STANDARDS FOR MATHEMATICAL PRACTICE (SMP)

SMP 1, 2, 3, 4, 5, and 6 are integrated into the Try-Discuss-Connect routine.*

This lesson provides additional support for:

- 7 Look for and make use of structure.
- 8 Look for and express regularity in repeated reasoning.
- * See page 1q to learn how every lesson includes these SMP.

Objectives

Content Objectives

- Understand that scale drawings are figures with side lengths in equivalent ratios.
- Find a scale factor.
- Use a scale factor to find an unknown length either in a scale drawing or in the object it represents.
- Apply the square of the scale factor to relate area in a scale drawing to the area of the object it represents.
- Use scale factors to redraw a scale drawing with a different scale.

Language Objectives

- Understand the term *scale drawing* and use it to describe figures with side lengths in equivalent ratios.
- Interpret word problems involving scale drawings and scale copies by identifying the scale and reasoning about the scale factor.
- Explain strategies for finding an unknown length in a scale drawing or in the object it represents using the lesson vocabulary.
- Explain how to use scale factors to make scale drawings, answer questions, and check for understanding during class discussion.

Prior Knowledge

- Write equivalent ratios.
- Calculate the unit rate for a given ratio.
- Use visual models, such as double number lines, to find values of quantities in equivalent ratios.
- Apply a unit rate to find unknown values.
- Find the areas of rectangles, parallelograms, and triangles.

Vocabulary

Math Vocabulary

scale tells the relationship between a length in a drawing, map, or model to the actual length.

scale drawing a drawing in which the measurements correspond to the measurements of the actual object by the same scale.

scale factor the factor you multiply all the side lengths in a figure by to make a scale copy.

Review the following key terms.

area the amount of space inside a closed two-dimensional figure. Area is measured in square units such as square centimeters.

dimension length in one direction. A figure may have one, two, or three dimensions.

unit rate the numerical part of a rate. For example, the rate 3 miles per hour has a unit rate of 3. For the ratio a : b, the unit rate is the quotient $\frac{a}{b}$.

Academic Vocabulary

actual real and existing, not a model or copy.

justify to explain why something is correct or incorrect by giving logical reasons.

Learning Progression

In Grade 6, students learned how to identify equivalent ratios, calculate rates, and represent these concepts using double number lines, tables, and other visual models. They applied the concept of unit rate to calculate unknown values of quantities in equivalent ratios. In this lesson, students apply the concepts of equivalent ratios and unit rates to recognize scale drawings and scale copies and to compare scale copies and scale drawings to the objects or figures they represent. They apply this knowledge to redraw a scale drawing at a new scale. Later in Grade 7, students will extend their knowledge by calculating unit rates for ratios of fractions. They will define proportional relationships and solve a variety of proportional relationship problems in mathematical and real-world contexts.

>

LESSON 1 **Overview**

Pacing Guide Items marked with 🔭 are available on the Teacher Toolbox .	MATERIALS	DIFFERENTIATION
SESSION 1 Explore Scale Drawings	(35–50 min)	
 Start (5 min) Try It (5–10 min) Discuss It (10–15 min) Connect It (10–15 min) Close: Exit Ticket (5 min) Additional Practice (pages 7–8) 	Math Toolkit double number lines, grid paper, ribbon, yarn Presentation Slides 🔖	PREPARE Interactive Tutorial RETEACH or REINFORCE Hands-On Activity Materials For each pair: scissors, Activity Sheet Rectangles, Squares, and Triangles
SESSION 2 Develop Using Scale to	Find Distances (45–60 min)	
 Start (5 min) Try It (10–15 min) Discuss It (10–15 min) Connect It (15–20 min) Close: Exit Ticket (5 min) Additional Practice (pages 13–14) 	Math Toolkit double number lines, grid paper, ribbon, yarn Presentation Slides ᇼ	 RETEACH or REINFORCE Hands-On Activity Materials For each pair: 1 ruler, a map of your region or state REINFORCE Fluency & Skills Practice EXTEND Deepen Understanding
SESSION 3 Develop Using Scale to	Find Areas (45–60 min)	
 Start (5 min) Try It (10–15 min) Discuss It (10–15 min) Connect It (15–20 min) Close: Exit Ticket (5 min) Additional Practice (pages 19–20) 	Math Toolkit double number lines, grid paper, ribbon, yarn Presentation Slides ᇼ	RETEACH or REINFORCE Hands-On Activity Materials For each pair: base-ten blocks (10 tens rods) REINFORCE Fluency & Skills Practice 📡 EXTEND Deepen Understanding
SESSION 4 Develop Redrawing a So	cale Drawing (45–60 min)	
 Start (5 min) Try It (10–15 min) Discuss It (10–15 min) Connect It (15–20 min) Close: Exit Ticket (5 min) Additional Practice (pages 25–26) 	Math Toolkit double number lines, grid paper, ribbon, rulers, yarn Presentation Slides 📡	RETEACH or REINFORCE Visual Model Materials For display: 1 meter stick REINFORCE Fluency & Skills Practice 🐂 EXTEND Deepen Understanding
SESSION 5 Refine Solving Problem	s Involving Scale (45–60 min)	
 Start (5 min) Monitor & Guide (15–20 min) Group & Differentiate (20–30 min) Close: Exit Ticket (5 min) 	Math Toolkit Have items from previous sessions available for students.	RETEACH Visual Model Materials For display: 3 rulers REINFORCE Problems 4–8
	Presentation Slides 🔖	EXTEND Challenge Materials For each pair: 1 ruler, 2 maps of different scales for the same region PERSONALIZE i -Ready
Lesson 1 Quiz 🔭 or Digital Comprehension Check		RETEACH Tools for Instruction 🐐 REINFORCE Math Center Activity 🔭 EXTEND Enrichment Activity 🍾

Connect to Culture

Use these activities to connect with and leverage the diverse backgrounds and experiences of all students. Engage students in sharing what they know about contexts before you add the information given here.

SESSION 1 🔳 🗆 🗆

Try It Ask students if they have ever seen or visited a geodesic dome or a dome-shaped playground structure and have them describe their impressions of the structure. Their spherical structure allows geodesic domes to enclose the greatest volume for a given amount of building material. The dome structure also allows air and energy to circulate without obstruction, making the space efficient to heat and cool. Although geodesic domes were once called "the houses of the future," they remain relatively uncommon in modern architecture. Discuss any other unusual structures students have seen or know about.

SESSION 2

Try It Ask students to describe any maps that they have seen or used, including maps that are published online. Cartography is the study of mapmaking, which dates back to ancient times. Today, cartographers rely on computer programs and satellite images, which help them produce extremely accurate and precise maps of places all over the Earth. A typical state road map in an atlas may have a scale of 1 inch representing between 10 miles and 25 miles. This means that at a scale of 1 in. to 25 mi, the entire state of Texas, with an area of 268,581 square miles, can be shown on a piece of paper measuring only 35 in. by 35 in. with room to spare.

SESSION 3

Try It Ask students to raise their hand if they have visited a museum. Then ask volunteers to say whether or not a map of the museum was a useful guide for their visit. Maps are especially helpful for exploring very large museums that have dozens of different galleries. One of the world's largest museums is the American Museum of Natural History in New York City, which spans 4 city blocks and includes 25 separate buildings. Its exhibit halls cover more than 2 million square feet!

SESSION 4

Try It Architects may design almost any type of building, including houses, apartment buildings, office plazas, theaters, and sports arenas. Today, architects develop two-dimensional scale drawings of new structures, such as floor plans and blueprints, and use computer software to develop three-dimensional models. Architects work on all aspects of a building, including its systems for heating, ventilation, electricity, and plumbing. Ask students if they are interested in a career in architecture or a related field.

CULTURAL CONNECTION

3c

Alternate Notation In the United States, a colon (:) separates the two quantities in a ratio. In Latin America, a colon can be used to indicate division. Encourage students who have experience with using a colon to express division to share what they know with the class.





Connect to Family and Community

After the Explore session, have students use the Family Letter to let their families know what they are learning and to encourage family involvement.

	LESSON 1 SOLVE PROBLEMS INVOLVING SCALE Activity Thinking About Scale Around You > Do this activity together to investigate scale in the
Dear Family, This week your student is learning about scale drawings. In a scale drawing, the size of an original figure changes, but its shape does not change. Here are some examples of scale drawings that you may be familiar with. • A floor plan is a scale drawing of the actual layout of space in a building. • A state road map is a scale drawing of the actual road in the state.	Store Have you ever taken a long road trip and come across some large roadside attractions? The world's largest convely boots are a sculpture in Texas. They are over 35 feet tall A convoly boot is normally just 12 Inches, or 1 foot, tall. Gift shops often have models of buildings that fit in the
Scale drawings are typically used when objects are either too small or too large to be shown at their actual sizes. Floor plans and maps are drawn smaller than actual size. Suppose a floor plan is drawn so that 1 inch on the floor plan represents an actual distance of 3 feet. For that floor plan, the scale is 1 in. to 3 ft. Your student will be solving scale drawing problems like the one below.	palm of your hand. In Washington, D.C., you can get a Lincoln Memorial model that is 65 finches tall. The actual memorial is 80 feet tall. These giant and tiny models are scale copies of real-life objects.
The scale from an actual volcano to a drawing of the volcano is 50 m to 5 cm. The height of the drawing of the volcano is 25 cm. How tall is the actual volcano? > ONE WAY to find the height is to use a double number line.	ng Scale
Height in Drawing (cm) 0 5 10 15 20 25 30 Image: Image of the state of th	
► ANOTHER WAY is to use a scale factor. The scale from the drawing to the actual volcano is 5 cm for every 50 m, so the scale factor from the drawing to the volcano is ⁵⁰ / ₅ , or 10. Multiply the height of the model by the scale factor: 25 × 10 = 250. Using either method, the height of the actual volcano is 250 m.	agetostarta
Conversation abo	bout scale. y scale 3 4 LESSON 1 Scale Phalama Smalling Scale Countralum Associane, LLC Copying In not permitted.

Connect to Language

For English language learners, use the Differentiation chart to scaffold the language in each session. Use the Academic Vocabulary routine for academic terms before Session 1.

DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Levels 1-3: Reading/Speaking

Help students make sense of Connect It problem 2. Using a **Co-Constructed Word Bank,** read the problem aloud and have students circle unknown words and phrases, like *larger, smaller, same exact shape*, and *original figure*. Review the selected terms with students. If appropriate, invite students to tell Spanish cognates. Then clarify the multiple meanings of *scale* in English.

Next, point out pairs of words with opposite meanings, like *smaller* and *larger* and *original figure* and *scale drawing*. Guide students to use these words to describe the triangles in the problem. Confirm understanding by asking students to identify pairs of corresponding sides in the original figure and scale drawing.

Levels 2–4: Reading/Speaking

Have students read Connect It problem 2 with partners and help them make sense of the text using a **Co-Constructed Word Bank.** If needed, suggest students include *scale*, *scale drawing*, and *scale factor*. Invite students to tell other meanings of *scale*.

Next, ask students to describe the figures in the problem and how they are related using *scale, scale drawing,* and *scale factor*. Ask:

- How are the figures alike? How are they different?
- What sides of the figures can you use to find the scale?

Encourage students to reword responses using terms from the word bank, when appropriate.

Use with Session 1 Connect It

Levels 3–5: Reading/Speaking

Have students read and make sense of Connect It problem 2 using a **Co-Constructed Word Bank.** Encourage students to include key words and phrases, like *scale, scale drawing, scale factor,* and *length of the original figure.* Then ask students to turn to partners and discuss the terms they selected. Have students read the definition of *scale* from the Interactive Glossary and use that definition to explain the meanings of *scale drawing* and *scale factor.* Then have students discuss other meanings of *scale.*

Next, have partners use **Say It Another Way** to confirm understanding of the problem. Encourage them to refer to the drawings to support their paraphrase.

Explore Scale Drawings

Purpose

- **Explore** the idea that rates and ratios can be applied to make scale drawings of shapes.
- **Understand** that scale drawings are figures with the same angles and with side lengths in equivalent ratios.



B and C both appear to be isosceles triangles.

D is the only triangle that appears to be a right triangle.

WHY? Support students' ability to describe and compare triangles.

SMP 1, 2, 4, 5, 6

SMP 2, 3, 6

TRY IT

Make Sense of the Problem

See **Connect to Culture** to support student engagement. Before students work on Try It, use **Three Reads** to help them make sense of the problem. Read the problem aloud and ask: *What is the problem about?* Record students' responses to each question in the routine so students may refer to them as they work. Next, ask a student to read the problem again and ask: *What are you trying to find out?* Have the class read the problem chorally for the third read and ask: *What are the important quantities and relationships in the problem?*

DISCUSS IT

5

Support Partner Discussion

After students work on Try It, have them respond to Discuss It with partners. Listen for understanding of:

- how to compare the angles of triangles.
- how to compare the side lengths of triangles.



Common Misconception Listen for students who argue that $\triangle B$ or $\triangle C$ has the same shape as triangle $\triangle A$ because of general appearance or orientation. As students share their strategies, ask them to define the terms that classify triangles according to their shape, such as *isosceles* and *equilateral*. Then encourage students to use these terms in their discussion.

Select and Sequence Student Strategies

Select 2–3 samples that represent the range of student thinking in your classroom. Here is one possible order for class discussion:

- classifying the triangles as equilateral or as isosceles, then comparing the isosceles triangles based on whether the two equal sides are longer or shorter than the third side
- (misconception) identifying $\triangle B$ or $\triangle C$ as similar to $\triangle A$ based on orientation or a vague sense of shape
- calculating and comparing ratios of side lengths between $\bigtriangleup A$ and the other triangles
- calculating and comparing ratios of side lengths within each triangle

Facilitate Whole Class Discussion

Call on students to share selected strategies. Ensure that students are listening carefully by asking listeners to repeat parts of each speaker's explanation.

Guide students to **Compare and Connect** the representations. Ask students to take individual think time and then turn and talk about the question before the class discusses.

ASK How do [student name]'s and [student name]'s strategies work to determine which triangle is the same shape as $\triangle A$?

LISTEN FOR Both strategies analyze and compare the side lengths of the triangles, and they use the relationships among side lengths to describe the triangles.

CONNECT IT

SMP 2, 4, 5

1 **Look Back** Look for understanding that shapes can be described or classified according to relationships among the side lengths.

DIFFERENTIATION | RETEACH or REINFORCE

Hands-On Activity Compare rectangles.

If students are unsure about the properties of rectangles, then have them compare examples of rectangles.

Materials For each pair: scissors, Activity Sheet *Rectangles, Squares, and Triangles* *****

- Distribute the Activity Sheet and have students cut out the shapes.
- Ask: There are 7 rectangles. Three are the exact same shape but different sizes. Which three? How do you know? [The squares because they are just as wide as they are long and the other rectangles are not.]
- Ask: Of the remaining rectangles, one is definitely not the same shape as the others. Which one? How do you know? [the skinny rectangle; the other rectangles look about twice as long as they are wide, but this one is more than twice as long as it is wide.]
- As time permits, repeat with the triangles. Students should see that two are equilateral and none of the others have the same shape.

LESSON 1 SESSION 1

CONNECT IT

- Look Back Which of the triangles could be the same shape as △A? Explain.
 △D; Possible explanation: Both triangles are isosceles triangles with one longer and two shorter side lengths.
- 2 Look Ahead △D is a scale drawing of △A. A scale drawing of a figure is larger or smaller than the original figure but has the same exact shape. The scale is the ratio between the side lengths of the original figure and the side lengths of the scale drawing. To make a scale drawing of a figure, you can multiply each length of the original figure by a scale factor to get the corresponding length in the scale drawing. You can think of the scale factor as a unit rate.



- a. Explain why the scale from △ABC to △DEF could be 1:3.
 Possible explanation: The ratio AB: DE is 1:3 and the ratio BC: EF is 2:6. Both of these ratios are equivalent to 1:3, so the scale from △ABC to △DEF could be 1:3.
- **b.** Explain why the scale factor from $\triangle ABC$ to $\triangle DEF$ is 3. Possible explanation: You can multiply each side length in $\triangle ABC$ by 3 to get the corresponding side length in $\triangle DEF$.
- c. What is the value of x? Describe two different ways you can find it.
 7.5; Possible answer: The ratio of AC to DF is 1 : 3. You can find an equivalent ratio for 2.5 : x, or you can multiply 2.5 by the scale factor 3.
- 3 **Reflect** Yukio says the scale from △*DEF* to △*ABC* is 3 : 1. Is Yukio correct? Explain. Yes; Possible explanation: The ratio *EF* : *BC* is 6 : 2, which is equivalent to 3 : 1.

2 Look Ahead Point out that the two triangles shown in problem 2 both have two acute angles and one larger angle and that their shapes are the same although their sizes are different. Students should recognize that a scale drawing has the same shape as the original figure but is either larger or smaller.

Ask volunteers to rephrase the definitions of *scale, scale factor,* and *scale drawing*. Support student understanding by asking students to use each term to describe the two triangles shown in problem 2.

CLOSE EXIT TICKET

6

3 **Reflect** Look for understanding that a scale is a ratio, so the order in which quantities are compared matters. Students should understand that they do not need to calculate to find the scale from $\triangle DEF$ to $\triangle ABC$, though they may do so to check their answers.

Common Misconception If students state that Yukio is incorrect because the problem states that the scale factor is 1 : 3, then remind students that a scale is a ratio. Have students explain what two quantities 1 : 3 is a ratio for. Ask them to consider what happens if they compare those quantities in a different order.

Prepare for Solving Problems Involving Scale

Support Vocabulary Development

Assign **Prepare for Solving Problems Involving Scale** as extra practice in class or as homework.

If you have students complete this in class, then use the guidance below.

Ask students to consider the term *unit rate*. Look over the graphic organizer with the class and call on students to explain what they think should be included in each of the sections. For *What Is It?*, students should consider the words *unit* and *rate*, as well as their understanding of unit rate. For *What I Know About It*, students should draw on prior experience with unit rates and rate problems.

Provide support as needed, perhaps suggesting that students identify the unit rate in simple examples, such as a package of 3 peppers that costs 75 cents.

Have students work in pairs to complete the graphic organizer. Invite pairs to share their completed organizers and prompt a whole-class comparative discussion of the definitions, examples, and descriptions of unit rates.

Have students discuss the situation presented in problem 2 with a partner. Encourage students to refer to their graphic organizers to review and apply their definitions of *unit rate* or to extend their examples to find the unit rates for the cost of the limes.

Problem Notes

- 1 Students should understand that a unit rate is different from a rate. A rate is the ratio that tells how many units of one quantity for one unit of the second quantity. A unit rate is the numerical part of a rate. It does not include a colon or a comparison phrase like *per* or *for every*, and it does not include units. Equivalent ratios have the same unit rate.
- 2 Students should recognize that the two unit rates for the limes are reciprocals of each other, because one is the unit rate for the rate of limes to dollars and one is the unit rate for the rate of dollars to limes.

LESSON 1 SESSION 1 Name:

Prepare for Solving Problems Involving Scale

1 Think about what you know about ratios and unit rates. Fill in each box. Use words, numbers, and pictures. Show as many ideas as you can. Possible answers:



2 Limes are on sale. The sale price is 8 limes for \$2.00. Why could the unit rate be 4 or 0.25?

Possible answer: You could find the rate of 4 limes per dollar, which has a unit rate of 4. Or you could find the rate 0.25 dollars per lime, which has a unit rate of 0.25.

REAL-WORLD CONNECTION

Every area has specific codes that architects must follow when designing buildings to ensure their safety. These codes are developed to cover the majority of buildings as safely as possible. However, when a special building such as a tall skyscraper is designed, engineers may recommend additional testing. One such test is a wind tunnel test. An exact model of the building is produced and placed in a wind tunnel that simulates the air pressure and forces that the building will face. It is important that the model is exact to ensure that the test is valid and the final building is safe. Ask students to think of other real-world examples of when a scale drawing or model might be useful.



7

LESSON 1 | SESSION 1 Additional Practice

25.251

painting of George Washington

3 Problem 3 provides another look at identifying scale drawings. This problem is similar to the problem about identifying the scale drawing for a triangle. In both problems, students compare the shape of an original figure to three other shapes, only one of which is a scale drawing of the original. This problem asks students to identify a scale drawing of a rectangle.

Students may want to solve the problem by using tables, double number lines, or ratios.

Suggest that students use **Three Reads**, asking themselves one of the following questions each time.

- What is the problem about?
- What is the question I am trying to answer?
- What information is important?

LESSON 1 SESSION 1

3 A museum sells postcards of famous paintings. The postcards must be the same shape as the painting. Below are three options for the size of the postcard.



 a. Which postcard could be the same shape as the painting? Show your work.
 Possible work:

The painting is a rectangle in which the long side is about 1.2 times as long as the short side.

Option A is a square.

Option B is a rectangle in which the long side is about 2 times as long as the short side.

Option C is a rectangle in which the long side is about 1.2 times as long as the short side.

SOLUTION Option C could be used for the postcard.

b. Check your answer to problem 3a. Show your work.
 Possible work:
 The ratio of the lengths is 30.25 in. to 6.05 in.
 The ratio of the widths is 20.25 in. to 5.05 in.

The scale is 5 in. to 1 in.

ENGLISH LANGUAGE LEARNERS

8

ACADEMIC VOCABULARY

DIFFERENTIATION

Suppose means to believe to be true.

In math, *find* means to calculate.

Levels 1–3: Reading/Writing

Help students interpret Connect It problem 4 and write responses. Review the Academic Vocabulary and the lesson vocabulary *actual*. Help students rephrase the problem using:

- I have a _____. I know the _____ of the scale drawing.
- I want to find the ____

Next, have students tell the steps they can use to find the actual length in their own words. Help students identify the number of steps and write each step in short sentences. Encourage students to use *find* and *actual length* in their writing.

Levels 2–4: Reading/Writing

Help students interpret Connect It problem 4 and write responses. Read the problem with students and help them discuss the meanings of *suppose, find,* and *actual.* Have students use **Say It Another Way** to paraphrase the problem. Then clarify that students need to write the steps for finding length. Encourage students to generate sequence words they can use when writing steps, like *first, next, then,* and *last.*

Have students draft responses using sequence words. Then have students work with partners to discuss and revise responses, as needed.

Use with Session 2 Connect It

Levels 3–5: Reading/Writing

Have students interpret Connect It problem 4 and craft written responses. Ask partners to discuss the meaning of *actual length* and tell what they should include in their written responses. Remind students that effective writing uses transition words, including sequence words, to show the flow of ideas.

Next, have students use **Stronger and Clearer Each Time** to draft responses independently and explain ideas to partners. Ask partners to discuss if the ideas make sense and are easy to follow. Have students revise their writing based on partner feedback.

Develop Using Scale to Find Distances

Purpose

- **Develop** strategies for finding actual distance using a scale drawing and scale factor.
- **Recognize** that a scale factor can be used to find both distances on a scale drawing and actual distances.

START CONNECT TO PRIOR KNOWLEDGE



Possible Solutions

All are expressions of ratios.

A is the only ratio expressed with a colon.

C and D are equivalent ratios.

B and D involve money.

WHY? Support students' ability to evaluate and compare ratios expressed in different ways.

- DEVELOP ACADEMIC LANGUAGE -

WHY? Reinforce understanding of *actual* through synonyms and antonyms.

HOW? Have students find the word in Model It. Ask them to tell the differences between a town represented on a map and the actual town. Then have them discuss the meaning of *actual* using synonyms and antonyms. (For example, *actual* is the same as *real* and the opposite of *unreal*.) Make a T-chart for students to list synonyms and antonyms as they work in the lesson.

SMP 1, 2, 4, 5, 6

TRY IT

Make Sense of the Problem

See **Connect to Culture** to support student engagement. Before students work on Try It, use **Three Reads** to help them make sense of the problem. Each time you or a student reads the problem, ask one of the following questions and record students' responses. After the first read, ask: *What is the problem about?* After the second, ask: *What are you trying to find out?* And after the third, ask: *What are the important quantities and relationships in the problem?*



Support Partner Discussion

After students work on Try It, have them explain their work and respond to Discuss It with a partner. If students need support in getting started, prompt them to ask each other questions such as:

- Did you find a scale factor that relates the map to the town? What was it?
- How do you know the units of the distances in the town?

Common Misconception Listen for students who calculate the distances by multiplying the distances along the map by 500 instead of finding that each centimeter represents 250 ft. As students share their strategies, encourage them to listen to their classmates' strategies and compare their work to the work of others to identify errors and revise their solutions as necessary.

Select and Sequence Student Strategies

Select 2–3 samples that represent the range of student thinking in your classroom. Here is one possible order for class discussion:

- making a double number line with centimeters on the map corresponding to feet in the town
- (misconception) using 500 as the scale factor instead of 250
- making a table of centimeters and corresponding feet
- writing equations that multiply lengths by the scale factor

Facilitate Whole Class Discussion

Call on students to share selected strategies. As students present, ask other students to rephrase key ideas so that students have an opportunity to hear the ideas more than once and expressed in different ways.

Guide students to **Compare and Connect** the representations. Have students turn and talk to rehearse their thinking with a partner before participating in whole-class discussion.

ASK How do the solutions of [student name] and [student name] both relate distance in the town to distance on the map?

LISTEN FOR Both of them use equivalent ratios or a unit rate based on the scale on the map.

Model It

If students presented these models, have students connect these models to those presented in class.

If no student presented at least one of these

models have students first analyze key features of the models and then connect them to the models presented in class.

ASK How does each model use the scale factor?

LISTEN FOR Both apply the known relationship between distances on the map and actual distances, which is 2 cm to 500 ft, to calculate the scale factor and to find unknown distances.

For the model that uses the double number line,

prompt students to describe how the double number line was constructed.

- Which pair of numbers do you think was written first on the double number line? Why?
- After the first number pair was written, how were the other numbers chosen and placed?

For the model that uses the scale factor, prompt students to explain how the scale factor was calculated and how it can be used.

• How does the table show the scale factor?

LESSON 1 SESSION 2

 Explore different ways to find actual lengths based on scale drawings.

This map is a scale drawing of the streets in a town. The scale from the town to the map is 500 ft to 2 cm. What are the actual distances, in feet, of the library, town hall, and train station from the school?



Model It

You can use a double number line to find the actual distances.



Model It

You can use a scale factor to find the actual distances.

Map (cm)	Actual (ft)
2	500
1	250

You can multiply each distance on the map by the scale factor, 250.

Library: 3.5×250 Train Station: 4×250 Town Hall: 4.5×250

10

DIFFERENTIATION | EXTEND

Deepen Understanding

Using Structure in Double Number Lines for Scales

Prompt students to consider that ratios always relate two quantities.

- **ASK** How can you read the pairs of numbers as ratios from top to bottom? What two quantities are they ratios of?
- **LISTEN FOR** You can read them as 1 to 250, 2 to 500, and so on. They are ratios of the number of centimeters on the map to the number of feet in the actual town.
- **ASK** How are those ratios like the scale on the map?
- LISTEN FOR One is identical to the scale on the map. The others are equivalent to it.
- **ASK** Suppose you read the pairs of numbers as ratios from bottom to top. How would this show the situation differently?

LISTEN FOR The double number line would compare the same quantities in a different order. You would read the ratios as 250 to 1, 500 to 2, and so on.

ASK When you found the ratio of centimeters to feet, you found the scale from the map to the town. When you find the ratio of feet to centimeters, what scale do you find? **LISTEN FOR** The scale from the town to the map.

SMP 7

Develop Using Scale to Find Distances

CONNECT IT

SMP 2, 4, 5, 6

Remind students that the quantities and the relationships between them are the same in each representation. Explain that they will now use those relationships to reason about using scale drawings and scale factors to find actual distances.

Before students begin to record and expand on their work in Model It, tell them that problem 3 will prepare them to provide the explanation asked for in problem 4.

Monitor and Confirm Understanding 1 – 2

- Find the actual distances in feet by multiplying the distances on the map in centimeters by the scale factor, 250.
- The double number line shows equivalent ratios. It shows that the ratio of 500 ft to 2 cm is equivalent to 250 ft to 1 cm. The scale factor is the unit rate for these ratios.

Facilitate Whole Class Discussion

3 Look for understanding of the concept of unit rate and how it relates to a scale factor.

ASK What is a unit rate, and why is a scale factor an example of a unit rate?

LISTEN FOR A unit rate relates a quantity to one unit, such as a speed of 50 miles per hour. A scale factor also relates a quantity to one unit, such as 250 ft per cm.

4 Look for the idea that multiplying by a scale factor is an efficient way to calculate actual distances.

ASK When you have a scale drawing, why is the scale factor a useful value to calculate?

LISTEN FOR You can multiply the scale factor by any distance on the drawing to find the actual distance.

5 Students should recognize that they do not need to find a new scale factor.

ASK How is the scale factor useful for finding an unknown distance on the map?

LISTEN FOR The scale factor relates distances on the map and actual distances, so it can be used to convert either one to the other.

6 **Reflect** Have all students focus on the strategies used to solve the Try It. If time allows, have students discuss their ideas with a partner.

CONNECT IT

- Use the problem from the previous page to help you understand how to find actual lengths from a scale drawing.
- 1 How far, in feet, are the library, town hall, and train station from the school? library: 875 ft; train station: 1,000 ft; town hall: 1,125 ft
- 2 Look at the first Model It. How does the double number line show the scale factor you can use to find an actual distance, given the distance on the map? Possible answer: The scale factor is the actual distance when the distance on the map is 1, and 1 cm on the map is the same as an actual distance of 250 ft.
- 3 Look at the second Model It. How is a scale factor like a unit rate? You can think of the scale as a rate for how many feet are represented by 1 cm on the map. The unit rate and the scale factor are just the numerical part of that relationship.
- Suppose you have a scale drawing and know its scale. How can you use the scale drawing to find an actual length?
 Possible answer: You can use the scale to find a scale factor. Then you can multiply the length on the scale drawing by the scale factor to find the actual length.
- The actual distance between the library and train station is 500 ft. Explain how you could find the distance between them in centimeters on the map.
 Possible explanation: Since you multiply by 250 to find the actual distance from a distance on the map, you can divide by 250 to use an actual distance to find the distance on the map.
- 6 Reflect Think about all the models and strategies you have discussed today. Describe how one of them helped you better understand how to solve the Try It problem.

Responses will vary. Check student responses.

11

DIFFERENTIATION | RETEACH or REINFORCE

Hands-On Activity

Use maps and scale factors to find actual distances.

If students are unsure about the relationship shown by a scale factor, then use this activity to show a real-life example of maps and the distances they represent.

Materials For each pair: 1 ruler, a map of your region or state

- Help students find the legend on the map, which shows the scale. If the scale factor is not shown, have students calculate the scale factor by identifying a unit of distance on the map, such as 1 cm or 1 in., and the actual distance it represents.
- Have students use the ruler to measure a distance along the map, such as between towns or landmarks. Ask: *How can you find the actual distance*? [Multiply the measured distance on the map by the scale factor.]
- Ask: Do you think all maps use the same scale and scale factor? Why or why not? [No; Other maps may have a lesser or greater scale factor. A greater scale factor allows a map to show a larger area, although generally with fewer details about the area.]

LESSON 1 | SESSION 2 Develop

Apply It

For all problems, encourage students to use a model to support their thinking. Allow some leeway in precision. For example, if students use a number line, they may approximate the distances between numbers, although the order of the numbers should be correct.

7 Students may also solve the problem by converting the mixed number and fraction to decimals and multiplying by the scale factor.

8 Students may also solve the problem by constructing a double number line to relate distances on the map and in the town or by making a table to compare the scale and actual distances.

LESSON 1 SESSION 2

Apply It

- > Use what you learned to solve these problems.
- A scientist who studies insects enlarges a photograph of an elm leaf beetle. Every 2 in. in the photograph represents 8 mm on the actual beetle. The length of the beetle in the photograph is 1¹/₂ in. The width of the beetle in the photograph is ³/₄ in. What are the length and width of the actual beetle? Show your work.
 Possible work:



The scale factor from the photograph to the actual beetle is $\frac{8}{2}$, or 4.

$$1\frac{1}{2} \times 4 = \frac{3}{2} \times 4$$
 $\frac{3}{4} \times 4 = 3$
= 6

SOLUTION The length of the beetle is 6 mm and the width is 3 mm.

- On a map, 2 cm represents 30 mi. The actual distance between two towns is 75 mi. What is the distance between the towns on the map? Show your work.
 Possible work:
 - There are 2 cm for every 30 mi. The scale factor $\frac{30}{2}$, or 15, means you can
 - divide the actual distance by 15 to get the distance on the map.

75 ÷ 15 = 5

SOLUTION The distance between the towns on the map is 5 cm.

9 Tyrone makes a scale drawing of his backyard. The scale from the backyard to the drawing is 2 ft to 1 in. The width of the patio on the drawing is 8 in. What is the width of Tyrone's actual patio? Show your work.
Possible work:

Every inch in the drawing represents 2 ft in the actual backyard. So, the scale factor from the drawing to the actual patio is 2.

8 × 2 = 16

12

SOLUTION The width of the patio is 16 ft.

CLOSE EXIT TICKET

- 9 Students' solutions should show an understanding of:
 - recognizing and applying a definition of scale.
 - calculating actual distance by multiplying the distance on the scale drawing by the scale factor.

Error Alert If students apply the reciprocal of the scale factor to find an actual distance of 4 ft, then ask them to review the definition of *scale factor*. As students discuss the calculations they performed, ask them to explain how they know whether they are using the correct scale factor or its reciprocal.

Practice Using Scale to Find Distances

Problem Notes

Assign **Practice Using Scale to Find Distances** as extra practice in class or as homework.

Students may also solve the problem by identifying the scale factor for converting distances in feet in the actual basement to lengths in inches in the scale drawing, which

is $\frac{1}{10}$, and multiplying this scale factor by the dimensions of the basement. **Basic**

2 Students may also first convert both heights on the drawing to the actual heights and find the difference. *Medium*

LESSON 1 SESSION 2 Name:

Practice Using Scale to Find Distances

Study the Example showing how to use a scale drawing to find an actual distance. Then solve problems 1–5.

Example

Colin makes a scale drawing of his bedroom. Every inch in his drawing represents 10 feet in his actual bedroom. The drawing is 1.25 in. wide and 1.5 in. long. How wide and long is his actual bedroom?

You can use a scale factor to find the dimensions.

The scale from the drawing to the bedroom is 1 in. to 10 ft, so the scale factor

from the drawing to the bedroom is $\frac{10}{1}$, or 10.

 $1.25 \times 10 = 12.5$ $1.5 \times 10 = 15$

Colin's bedroom is 12.5 ft wide and 15 ft long.

 A drawing of a basement uses the same scale as the Example. The basement is 28 ft wide and 35 ft long. How wide and long is the drawing? Show your work.
 Possible work:

The scale factor from the basement to the drawing is 10. $28 \div 10 = 2.8$ $35 \div 10 = 3.5$

SOLUTION The drawing is 2.8 in. wide and 3.5 in. long.

2 Efia draws this scale drawing of two famous landmarks. Each inch in the drawing represents 400 ft on the actual landmark. Approximately how much taller is the actual Eiffel Tower than the actual Space Needle? Show your work. Possible work: The drawing of the Space Needle is 1¹/₂ in. tall.

The drawing of the Eiffel Tower is $2\frac{5}{8}$ in. tall.

 $2\frac{5}{8} - 1\frac{1}{2} = 1\frac{1}{8}$

 $1\frac{1}{8} \times 400 = 450$

SOLUTION The Eiffel Tower is approximately 450 ft taller.

Fluency & Skills Practice 🍾

Using Scale to Find Distances

In this activity, students solve problems about scale drawings to determine actual lengths or distances, and they use actual lengths or distances to determine dimensions on scale drawings.



 π

Space Needle Eiffel Tower

. ___ 3 in

2 in

- 1 in.

13

3.25 cm

- 3 Students may divide 3.25 by 2, then multiply the quotient by 8 to find the distance across the coin in millimeters. *Medium*
- 4 Students may also solve the problem by using a visual model, such as a table or a double number line. *Medium*
- 5 Students may also solve the problem by multiplying the dimensions of the paper by the scale factor of 15 and comparing the actual distances represented by the paper with the dimensions of the memorial. *Challenge*

LESSON 1 SESSION 2

3 The photo shows a small coin. The scale from the actual coin to the photo is 8 mm to 2 cm. In the photo, the distance across the coin is 3.25 cm. What is the distance across the actual coin? Show your work. Possible work:

Since the scale from the photo to the actual coin is 2 cm for every 8 mm, the scale factor from the photo to the actual coin is $\frac{8}{2}$, or 4.

 $3.25 \times 4 = 13$

SOLUTION The distance across the coin is 13 mm.

In a photograph, Alison stands next to her brother Caleb. Alison is 4 cm tall in the photograph. Her actual height is 60 in. Caleb is 3.2 cm tall in the photograph. What is his actual height? Show your work.
 Possible work:

Since the scale from the photo to the actual height is 4 cm to 60 in., the scale factor from the photo to the actual height is $\frac{60}{4}$, or 15.

3.2 × 15 = 48

SOLUTION Caleb is 48 in. tall.

Adoncia makes a scale drawing of the front of the Lincoln Memorial. She uses a scale of 15 ft in the monument to 1 in. in the drawing. The front of the monument is about 80 ft high and 200 ft long. Will Adoncia's drawing fit on an 8¹/₂ in.-by-11 in. sheet of paper? Explain.



No; Possible explanation: The scale from the actual building to the drawing is 15 ft to 1 in., so the scale factor from the drawing to the actual building is $\frac{15}{1}$, or 15. The actual length is 200 ft, so the length of the scale drawing in inches is $200 \div 15$, or $13\frac{1}{3}$. Since $13\frac{1}{3} > 11$, a scale drawing using the scale 15 : 1 will not fit on the paper.

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DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Levels 1-3: Speaking/Writing

Have students solve Apply It problem 9 using models, diagrams, or equations. Then help students talk about the steps they used in their solution strategies. Display sentence frames students might use to describe their solution strategies. Possible descriptions might use these frames:

- My first step was _____.
- My next step was _____.

Help students tell how to complete the frames before turning to partners and describing their strategies. Ask partners to tell if their solutions and solution strategies were the same or not. Then have partners write their responses together.

Levels 2-4: Speaking/Writing

Have students solve Apply It problem 9 using models, diagrams, or equations. Have students turn to partners and describe the first step of their solution strategy using:

- I solved the problem by _____.
- I knew I had to _____, so my first step was _____.

Encourage partners to use sequence words to tell the order of the other steps in their solution strategies. Ask partners to review each others' steps and discuss:

• I think the _____ step is correct because ____

Provide time for students to revise strategies, if needed, before writing responses.

Use with Session 3 Apply It

Levels 3–5: Speaking/Writing

Have students solve Apply It problem 9 independently. Then have students discuss their solution strategies with partners. Have students take turns explaining the steps of their strategies to partners. Ask them to compare by telling how the steps were the same or different. If strategies differ, ask partners to review their work, check their solutions, and ask questions to clarify strategies. For example, display these possible questions:

- How did you find ____?
- Why was your first step ____?

Provide time for students to revise the steps in their strategies, if needed, before writing their responses in complete sentences.

Develop Using Scale to Find Areas

Purpose

- **Develop** strategies for using scale factors to find area.
- **Recognize** that an area on a scale drawing is multiplied by the square of the scale factor to find actual area.

START CONNECT TO PRIOR KNOWLEDGE

Always, Sometimes, Never

- A A scale drawing is larger than the original
- B The same scale factor applies to all the lengths in a scale drawing.
- C 1 in. on a scale drawing represents 0 ft
- A scale drawing of an object has the same measurements as the actual object

Solutions

- A is sometimes true.
- B is always true.
- C is never true.
- D is sometimes true.

WHY? Support students' understanding of scale, scale drawings, and scale factors.

- DEVELOP ACADEMIC LANGUAGE -

WHY? Understand and use phrases that describe relationships.

HOW? Display these frames: *The relationship between* _____ *and* _____ *is* *The ratio of* _____ *to* _____ *is* Tell students that they can use the structures to describe how two things are related or connected. Have students work in pairs to define *scale*. Then have them use one of the structures to talk about the relationship between scale and area in Reflect. Discuss other situations where these structures could be used.

SMP 1, 2, 4, 5, 6

TRY IT

Make Sense of the Problem

See **Connect to Culture** to support student engagement. Before students work on Try It, use **Three Reads** to help them make sense of the problem. Read the problem aloud and ask students to look at the picture as you read. Ask: *What is this problem about*? Then have a volunteer read the problem again and ask: *What are you trying to find out*? Have students pair up and read the problem again with a partner. Ask pairs to make a list of the important quantities and relationships in the problem. Then, work together to make a class list.



Support Partner Discussion

After students work on Try It, have them respond to Discuss It with a partner. If students need support in getting started, prompt them to ask each other questions such as:

- How is this problem similar to other problems about scale drawings that you have solved? How is it different?
- How did you use the scale for the blueprint?

Common Misconception Listen for students who calculate the area of the floor as shown on the scale drawing and multiply it by the scale factor of 20. As students share their strategies, have them explain the differences between length and area. Allow students to compare their results with others and make revisions as appropriate.

LESSON 1 | SESSION 3 Develop

Dinosaurs

Great Hall

1<u>3</u> in.

Special Exhibit

Hall of

The

Select and Sequence Student Strategies

Select 2–3 samples that represent the range of student thinking in your classroom. Here is one possible order for class discussion:

- using a double number line to find the actual dimensions and multiplying them to find the area
- (misconception) finding the area in the drawing and then multiplying by the scale factor
- multiplying by the scale factor to find the actual dimensions and then multiplying to find the area
- multiplying the area of the scale drawing by the square of the scale factor

Facilitate Whole Class Discussion

Call on students to share selected strategies. After each student shares, allow individual think time. Then encourage listeners to ask questions.

Guide students to **Compare and Connect** the representations. If the discussion lags, ask students to turn and talk about two of the strategies that were shared.

ASK How did [student name] and [student name] use the scale of the blueprint to find the area?

LISTEN FOR They both used it to find a scale factor and multiplied by that scale factor twice in some way.

Model It

If students presented these models, have students connect the models to those presented in class.

If no student presented these models, have students first analyze key features of the models and then connect them to the models presented in class.

ASK How are the two solution strategies alike? How are they different?

LISTEN FOR Both strategies begin by converting the distances in the blueprint to actual distances. Then they multiply to find the area. The first Model It uses a table to find the actual distances. The second Model It uses a scale factor, and it uses decimals instead of mixed fractions.

For the table, prompt students to think about the relationship between distances.

- How can you find an actual distance from a blueprint distance?
- Since both the length and width are to scale, what does this reveal about the relationship between the areas for the blueprint and the actual hall?

For the scale factor calculation, prompt students to look at the structure of the equation.

• Why does the factor 20 appear in two places in the expression for area?

LESSON 1 SESSION 3

Explore different ways to find actual areas from a scale drawing.

A blueprint for the floor of a natural history museum is shown. The scale of the museum to the blueprint is 20 yd to 1 in. What is the area of the floor of the actual Great Hall?

Model It

You can use a table of equivalent ratios to find the actual dimensions.

Blueprint (in.)	1	$\frac{1}{4}$	<u>3</u> 4	$1\frac{3}{4}$
Actual (yd)	20	5	15	35

The actual floor is 35 yd long and 15 yd wide.

Area of floor = 35×15

Model It

You can use a scale factor to find the area of the actual floor.

The scale from the blueprint to the museum is 20 : 1. So, the scale factor is 20.

Length of actual floor: 1.75 imes 20 Width of actual floor: 0.75 imes 20

Area = ℓw

 $= (1.75 \times 20)(0.75 \times 20)$



DIFFERENTIATION | EXTEND



Deepen Understanding

SMP 8

Express Regularity in Area Found from Scale Drawings

Prompt students to recognize how scale drawings and scale factors can be used to find actual distance and area.

- **ASK** Look at the expression shown in the last line of the second Model It. Why is the expression equal to the actual area of the exhibit hall?
- LISTEN FOR The expression is the product of the actual length and the actual width, each of which is the product of the dimension in the blueprint and the scale factor, 20.
- **ASK** Why can you also represent the actual area of the Great Hall with the expression $1.75 \times 0.75 \times 20^{2}$?
- LISTEN FOR You can multiply the factors from the expression on the page in any order, so you can put 1.75 and 0.75 before 20 \times 20. Then you can rewrite 20 \times 20 as 20².

Generalize Encourage students to describe how they might choose whether to multiply the area from a scale drawing by the square of the scale factor or multiply the scale factor by each dimension when finding the area.

Develop Using Scale to Find Areas

CONNECT IT

SMP 2, 4, 5, 6

Remind students that the quantities and the relationships between them are the same in each representation. Explain that they will now use those relationships to reason about area in scale diagrams.

Before students begin to record and expand on their work in Model It, tell them that problem 4 will prepare them to provide the explanation asked for in problem 5.

Monitor and Confirm Understanding 1 – 3

- The actual area can be calculated in two steps: first using the scale factor to find the actual length and the actual width, and then finding the product of the actual length and width.
- The scale factor is the unit rate for the ratio of actual distance to distance in the blueprint, which is 20.
- The scale factor represents a comparison of linear distance. Area is calculated as the product of two linear distances and can be found as the sum of individual square units.

Facilitate Whole Class Discussion

Have students discuss the idea that the scale factor is one-dimensional and area is two-dimensional.

ASK Suppose that a distance on the blueprint is $1\frac{5}{16}$ inches. How can you apply the scale factor of 20 to find the actual distance?

LISTEN FOR Multiply $1\frac{5}{16}$ by 20 to find the actual distance in yards.

ASK Now suppose that an area on the blueprint is $1\frac{5}{16}$ in.². How is using the scale factor different for finding area than for finding linear distances?

LISTEN FOR To find the actual area, multiply $1\frac{5}{16}$ by 20^2 , which is the square of the scale factor, instead of multiplying only once by the scale factor when finding the linear distance.

- 5 Look for the idea that converting an area on a scale drawing to an actual area involves multiplying by a linear scale factor two times, or multiplying by the scale factor squared.
- 6 Reflect Have all students focus on the strategies used to solve the Try It. If time allows, have students discuss their ideas with a partner.

CONNECT IT

- > Use the problem from the previous page to help you understand how to find actual areas from a scale drawing.
- 1 What is the area of the floor of the actual Great Hall? 525 yd²
- 2 Look at the second Model It. What does the scale factor mean in this situation? Possible answer: This scale factor means that every 1-in. distance in the blueprint represents a 20-yd distance in the museum.
- The scale from the museum to the drawing is 20 yd : 1 in. Another scale from the museum to the drawing is 400 yd² : 1 in.². Explain why.
 Possible explanation: A square in the drawing with 1-in. sides represents a square in the museum with 20-yd sides. Since the area of the square in the museum is 20 × 20, or 400 yd², every 1 in.² in the drawing represents 400 yd² in the museum.
- 4 The area of the Great Hall in the blueprint is $1\frac{5}{16}$ in.². Why does multiplying the blueprint area by the scale factor 20 not give the area of the floor of the actual Great Hall?

Possible explanation: Since you multiply each blueprint dimension by 20 to find the actual dimensions, you must multiply the blueprint area by the scale factor twice to find the actual area.

- How is finding an actual area from the area in a scale drawing similar to finding an actual length from a length in a scale drawing? How is it different?
 Possible answer: It is similar because you can multiply by the scale factor. But for the length, you only multiply by the scale factor once. For the area, you multiply by the scale factor twice.
- Reflect Think about all the models and strategies you have discussed today. Describe how one of them helped you better understand the relationship between an actual area and the area in a scale drawing. Responses will vary. Check student responses.
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DIFFERENTIATION | RETEACH or REINFORCE

Hands-On Activity Analyze scale models of a sidewalk and a patio.

If students are unsure about the use of scale factors in calculating area, have them build and analyze a scale model of a sidewalk and a patio.

Materials For each pair: base-ten blocks (10 tens rods)

- Tell students that the length of each unit cube in the tens rods represents 6 yd. Have students model a sidewalk by placing 10 rods end to end. Ask: *How can you find the length of the sidewalk this model represents?* [Multiply the number of units, 100, by the length in yards each unit represents, 6.] Ask: *Is 6 or 100 the scale factor? Why?* [6; You could multiply any number of units by it to get the distance they represent.]
- Next, have pairs model a patio by placing all 10 rods side by side to form a square 10 units long and 10 units wide. Ask: What is the area of the top of one unit cube? How do you know? [1 square unit, because both sides have length 1 unit and you multiply them] Ask: What is the area that the top of one unit cube represents? How do you know? [36 yd², because both sides have length 6 yd and you multiply them]
- Ask: *How can you use this to find the area of the patio?* [You can multiply 36 yd² by 100 cubes, or you can find that each side represents 60 yd and find 60 × 60.]

LESSON 1 | SESSION 3 Develop

Apply It

For all problems, encourage students to use a model to support their thinking. Allow some leeway in precision; for example, if students draw and label rectangles, the ratios of the side lengths do not need to be precise.

- Students may also solve the problem by calculating the area of the scale drawing, which is 20.14 square centimeters, and then multiplying by the square of the linear scale factor, which is 4², or 16.
- 8 D is correct. Students may solve this problem by dividing the actual area in square feet, which is 144, by the square of the scale factor, which is 6², or 36.
 - **A** is not correct. This answer may be the result of multiplying the actual area by the square of the scale factor instead of dividing.
 - **B** is not correct. This answer may be the result of dividing the actual area by the scale factor instead of the square of the scale factor.
 - **C** is not correct. This answer may be the result of interpreting the scale factor as representing area in the scale drawing.

Apply It Use what you learned to solve these problems. The scale from a playground to a scale drawing of the playground is 4 meters per centimeter. The length of the drawing of the playground is 5.3 cm and the width is 3.8 cm. What is the area of the actual playground? Show your work. Possible work: The scale factor from the drawing to the playground is $\frac{4}{1}$, or 4. 5.3 cn $Area = (5.3 \times 4)(3.8 \times 4)$ = 322.24**SOLUTION** ______ The area of the actual playground is 322.24 m². 3.8 cm scale drawing of a playground 8 A square has an area of 144 ft². In a scale drawing of the square, each inch represents 6 ft. What is the area of the square in the drawing? **B** 24 in.² A 5.184 in.² **C** 6 in.² (D) 4 in.² Below is a scale drawing of the side of a ramp at a skateboard park. The scale from the drawing to the actual ramp is 2 cm to 12 in. What is the area of the actual side of the ramp? Show your work. 2 cm 8 cm **Possible work:** Since the scale from the drawing to the actual ramp is 2 cm for every 12 in., the scale factor from the drawing to the actual ramp is $\frac{12}{2}$, or 6. Area = $\frac{1}{2}bh$ $=\frac{1}{2}(2 \times 6)(8 \times 6)$ = 288 **SOLUTION** The area of the side of the actual ramp is 288 in.².

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CLOSE EXIT TICKET

LESSON 1 SESSION 3

- 9 Students' solutions should show an understanding of:
 - calculating a scale factor from a verbal description of a scale by finding the unit rate for the ratio of the actual distance to the distance on the scale copy.
 - applying the scale factor to calculate actual area by multiplying by the scale factor twice.

Error Alert If students calculate the area of the triangular side of the ramp by multiplying $\frac{1}{2}$, 2, and 8, and multiplying by the scale factor of 6 (not 6²) to find the actual area, then have them calculate the actual base and height of the side of the ramp and use those dimensions to find the area. Ask students to explain why the results are different.

Practice Using Scale to Find Areas

Problem Notes

Assign **Practice Using Scale to Find Areas** as extra practice in class or as homework.

- 1 Students should recognize that the scale of 1 : 8 shows an enlargement of the logo, so if they are multiplying to solve the problem, they should use a scale factor of 8, not $\frac{1}{8}$. **Basic**
- 2 Students may also solve the problem by calculating the area of the rectangle on the scale drawing and multiplying by the square of the scale factor. *Basic*

LESSON 1 SESSION 3 Name:

Practice Using Scale to Find Areas

Study the Example showing how to find an actual area from a scale drawing. Then solve problems 1–4.

Example



The area of the logo on the sign is 4.5 ft².

Employees of the store in the Example wear shirts with the logo on the back. The scale from the original logo in inches to the shirt in inches is 1 : 8. What is the area of the logo on their shirts? Show your work. Possible work:

The scale factor from the logo to the shirt is $\frac{8}{1}$, or 8.

Area = $\frac{1}{2}bh$

 $=\frac{1}{2}(1\frac{1}{2}\times 8)(1\frac{1}{2}\times 8)$

SOLUTION The area of the logo on the shirt is 72 in.².

2 Dr. Gordon has a scale drawing of a building site. The drawing uses the scale 2 in. on the drawing for every 100 ft on the building site. Dr. Gordon marks a 6 in.-by-3.2 in. section of the drawing to show the section she will search. What is the area of the section she will search? Show your work. Possible work:

The scale factor from the drawing to the site is $\frac{100}{2}$, or 50.

Length: $6 \times 50 = 200$ Width: $3.2 \times 50 = 160$ Area: $200 \times 160 = 48,000$

SOLUTION The area of the section is 48,000 ft².

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Fluency & Skills Practice 🍾

Using Scale to Find Areas

In this activity, students solve problems in which the scale or a scale drawing is given and students calculate the actual area.



LESSON 1 | SESSION 3 Additional Practice

- 3 Students should recognize that the strategies for calculating an actual area from a scale diagram can be applied for any shape. *Medium*
 - a. Students may also support their answer by choosing possible dimensions for the park on the map, such as 3 cm and 2 cm, and then calculating the actual length and width of the park and the actual area. *Medium*
 - b. Students may also solve the problem by recognizing that the actual arena has length 100 m and width 100 m and dividing each dimension by the scale factor of 50.
 Challenge
 - **c.** Students should recognize that the scale factor for area is the square of the linear scale factor and that this relationship applies to all areas regardless of their shape. *Challenge*



c. The area of a parallelogram on the map is 5 cm². What is the area of the actual parallelogram? Explain why you can find the area without knowing the dimensions of the parallelogram.

12,500 m²; Possible explanation: To find the area of the actual parallelogram, you multiply by the scale factor twice, once for the base and once for the height. The actual base and height do not matter.

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DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Levels 1-3: Reading/Writing

Help students make sense of Apply It problem 7 using an adaptation of **Three Reads.** For Read 1, read sentences 1–2. Invite students to tell if they have seen or played four square or other similar games at a park. For Read 2, read sentences 3–4 and review the lesson academic vocabulary *actual* and *justify*. For Read 3, read the problem again and ask students to highlight any quantities and units. Encourage students to write the scales using numbers and symbols.

Ask students to tell if the new scale drawing is greater or less than the original. Ask students to refer to their calculations as they explain their reasoning in their own words. Then help students justify responses in writing.

Levels 2-4: Reading/Writing

Help students make sense of Apply It problem 7 using an adaptation of **Three Reads.** After Read 1, invite students to tell if they have seen or used any four-square courts and to describe the size of a typical court. After Read 2, have partners discuss strategies to find the dimensions of the new drawing. Call on students and record different possible strategies, rewording as needed. Then, after Read 3, have students identify the scale of the original drawing and draw and label the dimensions and scale of the new drawing.

Have partners prepare to write by first discussing their solution strategies and then explaining their strategies in writing using precise language.

Use with Session 4 Apply It

Levels 3–5: Reading/Writing

Have students interpret Apply It problem 7 by using **Three Reads.** After students read and discuss the problem, review with students the meaning of the lesson academic vocabulary *justify*. Have students describe ways to justify solution strategies in writing. Make sure students understand that one way to justify is to explain what they know about a problem and tell how the information helps them form logical conclusions.

Encourage students to justify their drawings by writing an explanation of how the information in the problem helped them find the dimensions of the new scale. Suggest they refer to calculations and their drawing as they write.

LESSON 1 | SESSION 4 **Develop** Redrawing a Scale Drawing

Purpose

- **Develop** strategies for redrawing a scale drawing with a new scale.
- Recognize that the scale factor between two scale drawings can be calculated as the guotient of the individual scale factors.

START **CONNECT TO PRIOR KNOWLEDGE**



Possible Solutions

- A uses a number that is not a power of 10.
- B has a decimal.
- C represents an enlargement.

D uses customary units instead of metric units.

WHY? Support students' understanding of scale, scale drawings, and scale factors.

DEVELOP ACADEMIC LANGUAGE

WHY? Support students as they explain their ideas clearly.

HOW? Explain that in order to explain ideas clearly, students need to speak loudly enough for others to hear. Share that it is helpful to pause occasionally to ask listeners for questions or comments. Provide sentence frames for students to share how they approached the problem. For example, I started off by _____ because ____

SMP 1, 2, 4, 5, 6

TRY IT

Make Sense of the Problem

See *Connect to Culture* to support student engagement. Before students work on Try It, use Three Reads to help them make sense of the problem. Read the problem and ask: *What is this* problem about? Read it again and ask: What are you trying to find out? After a third read, ask: What are the important quantities in the problem? Ensure that students understand there are three related rectangles: the original scale drawing, the actual pool, and the new scale drawing.

evelop Redrawing a Scale	Men's Locker Boom
vrawing	
Read and try to solve the problem below.	Women's Locker
The scale from the actual center to the drawing is 6 m to 1 cm. Make another scale drawing of the pool using 5 m to 1 cm as the scale from the recreation center to the drawing.	5 cm
TRY IT Math Toolkit double number lines, grid paper, ribbon, rulers, yarn	
Possible work:	
SAMPLE A 6 cm	
pool is 6.	
Since the original drawing is 2.5 cm wide and	3 cm
5 cm long, the actual pool is 15 m wide and	
30 m long.	
The scale factor from the new drawing to the pool is 5.	
$15 \pm 5 = 3$	
50 - C - 0	
SAMPLE B	
Since the scale from the original drawing to the pool is 1 : 6 and the	
new drawing is 6 : 5.	
	DISCUSS IT
6 cm	
5 cm	Ask: How did you start to figure out the
	dimensions of the nev
2.5 CM	drawing?
2.5 CM	snare: I knew
	sol

DISCUSS IT

Support Partner Discussion

After students work on Try It, have them respond to Discuss It with a partner. If students need support in getting started, prompt them to ask each other questions such as:

- How is this problem similar to other problems that you have solved about scale drawings? How is it different?
- How did you keep track of which dimensions go with each rectangle?

Common Misconception Listen for students who use a scale factor of 5 to make a scale drawing that has dimensions 5 times those of the original drawing, 12.5 cm wide and 25 cm long. As students share their strategies, have them listen to one another to recognize and address their misunderstandings about the actual dimensions, the original scale drawing, and the new scale drawing.

Select and Sequence Student Strategies

Select 2–3 samples that represent the range of student thinking in your classroom. Here is one possible order for class discussion:

- using a double number line to compare each scale drawing to the actual distance it represents
- (misconception) using a scale factor of 5 from the original drawing to the new drawing, not from the actual center to the new drawing
- applying a scale factor to calculate the actual dimensions of the pool, and then calculating the dimensions for the new scale diagram
- applying the quotient of the two scale factors to calculate the dimensions of the new diagram

Facilitate Whole Class Discussion

Call on students to share selected strategies. Remind students to listen actively by looking at the speaker and trying to understand the strategy that the speaker presents.

Guide students to **Compare and Connect** the representations. During the discussion, make sure each strategy is discussed and evaluated. Encourage students to ask questions about anything they do not understand.

ASK How did [student name] and [student name] apply the scale factors in different ways?

LISTEN FOR Scale factors are used to convert from scale distance to actual distance, or vice versa. The two scale factors can be applied in separate steps or combined into one step.

Model It & Analyze It

If students presented these models, have students connect these to those presented in class.

If no student presented either of these models, have students first analyze their key features and then connect them to the solutions presented in class.

ASK How is the reasoning in Analyze It supported by the double number lines in Model It?

LISTEN FOR It shows that 1 cm represents 6 m in the original drawing and 5 m in the new drawing.

For Model It, prompt students to describe how the double number lines were constructed.

• What pattern is shown in each double number line? How does this pattern show scale?

For Analyze It, prompt students to explain the reasoning about the two scales.

• Why is the scale expressed as 6:5, not 5:6?

LESSON 1 SESSION 4

> Explore different ways to make a new drawing with a different scale.

An architect makes a scale drawing of a recreation center. The scale from the actual center to the drawing is 6 m to 1 cm. The pool in the scale drawing is 2.5 cm wide and 5 cm long. Make another scale drawing of the pool using 5 m to 1 cm as the scale from the recreation center to the drawing.

Model It

You can use double number lines to find the dimensions.





Analyze It

You can reason about how the scales in each drawing are related.

Original Drawing Scale:

actual recreation center to drawing is 6 m to 1 cm

New Drawing Scale:

actual recreation center to drawing is 5 m to 1 cm

So, the scale from the original drawing to the new drawing is 6:5.

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DIFFERENTIATION | EXTEND



Deepen Understanding Expressing Regularity in Multiple Scale Factors

Prompt students to recognize how two scale factors can be used to redraw a scale diagram. Draw three rectangles, labeled *Original Scale Drawing*, *Actual Pool*, and *New Scale Drawing*. Label the sides of the first rectangle *2.5 cm* and *5 cm*.

ASK How can you find the length and width of the actual pool using the first drawing? LISTEN FOR Multiply by the scale factor, 6, and use meters as the unit.

Write \times 6 above the second rectangle and label its sides 15 m and 30 m.

- **ASK** How can you label the new scale drawing using the actual pool dimensions?
- **LISTEN FOR** Divide the labels for the second rectangle by 5 and use centimeters.

Write \div 5 above the third rectangle and label its sides 3 cm and 6 cm.

- **ASK** How could you combine multiplying by 6 and dividing by 5 into one operation?
- **LISTEN FOR** You can multiply by $\frac{6}{5}$.

Generalize Two scale factors can be written as a single scale factor to show a relationship between two scale drawings or copies of the same thing.

SMP 8

Develop Redrawing a Scale Drawing

CONNECT IT

SMP 2, 4, 5, 6

Remind students that the quantities and the relationships between them are the same in each representation. Explain that they will now use those relationships to reason about area in scale diagrams.

Before students begin to record and expand on their work in Model It, tell them that problem 3 will prepare them to provide the explanation asked for in problem 4.

Monitor and Confirm Understanding 1 – 3

• For both drawings, the ratio of any two corresponding values on the double number line is equivalent to the scale between that drawing and the pool. The scale is 1 cm to 6 m for the original drawing and 5 m to 1 cm for the new drawing.

Facilitate Whole Class Discussion

3 Look for the idea that a greater scale factor results in shorter lengths in the scale drawing.

ASK Are the lengths in the scale drawing longer when the scale factor is 6 or when the scale factor is 5? Why?

LISTEN FOR They are longer when the scale factor is 5, because you need 6 groups of 1 cm to show 30 m. When the scale factor is 6, you only need 5 groups of 1 cm to show 30 m.

Have students discuss the relationship between different scale diagrams of the same object.

ASK How can you find the scale factor from the original drawing to the new drawing?

LISTEN FOR The scale 6 : 5 is a ratio, so the scale factor is the unit rate for that ratio, $\frac{6}{5}$.

ASK How could you make another new diagram with a different scale factor?

LISTEN FOR Choose another scale factor and divide the first scale factor by the new scale factor to find the length and width for a different new drawing.

5 Reflect Have all students focus on the strategies used to solve the Try It. If time allows, have students discuss their ideas with a partner.

CONNECT IT

- > Use the problem from the previous page to help you understand how to reproduce a scale drawing using a different scale.
- What are the dimensions of the pool in the new drawing?
 3 cm wide and 6 cm long
- 2 Look at Model It. How are the two double number lines related? Possible answer: The first double number line shows how to use the dimensions of the original drawing and the scale to find the actual pool dimensions. The second shows how to use the actual dimensions and new scale to find the dimensions of the new drawing.
- Look at Analyze It. How do the dimensions of the pool in the original drawing compare to those in the new drawing?
 Possible answer: In the original drawing, 1 cm represents 6 m, and in the
 - new drawing, 1 cm represents 5 m. So, for every 5 cm of length in the original drawing, there are 6 cm in the new drawing.
- 4 The new drawing is a scale drawing of both the actual pool and the original drawing. Why?

Possible explanation: All three figures are the same shape. There are scale factors you can multiply all of the sides of the new drawing by to get the lengths of the sides of both the actual pool and the original drawing.

5 Reflect Think about all the models and strategies you have discussed today. Describe how one of them helped you better understand how to solve the Try It problem.

Responses will vary. Check student responses.

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DIFFERENTIATION | RETEACH or REINFORCE

Visual Model

Redraw a scale diagram of an object.

If students are unsure about using scale factors to redraw a scale diagram, have them analyze a strategy for drawing a scale diagram and redrawing it at a new scale.

Materials For display: 1 meter stick

- Have a volunteer use the meter stick to draw a rectangle on the board with dimensions 40 cm by 100 cm. Tell students that the rectangle is a scale diagram of a plaza and it has a scale of 6 m to 1 cm. Label the drawing with the scale factor. Have another volunteer calculate the dimensions of the actual plaza. [240 m by 600 m]
- Next, ask a student to draw a second scale diagram of the plaza, using a scale of 12 m to 1 cm. [20 cm by 50 cm] Label this new drawing with its scale factor.
- Ask: What relationship do you notice between the two scale diagrams? What is the scale between the two diagrams? [The dimensions of the new diagram are half the original dimensions. The scale from the first drawing to the second is 2 cm to 1 cm.]
- Ask: *How can you find the scale factor for this scale?* [Find the unit rate for the ratio 2 : 1, which is 2.]

LESSON 1 | SESSION 4 Develop

Apply It

For all problems, encourage students to use a model to support their thinking. Students may wish to draw more than one model for some questions to develop their ideas or support their thinking.

- ⁶ Students may also justify their scale drawing by calculating the scale factor from the original drawing to the new drawing. They may find a scale factor of 2 from the original drawing to the new drawing or a scale factor of $\frac{1}{2}$ from the new drawing to the original; both are correct.
- 7 Students should recognize that a scale of 12 ft to 1 cm will use fewer centimeters to depict the court than a scale of 8 ft to 1 cm does. They may also recognize that they can calculate the scale factor from the original drawing to the new drawing by dividing the original scale factor of 8 by the new scale factor of 12.

LESSON 1 SESSION 4

Apply It

- > Use what you learned to solve these problems.
- 6 The drawing at the right is a plan for an apartment. The length of each square on the grid represents 1 cm. The scale from the apartment to the drawing is 8 ft to 1 cm. Draw another scale drawing of the apartment using a scale from the apartment to the drawing of 4 ft to 1 cm. Justify why your drawing is accurate. Possible answer: Since each centimeter in the new plan represents 4 ft, it takes 2 cm in the new plan to represent 8 ft. That means the new plan uses 2 cm for each centimeter in the original, and the distances in the new plan are double the distances in the original plan.

7 The image at the right is a four-square court from a scale plan of a park. The scale from the actual court to the drawing is 8 ft to 1 cm. Draw another scale drawing of the court using a scale from the actual court to the new drawing of 12 ft to 1 cm. Justify why your drawing is accurate. Possible answer: In the original drawing, each

centimeter represents 8 ft. In the new drawing,

each centimeter represents 8 ft. So, the sides in the new drawing are $\frac{8}{12}$, or $\frac{2}{3}$, times as long as



2 cm

2 cm

side length 2 cm.	5		

the original. Since $3 \times \frac{2}{2}$ is 2, the new drawing is a square with

A design for a playground includes a sandbox. The length of a square on the grid represents 1 in. The scale from the playground to the drawing is 12 in. to 1 in. Draw another scale drawing of the sandbox using a scale from the playground to the new drawing of 20 in. to 1 in. Justify why your drawing is accurate. Possible answer: Since there are 12 in. in the sandbox for each inch in the original drawing and 5 × 12 is 60, the actual sandbox has side length 60 in. The scale for the new drawing is 1 in. to 20 in., and 60 ÷ 20 is 3, so the new drawing is a square with side length 3 in.

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CLOSE EXIT TICKET

- 8 Students' solutions should show an understanding of:
 - applying the scale to convert between the actual dimensions of a square sandbox and the dimensions of two scale diagrams of the sandbox.
 - applying the relationship between two scales to redraw a scale diagram of a square sandbox at a new scale.

Error Alert If students state that they do not have room to make the drawing at the new scale, then ask them to consider whether they need more squares with side length 12 or more squares with side length 20 to make up the length of one side of the sandbox.

Practice Redrawing a Scale Drawing

Problem Notes

Assign **Practice Redrawing a Scale Drawing** as extra practice in class or as homework.

1 Students may also solve the problem by applying the same process described in the Example, with the difference that the actual width of the fountain is calculated using Luke's scale. *Basic*

2 Students may also justify the answer by finding the measurements of the actual triangle and then finding the measurements of Jorge's drawing. *Basic*

LESSON 1 SESSION 4 Name:

Practice Redrawing a Scale Drawing

Study the Example showing how to redraw a scale drawing using a different scale. Then solve problems 1–5.

Example

Luke and Isabella make scale drawings of a fountain. The scale from the fountain to Isabella's drawing is 6 m to 3 cm. The height of the fountain in Isabella's drawing is 5 cm. The scale from the fountain to Luke's drawing is 5 m to 2 cm. What is the height of the fountain in Luke's drawing?

Isabella's scale is 6 m to 3 cm, so the scale factor from her drawing to the actual

fountain is $\frac{6}{3}$, or 2.

Since 5 imes **2** is 10, the actual fountain is 10 m tall.

Luke's scale is 5 m to 2 cm.

The actual fountain is 10 m tall. Since 10 is 5 \times 2, the scale factor from Luke's drawing to the actual fountain is 2.

Since 2 \times 2 is 4, the height of the fountain in Luke's drawing is 4 cm.

The width of the fountain in Luke's drawing from the Example is 2 cm. What is the width of the fountain in Isabella's scale drawing? Show your work. Possible work: The scale from the fountain to Luke's drawing is 5 m to 2 cm, so the fountain is 5 m wide. The scale factor from the fountain to Isabella's drawing is 2, and 5 ÷ 2 is 2.5.

SOLUTION The width in Isabella's drawing is 2.5 cm.

2 The scale from an actual triangle to the drawing at the right is 30 : 1. The scale from the same triangle to Jorge's drawing of the triangle is 10 : 1. Is this drawing or Jorge's drawing larger? Explain your answer.

Jorge's drawing; Possible explanation: Since the scale from the actual triangle to this drawing is 30 : 1 and the scale from the actual triangle to Jorge's drawing is 10 : 1, the scale from this drawing to Jorge's drawing is 10 : 30, or 1 : 3. All of the lengths in Jorge's drawing will be 3 times as long as the lengths in this drawing.



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- 3 a. Students should recognize that within each octagon, the four diagonal sides have one length and the horizontal and vertical sides have another. All the horizontal and vertical lengths are 2 units long in *A* and 4 units long in *B*, but the diagonal lengths in *B* are not twice the lengths of the diagonals in *A*. Students should conclude that these octagons are not scale diagrams of each other. *Medium*
 - **b.** Students should recognize that the vertical, horizontal, and diagonal lengths of *C* are all twice the corresponding lengths in *A*. *Medium*
- 4 Students may calculate the side lengths of the rectangle in the new drawing by multiplying the original side lengths by the ratio of the two scale factors, $\frac{24}{16}$, or $\frac{3}{2}$. *Medium*
- 5 Students should recognize that the scale factor for a scale diagram applies to any linear measurement, such as perimeter. They should calculate the scale factor as the ratio of the actual perimeter in feet to the perimeter of the diagram, or $\frac{150}{30} = 5$. *Challenge*

LESSON 1 SESSION 4

3 The drawing shows three octagons.

- a. Is octagon *B* a scale drawing of octagon *A*? Explain.
 No; Possible explanation: Octagon *B* has four sides that are different lengths than octagon *A* and four sides that are the same length as octagon *A*, so there is no scale factor that relates them.
- b. Is octagon C a scale drawing of octagon A? Explain.
 Yes; Possible explanation: All of the sides of octagon C are twice as long as the sides of octagon A. The scale is 1 : 2, so it is a scale drawing.
- The length of each square on the grid represents 1 cm. The scale from an actual rectangle to the drawing is 24 in. to 1 cm. Make a new scale drawing where each centimeter represents 16 in. Label the length and width of your drawing in centimeters.

5 The perimeter of a pool is 150 m. The rectangle at the right is a scale drawing of the pool. The length of each square on the grid

scale 25 m to 2 cm. Explain why your drawing is accurate.

represents 1 cm. Draw another scale drawing of the pool using the

Possible explanation: The perimeter of the drawing is 30 cm.

Since $150 \div 30 = 5$, each centimeter represents 5 m. So, the

pool's width is 25 m and its length is 50 m. Since the new drawing represents 25 m with 2 cm, and $\frac{25}{2} = 12.5$, the scale

factor from the pool to the new drawing is 12.5.





50 ÷ 12.5 = 4 25 ÷ 12.5 = 2

So, the side lengths of the new drawing are 4 cm and 2 cm.

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DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Levels 1–3: Reading/Speaking

Help students interpret Apply It problem 2. Read the problem aloud and ask students to tell other reasons people might make flags. Clarify that *fabric* is the material used to make flags, clothes, and other things. Then, ask students to point to the scale drawing. Say: *Kimani uses this scale drawing to make a flag.* Have students circle the scale and tell the scale factor using:

- I can use the scale to find the scale factor from the _____to the _____.
- The scale factor is _____.

Next, have students count the colors in the flag. Ask: *Is the amount for each color the same or different? How do you know?* Have partners work together to solve the problem.

Levels 2-4: Reading/Speaking

Help students interpret Apply It problem 2. Have students read the problem and discuss what they know about making flags. Then ask students to tell what they know about the flag in the problem. Ask students to reword statements using precise math language, like the lesson vocabulary terms *scale, scale factor, area, dimension,* and *unit rate,* when appropriate. Record statements for reference.

Next, read Consider This . . . with students. Ask students to turn to partners and discuss different ways they could solve the problem. Encourage students to take turns explaining their ideas. Suggest they check that explanations are clear by pausing and asking their partners for questions or comments.

Use with Session 5 Apply It

Levels 3–5: Reading/Speaking

Support students as they read and interpret Apply It problem 2. Have partners use **Say It Another Way** to paraphrase the problem and confirm understanding. Then ask partners to read Consider This . . . and discuss different strategies they can use to solve the problem. Remind students to use precise math language, like the lesson vocabulary terms *scale, scale factor, area, dimension,* and *unit rate,* as they discuss.

Have students work independently to solve the problem. Then have them turn to partners to explain their solution strategies. Prompt students to check that explanations are clear by pausing and asking for questions or comments.

Refine Solving Problems Involving Scale

Purpose

- **Refine** strategies for applying scale to compare scale diagrams with the objects they represent.
- **Refine** understanding of comparisons between two scale drawings.

The scale of a map is 20 mi to 4 in. What actual area does 2 in. ² on the map represent?	50 mi ²
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MONITOR & GUIDE

Before students begin to work, use their responses to the **Start** to determine those who will benefit from additional support. Use the **Error Analysis** table below to guide remediation.

Have all students complete the Example and problems 1–3, using Consider This and Pair/Share as appropriate. Observe and monitor their reasoning and guide or redirect students as needed.



S	IARI ERROR A	NALYSIS	
	If the error is	Students may	To support understanding
	0.08 mi ²	have divided the area on the map by the square of the scale factor instead of multiplying.	Ask students to write complete sentences about when to multiply by the chosen scale factor and when to divide by it. Have them select the approach for converting miles to inches and explain how to adapt it for area instead of length.
	2.5 mi ²	have divided the scale factor by the area on the map.	Ask students to explain how to use the scale factor to convert a linear distance on a map, such as 2 in., to the actual distance it represents. Then have them extend that explanation to finding an actual area.
	10 mi ²	have multiplied the area on the map by the scale factor, not the square of the scale factor.	Ask students to identify a possible length and width for an area that is 2 in. ² , such as 2 in. and 1 in. or 4 in. and $\frac{1}{2}$ in. Have students calculate the actual distances represented by these dimensions and use them to find the actual area.

scale were 1 cm for every 16 ft?

Example

Guide students in understanding the Example. Ask:

- What is the scale factor for the diagram?
- How can you use the diagram and scale factor to find the dimensions of the actual floor?
- How could you pack square tiles that are $\frac{1}{2}$ ft wide into a square that is 1 ft wide?

Help all students focus on the Example and responses to the questions by listening for understanding. Remind students that good listeners use engaged body language, such as looking at the speaker and nodding to show understanding.

Look for understanding that the dimensions in the scale diagram can each be multiplied by the scale factor to find the actual dimensions of the floor and that the number of tiles is 4 times the area of the floor.

Apply It

- 1 Students should recognize that the scale factor for each map can be found by dividing 12 by the number of centimeters 12 mi represents. Then they calculate the actual distances by multiplying the map distances by the scale factor. **DOK 2**
- Students may also solve the problem by calculating the area of the drawing, which is 2.25 in.², dividing the area by 4, and then multiplying by the square of the scale factor to find the area of each color on the flag. *DOK 2*
 - **D** is correct. Students may solve by multiplying each dimension of the original scale drawing by the quotient of the two scale factors: $\frac{8}{2}$, or 4.
 - A is not correct. This answer may be the result of dividing instead of multiplying by the appropriate scale factor for the two drawings.
 - **B** is not correct. This answer may be the result of dividing each of the dimensions of the original drawing by the scale factor for the second drawing.
 - **C** is not correct. This answer may be the result of multiplying each of the dimensions of the original drawing by the scale factor for the second drawing.

DOK 3

LESSON 1 SESSION 5



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GROUP & DIFFERENTIATE

Identify groupings for differentiation based on the **Start** and problems 1–3. A recommended sequence of activities for each group is suggested below. Use the resources on the next page to differentiate and close the lesson.

Approaching Proficiency

- **RETEACH** Visual Model
- **REINFORCE** Problems 4, 5, 7, 8

Meeting Proficiency

REINFORCE Problems 4–8

Extending Beyond Proficiency

- **REINFORCE** Problems 4–8
- EXTEND Challenge

Have all students complete the Close: Exit Ticket.

Resources for Differentiation are found on the next page.

Refine Solving Problems Involving Scale

Apply It

- 4 Students may also solve by using a visual model, such as a double number line, to represent the corresponding distances. *DOK 2*
 - **D** is correct. Students may solve by calculating the area of the square in the drawing and multiplying by the square of the scale factor.
 - **A** is not correct. This answer may be the result of finding the side length of the actual earring instead of the area.
 - **B** is not correct. This answer may be the result of multiplying the square of the scale factor by the side length of the earring instead of the area of the earring.
 - **C** is not correct. This answer may be the result of multiplying the area of the square in the drawing by the scale factor instead of by the square of the scale factor.

DOK 2

- 6 Students should recognize that the ratio of the actual perimeter in miles to the perimeter of the drawing in unit lengths is equal to the scale factor for the drawing. *DOK 3*
- 4 The scale from a map to actual distance is 2.5 cm to 620 mi. The distance on the map between Chicago and Boston is about 3.5 cm. What is the approximate distance, in miles, rea of detai between Chicago and Boston? Show your work. **Possible work:** Chicago The scale factor from the map to the actual distance Scale is $\frac{620}{25}$, or 248. 2.5 cm : 620 mi $3.5 \times 248 = 868$ **SOLUTION** The approximate distance is 868 mi. 5 On a scale drawing of the front of a square earring, each side of the earring is 3.2 cm. The scale from the earring to the drawing is 4 mm to 2 cm. What is the area of the front of the actual earring? **A** 6.4 mm² **B** 12.8 mm² C 20.48 mm² (D) 40.96 mm² 6 The image at the right is a scale drawing of a parking lot. The length of each square on the grid represents 0.5 cm. The actual parking lot has a perimeter of 84 m. Draw another scale drawing of the parking lot using a scale from the parking lot to the drawing of 2 m to 1 cm. Justify why your drawing is accurate. Possible answer: The original drawing has a perimeter of 14 units. Since 84 ÷ 14 is 6, each 0.5 cm in the original drawing represents 6 m. This is three times what each unit length will represent in the new drawing, so the length of each side of the new drawing needs to be triple the

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DIFFERENTIATION

RETEACH

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Visual Model Find patterns in scale diagrams.

Students approaching proficiency with analyzing scale diagrams will benefit from finding patterns in the scale diagrams of a rectangular area.

Materials For display: 3 rulers

• On the board, write the dimensions 60 ft and 90 ft for a rectangular patio. Have three pairs of students construct scale diagrams of the patio on the board, each using a different scale. Use the scales 6 ft to 1 in., 10 ft to 1 in., and 15 ft to 1 in. Have a different group of students calculate the length and width of the scale diagrams and draw and label the sides.

length of each side of the original drawing.

- Ask: *How are all three scale diagrams alike? How are they different?* [All are rectangles of the same shape, in which the length is 1.5 times the width. They have different sizes.]
- Ask: *How is the scale factor used to construct the scale diagram*? [Divide the dimensions of the patio by the scale factor to find the dimensions for the scale diagram.]
- Ask: Which scale drawing is the largest? The smallest? Why? [The largest scale drawing has a scale of 6 ft to 1 in., and the smallest scale drawing has a scale of 15 ft to 1 in. When you divide the actual length by a lesser number, which is the scale factor, the resulting scale drawing length is greater.]
- Ask: How can you find the scale factor between any two of the scale diagrams? Give an example. [Divide the scale factor for the first diagram by the scale factor for the second diagram. For example, to convert from the first diagram to the second diagram, the scale factor is $\frac{6}{10}$, or 0.6.]

LESSON 1 | SESSION 5

George Washington's head at Mt. Rushmor

- 7 Students may also calculate the actual difference between the two widths, 7 ft, and then divide by the scale factor of 6. DOK 2
- 8 Students may recognize that when the scale factor between distances is 5, the scale factor between areas is 5², or 25. *DOK 2*

CLOSE EXIT TICKET

9 Math Journal Look for understanding of scales and their relationship to the size of a scale drawing. Students should recognize that as the distance represented by one unit length increases, the size of the scale drawing decreases.

Common Misconception If students state that Jada is correct because scale is directly related to distance, then have them choose two different scales and make scale drawings of a simple shape to check their work.

End of Lesson Checklist

INTERACTIVE GLOSSARY Support students by suggesting that they review examples of scale drawings shown throughout the lesson, such as those in the Try It problems.

SELF CHECK Have students review and check off any new skills on the Unit 1 Opener.

LESSON 1 SESSION 5

Adela paints a mural of Mount Rushmore. She uses the scale 6 ft to 1 ft from the actual monument to the mural. On Adela's mural, how much wider is George Washington's mouth than his eye? Show your work. Possible work:

	Scale	Mouth (ft)	Eye (ft)	Difference (ft)
Actual	6	18	11	7
Mural	1	3	1 <u>5</u>	1 <u>1</u>

On the mural, George Washington's mouth is $1\frac{1}{6}$ ft wider.

⁸ The scale from a square park to a drawing of the park is 5 m to 1 cm. The actual park has an area of 1,600 m². What is the area of the drawing of the park? Show your work. Possible work:

To use the dimensions in the drawing to find the actual area, you can multiply by the scale factor 5 twice. So, to find the area of the drawing from the actual area, you can divide by 5 twice.

1,600 ÷ 5 = 320

320 ÷ 5 = 64

SOLUTION The area of the drawing is 64 cm².

9 Math Journal Jada will draw two scale drawings of the same object on 1-cm grid paper. Each centimeter will represent a greater distance in the second drawing than in the first. Jada claims that the lengths in the second drawing will be longer than the lengths in the first drawing. Do you agree? Give an example of a scale and a measurement to support your answer.

No; Possible answer: Suppose 1 cm in an original scale represents 4 ft and 1 cm in a new scale represents 8 ft. A length of 2 cm in the original scale drawing will be a length of 1 cm in the new scale drawing.

End of Lesson Checklist

INTERACTIVE GLOSSARY Find the entry for *scale drawing*. Rewrite the definition in your own words.

SELF CHECK Go back to the Unit 1 Opener and see what you can check off.

REINFORCE



Problems 4–8

Solve problems involving scale.

Students meeting proficiency will benefit from additional work with scale factors and scale drawings by solving problems in a variety of formats.

- Have students work on their own or with a partner to solve the problems.
- Encourage students to show their work.

EXTEND



Challenge Compare the accuracies of scale diagrams.

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Students extending beyond proficiency will benefit from calculating and comparing actual distances from various maps.

Materials For each pair: 1 ruler, 2 maps with different scales for the same region

- Have students identify pairs of places that are shown on both maps. Then have them measure the distance between the places on each map and use the scale to calculate the actual distance.
- Next, have students compare their calculations. Did they always find the same result for the distance between pairs of places? If not, have them come up with and evaluate possible explanations for the discrepancies.

PERSONALIZE

i-Ready[®]

Provide students with opportunities to work on their personalized instruction path with *i-Ready* Online Instruction to:

- fill prerequisite gaps.
- build up grade-level skills.

STANDARDS FOR MATHEMATICAL PRACTICE (SMP)

SMP 1, 2, 3, 4, 5, and 6 are integrated into the Try-Discuss-Connect routine.*

This lesson provides additional support for:

- 7 Look for and make use of structure.
- * See page 1q to learn how every lesson includes these SMP.

Objectives

Content Objectives

- Interpret verbal descriptions of problem situations.
- Use proportional relationship strategies and models to solve multi-step problems involving ratios.

Language Objectives

- Explain how to identify the constant of proportionality based on a verbal description.
- Interpret word problems about proportional relationships by identifying relationships among quantities.
- Agree or disagree and respectfully explain reasoning when listening to ideas about solution strategies in partner and class discussion.
- Demonstrate understanding of *constant* of proportionality and proportional relationship by using the terms when speaking and writing.

Prior Knowledge

- Calculate the constant of proportionality.
- Apply the constant of proportionality to solve problems.

Vocabulary

Math Vocabulary

There is no new vocabulary. Review the following key terms.

constant of proportionality the unit rate in a proportional relationship.

coordinate plane a two-dimensional space formed by two perpendicular number lines called axes.

proportional relationship the

relationship between two quantities where one quantity is a constant multiple of the other quantity. If the quantities x and y are in a proportional relationship, you can represent that relationship with the equation y = kx, where the value of k is constant (unchanging).

Academic Vocabulary

certain specific but not named.

verbal description an explanation of what something is or is like using words.

Learning Progression

Earlier in Grade 7, students learned to represent and interpret proportional relationships with tables, graphs, equations, and various diagrams. They learned to find the constant of proportionality for a proportional relationship using any of these models.

In this lesson, students use their knowledge of equivalent ratios, proportional relationships, and the constant of proportionality to interpret and solve multi-step problems. They also identify the constant of proportionality given a verbal description of a proportional relationship. Later in Grade 7, students will apply their understanding of proportional relationships to solve problems involving circles and problems involving percents. They will also use the results of a random sample to make inferences about the population the sample came from.

>

81a

LESSON 5 **Overview**

Pacing Guide Items marked with 🏷 are available on the Teacher Toolbox .	MATERIALS	DIFFERENTIATION						
SESSION 1 Explore Proportional Relationship Problems (35–50 min)								
 Start (5 min) Try It (5–10 min) Discuss It (10, 15 min) 	Math Toolkit double number lines, graph paper	 PREPARE Interactive Tutorial * RETEACH or REINFORCE Hands-On Activity Materials For each small group: at least 2 counters, at least 4 unit cubes, paper plate 						
 Discuss it (10–15 min) Connect It (10–15 min) Close: Exit Ticket (5 min) Additional Practice (pages 85–86) 	Presentation Slides 🍾							
SESSION 2 Develop Solving Multi-Step Ratio Problems (45–60 min)								
 Start (5 min) Try It (10–15 min) Discuss It (10–15 min) 	Math Toolkit double number lines, graph paper	RETEACH or REINFORCE Visual Model REINFORCE Fluency & Skills Practice 🔭						
 Connect It (15–20 min) Close: Exit Ticket (5 min) Additional Practice (pages 91–92) 	Presentation Slides 🍾	EXTEND Deepen Understanding						
SESSION 3 Refine Solving Proportional Relationship Problems (45–60 min)								
 Start (5 min) Monitor & Guide (15–20 min) 	Math Toolkit Have items from previous sessions available for	RETEACH Hands-On Activity Materials For each pair: a strip of paper, 2 rulers						
 Group & Differentiate (20–30 min) Close: Exit Ticket (5 min) 	students.	REINFORCE Problems 4–8						
	Presentation Slides 🍾	EXTEND Challenge						
		personalize 🗊 i-Ready						
Losson E Quiz * or								
Digital Comprehension Check								
		EXTEND Enrichment Activity						

Connect to Culture

Use these activities to connect with and leverage the diverse backgrounds and experiences of all students. Engage students in sharing what they know about contexts before you add the information given here.

SESSION 1 🗖 🗆

Try It Trail mix gets its name because it is a mixture of many ingredients that is easy for hikers to carry and eat. *Boy's Life* magazine listed 35 possible ingredients to include in trail mix, including nuts, dried fruits, seeds, and spices. Ask students if they have a favorite ingredient to include in trail mix or a similar mixture of foods.

SESSION 2

Try It Ask students if they have heard of or participated in a walk-a-thon or other fundraising effort. Have them describe their experiences. Walk-a-thons are used to raise money. Donors pledge money to individual participants, usually based on the distance or amount of time they walk. Walk-a-thons, bike-a-thons, and similar events are popular because they are a fun way to bring a community together for a common cause.



SESSION 3

Apply It Problem 1 Ask students if they have any experience with cooking. Ask if they have ever heard of marinades and what they are used for. A marinade is a type of sauce that is used to soak meat, fish, or other foods before cooking. The marinade adds flavor to the food and may make it more tender. Marinades can contain a wide variety of ingredients, including fruits such as lemon and pineapple, vegetables such as peppers and onions, and herbs such as rosemary and tarragon.

Apply It Problem 4 A gear is a wheel with teeth. Sets of interlocking gears are used in all sorts of devices, including clocks and watches, automobile transmissions, and can openers. Students may be most familiar with the gears that are used on a bicycle. Pedaling the bicycle turns the gears, which then rotate the wheels and tires. By changing gears, the cyclist can change the ratio of turns of the pedal to turns of the wheels, allowing for a reasonable pedaling speed over different types of terrain. Ask for a show of hands or have students stand up to prompts such as: *Have you ever ridden a bicycle? Have you ridden a bike with more than one gear? With more than 10 gears?* Then have students discuss their experiences with biking.



Connect to Family and Community

After the Explore session, have students use the Family Letter to let their families know what they are learning and to encourage family involvement.

DI INY ANA V		LESSON 5 SOLVE PROPORTIONAL RELATIONSHIP PROBLEMS			
	LESSON	Activity Thinking About	TODAY'S EXCHANGE RATE		
MOLECULA ANALAS A LA		Proportional Relationships			
Contraction of the second s		Around You			
Dear Family.		Do this activity together to investigate pr	oportional		
This week your student is learning how to solve proble relationships. Your student has already learned about a proportional relationship. A proportional relationshi a verbal description, a graph, an equation, or a diagrar number line).	ms that involve proportional different ways to represent to an be represented with the standard of the standar	relationships in the real world. Have you ever paid for something using mo than dollars and centra? Different countries u currencies. Mexico uses the peso and Japan the yen. You exchange dollars for another currency a	HUPPE OLDSS 136 rey other sedifferent uses OLDSS 136 Packass HL25609 Peso DL352 coording to FWASS		
Verbal Description Graph	Equation	the exchange rate. This tells you how many u	units of the		
Each pound of bananas costs \$0.50.	c = 0.5p c = total cost (\$) p = pounds	What are other situations where you use proportional relationships?			
C 100 050 0 1 2 3 4 5 6 Bananas (lb)	elation				
Your student will be solving problems like the one bel	ship P				
Charlotte and Sofia are 66 miles apart. They ride the other. They meet after 3 hours of riding. Each girl if Charlotte rides at 10 miles per hour. How far does S Tonether Sofia and Charlotte ride 66 miles. First find o	ir bikes toward each les at a constant rate. ofia ride?				
ONE WAY is to use a double number > ANOTHER	R WAY is to use an equation				
line to find how far Charlotte rides. to find h	ow far Charlotte rides.				
Time (h) 0 1 2 3 Let m =	niles and h = hours.				
	10(3)				
Distance (m) 0 10 20 30 =	30				
Charlotte rides 30 miles, so Sofia rides 66 miles – 30 n	iles = 36 miles.				
Both ways show that Sofia rides her bike 36 miles.					
	Use the next page to start a conversation about proportional relationships.				
CCurriculum Associates, LLC Copying is not permitted.	LESSON 5 Solve Proportional Relationship Problems 81	82 LESSON 5 Solve Proportional Relationship Problems	©Curriculum Associates, LLC Copying is not permitted		

Connect to Language

For English language learners, use the Differentiation chart to scaffold the language in each session. Use the Academic Vocabulary routine for academic terms before Session 1.

DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Levels 1-3: Reading/Speaking

Help students make sense of Connect It problem 1. Read the problem and ask students to underline *total amount* and *amount of each ingredient*. Display *quantity* and clarify that it has nearly the same meaning as *amount*.

Continue reading and have students highlight quantities. Ask students to tell partners the quantity in each description that is the *total amount* and the *amount of one ingredient*.

Rephrase problem 2d: *The two constants* of proportionality are different. Why? Have partners identify the constants of proportionality in problems 2b and 2c and turn and talk about whether each relationship is in terms of a total amount or an amount of one ingredient.

Levels 2–4: Reading/Speaking

Guide students to make sense of Connect It problem 2. Use **Say It Another Way** to help confirm students' understanding. Ensure they understand the meaning of *relationship between*.

Continue reading and have students underline phrases that describe the relationship between amounts, such as *for every* and *in terms of*. Provide think time for students to analyze how these phrases are used in the sentences. Then ask partners to discuss the relationships.

Read problem 2d. Have partners turn and talk about the constants of proportionality and the order of the quantities and the relationships. Encourage student to use *for every* or *in terms of* in their discussions.

Use with Session 1 Connect It

Levels 3–5: Reading/Speaking

Support students as they analyze how descriptions express relationships, paying close attention to the nuances of prepositions in Connect It problem 2. Ask students to circle *between, of,* and *from* and explain the usages.

Direct students to read each part of the problem. Have students use **Say It Another Way** to confirm understanding. Display the following phrases and have students discuss how each is used:

- the relationship between _____ and _____
- the relationship in terms of ____

After students respond in writing, have them meet with a partner to read each other's responses and discuss the prepositions they used.

Explore Proportional Relationship Problems

Purpose

- Explore how proportional relationships can be applied to solve a variety of problems.
- **Understand** that the constant of proportionality can be used to find the value of one quantity in a proportional relationship when the other quantity is known.

START CONNECT TO PRIOR KNOWLEDGE



Possible Solutions

All expressions are rates in miles per hour.

A and B show equivalent rates.

C and D show equivalent rates.

B and D give unit rates.

WHY? Support students' ability to apply proportional reasoning in a real-world context.

SMP 1, 2, 3, 4, 5, 6

SMP 1, 2, 3, 4, 6

TRY IT

Make Sense of the Problem

See **Connect to Culture** to support student engagement. Before students work on Try It, use **Three Reads** to help them make sense of the problem. After the first read, ask students to describe the problem situation. After the second read, ask them to explain the question they need to answer. For the third read, ask: *What information is needed in order to decide if Aniyah can make 14 cups of the trail mix?*

DISCUSS IT

Support Partner Discussion

After students work on Try It, have them respond to Discuss It. Remind them to provide reasons to justify their interpretation. Listen for understanding of:

- how to determine the proportional relationship between the cups of raisins and cups of trail mix.
- the need to solve the problem in multiple steps.



Common Misconception Listen for students who stop after finding the constant of proportionality in the relationship between cups of raisins and cups of trail mix, thinking that is the solution to the problem. As students share their strategies, ask them to listen to one another to confirm that they have fully answered the question that the problem poses.

Select and Sequence Student Strategies

Select 2–3 samples that represent the range of student thinking in your classroom. Here is one possible order for class discussion:

- using tables, double number lines, or graphs that compare the cups of raisins and trail mix and subtracting to find the extra amount of raisins that is needed
- (misconception) applying strategies that identify the constant of proportionality without subtracting to find the extra amount of raisins that is needed
- calculating the unit rate, or constant of proportionality, between cups of raisins and cups of trail mix and subtracting to find the extra amount of raisins that is needed

Facilitate Whole Class Discussion

Call on students to share selected strategies. After each strategy, ask a student to rephrase the key ideas and record them on the board.

Guide students to **Compare and Connect** the representations. Suggest that students take some individual think time to come up with ideas. Encourage students to listen to understand as they look at the speaker and attend to the message.

ASK How do the solutions of [student name] and [student name] each show the relationship between the cups of trail mix and raisins?

LISTEN FOR Both solutions involve the ratio of $\frac{1}{6}$ cup of raisins per cup of trail mix, which is then multiplied by the total amount of trail mix, which is 14 cups.

CONNECT IT

SMP 2, 4, 5

1 **Look Back** Look for understanding that because of the constant ratio of $\frac{1}{6}$ cup of raisins per cup of trail mix, Aniyah needs $14 \times \frac{1}{6}$, or $2\frac{1}{3}$ cups of raisins. This amount is $\frac{5}{6}$ cup more than the amount she has.

DIFFERENTIATION | RETEACH or REINFORCE

Hands-On Activity Model a proportional relationship.

If students are unsure about proportional relationships, then use this activity to help them visualize a real-world example.

Materials For each small group: at least 2 counters, at least 4 unit cubes, paper plate

- Prepare a plate for each group that includes some number of counters and twice as many cubes.
- Make a blank table on the board for each group to record the number of counters, the number of cubes, and the ratio of cubes to counters they have.
- Have students compare the sets of items on the plates. Ask: *How are all the sets alike? Different?* [The number of counters and cubes differs, but there are always twice as many cubes as counters.]
- Ask: *How does this activity show a proportional relationship?* [In each mixture, the ratio of cubes to counters is 2 to 1.]
- Ask: If you know that a plate has 40 cubes, how can you find the number of counters? [The ratio of cubes to counters is 2 : 1, so you can divide 40 by 2 to find the plate has 20 counters.]

LESSON 5 SESSION 1

CONNECT IT

- Look Back Does Aniyah need more raisins? If so, how much more? Yes, she needs ⁵/₆ cup more raisins.
- 2 Look Ahead There is a proportional relationship between the total amount of trail mix and the amount of each ingredient. You can identify constants of proportionality from descriptions of proportional relationships.
 - **a.** A recipe for 3 cups of snack mix calls for $\frac{1}{2}$ cup of pretzels. You can describe this as 3 cups of snack mix for every $\frac{1}{2}$ cup of pretzels and as $\frac{1}{2}$ cup of pretzels for every 3 cups of snack mix. Why?

You are not changing the quantities or the relationship between them. You are just changing which quantity is talked about first.

- b. How could you describe the relationship in terms of 1 cup of pretzels? What is this constant of proportionality?
 6 cups of snack mix per cup of pretzels; 6
- c. How could you describe the relationship in terms of 1 cup of snack mix? What is this constant of proportionality? $\frac{1}{6}$ cup of pretzels per cup of snack mix; $\frac{1}{6}$
- **d.** Why are the two constants of proportionality not the same? Possible answer: The constant of proportionality 6 is for the relationship in terms of 1 cup of pretzels. But $\frac{1}{6}$ is for the relationship in terms of 1 cup of snack mix.
- e. Suppose you are making 4 cups of snack mix. You want to find how many cups of pretzels you need. Which constant of proportionality would you use? Why? Possible answer: I would use $\frac{1}{6}$ because I can multiply 4 by $\frac{1}{6}$ to find the number of cups of pretzels I will need.
- 3 Reflect How does knowing the constant of proportionality help you solve problems that involve proportional relationships? Possible answer: The constant of proportionality tells how much of a quantity you need for each unit of another. This is useful when you have to find the amount of a quantity to keep the relationship proportional.
- 2 Look Ahead Point out that the constant of proportionality remains the same, no matter how much the total is. Students should recognize that ratios can be stated in either order.

Ask students: *How can there be two constants of proportionality for the same mixture?* Support student understanding that there are two different relationships, depending on what is being compared.

CLOSE EXIT TICKET

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3 Reflect Look for understanding that the constant of proportionality is the unit rate in a proportional relationship. It can be used to find unknown quantities.

Common Misconception If students think that a constant of proportionality such as $\frac{1}{8}$ means that the two parts of the ratio are always 1 and 8, then explain that the quantities can be any two numbers that form a ratio equivalent to 1 : 8.
Prepare for Proportional Relationship Problems

Support Vocabulary Development

Assign **Prepare for Proportional Relationship Problems** as extra practice in class or as homework.

If you have students complete this in class, then use the guidance below.

Ask students to consider the term *constant of proportionality* by discussing what *constant* means and what *proportional* means. Provide support as needed, helping students use previous knowledge of equivalent ratios and proportional relationships.

Have students work in pairs to complete the graphic organizer. Invite pairs to share their completed organizers and prompt a whole-class comparative discussion of their examples and understanding.

Have students look at problem 2 and discuss why a constant of proportionality can be a fraction. Encourage them to think of a situation involving a unit price that is not a whole number.

Problem Notes

1 Students should understand that the constant of proportionality is a unit rate relating the two quantities in a proportional relationship and it can be identified from any representation of the relationship, including a graph, table, or equation.

2 Students should recognize that a proportional relationship between two variables can be defined in either order of the variables. For example, if there are 8 red marbles for every white marble, then there is also 1 white marble for every 8 red marbles. The two constants of proportionality are reciprocals of each other, so if one is a whole number greater than 1, then the other is a fraction or decimal less than 1.

LESSON 5 SESSION 1 Name:

Prepare for Proportional Relationship Problems

Think about what you know about proportional relationships and constants of proportionality. Fill in each box. Use words, numbers, and pictures. Show as many ideas as you can. Possible answers:



2 Hiroaki says that a constant of proportionality must be a whole number and cannot be a fraction or a decimal. Explain why Hiroaki is incorrect. Possible answer: Hiroaki is incorrect because a constant of proportionality can be between 0 and 1. That means it has to be able to be a fraction or a decimal.

REAL-WORLD CONNECTION

Paint stores stock a relatively small number of colors of paint. However, by mixing the colors in specific ratios, or proportions, they can produce an almost infinite number of colors. The constants of proportionality between the colors in the mixture determine the final color. Painters study color ratios carefully to produce exactly the shades they want. By identifying the ratios, they can obtain exactly the same shade when they visit the store another time, no matter what quantity of paint they need. Ask students to think of other examples where constants of proportionality are useful.



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LESSON 5 | SESSION 1 Additional Practice

3 Problem 3 provides another look at applying proportional relationships. This problem is similar to the Try It problem about Aniyah and the trail mix. In both problems, a proportional relationship is specified between two quantities. The solution involves identifying the constant of proportionality to find an unknown quantity.

Students may want to use a table, a double number line, or equations to solve.

Suggest that students use **Three Reads**, asking themselves one of the following questions each time they read the problem:

- What is this problem about?
- What is the question I am trying to answer?
- What information is important?

LESSON 5 SESSION 1

- 3 A formula for 6 gallons of light green paint uses $\frac{3}{8}$ gallon of white paint. Liam has $\frac{9}{16}$ gallon of white paint.
 - Does Liam have enough white paint to make 8 gallons of light green paint? If not, how much more does he need? Show your work.
 Possible work:

Light Green Paint (gal)	White Paint (gal)
6	<u>3</u> 8
1	$\frac{3}{8} \div 6 = \frac{3}{48}$
8	$\frac{3}{48} \cdot 8 = \frac{24}{48}$ or $\frac{8}{16}$

Liam needs $\frac{8}{16}$ gallon of white paint for 8 gallons of light green paint.

 $\frac{8}{16} < \frac{9}{16}$

SOLUTION Liam has enough white paint.

b. Check your answer to problem 3a. Show your work. Possible work:

Constant of proportionality: $\frac{3}{6} = \frac{3}{8} \cdot \frac{1}{6}$ = $\frac{3}{48}$

$$y = \frac{24}{48}$$
 or $\frac{8}{8}$

$$y = \frac{1}{48}$$
 or $\frac{1}{1}$
 $\frac{1}{16} < \frac{9}{16}$

Liam has more than $\frac{8}{16}$ gallon of white paint, so he has enough.

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DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Levels 1-3: Speaking/Writing

Prepare students to respond in writing to Connect It problem 3. Review the terms *constant of proportionality, variable,* and *coefficient* and help students identify the parts of the equation. Have partners discuss the solution in their own words, and then use sentence frames to guide their writing:

- I can multiply the _____ by the _____ to find out how much money the girls raise together.
- The constant of proportionality is _____ dollars for every _____ hour.
- The variable _____ is the number of _____. The girls walk for _____ hours.

Levels 2–4: Speaking/Writing

Prepare students to respond in writing to Connect It problem 3. Display the terms *constant of proportionality, variable,* and *coefficient*. Call on volunteers to use the terms to explain the parts of the equation.

Have students turn and talk with a partner about the solution in their own words.

Adapt **Stronger and Clearer Each Time** by providing sentence frames to help students write their explanations:

- I used the equation to solve the problem because _____.
- The coefficient _____ shows that _____.

Use with Session 2 Connect It

Levels 3–5: Speaking/Writing

Prepare students to respond in writing to Connect It problem 3. Ask students to read the problem with a partner. Ask partners to explain how each part of the equation represents the situation. Encourage them to note important words and phrases that they use in their discussions.

Use **Stronger and Clearer Each Time** to help students draft and revise their responses. Remind them to use transitions to make their writing more coherent, such as *one way, another way,* and *then*.

LESSON 5 | SESSION 2 **Develop** Solving Multi-Step Ratio Problems

Purpose

- **Develop** facility with a variety of formats in which proportional relationships can be represented.
- **Recognize** that being able to identify the constant of proportionality in many representations lets you use the form most helpful for the problem.

START **CONNECT TO PRIOR KNOWLEDGE**



Possible Solutions

A is more money per hour than B.

B shows the greatest amount of money earned.

C offers the best hourly rate if the work takes 2 hours or less and the worst rate if the work takes longer than 4 hours.

WHY? Support students' ability to evaluate and compare rates and totals.

DEVELOP ACADEMIC LANGUAGE

WHY? Support students as they respectfully disagree with an idea during discussion.

HOW? Discuss with students how to disagree with an idea respectfully during discourse. Ask them to disagree with the idea, not the person. Model for students how understanding and working through disagreements is a way to learn. Suggest these sentence frames:

- ______said _____. I disagree because _____.
- I thought about this differently. I think _____

TRY IT

87

Make Sense of the Problem

See Connect to Culture to support student engagement. Before students work on Try It, use Say It Another Way to help them make sense of the problem. Ask a student to paraphrase the problem. Then ask the class to decide whether the paraphrase is complete and accurate. Listen for understanding that the girls walked for the same amount of time but earned money at different rates.

SMP 1, 2, 4, 5, 6



DISCUSS IT

Support Partner Discussion

After students work on Try It, encourage them respond to Discuss It with a partner. If students need support in getting started, remind them that as their partner shares their strategy, if they disagree, they should do so respectfully, disagreeing with the idea and not the person. They can ask each other questions, such as:

- How could you find the amount that each girl raises in 1 hour?
- After you find the amount that each girl raises in 1 hour, how can you find the amount each raises in 3 hours?

Common Misconception Listen for students who misinterpret the rates presented in this problem. They might mistakenly interpret \$25.50 and \$14.50 as the unit rates. As students share their strategies, ask them to listen to one another to see if they used the same rates. Have students give one another feedback to make sure that they use all of the information provided in the problem.

LESSON 5 | SESSION 2 Develop

Select and Sequence Student Strategies

Select 2–3 samples that represent the range of student thinking in your classroom. Here is one possible order for class discussion:

- using tables or double number lines that compare the dollars raised with walking time and then adding the two totals
- (misconception) misunderstanding the rates given in the problem as unit rates and using them to calculate the total amount raised
- plotting points and drawing a straight line through them and the origin and then using the line to find the individual totals
- writing an equation that applies the constant of proportionality of dollars per hour for each walker and then adding to find the total

Facilitate Whole Class Discussion

Call on students to share selected strategies. Remind students to speak clearly and pause for questions and comments. Allow think time for students to process the ideas they hear from classmates.

Guide students to Compare and Connect the representations.

ASK What do all of the solution methods have in common?

LISTEN FOR The constant of proportionality for each walker was used to find the amount of money the two walkers raised separately. Then the two individual amounts were added.

Model It

If students presented these models, have students connect these models to those presented in class.

If no student presented at least one of these models, have students first analyze key features of the models and then connect them to the models presented in class.

ASK How is the constant of proportionality useful in both models?

LISTEN FOR The constant of proportionality shows how much money each walker earns for walking one hour.

For the verbal description, prompt students to recognize that the constant of proportionality is the ratio of earnings in dollars to time in hours.

For the equations, prompt students to recognize that the constant of proportionality fills an important role in the equation. Ask: How do you know to substitute the 3 for h and not for m to find the money raised?

LESSON 5 SESSION 2

Explore different ways to solve problems that involve proportional relationships.

Francisca and Elizabeth are participating in a walk-a-thon fundraiser. For every $\frac{1}{2}$ hour Francisca walks she raises \$25.50. For every $\frac{1}{4}$ hour Elizabeth walks she raises \$14.50. Each girl walks for 3 hours. How much will they raise together?

Model It

You can identify the constant of proportionality from a verbal description. Francisca raises \$25.50 every 0.5 hour.

That means she raises \$51.00 per 1 hour.

constant of proportionality

Elizabeth raises \$14.50 every 0.25 hour.

That means she raises \$58.00 per 1 hour.

constant of proportionality

Model It

You can use equations to model how much Francisca and Elizabeth raise per hour. Let *m* represent dollars raised per hour. Let *h* represent time in hours.

You can use the equations to find how much money each girl raises in 3 hours.

Francisca	Elizabeth
m = 51h	m = 58h
= 51(3)	= 58(3)

DIFFERENTIATION | **EXTEND**

88



Deepen Understanding

Analyzing the Structure of Equations That Show **Proportional Relationships**

Ask students to make a table of values for each of the equations in the second Model It, showing the amount of money each walker earns for walking 1, 2, 3, 4, and 5 hours. Then have them graph the points from their table in the coordinate plane.

ASK How do the values in the table for each walker show that there is a proportional relationship between time walked and money earned?

LISTEN FOR The table for each walker shows equivalent ratios between the time walked and money earned.

ASK How do the graphs of the equation for each walker show that there is a proportional relationship between the time walked and money earned?

LISTEN FOR Each of the graphs is a straight line passing through the origin.



SMP 7

88

Develop Solving Multi-Step Ratio Problems

CONNECT IT

SMP 2, 4, 5, 6

Remind students that the constants of proportionality are the same in each representation. Explain that they will now use those ratio relationships to reason about solving problems involving proportional relationships.

Before students begin to record and expand on their work in Model It, tell them that problems 1, 2, and 3 will prepare them to provide the explanation asked for in problem 4.

Monitor and Confirm Understanding 1 – 2

- For each girl, earnings and time form a proportional relationship because the amount of money they earn is a constant multiple of the amount of time they walk. Students may conclude that if each girl walks 0 miles, then she earns \$0, which is another property of proportional relationships.
- The constant of proportionality is the number in the equation that you can multiply *h* by to find *m*.

Facilitate Whole Class Discussion

3 Look for understanding of equivalent strategies for solving the problem.

ASK Why is it possible to calculate the total of both girls' amounts raised with a single equation?

LISTEN FOR When the two girls walk for the same amount of time, their combined hourly rate is the sum of their separate rates.

4 Look for understanding of the meaning and uses of the constant of proportionality.

ASK Why is it useful to have more than one strategy to identify the constant of proportionality for a given relationship?

LISTEN FOR You can use the strategy that matches the information you have. If you have a graph, you can find the value of y when x = 1. If you have a table, you can find the ratio between corresponding values. If you have an equation, you can see the constant multiplied by one of the variables.

5 Reflect Have all students focus on the strategies used to solve the Try It. If time allows, have students discuss their ideas with a partner.

CONNECT IT

- Use the problem from the previous page to help you understand how to solve problems that involve proportional relationships.
- 1 Look at the first Model It. How can you identify the constant of proportionality from a verbal description?

Possible answer: You can find the two quantities that have a proportional relationship. Then you can divide the first quantity by the second quantity to find the unit rate. The constant of proportionality is the unit rate.

- Look at the second Model It. Where is the constant of proportionality in each equation? How can you use it to find how much money each girl raises in 3 hours? In both equations, the constant of proportionality is the coefficient of *h*. You can substitute 3 for *h* to find how much each girl raises in 3 hours.
- How much money do Francisca and Elizabeth raise together? You could use the equation m = (51 + 58)h to find the amount of money they raise together. Why? \$327; Possible answer: One way to solve the problem is to find the amount of money each girl raises alone and then add the amounts together. Another way is to find how much the girls will raise per hour together, then multiply that by the number of hours.
- 4 You have identified constants of proportionality in tables, verbal descriptions, graphs, and equations. Why is this helpful when solving problems that involve proportional relationships?

Possible answer: Being able to identify the constant of proportionality in many representations lets you use the form that you find most helpful for the problem.

5 Reflect Think about all the models and strategies you have discussed today. Describe how one of them helped you better understand how to solve the Try It problem.

Responses will vary. Check student responses.

DIFFERENTIATION | RETEACH or REINFORCE

Visual Model

Ш

Model a multi-step problem involving proportional relationships.

If students are unsure about the relationship between the equations, then use this activity to show how proportional relationships can be related.

- Write the following equations on the board: y = 2x and y = 3x. Have student volunteers plot points and graph for each equation.
- Ask: What pattern do you observe in each graph? [The points form a straight line that passes through the origin.] Ask: What kind of relationship is this? [proportional]
- Write the following equations on the board: y = 5x. Have student volunteers plot points and graph this equation.
- Ask: What kind of relationship is in the third graph? [proportional] Ask: What do you notice about the y-values of the three graphs? [The y-values for the third graph are the sum of the y-values from the first two graphs.] Ask: Why is this the case? [Because 5x is the sum of 2x and 3x.]
- Extend the activity by repeating the process with y = 7x or y = 8x. Have students discuss the relationship between the graphs.

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LESSON 5 | SESSION 2 Develop

7.4

Apply It

For all problems, encourage students to use a model to support their thinking. Students may choose to calculate the constant of proportionality and to use it in an equation. They may also support their work with visual models, including double number lines, graphs, tables, or diagrams.

6 Students may also solve the problem by identifying $\frac{5}{4}$ as the constant of proportionality between dollars of coupons and number of books. Because $5 \times 3 = 15$, the equivalent ratio is 15 : 12.

Students may also write and solve the reciprocal proportion of divers to racers to model the situation: $\frac{2}{6} = \frac{x}{18}$. Students should recognize that the solution to the equation, x = 6, is the number of divers, not the solution to the problem. To solve the problem, students can add the numbers of divers and racers: 6 + 18 = 24.

LESSON 5 SESSION 2 Apply It Use what you learned to solve these problems. At a certain bookstore, you get a \$5 coupon for every 4 books you buy. What is the least number of books you could buy to get \$15 in coupons? Show your work. Possible work: The constant of proportionality for books, b, to coupons, c, is $\frac{4}{5}$. $b=\frac{4}{5}c$ $b = \frac{4}{5}(15)$ *b* = 12 The least number of books you could buy is 12. SOLUTION Swim team members can race or dive. At a meet, 18 members race. The ratio of racers to divers is 6 : 2. How many members are on the team? Show your work. Possible work: Let x = the number of divers $\frac{6}{2} = \frac{18}{3}$ $\frac{6}{2}x = 18$ 3x = 186 + 18 = 24x = 6SOLUTION ______ There are 24 members of the swim team. 8 Roberto runs 25 miles. His average speed is 7.4 miles per hour. He takes a break after 13.9 miles. How many more hours does he run? Show your work. Possible work: The constant of proportionality is 7.4. d = 7.4h25 - 13.9 = 7.4h11.1 = 7.4h1.5 = hSOLUTION Roberto needs to run another 1.5 hours. 90



- 8 Students' solutions should show an understanding of:
 - the average speed of 7.4 miles per hour as the constant of proportionality between distance and time.
 - applying the constant of proportionality to find the running time for a distance of 25 13.9, or 11.1 miles.

Error Alert If students find the time needed to run either 25 miles or 13.9 miles, then encourage students to review the problem and identify all of the key information. Students can apply the constant of proportionality, which is 7.4 miles per hour, to develop a model of the situation. Have students calculate the time in hours that Roberto needs to run the first 13.9 miles [about 1.9 hours], the next 11.1 miles [1.5 hours], and the total time to run 25 miles [about 3.4 hours].

Practice Solving Multi-Step Ratio Problems

Problem Notes

Assign **Practice: Solving Multi-Step Ratio Problems** as extra practice in class or as homework.

a. Students should recognize that the constant of proportionality is identified in the problem statement, where Jasmine calculates the vinegar needed for 1 cup of spray cleaner: $\frac{1}{4}$ cup of vinegar divided by

2 cups of spray cleaner equals $\frac{1}{8}$. **Basic**

b. Students may also use a double number line or a table to show the proportional relationship between cups of water and cups of spray cleaner. *Medium* LESSON 5 SESSION 2 Name:

Practice Solving Multi-Step

Ratio Problems

Study the Example showing how to solve problems involving proportional relationships. Then solve problems 1–4.

Example

Jasmine is making a spray cleaner. She mixes $1\frac{1}{2}$ cups water, $\frac{1}{4}$ cup white vinegar, and $\frac{1}{4}$ cup rubbing alcohol. How much white vinegar would Jasmine need to make 5 cups of the spray cleaner? Find how much spray cleaner Jasmine makes.

Cups of cleaner: $1\frac{1}{2} + \frac{1}{4} + \frac{1}{4} = 2$

For every 2 cups of spray cleaner, Jasmine needs $\frac{1}{4}$ cup of white vinegar.

So, for 1 cup of spray cleaner, Jasmine needs $\frac{1}{4} \div 2 = \frac{1}{8}$ cup of white vinegar. Then find how much white vinegar Jasmine would need to make 5 cups of the spray cleaner.

$5 \cdot \frac{1}{8} = \frac{5}{8}$

Jasmine would need $\frac{5}{8}$ cup of white vinegar to make 5 cups of the spray cleaner.

- 1 a. In the Example, what is the constant of proportionality for cups of vinegar to cups of spray cleaner? $\frac{1}{8}$
 - b. Jasmine is making more of the spray cleaner. She only wants to use 1 cup of water. How much of the spray cleaner will Jasmine make? Show your work.
 Possible work: For every 2 cups of spray cleaner, you need 1¹/₂ cups of water.

$\frac{2}{\frac{3}{2}} = 2 \div \frac{3}{2}$ $= 2 \cdot \frac{2}{3}$ $= \frac{4}{3}$

Solving Multi-Step Ratio Problems In this activity, students use the skills they have learned about proportional relationships to solve multi-step word problems. In some problems students are given a ratio and have to calculate a missing value, and in others they have to calculate the ratio given

other information.

SOLUTION Jasmine will make $1\frac{1}{3}$ cups of spray cleaner.

Fluency & Skills Practice 🍾

FLUENCY AND SKILLS PRACTICE Name

Solving Multi-Step Ratio Problem

> Solve each problem.

for \$35. She spends \$5 on a large of of potato salad and the rest on turk sandwiches. Each sandwich is \$2.50 How many sandwiches does Kim bu 91

 $\frac{1}{4}$ cup

 $\frac{1}{4}$ cup

 $1\frac{1}{2}$ cups

vhite vinega

rubbing alcoho

A shark swims at a speed of 25 miles p hour. The shark rests after 40 miles. Ho long, in minutes, does the shark swim before resting?

al Ted is making trail mix for a party. He t Ali mixes $1\frac{1}{2}$ cups of nuts, $\frac{1}{4}$ cup of raisin at and $\frac{1}{4}$ cup of pretzels. How many cup of pretzels does not not be

> Dayren is driving to visit family. She drives at an average of 65 miles per hour. She drives 227.5 miles before lunch and then 97.5 miles after lunc How many hours did she spend driv

- 2 Students may reason that because Kadeem is driving at a slower speed, he will need more time to travel 25 miles or any other distance. Medium
- 3 Students may find it helpful to draw a diagram or picture to represent the problem. The diagram should show that when the two bikers meet, the sum of their distances is equal to the length of the bike path, which is 55 miles. After students find Ravi's distance, they have the information they need to calculate his speed. Challenge
- 4 Students should recognize that the problem includes information about the amount of corn syrup in the bubble solution but that this information is not needed or useful for solving the problem. The key information is the relationship between water and dish soap. Medium



4 Riley finds a recipe for bubble solution that uses 1 cup water, $\frac{1}{4}$ cup dish soap, and 1 tablespoon corn syrup. She uses 2 cups of dish soap. How much water should she use? Show your work. Possible work:

The ratios of water to dish soap are equivalent to 1: $\frac{1}{4}$, so the constant of proportionality is 4.

 $4 \cdot 2 = 8$

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SOLUTION <u>Riley should use 8 cups of water.</u>

DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Levels 1–3: Reading/Speaking

Help students make sense of Apply It problem 1. Read the problem and have partners work together to write a recipe for the marinade described in the problem. Have students read the question chorally. Call on a volunteer to explain what they need to find. Facilitate a group discussion about the steps to solving the problem. Provide sentence frames:

Have students work together to write steps they can follow to solve the problem using:

- First, we .
- Then, _____.
- Last, ____.

Levels 2-4: Reading/Speaking

Use Three Reads to help students make sense of Apply It problem 1. Adapt the routine by having partners use the routine together. Call on volunteers to paraphrase the problem to confirm understanding.

Review the lesson vocabulary with students and help them decide which they can use to talk about the problem. Have partners use these terms as they talk about the steps they will follow to solve the problem. Remind them to state the steps in order and use sequence words like first, next, then, and last.

Use with Session 3 Apply It

Levels 3–5: Reading/Speaking

Support students as they make sense of Apply It problem 1. Have students read the problem with a partner and use Say It **Another Way** to confirm understanding before they begin work.

Allow time for students to solve the problem individually. Ask students to review the lesson vocabulary and choose terms that might be used when they talk about their solution strategy with a partner. Compile the terms in a word bank for reference during partner discussion. Listen for examples of precise math language as partners discuss solution strategies.

Refine Solving Proportional Relationship Problems

Purpose

- **Refine** strategies for solving multi-step problems involving proportional relationships.
- **Refine** understanding of proportional relationships.

recipe for 4 servings of pudding calls for	Solution
cup of cream. How much cream is needed	2 cups
or 12 servings of pudding?	
· · · · · · · · · · · · · · · · · · ·	

MONITOR & GUIDE

Before students begin to work, use their responses to the **Start** to determine those who will benefit from additional support. Use the **Error Analysis** table below to guide remediation.

Have all students complete the Example and problems 1–3, using Consider This and Pair/Share as appropriate. Observe and monitor their reasoning and guide or redirect students as needed.



S	TART ERROR A	NALYSIS		
	If the error is	Students	may	To support understanding
	2 <u>9</u> cup	have appli between so	ed the reciprocal of the ratio erving sizes.	Ask students to consider if the amount of cream in 12 servings will be greater than or less than the amount of cream in 4 servings. When the number of servings increases, the amount of cream must increase as well.
	8 cups	have misid constant o	lentified the fraction $\frac{2}{3}$ as the froportionality.	Have students solve a related problem in which the quantity of 1 cup of cream replaces $\frac{2}{3}$ cup. Students should recognize that the constant of proportionality involves a relationship between two quantities. In this case, the two quantities are servings of pudding and cups of cream.
	72 cups	have misca the consta multiplying and then d	alculated or misapplied nt of proportionality by g 4 servings by 12 servings lividing by $\frac{2}{3}$.	Have students make a table of equivalent ratios. Encourage students to be mindful of headers and the quantities that go with each. Then have them identify the constant of proportionality between the number of servings and the amount of cream.

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LESSON 5 | SESSION 3 Refine

Example

Guide students in understanding the Example. Ask:

- The ratio of Uma's votes to Paloma's votes is to 4 : 3. How can you find a ratio for each candidate's votes to the total number of votes?
- How do you know to write the proportion $\frac{4}{7} = \frac{u}{217}$ instead of $\frac{4}{7} = \frac{217}{u}$?
- How does finding the values of u and p help you solve this problem?

Help all students focus on the Example and responses to the questions by asking them to identify strategies they agree with and build on a classmate's response by giving reasons why the strategy makes sense.

Look for understanding of the key steps of the problem: identifying the ratio of each candidate's votes to the total number of votes, writing and solving an equation to find each candidate's number of votes, and calculating the difference.

Apply It

1 See **Connect to Culture** to support student engagement. Students should recognize that solving this problem requires several steps, such as calculating the total number of cups in the recipe, determining the constant of proportionality between cups of olive oil and cups of marinade, and multiplying by 9 cups of marinade. **DOK 2**

2 Students should recognize that the problem identifies time in seconds but asks for a constant that includes minutes. DOK 2

- B is correct. The amount that Deyvi spends on tickets is \$20 \$2, or \$18. The price per ticket, or constant of proportionality, is \$1.50. Write and solve the equation 18 = 1.50t, where t is the number of tickets.
 - A is not correct. This answer may be the result of identifying \$2.00 as the constant of proportionality, or cost per ticket, as well as the admission fee.
 - **C** is not correct. This answer may be the result of ignoring the admission fee and thinking that Deyvi spent \$20.00 on tickets.
 - **D** is not correct. This answer may be the result of adding \$2.00 to \$20.00, instead of subtracting it, to calculate the amount that Deyvi spent on tickets.

DOK 3



GROUP & DIFFERENTIATE

Identify groupings for differentiation based on the **Start** and problems 1–3. A recommended sequence of activities for each group is suggested below. Use the resources on the next page to differentiate and close the lesson.

Approaching Proficiency

- **RETEACH** Hands-On Activity
- REINFORCE Problems 4, 5, 7

Meeting Proficiency

• **REINFORCE** Problems 4–8

Extending Beyond Proficiency

- **REINFORCE** Problems 4–8
- **EXTEND** Challenge

Have all students complete the Close: Exit Ticket.

Refine Solving Proportional Relationship Problems

Apply It

- See Connect to Culture to support student engagement. Students may reason that 60 turns of Gear A is equal to the product of 3 and 20 turns, so the 30 turns of Gear B should also be multiplied by 3. DOK 2
- 5 A and C are correct. 90 minutes is equivalent to 1.5 hours, or $\frac{3}{2}$ hours. 24 hours divided by 1.5 hours is equal to 16, and $\frac{1}{2}$ inch of rain falls per period. So the total amount of rain is 16 times $\frac{1}{2}$ inch. The constant of proportionality is $\frac{\frac{1}{2}}{\frac{3}{2}}$ or $\frac{1}{3}$ inch per hour. In 24 hours, the total rainfall in inches is 24 times $\frac{1}{3}$.
 - **B** is not correct. This answer may be the result of thinking that $\frac{1}{2}$ inch of rain per hour is the constant of proportionality.
 - **D** is not correct. This answer may be the result of multiplying 24 hours by 1.5 hours instead of dividing by it.
 - **E** is not correct. This answer may be the result of confusing the constant of proportionality with the 90-minute span of time expressed in hours.

DOK 3

Students may confirm their work by finding examples of *w* and *d* that make the equation true and then calculating their ratio. The ratio will always be equal to 1.5. *DOK 2* Gears A and B turn together. Gear A turns 20 times when Gear B turns 30 times. When Gear A turns 60 times, how many times does Gear B turn? Show your work.
 Possible work:
 20/30 = 60/x
 Since 60 = 20 • 3, then x = 30 • 3.
 x = 90

SOLUTION __Gear B turns 90 times when Gear A turns 60 times.

- In a certain town, in 90 minutes ¹/₂ inch of rain falls. It continues at the same rate for a total of 24 hours. Which of the following statements are true about the amount of rain in the 24-hour period? Select all that apply.
 - **A** The number of inches of rain is 16 times $\frac{1}{2}$.
 - **B** The number of inches of rain is 24 times $\frac{1}{2}$.
 - **C** The number of inches of rain is 24 times $\frac{1}{3}$.
 - **D** The number of inches of rain is $\frac{1}{2}$ times 36.
 - **E** The number of inches of rain is 24 times $1\frac{1}{2}$.
- Benjamin is planning to make scale drawings of bat wings. He wants all of his drawings to use the same scale. He says he can use the equation d = 1.5w to find the length of a scale drawing, d, based on the length of the bat wing, w. What is the scale factor of bat wing length to scale drawing length? Explain.
 1.5; Possible explanation: The scale factor is a constant of proportionality. In the equation, the coefficient of w is the constant of proportionality, so it gives the scale factor.

DIFFERENTIATION

RETEACH

Hands-On Activity

Model problems and see if they form proportional relationships.

Students approaching proficiency with solving proportional relationship problems will benefit from modeling examples of these problems.

Materials For each pair: a strip of paper, 2 rulers

- Pose the problem: A ribbon is 24 inches long. To the nearest centimeter, how many centimeters long is it?
- Give each pair of students at least one strip of paper. Have each student measure the length and width, one using centimeters and the other using inches.
- Have each pair choose a model to solve the problem and record their results.
- Ask: *Is there a proportional relationship between inches and centimeters? How do you know?* [Yes; the model shows that the ratio of centimeters to inches for the length and width are equivalent ratios.]
- Ask: A ribbon is 24 inches long. Suppose you want to know, to the nearest centimeter, how many centimeters long the ribbon is. How does knowing there is a proportional relationship between inches and centimeters help you find the length in centimeters? [Since there is a proportional relationship, you can find an equivalent ratio or use a model to find the length in centimeters.]
- Ask students to share other quantities that are in proportional relationships, such as cups and gallons. Extend the activity by posing problems that can be solved using these proportional relationships, such as: *You have a gallon of oil. You use 2 cups of oil for dinner. How much oil do you have left?*

LESSON 5 | SESSION 3 Refine

7 Students should recognize that the number of boxes that Adsila fills is 6 times 9, or 54 boxes. Subtract to find that Carlos fills 72 boxes. Carlos works for 6 hours, so his rate is $\frac{72}{6}$, or 12 boxes per hour. *DOK 2*

Students may recognize that 24 is a multiple of both 8 and 6, so they can use equivalent ratios to solve the problem. They can find $\frac{8}{60} = \frac{24}{180}$ and $\frac{6}{51} = \frac{24}{204}$. **DOK 3**

CLOSE EXIT TICKET

9 Math Journal Look for understanding of the ways in which additional steps are needed either before or after a proportional relationship is applied.

Error Alert If students write the reciprocal for the constant of proportionality, then have them write the rate in words with units. Discuss how proportional relationships relate two different quantities.

End of Lesson Checklist

INTERACTIVE GLOSSARY Support students by suggesting that they confirm their definition with examples and counterexamples of proportional relationships to record in their notebooks.

SELF CHECK Have students review and check off any new skills on the Unit 1 Opener.

LESSON 5 SESSION 3

- Adsila and Carlos volunteer to fill gift boxes for soldiers serving overseas. Both work at a constant rate. They work together for 6 hours and fill 126 boxes. Adsila fills 9 boxes every hour. How many boxes does Carlos fill every hour?
- 8 Aimee is in charge of buying tickets for a trip to the movies. She can buy tickets at the theater or online. At the theater, 8 tickets cost \$60. Online, 6 tickets cost \$51. Aimee needs to buy 24 tickets. How much money can Aimee save by buying tickets at the lower price? Show your work.

Possible work:

At the theater, 8 tickets cost \$60. The cost per ticket is \$60 \div 8, or \$7.50. So, 24 tickets will cost 24 \cdot \$7.50, or \$180.

Online, 6 tickets cost \$51. The cost per ticket is \$51 \div 6, or \$8.50. So, online, 24 tickets will cost 24 \cdot \$8.50, or \$204.

\$204 - \$180 = \$24

SOLUTION Aimee can save \$24.

Math Journal Write a word problem that involves a proportional relationship and needs more than one step to solve. Show how to solve the problem.
 Possible answer: Bao makes fruit punch. Her recipe calls for ¹/₂ cup orange

juice, $\frac{1}{4}$ cup pineapple juice, and $\frac{1}{4}$ cup apple juice. How much of that fruit punch will she make if she uses $\frac{3}{4}$ cup of apple juice?

$$\frac{3}{4} = \frac{1}{4} \cdot 3$$

That means Bao will make 3 times the recipe.
$$3(\frac{1}{2} + \frac{1}{4} + \frac{1}{4}) = 3(1)$$
$$= 3$$

Bao can make 3 cups of fruit punch.

🗹 End of Lesson Checklist -

☐ INTERACTIVE GLOSSARY Find the entry for *proportional relationship*. Rewrite the definition in your own words.

SELF CHECK Go back to the Unit 1 Opener and see what you can check off.

REINFORCE

Problems 4–8

Solve multi-step problems involving proportional relationships.

Students meeting proficiency will benefit from additional work with solving multi-step problems involving proportional relationships.

- Have students work on their own or with a partner to solve the problems.
- Encourage students to show their work.

EXTEND

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Challenge Solve multi-step problems involving proportional relationships.

Students extending beyond proficiency will benefit from multi-step problems involving proportional relationships.

- Have students work with a partner to solve: *Mia and José are planning a party. Mia suggests buying a pack of 15 invitations for \$21. José wants to buy \$7 of supplies and 20 blank invitations for 90 cents each to make invitations. Whose plan has the greater cost per invitation?*
- Students should find that José's rate is \$1.25 per invitation, which is less than Mia's rate of \$1.40 per invitation.
- Repeat, this time with partners suggesting other values for the quantities in the problem.

PERSONALIZE

i-Ready[®]

Provide students with opportunities to work on their personalized instruction path with *i-Ready* Online Instruction to:

- fill prerequisite gaps.
- build up grade-level skills.

STANDARDS FOR MATHEMATICAL PRACTICE (SMP)

SMP 1, 2, 3, 4, 5, and 6 are integrated into the Try-Discuss-Connect routine.*

This lesson provides additional support for:

- 2 Reason abstractly and quantitatively.
- **3** Construct viable arguments and critique the reasoning of others.
- 4 Model with mathematics.
- * See page 1q to learn how every lesson includes these SMP.

Objectives

Content Objectives

- Solve problems that include a single percent.
- Solve multi-step problems that include multiple percents.
- Solve problems involving markups and markdowns.
- Solve problems involving gratuities, tax, and commission.
- Calculate simple interest.

Language Objectives

- Justify solution strategies for problems that include percents by referring to models and expressions.
- Interpret word problems that include the lesson vocabulary and paraphrase the context of the problems using everyday language.
- Understand and use lesson vocabulary to discuss solution strategies for problems involving percent.
- Listen during partner and class discussion and check understanding by paraphrasing or summarizing a classmate's ideas.

Prior Knowledge

- Find what percent of a whole a number is.
- Find a part, given a percent and a whole.
- Find a whole, given a percent and a part.
- Understand that percents can be expressed as fractions and decimals.
- Use proportional relationships to solve problems.

Vocabulary

Math Vocabulary

commission a fee paid for services, often a percent of the total cost. A salesperson who earns a commission often gets a percent of the total sale.

gratuity an amount added on to the cost of a service, often a percent of the total cost. Gratuity is often referred to as a *tip*.

markdown an amount subtracted from the cost of an item to determine the final price. The amount subtracted is often a percent of the cost.

markup an amount added to the cost of an item to determine the final price. The amount added is often a percent of the cost.

simple interest a percent of an amount that is borrowed or invested.

tax a percent of income or of the cost of goods or services paid to the government.

Review the following key terms.

percent per 100. A percent is a rate per 100.

proportional relationship the relationship between two quantities where one quantity is a constant multiple of the other quantity.

rate a ratio that tells the number of units of one quantity for 1 unit of another quantity.

Academic Vocabulary

discount an amount taken off of an original price.

Learning Progression

In Grade 6, students saw that a percent is a rate with the whole divided into 100 equal parts. They represented percents with visual models and connected percents to fractions. They found a given percent of a number and found the whole when given a part and a percent.

Earlier in Grade 7, students used their knowledge of equivalent ratios and proportional relationships to interpret and solve multi-step problems.

In this lesson, students solve real-world problems involving both single and multiple percents. They learn about applications such as commissions, gratuities, markups, markdowns, simple interest, and tax. Students learn to recognize that finding a price after a discount of x% is the same as finding (100 - x)% of the original price and that finding a total after a markup of x% is the same as finding (100 + x)% of an amount.

>

Later in Grade 7, students will learn to find percent change and percent error.

In later grades, students will use their knowledge of percent to solve problems in math, science, social science, and real-world situations.

LESSON 20 Overview



Connect to Culture

Use these activities to connect with and leverage the diverse backgrounds and experiences of all students. Engage students in sharing what they know about contexts before you add the information given here.

SESSION 1 🗖 🗆 🗆 🗆

Try It Ask students to raise a hand if they play a musical instrument, and, if so, if they play in the school band. If there are any band members in class, ask them to describe their experiences. Some middle school bands are concert bands, while others may specialize in jazz or march at football games. Typical bands of this level include woodwinds, brass instruments, and percussion.

SESSION 2

Try It Since cars cost thousands of dollars, most people do not pay for a car all at once. Instead, they take out a loan and make monthly payments on that loan over time, often for 1 to 5 years. Banks and car dealerships offer these loans because when a borrower takes out a loan, they must pay back the amount borrowed plus interest. A shorter loan usually has higher monthly payments but a lower total cost, while a longer loan usually has lower monthly payments but a higher total cost. Ask students to discuss the pros and cons of shorter and longer car loans.

SESSION 3

Try It Ask students who have ever participated in a Nowruz (pronounced noh-rooz) celebration to share their experiences with the class. The Persian New Year, Nowruz, begins on the first day of spring. A special tablecloth, called the Haft Sin, or the cloth of seven dishes, is placed on a table in Persian households. These seven symbolic dishes consist of sabzeh, sprouts; samanu, a type of pudding; seeb, an apple; senjed, a sweet, dry fruit; seer, garlic; somaq, sumac berries; and serkeh, vinegar. Ask students to name other New Year celebrations with which they are familiar or other celebrations at which special food is served.

SESSION 4

Try It While stringed instruments have been played for thousands of years, the modern form of the acoustic guitar is only about 200 years old. An acoustic guitar player plucks or strums the guitar strings, which transmit vibrations into the body of the guitar. These vibrations travel into the surrounding air, where people can hear them as the sounds of the guitar. By contrast, an electric guitar makes electrical signals, and needs to be plugged into an amplifier or other electric device for those signals to turn into sound. Ask students about their experiences playing guitars and whether there are particular guitar players whose music they like.

Analysis Analysis



CULTURAL CONNECTION

Alternate Notation The Arabic language is written and read from right to left across the page, so in Arabic, the % symbol is written to the left of the numeral, rather than to the right, as in %7. Encourage students who have experience with Arabic mathematical notation to share what they know with the class.



Connect to Family and Community

After the Explore session, have students use the Family Letter to let their families know what they are learning and to encourage family involvement.



Connect to Language

For English language learners, use the Differentiation chart to scaffold the language in each session. Use the Academic Vocabulary routine for academic terms before Session 1.

DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Prepare for Connect It problem 2

by helping students activate their

Vocabulary and then invite students

to tell ways people can save, earn,

or spend money. If needed, provide

Next, read the problem as students

follow along. Guide them to draw

arrows to show if money increases

or decreases. Ask questions to help

For example, ask: Where can people

earn interest? [in a savings account]

students connect to their discussion.

prior knowledge of money and

percents. Review the Academic

suggestions. Record ideas for

reference.

ACADEMIC VOCABULARY

To *increase* is to become larger or greater in value.

To *decrease* is to become smaller or lesser in value.

To *earn* is to get something, such as money, for work or services.

Levels 1–3: Reading/Listening Levels 2–4: Reading/Listening

Help students make sense of Connect It problem 2 by activating their prior knowledge about money and percents. Have students read the first paragraph and discuss *percents*, *increase*, and *decrease*. Ask students to list real-world situations involving percents; record any ideas related to money.

Next, read the problem aloud as students follow along. Pause frequently to ask students to tell how words and phrases relate to money and percents and to help them connect or add to the ideas they listed earlier.

Use with Session 1 Connect It

Levels 3–5: Reading/Listening

Help students make sense of Connect It problem 2 by activating their prior knowledge. Ask partners to read the first paragraph and discuss real-world situations involving *percents*. Then ask students to think about ways people can earn or spend money and to consider if these situations relate to *percents*. Call on volunteers to share. To emphasize key ideas, ask other students to repeat and rephrase any statements related to *interest*, *markups*, *markdowns*, and *commission*.

Next, read the problem aloud as students listen for words and phrases related to *percents*. Allow students to ask questions and clarify meanings.

Explore Percents



- **Explore** the idea of simple interest and other applications of percent.
- **Understand** that tax, gratuities, commissions, discounts, and interest are all applications of percent.



Possible Solutions

All of the representations show the same value.

- A and C both use the digits 2 and 5.
- A is the only answer in percent form.
- B is the only answer in fraction form.
- C is the only answer in decimal form.
- D is the only representation without numbers.

WHY? Support students' understanding of equivalency of fractions, decimals, and percents.

TRY IT

SMP 1, 2, 4, 5, 6

SMP 2, 3, 6

Make Sense of the Problem

See **Connect to Culture** to support student engagement. Before students work on Try It, use **Three Reads** to help them make sense of the problem. After the first read, ask students what the situation is about. After the second read, ask students what they are being asked to find. Then after the third read, have students describe the important quantities in the problem and the relationships between them.

DISCUSS IT

Support Partner Discussion

After students work on Try It, have them respond to Discuss It with a partner. Listen for understanding that:

- 120% of a number is 100% of the number plus 20% of the number.
- there are more students in the band this year than last year.



Common Misconception Listen for students who think a percent cannot be greater than 100%, so they find 20% instead of 120%. As students share their strategies, ask them whether there will be more or fewer members in the band compared to last year. Have students compare their strategies and solutions to one another's and provide opportunities for revision and self-correction.

Select and Sequence Student Strategies

Select 2–3 samples that represent the range of student thinking in your classroom. Here is one possible order for class discussion:

- bar model showing 100% of 80, divided into 5 sections of 16
- (misconception) only finding 20% of 80
- double number line with percents up to 120 on one line and numbers up to 96 band members on the other
- an equation multiplying 80 by 1.2

Facilitate Whole Class Discussion

Call on students to share selected strategies. As students listen to the presenters, prompt them to build on ideas they agree with by adding details to help classmates understand more about the strategy.

Guide students to **Compare and Connect** the representations. Provide individual think time after posing the question below before starting the discussion.

ASK How did each representation show a percent greater than 100%?

LISTEN FOR All the visual models extended past one full model of 80. Any decimal or fractional representation used a number greater than 1.

CONNECT IT

SMP 2, 4, 5

1 **Look Back** Look for understanding that more than 100% of a starting number is greater than the starting number.

DIFFERENTIATION | RETEACH or REINFORCE

Visual Model

Find a percent of a number when the percent is greater than 100.

If students are unsure how a percent can be greater than 100, then use this activity to show them another way to think about percents greater than 100.

- Ask students what 100% of a quantity means and how they can write 100% as a decimal. [All of the quantity; 1.0 or equivalent, with any number of zeros after the decimal point.]
- Have students select a number. Have volunteers write expressions for 100% of that number using the percent form [100% \times the number] and the decimal form [1.0 \times the number].
- Display the expression 1.25 × the number. Display the expression (1 × the number) + (0.25 × the number) nearby. Ask: *Do these two expressions have the same value? Why*? [Yes; You can use the distributive property to rewrite the first expression as the second.]
- Ask a volunteer to rewrite the expression (1 \times the number) + (0.25 \times the number) using percents instead of decimals. [(100% \times the number) + (25% \times the number)]
- Ask: Can you use the distributive property to rewrite this expression? What happens? [Yes; You get 125% × the number.]
- Repeat, using other rational numbers and percents between 100% and 1,000%.

LESSON 20 SESSION 1

CONNECT IT

- Look Back How many students are in the band this year? How do you know?
 96; Possible explanation: Since 100% of 80 is 80, 20% of 80 is 16, and 80 + 16 = 96, there are 96 students in the band this year.
- 2 Look Ahead You can think of 120% as 100% + 20%. Similarly, you can think of 80% as 100% 20%. Percents are used in calculating simple interest. You can owe interest on a loan or earn interest on a bank account or investment.

Simple interest formula: *I* = *Prt*

The amount of interest, *I*, is based on the principal, *P*, the interest rate, *r*, and the time you borrow or invest the money for, *t*. The principal is the starting amount. The rate is written as a decimal. For simple interest, time is measured in years.

a. Suppose you borrow \$300 at a yearly, or annual, simple interest rate of 3.4% for 3 years. What values would you use for *P*, *r*, and *t*?

P = 300, r = 0.034, t = 3

b. Suppose you borrow \$400 at a yearly simple interest rate of 3% for 3 months. What values would you use for *P*, *r*, and *t*?

 $P = 400, r = 0.03, t = \frac{1}{4}, \text{ or } 0.25$

- c. A markdown decreases the cost of an item. A markup increases the cost. Suppose a store puts an item on sale for 25% off. Is that an example of a markup or a markdown? markdown
- d. Often when you buy something, you pay a percent of the price as a tax.
 Suppose you pay a 7% tax on an item. What percent of the price of the item will you pay? 107%
- e. Many people gives tips, or gratuities, for good service. Many salespeople earn commission on their sales. Jason earns a 9% commission on a \$1,000 sale. How much is Jason's commission? \$90
- Reflect Is it possible for a single price change to be both a markup and a markdown? Explain your thinking.
 No; Possible explanation: A markup is an increase in the price and a markdown is a decrease, so one change cannot be both.

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2 Look Ahead Point out that interest, markdown, markup, taxes, gratuities, and commissions are all common uses of percent and that they may add to or take away from a quantity. Students should recognize that finding these values always involves a part, a percent, and a whole. When two of these are known, the third can be found. For problem 2a, have students use what they know about the decimal form of 3% to find the decimal form of 3.4%.

Ask volunteers to rephrase the definitions of *simple interest, markdown, markup, taxes, gratuities,* and *commission*. Support student understanding by encouraging them to give a real-world example including each term.

CLOSE EXIT TICKET

Reflect Look for understanding that markup and markdown are different because one is an increase while the other is a decrease.

Common Misconception If students think a single price change can be both a markup and a markdown, then challenge them to give an example of a single change to the price of a specific item, such as a smartphone app, that represents both an increase and a decrease.

Prepare for Solving Problems Involving Percents

Support Vocabulary Development

Assign **Prepare for Solving Problems Involving Percents** as extra practice in class or as homework.

If you have students complete this in class, then use the guidance below.

Ask students to consider the term *percent*. Remind them to draw on anything they know about representing percent, for example, representing percent on a hundreds grid.

Have students work individually to complete the graphic organizer. Invite students to share their completed organizers and prompt a whole-class comparative discussion of the definitions, known facts, and examples students provided.

Have students look at the statement in problem 2 and discuss with a partner how the numbers given relate to each other. Have them discuss how they are the same and how they are different.

Problem Notes

1 Students should understand that percent is a rate per 100. Student responses might include that percents can be converted to decimals and fractions. Examples given might include a percent problem where a percent of a number is found, fractions written as percent, and decimals written as percent. Students may note that a percent can be less than 1 or greater than 100.

2 The decimal 0.02 is 2 hundredths. A percent is a rate per 100, so 2 hundredths are equivalent to 2%. Isabel could express the decimal 0.2 or 0.20 as 20%.

LESSON 20 SESSION 1 Name:

Prepare for Solving Problems Involving Percents

- Think about what you know about percents. Fill in each box. Use words, numbers, and pictures. Show as many ideas as you can. **Possible answers:** What Is It? What I Know About It Percent is a rate per 100. You can use the percent symbol, %, to show a percent. You can rewrite a percent as a fraction or a decimal. percent Examples Examples Examples 50% of 250 is 125. <u>10</u> is 100%. $0.3 = \frac{30}{100} = 30\%$
- Isabel says that 0.02 can be expressed as 20%. Is she correct? Explain why or why not.

No; Possible explanation: The decimal 0.02 is the same as $\frac{2}{100}$, or 2%, not 20%.

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REAL-WORLD CONNECTION

When you dine at a restaurant, the bill you receive from your server includes the total cost of your food and beverages, plus tax. This sales tax is a fixed percent of the total bill. Different cities and states have sales taxes that are different percents. When you receive your bill, you also calculate the tip, or gratuity, that you wish to give your server for their service. The amount of the tip is up to you; however, it is typically between 15% and 20% of the total bill. Ask students to think of other real-world examples when finding a percent of a number might be useful.



LESSON 20 | SESSION 1 **Additional Practice**

Problem 3 provides another look at finding a percent greater than 100% of a given number. This problem is similar to the problem about finding the number of members in a middle school band. Both problems involve using a percent greater than 100 to find a year-over-year increase in a quantity. This problem asks for 125% of 40 to find the number of times a rapper performs this year.

Students may want to use grid paper, hundreds grids, bar models, or number lines to solve.

Suggest that students use Three Reads to help them understand what the numbers in the problem represent and what they are trying to determine.



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DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

MATH TERM

The constant of proportionality is the unit rate in a proportional relationship.

Levels 1–3: Speaking/Writing

Prepare students to respond to Connect It problem 4. Read the problem aloud and review the terms proportional relationship and constant of proportionality. Display the formula and ask partners to identify the defined variables and the constant of proportionality. Reread the question and have partners write responses using:

- The formula shows that interest is the product of _____, ____, and _
- In this problem, the constant of proportionality is _____.
- This means interest is to time.

Levels 2–4: Speaking/Writing

Prepare students to respond in writing to Connect It problem 4. Read the problem aloud and review the meaning of proportional relationship and constant of proportionality. Display the formula, point to each variable, and ask: What does the variable _ represent? Which quantity is a constant multiple of another quantity?

Next, have students use Stronger and Clearer Each Time to draft, discuss, and revise their responses to the problem. Ask partners to work together to revise writing for clarity of ideas.

Use with Session 2 Connect It

Levels 3–5: Speaking/Writing

Prepare students to respond in writing to Connect It problem 4. First, have students read the problem in pairs. Ask partners to discuss the meaning of proportional relationship and constant of proportionality, define the variables in the formula, and tell why the relationship between simple interest and time is proportional.

Next, have students use Stronger and Clearer Each Time.

Remind students to ask questions about ideas that are not clear during structured pairings. Have students revise their writing to clarify ideas, if needed.

Develop Finding Simple Interest



- **Develop** strategies for finding simple interest and any of the components of the simple interest formula.
- **Recognize** that interest is an amount of money that is earned (savings) or paid back (loan) on an amount of money (principal) over a fixed amount of time.





Possible Solutions

- A is the only single-digit percent.
- C is the only one not equal to 30.

D is the only one with a percent greater than 100.

WHY? Support students' facility with calculating and comparing percents.

- DEVELOP ACADEMIC LANGUAGE -

WHY? Develop understanding of the multiple-meaning word *principal*.

HOW? Students are likely familiar with the word *principal* as the leader of a school. Explain that in the context of money, *principal* refers to the amount invested or borrowed. Have students turn and talk about what the two meanings might have in common. Encourage students to use *principal* as they discuss interest in the Apply It problems and upcoming lessons. If time allows, you might show and discuss the meaning of the commonly confused term *principle*, which refers to a basic belief or value that other beliefs or values are built upon.

TRY IT

Make Sense of the Problem

See **Connect to Culture** to support student engagement. Before students work on Try It, use **Three Reads** to help them make sense of the problem. Have students turn and talk with a partner before they discuss the answer to each question as a class.

SMP 1, 2, 4, 5, 6

 Read and try to solve the problem below. 	NEW CAP
Dario borrows \$12,000 to buy a car. He borrows the money at a yearly, or annual, simple interest rate of 4.2%. How much more interest will Dario owe if he borrows the money for 5 years instead of 1 year?	4.2% annual simple interest rate
Math Toolkit double number lines, grid paper	
Possible work:	
SAMPLE A	
Interest for 1 year:	
$12,000(\frac{4.2}{100}) = 504$	
Interest for 5 years:	
$12,000\left(\frac{4.2}{100}\right)(5) = 2,520$	
2,520 - 504 = 2,016	
Dario will owe \$2,016 more in interest.	
SAMPLE B	
Interest for 4 years = Interest for 5 years $-$ Interest for 1 year	
= (12,000)(4.2%)(5) - (12,000)(4.2%)(1)	
= (12,000)(0.042)(5) - (12,000)(0.042)(1)	
= 2,520 - 504	DISCUSS
Dario will owe \$2,016 more in interest.	Ask: Why did you choose that strate
	Share: I knew so I

Support Partner Discussion

After students work on Try It, have them explain their work and then respond to Discuss It with a partner. Listen for understanding that:

- the interest can be found by multiplying the product of the principal, 12,000, and the rate, 0.042, by the time in years.
- the interest for *n* years is *n* times the interest for 1 year.

Error Alert If students make errors when writing 4.2% as a decimal, then have them write the percent as a fraction before writing it as a decimal.

Select and Sequence Student Strategies

Select 2-3 samples that represent the range of student thinking in your classroom. Here is one possible order for class discussion:

- make a table of values for years 1 through 5
- multiply by the interest rate in fraction form to find interest for 1 year, and then multiply by 5
- multiply by the interest rate in decimal form to find interest for 1 year, and then multiply by 5
- use the simple interest formula for 1 year and for 5 years, and then subtract

Facilitate Whole Class Discussion

Call on students to share selected strategies. As they listen to the presentations, review with students that one way to listen to understand is to paraphrase the speaker's ideas and to check with the speaker to confirm.

Guide students to Compare and Connect the representations. Use turn and talk to help students think through their responses before sharing with the group.

ASK How does each strategy show how much more interest Dario owes for 5 years than for 1 year?

LISTEN FOR Different methods were used to find the interest, but in each strategy the interest for 1 year was subtracted from the interest for 5 years to determine how much extra was paid.

Model It

If students presented these models, have students connect these models to those presented in class.

If no student presented at least one of these

models, have students first analyze key features of the models and then connect them to the models presented in class.

ASK How are the models alike, and how are they different?

LISTEN FOR Each finds the interest for different numbers of years. The first uses a table to find the interest for 1, 2, 3, 4, and 5 years. The second uses the simple interest formula to find the interest for just 1 year and 5 years.

For the table, prompt students to look for a pattern.

• What patterns do you notice in the table?

For the simple interest formula, prompt students to compare and contrast the values substituted for the variables.

- What is the same about both calculations?
- How does the interest for 5 years compare to the interest for 1 year?

LESSON 20 SESSION 2	
Explore different ways to find simple interest.	
Dario borrows \$12,000 to buy a car. He borrows the money at a yearly, or annual, simple interest rate of 4.2%. How much more interest will Dario owe if he borrows the money for 5 years instead of 1 year?	S12,000

Model It

You can use the relationship between time and interest.

After one year, Dario will owe 4.2% of \$12,000 in interest.

0.042(12,000) = 504

Year	Total Interest
1	\$504
2	\$1,008
3	\$1,512
4	\$2,016
5	\$2,520

Model It

You can use the simple interest formula to find the interest.

<i>l</i> = <i>Prt</i>			
<i>I</i> = interest	P = principal	r = interest rate	<i>t</i> = time (in years)
1 Year		5 Years	
I = Prt		I = Pr	t
= (12,000)(0.0	042)(1)	= (12	<mark>2,000</mark>)(0.042)(5)
= 504(1)		= 50	4(5)
= 504		= 2,5	520
After 1 year Darie	will awa CEOA in int	arast	

After 1 year, Dario will owe \$504 in interest.

After 5 years, Dario will owe \$2,520 in interest.

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DIFFERENTIATION | **EXTEND**

Deepen Understanding Reasoning Abstractly About Simple Interest Problems

Draw students' attention to the variable *t*, time (in years), in the second Model It.

- **ASK** Suppose the value of t is $\frac{3}{4}$. What period of time does that mean?
- LISTEN FOR It means three quarters of a year, or nine months.
- **ASK** What value can you use for t to find simple interest for a period of five months? Why? **LISTEN FOR** You can use $\frac{5}{12}$, because each month is $\frac{1}{12}$ of the year.
- **ASK** What value can you use for t to find simple interest for a period of 100 days? Why? **LISTEN FOR** You can use $\frac{100}{365}$, because each day is $\frac{1}{365}$ of the year. (Students may also argue for $\frac{100}{366}$, or $\frac{50}{183}$, in the case of leap years.)
- **Generalize** t can be written as any fraction of a year or years.

SMP 2

CONNECT IT

SMP 2, 4, 5, 6

Remind students that the amount of interest each year is the same in each representation. Explain that they will now use those representations to reason about the simple interest formula.

Before students begin to record and expand on their work in Model It, tell them that problem 3 will prepare them to provide the explanation asked for in problem 4.

Monitor and Confirm Understanding 1 – 2

- The amount of interest each year is constant.
- The amount of interest for a 4-year loan is the same as the difference between the amounts of interest for 5 years and 1 year.

Facilitate Whole Class Discussion

3 Look for understanding that the principal and interest rate do not change, but the time and amount of interest do change.

ASK Why do some values stay the same and some change?

LISTEN FOR The principal amount and interest rate do not change because those are fixed amounts for a loan. The amount of time changes, and so the amount of interest changes because it is dependent on time.

Look for the idea that there is a proportional relationship between the amount of simple interest and the amount of time.

ASK Once you have values for P and r, does the product of P and r ever change?

LISTEN FOR No, the product of *P* and *r* remains constant.

ASK What key element of a proportional relationship is present in the simple interest formula?

LISTEN FOR The formula has a constant of proportionality that is represented by the product of the two non-changing values, principal and interest rate.

- 5 Look for the idea that a proportional relationship has a constant multiplier, so a relationship with a sum is not proportional. Students may also reason that the amount owed is not equal to 0 at time 0.
- 6 Reflect Have all students focus on the strategies used to solve the Try It. If time allows, have students discuss their ideas with a partner.

CONNECT IT

- Use the problem from the previous page to help you understand how to find simple interest.
- Look at the table in the first Model It. How does the interest change over time? The amount of interest for each year is \$504. Each year, the total amount of interest increases by \$504.
- 2 How much more interest will Dario owe for 5 years than for 1 year? How does this difference compare to the amount of interest Dario would owe for borrowing the money for 4 years?
 - \$2,016; It is the same.
- 3 Look at the second Model It. Which values stay the same when you use the formula to find the interest for 1 year and 5 years? Which values change? The interest rate and the principal do not change; The length of time and the amount of interest change.
- 4 How does the formula *I* = *Prt* show a proportional relationship between simple interest and time?

The formula shows that the simple interest, *I*, is a constant multiple of time, *t*. The constant of proportionality is *Pr*, the product of the principal and the interest rate.

- The total amount Dario owes is the sum of the interest and the principal. Is the relationship between total amount owed and time proportional? Explain.
 No; Possible explanation: For two quantities to be proportional, one quantity must be a constant multiple of the other. If A is the total amount owed, then A = P + Prt. A is not equal to a constant times t.
- 6 Reflect Think about all the models and strategies you have discussed today. Describe how one of them helped you better understand how to think about and find simple interest.

Responses will vary. Check student responses.

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DIFFERENTIATION | RETEACH or REINFORCE

Visual Model

Use a model to identify the constant of proportionality.

If students are unsure about why I = Prt shows a proportional relationship, then use this model to connect the simple interest formula to other proportional relationships.

- Display the equation y = 504x. Ask students if this is a proportional relationship and how they know. [It is; One quantity is a constant multiple of the other.]
- Ask: What parts of this equation are variables? [y and x] What parts are constants? [504] Ask: When you change the value you use for one variable, what happens to the other variable? [It also changes.] What happens to the constant? [It stays the same.]
- Display *I* = *Prt*. Call on volunteers to take turns writing expressions with the values from the Try It problem substituted for each variable. Use 1, 2, 3, 4, and 5 for *t*.
- Ask students to look at and compare the expressions. Ask: *Which values varied*? [interest and time] *Which values were constant*? [the principal and the rate]
- Ask: You have a multiplication expression that has two constant values in it. Is there a more efficient way to write this? [Yes; You can multiply Pr and use that product as the constant.] Ask: Is this an equation for a proportional relationship? Why? [Yes; You have one variable that is a constant multiple of another variable.]

LESSON 20 | SESSION 2 Develop

Apply It

For all problems, encourage students to use a model to support their thinking. Encourage students to show all steps to minimize mathematical errors, but allow some leeway for students who are able to explain their work.

- 7 Students may solve the problem by finding the interest for one year and using repeated addition to find the total amount of interest.
- 8 Students may divide 180 by 2 to determine that the amount of interest for one year is \$90. Then they may divide 90 by 1,200 to determine that the interest rate is 7.5%.

Apply It		
Use what you learned to	solve these problems.	
 Ava borrows \$600 to buy rate of 2.25%. Ava borrow much does Ava pay in sim pay in all? Show your work Possible work: I = Prt 	a bike at a yearly simple interest is the money for 3 years. How uple interest? How much does Ava k. Total = $P + I$	Ava's Bike Purchase Borrows to Buy a Bike \$600
= (600)(0.0225)(3)) = 600 + 40.50	
= 40.50	= 640.50	
 8 Zhen borrows \$1,200. She interest. What is the yearly people work 	borrows the money for 2 years and y simple interest rate on Zhen's loan	al. I owes \$180 in simple ? Show your work.
8 Zhen borrows \$1,200. She interest. What is the yearly Possible work: I = Prt 180 = (1,200)r(180 = 2,400r $\frac{180}{2,400} = r$	e borrows the money for 2 years and y simple interest rate on Zhen's loan	al. I owes \$180 in simple ? Show your work.
8 Zhen borrows \$1,200. She interest. What is the yearly Possible work: $I = Prt$ $180 = (1,200)r($ $180 = 2,400r$ $\frac{180}{2,400} = r$ $0.075 = r$	e borrows the money for 2 years and y simple interest rate on Zhen's loan	al. I owes \$180 in simple ? Show your work.
8 Zhen borrows \$1,200. She interest. What is the yearly Possible work: $I = Prt$ $180 = (1,200)r($ $180 = 2,400r$ $\frac{180}{2,400} = r$ $0.075 = r$ SOLUTION The simple	e borrows the money for 2 years and y simple interest rate on Zhen's loan (2) e interest rate on Zhen's loan is 7.	al. I owes \$180 in simple ? Show your work. 5%.
 8 Zhen borrows \$1,200. She interest. What is the yearly Possible work: I = Prt 180 = (1,200)r(180 = 2,400r) 180 = 2,400r) 180 = 2,400r 180 = 2,4	e borrows the money for 2 years and y simple interest rate on Zhen's loan (2) e interest rate on Zhen's loan is 7. count with a yearly simple interest r avings account. How much simple ir 6 months? Show your work.	al. I owes \$180 in simple ? Show your work. 5%. Fate of 2%. Suppose Interest will you earn
 8 Zhen borrows \$1,200. She interest. What is the yearly Possible work: I = Prt 180 = (1,200)r(180 = 2,400r) 180 = 2,400r 180 = 2,400r 180 = 2,400r 180 = r 0.075 = r SOLUTION The simple 9 A bank offers a savings ac you deposit \$550 into a sa in 4 years? In 4 years and 6 Possible work: 4 years and 6 months is 	e borrows the money for 2 years and y simple interest rate on Zhen's loan (2) e interest rate on Zhen's loan is 7. count with a yearly simple interest r avings account. How much simple in 6 months? Show your work. 4.5 years.	al. I owes \$180 in simple ? Show your work. 5%. Fate of 2%. Suppose Interest will you earn
 8 Zhen borrows \$1,200. She interest. What is the yearly Possible work: I = Prt 180 = (1,200)r(180 = 2,400r) 180 = 2,400r) 180 = 2,400r) 180 = r 0.075 = r SOLUTION The simple 9 A bank offers a savings ac you deposit \$550 into a sa in 4 years? In 4 years and 6 Possible work: 4 years and 6 months is I = Prt 	e borrows the money for 2 years and y simple interest rate on Zhen's loan (2) e interest rate on Zhen's loan is 7.1 count with a yearly simple interest r avings account. How much simple in 6 months? Show your work. 4.5 years. I = Prt	al. I owes \$180 in simple ? Show your work. 5%. 5%. rate of 2%. Suppose interest will you earn
 Solution <u>Availables s</u> Zhen borrows \$1,200. She interest. What is the yearly Possible work: I = Prt 180 = (1,200)r(180 = 2,400r) 180 = 2,400r 1	(2) e borrows the money for 2 years and y simple interest rate on Zhen's loan (2) e interest rate on Zhen's loan is 7. count with a yearly simple interest rate avings account. How much simple interest rate of months? Show your work. 4.5 years. I = Prt = (550)(0.02)(4.5)	al. I owes \$180 in simple ? Show your work. 5%. Fate of 2%. Suppose Interest will you earn

CLOSE EXIT TICKET

- 9 Students' solutions should show an understanding that:
 - the simple interest formula can be used to calculate the amount earned in an interest-bearing account.
 - partial years can be used in the simple interest formula.

Error Alert If students use the number of months in the simple interest formula rather than the number of years, then remind them that the variable *t*, in the simple interest formula, represents the number of years. If time is given in months, then they will need to convert the time to years to use the formula.

Practice Finding Simple Interest

Problem Notes

Assign **Practice Finding Simple Interest** as extra practice in class or as homework.

1 Students may also make a two-column table with *Number of Years* in one column and *Interest* in the other. Then they may methodically substitute 1, 2, 3, 4, and 5 for *t* into the simple interest formula until they find \$26 as the interest. *Medium*

2 Students may solve the problem by using the simple interest formula to find the amount of interest owed for 1 year and for 2 years, and then adding the interest for 2 years to the principal to find the total amount. *Basic*

LESSON 20 | SESSION 2

Name:

Practice Finding Simple Interest

Study the Example showing how to use the simple interest formula. Then solve problems 1–6.

Example

Pablo deposits \$750 into a bank account. The account earns yearly simple interest at a rate of $3\frac{1}{2}$ %. How many years will it take Pablo to earn a total of \$105 in simple interest? Use the simple interest formula and solve for *t*. P = \$750, r = 3.5%, and I = \$105

105 = (750)(3.5%)(t)

I = Prt

105 = (750)(**0.035**)(*t*)

105 = 26.25t4 = t

It will take 4 years for Pablo to earn \$105 in interest.

1 Suppose you deposit \$1,200 into a bank account. The account earns yearly simple interest at a rate of $1\frac{3}{4}$ %. How many years will it take to earn a total of \$126 in simple interest? Show your work. Possible work:

 $1\frac{3}{4} = 1.75$ I = Prt 126 = (1,200)(0.0175)(t) 126 = 21t6 = t

SOLUTION <u>It will take 6 years.</u>

Conan borrows \$3,000 at a yearly simple interest rate of 1.6% for 2 years.
 He owes <u>\$96</u> in interest. He needs to pay back <u>\$3,096</u> in all.

Vocabulary simple interest a percent of an amount that is borrowed or invested.

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GRADE 7 + LESSON 20 Page 1 of 2

LESSON 20 | SESSION 2 **Additional Practice**

2.65%

LESSON 20 SESSION 2 3 Jamila deposits \$800 in an account that earns yearly simple interest at a rate Jamila of 2.65%. How much money is in the account after 3 years and 9 months? Show your work. Possible work: DEPOSIT would then be the sum of the principal, 3 times I = Prt \$800.00 the interest for one year, and the interest for the = (800)(0.0265)(3.75) YEARLY SIMPLE = 79.5 INTEREST RATE 800 + 79.5 = 879.5 calculating the interest for 2 years and 4 years SOLUTION ______ There is \$879.50 in the account 4 Carmela borrows \$400 and will pay 5.25% yearly simple interest. How much more interest will Carmela owe if she borrows the money for 4 years instead of 2 years? Show your work. Possible work: revise method by substituting different values She will owe interest for 2 more years. for *P* into the formula 39 = P(0.065)(4) until they One year of interest: (400)(0.0525) = 21 Two years of interest: (21)(2) = 42**SOLUTION** Carmela will owe \$42 more interest. principal + (principal)(interest rate)(time). Then 5 Ellie borrows money at a yearly simple interest rate of $6\frac{1}{2}$ %. After 4 years, Ellie they may substitute known values. When they owes \$39 in interest. How much money did Ellie borrow? Show your work. Possible work: compare their answer to Lilia's, they will see the I = Prt39 = (P)(0.065)(4)39 = 0.26P 150 = *P* SOLUTION Ellie borrowed \$150. 6 Lilia borrows \$400 at a yearly simple interest rate of 6%. She writes the expression 400 + (0.6 imes 400) to represent the total amount of money she will pay back for borrowing the money for 1 year. Is Lilia's expression correct? Explain your answer

> and determine the amount of money Lilia will need to pay back after 1 year. No; Lilia wrote the interest rate incorrectly. The expression should be $400 + (0.06 \times 400)$. She will need to pay back \$424 after one year.

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DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Levels 1–3: Speaking/Listening

3 Students may solve the problem by finding

the interest for one year, then finding $\frac{3}{4}$ of

that amount to account for the interest for

9 months. The total money in the account

Students may also solve the problem by

and finding the difference between them.

Students may also use a guess, check, and

determine that the value of P is 150. Medium

6 Students may solve the problem by

writing the equation in words: total =

interest rate is different. Challenge

9 months. Medium

Medium

5

Help students make sense of Model It by discussing and connecting the representations. Display the two bar models and ask students to share what they notice in their own words. Record responses for reference. Guide students to rephrase the statements using mathematical language, like original cost, percent off, product, sum, and difference.

Next, help students make connections between the bar models and the equations. Display the equation that relates to each bar model. Ask: What is the same about the bar model and the equation? What is different?

- Both have .
- The bar models . The equations

Levels 2-4: Speaking/Listening

Have students make sense of Model It by discussing and connecting the representations. Display the bar model and equation for the cost of groceries. Prompt students to connect the models. Ask: What information is shown in the bar model and in the equation? How are the bar model and equation alike? How are they different?

Encourage students to include math terms, like percent, product, sum, and difference. Next, display the bar model and equation for the cost of flowers. Help students compare the models:

- Both the bar models and equations _____.
- The bar models , but the equations .

Use with Session 3 Model It

Levels 3–5: Speaking/Listening

Have students make sense of Model It by discussing and connecting the representations. First, ask partners to discuss what they notice about the bar models, and then call on students to share their ideas. Next, have partners discuss how the equations connect to the bar models. Prompt students to make connections by explaining how each strategy provides the same information.

Reinforce that students should listen to their partners and build on to ideas they agree with or introduce different ideas:

- I think you are right because .
- I have a different idea. I think

Develop Solving Problems Involving a Single Percent

Purpose

- **Develop** strategies for solving problems involving a single percent.
- **Recognize** that an increase by n% is the same as multiplying by (100 + n)% and a decrease by n% is the same as multiplying by (100 n)%.

START CONNECT TO PRIOR KNOWLEDGE



Possible Solutions

A, if you want the lowest interest rate because you need more time to pay off the loan. You would pay \$210 in interest.

B, because you pay it off in the shortest time. You would pay \$140 in interest.

C, because it has the least total interest, only \$127.50.

WHY? Support students' facility making calculations for problems involving simple interest.

DEVELOP ACADEMIC LANGUAGE -

WHY? Help students synthesize information.

HOW? Display Connect It problem 3. Ask: *What is the question about Allen's expression? How do you know?* If needed, think aloud as you circle the parts in the first question that complete the second question. Explain that often readers need to refer to other sentences to get all the information. Ask students to find another question that does not contain all of the information. [problem 4]

TRY IT

SMP 1, 2, 4, 5, 6

Make Sense of the Problem

See **Connect to Culture** to support student engagement. Before students work on Try It, use **Notice and Wonder** to help them make sense of the problem. Return to the students' Notice and Wonder ideas after displaying the question to see which ideas relate to the given problem.



DISCUSS IT

Support Partner Discussion

After students work on Try It, have them explain their work and respond to Discuss It with a partner. If students need support in getting started, prompt them to ask each other questions such as:

- What does it mean for something to be 10% off?
- What does it mean for something to be taxed at a rate of 6.25%?
- How are a discount and sales tax the same or different?

Error Alert If students find the increase and decrease but not the total amount, then remind them to read the problem carefully and think about what information they are trying to find.

SMP 2, 3, 6

Select and Sequence Student Strategies

Select 2–3 samples that represent the range of student thinking in your classroom. Here is one possible order for class discussion:

- use double number lines to find the discount on groceries and the sales tax on the flowers
- use bar models to find the discount and tax
- calculate final percents by subtracting them from or adding them to 100% before multiplying
- use equations to calculate each final cost and find their sum

Facilitate Whole Class Discussion

Call on students to share selected strategies. Before they present, review with students that clear explanations include details about what they noticed and assumed about the problem, what strategies they tried to solve the problem, and why.

Guide students to Compare and Connect the

representations. Call on several students to rephrase important ideas so that everyone hears them more than once and in more than one way.

ASK How were [student name's] and [student name's] strategies the same? Different?

LISTEN FOR Both subtracted 10% and added 6.25% in some way. One found what 10% of the cost of groceries and 6.25% of the cost of flowers were. The other subtracted 10% from 100% and added 6.25% to 100% and found what 90% and 106.25% of the respective costs were.

Model It

If students presented these models, have students connect these models to those presented in class.

If no student presented at least one of these

models, have students first analyze key features of the models, and then connect them to the models presented in class.

ASK What does 100% represent in the problem? Where do you see it in each model?

LISTEN FOR It represents the original costs. It is a label on the bars in the bar models and it is written as 100% in the equations.

For the bar models, prompt students to focus on how the bars are used to find the correct percent.

- How does the percent of the original cost connect to the context?
- How is each bar divided?

For the equations, prompt students to focus on the structure of the equations.

• How do you know whether to subtract or add?

LESSON 20 SESSION 3

 Explore different ways to solve a problem with a single percent.

Cyrus is hosting a dinner to celebrate Nowruz, the Persian New Year. His groceries cost \$150 before he uses a 10%-off coupon. He also orders \$60 worth of flowers. Sales tax on the flowers is 6.25%. What is the total amount Cyrus spends?

Model It

You can draw bar models to find the percents.



Cyrus pays **90%** of the original cost for groceries.

Original Cost of Flowers

3.75	3.75	7.5	15	30	3.75
%	12.	.5% 25	i% 50	0%	100%106.2
6.2	5%				Sales Tax

Cyrus pays 106.25% of the original cost for flowers.

Model It

You can write equations to solve the problem.

Cost of Groceries: (100% - 10%)150 = (90%)150 Cost of Flowers: (100% + 6.25%)60 = (106.25%)60 = (1.0625)(60)

 $\label{eq:cost} \mbox{Total Cost} = \mbox{Cost of Groceries} + \mbox{Cost of Flowers}$

= (**0.90**)(150) + (**1.0625**)(60)

= (0.90)(150)

= 135 + 63.75

DIFFERENTIATION | EXTEND



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Deepen Understanding Modeling Interest Problems with Bar Models

Prompt students to compare the parts of the bar models.

ASK Why are bar models an appropriate tool for solving this problem?

LISTEN FOR The whole bars represent the original costs. The individual sections of the bars show the corresponding percents of the whole that apply to each situation.

ASK Why does the first bar model have a section that is not shaded, while the second bar model is completely shaded?

LISTEN FOR The first bar model represents a discount, so the shaded area is showing the amount of the original cost that is being paid after the discount. The second bar model has tax, which is an additional cost, so the bar model is showing that both all of the original amount and some extra are being paid.

ASK Why are the bars divided differently? How do you determine how to divide them?

LISTEN FOR Each bar represents a different percent. The first bar represents a 10% discount, so the bar is divided into 10 equal groups of 10%. The second bar is repeatedly divided in half until 6.25% is reached.



Nowruz celebration

SMP 4

CONNECT IT

SMP 2, 4, 5, 6

Remind students that the quantities and the relationships between them are the same in each representation. Explain that they will now use those relationships to reason about different ways to represent discounts and additional costs.

Before students begin to record and expand on their work in Model It, tell them that problems 2–3 will prepare them to provide the explanation asked for in problem 4.

Monitor and Confirm Understanding 1 – 3

- The total spent is the cost of the discounted groceries plus the cost of the taxed flowers.
- A discounted amount can be found in one step by subtracting the discount percent from 100% and multiplying the difference by the original cost.

Facilitate Whole Class Discussion

4 Look for the idea that adding a percent of a number to that same number is the same as multiplying the original number by 100% plus the percent amount.

ASK How does each expression show 5% of a? 100% of a?

LISTEN FOR Hiroaki's expression uses 0.05*a* to represent 5% of *a* and *a* to represent 100% of *a*. In Allen's expression, the 0.05 in 1.05*a* represents 5% of *a* and the 1 represents 100% of *a*.

5 Look for understanding that 0.80 represents 80% of the original amount. So, 100% - 80% = 20% is the discount amount.

6 Reflect Have all students focus on the strategies used to solve the Try It. If time allows, have students discuss their ideas with a partner.

CONNECT IT

- Use the problem from the previous page to help you understand how to solve a problem with a single percent.
- 1 What is the total amount Cyrus spends? \$198.75
- 2 Look at the Model Its. How do they show that you can multiply 150 by 0.90 to find the discounted cost of the groceries?
 - Possible explanation: They show that the discounted amount is 10% less than the original, which is 90% of the original. Since 90% = 0.90, you can multiply the original amount by 0.90 to find the discounted amount.
- Hiroaki uses the expression a 0.1a to represent a 10% discount on an amount a. Allen uses the expression 0.9a. Is Hiroaki's expression correct? Is Allen's? Explain. Yes; Yes; Possible explanation: Since 10% = 0.1, the first expression shows finding 10% less than an amount. That is the same as multiplying the amount by 90%, or 0.9.
- Hiroaki uses the expression a + 0.05a to represent an amount increasing by 5%.
 Allen uses the expression 1.05a. Explain why both Hiroaki's and Allen's expressions are correct.

Possible explanation: Since 5% = 0.05, the first expression shows finding the sum of the amount and 5% of the amount. That is the same as multiplying the amount by 105%, or 1.05.

5 The expression (110)(0.80) can be used to find the sale price of an item that has an original price of \$110. By what percent is the original price marked down? How do you know?

20%; Possible explanation: You can think of 0.8 as 1 - 0.2. Since 0.80 is a way to express 80%, that means 1 - 0.2 is a way to express 100% - 20%, or a markdown of 20%.

6 Reflect Think about all the models and strategies you have discussed today. Describe how one of them helped you better understand how to solve the Try It problem.

Responses will vary. Check student responses.

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DIFFERENTIATION | RETEACH or REINFORCE

Hands-On Activity

Use unit cubes to understand percent decrease.

If students are unsure about percent increase and decrease, then use this activity to have students practice modeling percent decrease.

Materials For each student: 10 unit cubes

- Tell students that each cube represents \$1. Have students use the cubes to model \$10. Ask: *When you take away 50% of anything, what percent is left? How do you know?* [50%; 50% is half. When you take away half of something, half remains.] If needed, remind students that 100% of any amount is that whole amount.
- Ask students to model taking away 50% of \$10. Ask: *How many cubes did you take away? How many dollars does that represent?* [5; 5] Ask: *You took away 50% of \$10, so you found a 50% decrease from \$10. What is the result?* [You have 5 cubes representing \$5 left.]
- Ask: If you wanted to find a 10% discount, how many cubes would you take away? Why? [1 cube; Each cube represents one tenth or 10% of the whole.]
- Repeat the activity using \$20 and \$12. You can also use the model to have students explore percent increase.

LESSON 20 | SESSION 3 Develop

Apply It

For all problems, encourage students to use a model to support their thinking. Allow some leeway in precision when students use bar models or double number lines. It is not critical that the bar models or number lines are drawn precisely to scale as long as they are correctly labeled with the amounts they represent.

- 7 Students may also solve the problem by using a double number line to find the amount of commission.
- 8 C is correct. Students may solve the problem by using a bar model to find the amount of the tip and then add it to the original amount to get the total amount.
 - **A** is not correct. This answer represents only the tip amount.
 - **B** is not correct. This answer represents the cost of Heidi's lunch minus the amount of the tip.
 - **D** is not correct. This answer represents the cost of Heidi's lunch plus a tip of 82%.

	pply it
>	Use what you learned to solve these problems.
	 Alanna earns a commission of 8% on her sales. How much commission does Alanna earn on a sale of \$32,000? Show your work. Possible work: 8% of \$32,000 = (0.08)(32,000)
	= 2,560
	SOLUTION Alanna earns \$2,560 in commission.
(Heidi's lunch costs \$12.50. Heidi wants to leave a tip of 18%. How much money does Heidi need to pay for lunch, including the tip?
	A \$2.25
	B \$10.25
	C \$14.75
	D \$22.75
	Before hibernation, a bear weighs 990 pounds. Its weight decreases by 32% during hibernation. How much does the bear weigh when it comes out of hibernation? Show your work.
	A 32% decrease means the bear will weigh 100% – 32%,
	or 68%, of its original weight.
	= 673.2
	SOLUTION The bear weighs 673.2 pounds when it comes out of hibernation.

- a percent decrease in the bear's weight also means an amount of decrease in the bear's weight
- a 32% decrease from 990 pounds may be found by subtracting 32% of 990 from 990 or by finding 68% of 990, as 100% 32% = 68%.

Error Alert If students find a 32% increase from 990 and respond that the bear weighs 1,306.8 pounds, then ask them to compare their answer to the bear's starting weight and check if they found an increase or a decrease.

Practice Solving Problems Involving a Single Percent

Problem Notes

Assign **Practice Solving Problems Involving a Single Percent** as extra practice in class or as homework.

- 1 Students may also solve the problem by finding 118% of the additional \$20 and adding the result to the total cost of the standard room from the example. *Basic*
- 2 Students may also solve the problem by finding 5% of the average playing time, subtracting the result from the original amount, and then multiplying by 7 days. *Medium*

LESSON 20 SESSION 3

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Name:
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Practice Solving Problems Involving a Single Percent

Study the Example showing how to solve a problem with a percent. Then solve problems 1–5.



GRADE 7 + LESSON 20 Page 1 of 2

LESSON 20 | SESSION 3 **Additional Practice**



DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Levels 1–3: Reading/Speaking

from 40. Medium

50 jars. Medium

Support students as they make sense of Apply It problem 9. Read the problem aloud. Then summarize the first three sentences of the problem using the sequencing words first, next, and then. Represent each sentence with numbers, symbols, and/or pictures. For example, you might draw binoculars and write: first \$45; next \uparrow 40%; then \downarrow 10%.

Have students Act It Out by role playing the situation with a partner. One partner can play the role of the store manager buying and marking up the binoculars. The other partner can role play a customer buying the binoculars at the sale price. Call on volunteers to share their experiences with the class.

Levels 2-4: Reading/Speaking

Support students as they make sense of Apply It problem 9. Adapt **Three Reads** by reading the problem aloud each time and having students turn and talk with a partner to answer each question before discussing with the whole group. Encourage students to take notes during the discussion.

After discussing the context of the problem, what students are asked to find, and the important quantities and relationships in the problem, ask students to use their notes to paraphrase the problem using the sequencing words first, next, and then. Listen for and highlight ideas that demonstrate understanding that markups and markdowns are not additive.

Use with Session 4 Apply It

Levels 3–5: Reading/Speaking

Support students as they make sense of Apply It problem 9. Adapt **Three Reads** by having students read the problem independently each time and take notes before discussing the questions with a partner.

Ask students to use their notes to determine the steps for solving the problem. Have them list the steps in order using the sequencing words first, next, and then. Next have partners compare the steps they have written. Encourage students to ask clarifying questions as needed. Listen for and highlight ideas that demonstrate understanding that markups and markdowns are not additive.

Develop Solving Problems Involving Multiple Percents

Purpose

- Develop strategies for solving problems involving multiple percents.
- **Recognize** that multiple percent changes involve multiplicative rather than additive relationships.

START CONNECT TO PRIOR KNOWLEDGE



Possible Solutions

All represent percent changes.

A is a markup.

B is a discount.

C is a tax.

A and C both result in a price greater than 100% of the original amount.

B results in a price less than 100% of the original amount.

WHY? Support students' facility with the concept of markups and markdowns.

DEVELOP ACADEMIC LANGUAGE -

WHY? Model effective listening skills by paraphrasing and confirming understanding.

HOW? Introduce the idea that listeners can take notes as a speaker talks and use the notes to paraphrase the explanation. Model by writing key words and phrases as a volunteer explains his or her ideas. Confirm understanding by asking, *Did I understand what you said correctly?* Have partners practice good listening skills as they share explanations in Discuss It.

TRY IT

SMP 1, 2, 4, 5, 6

Make Sense of the Problem

See **Connect to Culture** to support student engagement. Before students work on Try It, use **Co-Craft Questions** to help them make sense of the problem. Prompt students to incorporate information from both the picture and the problem when they write their questions.



Support Partner Discussion

After students work on Try It, encourage them to respond to Discuss It with a partner. To support students in extending the conversation, prompt them to discuss this question:

• Does order matter when applying multiple discounts?

Common Misconception Listen for students who treat percents as quantities that may be added or subtracted without reference to the wholes they are percents of. As students share their strategies, present an example with easier numbers, such as \$100. Have students work through the multiple-discount scenario in two ways. First, have them add the discounts and find 80% off \$100. Then have them find 50% off \$100, and then find 30% off the remaining \$50. Discuss how the results are different because, in the second scenario, the additional 30% discount is taken off the already-discounted amount.

Select and Sequence Student Strategies

Select 2–3 samples that represent the range of student thinking in your classroom. Here is one possible order for class discussion:

- use double number lines to find the price at each store and subtract to find the difference
- (misconception) add the discount percents for Store B and find the discounted price as 80% off the original price
- use equations to find each sale price, including two steps to find Store B, and then subtract
- use equations to find each sale price, including calculating multiple percents at one time for Store B, and then subtract to find the difference

Facilitate Whole Class Discussion

Call on students to share selected strategies. As they listen, remind students that one way to agree and build on ideas is to add details to help listeners understand more about the strategy or solution.

Guide students to **Compare and Connect** the representations. Ask students to take individual think time and then turn and talk to a partner to confirm their understanding of the strategies.

ASK How did the strategy [student name] used differ from the strategy [student name] used?

LISTEN FOR One solved the problem by finding all the discounts and subtracting them from the original prices, and one solved the problem by finding the sale prices rather than the discounts.

Model It

If students presented these models, have students connect these models to those presented in class.

If no student presented at least one of these models, have students first analyze key features of the models, and then connect them to the models presented in class.

ASK How does each Model It find the Store B price?

LISTEN FOR Both models take into account two rounds of discounts. The first Model It finds them in two steps, and the second Model It combines the steps into one equation.

For the model that finds percents separately,

prompt students to compare the discounts being offered at Store A and Store B.

What changes at Store B between discounts?

For the model that finds multiple percents at

once, prompt students to compare the final amount to the original amount.

• What percent of the original amount is 56?

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LESSON 20 SESSION 4
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> Explore different ways to solve a problem with multiple percents.

The guitar Francisca wants is on sale at two different stores. The original price of the guitar at both stores is \$160. At Store A, the guitar is 75% off. At Store B, the guitar is 50% off, with an additional 30% off the discounted price. At which store is the guitar less expensive? How much less expensive?

Model It

You can find each percent separately.

Store A

A discount of 75% is the same as paying 25% of the original price.

25% of 160 = (**0.25**)(160)

= 40

Store B

The 50%-off price is 100% - 50%, or 50%, of the original price.

- **50%** of 160 = (**0.5**)(160)
 - = 80

An additional 30% discount is applied to the 50%-off price of the guitar.

A discount of 30% is the same as paying 70% of the price.

70% of 80 = (**0.7**)(80)

= 56

Model It

You can find multiple percents at one time. **Store A**

25% of 160 = (0.25)(160)

= 40

Store B

70% of 50% of 160 = (0.7)(0.5)(160)





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DIFFERENTIATION | EXTEND

Deepen Understanding

SMP 3

Constructing Arguments About Percent Discounts

Prompt students to consider the equations in the second Model It.

ASK One person at Store A thinks that a 75% discount is the same as offering "70% off and an additional 5% off the sale price." Do you agree? Why or why not?

LISTEN FOR No. That would mean 70% off applies to the original price and 5% off applies to the discounted price. 75% off means that a 75% discount applies to the original price.

ASK Someone else at Store A wants to put up a sign that says "50% off and an additional 50% off the sale price." Is this the same as a 75% discount? Why or why not?

LISTEN FOR Yes. That would be the same as finding half of the original price and then finding half of that amount. You would find one quarter of the original price, which is the same as a 75% discount.

Generalize You can express two stacked percent discounts as a single percent discount, but it will not be the sum of the percents.

CONNECT IT

SMP 2, 4, 5, 6

Remind students that the percents and final amounts at each store are the same in each representation. Explain that they will now use those percents to reason about problems with multiple percents.

Before students begin to record and expand on their work in Model It, tell them that problem 3 will prepare them to provide the explanation asked for in problem 4.

Monitor and Confirm Understanding 1 – 2

- The price of \$40 at Store A is \$16 less than the price of \$56 at Store B.
- The initial discount on the guitar can be modeled as 0.5(160) and the additional discount as 0.7(0.5)(160).

Facilitate Whole Class Discussion

3 Look for understanding that multiplying the two percent discounts results in a 65% discount and/or that multiplying the percents paid results in 35% of the original price.

ASK How does the total discount compare to finding just a 50% or a 30% discount from the original price? How does the total discount compare to the sum of the original discounts?

LISTEN FOR Applying a 50% discount and an additional 30% discount results in a price that is (1 - 0.50)(1 - 0.30) = 0.35, or 35% of the original price. The total discount is greater than either of the discounts taken alone, but it is not equal to their sum.

Look for the idea that taking the markdowns is multiplicative, so the result of taking the discounts in reverse order is the same.

ASK How could you model 50% off 30% off \$160? How does that compare to modeling 30% off 50% off \$160?

LISTEN FOR You could model it as 0.5(0.7)(160). That has all the same factors as 0.7(0.5)(160), just in a different order.

Look for understanding that multiple markups and markdowns are not additive.

6 Reflect Have all students focus on the strategies used to solve the Try It. If time allows, have students discuss their ideas with a partner.

CONNECT IT

- Use the problem from the previous page to help you understand how to solve a problem involving multiple percents.
- 1 At which store is the sale price of the guitar less? How much less? Store A; \$16 less
- 2 Look at the Model Its. Why does the expression (0.7)(0.5)(160) represent the price of a guitar at Store B?
- Possible explanation: A 30% discount is the same as paying 70% of the price. A 50% discount is the same as paying 50% of the price. So, you can find 70% of 50% of the price with the expression (0.7)(0.5)(160).
- 3 Look at the second Model It. Explain why the sale price at Store B is 35% of the original price.

Possible explanation: Since (0.7)(0.5) = 0.35, finding 70% of 50% of an amount is the same as finding $\frac{35}{100}$, or 35%, of that amount.

 Would the amount Francisca would pay at Store B change if the sale were 30% off the price of the guitar, with an additional 50% off all sale prices? Explain.
 No; Possible explanation: You can represent 50% off 30% off 160 with the expression (0.5)(0.7)(160). That is equivalent to (0.7)(0.5)(160), so the end price will be the same.

5 Explain why a 75% discount followed by an additional 25% discount is not the same as a 100% discount.
 Possible explanation: The 25% discount is taken from the already discounted amount, not the original amount. Since 25% of 25% is 6.25%, the total discount is 75% + 6.25% = 81.25%, not 100%.

6 Reflect Think about all the models and strategies you have discussed today. Describe how one of them helped you better understand how to solve the Try It problem.

Responses will vary. Check student responses.

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DIFFERENTIATION | RETEACH or REINFORCE

Hands-On Activity

Use a model to understand multiple markdowns and markups.

If students are unsure about multiple markdowns, then use this activity to connect the concepts visually and kinesthetically.

Materials For each student: 10 unit cubes

- Display: A \$200 video game system is marked down by 20%. Then the store advertises an additional discount of 50%. How much does the game system cost?
- Have students use 10 unit cubes to represent \$200. Ask: *How many dollars does each cube represent? What percent of \$200 is that?* [\$20; 10%]
- Have students model the 20% discount. Ask: How many cubes are left? [8]
- Have students take away half their remaining cubes. Ask: *How many did you remove? Why does this model a 50% discount?* [4; I took away half of what was left.] Ask: *How is this different from finding 50% off the original cost?* [50% off the original cost would be 5 cubes, not 4.] Ask: *What is the final cost of the video game system?* [\$80]
- Prompt students to model the discounts again, this time with the 50% discount first, and compare the results.

Apply It

For all problems, encourage students to use a model to support their thinking. Allow some leeway in precision in drawing bar models or number lines, but encourage students to show their steps so that errors in procedure or understanding can be easily identified.

Students may also solve the problem by finding 70% of 125% of 120.

Students may also solve by finding 90% of 80% of 95% of 500 vegetable plant sprouts, multiplying all of the percentages together to obtain the equation (0.684)(500) = 342.

LESSON 20 SESSION 4

Apply It

- > Use what you learned to solve these problems.
- A bookstore has 120 science fiction books. It has 30% fewer mysteries than science fiction books. It has 25% more biographies than mysteries. How many biographies are in the bookstore? Show your work. Possible work: Mysteries: 70% of 120 = (0.7)(120)

= 84 Biographies: 125% of 84 = (1.25)(84) = 105

SOLUTION There are 105 biographies.



SOLUTION The sale price of the binoculars is \$56.70.

CLOSE EXIT TICKET

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- 9 Students' solutions should show an understanding that:
 - the markup is equivalent to 140% of the original price.
 - the sale price of 10% less is equivalent to 90% of the markup price.
 - combinations of markups and markdowns can be found using multiple percents at one time.

Error Alert If students use 60% for the first percent and 110% for the second percent, then point out the specific mentions of markup and reduced price (markdown) and have students review and correct their work.
Practice Solving Problems Involving Multiple Percents

Problem Notes

Assign **Practice Solving Problems Involving Multiple Percents** as extra practice in class or as homework.

- Students may find the discount amount, subtract it from the original price, and then multiply that result by 105.4%, for a total price at Store B of \$1,096.16. Then they subtract 1,075.08 from 1,096.16 to determine how much less the computer costs at Store A. *Medium*
- 2 Students may first calculate the number of teams after the 20% increase and use that result to calculate the number of teams after the 25% decrease. *Basic*

LESSON 20 SESSION 4

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N 4 Name:
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Practice Solving Problems Involving Multiple Percents

Study the Example showing how to solve a problem involving multiple percents. Then solve problems 1–5.

Example

Store A sells a computer for \$1,200. The computer is on sale for 15% off. The sales tax is 5.4%. What is the total cost of the computer?

A discount of 15% is the same as paying 85%.

Paying a 5.4% sales tax is the same as paying **105.4%**.

(1.054)(0.85)(1,200) = 1,075.08

The total cost is \$1,075.08.

Store B sells the same computer as in the Example for \$1,300. Store B offers a 20% discount on the computer. The tax rate is the same. Which store has the lower total price? How much lower? Show your work. Possible work:
 A discount of 20% is the same as paying 80% of the price.

Paying a 5.4% sales tax is the same as paying 105.4%

(1.054)(0.8)(1,300) = 1.054(1,040)

= 1,096.16

1,096.16 - 1,075.08 = 21.08

SOLUTION The computer costs \$21.08 less at Store A.

- A lacrosse league has 20 teams in its first year. The number of teams in the league increases by 20% in its second year. In the third year, the number of teams decreases by 25% from the second year. How many teams are in the league in the third year? Show your work. Possible work:
 A 20% increase is the same as 120%, or 1.2.
 - A 25% decrease is the same as 75%, or 0.75.

(0.75)(1.2)(20) = 18

SOLUTION The third year there are 18 teams in the league.

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3 Students may also solve the problem by LESSON 20 SESSION 4 multiplying 80%, 95%, and 105.75% by \$50 in a single equation. Medium Galeno wants to buy a video game at a store having a 20%-off storewide sale. The regular price of the video game is \$50. 4 Students should recognize that they are being Galeno also has a coupon for an extra 5% off the sale price of any video game. Sales tax on the video game is 5.75%. How asked to calculate the amount of Jesse's bonus, much does Galeno pay for the video game? Show your work. not the total amount of money Jesse receives. **Possible work:** The raise is calculated as a percent increase, but 50(0.8) = 40 the bonus is a percent of the new salary. To find 40(0.95) = 38 the amount of the bonus, they must multiply by 38(1.0575) = 40.185 0.1, not 1.1. Challenge 5 Students should understand that 50% more refers to a 50% increase, or 150% of the original SOLUTION _____Galeno pays \$40.19. amount, and 110% more refers to a 110% increase, or 210% of the original amount. 4 Jesse's starting salary is \$30,000 a year. He gets a 3% raise after his first year. Then he gets a 10% bonus on his second-year salary. How much is Jesse's bonus? Challenge Show your work. **Possible work:** 30,000(1.03)(0.1) = 3,090

SOLUTION Jesse's bonus is \$3,090.

S Volunteers collect and remove litter from a park. They collect 20 cans. They also collect 50% more glass bottles than cans and 110% more plastic bottles than glass bottles. How many plastic bottles do the volunteers collect? Show your work. Possible work:

150% of 20 = (1.5)(20)= 30210% of 30 = (2.1)(30)

= 63

SOLUTION Volunteers collect 63 plastic bottles.

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Use with Session 5 Apply It

Levels 1–3: Reading/Speaking

Help students interpret Apply It problem 6. First, read the problem as students follow along. Ask them to underline *less than \$22* and *budget*. Rephrase the problem. Say: *Jade has \$22 to spend*. Draw a number line and ask students to identify Jade's budget. Highlight the number line to show amounts Jade can spend.

DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Help students read the answer choices and underline *plus* and *off*. Remind students that *plus* is used for addition and % *off* makes an amount smaller. Ask students to write expressions they can use to find the cost in each answer choice. Then have students turn to partners and use *more than \$22* or *less than \$22* to describe the answer choices.

Levels 2-4: Reading/Speaking

Guide students to interpret Apply It problem 6. First, have students read the problem and underline *less than \$22* and *budget*. Explain that a *budget* is the maximum amount that a person wants to spend. Have students summarize the problem using:

• Jade's budget is \$_____. She wants to spend _____ than _____.

Explain that the answer choices show costs with some markups and markdowns. Then have students read the answer choices and use **Say It Another Way** to confirm understanding. Give students time to select their answers individually. Then have students explain their selections to partners.

Levels 3–5: Reading/Speaking

Support students as they interpret and discuss Apply It problem 6. First have partners read the problem and answer choices. Encourage them to use a dictionary or the Interactive Glossary to find the meanings of any unfamiliar terms. Then ask students to **Say It Another Way** to confirm understanding.

Ask students to select all the possible choices that are less than \$22. Then have students turn to partners and explain their reasoning. Remind students that there may be more than one way to reach an answer. Suggest students refer to their calculations or models to provide reasons why a possible answer is correct or incorrect.

Refine Solving Problems Involving Percents

Purpose

- **Refine** strategies for solving problems with percents.
- **Refine** understanding of how to solve a variety of real-world problems involving single and multiple percents.



WHY? Confirm students' understanding of solving a real-world problem involving multiple percents, identifying common errors to address as needed.

MONITOR & GUIDE

Before students begin to work, use their responses to the **Start** to determine those who will benefit from additional support. Use the **Error Analysis** table below to guide remediation.

Have all students complete the Example and problems 1–3, using Consider This and Pair/Share as appropriate. Observe and monitor their reasoning and guide or redirect students as needed.

Example	You can describe the
Ethan pays \$31.50 for a jacket. The amount includes a sales tax of 5%. What is the price of the jacket without the sales tax?	price without tax as 100%.
Look at how you could show your work using a diagram.	
p = price of the jacket without tax	
\$31.50	
Price of Jacket: 100% of p I ax: 5% of p	
105% = 1.05 1.05p = 31.50 SOLUTION The price of the jacket is \$30.	PAIR/SHARE How would the proble change if \$31.50 included a discount of 5% instead of a sales to of 5%?
oly It	•
The total simple interest owed on a loan of \$8,000 after 60 months is \$1,280. What is the yearly interest rate on the loan? Show your work. Possible work:	CONSIDER THIS . There are 12 months in a year.
60 months is the same as 5 years.	
r = yearly interest rate	
1,280 = (8,000)r(5)	
1,280 = 40,000r	
0.032 = r	PAIR/SHARE
0.052 - 1	How would you solve
	the problem if you wanted to know the

S	TART ERROR A	T ERROR ANALYSIS				
	If the error is	Students may	To support understanding			
	\$64	have only applied the discount.	Ask students to carefully read the question and apply both percents.			
	\$105.60	have multiplied the discounted price by 1.65 instead of 1.065.	Have students write 6.5% as a fraction and a decimal. Then have them express an increase of 6.5% using either a fraction or a decimal.			
	\$17.04	have found the discount or decrease of 20% by multiplying the cost by 20%, or 0.2, instead of 100% – 20%, or 0.8.	Ask students to discuss what a 20% discount means. Elicit from students that a 20% discount is the same as paying 80%. Then prompt students to find each percent change separately by writing equations.			

CONSIDER THIS ...

difference between the

amount the owner pays

and the selling price of

The profit is the

PAIR/SHARE

Does it matter if you find the markup price or

the sale price first?

CONSIDER THIS

How can you find the amount of the discount

for each item?

the bench.

Example

Guide students in understanding the Example. Ask:

- What percent represents the price of the jacket and the sales tax? What is this percent written as a decimal?
- If the original price of the jacket including the sales tax was known, how could you find the total amount Ethan pays for the jacket? How can you apply this process to the given information?
- How can you write and solve an equation to find the price of the jacket before sales tax? How else could you solve this problem?

Help all students focus on the Example and responses to the questions by prompting students to check that their explanations are clear by reminding them to pause and ask classmates for questions or comments.

Look for understanding that the amount Ethan pays is the sum of the price of the jacket and the sales tax, which is represented by 105% of the jacket price or 1.05 times the jacket price.

Apply It

- Students should realize that the amount of \$1,280 is the amount of interest earned after 5 years, so *l* = 1,280. *DOK 2*
- 2 Students may find each percent separately, using bar models or another visual model. Students should understand that the end-ofseason discount is 30% off the marked-up price, not 30% off \$50. DOK 2
 - **D** is correct. Students may write and solve an equation to find the percent discount for each item.
 - A is not correct. This answer may be the result of finding two items that have the same percent discount and not checking the remaining items.
 - **B** is not correct. This answer is the result of selecting the three items with the least sale prices.
 - **C** is not correct. This answer is the result of comparing the difference in the original price and sale price instead of comparing the percent of the discount.

DOK 3



2 The owner of a store buys wooden benches for \$50 each. She marks up the price by 75%. At the end of the season, she sells the remaining benches for 30% off. How much profit does the owner make on each bench at the end of the season? Show your work.

Possible work: Price with markup: 175% of 50 = 1.75(50) = 87.50 Sale price: 70% of 87.50 = 0.70(87.50) = 61.25 Profit: 61.25 - 50 = 11.25

SOLUTION The owner makes a profit of \$11.25.

3 Which items have the same percent discount?

ltem	Original Price	Sale Price
Sweater	\$40	\$32
Shorts	\$30	\$24
Jeans	\$50	\$40
Shirt	\$45	\$35

A sweater and shorts only

- B sweater, shorts, and shirt only
- **C** jeans and shirt only
- **D** sweater, shorts, and jeans only

Elias chose C as the correct answer. How might he have gotten that answer? Possible answer: Elias might have compared the difference in the original price and sale price instead of comparing the percent of the discount.

> PAIR/SHARE How can you check your answer?

GROUP & DIFFERENTIATE

Identify groupings for differentiation based on the **Start** and problems 1–3. A recommended sequence of activities for each group is suggested below. Use the resources on the next page to differentiate and close the lesson.

Approaching Proficiency

- **RETEACH** Hands-On Activity
- REINFORCE Problems 5, 6, 7

Meeting Proficiency

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• **REINFORCE** Problems 4–8

Extending Beyond Proficiency

- **REINFORCE** Problems 4–8
- **EXTEND** Challenge

Have all students complete the **Close: Exit Ticket**.

Resources for Differentiation are found on the next page.

Refine Solving Problems Involving Percents

Apply It

- Students may note that an increase of 20% is the same as 100% + 20% = 120%, or 1.2. They can continue to multiply the amount of time from the previous day by 1.2 until it exceeds 60 seconds, or 1 minute. DOK 3
- 5 Students should realize that \$37.80 is the price of the ticket including sales tax. They may draw a bar model to represent the problem and use it to help them write an equation. **DOK 2**
- A is correct. 15% off is the same as paying 85% of \$25, or \$21.25.
 - **D** is correct. To find the cost with the shipping fee, multiply 20.45 by 1.05. The total cost is \$21.47.
 - **F** is correct. The price can be modeled as 0.95(0.7)(32), or \$21.28.
 - **B** is not correct. A 30% discount on \$32 would result in a cost of \$22.40.
 - **C** is not correct. A 15% shipping fee requires multiplying the cost by 1.15, yielding \$22.43.
 - **E** is not correct. Applying the discount and the shipping fee gives 25(0.85)(1.05), or \$22.31, which is too expensive.

DOK 2

On the first day of training, Aretha holds a plank position for 30 seconds. She increases her time by 20% each day. What is the first day on which Aretha holds a plank for over a minute? Show your work. Possible work:





5 Gabriel pays \$37.80 for a ticket to a show. The amount includes an 8% sales tax. What is the price of the ticket without sales tax? Show your work. Possible work:

SOLUTION Day 5 is the first day Aretha holds a plank for over a minute.

p represents the price of the ticket.

So, 108% of the price, *p*, is 37.80.

1.08p = 37.80p = 35

SOLUTION The price of the ticket is \$35.

6 Jade wants to spend less than \$22 on a board game. Which of the following prices are in Jade's budget? Select all that apply.

(A) 15% off \$25

- **B** 30% off \$32
- C \$19.50 plus a 15% shipping fee
- **D** \$20.45 plus a 5% shipping fee
- E 15% off \$25 plus a 5% shipping fee
- **F** 30% off \$32 plus an additional 5% off the discounted price

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DIFFERENTIATION

RETEACH



Hands-On Activity Model discounts.

Students approaching proficiency with solving a real-world problem involving a discount will benefit from using base-ten blocks to model the discount.

Materials For each pair: base-ten blocks (2 hundreds flats, 20 tens rods, 20 ones units)

- Display: A printer costs \$120. Now it is on sale for 20% off. How much does the printer cost?
- Instruct students to model the 120 using the base-ten blocks. [1 flat and 2 tens rods] Explain that these base-ten blocks represent 100% of the cost of the printer, \$120.
- Ask: What dollar amount does each block represent? [The flat represents \$100, and each rod represents \$10.]
- Ask: What fraction is the same as 20%? $\left\lfloor \frac{1}{5} \right\rfloor$ If students need help seeing this, draw a number line or bar model to demonstrate. Instruct students to make 5 equal piles of the base-ten blocks in their model, trading the flat for tens rods and tens rods for ones units as needed.
- Ask: What blocks are in each pile? [2 tens rods and 4 ones units] Ask: What dollar amount does each pile represent? [\$24]
- Ask: How can you demonstrate the discount? [Remove one of the five piles.]
- Explain that removing one pile represents 20% off, or subtracting \$24. Ask: *What do the remaining four piles represent?* [The sale price, which is $4 \times 24 , or \$96]
- If time permits, repeat the activity for a printer that costs \$210 and is on sale for 30% off.

LESSON 20 | SESSION 5 Refine

Students may use the simple interest formula to find the amount of interest earned in 1 year. Then they may multiply that amount by 6 and add it to the initial deposit. DOK 2

Students may subtract the amount Nikia pays with the discount, \$160, from the amount she would have paid without the discount, \$200. Then they may find what percent 40 is of 200 to find the percent discount on her total purchase. DOK 2

CLOSE EXIT TICKET

9 Math Journal Look for understanding that 108% of 2 can be found by multiplying 2 by 1.08 and that 1.08% of 200 can be found by multiplying 200 by 0.0108.

Error Alert If students say that 1.08% of 200 is greater because 200 is greater than 2, then have them look at the percent carefully. Ask: *Is 108% the same as 1.08%?* [No]

End of Lesson Checklist

INTERACTIVE GLOSSARY Support students by suggesting they work with a partner to describe each variable in the simple interest formula.

SELF CHECK Have students review and check off any new skills on the Unit 5 Opener.

LESSON 20 SESSION 5



= 680 + (680)(0.035)(6)

- = 680 + 142.80
- = 822.80

SOLUTION ______ There will be \$822.80 in Anne's account after 6 years.

8 A store has a sale. Customers can buy one item at full price and take 50% off the cost of a second item with a lesser price. Nikia buys one item with a price of \$80 and another item with a price of \$120. With the sale, what percent discount does Nikia receive on her total purchase? Show your work. Possible work: The original cost of the items is 80 + 120 = 200.

The original cost of the items is $30 \pm 120 = 200$.

She can take 50% off the item with the lesser price.

50% off \$80 = (0.5)(80) = 40

So, she saves \$40 off the original cost.

```
\frac{40}{200} = \frac{20}{100}
```

SOLUTION ______ The percent discount on Nikia's total purchase is 20%.

 Math Journal Is 108% of 2 greater than, less than, or equal to 1.08% of 200? Explain your reasoning.
 equal to; Possible explanation:

```
108% of 2: 1.08(2) = 2.16
```

1.08% of 200: 0.0108(200) = 2.16

Since 2.16 = 2.16, 108% of 2 is equal to 1.08% of 200.

End of Lesson Checklist

INTERACTIVE GLOSSARY Find the entry for *simple interest*. Add two important

things you learned about simple interest in this lesson.
SELF CHECK Go back to the Unit 5 Opener and see what you can check off.

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REINFORCE



Problems 4–8

Solve real-world problems involving multiple percents.

Students meeting proficiency will benefit from additional work with single and multiple percents by solving problems in a variety of formats.

- Have students work on their own or with a partner to solve the problems.
- Encourage students to show their work.

EXTEND



Solve a credit card problem.

Students extending beyond proficiency will benefit from solving a problem involving a credit card balance.

- Have students work with a partner to solve this problem: Tara used a credit card to pay for a jacket that costs \$150. She did not purchase any other items using the credit card. The credit card company charges 25% interest each month on the remaining balance. Tara pays \$40 per month after the monthly interest has been applied. How long will it take Tara to pay off the balance? How much does she pay in all?
- Some students may make a table to show the information.
- Repeat, this time with a jacket that costs \$200, a 20.5% monthly interest charge, and a \$50 monthly payment.

PERSONALIZE

i-Ready[®]

Provide students with opportunities to work on their personalized instruction path with *i-Ready* Online Instruction to:

- fill prerequisite gaps.
- build up grade-level skills.