LESSON 1 SESSION 1

Explore Scale Drawings

Previously, you learned about ratios and rates. In this lesson, you will learn about scale drawings.

Use what you know to try to solve the problem below.



Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

LESSON 1 SESSION 1

CONNECT IT

1 Look Back Which of the triangles could be the same shape as $\triangle A$? Explain.

2 Look Ahead △D is a scale drawing of △A. A scale drawing of a figure is larger or smaller than the original figure but has the same exact shape. The scale is the ratio between the side lengths of the original figure and the side lengths of the scale drawing. To make a scale drawing of a figure, you can multiply each length of the original figure by a scale factor to get the corresponding length in the scale drawing. You can think of the scale factor as a unit rate.



a. Explain why the scale from $\triangle ABC$ to $\triangle DEF$ could be 1 : 3.

- **b.** Explain why the scale factor from $\triangle ABC$ to $\triangle DEF$ is 3.
- c. What is the value of x? Describe two different ways you can find it.

3 Reflect Yukio says the scale from $\triangle DEF$ to $\triangle ABC$ is 3 : 1. Is Yukio correct? Explain.

Prepare for Solving Problems Involving Scale

Think about what you know about ratios and unit rates. Fill in each box. Use words, numbers, and pictures. Show as many ideas as you can.



2 Limes are on sale. The sale price is 8 limes for \$2.00. Why could the unit rate be 4 or 0.25? A museum sells postcards of famous paintings.
The postcards must be the same shape as the painting.
Below are three options for the size of the postcard.



a. Which postcard could be the same shape as the painting? Show your work.



painting of George Washington

SOLUTION __

b. Check your answer to problem 3a. Show your work.

Develop Using Scale to Find Distances

> Read and try to solve the problem below.

This map is a scale drawing of the streets in a town. The scale from the town to the map is 500 ft to 2 cm. What are the actual distances, in feet, of the library, town hall, and train station from the school?





Math Toolkit double number lines, grid paper, ribbon, yarn



Ask: What did you do first to find the actual distances?

Share: I knew that ... so I decided to ...

Explore different ways to find actual lengths based on scale drawings.

This map is a scale drawing of the streets in a town. The scale from the town to the map is 500 ft to 2 cm. What are the actual distances, in feet, of the library, town hall, and train station from the school?



Model It

You can use a double number line to find the actual distances.



Model It

You can use a scale factor to find the actual distances.

Map (cm)	Actual (ft)
2	500
1	250

You can multiply each distance on the map by the scale factor, 250.

Library: 3.5 × **250**

Train Station: 4×250

Town Hall: 4.5 imes 250

CONNECT IT



1 How far, in feet, are the library, town hall, and train station from the school?

2 Look at the first Model It. How does the double number line show the scale factor you can use to find an actual distance, given the distance on the map?

3 Look at the second **Model It**. How is a scale factor like a unit rate?

4 Suppose you have a scale drawing and know its scale. How can you use the scale drawing to find an actual length?

5 The actual distance between the library and train station is 500 ft. Explain how you could find the distance between them in centimeters on the map.

6 Reflect Think about all the models and strategies you have discussed today. Describe how one of them helped you better understand how to solve the Try It problem.

Apply It

Use what you learned to solve these problems.

7 A scientist who studies insects enlarges a photograph of an elm leaf beetle. Every 2 in. in the photograph represents 8 mm on the actual beetle. The length of the beetle in the photograph is $1\frac{1}{2}$ in. The width of the beetle in the photograph is $\frac{3}{4}$ in. What are the length and width of the actual beetle? Show your work.



SOLUTION

8 On a map, 2 cm represents 30 mi. The actual distance between two towns is 75 mi. What is the distance between the towns on the map? Show your work.

SOLUTION

9 Tyrone makes a scale drawing of his backyard. The scale from the backyard to the drawing is 2 ft to 1 in. The width of the patio on the drawing is 8 in. What is the width of Tyrone's actual patio? Show your work.

Practice Using Scale to Find Distances

Study the Example showing how to use a scale drawing to find an actual distance. Then solve problems 1–5.

Example

Colin makes a scale drawing of his bedroom. Every inch in his drawing represents 10 feet in his actual bedroom. The drawing is 1.25 in. wide and 1.5 in. long. How wide and long is his actual bedroom?

You can use a scale factor to find the dimensions.

The scale from the drawing to the bedroom is 1 in. to 10 ft, so the scale factor

from the drawing to the bedroom is $\frac{10}{1}$, or 10.

 $1.25 \times 10 = 12.5$ $1.5 \times 10 = 15$

Colin's bedroom is 12.5 ft wide and 15 ft long.

1 A drawing of a basement uses the same scale as the Example. The basement is 28 ft wide and 35 ft long. How wide and long is the drawing? Show your work.

SOLUTION

2 Efia draws this scale drawing of two famous landmarks. Each inch in the drawing represents 400 ft on the actual landmark. Approximately how much taller is the actual Eiffel Tower than the actual Space Needle? Show your work.



The photo shows a small coin. The scale from the actual coin to the photo is 8 mm to 2 cm. In the photo, the distance across the coin is 3.25 cm. What is the distance across the actual coin? Show your work.



3.25 cm

SOLUTION

In a photograph, Alison stands next to her brother Caleb. Alison is 4 cm tall in the photograph. Her actual height is 60 in. Caleb is 3.2 cm tall in the photograph. What is his actual height? Show your work.

SOLUTION

5 Adoncia makes a scale drawing of the front of the Lincoln Memorial. She uses a scale of 15 ft in the monument to 1 in. in the drawing. The front of the monument is about 80 ft high and 200 ft long. Will Adoncia's drawing fit on an 8¹/₂ in.-by-11 in. sheet of paper? Explain.



Lincoln Memorial

Develop Using Scale to Find Areas

Read and try to solve the problem below.

A blueprint for the floor of a natural history museum is shown. The scale of the museum to the blueprint is 20 yd to 1 in. What is the area of the floor of the actual Great Hall?





Math Toolkit double number lines, grid paper, ribbon, yarn

DISCUSS IT

Ask: Why did you choose that strategy to find the area?

Share: The strategy I used was . . . because . . .

Explore different ways to find actual areas from a scale drawing.

A blueprint for the floor of a natural history museum is shown. The scale of the museum to the blueprint is 20 yd to 1 in. What is the area of the floor of the actual Great Hall?

Model It

You can use a table of equivalent ratios to find the actual dimensions.

Blueprint (in.)	1	$\frac{1}{4}$	<u>3</u> 4	1 <u>3</u>
Actual (yd)	20	5	15	35

The actual floor is 35 yd long and 15 yd wide.

Area of floor = 35×15

Model It

You can use a scale factor to find the area of the actual floor.

The scale from the blueprint to the museum is 20:1. So, the scale factor is 20.

Length of actual floor: 1.75 \times 20

Width of actual floor: 0.75 imes 20

Area = ℓw

= (1.75 × **20**)(0.75 × **20**)





CONNECT IT

- Use the problem from the previous page to help you understand how to find actual areas from a scale drawing.
- 1 What is the area of the floor of the actual Great Hall?
- 2 Look at the second Model It. What does the scale factor mean in this situation?
- 3 The scale from the museum to the drawing is 20 yd : 1 in. Another scale from the museum to the drawing is 400 yd² : 1 in.². Explain why.

4 The area of the Great Hall in the blueprint is $1\frac{5}{16}$ in.². Why does multiplying the blueprint area by the scale factor 20 not give the area of the floor of the actual Great Hall?

5 How is finding an actual area from the area in a scale drawing similar to finding an actual length from a length in a scale drawing? How is it different?

6 **Reflect** Think about all the models and strategies you have discussed today. Describe how one of them helped you better understand the relationship between an actual area and the area in a scale drawing.

Apply It

Use what you learned to solve these problems.

The scale from a playground to a scale drawing of the playground is 4 meters per centimeter. The length of the drawing of the playground is 5.3 cm and the width is 3.8 cm. What is the area of the actual playground? Show your work.



3.8 cm scale drawing of a playground

SOLUTION _

- 8 A square has an area of 144 ft². In a scale drawing of the square, each inch represents 6 ft. What is the area of the square in the drawing?
 - **A** 5,184 in.² **B** 24 in.²
 - **C** 6 in.^2 **D** 4 in.^2
- 9 Below is a scale drawing of the side of a ramp at a skateboard park. The scale from the drawing to the actual ramp is 2 cm to 12 in. What is the area of the actual side of the ramp? Show your work.



Practice Using Scale to Find Areas

Study the Example showing how to find an actual area from a scale drawing. Then solve problems 1–4.

Example

A camping supply store uses a tent as its logo. The store makes a sign with the logo on it. The scale from the actual logo to the logo on the sign is 1 in. to 2 ft. What is the area of the logo on the sign?

You can use the scale factor to find the actual dimensions. The scale factor from the logo to the sign is $\frac{2}{1}$, or 2.

The area of the logo on the sign is 4.5 ft².

Employees of the store in the Example wear shirts with the logo on the back. The scale from the original logo in inches to the shirt in inches is 1 : 8. What is the area of the logo on their shirts? Show your work.

SOLUTION

2 Dr. Gordon has a scale drawing of a building site. The drawing uses the scale 2 in. on the drawing for every 100 ft on the building site. Dr. Gordon marks a 6 in.-by-3.2 in. section of the drawing to show the section she will search. What is the area of the section she will search? Show your work.

SOLUTION

1.5 in.

1.5 in.

 $=\frac{1}{2}(1.5 \times 2)(1.5 \times 2)$

 $A=\frac{1}{2}bh$

= 4.5

3 An artist makes a scale drawing of a parallelogram-shaped sculpture. The scale is 10 cm on the drawing for every 8 m on the sculpture. What is the area of the scale drawing? Show your work.



SOLUTION

4) On a map, each centimeter represents 50 m.

a. The area of a rectangular park on the map is 6 cm². Tameka says that to find the area of the actual park, she can multiply the area of the park on the map by 2,500. Do you agree or disagree? Explain.

b. The area of a square sports arena is 10,000 m². What are the dimensions of the sports arena on the map? Show your work.

SOLUTION _

c. The area of a parallelogram on the map is 5 cm². What is the area of the actual parallelogram? Explain why you can find the area without knowing the dimensions of the parallelogram.

Develop Redrawing a Scale

Drawing

> Read and try to solve the problem below.

An architect makes a scale drawing of a recreation center. The scale from the actual center to the drawing is 6 m to 1 cm. Make another scale drawing of the pool using 5 m to 1 cm as the scale from the recreation center to the drawing.







Ask: How did you start to figure out the dimensions of the new drawing?

Share: I knew . . . so I . . .

> Explore different ways to make a new drawing with a different scale.

An architect makes a scale drawing of a recreation center. The scale from the actual center to the drawing is 6 m to 1 cm. The pool in the scale drawing is 2.5 cm wide and 5 cm long. Make another scale drawing of the pool using 5 m to 1 cm as the scale from the recreation center to the drawing.

Model It

You can use double number lines to find the dimensions.





Analyze It

You can reason about how the scales in each drawing are related.

Original Drawing Scale:

actual recreation center to drawing is 6 m to 1 cm

New Drawing Scale:

actual recreation center to drawing is 5 m to 1 cm

So, the scale from the original drawing to the new drawing is 6:5.

CONNECT IT



5 Reflect Think about all the models and strategies you have discussed today. Describe how one of them helped you better understand how to solve the Try It problem.

Apply It

> Use what you learned to solve these problems.

⁶ The drawing at the right is a plan for an apartment. The length of each square on the grid represents 1 cm. The scale from the apartment to the drawing is 8 ft to 1 cm. Draw another scale drawing of the apartment using a scale from the apartment to the drawing of 4 ft to 1 cm. Justify why your drawing is accurate.

7 The image at the right is a four-square court from a scale plan of a park. The scale from the actual court to the drawing is 8 ft to 1 cm. Draw another scale drawing of the court using a scale from the actual court to the new drawing of 12 ft to 1 cm. Justify why your drawing is accurate.









Practice Redrawing a Scale Drawing

Name:

Study the Example showing how to redraw a scale drawing using a different scale. Then solve problems 1–5.

Example

Luke and Isabella make scale drawings of a fountain. The scale from the fountain to Isabella's drawing is 6 m to 3 cm. The height of the fountain in Isabella's drawing is 5 cm. The scale from the fountain to Luke's drawing is 5 m to 2 cm. What is the height of the fountain in Luke's drawing?

Isabella's scale is 6 m to 3 cm, so the scale factor from her drawing to the actual

fountain is $\frac{6}{3}$, or 2.

Since 5 \times **2** is 10, the actual fountain is 10 m tall.

Luke's scale is 5 m to 2 cm.

The actual fountain is 10 m tall. Since 10 is 5 \times 2, the scale factor from Luke's drawing to the actual fountain is 2.

Since 2 \times 2 is 4, the height of the fountain in Luke's drawing is 4 cm.

1 The width of the fountain in Luke's drawing from the Example is 2 cm. What is the width of the fountain in Isabella's scale drawing? Show your work.

SOLUTION

2 The scale from an actual triangle to the drawing at the right is 30 : 1. The scale from the same triangle to Jorge's drawing of the triangle is 10 : 1. Is this drawing or Jorge's drawing larger? Explain your answer.



LESSON 1 SESSION 4

- 3 The drawing shows three octagons.
 - **a.** Is octagon *B* a scale drawing of octagon *A*? Explain.

b. Is octagon *C* a scale drawing of octagon *A*? Explain.

The length of each square on the grid represents 1 cm. The scale from an actual rectangle to the drawing is 24 in. to 1 cm. Make a new scale drawing where each centimeter represents 16 in. Label the length and width of your drawing in centimeters.

5 The perimeter of a pool is 150 m. The rectangle at the right is a scale drawing of the pool. The length of each square on the grid represents 1 cm. Draw another scale drawing of the pool using the scale 25 m to 2 cm. Explain why your drawing is accurate.







Refine Solving Problems Involving Scale

Complete the Example below. Then solve problems 1–9.



Apply It

 On one map of a town, the scale from the town to the map is 12 mi to 3 cm. The school is 2.5 cm from the grocery store on this map. On a different map of the same town, the scale from the town to the map is 12 mi to 2 cm. The school is 1.5 cm from the library on that map. Is the grocery store or library closer to the school? Show your work.

CONSIDER THIS ... How might finding a unit rate help you?

PAIR/SHARE

How else could you solve this problem?

LESSON 1 SESSION 5

2 Nautical flags are used to communicate on ships. Kimani uses the scale drawing at the right to sew a nautical Z flag. The scale from her flag to the drawing is 20 : 1. Each color is ¹/₄ of the flag. How many square inches of each color of fabric does Kimani need? Show your work.



3.6 cm

2.4 cm

CONSIDER THIS... There is more than one way to solve this problem.

SOLUTION

- The scale from a garden to this drawing is 8 ft to 1 cm. The scale from the same garden to another drawing is 2 ft to 1 cm. What are the lengths of the base and height of the garden in the other scale drawing?
 - **A** 0.6 cm and 0.9 cm
- **B** 1.2 cm and 1.8 cm
- **C** 4.8 cm and 7.2 cm **D** 9.6 cm and 14.4 cm

Kamal chose A as the correct answer. How might he have gotten that answer?

PAIR/SHARE

How can you convert the area to square feet?



In the other scale drawing, how many centimeters represent 8 ft?

PAIR/SHARE

What would be the length of the base and the height if the new scale were 1 cm for every 16 ft?

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4 The scale from a map to actual distance is 2.5 cm to 620 mi. The distance on the map between Chicago and Boston is about 3.5 cm. What is the approximate distance, in miles, between Chicago and Boston? Show your work.



SOLUTION _

- 5 On a scale drawing of the front of a square earring, each side of the earring is 3.2 cm. The scale from the earring to the drawing is 4 mm to 2 cm. What is the area of the front of the actual earring?
 - **A** 6.4 mm²

B 12.8 mm²

C 20.48 mm²

D 40.96 mm²

⁶ The image at the right is a scale drawing of a parking lot. The length of each square on the grid represents 0.5 cm. The actual parking lot has a perimeter of 84 m. Draw another scale drawing of the parking lot using a scale from the parking lot to the drawing of 2 m to 1 cm. Justify why your drawing is accurate.



Adela paints a mural of Mount Rushmore. She uses the scale 6 ft to 1 ft from the actual monument to the mural. On Adela's mural, how much wider is George Washington's mouth than his eye? Show your work.



SOLUTION _

8 The scale from a square park to a drawing of the park is 5 m to 1 cm. The actual park has an area of 1,600 m². What is the area of the drawing of the park? Show your work.

George Washington's head at Mt. Rushmore

SOLUTION

9 Math Journal Jada will draw two scale drawings of the same object on 1-cm grid paper. Each centimeter will represent a greater distance in the second drawing than in the first. Jada claims that the lengths in the second drawing will be longer than the lengths in the first drawing. Do you agree? Give an example of a scale and a measurement to support your answer.

End of Lesson Checklist

INTERACTIVE GLOSSARY Find the entry for *scale drawing*. Rewrite the definition in your own words.

SELF CHECK Go back to the Unit 1 Opener and see what you can check off.

Explore Proportional Relationship

Problems

TRY

Previously, you learned to identify and model proportional relationships. In this lesson, you will use proportional relationships to solve multi-step problems.

> Use what you know to try to solve the problem below.

A recipe for 4 cups of trail mix calls for $\frac{2}{3}$ cup of raisins. Aniyah has $1\frac{1}{2}$ cups of raisins. Does Aniyah have enough raisins to make 14 cups of the trail mix? If not, how many more cups of raisins does Aniyah need?

Math Toolkit double number lines, graph paper



Ask: How can you explain what the problem is asking in your own words?

Share: The problem is asking . . .

Learning Targets SMP 1, SMP 2, SMP 3, SMP 4, SMP 5, SMP 6

Identify the constant of proportionality (unit rate) in verbal descriptions of proportional relationships.

• Use proportional relationships to solve multistep ratio and percent problems.

LESSON 5 SESSION 1

CONNECT IT

- **1** Look Back Does Aniyah need more raisins? If so, how much more?
- 2 Look Ahead There is a proportional relationship between the total amount of trail mix and the amount of each ingredient. You can identify constants of proportionality from descriptions of proportional relationships.
 - **a.** A recipe for 3 cups of snack mix calls for $\frac{1}{2}$ cup of pretzels. You can describe this as 3 cups of snack mix for every $\frac{1}{2}$ cup of pretzels and as $\frac{1}{2}$ cup of pretzels for every 3 cups of snack mix. Why?
 - **b.** How could you describe the relationship in terms of 1 cup of pretzels? What is this constant of proportionality?
 - **c.** How could you describe the relationship in terms of 1 cup of snack mix? What is this constant of proportionality?
 - d. Why are the two constants of proportionality not the same?
 - **e.** Suppose you are making 4 cups of snack mix. You want to find how many cups of pretzels you need. Which constant of proportionality would you use? Why?
- **3 Reflect** How does knowing the constant of proportionality help you solve problems that involve proportional relationships?

Prepare for Proportional Relationship Problems

Think about what you know about proportional relationships and constants of proportionality. Fill in each box. Use words, numbers, and pictures. Show as many ideas as you can.



2 Hiroaki says that a constant of proportionality must be a whole number and cannot be a fraction or a decimal. Explain why Hiroaki is incorrect.

LESSON 5 SESSION 1

3 A formula for 6 gallons of light green paint uses $\frac{3}{8}$ gallon of white paint. Liam has $\frac{9}{16}$ gallon of white paint.

a. Does Liam have enough white paint to make 8 gallons of light green paint? If not, how much more does he need? Show your work.



SOLUTION

b. Check your answer to problem 3a. Show your work.

Develop Solving Multi-Step Ratio Problems

Read and try to solve the problem below.

Francisca and Elizabeth are participating in a walk-a-thon fundraiser. Each girl walks for 3 hours. How much will they raise together?





Walk-a-thon Fundraiser

every $\frac{1}{4}$ hour

every $\frac{1}{2}$ hour

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DISCUSS IT

Ask: What did you do first to find how much the girls raise together?

Share: First, I ...

Explore different ways to solve problems that involve proportional relationships.

Francisca and Elizabeth are participating in a walk-a-thon fundraiser. For every $\frac{1}{2}$ hour Francisca walks she raises \$25.50. For every $\frac{1}{4}$ hour Elizabeth walks she raises \$14.50. Each girl walks for 3 hours. How much will they raise together?

Model It

You can identify the constant of proportionality from a verbal description.

Francisca raises \$25.50 every 0.5 hour.

That means she raises \$51.00 per 1 hour.

constant of proportionality

Elizabeth raises \$14.50 every 0.25 hour. That means she raises \$58.00 per 1 hour.

constant of proportionality

Model It

You can use equations to model how much Francisca and Elizabeth raise per hour.

Let *m* represent dollars raised per hour. Let *h* represent time in hours.

You can use the equations to find how much money each girl raises in 3 hours.

Francisca	Elizabeth
<i>m</i> = 51 <i>h</i>	m = 58h
= 51(3)	= 58(3)



CONNECT IT

- Use the problem from the previous page to help you understand how to solve problems that involve proportional relationships.
- Look at the first Model It. How can you identify the constant of proportionality from a verbal description?

- 2 Look at the second Model It. Where is the constant of proportionality in each equation? How can you use it to find how much money each girl raises in 3 hours?
- 3 How much money do Francisca and Elizabeth raise together? You could use the equation m = (51 + 58)h to find the amount of money they raise together. Why?

4 You have identified constants of proportionality in tables, verbal descriptions, graphs, and equations. Why is this helpful when solving problems that involve proportional relationships?

5 **Reflect** Think about all the models and strategies you have discussed today. Describe how one of them helped you better understand how to solve the **Try It** problem.

Apply It

> Use what you learned to solve these problems.

6 At a certain bookstore, you get a \$5 coupon for every 4 books you buy. What is the least number of books you could buy to get \$15 in coupons? Show your work.

SOLUTION _____

7 Swim team members can race or dive. At a meet, 18 members race. The ratio of racers to divers is 6 : 2. How many members are on the team? Show your work.

SOLUTION _

8 Roberto runs 25 miles. His average speed is 7.4 miles per hour. He takes a break after 13.9 miles. How many more hours does he run? Show your work.



Practice Solving Multi-Step Ratio Problems

Study the Example showing how to solve problems involving proportional relationships. Then solve problems 1–4.

Example

Jasmine is making a spray cleaner. She mixes $1\frac{1}{2}$ cups water, $\frac{1}{4}$ cup white vinegar, and $\frac{1}{4}$ cup rubbing alcohol. How much white vinegar would Jasmine need to make 5 cups of the spray cleaner?

Find how much spray cleaner Jasmine makes.

Cups of cleaner: $1\frac{1}{2} + \frac{1}{4} + \frac{1}{4} = 2$

For every 2 cups of spray cleaner, Jasmine needs $\frac{1}{4}$ cup of white vinegar.

So, for 1 cup of spray cleaner, Jasmine needs $\frac{1}{4} \div 2 = \frac{1}{8}$ cup of white vinegar. Then find how much white vinegar Jasmine would need to make 5 cups of the spray cleaner.

$$5 \cdot \frac{1}{8} = \frac{5}{8}$$

Jasmine would need $\frac{5}{8}$ cup of white vinegar to make 5 cups of the spray cleaner.

- **a.** In the Example, what is the constant of proportionality for cups of vinegar to cups of spray cleaner?
- **b.** Jasmine is making more of the spray cleaner. She only wants to use 1 cup of water. How much of the spray cleaner will Jasmine make? Show your work.



 $\frac{1}{4}$ cup

cup

 $\frac{1}{2}$ cups

white vinegar

rubbing alcohol

water

2 Kadeem and Quinn both drive 25 miles. Kadeem drives at a constant speed of 50 miles an hour. Quinn drives at a constant speed of 75 miles an hour. Who takes longer to drive the 25 miles? How much longer? Show your work.

SOLUTION

Pilar and Ravi start at opposite ends of a 55-mile bike trail. They start riding their bikes toward each other at the same time. After 3 hours, they meet. Pilar rides 34 miles before they meet. What is Ravi's average speed? Show your work.

SOLUTION _

Riley finds a recipe for bubble solution that uses 1 cup water, ¹/₄ cup dish soap, and 1 tablespoon corn syrup. She uses 2 cups of dish soap. How much water should she use? Show your work.

Refine Solving Proportional Relationship Problems

Complete the Example below. Then solve problems 1–9.

Example In the student council election, 217 students vote. Uma receives 4 votes for every 3 that Paloma receives. How many more votes does Uma receive than Paloma? Look at how you could use proportional relationships. Find how many votes Uma, u, and Paloma, p, each receive. Uma: Paloma: $\frac{4}{7} = \frac{u}{217}$ $\frac{3}{7} = \frac{p}{217}$ $217\left(\frac{4}{7}\right) = u$ $217\left(\frac{3}{7}\right) = p$ 124 = u93 = pThen find the difference. **PAIR/SHARE** How can you check that your answer makes SOLUTION sense?

CONSIDER THIS

This problem involves more than one proportional relationship.

Apply It

1 Vinh has a recipe for a marinade. The recipe says to mix $\frac{3}{8}$ cup olive oil, $\frac{1}{4}$ cup soy sauce, and $\frac{1}{8}$ cup lime juice. How much olive oil does he need to make 9 cups of the marinade? Show your work.

CONSIDER THIS... A marinade is often used to add a different flavor when cooking.

PAIR/SHARE

Is there a different way you could solve this problem?

LESSON 5 SESSION 3

2 Neena is listening to a song. It has a consistent beat. She counts 11 beats in 5 seconds. What is the constant of proportionality for the relationship of beats to minutes? Show your work.

CONSIDER THIS... There are 60 seconds in 1 minute.

SOLUTION

- 3 Deyvi goes to a carnival with \$20.00. He spends \$2.00 to get in and the rest on ride tickets. Each ticket is \$1.50. How many tickets does Deyvi buy?
 - A 9 tickets
 - B 12 tickets
 - C 13 tickets
 - D 14 tickets

Bruno chose C as the correct answer. How might he have gotten that answer?

PAIR/SHARE Explain why you think your answer is reasonable.

CONSIDER THIS ...

Can you use the tickets without paying for admission?

PAIR/SHARE
How can you check your
answer?

4 Gears A and B turn together. Gear A turns 20 times when Gear B turns 30 times. When Gear A turns 60 times, how many times does Gear B turn? Show your work.

- 5 In a certain town, in 90 minutes $\frac{1}{2}$ inch of rain falls. It continues at the same rate for a total of 24 hours. Which of the following statements are true about the amount of rain in the 24-hour period? Select all that apply.
 - **A** The number of inches of rain is 16 times $\frac{1}{2}$.
 - **B** The number of inches of rain is 24 times $\frac{1}{2}$.
 - **C** The number of inches of rain is 24 times $\frac{1}{3}$.
 - **D** The number of inches of rain is $\frac{1}{2}$ times 36.
 - **E** The number of inches of rain is 24 times $1\frac{1}{2}$.
- 6 Benjamin is planning to make scale drawings of bat wings. He wants all of his drawings to use the same scale. He says he can use the equation d = 1.5w to find the length of a scale drawing, d, based on the length of the bat wing, w. What is the scale factor of bat wing length to scale drawing length? Explain.

LESSON 5 SESSION 3

7 Adsila and Carlos volunteer to fill gift boxes for soldiers serving overseas. Both work at a constant rate. They work together for 6 hours and fill 126 boxes. Adsila fills 9 boxes every hour. How many boxes does Carlos fill every hour?

8 Aimee is in charge of buying tickets for a trip to the movies. She can buy tickets at the theater or online. At the theater, 8 tickets cost \$60. Online, 6 tickets cost \$51. Aimee needs to buy 24 tickets. How much money can Aimee save by buying tickets at the lower price? Show your work.

SOLUTION

9 Math Journal Write a word problem that involves a proportional relationship and needs more than one step to solve. Show how to solve the problem.



End of Lesson Checklist

INTERACTIVE GLOSSARY Find the entry for proportional relationship. Rewrite the definition in your own words.

SELF CHECK Go back to the Unit 1 Opener and see what you can check off.



LESSON 20 SESSION 1

Last Year's Middle School Band Students



Explore Percents

Previously, you learned how to find the percent of a number. In this lesson, you will learn about solving problems that involve percents.

> Use what you know to try to solve the problem below.

Last year, a middle school band had 80 students. This year, the band has 120% of that number of students. How many students are in the band this year?

Math Toolkit double number lines, grid paper, hundredths grids



Ask: How did you get started finding the number of students?

Share: I got started by ...

Learning Target SMP 1, SMP 2, SMP 3, SMP 4, SMP 5, SMP 6 Use proportional relationships to solve multistep ratio and percent problems.

lacksquare

CONNECT IT

1 Look Back How many students are in the band this year? How do you know?

2 Look Ahead You can think of 120% as 100% + 20%. Similarly, you can think of 80% as 100% - 20%. Percents are used in calculating simple interest. You can owe interest on a loan or earn interest on a bank account or investment.

Simple interest formula: *I* = *Prt*

The amount of interest, *I*, is based on the principal, *P*, the interest rate, *r*, and the time you borrow or invest the money for, *t*. The principal is the starting amount. The rate is written as a decimal. For simple interest, time is measured in years.

- **a.** Suppose you borrow \$300 at a yearly, or annual, simple interest rate of 3.4% for 3 years. What values would you use for *P*, *r*, and *t*?
- **b.** Suppose you borrow \$400 at a yearly simple interest rate of 3% for 3 months. What values would you use for *P*, *r*, and *t*?
- **c.** A **markdown** decreases the cost of an item. A **markup** increases the cost. Suppose a store puts an item on sale for 25% off. Is that an example of a *markup* or a *markdown*?
- **d.** Often when you buy something, you pay a percent of the price as a **tax**. Suppose you pay a 7% tax on an item. What percent of the price of the item will you pay?
- Many people gives tips, or gratuities, for good service. Many salespeople earn commission on their sales. Jason earns a 9% commission on a \$1,000 sale. How much is Jason's commission?

3 **Reflect** Is it possible for a single price change to be both a markup and a markdown? Explain your thinking.

Prepare for Solving Problems Involving Percents

1 Think about what you know about percents. Fill in each box. Use words, numbers, and pictures. Show as many ideas as you can.



Isabel says that 0.02 can be expressed as 20%. Is she correct? Explain why or why not.

- 3 Last year, a rapper performed 40 times. This year, the rapper performs 125% of that number of times.
 - **a.** How many times does the rapper perform this year? Show your work.



SOLUTION

b. Check your answer to problem 3a. Show your work.

Develop Finding Simple Interest

> Read and try to solve the problem below.

Dario borrows \$12,000 to buy a car. He borrows the money at a yearly, or annual, simple interest rate of 4.2%. How much more interest will Dario owe if he borrows the money for 5 years instead of 1 year?







Math Toolkit double number lines, grid paper



Ask: Why did you choose that strategy to find the interest?

Share: I knew . . . so I . . .

Explore different ways to find simple interest.

Dario borrows \$12,000 to buy a car. He borrows the money at a yearly, or annual, simple interest rate of 4.2%. How much more interest will Dario owe if he borrows the money for 5 years instead of 1 year?

Model It

You can use the relationship between time and interest.

After one year, Dario will owe 4.2% of \$12,000 in interest.

0.042(12,000) = 504

Year	Total Interest	
1	\$504	
2	\$1,008	
3	\$1,512	
4	\$2,016	
5	\$2,520	

Model It

You can use the simple interest formula to find the interest.

l = *Prt*

```
I = interestP = principalr = interest ratet = time (in years)1 Year5 YearsI = PrtI = Prt= (12,000)(0.042)(1)= (12,000)(0.042)(5)= 504(1)= 504(5)= 504= 2,520
```

After 1 year, Dario will owe \$504 in interest.

After 5 years, Dario will owe \$2,520 in interest.

CONNECT IT

Use the problem from the previous page to help you understand how to find simple interest.

1 Look at the table in the first Model It. How does the interest change over time?

2 How much more interest will Dario owe for 5 years than for 1 year? How does this difference compare to the amount of interest Dario would owe for borrowing the money for 4 years?

3 Look at the second **Model It**. Which values stay the same when you use the formula to find the interest for 1 year and 5 years? Which values change?

4 How does the formula *I* = *Prt* show a proportional relationship between simple interest and time?

5 The total amount Dario owes is the sum of the interest and the principal. Is the relationship between total amount owed and time proportional? Explain.

6 **Reflect** Think about all the models and strategies you have discussed today. Describe how one of them helped you better understand how to think about and find simple interest.

Apply It

- Use what you learned to solve these problems.
- Ava borrows \$600 to buy a bike at a yearly simple interest rate of 2.25%. Ava borrows the money for 3 years. How much does Ava pay in simple interest? How much does Ava pay in all? Show your work.



SOLUTION _____

8 Zhen borrows \$1,200. She borrows the money for 2 years and owes \$180 in simple interest. What is the yearly simple interest rate on Zhen's loan? Show your work.

SOLUTION _

9 A bank offers a savings account with a yearly simple interest rate of 2%. Suppose you deposit \$550 into a savings account. How much simple interest will you earn in 4 years? In 4 years and 6 months? Show your work.

Practice Finding Simple Interest

Study the Example showing how to use the simple interest formula. Then solve problems 1–6.

Example

Pablo deposits \$750 into a bank account. The account earns yearly simple interest at a rate of $3\frac{1}{2}$ %. How many years will it take Pablo to earn a total of \$105 in simple interest? Use the simple interest formula and solve for *t*. P = \$750, r = 3.5%, and l = \$105l = Prt105 = (750)(3.5%)(t)105 = (750)(0.035)(t)105 = 26.25t4 = tIt will take 4 years for Pablo to earn \$105 in interest.

Suppose you deposit \$1,200 into a bank account. The account earns yearly simple interest at a rate of $1\frac{3}{4}$ %. How many years will it take to earn a total of \$126 in simple interest? Show your work.

SOLUTION

2 Conan borrows \$3,000 at a yearly simple interest rate of 1.6% for 2 years.

He owes ______ in interest. He needs to pay back ______ in all.

Vocabulary simple interest

a percent of an amount that is borrowed or invested. Jamila deposits \$800 in an account that earns yearly simple interest at a rate of 2.65%. How much money is in the account after 3 years and 9 months? Show your work.

Centre Control 1000 Control 100

SOLUTION

4 Carmela borrows \$400 and will pay 5.25% yearly simple interest. How much more interest will Carmela owe if she borrows the money for 4 years instead of 2 years? Show your work.

SOLUTION

Ellie borrows money at a yearly simple interest rate of $6\frac{1}{2}$ %. After 4 years, Ellie owes \$39 in interest. How much money did Ellie borrow? Show your work.

SOLUTION _

6 Lilia borrows \$400 at a yearly simple interest rate of 6%. She writes the expression 400 + (0.6 × 400) to represent the total amount of money she will pay back for borrowing the money for 1 year. Is Lilia's expression correct? Explain your answer and determine the amount of money Lilia will need to pay back after 1 year.

Develop Solving Problems Involving a Single Percent

Read and try to solve the problem below.

Cyrus is hosting a dinner to celebrate Nowruz, the Persian New Year. His groceries cost \$150 before he uses a 10%-off coupon. He also orders \$60 worth of flowers. Sales tax on the flowers is 6.25%. What is the total amount Cyrus spends?





Math Toolkit double number lines, grid paper



Ask: How would you explain what the problem is asking in your own words?

Share: The problem is asking . . .

.

Explore different ways to solve a problem with a single percent.

Cyrus is hosting a dinner to celebrate Nowruz, the Persian New Year. His groceries cost \$150 before he uses a 10%-off coupon. He also orders \$60 worth of flowers. Sales tax on the flowers is 6.25%. What is the total amount Cyrus spends?

Model It

You can draw bar models to find the percents.



Nowruz celebration



Cyrus pays 90% of the original cost for groceries.



Cyrus pays 106.25% of the original cost for flowers.

Model It

You can write equations to solve the problem.

Cost of Groceries:

Cost of Flowers:

(100% - 10%)150 = (90%)150= (0.90)(150) (100% + 6.25%)60 = (106.25%)60

= (**1.0625**)(60)

Total Cost = Cost of Groceries + Cost of Flowers

= (0.90)(150) + (1.0625)(60)= 135 + 63.75

CONNECT IT

- Use the problem from the previous page to help you understand how to solve a problem with a single percent.
- 1 What is the total amount Cyrus spends?
- 2 Look at the **Model Its**. How do they show that you can multiply 150 by 0.90 to find the discounted cost of the groceries?

3 Hiroaki uses the expression a – 0.1a to represent a 10% discount on an amount a. Allen uses the expression 0.9a. Is Hiroaki's expression correct? Is Allen's? Explain.

4 Hiroaki uses the expression a + 0.05a to represent an amount increasing by 5%. Allen uses the expression 1.05a. Explain why both Hiroaki's and Allen's expressions are correct.

5 The expression (110)(0.80) can be used to find the sale price of an item that has an original price of \$110. By what percent is the original price marked down? How do you know?

6 Reflect Think about all the models and strategies you have discussed today. Describe how one of them helped you better understand how to solve the Try It problem.

Apply It

> Use what you learned to solve these problems.

Alanna earns a commission of 8% on her sales. How much commission does Alanna earn on a sale of \$32,000? Show your work.

SOLUTION _____

8 Heidi's lunch costs \$12.50. Heidi wants to leave a tip of 18%. How much money does Heidi need to pay for lunch, including the tip?

- **A** \$2.25
- **B** \$10.25
- **C** \$14.75
- **D** \$22.75

9 Before hibernation, a bear weighs 990 pounds. Its weight decreases by 32% during hibernation. How much does the bear weigh when it comes out of hibernation? Show your work.



Practice Solving Problems Involving a Single Percent

Study the Example showing how to solve a problem with a percent. Then solve problems 1–5.

Ex Th W	cample e sales tax at a hotel is 18%. A standard room c nat is the total cost of a standard room?	osts \$98 before tax.
	Total Cost	
	Price of Room: 100% of \$98	Tax: 18% of \$98
	118% of 98 = (1.18)(98)	
	= 115.64	
Th	e total cost of a standard room is \$115.64.	

The hotel in the Example offers a double room for \$20 more than a standard room, before tax. What is the total cost of a double room, including tax? Show your work.

SOLUTION

Darnell wants to limit his screen time. Last week, he spent an average of 3¹/₂ h on his phone each day. This week, he reduces his screen time by 5%. To the nearest hour, how much time does Darnell spend on his phone this week? Show your work.

Vocabulary markup

an amount added to the cost of an item to determine the final price. The amount added is often a percent of the cost.

tax

a percent of income or of the cost of goods or services paid to the government.

Alexis is training for her next 5k race. Her current 5k race time is 40 min. She wants to decrease her time by 20%. What does Alexis want her next 5k race time to be? Show your work.

SOLUTION _

4 An art store manager buys and sells art supplies.

a. The store manager buys easels for \$10.20 each. He marks up the cost by 75% to get the selling price. What is the selling price of each easel? Show your work.

SOLUTION _

b. The original price of a jar of paint is \$1.20. The store manager gets a 10% discount on orders of at least 50 jars of paint. How much does the store manager pay for 50 jars of paint? Show your work.



SOLUTION _

A store is having a buy-one-get-one-30%-off sale. During the sale, Christopher buys two pairs of shoes that each have a regular price of \$48. How much does Christopher pay for the two pairs of shoes? Show your work.

Develop Solving Problems Involving Multiple Percents

> Read and try to solve the problem below.

The guitar Francisca wants is on sale at two different stores. The original price of the guitar at both stores is \$160. At which store is the guitar less expensive? How much less expensive?



Math Toolkit double number lines, grid paper

STORE B

STORE A

75%

GUITARS



DISCUSS IT

Ask: How is each percent represented in your solution?

Share: In my solution each percent is represented by . . .

Explore different ways to solve a problem with multiple percents.

The guitar Francisca wants is on sale at two different stores. The original price of the guitar at both stores is \$160. At Store A, the guitar is 75% off. At Store B, the guitar is 50% off, with an additional 30% off the discounted price. At which store is the guitar less expensive? How much less expensive?

Model It

You can find each percent separately.

Store A

A discount of 75% is the same as paying 25% of the original price.

$$25\%$$
 of $160 = (0.25)(160)$

= 40

Store B

The 50%-off price is 100% - 50%, or 50%, of the original price.

50% of 160 = (0.5)(160)

= 80

An additional 30% discount is applied to the 50%-off price of the guitar.

A discount of 30% is the same as paying 70% of the price.

70% of 80 =
$$(0.7)(80)$$

= 56

Model It

You can find multiple percents at one time.

Store A

25% of 160 = (0.25)(160)

Store B

70% of **50%** of 160 = (0.7)(0.5)(160)

$$= (0.7)(80)$$

= 56



CONNECT IT

- Use the problem from the previous page to help you understand how to solve a problem involving multiple percents.
- 1 At which store is the sale price of the guitar less? How much less?
- 2 Look at the **Model Its**. Why does the expression (0.7)(0.5)(160) represent the price of a guitar at Store B?
- 3 Look at the second **Model It**. Explain why the sale price at Store B is 35% of the original price.
- Would the amount Francisca would pay at Store B change if the sale were 30% off the price of the guitar, with an additional 50% off all sale prices? Explain.

5 Explain why a 75% discount followed by an additional 25% discount is not the same as a 100% discount.

6 **Reflect** Think about all the models and strategies you have discussed today. Describe how one of them helped you better understand how to solve the **Try It** problem.

Apply It

Use what you learned to solve these problems.

7 A bookstore has 120 science fiction books. It has 30% fewer mysteries than science fiction books. It has 25% more biographies than mysteries. How many biographies are in the bookstore? Show your work.

SOLUTION _

8 Members of a community garden grow 500 vegetable plant sprouts. They donate 10% of the sprouts to a school. They sell 20% of the remaining sprouts to a local park. They plant 5% of those left in a greenhouse. Then they plant the rest of the sprouts outside. How many sprouts do the members plant outside? Show your work.



SOLUTION

9 A store manager buys binoculars for \$45 each. He marks up the cost by 40% to get the store price. Then the store has a sale and the store price is reduced by 10%. What is the sale price of the binoculars? Show your work.

Practice Solving Problems Involving Multiple Percents

Study the Example showing how to solve a problem involving multiple percents. Then solve problems 1–5.

Example

Store A sells a computer for \$1,200. The computer is on sale for 15% off. The sales tax is 5.4%. What is the total cost of the computer?

A discount of 15% is the same as paying 85%.

Paying a 5.4% sales tax is the same as paying 105.4%.

(1.054)(0.85)(1,200) = 1,075.08

The total cost is \$1,075.08.

Store B sells the same computer as in the Example for \$1,300. Store B offers a 20% discount on the computer. The tax rate is the same. Which store has the lower total price? How much lower? Show your work.

SOLUTION

2 A lacrosse league has 20 teams in its first year. The number of teams in the league increases by 20% in its second year. In the third year, the number of teams decreases by 25% from the second year. How many teams are in the league in the third year? Show your work.

Galeno wants to buy a video game at a store having a 20%-off storewide sale. The regular price of the video game is \$50. Galeno also has a coupon for an extra 5% off the sale price of any video game. Sales tax on the video game is 5.75%. How much does Galeno pay for the video game? Show your work.



SOLUTION

Jesse's starting salary is \$30,000 a year. He gets a 3% raise after his first year. Then he gets a 10% bonus on his second-year salary. How much is Jesse's bonus? Show your work.

SOLUTION _

5 Volunteers collect and remove litter from a park. They collect 20 cans. They also collect 50% more glass bottles than cans and 110% more plastic bottles than glass bottles. How many plastic bottles do the volunteers collect? Show your work.

LESSON 20 SESSION 5

Refine Solving Problems Involving Percents

Complete the Example below. Then solve problems 1–9.

Example Ethan pays \$31.50 for a jacket. The amount includes a sales tax of 5%. What is the price of the jacket without the sales tax?			CONSIDER THIS You can describe the
			price without tax as 100%.
Look at how you could show your work using	a diagram.		
<i>p</i> = price of the jacket without tax			
\$31.50			
Price of Jacket : 100% of p	Tax : 5% of p		
The amount Ethan pays, \$31.50, is equal t the jacket, <i>p</i> .	to 105% of the price of		
105% = 1.05			/
1.05p = 31.50			PAIR/SHARE How would the problem
SOLUTION			included a discount of 5% instead of a sales tax of 5%?

Apply It

1 The total simple interest owed on a loan of \$8,000 after 60 months is \$1,280. What is the yearly interest rate on the loan? Show your work.

CONSIDER THIS... There are 12 months in a year.

PAIR/SHARE

How would you solve the problem if you wanted to know the monthly interest rate?

2 The owner of a store buys wooden benches for \$50 each. She marks up the price by 75%. At the end of the season, she sells the remaining benches for 30% off. How much profit does the owner make on each bench at the end of the season? Show your work.

CONSIDER THIS...

The profit is the difference between the amount the owner pays and the selling price of the bench.

PAIR/SHARE

Does it matter if you find the markup price or the sale price first?

SOLUTION _

3 Which items have the same percent discount?

ltem	Original Price	Sale Price
Sweater	\$40	\$32
Shorts	\$30	\$24
Jeans	\$50	\$40
Shirt	\$45	\$35

- **A** sweater and shorts only
- **B** sweater, shorts, and shirt only
- **C** jeans and shirt only
- **D** sweater, shorts, and jeans only

Elias chose C as the correct answer. How might he have gotten that answer?

CONSIDER THIS ...

How can you find the amount of the discount for each item?

PAIR/SHARE How can you check your answer? On the first day of training, Aretha holds a plank position for 30 seconds. She increases her time by 20% each day. What is the first day on which Aretha holds a plank for over a minute? Show your work.



SOLUTION

5 Gabriel pays \$37.80 for a ticket to a show. The amount includes an 8% sales tax. What is the price of the ticket without sales tax? Show your work.

SOLUTION _

6 Jade wants to spend less than \$22 on a board game. Which of the following prices are in Jade's budget? Select all that apply.

A 15% off \$25

- **B** 30% off \$32
- C \$19.50 plus a 15% shipping fee
- **D** \$20.45 plus a 5% shipping fee
- E 15% off \$25 plus a 5% shipping fee
- F 30% off \$32 plus an additional 5% off the discounted price

Anne deposits \$680 in an account that pays 3.5% yearly simple interest. She neither adds more money nor withdraws any money. How much will be in Anne's account after 6 years? Show your work.

SOLUTION _

8 A store has a sale. Customers can buy one item at full price and take 50% off the cost of a second item with a lesser price. Nikia buys one item with a price of \$80 and another item with a price of \$120. With the sale, what percent discount does Nikia receive on her total purchase? Show your work.

SOLUTION _

9 Math Journal Is 108% of 2 greater than, less than, or equal to 1.08% of 200? Explain your reasoning.

End of Lesson Checklist

INTERACTIVE GLOSSARY Find the entry for *simple interest*. Add two important things you learned about simple interest in this lesson.

SELF CHECK Go back to the Unit 5 Opener and see what you can check off.