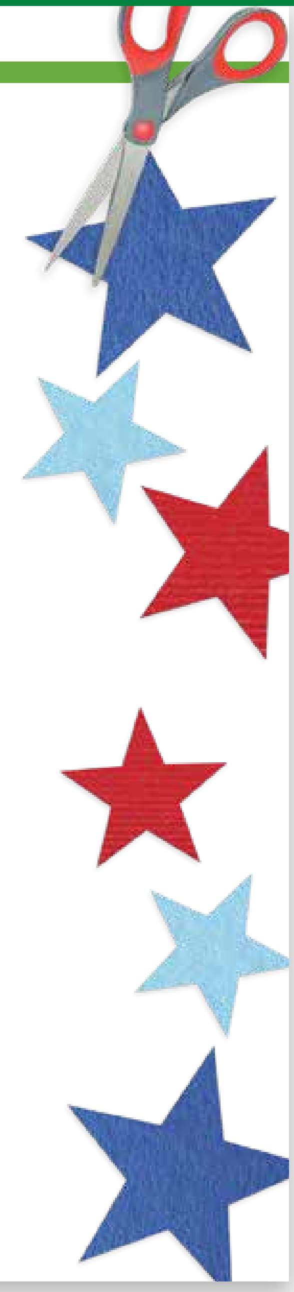
$72 + \frac{1}{2} - 24 - 8.5 + 12 + 8$

$72.5 - 24 - 8.5 + 12 + 8$

$48.5 - 8.5 + 12 + 8$

$40 + 12 + 8$

60



DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Use with **Session 2** **Connect It**

Levels 1–3: Speaking/Writing

Facilitate discussion about Connect It problem 4 to prepare students to respond in writing. Rephrase the problem. Say: *Compare the steps to expand an expression with negative terms and the steps to expand an expression with all positive terms. How are the steps the same? How are the steps different?*

Clarify *expand, expression, positive,* and *negative.* Use a **Co-Constructed Word Bank** to help students generate words they can use to orally describe and compare the steps before writing responses. Sample terms might include *multiply, order of operations*, *same,* and *different.* Support students as they use the word bank to write responses.

Levels 2–4: Speaking/Writing

Facilitate a discussion about Connect It problem 4 to prepare students to respond in writing. Read the problem with students. Have them identify the two types of expressions and provide examples of each.

Provide think time for students to consider the sequence of steps to expand both types of expressions. Have partners turn and talk about how the sequences are the same and different. Ask partners to make a **Co-Constructed Word Bank** of terms they might use in their responses. Encourage them to include comparison words. Compile the responses into a class word bank for students to reference as they respond in writing.

Levels 3–5: Speaking/Writing

Facilitate partner discussion about Connect It problem 4 to prepare students to respond in writing. Read the problem with students. Have partners identify words that signal a comparison and explain what is being compared. Then have students **Say It Another Way** to paraphrase the problem. Ask students who gave thumbs down to share alternative words than could be used.

Give students think time to consider the sequence of steps to expand both types of expressions. Have partners discuss and compare the sequences before writing their responses independently. Remind students to use comparison words in their written responses.

Develop Expanding Expressions

Purpose

- **Develop** fluency with using the distributive property to expand expressions with rational terms.
- **Recognize** that the strategies for multiplying positive and negative rational numbers apply when using the distributive property to expand expressions with rational terms.

START CONNECT TO PRIOR KNOWLEDGE

Which One Doesn't Belong?

A: $5 - 2c$	B: $2.5 + c + 2.5 + c$
C: $2c + 5$	D: 7

Possible Solutions

A is the only expression with subtraction.

B is the only expression that has terms that can be combined.

C is the only expression in which the variable term is listed first.

D is the only expression without a variable.

WHY? Support students' ability to compare expressions and identify equivalent expressions.

DEVELOP ACADEMIC LANGUAGE

WHY? Confirm understanding of *all that apply* when used in mathematical problems.

HOW? Direct students to Apply It problem 7. Ask students what *all that apply* tells about answering the problem. [There is probably more than one correct answer.] Ask students to suggest other words or phrases that books and tests sometimes use to show there can be more than one correct answer.

TRY IT

SMP 1, 2, 4, 5, 6

Make Sense of the Problem

Before students work on Try It, use **Notice and Wonder** to help them make sense of the problem. Invite students to talk about what they notice about the terms in the two expressions and what they wonder about that mathematics can answer. If necessary, encourage students to wonder whether the two expressions are equivalent.

LESSON 15 | SESSION 2

Develop Expanding Expressions

➤ **Read and try to solve the problem below.**

Are $-\frac{1}{3}(-3m + 6 - 12 + 15m)$ and $2(1 - 2m)$ equivalent expressions? Show why or why not.

TRY IT

Math Toolkit grid paper, sticky notes

Possible work:

SAMPLE A

$-\frac{1}{3}(-3m + 6 - 12 + 15m)$

$-\frac{1}{3}(-3m) + \left(-\frac{1}{3}\right)(6) - \left(-\frac{1}{3}\right)(12) + \left(-\frac{1}{3}\right)(15m)$

$m + (-2) - (-4) + (-5m)$

$-4m + 2$

$2 - 4m$

$2(1 - 2m)$

$2(1) - 2(2m)$

$2 - 4m$

The expressions are equivalent.

SAMPLE B

$-\frac{1}{3}(-3m + 6 - 12 + 15m)$

$-\frac{1}{3}(12m - 6)$

$-4m + 2$

$2(-2m + 1)$

$2(1 - 2m)$

The expressions are equivalent.

DISCUSS IT

Ask: How did you get started figuring out whether the expressions are equivalent?

Share: I started by . . .

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DISCUSS IT

SMP 2, 3, 6

Support Partner Discussion

After students work on Try It, have them respond to Discuss It with a partner. To support students in extending the conversation, prompt them to discuss this question:

- *How are the two expressions alike and different?*

Common Misconception Listen for students who combine unlike terms, such as a constant term and a variable term. As students share their strategies, ask them to identify the terms they are working with and justify the operations they are applying in each step. As students listen to one another's presentations, allow them to revisit their own strategy and solution.

Select and Sequence Student Strategies

Select 2–3 samples that represent the range of student thinking in your classroom. Here is one possible order for class discussion:

- evaluating the expressions for specific values of the variable and comparing
- **(misconception)** combining unlike terms
- distributing and combining like terms
- combining like terms and applying the distributive property

Facilitate Whole Class Discussion

Call on students to share selected strategies.

Guide students to **Compare and Connect** the representations. Ask students to take individual think time and then turn and talk to discuss their strategies for determining if the expressions are equivalent. Have students build on one another's ideas by suggesting alternatives or by proposing ways to confirm that others' strategies are accurate.

ASK *How can you follow different steps and come to the same conclusion about the two expressions?*

LISTEN FOR Changing the grouping of the terms and the order that the steps are done still keeps the expressions equivalent.

Solve It

If students presented these solutions, have students connect these solutions to those presented in class.

If no student presented at least one of these solutions, have students first analyze key features of the solutions. Then have them connect them to the solutions presented in class.

ASK *How are the two solutions similar to each other? How are they different?*

LISTEN FOR Both solutions involve combining like terms and applying the distributive property, but they do so in a different order.

For the first solution, prompt students to notice and explain the like terms.

- *How does this solution use like terms? How is identifying like terms useful in analyzing the solution?*

For the second solution, prompt students to justify some of the steps.

- *How do you know that $2 - 4m$ is equivalent to $-4m + 2$?*

LESSON 15 | SESSION 2

➤ **Explore different ways to find equivalent expressions.**

Are $-\frac{1}{3}(-3m + 6 - 12 + 15m)$ and $2(1 - 2m)$ equivalent expressions? Show why or why not.

Solve It

You can combine like terms, then expand.

$-\frac{1}{3}(-3m + 6 - 12 + 15m)$

$-\frac{1}{3}(12m - 6)$

$-\frac{1}{3}(12m) - \left(-\frac{1}{3}\right)(6)$

$-4m + 2$

$2(1 - 2m)$

$2(1) - 2(2m)$

$2 - 4m$

$-4m + 2$

Solve It

You can expand, then combine like terms.

$-\frac{1}{3}(-3m + 6 - 12 + 15m)$

$-\frac{1}{3}(-3m) + \left(-\frac{1}{3}\right)(6) - \left(-\frac{1}{3}\right)(12) + \left(-\frac{1}{3}\right)(15m)$

$m - 2 + 4 - 5m$

$-4m + 2$

$2(1 - 2m)$

$2(1) - 2(2m)$

$2 - 4m$

$-4m + 2$

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DIFFERENTIATION | EXTEND

SMP 2

Deepen Understanding

Applying the Properties of Operations to Quantitative Reasoning

Prompt students to think about the properties of operations they are using when evaluating expressions and writing equivalent expressions.

ASK *Look at the expression $2(4n - 7 + 3n - 3)$. Could you reorder the four terms in parentheses to form an equivalent expression? Why or why not?*

LISTEN FOR Yes. You can rewrite -7 and -3 as $+ (-7)$ and $+ (-3)$. Then you can add the terms in any order because of the commutative property of addition.

ASK *Which terms can you combine? Explain.*

LISTEN FOR Only like terms can be added together. A term with a variable, such as $4n$, cannot be added to a term without a variable, such as -7.

Generalize Have each student write an equivalent expression to the one above. Then have them compare their expressions to ones that others wrote. Encourage students to describe how they might choose which properties of operations to use when writing an equivalent expression.

Develop Expanding Expressions

CONNECT IT

SMP 2, 4, 5, 6

Remind students that the quantities and the relationships between them are the same in each representation. Explain that they will now use those relationships to reason about equivalent expressions.

Before students begin to record and expand on their work in Solve It, tell them that problem 3 will prepare them to provide the explanation asked for in problem 4.

Monitor and Confirm Understanding 1 – 2

- Simplifying or expanding the expressions can help show that they are equivalent.
- If two expressions are both equivalent to another expression, they are also equivalent to each other.

Facilitate Whole Class Discussion

3 Students should recognize that the expressions in this problem are related to, but slightly different from, expressions in the Solve It problem. They should compare and contrast the three expressions, identifying their similarities and differences.

> **ASK** *What is the key difference between* $-1 + \frac{1}{3}m$ and $-1 - \frac{1}{3}m$?
> **LISTEN FOR** One is a sum and one is a difference.

4 Look for the idea that the same strategies and properties apply to expanding all expressions, regardless of the signs of the terms. When the terms are negative, however, the expanded terms could be positive or negative.

5 **Reflect** Have all students focus on the strategies used to solve the Try It. If time allows, have students discuss their ideas with a partner.

CONNECT IT

Use the problem from the previous page to help you understand how to find equivalent expressions.

1 How do the **Solve Its** show that the expressions are equivalent?
Each expression is equivalent to $-4m + 2$, so they are equivalent to each other.

2 How can expanding two expressions help show that the expressions are equivalent?
Possible answer: It can help you transform one expression into the other or both expressions into the same third expression.

3 Is $-\frac{1}{3}(3 - m)$ equivalent to $-1 + \frac{1}{3}m$ or to $-1 - \frac{1}{3}m$? Explain.
$-1 + \frac{1}{3}m$; Possible explanation: $(3 - m)$ is another way of writing $[3 + (-m)]$. When you distribute $-\frac{1}{3}$, you multiply it by $-m$. That means the term $\frac{1}{3}m$ is added, not subtracted, in the equivalent expression.

4 How is expanding an expression with negative terms like expanding an expression with positive terms? How is it different?
Possible answer: Expanding an expression with negative terms is like expanding one with positive terms because you use order of operations and the properties of operations the same way. It is different because you have to remember how to multiply with negative numbers and whether products are positive or negative.

5 **Reflect** Think about all the models and strategies you have discussed today. Describe how one of them helped you better understand how to determine whether expressions are equivalent.
Responses will vary. Check student responses.

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DIFFERENTIATION | RETEACH or REINFORCE

Visual Model

Evaluate equivalent expressions.

If students are unsure about expanding expressions with negative factors, then use this activity to reinforce the concept.

- Display a table with column headers matching the three expressions in problem 3: $-\frac{1}{3}(3 - m)$; $-1 + \frac{1}{3}m$; and $-1 - \frac{1}{3}m$. Label the rows $m = 0$, $m = 3$, and $m = -3$.
- Have volunteers evaluate each expression for the values of m.
- Ask: *How does the table show you which expressions may be equivalent?* [Since equivalent expressions represent the same value and only the values in the first two columns are equal to each other, only the first two expressions may be equivalent.]
- Ask: *How can you show that* $-\frac{1}{3}(3 - m)$ *and* $-1 + \frac{1}{3}m$ *are equivalent?* [Applying the distributive property can show that the expressions are equivalent.]

Apply It

For all problems, encourage students to use a model to support their thinking. Allow some leeway in precision; remind students that two expressions are equivalent if they always name the same value, no matter what number is substituted for the variable, and that showing two expressions name the same value in one case does not prove that the expressions are equivalent.

6 Students may also solve the problem by expanding the expression on their own and comparing their expression with Raúl's expression. Students might also demonstrate that Raúl has made an error by evaluating the original expression and Raúl's expression for a specific value of x, such as $x = 100$.

7 **B is correct.** Students may find this expression by combining like terms and expanding the expression, in either order.

C is correct. Students may recognize that -4 should be distributed to each term and the order and grouping of the terms can change.

E is correct. Students should recognize that equivalent expressions may present terms in any order and that the negative sign before the first parenthesis can be interpreted as a factor of -1.

A is not correct. This answer may be the result of forgetting to multiply the factor -4 by each term inside the parentheses.

D is not correct. This answer may be the result of incorrectly applying the commutative property to the expression $-100y + 16$.

F is not correct. This answer may be the result of incorrectly finding only negative products when distributing.

Apply It

➤ **Use what you learned to solve these problems.**

6 Raúl says the expression $-25\left(\frac{1}{5}x - 20\right)$ is equivalent to $-5x - 500$. Do you agree? Explain why or why not.

No; Possible explanation: When you expand, you multiply -25 and -20, which is 500, not -500. So, an equivalent expression is $-5x + 500$.

7 Which expressions are equivalent to $-4(-25y + 4 + 50y - 8)$? Select all that apply.

A $-100y - 4$

B $-2(23y - 5 + 27y - 3)$

C $(100y + 32) - (200y + 16)$

D $-16 - 100y$

E $-(-100y + 16 + 200y - 32)$

F $-300y - 48$

8 Are the expressions $2(3x - 6)$ and $2(3x) - 6$ equivalent? Explain your reasoning.

No; Possible explanation: When you expand $2(3x - 6)$, you get $6x - 12$. When you expand $2(3x) - 6$, you get $6x - 6$. These two expressions will never have the same value.

CLOSE EXIT TICKET

8 Students' solutions should show an understanding of:

- multiplying the term outside the parentheses by each term inside the parentheses when expanding.
- how to multiply a term that is being subtracted.
- confirming when two expressions cannot be equivalent, such as by evaluating both expressions for the same value of the variable and getting different results.

Error Alert If students conclude that the two expressions are equivalent, then ask them to support their conclusion by evaluating the two expressions for a few specific values of x, such as $x = 0$ or $x = 1$.

Practice Expanding Expressions

Problem Notes

Assign **Practice Expanding Expressions** as extra practice in class or as homework.

1. Students should recognize that the product of two negative rational numbers is equal to a positive rational number and that they can apply the properties of operations to expand expressions with negative numbers. ***Basic***

2. Students may find Bianca's error by substituting the same value of y into each expression and evaluating them. ***Medium***

Practice Expanding Expressions

➤ **Study the Example showing how to decide whether two expressions are equivalent. Then solve problems 1–5.**

Example

Is $-\frac{3}{4}(-8a - 12)$ equivalent to $6a + 9$?

You can start by expanding $-\frac{3}{4}(-8a - 12)$.

$-\frac{3}{4}(-8a - 12)$

$-\frac{3}{4}[-8a + (-12)]$

$\left(-\frac{3}{4}\right)(-8a) + \left(-\frac{3}{4}\right)(-12)$

$6a + 9$

You can rewrite $-\frac{3}{4}(-8a - 12)$ as $6a + 9$. So, the two expressions are equivalent.

1. Look at the Example. How do you know that $\left(-\frac{3}{4}\right)(-8a) + \left(-\frac{3}{4}\right)(-12)$ is equivalent to both $-\frac{3}{4}(-8a - 12)$ and $6a + 9$?

   Since you can rewrite $-\frac{3}{4}(-8a - 12)$ as $\left(-\frac{3}{4}\right)(-8a) + \left(-\frac{3}{4}\right)(-12)$, they are equivalent. Since you can rewrite $\left(-\frac{3}{4}\right)(-8a) + \left(-\frac{3}{4}\right)(-12)$ as $6a + 9$, they are equivalent, too.

2. Bianca makes an error when she tries to write an expression equivalent to $12 + 15(3 - y) - 10y$. What is the error? Fix Bianca's error.

   $12 + 15(3 - y) - 10y$

   $12 + 45 + 15y - 10y$

   $57 + 5y$

   She wrote $15y$ instead of $-15y$ when she expanded. The equivalent expression should be $57 - 25y$.

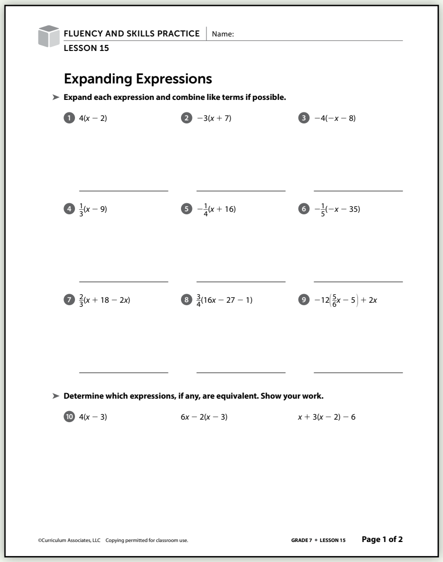
Vocabulary

equivalent expressions
two or more different expressions that always name the same value.

Fluency & Skills Practice

Expanding Expressions

In this activity, students practice expanding linear expressions and identifying whether expressions are equivalent.


FLUENCY AND SKILLS PRACTICE | Name:
LESSON 15

Expanding Expressions

➤ Expand each expression and combine like terms if possible.

1. $4(x - 2)$
2. $-3(x + 7)$
3. $-4(-x - 8)$
4. $\frac{1}{3}(x - 9)$
5. $-\frac{1}{4}(x + 16)$
6. $-\frac{1}{5}(-x - 35)$
7. $\frac{2}{3}(x + 18 - 2x)$
8. $\frac{3}{4}(16x - 27 - 1)$
9. $-12\left(\frac{5}{6}x - 5\right) + 2x$

➤ Determine which expressions, if any, are equivalent. Show your work.

10. $4(x - 3)$ $6x - 2(x - 3)$ $x + 3(x - 2) - 6$

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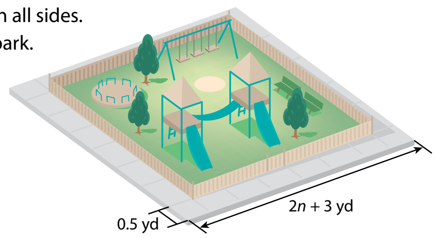
Additional Practice

3. Students may draw a picture or diagram as a model of the problem. Students may identify two of a variety of equivalent expressions for the perimeter of the playground, such as the sum of four identical addends of $2n + 2$. ***Challenge***

4. Students may choose to distribute before combining like terms to show that the expressions are equivalent. ***Basic***

5. Students should recognize that both expressions can be expanded to two terms and that the expanded expressions are the same. ***Basic***

3. A square playground is surrounded by a sidewalk on all sides. The sidewalk is $2n + 3$ yd long on each side of the park. The sidewalk is 0.5 yd wide. Write two equivalent expressions for the perimeter of the playground. Show your work.



Possible work:

Each side: $2n + 3 - 0.5 - 0.5 = 2n + 2$

Whole playground: $4(2n + 2)$ or $8n + 8$

SOLUTION **$4(2n + 2)$ and $8n + 8$, or any equivalent expression**

4. Is $-\frac{2}{3}(-12b - 6 + 9b - 18)$ equivalent to $2(b + 8)$? Show your work.

Possible work:

$-\frac{2}{3}(-12b - 6 + 9b - 18)$

$-\frac{2}{3}(-12b + 9b - 6 - 18)$

$-\frac{2}{3}(-3b - 24)$

$-\frac{2}{3}(-3b) - \left(-\frac{2}{3}\right)(24)$

$2b + 16$

$2(b + 8)$

$2b + 16$

SOLUTION **$-\frac{2}{3}(-12b - 6 + 9b - 18)$ is equivalent to $2(b + 8)$.**

5. Juanita says that $3.5[4d - (2)(1.5)]$ and $2[7d - (5)(1.05)]$ are equivalent. Is Juanita correct? Explain your reasoning.

Yes; Possible explanation: The two expressions are equivalent because both expressions are equivalent to $14d - 10.5$.

$3.5[4d - (2)(1.5)]$

$3.5(4d - 3)$

$14d - 10.5$

$2[7d - (5)(1.05)]$

$2(7d - 5.25)$

$14d - 10.5$

DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Use with **Session 3** **Apply It**

ACADEMIC VOCABULARY

To *model* (verb) is to make a representation of something.

A *model* (noun) is a representation of a system, process, or relationship.

A *model* (noun) of a piece of technology is a particular version of a product.

Levels 1–3: Reading/Listening

Support understanding of Apply It problem 6. Read the problem as students follow along. Begin a **Co-Constructed Word Bank** to clarify meanings of unknown words. Guide students to understand the meaning of *total cost* in this problem context. Review the question and discuss what *that* refers to.

Direct students' attention to the diagram and ask: *How does this model represent the problem?* Provide individual think time. Then guide students to understand the expression and connect the terms to the diagram.

Levels 2–4: Reading/Listening

Support understanding of Apply It problem 6. Read the problem and discuss the question. Discuss and clarify what *that* refers to in the question. Ask partners to make a **Co-Constructed Word Bank** of important terms that might be used in discussion. Compile the terms into a class Word Bank. Have partners discuss the significance of the phrases *total price, per month,* and *equivalent.*

Have partners turn and talk about the diagram. Then have them connect the terms in the expression to the diagram.

Levels 3–5: Reading/Listening

Support understanding of Apply It problem 6 by having partners work together to use **Three Reads** to interpret the problem. After the first read, have students tell what the problem is about. Ask them to rephrase the question. After the third read, have partners turn and talk about the total cost for three phones.

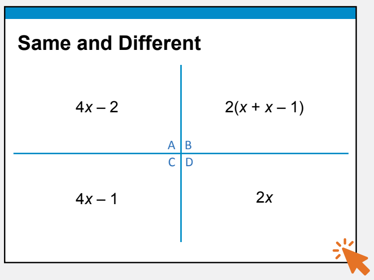
Ask partners to talk about how the diagram represents the expression. Call on students to share ideas.

Develop Factoring Expressions

Purpose

- **Develop** skills for factoring expressions with rational terms.
- **Recognize** that factoring, expanding, and combining like terms can all be applied to find equivalent expressions.

START CONNECT TO PRIOR KNOWLEDGE



Possible Solutions

All the expressions include the variable *x*.

A and B are equivalent expressions.

C and D have the same value when $x = 1$.

D has only one term.

WHY? Support students' ability to analyze expressions and identify equivalent expressions.

DEVELOP ACADEMIC LANGUAGE

WHY? Support students as they craft clear explanations using precise language.

HOW? Remind students to use precise mathematical language in their explanations. Work with students to list precise terms that might be used in discourse about factoring expressions, such as *equivalent expressions, like terms, factoring* and *variable*. During Connect It, collect and display authentic examples of clear explanations.

TRY IT

SMP 1, 2, 4, 5, 6

Make Sense of the Problem

Before students work on Try It, use **Notice and Wonder** to help them make sense of the problem. Ask students what they notice, especially about the terms and signs in the two expressions. Then ask what they wonder about that mathematics can help answer. If no student raises the issue, you may want to wonder if the expressions are equivalent.

Develop Factoring Expressions

➤ **Read and try to solve the problem below.**

Are the expressions $24c - 36 + 12 - 12c + 36 - 48c$ and $-2(18c - 6)$ equivalent? Show why or why not.

TRY IT Math Toolkit algebra tiles, grid paper

Possible work:

SAMPLE A

$24c - 36 + 12 - 12c + 36 - 48c$
$24c + (-36 + 12) - 12c + 36 - 48c$
$24c - 24 - 12c + 36 - 48c$
$(24c - 12c) - 24 + 36 - 48c$
$12c + (-24 + 36) - 48c$
$12c + 12 - 48c$
$-36c + 12$

$-2(18c - 6)$
$-2(18c) - (-2)(6)$
$-36c - (-12)$
$-36c + 12$

The expressions are equivalent.

SAMPLE B

$24c - 36 + 12 - 12c + 36 - 48c$
$24c - 12c - 48c - 36 + 12 + 36$
$(24c - 12c - 48c) + (-36 + 12 + 36)$
$-36c + 12$

$24c - 36 + 12 - 12c + 36 - 48c$
$24c + (-36 + 12) - 12c + 36 - 48c$
$24c - 24 - 12c + 36 - 48c$
$(24c - 12c) - 24 + 36 - 48c$
$12c + (-24 + 36) - 48c$
$12c + 12 - 48c$
$-36c + 12$

The expressions are equivalent.

DISCUSS IT

Ask: How did you reach that conclusion?

Share: First, I . . .

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DISCUSS IT

SMP 2, 3, 6

Support Partner Discussion

After students work on Try It, have them explain their work and respond to Discuss It with a partner.

Listen for understanding that:

- *equivalent expressions can differ in length and number of terms.*
- *combining like terms, factoring, and expanding are tools to generate equivalent expressions.*
- *equivalent expressions have the same value as each other, regardless of any given value of a variable.*

Common Misconception Listen for students who evaluate both expressions for one value of *c* and conclude that the expressions must be equivalent because they have the same value. As students share their strategies, ask how they know that the expressions will always name the same value.

Select and Sequence Student Strategies

Select 2–3 samples that represent the range of student thinking in your classroom. Here is one possible order for class discussion:

- combining like terms of the first expression and expanding the second expression
- (misconception) evaluating both expressions for one value of c
- combining like terms and then factoring

Facilitate Whole Class Discussion

Call on students to share selected strategies. Ask students to rephrase other students' explanations to demonstrate their understanding.

Guide students to **Compare and Connect** the representations. Reinforce with students that clear explanations use complete sentences and precise vocabulary.

ASK *How are [student's] and [student's] solutions the same?*

LISTEN FOR Both solutions factor, expand, and/or combine like terms to form equivalent expressions.

Solve It

If students presented these solutions, have students connect these solutions to those presented in class.

If no student presented at least one of these solutions, have students first analyze key features of the solutions and then connect them to the models presented in class.

ASK *What do all three solutions have in common? How are they different?*

LISTEN FOR All of the solutions combined like terms and applied the distributive property, but they did these things in a different sequence.

For the first solution, prompt students to explain the property used in the first step.

- *How do you know that the first two forms of the first expression are equivalent?*

For the second solution, prompt students to justify the last step of the solution.

- $-36c + 12$ is equivalent to $-2(18c - 6)$?

For the third solution, prompt students to recognize how each expression is factored.

- *What do the numbers in green in the first expression have in common? How does this help you to factor?*

➤ **Explore different ways to determine whether expressions are equivalent.**

Are the expressions $24c - 36 + 12 - 12c + 36 - 48c$ and $-2(18c - 6)$ equivalent? Show why or why not.

Solve It

You can combine like terms and then expand the expression that has parentheses.

$24c - 36 + 12 - 12c + 36 - 48c$	$-2(18c - 6)$
$(24c - 12c - 48c) + (-36 + 12 + 36)$	$-36c + 12$
$-36c + 12$	

Solve It

You can combine like terms and then factor.

$24c - 36 + 12 - 12c + 36 - 48c$	$-2(18c - 6)$
$(24c - 12c - 48c) + (-36 + 12 + 36)$	
$-36c + 12$	
$-2(18c - 6)$	

Solve It

You can factor each expression before combining like terms.

$24c - 36 + 12 - 12c + 36 - 48c$	$-2(18c - 6)$
$12(2c - 3 + 1 - c + 3 - 4c)$	$-2[-6(-3c + 1)]$
$12(-3c + 1)$	$12(-3c + 1)$

DIFFERENTIATION | EXTEND

SMP 7

Deepen Understanding

Using the Structure of an Expression to Help Rewrite It by Factoring

Prompt students to recognize how the structure of the expression can be used to generate an equivalent expression.

ASK *How were both expressions factored as a first step?*

LISTEN FOR Both expressions were factored by the greatest common factor of their terms, 12 for the first expression and 6 for the second.

ASK *Does factoring alone show that the two expressions are equivalent? Explain.*

LISTEN FOR No, because the factors of each expression were different.

ASK *Can factoring be applied when terms do not have a common factor greater than 1, such as in the expression $2x + 3$? Explain.*

LISTEN FOR Yes, though the rewritten expression could include fractions or decimals. For example, the expression $2x + 3$ could be factored as $2(x + 1.5)$.

Generalize Guide students to see that factoring is a choice that may or may not be helpful.

Develop Factoring Expressions

CONNECT IT

SMP 2, 4, 5, 6

Remind students that the quantities and the relationships between them are the same in each representation. Explain that they will now use those relationships to reason about factoring expressions.

Before students begin to record and expand on their work in Solve It, tell them that problem 3 will prepare them to provide the explanation asked for in problem 4.

Monitor and Confirm Understanding 1 – 2

- In all three solutions, the same properties of operations are applied, including the combining of like terms and the distributive property. The difference is in the order of the applications.
- In the second expression, there are two terms ($18c$ and -6) that have a greatest common factor of 6, so 6 can be factored out of their sum.

Facilitate Whole Class Discussion

3. Look for the idea that the same properties of operations apply to positive and negative numbers but the signs of the numbers may change with negative numbers. You may have students turn and talk to share their understanding and explore possible answers.

4. Look for the idea that all of these strategies are useful and can lead to equivalent expressions but that specific strategies are more efficient or reasonable to use in specific situations.

   ASK *Why is it helpful to analyze an expression before choosing a strategy?*

   LISTEN FOR You can analyze an expression to find like terms or common factors that might be helpful to form an equivalent expression.

5. **Reflect** Have all students focus on the strategies used to solve the Try It. If time allows, have students discuss their ideas with a partner.

CONNECT IT

➤ **Use the problem from the previous page to help you understand how to use the distributive property to find equivalent expressions.**

1. How does each of the **Solve Its** show that the expressions are equivalent?
   Possible answer: The first and third Solve Its transform both expressions into the same third expression. The second Solve It shows that you can rewrite the first expression as the second.

2. In the third **Solve It**, the second expression is already in factored form. Why can it be factored again?
   It can be factored again because the two terms inside the parentheses still have a common factor.

3. How is factoring an expression with negative terms similar to factoring an expression with positive terms? How is it different?
   Possible answer: Factoring with negative terms is like factoring with positive terms because you divide terms by a factor in both cases. It is different because you have to pay attention to the signs of the original number and the factors when factoring with negative terms.

4. Combining like terms, expanding, and factoring are strategies you can use to write equivalent expressions. When might you want to use each of these strategies?
   Possible answer: You might want to combine like terms when there are like terms in an expression. You might want to expand when there are terms inside parentheses that cannot be combined with other like terms until they are out of the parentheses. You might want to factor when terms have a common factor.

5. **Reflect** Think about all the models and strategies you have discussed today. Describe how one of them helped you better understand how to determine whether expressions are equivalent.
   Responses will vary. Check student responses.

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DIFFERENTIATION | RETEACH or REINFORCE

Visual Model

Factor expressions with negative coefficients.

If students are unsure about factoring expressions with negative coefficients, then use this activity to build understanding of the concept.

- Display the expression $-3\left(\frac{2}{3}x - 9\right)$. Draw a box with two columns. Ask: *Each box represents the product of the terms on the left side and the top. What terms go on the left side and the top of the box?* [-3 on the left side, $\frac{2}{3}x$ and -9 on top] Discuss why the term is -9 and not 9.
- Have volunteers find the value of each box. [$-2x$; 27.] Ask: *What is the total value of the boxes?* [the sum, $2x + 27$]
- Repeat with another example, such as $9\left(\frac{-2}{9}x + 3\right)$. Have students discuss why the expressions are equivalent.

Apply It

For all problems, encourage students to use a model to support their thinking. Remind them that an expression is a model and that there is often more than one expression that can be used.

6 See ***Connect to Culture*** to support student engagement.

A **is correct.** Students may recognize that this expression can be found by factoring 12 from each term and converting fractions to decimals before or after combining like terms.

E **is correct.** Students may recognize that this expression can be found by multiplying 12 by $\frac{3}{4}c$ to equal $9c$ and 12 by $\frac{1}{2}c$ to equal $6c$. Then factor c from the three terms.

B is not correct. This answer may be the result of confusing a multiplication expression, such as $12\left(\frac{3}{4}c\right)$, with an expression that includes a mixed number, such as $12\frac{3}{4}c$.

C is not correct. This answer may be the result of finding the product of 12 and $\left(\frac{3}{4}c\right)$ is $8c$, not $9c$.

D is not correct. This answer may be the result of adding the terms $12c$, $12\frac{3}{4}c$, and $12\frac{1}{2}c$ instead of recognizing the second and third terms as products instead of mixed numbers.

7 Students may also transform $(9g - 11 + 10g) - (12 - 11g + 13)$ into $-3(-10g + 12)$.

Apply It

➤ **Use what you learned to solve these problems.**

6 A cell phone company is having a sale. The expression $12c + 12\left(\frac{3}{4}c\right) + 12\left(\frac{1}{2}c\right)$ shows the total cost for buying 3 phones that each cost c dollars per month for 12 months.

Which expressions are equivalent to that expression? Select all that apply.

A $12(2.25c)$

B $12c + 12\frac{3}{4}c + 12\frac{1}{2}c$

C $12c + 8c + 6c$

D $37\frac{1}{4}c$

E $c(12 + 9 + 6)$

7 Are $(9g - 11 + 10g) - (12 - 11g + 13)$ and $-3(-10g + 12)$ equivalent expressions? Explain.

$(9g - 11 + 10g) - (12 - 11g + 13)$	$-3(-10g + 12)$
$9g - 11 + 10g - 12 + 11g - 13$	$-3(12 - 10g)$
$9g + 10g + 11g - 11 - 12 - 13$	$-36 + 30g$
$30g - 36$	$6(-6 + 5g)$
$6(5g - 6)$	$6(5g - 6)$

Yes; Possible explanation: You can combine like terms and factor the first expression and expand and factor the second expression to get two expressions that are the same.

8 Is $2(6 - 3x) + x$ equivalent to $2(3x) + x + 12$? Show your work. Possible work:

$2(6 - 3x) + x$

$12 - 2(3x) + x$

$-2(3x) + x + 12$

SOLUTION No, $2(6 - 3x) + x$ is not equivalent to $2(3x) + x + 12$.

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CLOSE **EXIT TICKET**

8 Students' solutions should show an understanding of:

- combining like terms to generate an equivalent expression.
- factoring positive or negative factors to generate an equivalent expression.

Error Alert If students state the two expressions are equivalent because they have the same value when $x = 0$, then ask students to substitute another value of x. Remind students that equivalent expressions must always name the same value.

Practice Factoring Expressions

Problem Notes

Assign **Practice Factoring Expressions** as extra practice in class or as homework.

1. Students may also choose to factor 3 or -3 from the expression. These yield the equivalent expressions $3(-3t + 3)$ and $-3\ (3t - 3)$. ***Basic***

2. Students may incorrectly find $-12 - 24m$ when expanding the expression. Students are likely to choose any positive or negative factor of 6 to factor the expanded expression and may write the factors in any order. ***Medium***

3. Some students may incorrectly reason that the two expressions cannot be equivalent because the first contains a variable and the second does not. ***Medium***

LESSON 15 | SESSION 3 Name:

Practice Factoring Expressions

➤ **Study the Example showing how to use factoring to write an equivalent expression. Then solve problems 1–6.**

Example

Consider the expression $(-3t + 3) + (2 - 2t) + (-4t + 4)$. Write an equivalent expression that is the product of two factors.

You can combine like terms. Then find a common factor.

$(-3t + 3) + (2 - 2t) + (-4t + 4)$

$(-3t - 2t - 4t) + (3 + 2 + 4)$

$-9t + 9$

$-9[t + (-1)]$

An equivalent expression is $-9(t - 1)$.

1. Write $-9t + 9$ as the product of two factors in a way that is not shown in the Example. Explain how you found it.

   Possible answer: $9(-t + 1)$; you can factor out 9 instead of -9 from $-9t + 9$.

2. Write an expression equivalent to $6 - 4(3 - 6m) + 12m$ that is the product of two factors. Show your work.

   Possible work:

   $6 - 4(3 - 6m) + 12m$

   $6 - 12 + 24m + 12m$

   $36m - 6$

   $6(6m - 1)$

   SOLUTION $6(6m - 1)$, $-6(1 - 6m)$, or equivalent

3. Is $1 + 4(3x - 10) - 12x$ equivalent to -39? Explain.

   Yes; Possible explanation: When you expand the first expression you get $1 + 12x - 40 - 12x$. When you combine like terms you get $1 - 40$, or -39.

Vocabulary

equivalent expressions
two or more different expressions that always name the same value.

factor (noun)
a number, or expression within parentheses, that is multiplied.

factor (verb)
to rewrite an expression as a product of factors.

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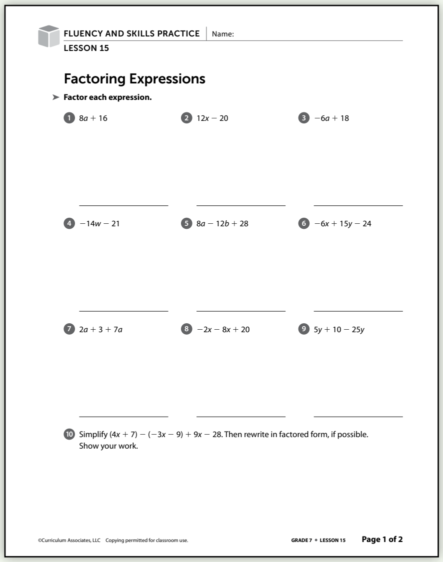
Fluency & Skills Practice

Factoring Expressions

In this activity, students practice factoring linear expressions and identifying whether expressions are equivalent.



FLUENCY AND SKILLS PRACTICE | Name:
LESSON 15

Factoring Expressions

➤ Factor each expression.

1. $8a + 16$
2. $12x - 20$
3. $-6a + 18$
4. $-14w - 21$
5. $8a - 12b + 28$
6. $-6x + 15y - 24$
7. $2a + 3 + 7a$
8. $-2x - 8x + 20$
9. $5y + 10 - 25y$
10. Simplify $(4x + 7) - (-3x - 9) + 9x - 28$. Then rewrite in factored form, if possible. Show your work.

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Additional Practice

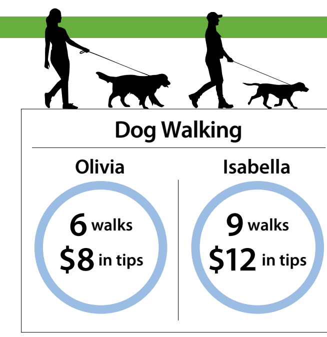
4 **a.** Students may also factor Olivia's expression to generate Isabella's expression. ***Medium***

b. Students may demonstrate their argument by evaluating each expression when $d = 12$. Both expressions show that they will earn a total of \$200. ***Medium***

5 Students may also show that the expressions are not equivalent by evaluating them for a sample value of x, such as $x = 0$. ***Challenge***

6 Students may also convert the decimal -0.75 to the fraction $\frac{3}{4}$ and apply the distributive property to expand the first expression. ***Basic***

LESSON 15 | SESSION 3



4 Olivia and Isabella each earn d dollars for each dog she walks. One day, Olivia walks 6 dogs and gets \$8 in tips. That same day, Isabella walks 9 dogs and gets \$12 in tips.

a. Olivia writes $6d + 8 + 9d + 12$ to represent the amount they earn together. Isabella writes $5(3d + 4)$. Are their expressions equivalent? Show your work.

Possible work:

$6d + 8 + 9d + 12$	$5(3d + 4)$
$6d + 9d + 8 + 12$	$5(3d) + 5(4)$
$15d + 20$	$15d + 20$

SOLUTION Their expressions are equivalent.

b. Olivia and Isabella want to find out how much money they earn altogether. Suppose they earn \$12 for each dog walk. Will you get a different amount if you evaluate Isabella's expression instead of Olivia's? Explain.

No; Possible explanation: Since the expressions are equivalent, they will have the same value when you substitute 12 for d.

5 Are $\frac{1}{2}x + \frac{3}{4} - \frac{5}{8}x - \frac{7}{8}$ and $\frac{1}{8}(x + 1)$ equivalent expressions? Show your work. Possible work:

$\frac{1}{2}x + \frac{3}{4} - \frac{5}{8}x - \frac{7}{8}$

$-\frac{1}{8}x - \frac{1}{8}$

$-\frac{1}{8}(x + 1)$

You cannot rewrite $\frac{1}{2}x + \frac{3}{4} - \frac{5}{8}x - \frac{7}{8}$ as $\frac{1}{8}(x + 1)$.

SOLUTION They are not equivalent.

6 Show that $-0.75(-4f + 12)$ and $(5f + 9) - (2f + 18)$ are equivalent expressions.

Possible answer:

$-0.75(-4f + 12)$	$(5f + 9) - (2f + 18)$
$(-0.75)(-4f) + (-0.75)(12)$	$5f + 9 - 2f - 18$
$3f - 9$	$(5f - 2f) + (9 - 18)$
	$3f - 9$

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DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Use with **Session 4** **Apply It**

Levels 1–3: Reading/Speaking

Help students interpret and discuss Apply It problem 4. Use gestures to help clarify unfamiliar words and phrases such as *fit in* and *same space*. Use classroom objects to model fitting several objects of differing sizes into the same space. Then have partners point to the model and discuss using:

- *A big/small car fits in a ____ space.*

Ask students what they need to do to solve the problem. Record the steps, rewording as appropriate. Help students build on to each other's ideas:

- *I agree. The idea makes sense because ____.*
- *I disagree. I thought about this differently. I think ____.*

Levels 2–4: Reading/Speaking

Have partners read Apply It problem 4 and use **Say It Another Way** to confirm their understanding. Ask students to identify the words used to describe the parking spaces in the problem. Then have students discuss how these words relate to the *compact space* and *full size space* shown in the model.

Have partners discuss what they need to do to solve the problem. Ask them to take turns explaining the steps and building on each other's ideas. Provide these sentence starters to support discussions.

- *First, we need to ____ Then, we can decide if ____.*
- *I agree. Your idea makes sense because ____.*
- *I disagree because ____.*

Levels 3–5: Reading/Speaking

Have students read Apply It problem 4. Have partners discuss the sizes of the parking spaces shown in the model and connect them with the language used in the problem. Then have partners **Say It Another Way** to confirm their understanding.

Ask students what steps they will use to solve the problem. Provide individual think time. Then have students turn and talk to partners about the steps. Have students take turns explaining their ideas while their partner listens actively and then builds on to the explanation. Remind students that one way to build on is to explain why they think an idea is correct or incorrect and to suggest how it can be corrected, if needed.

Refine Writing Equivalent Expressions Involving Rational Numbers

Purpose

- **Refine** strategies for using addition, subtraction, factoring, and expanding to generate equivalent expressions that have terms with rational coefficients.
- **Refine** understanding of using properties of operations for simplifying, expanding, and factoring expressions.

START CHECK FOR UNDERSTANDING

Consider this expression:

$2.5y - 6(y - 2) - 18 + 3.5y$

What equivalent expression has only one term?

Solution

-6

WHY? Confirm students' understanding of applying properties of operations to generate equivalent expressions, identifying common errors to address as needed.

MONITOR & GUIDE

Before students begin to work, use their responses to the **Start** to determine those who will benefit from additional support. Use the **Error Analysis** table below to guide remediation.

Have all students complete the Example and problems 1–3, using Consider This and Pair/Share as appropriate. Observe and monitor their reasoning and guide or redirect students as needed.

LESSON 15 | SESSION 4

Refine Writing Equivalent Expressions Involving Rational Numbers

➤ **Complete the Example below. Then solve problems 1–9.**

Example

Consider the expression $-\frac{3}{4}(-4f + 3g + 8f - 5g)$. Write an equivalent expression that is the sum of two terms.

Look at how you could rewrite an expression with more than one variable.

$-\frac{3}{4}(-4f + 3g + 8f - 5g)$

$-\frac{3}{4}(-4f + 8f + 3g - 5g)$

$-\frac{3}{4}(4f - 2g)$

SOLUTION $-3f + \frac{3}{2}g$ or $\frac{3}{2}g - 3f$

CONSIDER THIS . . . To be like terms, the terms must have the same variables.

PAIR/SHARE What would happen if you factored 4 out of $(4f - 2g)$?

Apply It

1. Are $0.5 - 3(-2x - \frac{1}{3} + 4 + 8x)$ and $-1.5(12x + 7)$ equivalent expressions? Show your work.

Possible work:

$0.5 + (-3)\left(-2x - \frac{1}{3} + 4 + 8x\right)$

$0.5 + (-3)(-2x) + (-3)\left(-\frac{1}{3}\right) + (-3)(4) + (-3)(8x)$

$0.5 + 6x + 1 - 12 - 24x$

$0.5 + 1 - 12 + 6x - 24x$

$-10.5 - 18x$

$-1.5(7 + 12x)$ or $-1.5(12x + 7)$

SOLUTION Yes, they are equivalent expressions.

CONSIDER THIS . . . Sometimes it is easier to see common factors after you combine like terms.

PAIR/SHARE What is another way you could show that your answer is correct?

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START ERROR ANALYSIS

If the error is . . .	Students may . . .	To support understanding . . .
-30	not have correctly accounted for the negative factor when they expanded the product $-6(y - 2)$.	Ask students to expand similar expressions, such as $6(y - 2)$ and $-6(y + 2)$. As they discuss their strategies and solutions, ask them to explain how they determine the sign of a product with negative factors.
$-6y$	have combined unlike terms.	Ask students to explain how they combined the terms in the expression to form a single term and to identify the variable factors for each term.
-20	have ignored the parentheses in the term $-6(y - 2)$.	Ask students to focus on the expression $-6(y - 2)$ and to identify a strategy for rewriting it. Point out that the expression is in factored form, which means that it can be rewritten by expansion.

Example

Guide students in understanding the Example. Ask:

- *How is rewriting this expression similar to rewriting an expression with only one variable? How is it different?*
- *Why is it useful to rewrite the four terms of the second factor in a new order?*
- *In the third line of the solution, can the terms 4f and 2g be combined? Why or why not?*

Help all students focus on the Example and responses to the questions by justifying each step of the solution process with the properties of operations, including the concepts of factoring and expanding expressions.

Look for understanding that rewriting expressions of two variables involves applying the same principles and strategies as rewriting expressions of one variable.

Apply It

1. Students may also expand the second expression to show that it is equivalent to the first expression. *DOK 2*

2. Students may begin by expanding the expression $-16(3x - 10)$ to form the equivalent expression $-48x + 160$ and then show that both of the given expressions are equivalent to $-48x + 160$. *DOK 2*

3. **A is correct.** Students may have expanded the product $-3(10m - 2)$ and then added $6m$ to the result.

   B is not correct. This answer may be the result of incorrectly distributing the factor -3 to the second term of the product.

   C is not correct. This answer may be the result of misinterpreting the parentheses and adding $10m$ and $6m$ and multiplying by -3.

   D is not correct. This answer may be the result of ignoring the expression in parentheses and instead multiplying -3 by $6m$.

   DOK 3

2 Are the expressions $8(9 - 6x + 11)$ and $15 + \frac{3}{2}(-32x + 120) - 35$ both equivalent to $-16(3x - 10)$? Show your work.

Possible work:

$8(9 - 6x + 11)$	$15 + \frac{3}{2}(-32x + 120) - 35$
$8(20 - 6x)$	$\frac{3}{2}(-32x + 120) - 20$
$-16(-10 + 3x)$	$-48x + 180 - 20$
$-16(3x - 10)$	$-48x + 160$
	$-16(3x - 10)$

SOLUTION Yes, they are both equivalent to $-16(3x - 10)$.

CONSIDER THIS . . .
You can start by expanding, factoring, or combining like terms.

PAIR/SHARE
Is it possible for both expressions to be equivalent to $-16(3x - 10)$ but not to each other? Why or why not?

3 Which expression is equivalent to $-3(10m - 2) + (3 + 6m - 3)$?

A $-24m + 6$

B $-24m - 6$

C $-48m + 6$

D $-18m$

Evelyn chose A as the correct answer. How might she have gotten that answer?

Possible answer: Evelyn distributed the -3 to the terms in the first set of parentheses to get $-30m + 6$. Then she added within the second set of parentheses to get $6m$. Then she added $6m$ to $-30m + 6$.

CONSIDER THIS . . .
The product of two negative numbers is positive.

PAIR/SHARE
How could you check your answer?

GROUP & DIFFERENTIATE

Identify groupings for differentiation based on the **Start** and problems 1–3. A recommended sequence of activities for each group is suggested below. Use the resources on the next page to differentiate and close the lesson.

Approaching Proficiency
- **RETEACH** Hands-On Activity
- **REINFORCE** Problems 4, 6, 7

Meeting Proficiency
- **REINFORCE** Problems 4–8

Extending Beyond Proficiency
- **REINFORCE** Problems 4–8
- **EXTEND** Challenge

Have all students complete the **Close: Exit Ticket**.

Resources for Differentiation are found on the next page.

Refine Writing Equivalent Expressions Involving Rational Numbers

Apply It

4 See ***Connect to Culture*** to support student engagement. Students should recognize that they can write one expression in terms of w to describe the current arrangement of parking spaces and another to describe the proposed new arrangement. Then they can check whether those expressions are equivalent.
DOK 2

5 Students may support their answer by evaluating both expressions for possible values.
DOK 3

6 **A** **is correct.** Students may show the equivalent expressions by combining the like terms in the second factor and by distributing x, but not $\frac{1}{5}$, to the original expression.

F **is correct.** Students may distribute $\frac{1}{5}$, but not x, to the original expression.

B is not correct. This answer may be the result of distributing $\frac{1}{5}x$ to the first term but only x to the second term.

C is not correct. This answer may be the result of forgetting the factor x in the original expression.

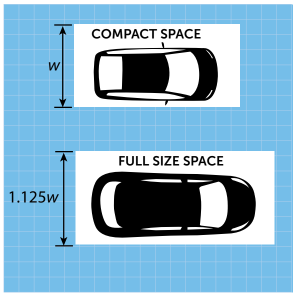
D is not correct. This answer may be the result of replacing the factor $\frac{1}{5}x$ in the original expression with $5x$.

E is not correct. This answer may be the result of combining unlike terms.

DOK 3

4 In front of a store, there is a row of parking spaces. Cars park parallel to one another, with the front of each car facing the store. Currently there are 10 compact spaces and 12 full size spaces. The store owners think they can repaint in the same space to fit 16 compact spaces and 9 full size spaces. The width of each type of space will not change. Are the store owners correct? Explain.

PARKING SPACE PLAN



No; Possible explanation: The width of the current row of parking spaces can be represented as $10w + 12(1.125w)$, or $23.5w$. Under the new plan, the width of the row of parking spaces is $16w + 9(1.125w)$, or $26.125w$. Since $23.5w$ and $26.125w$ are not equivalent expressions, the space needed for the new plan is not the same. Since there is no positive value of w where $26.125w$ will be less than $23.5w$, the plan will not work.

5 The variable z represents a positive integer. Does $4 + 3(2z - 5)$ represent a number that is *greater than*, *less than*, or *equal to* $2(3z - 4)$? Show your work.

Possible work:

$4 + 3(2z - 5)$	$2(3z - 4)$
$4 + 6z - 15$	$6z - 8$
$6z - 11$	

When z is positive, $6z - 11$ has to be less than $6z - 8$.

SOLUTION $4 + 3(2z - 5)$ is less than $2(3z - 4)$.

6 Which expressions are equivalent to $\frac{1}{5}x(5y + 60)$? Select all that apply.

A $\frac{1}{5}(2xy + 20x + 3xy + 40x)$

B $xy + 60x$

C $y + 12x$

D $25xy + 300x$

E $13xy$

F $x(y + 12)$

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DIFFERENTIATION

RETEACH

Hands-On Activity

Apply a model to generate equivalent expressions in two variables.

Students approaching proficiency with generating equivalent expressions with two variables will benefit from applying a model of the process.

Materials For each pair: 10 counters, 10 unit cubes

- Write these instructions on the board and reveal one step at a time as pairs follow along: Start with 8 counters. Add 4 cubes. Remove 3 counters. Remove 2 cubes. Add 1 counter. Finally, remove half of each type of object.
- Have students write an algebraic expression to represent the set of instructions they followed. Have them use x and y to represent the counters and cubes. Have volunteers write their expressions on the board. [$(8x + 4y - 3x - 2y + x) \div 2$]
- Next, ask students to apply the properties of operations to write an equivalent expression in only two terms. [$3x + y$]
- Ask: *How does the activity model the equivalent expression you wrote?* [The end result of the instructions is 3 counters and 1 cube as expressed by $3x + y$.]
- Have students suggest revised numbers of counters and cubes and repeat the activity. Ask: *Will the equivalent expression always match the end result of the instructions? Why or why not?* [Yes, because the original expression represents the instructions and the equivalent expression is the simplified form of the instructions.]

7 **a.** Both expressions are equivalent to $-30a + 24$.

b. The first expression is equivalent to $2\frac{1}{2}a - 6$ and the second to $-2\frac{1}{2}a - 6$.

c. The second expression is equivalent to $-3a - 0.75$.

d. Both expressions are equivalent to $-12a - 16$.

DOK 2

8 Students may evaluate the expressions for other values of x. *DOK 3*

CLOSE EXIT TICKET

9 **Math Journal** Look for an accurate application of the properties of operations, including combining like terms and the distributive property.

Error Alert If students combine $-\frac{2}{3}$ and $\frac{2}{3}$ to form either $\frac{4}{3}$ or $-\frac{4}{3}$, then have them represent the sum with a model.

End of Lesson Checklist

INTERACTIVE GLOSSARY Support students by suggesting an example of the use of *factor* as a verb, such as factoring the expression $5x - 10$.

SELF CHECK Have students review and check off any new skills on the Unit 3 Opener.

7 Tell whether each statement is *True* or *False*.

	True	False
a. $6(-5a + 4)$ and $3(-10a + 8)$ are equivalent expressions.	●	○
b. $\frac{1}{6}(15a - 36)$ and $\frac{1}{2}a - (3a + 6)$ are equivalent expressions.	○	●
c. $-3a + 1.5$ and $-0.5(6a + 1.5)$ are equivalent expressions.	○	●
d. $-4(a - 4 + 2a + 8)$ and $-4(a - 4) - 4(2a + 8)$ are equivalent expressions.	●	○

8 Kazuko says the expressions $5x$ and $6 - x$ are equivalent expressions, because you can substitute 1 for x in both expressions and get the same result. Is Kazuko's reasoning correct? Explain.

No; Possible explanation: If you substitute any other number for x, the expressions do not have the same value. For example, if you substitute 2 for x, you get 10 for the first expression and 4 for the second. Equivalent expressions always name the same value no matter what value you substitute for the variable.

9 **Math Journal** Start with the expression $12 - \frac{3}{4}\left(8f - \frac{5}{3} + 4f + \frac{2}{3}\right)$. Write an equivalent expression, and explain why it is equivalent. Then write an expression that is not equivalent and explain why it is not equivalent.

Possible answer:

Equivalent: $12 - \frac{3}{4}(12f - 1)$

Not equivalent: $12 - 6f - \frac{5}{3} + 4f + \frac{2}{3}$

The first expression is equivalent because the like terms within the parentheses were combined. The second expression is not equivalent because only the first term within the parentheses was multiplied by $-\frac{3}{4}$.

End of Lesson Checklist

- ☐ **INTERACTIVE GLOSSARY** Find the entry for *factor* (*verb*). Add two things you learned about factoring in this lesson.
- ☐ **SELF CHECK** Go back to the Unit 4 Opener and see what you can check off.

REINFORCE

Problems 4–8

Find equivalent expressions.

Students meeting proficiency will benefit from additional work with generating equivalent expressions by solving problems in a variety of formats.

- Have students work on their own or with a partner to solve the problems.
- Encourage students to show their work.

EXTEND

Challenge

Generate equivalent expressions.

Students extending beyond proficiency will benefit from generating equivalent expressions that meet specific criteria.

- Have students write an expression of only one term, such as 5, $13x$, or $20y$.
- Challenge students to write sets of equivalent expressions that have various numbers of terms, from 2 to 5, as well as equivalent expressions that include one, two, or three sets of parentheses.

PERSONALIZE

i-Ready

Provide students with opportunities to work on their personalized instruction path with *i-Ready* Online Instruction to:

- fill prerequisite gaps.
- build up grade-level skills.

Overview | Write and Solve Multi-Step Equations

STANDARDS FOR MATHEMATICAL PRACTICE (SMP)

SMP 1, 2, 3, 4, 5, and 6 are integrated into the Try-Discuss-Connect routine.*

This lesson provides additional support for:

7 Look for and make use of structure.

* See page 1q to learn how every lesson includes these SMP.

Objectives

Content Objectives

- Represent mathematical and real-world problems with equations of the form $px + q = r$ and $p(x + q) = r$ (where p, q, and r are nonzero rational numbers).
- Solve equations of the form $px + q = r$ and $p(x + q) = r$ algebraically and arithmetically.
- Solve problems involving rational numbers.

Language Objectives

- Interpret word problems by analyzing relationships between quantities and representing them with equations.
- Compare algebraic and arithmetic solutions using the lesson vocabulary.
- Respond to clarifying questions about problems involving rational numbers using complete sentences.
- Describe a way to test whether a strategy is true using sentence frames such as "I could test this strategy by _____."

Prior Knowledge

- Write and solve one-step equations.
- Understand that both sides of an equation are equal, and whatever operation is performed on one side of the equation must be done on the other side to maintain equality.
- Expand and factor linear expressions by applying the distributive property.
- Simplify expressions by combining like terms.

Vocabulary

Math Vocabulary

There is no new vocabulary. Review the following key terms.

coefficient a number that is multiplied by a variable.

like terms two or more terms that have the same variable factors.

product the result of multiplication.

unknown the value you need to find to solve a problem.

variable a letter that represents an unknown number. In some cases, a variable may represent more than one number.

Academic Vocabulary

represent to use a sign, symbol, or example for something.

Learning Progression

In Grade 6 students learned to solve one-step equations and operate with positive rational numbers.

Earlier in Grade 7 students applied the properties of operations to analyze and identify equivalent expressions. They also wrote expressions to represent situations.

In the previous lesson students developed an understanding of how to use hanger diagrams and reasoning to solve multi-step equations with integer coefficients.

In this lesson students will build on their understanding of using reasoning to solve equations of the form $px + q = r$ and $p(x + q) = r$ to solve equations algebraically. They will also write equations to represent real-world and mathematical problems. They will explore multiple ways to approach solving multi-step equations based on the numbers in the problems. They also compare solving multi-step problems arithmetically and algebraically.

Later in Grade 7 students will extend their understanding of writing and solving multi-step equations to write and solve inequalities that represent real-world and mathematical problems. They will also solve problems involving percents and unknown measurements in geometric figures.

In Grade 8 students will solve equations that have one, infinitely many, or no solutions. They will also solve equations that involve exponents and solve systems of equations that involve more than one variable.

Pacing Guide

Items marked with are available on the **Teacher Toolbox**.

	MATERIALS	DIFFERENTIATION
SESSION 1 **Explore** Solving Multi-Step Equations (35–50 min)		
• **Start** (5 min) • **Try It** (5–10 min) • **Discuss It** (10–15 min) • **Connect It** (10–15 min) • **Close: Exit Ticket** (5 min) **Additional Practice** (pages 357–358)	**Math Toolkit** algebra tiles, grid paper, number lines, sticky notes Presentation Slides	**PREPARE** Interactive Tutorial **RETEACH or REINFORCE** Visual Model
SESSION 2 **Develop** Writing and Solving Equations With Two or More Addends (45–60 min)		
• **Start** (5 min) • **Try It** (10–15 min) • **Discuss It** (10–15 min) • **Connect It** (15–20 min) • **Close: Exit Ticket** (5 min) **Additional Practice** (pages 363–364)	**Math Toolkit** algebra tiles, grid paper, number lines, sticky notes Presentation Slides	**RETEACH or REINFORCE** Hands-On Activity **Materials** For each pair: algebra tiles (6 x-tiles, 26 1-tiles) **REINFORCE** Fluency & Skills Practice **EXTEND** Deepen Understanding
SESSION 3 **Develop** Solving Equations with Grouping Symbols (45–60 min)		
• **Start** (5 min) • **Try It** (10–15 min) • **Discuss It** (10–15 min) • **Connect It** (15–20 min) • **Close: Exit Ticket** (5 min) **Additional Practice** (pages 369–370)	**Math Toolkit** grid paper, number lines, sticky notes Presentation Slides	**RETEACH or REINFORCE** Hands-On Activity **Materials** For each pair: algebra tiles (10 x-tiles, 21 1-tiles) **REINFORCE** Fluency & Skills Practice **EXTEND** Deepen Understanding
SESSION 4 **Refine** Writing and Solving Multi-Step Equations (45–60 min)		
• **Start** (5 min) • **Monitor & Guide** (15–20 min) • **Group & Differentiate** (20–30 min) • **Close: Exit Ticket** (5 min)	**Math Toolkit** Have items from previous sessions available for students. Presentation Slides	**RETEACH** Hands-On Activity **Materials** For each pair: algebra tiles (12 x-tiles, 48 1-tiles) **REINFORCE** Problems 4–8 **EXTEND** Challenge **PERSONALIZE** i-Ready
Lesson 18 Quiz or **Digital Comprehension Check**		**RETEACH** Tools for Instruction **REINFORCE** Math Center Activity **EXTEND** Enrichment Activity

Overview | Write and Solve Multi-Step Equations

Connect to Culture

Use these activities to connect with and leverage the diverse backgrounds and experiences of all students. Engage students in sharing what they know about contexts before you add the information given here.

SESSION 1

Try It Ask students to share their stories about the Purim holiday or ask questions they may have about the holiday. The Jewish holiday of Purim is celebrated on a specific date in the Hebrew calendar that generally falls in early spring. Purim commemorates a story of a triumph of the Jewish people in ancient Persia. An evil prime minister named Haman was plotting to kill the Jews of the country. Haman was defeated by the heroes Esther and Mordechai. Today, Jews celebrate Purim with carnivals and pageants. One of the traditional Purim delicacies is a triangle-shaped pastry called a hamentashen, named after the villain of the story. Ask students if they know of other holidays celebrated with pageants or special foods.

SESSION 2

Try It A lot of what you see on the stage of a play or in a movie may look real but is actaully fake. Houses and other buildings are usually just a façade with nothing behind them. Lots of things, from walls and furniture to rocks and trees, are made of styrofoam. Canned frosting is used in place of ice cream as it doesn't melt as quickly. Windows are sometimes made of sheets of sugar candy. That way if someone falls through a window, it shatters like glass but without having sharp or dangerous pieces. Ask for a show of hands of students who have either seen a play performed at a theater acted in a play at school or elsewhere, and/or worked behind the scenes on a play, maybe on props or scenery!

SESSION 3

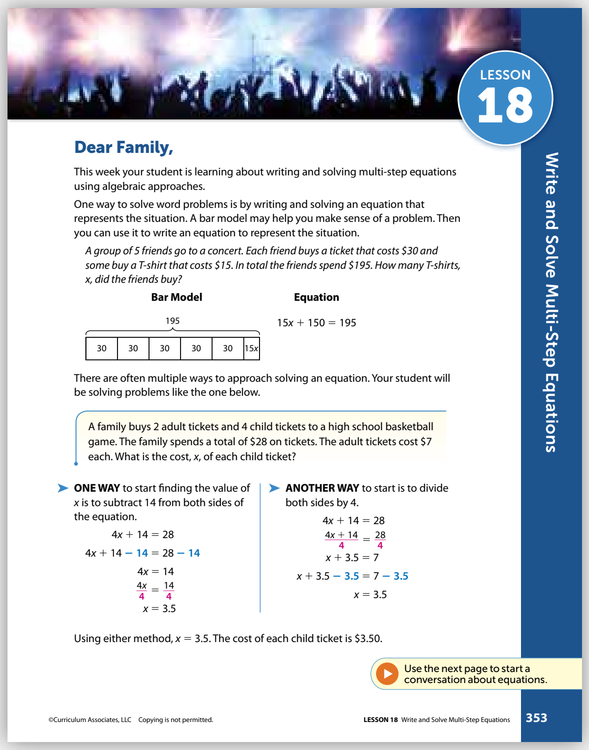
Try It In 1714, German physicist Daniel Fahrenheit developed the first modern thermometer, which used a column of mercury to indicate temperature. He also devised the temperature scale that is now named after him. The scale sets the freezing point of water at 32°F and the boiling point of water at 212°F. Later the Swedish astronomer Anders Celsius developed the Celsius scale. On the Celsius scale these two values are 0°C and 100°C. Today, most nations use the Celsius scale to describe the weather and other temperatures, and scientists use this scale as well. The Fahrenheit scale, however, remains popular and widely used in the United States. Ask students if they have used the Celsius scale in science classes or for other purposes, and how they think using it compares to using the Fahrenheit scale.

SESSION 4

Apply It **Problem 7** On a long hike through the wilderness, a hiker will need to carry food to eat, water to drink, and maybe camping equipment too! The heavier the backpack, the more energy needed to carry it—especially uphill. Experts advise hikers to carry along only the things they need, and to try to keep the weight to a minimum. One suggestion is to keep the weight of the backpack less than 20 percent of your body weight for a long hike, or less than 10 percent for a day trip. The good news for hikers is that very lightweight gear is available. Ask students to describe their experiences on hikes or walks, including any unusual or cumbersome objects that they carried.

Connect to Family and Community

➤ **After the Explore session, have students use the Family Letter to let their families know what they are learning and to encourage family involvement.**



LESSON 18

Write and Solve Multi-Step Equations

Dear Family,

This week your student is learning about writing and solving multi-step equations using algebraic approaches.

One way to solve word problems is by writing and solving an equation that represents the situation. A bar model may help you make sense of a problem. Then you can use it to write an equation to represent the situation.

A group of 5 friends go to a concert. Each friend buys a ticket that costs $30 and some buy a T-shirt that costs $15. In total the friends spend $195. How many T-shirts, x, did the friends buy?

Bar Model

195

30	30	30	30	30	15x

Equation

$15x + 150 = 195$

There are often multiple ways to approach solving an equation. Your student will be solving problems like the one below.

A family buys 2 adult tickets and 4 child tickets to a high school basketball game. The family spends a total of $28 on tickets. The adult tickets cost $7 each. What is the cost, x, of each child ticket?

➤ **ONE WAY** to start finding the value of x is to subtract 14 from both sides of the equation.

$$4x + 14 = 28$$
$$4x + 14 - 14 = 28 - 14$$
$$4x = 14$$
$$\frac{4x}{4} = \frac{14}{4}$$
$$x = 3.5$$

➤ **ANOTHER WAY** to start is to divide both sides by 4.

$$4x + 14 = 28$$
$$\frac{4x + 14}{4} = \frac{28}{4}$$
$$x + 3.5 = 7$$
$$x + 3.5 - 3.5 = 7 - 3.5$$
$$x = 3.5$$

Using either method, $x = 3.5$. The cost of each child ticket is $3.50.

Use the next page to start a conversation about equations.

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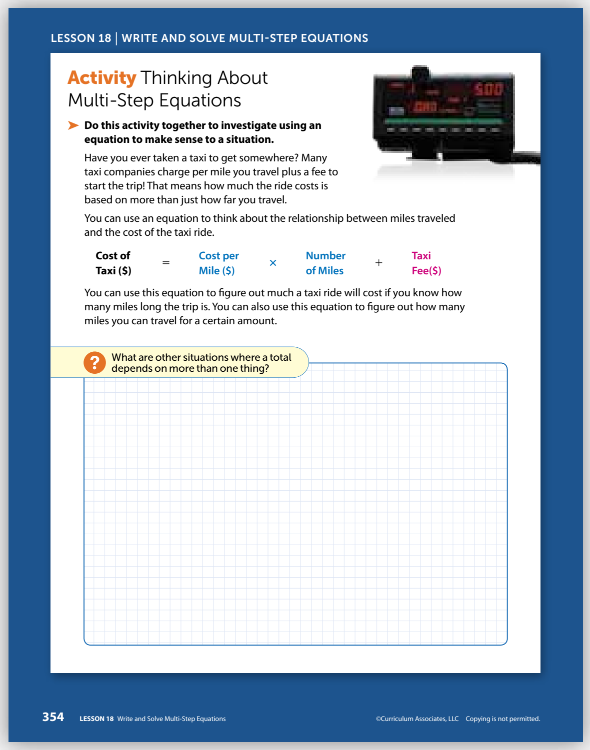

LESSON 18 | WRITE AND SOLVE MULTI-STEP EQUATIONS

Activity Thinking About Multi-Step Equations

➤ **Do this activity together to investigate using an equation to make sense to a situation.**

Have you ever taken a taxi to get somewhere? Many taxi companies charge per mile you travel plus a fee to start the trip! That means how much the ride costs is based on more than just how far you travel.

You can use an equation to think about the relationship between miles traveled and the cost of the taxi ride.

Cost of Taxi ($) = Cost per Mile ($) × Number of Miles + Taxi Fee($)

You can use this equation to figure out much a taxi ride will cost if you know how many miles long the trip is. You can also use this equation to figure out how many miles you can travel for a certain amount.

? What are other situations where a total depends on more than one thing?

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Connect to Language

➤ **For English language learners, use the Differentiation chart to scaffold the language in each session. Use the Academic Vocabulary routine for academic terms before Session 1.**

DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS Use with **Session 1** Connect It

ACADEMIC VOCABULARY

An *approach* is a procedure or a way to do something. An approach can be a sequence of steps.

To *reason* is to think about something using logic in order to come to a conclusion.

Levels 1–3: Reading/Speaking

Before reading Connect It problem 2a, display the arithmetic and algebraic approaches. Have students state what they **Notice and Wonder.** Allow them to support their statements by pointing to the approaches and using markers to color code the numbers and symbol in the equations, when needed. Reword unclear statements and record.

Read the problem with students. Define the Academic Vocabulary and clarify *arithmetically* and *algebraically*. Help students connect the questions to their Notice and Wonder statements.

Levels 2–4: Reading/Speaking

Before reading Connect It problem 2a, display the arithmetic and algebraic approaches. Review *unknown, variable,* and the Academic Vocabulary. Have students use these words to state what they **Notice and Wonder** about the approaches. Record statements for later reference. Encourage students to compare the approaches using:

- *I notice that _____, but _____.*
- *I notice that both approaches _____.*

Read the problem with students. Clarify the terms *arithmetically* and *algebraically*. Help students connect statements from **Notice and Wonder** to the problem.

Levels 3–5: Reading/Speaking

Before students read Connect It problem 2a, ask them to use **Notice and Wonder** to compare the two approaches. Provide think time before calling on students to share their ideas. Record statements for later reference.

Have partners read the problem and discuss the meanings of the Academic Vocabulary. Ask students to discuss how identifying base words and word parts helps with *arithmetically, algebraically, unknown,* and *variable.*

Have partners read the **Notice and Wonder** statements and discuss how the ideas connect to the problem.

Explore Solving Multi-Step Equations

Purpose

- **Explore** the idea that an equation can be solved using either arithmetic or algebra reasoning.
- **Understand** that different ways of reasoning about solving a problem can lead to different approaches to solving the equation.

START CONNECT TO PRIOR KNOWLEDGE

Same and Different

A: $4a = 24$	B: $0.5b = 3$
C: $c - 1.2 = 4.8$	D: $d + 5.5 = 11.5$

Possible Solutions

All are equations with solution 6.

All are equations that can be solved with one step.

All but A have decimals.

A and B can both be solved using multiplication or division.

WHY? Support students' ability to compare and solve one-step equations.

TRY IT

SMP 1, 2, 4, 5, 6

Make Sense of the Problem

See ***Connect to Culture*** to support student engagement. Before students work on Try It, use **Co-Craft Questions** to help them make sense of the problem. Allow think time before asking for possible questions, and list students' suggestions on the board. If time permits at the end of this session, have students choose one or two questions from the list to answer.

DISCUSS IT

SMP 2, 3, 6

Support Partner Discussion

After students work on Try It, have them respond to Discuss It with a partner. Listen for understanding of:

- the multiplicative relationship between the number of pictures taken by Santo and Adela, and the additive relationship between the number of pictures taken by Adela and Rachel.
- strategies for using those relationships to find the number of pictures Rachel takes.

LESSON 18 | SESSION 1 ■ □ □ □

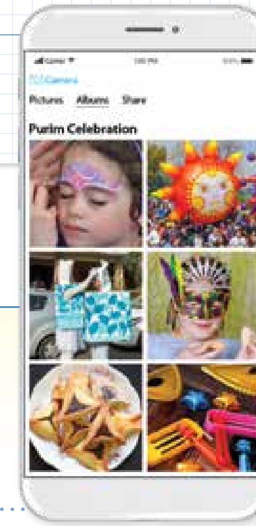
Explore Solving Multi-Step Equations

Previously, you learned how to reason about equations to find unknown values. In this lesson, you will learn about solving equations algebraically.

➤ **Use what you know to try to solve the problem below.**

Adela, Rachel, and Santo take pictures at a Purim celebration.
- Adela takes 7 more pictures than Rachel.
- Santo takes 4 times as many pictures as Adela.
- Santo takes 48 pictures.

How many pictures does Rachel take?



TRY IT Math Toolkit algebra tiles, grid paper, number lines, sticky notes

Possible work:

SAMPLE A

x = number of pictures Rachel takes

$x + 7$ = number of pictures Adela takes

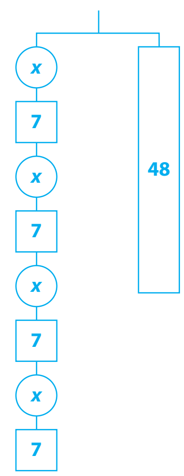
$4(x + 7)$ = number of pictures Santo takes

48 = number of pictures Santo takes

Since $4(x + 7) = 48$ and $4 \cdot 12 = 48$, $x + 7$ must be 12.

Since $x + 7 = 12$ and $5 + 7 = 12$, x must be 5.

Rachel takes 5 pictures.



SAMPLE B

Santo takes 48 pictures. This is 4 times as many as Adela. Since $48 \div 4 = 12$, Adela takes 12 pictures.

Adela takes 7 more pictures than Rachel.

Since $12 - 7 = 5$, Rachel takes 5 pictures.

DISCUSS IT

Ask: What did you do first to find the number of pictures Rachel takes? Why?

Share: I started by . . . because . . .

Learning Targets SMP 1, SMP 2, SMP 3, SMP 4, SMP 5, SMP 6, SMP 7

Use variables to represent quantities and construct simple equations to solve problems.
- Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach.

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Common Misconception Listen for students who confuse additive and multiplicative comparisons, interpreting the phrase *4 times as many* as *4 more than*, or interpreting *7 more than* as *7 times as many*. As students share their strategies, ask them to compare their answers with other students' and to think about what they might have done differently. Facilitate discussion to support students in helping one another clearly differentiate additive and multiplicative comparisons.

Select and Sequence Student Strategies

Select 2–3 samples that represent the range of student thinking in your classroom. Here is one possible order for class discussion:

- drawing a hanger diagram to represent the known and unknown values and the relationships among them
- (misconception) solving for the number of Santo's pictures as the number of Adela's pictures plus 4, instead of times 4
- reasoning arithmetically to identify the unknown quantities
- writing equations with variables to represent unknowns

Facilitate Whole Class Discussion

Call on students to share selected strategies. As they listen to presentations, remind students that they can add to a strategy they agree with by giving additional reasons that the strategy is successful or by describing ways the strategies are alike or different.

Guide students to **Compare and Connect** the representations. Listen for understanding of inverse relationships, i.e., how a relationship of *4 times greater* could also be expressed using division, and that a relationship of *4 more* could also be expressed using subtraction.

ASK *How do the strategies of [student name] and [student name] compare with each other?*

LISTEN FOR All successful strategies show how the number of pictures each person takes relate to each other. But may take different approaches toward answering the question.

CONNECT IT

SMP 2, 4, 5

1 **Look Back** Look for understanding that identifying the number of pictures Adela took, based on the relationship to the number of photos Santo took, is a step in finding the number of pictures Rachel took.

DIFFERENTIATION | RETEACH or REINFORCE

Visual Model

Identify and model mathematical relationships.

If students are unsure about making connections between related quantities, then use this activity to support their thinking.

- Display this problem: *A store has 2 fewer frogs than lizards, and 3 times as many snails as frogs. There are 6 snails. How many frogs and lizards are there?*
- Ask a volunteer to underline two facts about frogs.
- Draw a bar model to show the relationship between frogs and snails. Ask students how the model shows the relationship.
- Ask: *If you know the number of snails, how can you find the number of frogs?* [Divide the number of snails by 3.] *If you know the number of frogs, how can you find the number of snails?* [Multiply the number of frogs by 3.]
- Add the fact about snails to the bar model. Ask: *What equation can you write to relate the number of frogs, f, and snails?* [$3f = 6$ or $6 \div 3 = f$]
- Repeat these steps for the facts about lizards.

LESSON 18 | SESSION 1

CONNECT IT

1 **Look Back** How many pictures do Adela and Rachel each take? How do you know?

Adela takes 12 pictures and Rachel takes 5; Possible explanation: Santo takes 48 pictures, which is 4 times as many as Adela. Since $4 \cdot 12 = 48$, Adela takes 12 pictures. Adela takes 7 pictures more than Rachel, and $12 - 7 = 5$, so Rachel takes 5 pictures.

2 **Look Ahead** One way to find the number of photos Adela and Rachel each take is to reason about the quantities arithmetically. Another way is to solve an equation algebraically. Look at two ways you could find the unknown in the statement *the product of 6 and a number, n, plus 4 is 22.*

Arithmetic Approach	Algebraic Approach
Think: What number is 4 less than 22?	$6n + 4 = 22$
Step 1: $22 - 4 = 18$	**Step 1:** $6n + 4 - 4 = 22 - 4$
Think: What number times 6 is 18?	$6n = 18$
Step 2: $18 \div 6 = 3$	**Step 2:** $6n \div 6 = 18 \div 6$
The number is 3.	$n = 3$

a. How is Step 1 in the arithmetic approach like Step 1 in the algebraic approach?
You subtract 4 from 22 in both approaches.

b. How is Step 2 in the arithmetic approach like Step 2 in the algebraic approach?
You divide 18 by 6 in both approaches.

c. Why do both approaches lead to the same solution?
Possible answer: In both approaches you use the same operations in the same order to solve for the unknown.

3 **Reflect** How is the algebraic approach similar to the arithmetic approach? How is it different?

Possible answer: Both approaches work backward using inverse operations to solve the problem. The algebraic approach uses a variable to stand for the unknown.

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2 **Look Ahead** As students review the two solutions to the problem, point out the pairs of identical operations between them. Students should begin to recognize the similarities between the algebraic and arithmetic approaches to solving equations and problems.

CLOSE EXIT TICKET

3 **Reflect** Look for understanding the use of inverse operations to find solutions in both approaches and of the use of variables in the algebraic approach but not in the arithmetic approach.

Common Misconception If students do not recognize the pairs of inverse operations presented in the solutions (addition and subtraction; multiplication and division), then ask students to compare the use of the phrases *plus 4* and *product of 6 and a number* in the problem statement with the corresponding phrases in the solution. Ask students to explain why they think that the solution uses subtraction and division, while the problem statement includes addition and multiplication.

Prepare for Writing and Solving Multi-Step Equations

Support Vocabulary Development

Assign **Prepare for Writing and Solving Multi-Step Equations** as extra practice in class or as homework.

If you have students complete this in class, then use the guidance below.

Ask students to consider the term *like terms.* Encourage them to think of specific examples of like terms to clarify their definitions or improve their explanations.

Have students work in pairs to complete the graphic organizer. Invite pairs to share their completed organizers, and prompt a whole-class comparative discussion of the definitions, examples, and non-examples given.

Have students review Rosa's and Tiffany's statements in problem 2 and discuss with a partner whether the examples in the problem are like terms that can be combined. Encourage students to refer to the examples they listed in the graphic organizer to help them construct their response.

Problem Notes

1. Students should understand that a term may include a number, a variable, or a product of numbers and variables. Two or more terms are classified as *like terms* when their variable components are exactly the same. This allows the like terms to be combined.
2. Students should recognize that $4x$ and $-6x$ each include the same variable, x, so they are like terms that could be combined. The fact that one coefficient is positive and the other is negative does not affect their classification as like terms. Students should also recognize that $5a$ and $5b$ are not like terms because the variables are different.

LESSON 18 | SESSION 1 Name:

Prepare for Writing and Solving Multi-Step Equations

1. Think about what you know about the like terms in an expression. Fill in each box. Use words, numbers, and pictures. Show as many ideas as you can. Possible answers:

like terms

What Is It?	What I Know About It
Terms that have the same variable or variables	The terms b and $2b$ are like terms because the only thing different is their coefficients. The terms b and b^2 are not like terms because they have different exponents.

Examples	Non-Examples
$2, -9, 1\frac{5}{6}$, and -0.37 $6x$ and $12x$ $\frac{1}{2}b$ and $-b$	7 and m $2x$ and $2y$ -1 and $-n$

2. Rosa says $4x$ and $-6x$ are like terms, so they can be combined. Tiffany says $5a$ and $5b$ are like terms, so they can be combined. Is Rosa correct? Is Tiffany correct? Why or why not?

   No, only Rosa is correct; Possible explanation: Both $4x$ and $-6x$ have the same variable, x. Since $5a$ and $5b$ have different variables, they are not like terms.

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REAL-WORLD CONNECTION

Merchants can use algebraic expressions to calculate or predict their profit, which is the difference between income and expenses. The break-even point is where the profit and expenses are equal. For example, suppose it costs an online merchant \$9 to make a hat and \$40 a week to run the business. If one week the merchant gets orders for 9 hats, the merchant could use the equation $0 = 9(8) + 40 + 8p$ to find what the selling price, p, would need to be to break even that week.

Ask students to think of other examples where using a multi-step equations would be helpful.

Additional Practice

3 Problem 3 provides another look at identifying an unknown quantity based on mathematical relationships. This problem is similar to the problem about the three friends and their pictures. In both problems, three quantities are related by an additive or multiplicative relationship, and students are asked to determine one of the unknown quantities based on the information provided.

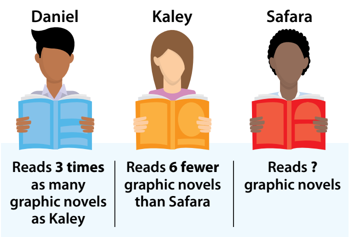
Students may solve the problem by writing and solving algebraic equations, applying arithmetic reasoning, or modeling the quantities with counters, algebra tiles, or a drawing.

Suggest that students use **Say It Another Way** to make sense of the relationships among the three readers, the meanings of the numbers in the problem, and the question being asked.

LESSON 18 | SESSION 1

3 Kaley, Safara, and Daniel keep track of how many graphic novels they read over the summer.

- Kaley reads 6 graphic novels fewer than Safara.
- Daniel reads 3 times as many as Kaley.
- Daniel reads 30 graphic novels.



a. How many graphic novels does Safara read? Show your work.

Possible work:

x = the number of graphic novels Safara reads

$x - 6$ = the number of graphic novels Kaley reads

$3(x - 6)$ = the number of graphic novels Daniel reads

30 = the number of graphic novels Daniel reads

Since $3(x - 6) = 30$ and $3 \cdot 10 = 30$, $x - 6$ must equal 10.

Since $x - 6 = 10$ and $16 - 6 = 10$, x must equal 16.

SOLUTION Safara reads 16 graphic novels.

b. Check your answer to problem 3a. Show your work.

Possible work:

You can check if Safara reads 16 graphic novels by checking if that means Daniel reads 30 graphic novels.

If Safara reads 16 graphic novels, then Kaley reads $16 - 6$, or 10 graphic novels.

Daniel reads 3 times as many graphic novels as Kaley. So, if Kaley reads 10 graphic novels, then Daniel reads $3 \cdot 10$, or 30 graphic novels.

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DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Use with **Session 2** **Connect It**

Levels 1–3: Speaking/Writing

Prepare students to talk and write about Connect It problem 6. Give each student two note cards. Have them write *subtract* on one and *divide* on the other. Display the equation $2x + 12 = 8$. Ask students to sequence the note cards to show the steps for solving the equation. Guide them to **Compare and Connect** strategies with a partner. Provide sentence frames that students can complete using words from the note cards:

- *One way to solve the equation is to* ____ *and then* ____.
- *Another way to solve the equation is to* ____ *and then* ____.

Call on volunteers to explain the steps in their own words.

Levels 2–4: Speaking/Writing

Prepare students to talk and write about Connect It problem 6. Have students read the problem with a partner, then look back at the Model Its to locate words and phrases that might be useful for writing about two ways to solve the equation $2x + 12 = 8$. Ask partners to begin a **Co-Constructed Word Bank**. Compile the terms into a class word bank. Sample terms might include *isolate, x-term, subtract, divide, same value, both sides.*

Adapt **Stronger and Clearer Each Time** by encouraging students to use terms from the word bank during the prewriting step. Have students participate in two structured pairings, and then allow time for them to revise their descriptions.

Levels 3–5: Speaking/Writing

Prepare students to write about Connect It problem 6 by having them talk about ways to solve the equation with a partner. First, have students read the problem and review the Model Its as needed. Then organize them into pairs and give each pair a white board. Have students take turns solving the equation in different ways while verbally explaining each step. Ask the speaker to use the white board to show each step. Encourage listeners to ask clarifying questions as needed.

Allow time for students to draft their answers individually. Remind them to use complete sentences, transition words, and precise math terms.

Develop Writing and Solving Equations With Two or More Addends

Purpose

- **Develop** strategies for writing and solving equations of the form $px + q = r$.
- **Recognize** that the numbers in equations of the form $px + q = r$ can influence the strategy used to solve the equation.

START CONNECT TO PRIOR KNOWLEDGE

Which One Doesn't Belong?

A: multiply x by 2, then add 2	B: multiply y by 5, then add 2
C: multiply 5 by the sum of x and 2	D: multiply x by 7

Possible Solutions

A uses the same number twice.

B uses a different variable.

C will need parentheses to write algebraically.

D can be expressed as a single term.

WHY? Support students' ability compare the structure of algebraic expressions.

DEVELOP ACADEMIC LANGUAGE

WHY? Develop understanding of the phrase *isolate the variable*.

HOW? In the second Model It, students explore solving an equation by isolating the x-term. Ask students to use prior knowledge to give a rough definition for *isolate*. Provide the synonym *separate*. Read the second Model It and have students turn and talk with a partner about the steps used to isolate the x-term.

TRY IT

SMP 1, 2, 4, 5, 6

Make Sense of the Problem

See ***Connect to Culture*** to support student engagement. Before students work on Try It, use **Three Reads** to help them make sense of the problem. Read the problem and ask: *What is the problem about?* After the second read ask: *What are you trying to find out?* After the third read, ask: *What are the important quantities and relationships in in the problem?* Ensure students notice all three different uses for the bricks.

LESSON 18 | SESSION 2

Develop Writing and Solving Equations With Two or More Addends

Read and try to solve the problem below.

Noah is designing a set for a school theater production. He has 150 cardboard bricks. He needs to use some of the bricks to make a chimney and 4 times as many bricks to make an arch. He also saves 15 bricks in case some get crushed. How many cardboard bricks can he use to make the arch?

TRY IT

Math Toolkit algebra tiles, grid paper, number lines, sticky notes

Possible work:

SAMPLE A

He has 150 bricks and wants to save 15. So, he has $150 - 15 = 135$ bricks to use.

For every 1 brick in the chimney, he needs 4 bricks in the arch. That means he needs to split the remaining bricks into 5 groups.

$135 \div 5 = 27$

He wants 4 times as many bricks in the arch as the chimney.

$4(27) = 108$

He can use 108 bricks to make the arch.

SAMPLE B

He has 150 bricks and wants to save 15. So, he has $150 - 15 = 135$ bricks to use.

x = number of bricks to make the chimney

$4x$ = number of bricks to make the arch

$x + 4x = 135$

$5x = 135$

$x = 27$

Since $4 \cdot 27 = 108$, Noah can use 108 bricks to make the arch.

DISCUSS IT

Ask: How would you explain what the problem is asking in your own words?

Share: The problem is asking . . .

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DISCUSS IT

SMP 2, 3, 6

Support Partner Discussion

After students work on Try It, have them explain their work and respond to Discuss It with a partner.

To support students in extending the conversation, prompt them to discuss:

- the relationship between the three different groups of bricks.

Error Alert Listen for students who divide 135 bricks by 4, or who divide 150 bricks by 5 to calculate the number of bricks used for the chimney. As students share their strategies, have them explain how their model shows the number of bricks for the chimney, the arch, and those saved. Then have partners check one another's work by describing how the 150 bricks are split among the three different uses.

Select and Sequence Student Strategies

Select 2–3 samples that represent the range of student thinking in your classroom. Here is one possible order:

- drawing pictures of the bricks and grouping to find specific numbers for each purpose
- using a visual model, such as a bar diagram, to represent and solve for the unknown quantity
- using arithmetic methods to reason
- representing and solving algebraically by assigning a variable to the unknown quantity

Facilitate Whole Class Discussion

Call on students to share selected strategies. As students present, call on others to repeat or rephrase key ideas so that everyone is engaged and hears the ideas more than once.

Guide students to **Compare and Connect** the representations. During the class discussion, remind students to provide reasons to explain how they know the solutions are correct.

ASK *What number do you divide the number of bricks used by? Why?*

LISTEN FOR For every 1 brick in the chimney there are 4 in the arch, so you need to divide the number of bricks used by 5.

Model It

If students presented these models, have students connect these models to those presented in class.

If no student presented at least one of these models, have students first analyze key features of the models, and then connect them to the models presented in class.

ASK *Why is it useful to use x to represent the number of bricks in the chimney when the problem asks for the number of bricks in the arch?*

LISTEN FOR Since there are 4 times as many bricks in the arch as the chimney, you can use $4x$ for the number of bricks in the arch.

For the bar model, prompt students to explain how the relationships in the problem are modeled.

- *How are the numbers 150, 4, and 15 from the problem shown in the model?*

For the model using subtraction, prompt students to justify the steps.

- *To use this method, how do you know what operation and number to use?*

For the model using division, prompt students to justify the steps.

- *Why were both 5x and 15 divided by 5?*

LESSON 18 | SESSION 2

➤ **Explore different ways to find an unknown value in an equation that has two or more addends.**

Noah is designing a set for a school theater production. He has 150 cardboard bricks. He needs to use some of the bricks to make a chimney and 4 times as many bricks to make an arch. He also saves 15 bricks in case some get crushed. How many cardboard bricks can he use to make the arch?

Model It

You can draw a bar model to make sense of the problem.

Let x represent the number of bricks in the chimney.

$$\overbrace{\boxed{x}\boxed{\quad\quad 4x \quad\quad}\boxed{15}}^{150}$$

Use the model to write an equation.

$$x + 4x + 15 = 150$$
$$5x + 15 = 150$$

Model It

You can start solving the equation by isolating the x-term.

$$5x + 15 = 150$$
$$5x + 15 - 15 = 150 - 15$$
$$5x = 135$$

Model It

You can start solving the equation by dividing both sides by the same value.

$$5x + 15 = 150$$
$$\frac{(5x + 15)}{5} = \frac{150}{5}$$
$$x + 3 = 30$$

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DIFFERENTIATION | EXTEND

SMP 7

Deepen Understanding

Reason About How the Form of an Equation Drives Solution Strategies

Prompt students to think about the operations they choose when writing and solving algebraic equations.

ASK *Suppose that Noah no longer wants to save the 15 bricks at the beginning. How does that change the task of finding the number of bricks in the arch?*

LISTEN FOR You can still use x for the number of bricks in the chimney and $4x$ for the bricks in the arch, but now the sum of x and $4x$ is 150.

ASK *How is this equation different from the one in the problem?*

LISTEN FOR After combining like terms, you can solve this equation in one step by dividing both sides by 5, to get $x = 30$. The equation in the original problem takes more than one step because it has both multiplication and addition.

ASK *How would the equation change if Noah also wanted to use 10 bricks to build a staircase? How does that change the task of finding the number of bricks in the arch?*

LISTEN FOR The equation would change to $x + 4x + 15 + 10 = 150$. You could add 15 and 10, then you follow the same steps as before.

Develop Writing and Solving Equations With Two or More Addends

CONNECT IT

SMP 2, 4, 5, 6

Remind students that the quantities and the relationships between them are the same in each representation. Explain that they will now use these relationships to reason about writing and solving equations with two or more addends.

Before students begin to record and expand on their work in Model It, tell them that problems 3-5 will prepare them to provide the explanation asked for in problem 6.

Monitor and Confirm Understanding 1 – 4

- Solving the equation shows that $x = 27$. This is the number of bricks in the chimney. The arch has 4 times as many bricks, or 108.
- The bar is divided into 3 sections. The x-section represents the number of bricks in the chimney, the $4x$-section the number of bricks in the arch, and the 15-section the number of bricks saved.
- After subtracting 15, the equation shows a product equal to 135. Division is needed to undo the multiplication.
- Students should recognize that all the terms on the left side of the equation must be divided by 5 to maintain equality.

Facilitate Whole Class Discussion

5 Look for the idea that the equation includes multiplication and addition. Subtraction and division are operations that can be used to solve the equation, but can be done in either order as long as you divide all of the terms on both sides of the equation.

> **ASK** *What goal do the methods shown in the second and third Model Its have in common? How does the first step in each method help?*
>
> **LISTEN FOR** Both methods are working to isolate x on one side of the equation. The first method isolates the x-term. The second method makes the coefficient of x equal to 1.

6 Encourage students to generalize from their earlier work on the second and third Model Its.

> **ASK** *What operations could you choose to use for your first step?*
>
> **LISTEN FOR** I could start by subtracting by 12 or dividing all the terms by 2.

7 **Reflect** Have all students focus on the strategies used to solve the Try It. If time allows, have students discuss their ideas with a partner.

CONNECT IT

➤ **Use the problem from the previous page to help you understand how to solve an equation that has two or more addends.**

1 How many bricks can Noah use to make the arch? 108 bricks

2 Look at the first **Model It**. How does the bar model represent the situation?
Possible answer: It shows that the sum of the number of bricks in the chimney, in the arch, and the extra 15 is 150.

3 Look at the second **Model It**. Why do you subtract 15 from both sides? What do you need to do next to find the value of x?
Subtracting 15 from both sides gets $5x$ alone on one side of the equal sign; Divide both sides of the equation by 5.

4 Look at the third **Model It**. Why do you divide all of the terms by 5?
The expression $5x + 15$ has two terms, so you need to divide both terms by 5 to keep the equation balanced.

5 Look at the second and third **Model Its**. How are the strategies for solving $5x + 15 = 150$ similar? How are they different?
Possible answer: In both strategies you want to isolate x. In the first method you start by subtracting to get the x-term alone. In the other method you start by dividing to get the coefficient of x to be 1.

6 Describe two ways you could solve the equation $2x + 12 = 8$.
You could subtract 12 from both sides and then divide both sides by 2. You could also start by dividing both sides by 2 and then subtract 6 from both sides.

7 **Reflect** Think about all the models and strategies you have discussed today. Describe how one of them helped you better understand how to solve the **Try It** problem.
Responses will vary. Check student responses.

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DIFFERENTIATION | RETEACH or REINFORCE

Hands-On Activity

Modeling a solution method for solving an equation.

Materials For each pair: algebra tiles (6 x-tiles, 26 1-tiles)

If students are unsure about solving an equation of the form $px + q = r$, then use this activity to demonstrate useful methods.

- Have students use algebra tiles to model the equation $2x - 6 = 8$. Do this by placing two x-tiles and six -1-tiles to the left of the equal sign and eight 1-tiles to the right.
- Have students use the algebra tiles to generate equivalent equations that lead to a solution, in which x is isolated on one side of the equation and a number is on the other side.
- Ask: *Do you recommend a first step of addition, subtraction, multiplication, or division? Explain why.* [Addition; add 6 to both sides by adding 6 1-tiles. and remove the zero pairs. This leads to the equation $2x = 14$. You could also start by dividing both sides by 2 to get the equation $x - 3 = 4$.]
- Ask: *What is the next step?* [Divide both sides by 2 or add 3 to both sides. Both ways isolates x and leads to the solution $x = 7$.]

Apply It

For all problems, encourage students to use a model to support their thinking. Allow some leeway in precision; if a bar diagram is used, pieces representing equal quantities may not be precisely the same size, but the diagram can still help the student think through a solution.

8 Students may also choose to solve the equation by subtracting 6 from both sides, which results in the equation $-27 = -\frac{1}{4}y$. Then they can multiply both sides by -4 to solve.

9 **A** **is correct.** The fence includes two sides perpendicular to the house that each measure w meters and a third side parallel to the house that measures 9 meters, for a total of 21.5 meters.

B is not correct. This equation includes the lengths of 2 sides of 9 meters. The garden uses the side of the house as one side of its border, so there is only 1 side of 9 meters.

C is not correct. This equation is equivalent to subtracting the length of fence parallel to the house from the sum of the two lengths perpendicular to the house. This length should be added.

D is not correct. This equation reverses the roles of the total length of the fence and the length of the side of the garden that is parallel to the house.

Apply It

➤ **Use what you learned to solve these problems.**

8 Solve $-21 = -\frac{1}{4}y + 6$. Show your work. **Possible work:**

$$-21 = -\frac{1}{4}y + 6$$
$$-4(-21) = -4\left(-\frac{1}{4}y + 6\right)$$
$$84 = y - 24$$
$$84 + 24 = y + (-24) + 24$$
$$108 = y$$

SOLUTION $y = 108$

9 A rectangular garden sits next to a house. There is fencing on three sides of the garden and the fourth side is the house. There is a total of 21.5 meters of fencing around the garden. The length of the garden along the house is 9 meters. Which equation can be used to find the width, w, of the garden in meters?

A $2w + 9 = 21.5$

B $2w + 18 = 21.5$

C $2w - 21.5 = 9$

D $2w + 21.5 = 9$

10 The total cost of a sketchpad and 6 pencils is \$22.53. The sketchpad costs \$9.99. Each pencil costs the same amount. How much does each pencil cost? Show your work. **Possible work:**

p = the cost of each pencil

$$6p + 9.99 = 22.53$$
$$6p + 9.99 - 9.99 = 22.53 - 9.99$$
$$6p = 12.54$$
$$\frac{6p}{6} = \frac{12.54}{6}$$
$$p = 2.09$$

SOLUTION Each pencil costs \$2.09.

CLOSE **EXIT TICKET**

10 Students' solutions should show an understanding of:
- using a coefficient of 6 with a variable to represent the cost of the 6 pencils.
- adding the cost of the sketchbook to express the total cost.
- using division and subtraction to solve the equation.

Error Alert If students use division as a first step in solving the equation, but neglect to divide both terms in the sum, then ask the student to check the solution by substituting into the original equation. It will not make the equation a true statement. Next, have the student justify each step of the solution with a property of operations or equality. This should help the student see which step led to the error, how to correct it, and connect division to the distributive property.

Practice Writing and Solving Equations With Two or More Addends

Problem Notes

Assign **Practice Writing and Solving Equations With Two or More Addends** as extra practice in class or as homework.

1. Students may start by finding there is $6 - 3$, or 3, yards of fabric that will be cut into strips that are $\frac{3}{4}$ yard long. They may then write and solve the equation $\frac{3}{4}x = 3$. ***Basic***

2. Students may choose to write an equivalent equation, $12x - 16 = -7$, by exchanging the left and right sides of the original equation. Doing so places the variable term in a more familiar position on the left side of the equation. ***Medium***

LESSON 18 | SESSION 2 Name:

Practice Writing and Solving Equations With Two or More Addends

➤ **Study the Example showing how to solve a problem using an equation. Then solve problems 1–5.**

Example

Chloe is making a mural. She spends 6 hours designing it. She paints it during 3 sessions. Each session is the same number of hours long. In all, Chloe spends 24 hours making the mural. How many hours long, h, is each painting session?

You can represent the situation with an equation.

$$3h + 6 = 24$$
$$\frac{3h + 6}{3} = \frac{24}{3}$$
$$h + 2 = 8$$
$$h + 2 - 2 = 8 - 2$$
$$h = 6$$

Each painting session is 6 hours long.

1. Demarco has a piece of fabric 6 yd long. He uses a piece 3 yd long. He cuts the rest into strips that are each $\frac{3}{4}$ yd long. How many $\frac{3}{4}$ yd long strips are there? Show your work.

   Possible work:

   x = the number of strips

   $$3 + \frac{3}{4}x = 6$$
   $$\frac{4}{3}\left(3 + \frac{3}{4}x\right) = 6\left(\frac{4}{3}\right)$$
   $$4 + x = 8$$
   $$4 - 4 + x = 8 - 4$$
   $$x = 4$$

   SOLUTION There are 4 strips of fabric.

2. Solve $-7 = 12x - 16$. Show your work.

   Possible work:

   $$-7 = 12x - 16$$
   $$-7 + 16 = 12x - 16 + 16$$
   $$9 = 12x$$
   $$\frac{9}{12} = \frac{12x}{12}$$
   $$0.75 = x$$

   SOLUTION $x = 0.75$

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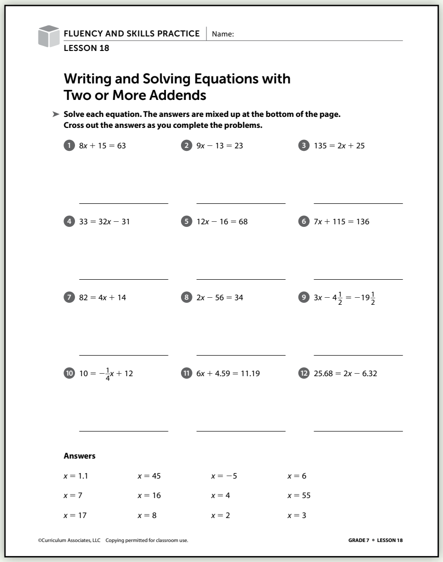
Fluency & Skills Practice

Writing and Solving Equations with Two or More Addends

In this activity, students solve equations in the form $px + q = r$, and check their work by finding each answer in an answer bank at the bottom of the page.


FLUENCY AND SKILLS PRACTICE | Name:
LESSON 18

Writing and Solving Equations with Two or More Addends

➤ Solve each equation. The answers are mixed up at the bottom of the page. Cross out the answers as you complete the problems.

1. $8x + 15 = 63$
2. $9x - 13 = 23$
3. $135 = 2x + 25$
4. $33 = 32x - 31$
5. $12x - 16 = 68$
6. $7x + 115 = 136$
7. $82 = 4x + 14$
8. $2x - 56 = 34$
9. $3x - 4\frac{1}{2} = -19\frac{1}{2}$
10. $10 = -\frac{1}{4}x + 12$
11. $6x + 4.59 = 11.19$
12. $25.68 = 2x - 6.32$

Answers

$x = 1.1$	$x = 45$	$x = -5$	$x = 6$
$x = 7$	$x = 16$	$x = 4$	$x = 55$
$x = 17$	$x = 8$	$x = 2$	$x = 3$

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Additional Practice

3 Students may also solve the problem by writing the fraction $\frac{3}{8}$ as the decimal 0.375, or by multiplying both sides of the equation by 8 to eliminate the fractions. ***Medium***

4 Students may also start by multiplying all terms of the equation by -1. Doing so reduces the number of negative terms in the equation and avoids the need to divide by a negative coefficient. ***Basic***

5 Students may also use a variable, such as c, to represent the cost of a stamp and $2c$ to represent the cost of a postcard. Then they may write and solve the equation $12(2c + c) = 14.10$ and find $2c$. ***Challenge***

LESSON 18 | SESSION 2

3 Liam makes soap sculptures of sea turtles. Each sculpture weighs $\frac{3}{8}$ pound. He ships them in a wooden box that weighs 2 pounds. The total weight of the box filled with the t sea turtles is 5 pounds. How many sea turtles are in the box? Show your work. **Possible work:**

$$\frac{3}{8}t + 2 = 5$$
$$\frac{3}{8}t + 2 - 2 = 5 - 2$$
$$\frac{3}{8}t = 3$$
$$\frac{8}{3} \cdot \frac{3}{8}t = 3 \cdot \frac{8}{3}$$
$$t = 8$$

SOLUTION There are 8 sea turtles in the box.

4 Solve $-0.4k - 6 = 1.2$. Show your work. **Possible work:**

$$-0.4k - 6 = 1.2$$
$$-0.4k - 6 + 6 = 1.2 + 6$$
$$-0.4k = 7.2$$
$$-0.4k \div (-0.4) = 7.2 \div (-0.4)$$
$$k = -18$$

SOLUTION $k = -18$

5 Claudia buys 12 postcards, 12 stamps, and 1 pen. The postcards cost twice as much as the stamps. The pen costs \$1.50. The total cost is \$14.10. How much does each postcard cost? Show your work.

Possible work: p = cost of 1 postcard and $0.5p$ = cost of 1 stamp

$$12(p + 0.5p) + 1.50 = 14.10$$
$$12(1.5p) + 1.50 = 14.10$$
$$18p + 1.50 - 1.50 = 14.10 - 1.50$$
$$18p = 12.60$$
$$p = 0.7$$

SOLUTION Each postcard costs \$0.70.

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DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Use with **Session 3** **Connect It**

MATH TERM

Distributing refers to multiplying each term in a sum or difference by a common factor.

ACADEMIC VOCABULARY

An *advantage* is a benefit.

Levels 1–3: Reading/Writing

Guide students to understand and write about Connect It problem 4. Read the problem aloud. Present the terms *distributing* and *advantage*. If applicable, connect to the Spanish cognates, *distribuido* and *ventaja*.

Have partners work together to identify advantages of each strategy shown in the Model Its. Give partners two sticky notes to label the models. Have them write sentence starters on each one to complete together:

- *One advantage of distributing first is _____.*
- *One advantage of dividing first is _____.*

Levels 2–4: Reading/Writing

Guide students to interpret Connect It problem 4 and respond in writing. Read the problem aloud. Use **Say It Another Way** to make sure students hear the problem in a variety of ways.

Have students refer back to the Model Its and tell a partner which strategy they prefer and why. Call on volunteers to justify their choices by explaining the advantage(s) of the strategy. Provide a sentence frame to support speaking and writing:

- *One advantage of _____ first is _____.*

Allow time for students to write their answers individually, then compare with their partner.

Levels 1–3: Reading/Writing

Guide students to interpret Connect It problem 4 and respond in writing. Read the problem with students. Adapt **Say It Another Way** by having students turn and talk with a partner to paraphrase the problem.

Have partners refer back to the Model Its and discuss the advantages of each strategy. Encourage them to work together to develop a **Co-Constructed Word Bank** of terms that might be used to write about the advantages of each strategy. Compile the words in a class bank. Have students use the word bank when writing their answers individually.

Develop Writing and Solving Equations with Grouping Symbols

Purpose

- **Develop** multiple strategies for solving equations that include grouping symbols.
- **Recognize** that the numbers in equations of the form $p(x + q) = r$ can influence the strategy used to solve the equation.

START CONNECT TO PRIOR KNOWLEDGE

Same and Different

A: $-3(6 + x)$	B: $-18 + 3x$
C: $-20 - 5x$	D: $-3(6 - x)$

Possible Solutions

All of the expressions include multiplication.

A and B are equivalent expressions.

A and D are both expressed as the product of two factors where one factor is inside parentheses.

B and C do not include parentheses.

WHY? Support students' ability to interpret grouping symbols.

DEVELOP ACADEMIC LANGUAGE

WHY? Guide students to test strategies to see if they agree or disagree.

HOW? Tell students that an effective way to decide whether they agree or disagree with another's strategy is to come up with a way to test whether it makes sense. Some good tests are to refer to a model or to try the strategy with different quantities. Have students use the sentence frame *I could test this strategy by* ____.

TRY IT SMP 1, 2, 4, 5, 6

Make Sense of the Problem

See ***Connect to Culture*** to support student engagement. Before students work on Try It, use **Notice and Wonder** to help them make sense of the problem. Ask students what they notice about the information and the equation in the problem and what questions they wonder about that math could help them to answer. If no one mentions it, ask students what they notice or wonder about the 32 in the equation.

Develop Writing and Solving Equations with Grouping Symbols

➤ **Read and try to solve the problem below.**

Hugo is traveling in Toronto, Canada. His weather app shows the temperature is 25°C. Hugo writes the equation $25 = \frac{5}{9}(F - 32)$ to find the temperature in degrees Fahrenheit, F. What is the temperature in degrees Fahrenheit?

TRY IT **Math Toolkit** grid paper, number lines, sticky notes

Possible work:

SAMPLE A

$25 = \frac{5}{9}(F - 32)$ means 25 is $\frac{5}{9}$ of some value, $F - 32$. Since $\frac{1}{5}$ of $\frac{5}{9}$ is $\frac{1}{9}$ and $\frac{1}{5}$ of 25 is 5, the value of $F - 32$ is 5 times 9, or 45.

Since $F - 32$ is 45, then $F = 45 + 32$, or 77.

The temperature is 77°F.

SAMPLE B

$$25 = \frac{5}{9}(F - 32)$$
$$9 \cdot 25 = 9 \cdot \frac{5}{9}(F - 32)$$
$$225 = 5(F - 32)$$
$$225 = 5F - 160$$
$$225 + 160 = 5F - 160 + 160$$
$$385 = 5F$$
$$77 = F$$

The temperature is 77°F.

DISCUSS IT

Ask: Why did you choose that strategy to find the temperature in degrees Fahrenheit?

Share: I knew . . . so I . . .

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DISCUSS IT SMP 2, 3, 6

Support Partner Discussion

After students work on Try It, encourage them to respond to Discuss It with a partner. Listen for understanding of:

- *the two operations, multiplication and subtraction, in the equation.*
- *the role of the parentheses in the equation.*

Error Alert Listen for students who multiply both sides of the equation by 9 to eliminate the fractions, but then neglect to multiply both terms in the parentheses by 5 to correctly use the distributive property. Encourage students to check that they have maintained the equality of the expressions on either side of the equal sign. Encourage students to make a habit of substituting their final answer back into the original equation to check their work.

Select and Sequence Student Strategies

Select 2–3 samples that represent the range of student thinking in your classroom. Here is one possible order for class discussion:

- using guess and check by substituting values for F to hone in on the correct value
- expanding to remove parentheses as a first step
- multiplying both sides by 9 as a first step, and then multiplying $(F - 32)$ by 5
- multiplying both sides by $\frac{9}{5}$ as a first step

Facilitate Whole Class Discussion

Call on students to share selected strategies.

Guide students to **Compare and Connect** the representations. Provide students with think time before they share and evaluate one another's ideas. To properly evaluate another student's strategy, remind students to test it, perhaps by identifying the properties of equality and operations that are used, such as the distributive property.

ASK *How do all the strategies address the fraction?*

LISTEN FOR Each strategy uses multiplication and/or division to eliminate the fraction, but when that occurs is different.

Model It

If students presented these models, have students connect these models to those presented in class.

If no student presented at least one of these models, have students first analyze key features of the models, and then connect them to the models presented in class.

ASK *Do both models require you to find a common denominator? Explain.*

LISTEN FOR Only the first model requires a common denominator to add 25 and $\frac{160}{9}$. Dividing by the fractional coefficient eliminates fractions from the equation.

For the model using the distributive property, prompt students to describe the next steps for solving the equation.

- *Would you add $\frac{160}{9}$ to both sides, or multiply both sides by 9? Explain your choice.*

For the model dividing by the coefficient, prompt students to explain the steps of the method.

- *When both sides of the equation are multiplied by $\frac{9}{5}$, why is $\frac{9}{5}$ not distributed to the terms inside the parentheses, F and 32?*

LESSON 18 | SESSION 3

➤ **Explore different ways to find an unknown value in an equation with grouping symbols.**

Hugo is traveling in Toronto, Canada. His weather app shows the temperature is 25°C. Hugo writes the equation $25 = \frac{5}{9}(F - 32)$ to find the temperature in degrees Fahrenheit, F. What is the temperature in degrees Fahrenheit?

Model It

You can use the distributive property to expand.

$$25 = \frac{5}{9}(F - 32)$$
$$25 = \frac{5}{9}(F) - \frac{5}{9}(32)$$
$$25 = \frac{5}{9}F - \frac{160}{9}$$
$$25 + \frac{160}{9} = \frac{5}{9}F - \frac{160}{9} + \frac{160}{9}$$
$$\frac{385}{9} = \frac{5}{9}F$$

Model It

You can divide each side by the coefficient $\frac{5}{9}$.

$$25 = \frac{5}{9}(F - 32)$$
$$25 \div \frac{5}{9} = \frac{5}{9}(F - 32) \div \frac{5}{9}$$
$$25 \cdot \frac{9}{5} = \frac{5}{9}(F - 32) \cdot \frac{9}{5}$$
$$45 = F - 32$$

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DIFFERENTIATION | EXTEND

SMP 7

Deepen Understanding

Making Use of Structure to Solve an Equation

Prompt students to think about the equation in a different way. Display the equations $25 = \frac{5}{9}(F - 32)$ and $25 = \frac{5(F - 32)}{9}$.

ASK *How do you know these are equivalent equations?*

LISTEN FOR Multiplying $(F - 32)$ by $\frac{5}{9}$ is the same as multiplying $(F - 32)$ by 5 and then dividing by 9.

ASK *Suppose the equation was given as $25 = \frac{5(F - 32)}{9}$. How could you think about solving that equation? How would that method be different from the Model Its?*

LISTEN FOR You could think of this as 5 times $(F - 32)$ divided by 9 equals 25. So you could start by multiplying both sides by 9 and then the equation would be 25 times 9 (or 225) is equal to 5 times $(F - 32)$. Then you would not need to multiply or divide by a fraction.

Generalize Encourage students to describe how the given form of an equation, or the types of numbers in the equation, might influence how they approach solving it.

Develop Writing and Solving Equations with Grouping Symbols

CONNECT IT

SMP 2, 4, 5, 6

Remind students that the quantities and the relationships between them are the same in each representation. Explain that they will now use these relationships to draw conclusions about solving equations that include parentheses.

Before students begin to record and expand on their work in Model It, tell them that problems 2 and 3 will prepare them to provide the comparison asked for in problem 4.

Monitor and Confirm Understanding 1 – 3

- Equations can often be solved in different ways. You can choose the strategy you prefer.
- One of the properties of equality is that multiplying both sides of an equation by the same non-zero factor maintains equivalence.
- Dividing by $\frac{5}{9}$ forms an equivalent equation that can be solved with one more step.
- Both solution methods are equally valid, and both produce the same solution.

Facilitate Whole Class Discussion

4 Look for the idea that each strategy has advantages and disadvantages and that student preferences can guide their choice of strategy.

> **ASK** *Is it possible to get a different solution depending on the strategy you choose for solving an equation? Explain.*
>
> **LISTEN FOR** No. If you always perform the same operation to both sides of an equation, you will produce equivalent equations. Equivalent equations have the same solution.

5 Help students to understand they are not solving for b but considering different possible equations. Look for the idea that values for b that are even numbers will eliminate decimals from the equation once the expression on the right is expanded. When the values of b are fractions, dividing by the fraction first will eliminate fractions from the left side.

6 **Reflect** Have all students focus on the strategies used to solve the Try It. If time allows, have students discuss their ideas with a partner.

CONNECT IT

➤ **Use the problem from the previous page to help you understand how to solve an equation with grouping symbols.**

1 What is 25°C in degrees Fahrenheit? 77°F

2 Look at the first **Model It**. Describe the steps shown for solving the equation. What do you still need to do to find the value of F?

First $\frac{5}{9}$ is distributed. Then $\frac{160}{9}$ is added to both sides to get $\frac{5}{9}F$ alone on one side of the equation; To find the value of F, divide both sides of the equation by $\frac{5}{9}$.

3 Look at the second **Model It**. Describe the steps shown for solving the equation. What do you still need to do to find the value of F?

First both sides are divided by $\frac{5}{9}$; To find the value of F, add 32 to both sides.

4 Look at the **Model Its**. What was one advantage of distributing first? What was one advantage of dividing first?

Possible answer: When you distribute first, you end up with an equation that has two addends on one side. When you divide first, you eliminate the fractions and just have integers.

5 Consider the equation $12 = b(2.5x + 15)$. What values of b might make you want to start solving the equation by distributing b? What values of b might make you want to start solving the equation by dividing by b?

Possible answer: When b is a multiple of 2, distributing first would mean there would no longer be a decimal coefficient. When b is a factor of 12, if you start by dividing by b, you end with just $2.5x + 15$ on the right side.

6 **Reflect** Think about all the models and strategies you have discussed today. Describe how one of them helped you better understand how to write and solve an equation that includes grouping symbols.

Responses will vary. Check student responses.

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DIFFERENTIATION | RETEACH or REINFORCE

Hands-On Activity

Use algebra tiles to model equations that include parentheses.

If students are unsure solving equations that include parentheses, then use this activity to help them visualize how to solve equations using the distributive property.

Materials For each pair: algebra tiles (9 x-tiles, 21 1-tiles)

- Have students work with a partner to model $3(x - 2) = 3$ with their tiles. If necessary, suggest that they show the multiplication on the left by making an array of width $x - 2$ and height 3 with their tiles.
- Have students look at their models a new way. Ask: *Can you write a different equation that still represents these tiles? If so, how is it related to the first?* [Yes. $3x - 6 = 3$. It is the expanded form.]
- Encourage students to try different strategies to solve the equation. Ask: *How could you show the process of dividing both sides of the equation by three with your tiles?* [Make three equal groups on both sides. On the left, each of the three rows in the array has one x-tile and two -1-tiles. On the right, each group would have one 1-tile.]
- Ask: *What equation does this represent? What does x equal?* [$x - 2 = 1$, $x = 3$]

Apply It

For all problems, encourage students to use a model to support their thinking. Help them to think of an equation as a mathematical model, especially when it represents a real-world relationship.

7 **B and C are correct.** Students may solve this problem by writing and solving the equation $5(c + 20) = 475$ or may use its expanded form, $5c + 100 = 475$.

A is not correct. This answer may be the result of multiplying the number of crates by the cost of the leash.

D is not correct. This answer may be the result of neglecting to multiply the price of a leash by the number of puppies, although the total cost for the crates is correct.

E is not correct. This answer may be the result of misunderstanding that the $475 total cost is for 5 crates and 5 leashes, and not just one of each.

8 Students may not recognize the fraction bar as a grouping symbol. They may start by rewriting $\frac{k-4}{-6}$ as $\frac{k}{-6} + \frac{-4}{-6}$ or $-\frac{k}{6} + \frac{4}{6}$. Then they may subtract $\frac{4}{6}$ from both sides before multiplying both sides by -6.

Apply It

➤ **Use what you learned to solve these problems.**

7 Carolina fosters 5 puppies. For each puppy she buys a crate that costs c dollars and a leash that costs $20. She spends $475 total. Which equations model the situation? Select all that apply.

A $5c + 20c = 475$

B $5(c + 20) = 475$

C $5c + 100 = 475$

D $5c + 20 = 475$

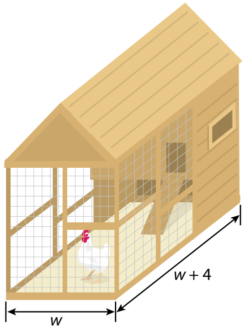
E $c + 20 = 475$

8 Solve $-8 = \frac{k-4}{-6}$. Show your work. Possible work:

$$-8 = \frac{k-4}{-6}$$
$$-8(-6) = \left(\frac{k-4}{-6}\right)(-6)$$
$$48 = k - 4$$
$$48 + 4 = k - 4 + 4$$
$$52 = k$$

SOLUTION $k = 52$

9 The perimeter of a rectangular chicken coop is 30 feet. The width is w feet and the length is $w + 4$ feet. What are the length and width of the coop? Show your work.



Possible work:

$$2(w + w + 4) = 30$$
$$2(2w + 4) = 30$$
$$4w + 8 = 30$$
$$4w + 8 - 8 = 30 - 8$$
$$4w = 22$$
$$\frac{4w}{4} = \frac{22}{4}$$
$$w = 5.5$$
$$w + 4 = 5.5 + 4, \text{ or } 9.5$$

SOLUTION The chicken coop is 5.5 ft wide and 9.5 ft long.

CLOSE EXIT TICKET

9 Students' solutions should show an understanding of:
- writing an a multi-step equation to represent a real-world problem.
- solving a multi-step equation by strategically applying division and subtraction.

Error Alert If students struggle to get started solving a equation when the unknown is within parentheses, then encourage students to cover up the expression inside the parentheses and ask them how they would solve the simpler equation.

Prepare for Writing and Solving Equations with Grouping Symbols

Problem Notes

Assign **Practice Writing and Solving Equations with Grouping Symbols** as extra practice in class or as homework.

1 **a.** Students should use the fact that d is the amount of Lillie's donation and reread the problem statement to find the quantity that $d + 3$ represents. ***Basic***

b. Students may look back at the problem statement to see that the factor 4.5 refers to the relationship between the amount of money Lillie's parents donate and the amount of money her brother donates. ***Basic***

c. Students may use their answer to problem 1c to relate the expression $d + 3$ to Lillie's brother's donation. ***Basic***

2 Students could also solve the problem arithmetically, first by dividing the payment of \$49.50 by 3 to calculate the discounted monthly payment, and then adding \$2. ***Medium***

LESSON 18 | SESSION 3 Name:

Practice Writing and Solving Equations with Grouping Symbols

➤ **Study the Example showing how to use an equation with grouping symbols to solve a problem. Then solve problems 1–5.**

Example

Lillie and her family donate money to charity at the end of each year. Lillie's brother donates \$3 more than Lillie. Her parents donate 4.5 times as much as Lillie's brother. Lillie's parents donate \$45. How much does Lillie donate?

You can represent the situation with an equation.

d = Lillie's donation in dollars

$$4.5(d + 3) = 45$$

$$\frac{4.5(d + 3)}{4.5} = \frac{45}{4.5}$$

$$d + 3 = 10$$

$$d + 3 - 3 = 10 - 3$$

$$d = 7$$

Lillie donates \$7.

1 Look at $4.5(d + 3) = 45$ from the Example.

a. What does $(d + 3)$ represent?

The amount of Lillie's brother's donation.

b. Why is $(d + 3)$ multiplied by 4.5?

Lillie's parents give 4.5 times her brother's donation.

c. How much does Lillie's brother donate?

\$10

2 Malik joins a gym. He gets \$2 per month off the regular monthly rate for 3 months. Malik pays \$49.50 for 3 months. What is the gym's regular monthly rate, r?

Show your work. Possible work:

$$3(r - 2) = 49.5$$

$$\frac{3(r - 2)}{3} = \frac{49.5}{3}$$

$$r - 2 = 16.5$$

$$r - 2 + 2 = 16.5 + 2$$

$$r = 18.5$$

SOLUTION The regular monthly rate is \$18.50.

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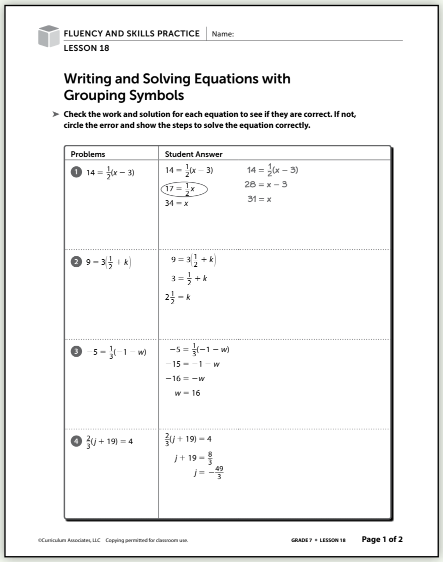
Fluency & Skills Practice

Writing and Solving Equations with Grouping Symbols

In this activity, students practice solving equations by analyzing and correcting given solutions.


FLUENCY AND SKILLS PRACTICE Name:
LESSON 18

Writing and Solving Equations with Grouping Symbols

➤ Check the work and solution for each equation to see if they are correct. If not, circle the error and show the steps to solve the equation correctly.

Problems	Student Answer
1 $14 = \frac{1}{2}(x - 3)$	$14 = \frac{1}{2}(x - 3)$; $17 = \frac{1}{2}x$; $34 = x$
2 $9 = 3(\frac{1}{2} + k)$	$9 = 3(\frac{1}{2} + k)$; $3 = \frac{1}{2} + k$; $2\frac{1}{2} = k$
3 $-5 = \frac{1}{3}(-1 - w)$	$-5 = \frac{1}{3}(-1 - w)$; $-15 = -1 - w$; $-16 = -w$; $w = 16$
4 $\frac{2}{3}(j + 19) = 4$	$\frac{2}{3}(j + 19) = 4$; $j + 19 = \frac{8}{3}$; $j = -\frac{49}{3}$

Additional Practice

3 Students may also solve the problem by reasoning that Luis paid \$9 for each mug, because 6 mugs times \$9 per mug equals \$54. Then they add \$4 to \$9 to calculate the regular price. ***Medium***

4 Students should recognize that the equation contains one more term than the other equations in this session. Some students might see that after subtracting 8 from both sides, the equation takes on a more familiar structure. Other students may choose to expand the expression and combine like terms, although this could involve additional calculations with factions. ***Challenge***

5 Students may also solve this equation by expanding the right side, to get $-72 = 8y - 24$. Then they can add 24 to both sides to get $-48 = 8y$ and last divide both sides by 8. ***Medium***

3 Luis is shopping for gifts. Mugs are on sale for \$4 off the regular price, p. Luis buys 6 mugs. He pays a total of \$54. What is the regular price of a mug? Show your work.

Possible work:

$$6(p - 4) = 54$$
$$6p - 24 = 54$$
$$6p - 24 + 24 = 54 + 24$$
$$6p = 78$$
$$\frac{6p}{6} = \frac{78}{6}$$
$$p = 13$$

SOLUTION The regular price of a mug is \$13.

4 Solve $\frac{3}{4}(5x - 3) + 8 = 17$. Show your work. **Possible work:**

$$\frac{3}{4}(5x - 3) + 8 = 17$$
$$\frac{3}{4}(5x - 3) + 8 - 8 = 17 - 8$$
$$\frac{3}{4}(5x - 3) = 9$$
$$\left(\frac{4}{3} \cdot \frac{3}{4}\right)(5x - 3) = 9 \cdot \frac{4}{3}$$
$$5x - 3 = 12$$
$$5x - 3 + 3 = 12 + 3$$
$$5x = 15$$

SOLUTION $x = 3$

5 Solve $-72 = 8(y - 3)$. Show your work. **Possible work:**

$$-72 = 8(y - 3)$$
$$\frac{-72}{8} = \frac{8(y - 3)}{8}$$
$$-9 = y - 3$$
$$-9 + 3 = y - 3 + 3$$
$$-6 = y$$

SOLUTION $y = -6$

DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Use with **Session 4** Apply It

MATH TERMS

An *isosceles triangle* is a triangle that has at least two sides the same length.

Perimeter is the distance around a two-dimensional shape. The perimeter is equal to the sum of the lengths of the sides.

Levels 1–3: Reading/Speaking

Support students as they make sense of and discuss Apply It problem 5. Read the problem aloud. Display and discuss the Math Terms. Call on a volunteers to sketch an isosceles triangle and share which two sides are the same length. Read the first two sentences again. Ask a volunteer to label the lengths of the sides. Read the third sentence, then ask: *What do we need to find?* Have students work with a partner to set up an equation. Call on volunteers to share their equations. Allow time for students to solve individually.

Levels 2–4: Reading/Speaking

Modify **Three Reads** to help students read Apply It problem 5. Before each read, display the question and help students craft a sentence starter they can use to answer.

After the first read, review the Math Terms and have students draw an isosceles triangle. After the second read, have them use their drawing to show what they need to find. After the third read, have students label the drawing with the expressions and the perimeter. Have students turn and talk about how to set up the equation and then solve the problem individually.

Levels 3–5: Reading/Speaking

Modify **Three Reads** to help students read Apply It problem 5. Before each read, have students discuss the focus for each read.

After the first read, have partners discuss the Math Terms and the term *represented by*. Have them clarify other terms, as needed. After the third read, have partners discuss possible approaches they could take to solve the problem. Encourage students to sketch and label the triangle to organize information.

Refine Writing and Solving Multi-Step Equations

Purpose

- **Refine** strategies for solving multi-step equations.
- **Refine** understanding of how the structure of an equation informs a solution strategy.

START CHECK FOR UNDERSTANDING

Solve this equation:

$2(t - 2) + 2 = 22$

Solution

$t = 12$

WHY? Confirm students' ability to solve a multi-step equation.

MONITOR & GUIDE

Before students begin to work, use their responses to the **Start** to determine those who will benefit from additional support. Use the **Error Analysis** table below to guide remediation.

Have all students complete the Example and problems 1–3, using Consider This and Pair/Share as appropriate. Observe and monitor their reasoning and guide or redirect students as needed.

LESSON 18 | SESSION 4 ■ ■ ■ ■

Refine Writing and Solving Multi-Step Equations

➤ **Complete the Example below. Then solve problems 1–8.**

Example

Solve $-0.25x + 7.5 = 15$.

Look at how you could show your work using multiplication.

$$-0.25x + 7.5 = 15$$
$$100(-0.25x + 7.5) = (100)15$$
$$-25x + 750 = 1{,}500$$
$$-25x + 750 - 750 = 1{,}500 - 750$$
$$-25x = 750$$

SOLUTION $x = -30$

CONSIDER THIS . . . You can multiply both sides by a power of 10 to eliminate the decimals.

PAIR/SHARE What is another way you could solve this problem?

Apply It

1 Solve $0 = -1.8y + 0.72$. Show your work.

Possible work:

$$0 = -1.8y + 0.72$$
$$0 - 0.72 = -1.8y + 0.72 - 0.72$$
$$-0.72 = -1.8y$$
$$\frac{-0.72}{-1.8} = \frac{-1.8y}{-1.8}$$
$$0.4 = y$$

SOLUTION $y = 0.4$

CONSIDER THIS . . . You can think of $0 = -1.8y + 0.72$ as having two addends.

PAIR/SHARE How can you check your answer?

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START ERROR ANALYSIS

If the error is . . .	Students may . . .	To support understanding . . .
11	have attempted to divide both sides of the equation by 2 but did not divide every term.	Ask students to explain each step of the solution process that they applied and to identify the property that they are using. If students argue in support of an incorrect solution, have them substitute the solution into the original equation and evaluate it.
13	have incorrectly applied the distributive property to expand the left side of the equation.	Ask to students to model the equation with algebra tiles to better see how to expand the expression using the distributive property. Or ask students to describe a process for confirming their answer, such as substituting it into the original equation.
14	have started by adding 2 to both sides of the equation instead of subtracting 2.	Ask students to solve a related equation that involves addition only, such as $w + 2 = 22$. Have students explain why subtracting 2 from both sides leads to the solution. Then have students apply the same logic to the original equation.

Example

Guide students in understanding the Example. Ask:

- *Why is it useful to start by multiplying both sides of the equation by 100?*
- *Why do you need to multiply both sides of the equation by 100, not just the side with decimals?*
- *Could you start by multiplying by 10 or 1,000? How would that change the solution?*

Help all students focus on the Example and responses to the questions by justifying each step of the solution process with the properties of equality or operations, including the distributive property.

Look for understanding that all of the methods for solving equations apply the same set of properties of equality. When one side of an equation is multiplied or divided by a number or a number is added to or subtracted from that side, the same operation must be performed on the other side of the equation to maintain equality.

Apply It

1 Students may recognize that they can exchange the left side and right side of an equation at any time while solving the equation without changing the solution. *DOK 2*

2 Students may also notice that the denominators are both 8 and realize that means the numerators must be equal. So, they might start by rewriting the equation as $2(n + 17) = 3$ and then divide both sides by 2 before subtracting 17 from both sides. *DOK 2*

3 **C is correct.** Students may model the problem with the equation $x + (x + 1) + (x + 2) = 42$ and recognize that $x + 2$ represents the age of the oldest sibling.

A is not correct. This answer may be the result of assigning a variable to represent the age of the youngest sibling, but not completing the solution by adding 2 to find the age of the oldest sibling.

B is not correct. This answer may be the result of assigning a variable to represent the age of the middle sibling, or of dividing 42 by 3.

D is not correct. This answer may be the result of not understanding or misinterpreting the meaning of the word *consecutive*.

DOK 3

2 Solve $\frac{2(n+17)}{8} = \frac{3}{8}$. Show your work.

Possible work:

$$\frac{2(n+17)}{8} = \frac{3}{8}$$
$$\frac{8}{1} \cdot \frac{2(n+17)}{8} = \frac{3}{8} \cdot \frac{8}{1}$$
$$2(n + 17) = 3$$
$$2n + 34 = 3$$
$$2n + 34 - 34 = 3 - 34$$
$$2n = -31$$
$$\frac{2n}{2} = \frac{-31}{2}$$
$$n = -15.5$$

SOLUTION $n = -15.5$

CONSIDER THIS . . . There is more than one way to think about this problem.

PAIR/SHARE How did you choose your first step?

3 Three siblings are born on the same date in consecutive years. The sum of their ages is 42. What is the age of the oldest sibling?

A 13

B 14

C 15

D 16

Victoria chose A as the correct answer. How might she have gotten that answer?

Possible answer: She may have solved for x, the age of the youngest sibling and forgot to add 2 to find the age of the oldest sibling.

CONSIDER THIS . . . Consecutive integers follow each other, like 4, 5, 6. If the first integer is x, the next is $x + 1$, then $x + 2$, and so on.

PAIR/SHARE How would the answer change if there were four siblings?

GROUP & DIFFERENTIATE

Identify groupings for differentiation based on the **Start** and problems 1–3. A recommended sequence of activities for each group is suggested below. Use the resources on the next page to differentiate and close the lesson.

Approaching Proficiency
- **RETEACH** Hands-On Activity
- **REINFORCE** Problems 4, 6, 7

Meeting Proficiency
- **REINFORCE** Problems 4–8

Extending Beyond Proficiency
- **REINFORCE** Problems 4–8
- **EXTEND** Challenge

Have all students complete the **Close: Exit Ticket.**

Resources for Differentiation are found on the next page.

Refine Writing and Solving Multi-Step Equations

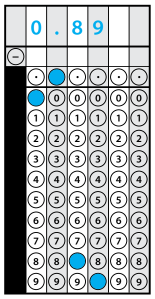
Apply It

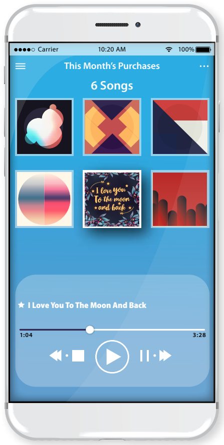
4 Students may solve this problem by writing and solving an equation, such as $12.50 + 6x = 17.84$, or by reasoning that the difference between this month's fee and the regular fee, which is \$5.34, represents the cost of the 6 songs. ***DOK 2***

5 Students should recognize that they can write and solve an equation for x but that the solution to the problem involves additional steps. The value of x must be substituted into each expression to find the side lengths of the triangle. ***DOK 3***

6 Students may also explain that Khalid can subtract 8.5 from both sides and then divide both sides by -1. Then he reversed the sides of the equation. ***DOK 3***

4 Leon pays \$12.50 per month for a music subscription service. One month he also buys 6 songs from the service. Each song costs the same. His bill for that month is \$17.84. In dollars, how much does he pay for each song?





5 One side of an isosceles triangle is $2x + 1$ ft long. The other two sides are both $3x - 1$ ft long. The perimeter of the triangle is 55 ft. What is the length of each side? Show your work. Possible work:

$$2(3x - 1) + 2x + 1 = 55$$
$$6x - 2 + 2x + 1 = 55$$
$$8x - 1 = 55$$
$$8x - 1 + 1 = 55 + 1$$
$$8x = 56$$
$$\frac{8x}{8} = \frac{56}{8}$$
$$x = 7$$

$$3(7) - 1 = 21 - 1 = 20$$
$$2(7) + 1 = 14 + 1 = 15$$

SOLUTION The lengths of the sides are 20 ft, 20 ft, and 15 ft.

6 Khalid is solving the equation $8.5 - 1.2y = 6.7$. He gets to $1.8 = 1.2y$. Explain what he might have done to get to this equation.

Possible explanation: Khalid subtracted 6.7 and added $1.2y$ to both sides of the equation.

DIFFERENTIATION

RETEACH

Hands-On Activity

Apply a model to write and solve multi-step equations

Students approaching proficiency with writing and solving multi-step equations will benefit from applying a model of the process.

Materials For each pair: algebra tiles (12 x-tiles, 48 1-tiles)

- Display the equation: $4(x - 3) - 2 = 6$
- Have partners use algebra tiles to model the equation and its solution.
- They may also solve the equation algebraically, without tiles.
- When pairs are ready, have them explain to another pair how they solved the equation and how they can confirm that their solution is correct. In this discussion, encourage students to ask questions of one another and to explore alternative solution methods.
- Ask those who solved the problem algebraically as well as with algebra tiles to make connections between the two models
- Ask: *After you solve an equation, how can you check to make sure it is correct? Why is confirming the solution useful to do?* [The solution can be confirmed by substituting it into the original equation and then evaluating. This is useful because a solution process can have many steps, and arithmetic mistakes can be made in any of the steps.]
- As time allows, have partners write and then solve multi-step equations of their own design.

7 See ***Connect to Culture*** to support student engagement. Students comfortable with fraction calculation may not begin by multiplying by 16 to eliminate fractions from the equation. ***DOK 2***

8 Students may choose to continue the solution with fraction calculations rather than multiplying by 6. Subtracting $\frac{1}{2}$ from both sides results in the equation $\frac{1}{3}w = -\frac{1}{3}$. Multiplying both sides of the equation by 3 then yields $w = -1$. ***DOK 2***

CLOSE EXIT TICKET

9 **Math Journal** Look for an accurate application of the properties of equality and of operations, expressing the decimals as fractions or the fractions as decimals.

Error Alert If students struggle to see the equations are equivalent, then ask students to solve each equation one step at a time, looking for equivalent equations as they go.

End of Lesson Checklist

INTERACTIVE GLOSSARY Support students by asking if they can use the word *represent* with the word *variable* to explain what *represent* means in the context of equations.

SELF CHECK Have students review and check off any new skills on the Unit 4 Opener.

LESSON 18 | SESSION 4

7 Mora preparing her pack for a hike. Her empty pack weighs $\frac{15}{16}$ pound. She adds some water bottles that each weigh $1\frac{1}{8}$ pound. Now Mora's pack weighs $6\frac{9}{16}$ pounds. How many bottles, b, does Mora add to her pack? Show your work.

Possible work:

$$\frac{15}{16} + 1\frac{1}{8}b = 6\frac{9}{16}$$
$$16\left(\frac{15}{16} + \frac{9}{8}b\right) = \frac{105}{16} \cdot 16$$
$$15 + 18b = 105$$
$$15 - 15 + 18b = 105 - 15$$
$$18b = 90$$
$$b = 5$$

SOLUTION Mora adds 5 water bottles to her pack.

8 Solve $\frac{1}{2} + \frac{1}{3}w = \frac{1}{6}$. Show your work. Possible work:

$$\frac{1}{2} + \frac{1}{3}w = \frac{1}{6}$$
$$6\left(\frac{1}{2} + \frac{1}{3}w\right) = 6\left(\frac{1}{6}\right)$$
$$3 + 2w = 1$$
$$3 - 3 + 2w = 1 - 3$$
$$2w = -2$$

SOLUTION $w = -1$

9 **Math Journal** Damita says the equations $0.8x - 0.8 = 1.6$ and $\frac{4}{5}(x - 1) = 1\frac{3}{5}$ are the same. How can she show this, without solving the equations?

Possible answer: She could rewrite the fractions as decimals to get the equation $0.8(x - 1) = 1.6$. Then she could distribute 0.8 to get the equation $0.8x - 0.8 = 1.6$. This is the same as the other equation.

End of Lesson Checklist

- [ ] **INTERACTIVE GLOSSARY** Write a new entry for *represent*. Write at least one synonym for *represent*.
- [ ] **SELF CHECK** Go back to the Unit 4 Opener and see what you can check off.

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REINFORCE

Problems 4–8
Solve multi-step equations

Students meeting proficiency will benefit from additional work with writing and solving multi-step equations by solving problems in a variety of formats.

- Have students work on their own or with a partner to solve the problems.
- Encourage students to show their work.

EXTEND

Challenge
Generate and solve multi-step equations

Students extending beyond proficiency will benefit from generating and solving equations with many terms.

- Ask individual students to write and then solve a multi-step equation that includes the variable x and the fractions $\frac{1}{2}$ and $\frac{1}{3}$. Then have students exchange equations and solutions with a partner to check their work.
- Challenge students to revise their equation by adding another term that uses $\frac{1}{4}$ as either a constant or coefficient. Have them solve the revised equation. Then repeat by including terms using $\frac{1}{5}$ and then $\frac{1}{6}$.

PERSONALIZE

i-Ready

Provide students with opportunities to work on their personalized instruction path with *i-Ready* Online Instruction to:

- fill prerequisite gaps.
- build up grade-level skills.