Overview | Interpret Median and Interquartile Range in Box Plots

STANDARDS FOR MATHEMATICAL PRACTICE (SMP)

SMP 1, 2, 3, 4, 5, and 6 are integrated into the Try-Discuss-Connect routine.*

This lesson provides additional support for:

5 Use appropriate tools strategically.

6 Attend to precision.

* See page 1q to learn how every lesson includes these SMP.

Objectives

Content Objectives

- Understand median as a measure of center.
- Calculate the median and quartiles of a data set.
- Construct box plots and use the IQR to measure variability of a data set.
- Interpret the median and IQR in a given context.

Language Objectives

- Explain in writing why the median can be used as a measure of center.
- Summarize a data set using lesson vocabulary, including *lower quartile (Q1), median (Q2),* and *upper quartile (Q3).*
- Describe the variability of a data set by explaining how box plots and the IQR represent a data distribution in whole-class discussion.
- Demonstrate understanding of word problems by explaining how the median and IQR connect to the problem context.

Prior Knowledge

- Make visual representations of data sets.
- Describe a data set by its center, range, and shape.
- Interpret data sets.

Vocabulary

Math Vocabulary

box plot a visual display of a data set on a number line that shows the minimum, the lower quartile, the median, the upper quartile, and the maximum. The sides of the box show the lower and upper quartiles and the line inside the box shows the median.

interquartile range (IQR) the difference between the upper quartile and lower quartile.

lower quartile the middle number between the minimum and the median in an ordered set of numbers. The lower quartile is also called the 1st quartile or Q1.

measure of center a single number that summarizes what is typical for all the values in a data set. Median is a measure of center.

measure of variability a single number that summarizes how much the values in a data set vary. Interquartile range is a measure of variability.

median the middle number, or the halfway point between the two middle numbers, in an ordered set of values.

upper quartile the middle number between the median and the maximum in an ordered set of numbers. The upper quartile is also called the 3rd quartile or Q3.

Academic Vocabulary

consistent the same or similar in different situations.

Learning Progression

In Grade 5, students made line plots for data sets. They used measurements in data sets and interpreted data.

Earlier in Grade 6, students asked statistical questions about data and understood differences in data distribution. They displayed data distributions with dot plots and histograms and described the overall shape of a data distribution.

In this lesson, students display and describe data distributions, finding the median as a measure of center (a single number that is a summary of all the data values) and the IQR as a measure of variability (a single number that describes how much the values vary). They construct box plots to visualize the distribution. Given different contexts, they use the median and IQR in their interpretation of the data set.

Later in Grade 6, students will use the mean and mean absolute deviation to describe data. They will determine which measure of center and variability is better in different contexts, and they will summarize data sets with all of the measures they will have learned.

In Grade 7, students will use measures of center and variability to describe random samples and populations.

Overview

Pacing Guide

Items marked with ↖ are available on the **Teacher Toolbox**.

	MATERIALS	DIFFERENTIATION
SESSION 1 **Explore** The Median of a Data Set (35–50 min)		
• **Start** (5 min) • **Try It** (5–10 min) • **Discuss It** (10–15 min) • **Connect It** (10–15 min) • **Close: Exit Ticket** (5 min) **Additional Practice** (pages 697–698)	**Math Toolkit** graph paper, number lines, sticky notes Presentation Slides	**PREPARE** Interactive Tutorial **RETEACH or REINFORCE** Hands-On Activity **Materials** For each pair: 11 sticky notes, 15 counters, Activity Sheet *Number Lines*
SESSION 2 **Develop** Finding the Median and Quartiles (45–60 min)		
• **Start** (5 min) • **Try It** (10–15 min) • **Discuss It** (10–15 min) • **Connect It** (15–20 min) • **Close: Exit Ticket** (5 min) **Additional Practice** (pages 703–704)	**Math Toolkit** graph paper, number lines, sticky notes Presentation Slides	**RETEACH or REINFORCE** Hands-On Activity **Materials** For each pair: scissors, Activity Sheet *1-Inch Grid Paper* **REINFORCE** Fluency & Skills Practice **EXTEND** Deepen Understanding
SESSION 3 **Develop** Using Box Plots and IQR to Describe Variability (45–60 min)		
• **Start** (5 min) • **Try It** (10–15 min) • **Discuss It** (10–15 min) • **Connect It** (15–20 min) • **Close: Exit Ticket** (5 min) **Additional Practice** (pages 709–710)	**Math Toolkit** graph paper, number lines, sticky notes Presentation Slides	**RETEACH or REINFORCE** Hands-On Activity **Materials** For each student: Activity Sheet *Number Lines* **REINFORCE** Fluency & Skills Practice **EXTEND** Deepen Understanding
SESSION 4 **Refine** Interpreting Median and Interquartile Range in Box Plots (45–60 min)		
• **Start** (5 min) • **Monitor & Guide** (15–20 min) • **Group & Differentiate** (20–30 min) • **Close: Exit Ticket** (5 min)	**Math Toolkit** Have items from previous sessions available for students. Presentation Slides	**RETEACH** Hands-On Activity **Materials** For each pair: 2 number cubes, Activity Sheet *Number Lines* **REINFORCE** Problems 4–8 **EXTEND** Challenge **PERSONALIZE** i-Ready
Lesson 31 Quiz ↖ or **Digital Comprehension Check**		**RETEACH** Tools for Instruction **REINFORCE** Math Center Activity **EXTEND** Enrichment Activity

Overview | Interpret Median and Interquartile Range in Box Plots

Connect to Culture

➤ **Use these activities to connect with and leverage the diverse backgrounds and experiences of all students. Engage students in sharing what they know about contexts before you add the information given here.**

SESSION 1 ■ □ □ □

Try It Ask students if they have a favorite yogurt flavor. Yogurt is produced by the bacterial fermentation of milk or other products, such as almond, soy, or coconut. The bacteria used to make yogurt are not the kind that make you sick, though; they help keep you healthy! They are actually good bacteria that live in your digestive system and help you in multiple ways. Eating yogurt may help prevent heart disease and osteoporosis. It may even improve your immune system and prevent you from getting sick!

SESSION 2 ■ ■ □ □

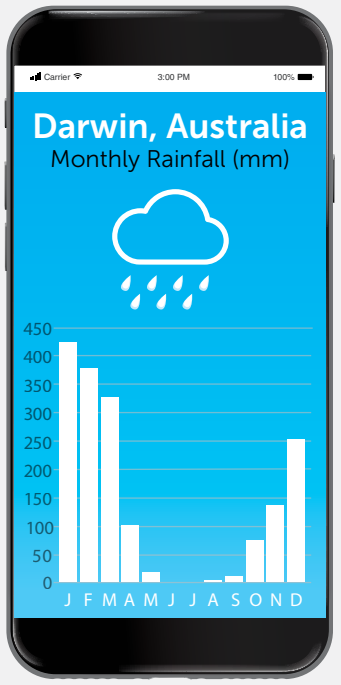
Try It Poll students to see how many prefer rainy days or sunny days. The city of Darwin, Australia, is home to an annual rainy season that extends from the month of November all the way through April. The lightning storms that highlight the beginning of the rainy season are so brilliant that locals observe them as if they are watching a fireworks show. Throughout the rainy season, thriving plants turn the landscape green and animals are better able to find food. During the rest of the year, the region experiences a dry season and noticeably cooler temperatures, especially at night. Some areas that are accessible in the dry season are unreachable during the wet season. Invite students to describe any other unique weather in their area or other parts of the world that they know about.

SESSION 3 ■ ■ ■ □

Try It Ask students if they have ever had the battery run out on a device that they were using. Charging the batteries that power devices has become an everyday occurrence for many people. Some schools distribute electronic devices to students and expect that they will be fully charged and ready for class. Frustration stemming from low or dead batteries is so common that some people have resorted to carrying external battery packs for their devices. Interestingly, although some people use electronic devices such as cell phones more than other electronics in their home, the electricity used to recharge a cell phone typically costs less than $1 per year.

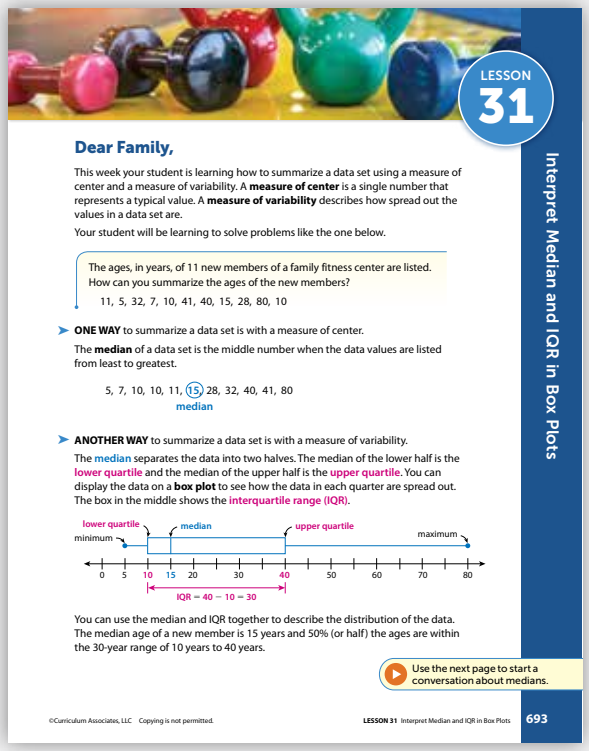
SESSION 4 ■ ■ ■ ■

Apply It **Problem 2** Harmful chemicals are usually stored in regulated containers, but sometimes dangerous chemicals can fall from the sky! When normal rainfall mixes with man-made pollutants, or even naturally occurring pollutants in the air, such as volcanic dust, it can turn into acid rain. This acid rain is damaging to plants and harmful to animals, especially animals that live in the water. To help prevent acid rain, individuals can reduce their use of products that cause pollution during production. Companies can make a difference by utilizing methods and materials that cause as little pollution as possible.



Connect to Family and Community

➤ **After the Explore session, have students use the Family Letter to let their families know what they are learning and to encourage family involvement.**



LESSON 31

Interpret Median and IQR in Box Plots

Dear Family,

This week your student is learning how to summarize a data set using a measure of center and a measure of variability. A **measure of center** is a single number that represents a typical value. A **measure of variability** describes how spread out the values in a data set are.

Your student will be learning to solve problems like the one below.

> The ages, in years, of 11 new members of a family fitness center are listed. How can you summarize the ages of the new members?
>
> 11, 5, 32, 7, 10, 41, 40, 15, 28, 80, 10

➤ **ONE WAY** to summarize a data set is with a measure of center.

The **median** of a data set is the middle number when the data values are listed from least to greatest.

5, 7, 10, 10, 11, (15), 28, 32, 40, 41, 80
median

➤ **ANOTHER WAY** to summarize a data set is with a measure of variability.

The median separates the data into two halves. The median of the lower half is the lower quartile and the median of the upper half is the upper quartile. You can display the data on a **box plot** to see how the data in each quarter are spread out. The box in the middle shows the interquartile range (IQR).

You can use the median and IQR together to describe the distribution of the data. The median age of a new member is 15 years and 50% (or half) the ages are within the 30-year range of 10 years to 40 years.

Use the next page to start a conversation about medians.

Activity Thinking About Medians Around You

➤ **Do this activity together to investigate medians in the real world.**

Online videos are a popular form of entertainment. Sometimes videos with a lot of views can earn money from showing advertisements. There are videos that have as many as one billion views!

Did you know that the median number of views for all videos uploaded in 2016 was only 89? This means half of these videos were viewed fewer than 89 times. That view count is much too low to earn any money!

Describe a situation where you might want to know the median of a data set.

Connect to Language

➤ **For English language learners, use the Differentiation chart to scaffold the language in each session. Use the Academic Vocabulary routine for academic terms before Session 1.**

DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS Use with **Session 1** Connect It

Levels 1–3: Reading/Speaking

Support students as they interpret and discuss Connect It problems 2a and 2d. Display the term *median*. Use **Act It Out** to illustrate the meaning of *middle* and *halfway*. Then read problem 2a, pausing after each sentence. Allow time for partners to complete each step. Then help partners restate the steps to find the median:

- *First, we ____. Then, we ____.*
- *The median is ____.*

Support problem 2d by having students count the values. Ask: *How many middle values are there? What is the median?*

- *The median is the ____ between ____ and ____.*

Levels 2–4: Reading/Speaking

Support students as they interpret and discuss Connect It problems 2a and 2d. Display the lesson term *median*. Make a sketch to illustrate the meaning of *middle* and *halfway*. Then read problem 2a, pausing after each sentence. Allow time for partners to complete each step. Then help partners restate the steps to find the median. Ask: *What do you do first? And next? What is the median?*

Call on volunteers to give examples of even numbers. Invite other volunteers to act out or explain the meaning of *between*. Then read problem 2d with students. Ask: *How many middle values are there? What is the median?* Encourage students to use *halfway* and *between* to answer.

Levels 3–5: Reading/Speaking

Support students as they interpret and discuss Connect It problems 2a and 2d. Call on volunteers to read problem 2a aloud. Then have students work with a partner to **Say It Another Way**. Monitor as students paraphrase each sentence. Encourage students to ask clarifying questions as needed. Then have students follow the steps and find the median.

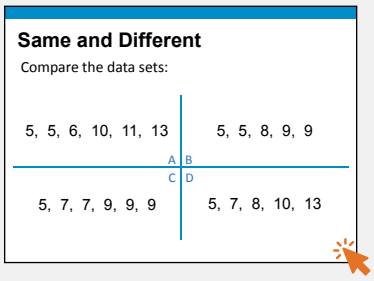
Next, have students turn to partners to read problem 2d and find the median. Ask questions to help them connect: *What steps did you follow to find the median in problem 2a? And in problem 2d? How are the steps the same? How are they different?*

Explore The Median of a Data Set

Purpose

- **Explore** the idea that you can choose one number to represent a typical value of a data set.
- **Understand** that the median is one way to summarize a data set with a single number.

START CONNECT TO PRIOR KNOWLEDGE



All data sets are in order from least to greatest.

A and C have an even number of values.

B and D have an odd number of values.

A and D are spread from 5 to 13.

B and C are spread from 5 to 9.

WHY? Support students' ability to analyze data sets.

TRY IT SMP 1, 2, 4, 5, 6

Make Sense of the Problem

See ***Connect to Culture*** to support student engagement. Before students work on Try It, use **Three Reads** to help them make sense of the problem. After each read, have students turn and talk with a partner before discussing as a class.

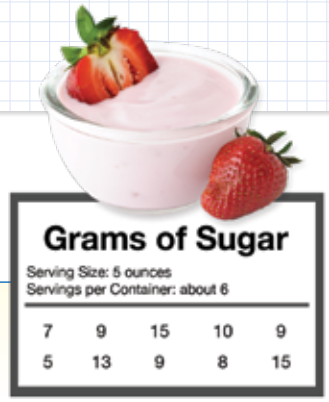
DISCUSS IT SMP 2, 3, 6

Support Partner Discussion

After students work on Try It, have them respond to Discuss It with a partner. Listen for understanding that:

- the data can be ordered from least to greatest.
- there is more than one way to choose and justify a number of grams of sugar to represent the data.

Explore The Median of a Data Set



Previously, you learned about displaying data distributions with dot plots and histograms. In this lesson, you will learn about displaying data distributions with box plots.

➤ **Use what you know to try to solve the problem below.**

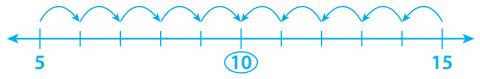
The data show the number of grams of sugar in one serving of different types of yogurt. Suppose you are given a serving of yogurt. About how many grams of sugar would you expect it to have?

TRY IT

Math Toolkit graph paper, number lines, sticky notes

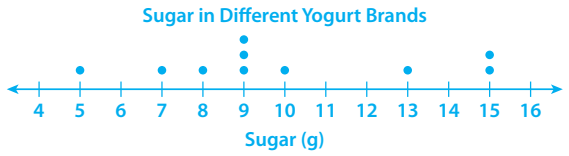
Possible work:

SAMPLE A



The data are spread from 5 to 15 g. The number 10 is in the middle. I would expect the serving of yogurt to have about 10 g of sugar.

SAMPLE B



The most common value is 9 g. I expect that the serving of yogurt would have about 9 g of sugar.

DISCUSS IT

Ask: How is your strategy similar to mine? How is it different?

Share: My strategy shows . . .

Learning Targets SMP 1, SMP 2, SMP 3, SMP 4, SMP 5, SMP 6

- Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.
- Display numerical data in plots on a number line, including dot plots, histograms, and box plots.
- Summarize numerical data sets in relation to their context by giving quantitative measures of center and variability.

695

Common Misconception Listen for students who think that there cannot be more than one correct choice for a typical value of a data set. As students share their strategies, have them consider and discuss why different choices of the amount of sugar to expect in one serving could be valid.

Select and Sequence Student Strategies

Select 2–3 samples that represent the range of student thinking in your classroom. Here is one possible order for class discussion:

- strategies that identify a value of 10 because it is halfway between the least and greatest values.
- strategies that identify a value of 9 because it is the most common value.
- strategies that identify a value of 9 because there are two values of 9 in the middle when the data are ordered from least to greatest.
- **(misconception)** there is only one possible choice for a typical value for a data set.

Facilitate Whole Class Discussion

Call on students to share selected strategies. Prompt students to justify their solutions by providing reasons that they are reasonable in the situation.

Guide students to **Compare and Connect** the representations. After each strategy, allow individual think time for students to process the ideas.

> **ASK** *How [student name]'s and [student name]'s strategies show reasonable solutions?*
>
> **LISTEN FOR** Each strategy shows a number that is within the spread of the data (5 to 15) and uses information about the distribution to justify the choice.

CONNECT IT

SMP 2, 4, 5

1 **Look Back** Look for understanding that you can use information about a data distribution (such as the range of the data, the least and greatest values, and the frequency of the values) to choose a single value to represent the data set.

DIFFERENTIATION | RETEACH or REINFORCE

Hands-On Activity

Use counters to explore dot plots.

If students are unsure about choosing one value as a typical value for a data set, then use this activity to have them gain experience with the concept.

Materials For each pair: 11 sticky notes, 15 counters, Activity Sheet *Number Lines*

- Tell students they will make a data set where: The data are all whole numbers; there are 15 values; the range is 10; a value of 5 could be considered a good choice as a typical value.
- Pairs can line up their sticky notes in a row with one number written on each to form a number line. They can experiment with placing the counters as dots above the sticky notes to make their data set. Have them record their final data set in a dot plot.
- Have several pairs display their dot plots. Ask each pair to describe their data set and to explain why they feel 5 is a good choice as a typical value. Compare and contrast them.
- Remind students that there may be more than one good choice for a typical value. Ask students to look at the data sets and offer explanations for choosing a value other than 5.
- Extend the activity by having pairs modify their data set so that a value other than 5 could be considered a typical value, keeping the range 10 and the number of values 15.

CONNECT IT

1 **Look Back** About how many grams of sugar would you expect there to be in one serving of yogurt? Explain how you can answer this question.

Possible answer: 10 g; You can find the halfway point between the least and greatest data values.

2 **Look Ahead** In the **Try It** problem, you chose one number to represent a typical amount of sugar in a serving of yogurt. When you use a single value to summarize a data set, you are using a **measure of center**. One measure of center is the **median**, or middle value when the data values are listed in order.

a. The table shows the number of grams of sugar in one serving of different drinks. List the values in order from least to greatest. What is the middle value of the data set? Describe how you found it.

Grams of Sugar				
14	15	5	30	13
6	8	9	12	7
12	8	13	16	8
24	5			

~~5~~, ~~5~~, ~~6~~, ~~7~~, ~~8~~, ~~8~~, ~~8~~, ~~9~~, 12, ~~12~~, ~~13~~, ~~13~~, ~~14~~, ~~15~~, ~~16~~, ~~24~~, ~~30~~

12; Possible answer: I crossed off pairs of values, one at each end, until there was one value left in the middle.

b. What is the median number of grams of sugar in a drink? How do you know?

12 g of sugar; There are 8 values on both sides of 12, which means 12 is the middle value.

c. The median splits the data into two halves. Complete the statements.

About half of the drinks have less than 12 grams of sugar per serving.

About 50% of the drinks have more than 12 grams of sugar per serving.

d. When a data set has an even number of values, there are two middle values. The number halfway between these two values is the median. What is the median of the data set shown below?

4, 4, 5, (5, 7,) 8, 10, 11 The median is 6.

3 **Reflect** Explain why the median could be a good value to use to summarize all the values of a data set, or to represent a typical value.

Possible explanation: The median is the middle value of the data set, which means that half the values are less than the median and half are greater.

2 **Look Ahead** Point out that using a single number to represent a typical value is a way of summarizing a data set. Ask a volunteer to rephrase the definitions of *measure of center* and *median*. Support students in understanding that the word *center* in the term *measure of center* reflects the fact that a typical value may be in the "middle" of the data. If students mention finding the average of the data values (the *mean*) or finding the value that occurs most often (the *mode*), acknowledge that these are also ways of choosing a value that may be seen as typical of a data set. Students will learn about the mean in the next lesson.

CLOSE EXIT TICKET

3 **Reflect** Look for understanding that half the data are greater than the median of a data set and half are less than the median.

Common Misconception If students describe the median as the number halfway between the least and greatest data values, then have them use a dot plot of the grams of sugar data in Try It to see that 10 is halfway between 5 and 15, but the median of the data is 9. For a perfectly symmetric data set, these two numbers must be the same, but otherwise they can be different.

Prepare for Interpreting Median and Interquartile Range in Box Plots

Support Vocabulary Development

Assign **Prepare for Interpreting Median and Interquartile Range in Box Plots** as extra practice in class or as homework.

If you have students complete this in class, then use the guidance below.

Ask students to consider the term *range*. Prompt students to think about past data sets they have studied and use those to help write the definition and examples.

Have students work in pairs to complete the graphic organizer. Invite pairs to share their completed organizers, and prompt a whole-class comparative discussion of the definition, what they know about range, and examples of range.

Have students look at the dot plots in problem 2 and discuss with a partner how the data is organized on each dot plot. Encourage students to use the term *range* to discuss the difference between the least and greatest values in the data sets.

Problem Notes

1. Students should understand that range is the difference between the greatest and least values in a data set. Student responses may include referencing a dot plot or table with a data set to illustrate the range of that data set. Students may recognize that the range of a data set gives you information about how spread out the data are.

2. Students should recognize that the data shown on each dot plot are different, even though the number lines and number of data values are the same for each data set. To find the range, students find the difference between the greatest and least data values, not the greatest and least values on the number lines.

Prepare for Interpreting Median and IQR in Box Plots

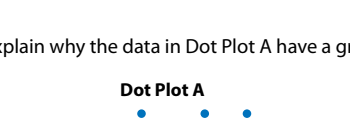
1. Think about what you know about data distributions. Fill in each box. Use words, numbers, and pictures. Show as many ideas as you can. **Possible answers:**

What Is It?
The range is the difference between the greatest and least values in a data set.

What I Know About It
The range of a data set gives you information about how spread out the data are.

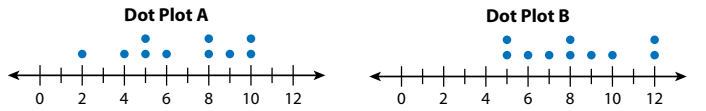
range

Examples
The range of the data set is 6 − 1 = 5.



Examples
The range of the data set is 4 − 0 = 4.
0, 0, 1, 1, 3, 4, 4, 4

2. Explain why the data in Dot Plot A have a greater range than the data in Dot Plot B.



Possible answer: The range for Dot Plot A is 10 − 2 = 8. The range for Dot Plot B is 12 − 5 = 7. So, the range for Dot Plot A is greater.

REAL-WORLD CONNECTION

When you purchase something with a high value such as a house or a car, you are required to buy insurance in case it becomes damaged. Insurance companies employ actuaries, who use mathematics to calculate how much the insurance should cost. Using figures such as the median, they determine how much damage is likely to occur over a certain period of time. Then, while taking into consideration that the price must be high enough so they will make a profit when customers purchase insurance, and low enough that customers choose to buy their insurance, the company will set a price. Different companies have various methods of applying the median in different situations, so they will have varying costs. Ask students to think of other real-world examples when knowing how to find the median would be useful.

3 Problem 3 provides another look at finding a typical value of a data set. This problem is similar to the problem about the number of grams of sugar in a serving of yogurt. In both problems, a data set is given and students are asked to identify a typical value. This problem asks for the number of grams of protein that you would expect to find in a typical serving of yogurt.

Students may choose to use a dot plot or number line to solve.

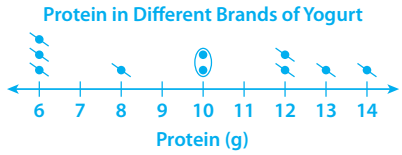
Suggest that students use **Three Reads**, asking themselves one of the following questions each time:

- *What is this problem about?*
- *What is the question I am trying to answer?*
- *What information is important?*

3 The list shows how many grams of protein there are in one serving of different brands of yogurt.

8, 10, 6, 12, 14, 6, 10, 12, 13, 6

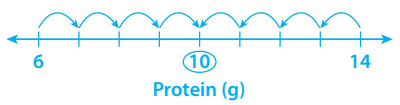
a. About how many grams of protein would you expect a typical serving of yogurt to have? Show your work.



SOLUTION A typical serving of yogurt has about 10 g of protein.

b. Justify why your answer is reasonable. Show your work.

Possible answer: The least value is 6 and the greatest value is 14.



The number halfway between 6 and 14 is 10. So, my answer is reasonable.

DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Use with **Session 2** Apply It

Levels 1–3: Listening/Speaking

Help students with Apply It problem 8 by having them summarize the data set and describe its quartiles. Display the data and ask: *What are some ways to summarize the data?* If needed, suggest students construct a dot plot or number line, rewrite the values in order, or identify the median and quartiles. Clarify that *quartile* is 1 of 4 equal groups and connect to *quarter* and *fourth*. Ask for cognates, like *cuartilla* and *cuarto* in Spanish.

Give students time to summarize the data before asking them to tell what they notice about the values and quartiles. Record models and statements for reference. Next, help students read the problem and connect words in the answer choices to the recorded ideas.

Levels 2–4: Listening/Speaking

Help students make sense of Apply It problem 8. Without showing the problem, display the answer choices for students to preview. Ask: *What do you think the problem is about?* Have partners discuss the answer choices and identify statements that relate to *lower quartile, median,* or *upper quartile*. Guide discussion using:

- *Do you think ____ connects to the median or one of the quartiles?*
- *I think ____ because ____. Do you agree?*

Then have students read and solve the problem by summarizing the data set.

Levels 3–5: Listening/Speaking

Have students make sense of Apply It problem 8. Without showing the problem, display the answer choices for students to preview. Have students discuss what they think the problem is about and how they might determine which statement is true. Explain that the statements give information about the possible *lower quartile, median,* and *upper quartile* of the data set. Have students turn and talk about what information each statement might provide. Reinforce that partners should tell if they agree or disagree and explain their reasons.

Have students read and solve the problem individually and then meet with partners to discuss responses.

Develop Finding the Median and Quartiles

Purpose

- **Develop** strategies for summarizing a data set using medians.
- **Recognize** that the upper and lower quartiles and the median provide information about the center and symmetry of a data set.

START CONNECT TO PRIOR KNOWLEDGE

Same and Different

10, 12, 15, 14, 18	0, 15, 15, 15, 15
9, 11, 15, 29, 31	0, 2, 15, 20, 95

Possible Solutions

All data sets are in order from least to greatest and have the same middle value.

A has the least range.

B has a low outlier.

C has only odd values.

D has a high outlier.

WHY? Support students' ability to analyze data sets.

DEVELOP ACADEMIC LANGUAGE

WHY? Support understanding of *quartile*.

HOW? Explain that *quartile* comes from the Latin word *quartus* that means "fourth." Ask students to list related terms or phrases such as *quarter, quart*, and *quarter past the hour* and discuss how the terms express *fourth*. During discussion of the second Model It, ask: *How does the lower quartile divide the data? And the upper quartile? How many groups or parts in all?*

TRY IT

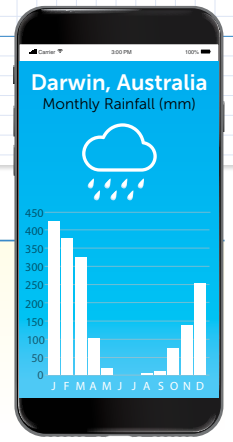
SMP 1, 2, 4, 5, 6

Make Sense of the Problem

See ***Connect to Culture*** to support student engagement. Before students work on Try It, use **Notice and Wonder** to help them make sense of the problem. After students share what they notice, call attention to the data set. If no one mentions that the data is not ordered, discuss how the order of the data would affect finding the median.

LESSON 31 | SESSION 2 ■ ■ □ □

Develop Finding the Median and Quartiles



➤ **Read and try to solve the problem below.**

The town of Darwin, Australia, has a rainy season and a dry season. The data show the monthly amount of rainfall in one year. Each amount is rounded to the nearest 5 millimeters.

425, 375, 320, 100, 20, 0, 0, 5, 15, 70, 140, 250

Show that the median rainfall per month is 85 mm. How much rain does Darwin typically get per month during the drier 6 months of the year?

TRY IT

Math Toolkit graph paper, number lines, sticky notes

Possible work:

SAMPLE A

Whole year: ~~0~~, ~~0~~, ~~5~~, ~~15~~, ~~20~~, 70, 100, ~~140~~, ~~250~~, ~~320~~, ~~375~~, ~~425~~

The median is halfway between 70 and 100, which is 85.

Drier 6 months: ~~0~~, ~~0~~, 5, 15, ~~20~~, ~~70~~

The median is halfway between 5 and 15, which is 10.

Darwin typically gets about 10 mm of rain per month in the drier 6 months.

SAMPLE B

There are 12 data values, so the 6th and 7th values are in the middle.

0, 0, 5, 15, 20, 70, 100, 140, 250, 320, 375, 425

85 is the median because it is halfway from 70 to 100.

Dot plot for the drier 6 months:

10 is the median because it is halfway from 5 to 15.

A typical amount of rainfall per month in the drier 6 months is 10 mm.

DISCUSS IT

Ask: How did you use your model to find a typical value?

Share: In my model, . . .

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DISCUSS IT

SMP 2, 3, 6

Support Partner Discussion

After students work on Try It, have them respond to Discuss It with a partner. Listen for understanding that:

- the data must first be ordered from least to greatest.
- when there are an even number of data values in a set, the median marks the value that is halfway between the two middle values.
- the drier 6 months are represented by the 6 lesser values in the data set.
- a number within the range of the 6 drier months could be used to describe the typical rainfall during the dry season.

Common Misconception Listen for students who think that they should not include 70 mm in the data for the drier six months, because it is so much greater than the other five values. They may think that they should remove outliers from a data set before analyzing the data. As students share their strategies, remind them that unless directed otherwise, all data must be included in statistical calculations.

Select and Sequence Student Strategies

Select 2–3 samples that represent the range of student thinking in your classroom. Here is one possible order for class discussion:

- the data is listed in order from least to greatest, then divided into halves, and the medians are found by locating the value halfway between the two middle values data points divided by 2.
- **(misconception)** removing the outlier 70 from the data for the drier 6 months.
- a dot plot that represents the median for the entire data set and a second dot plot that represents the median for the drier months.

Facilitate Whole Class Discussion

Call on students to share selected strategies. Review that one way to justify a solution is to use definitions, properties, and what you already know to support your strategy and thinking.

Guide students to **Compare and Connect** the representations. Allow time for students to think by themselves before starting the discussion.

ASK *How do the models show the amounts of rain that fall each month? During the drier 6 months?*

LISTEN FOR The models show the data in order from least to greatest. A middle value is determined for the entire data set. Then, a middle value is determined for the drier 6 months.

Model It & Analyze It

If students presented these models, have students connect these models to those presented in class.

If no student presented at least one of these models, have students first analyze key features of the models and then connect them to the models presented in class.

ASK *Where is the median in each model?*

LISTEN FOR The median is the middle value of the data set. Since the data set has an even number of values, the median is between the sixth and seventh values.

For Model It, prompt students to note how the median is calculated.

- *What calculations are used to find the median?*

For Analyze It, prompt students to notice how the data are grouped.

- *How many values are in each half?*
- *How many values are less than and greater than Q1? Q3?*

➤ **Explore different ways to use medians to describe a data distribution.**

The town of Darwin, Australia, has a rainy season and a dry season. The data show the monthly amount of rainfall in one year. Each amount is rounded to the nearest 5 millimeters.

425, 375, 320, 100, 20, 0, 0, 5, 15, 70, 140, 250

Show that the median rainfall per month is 85 mm. How much rain does Darwin typically get per month during the drier 6 months of the year?

Model It

You can use the median to describe the center of a data distribution.

To find the median, first list the values in order from least to greatest. Then find the middle value, or the two middle values.

0, 0, 5, 15, 20, 70, 100, 140, 250, 320, 375, 425

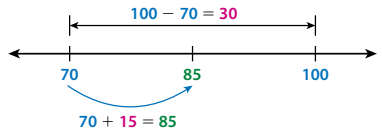
The median is halfway between the 6th and 7th values.

You can use a number line to find the halfway point between 70 and 100.

The distance from 70 to 100 is 30 and half of this distance is 15.

Add half of the distance from 70 to 100 to 70.

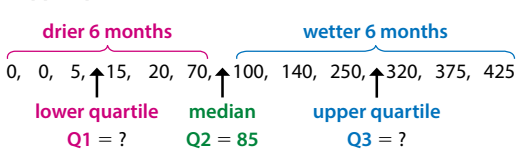
The median is 85.



Analyze It

You can find the median of each half of a data distribution.

The median, 85, separates the rainfall data into two halves. The median of the lower half of the data is the **lower quartile (Q1)**. The median of the upper half of the data is the **upper quartile (Q3)**. The median of the whole data set is Q2.



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DIFFERENTIATION | EXTEND

SMP 6

Deepen Understanding

Use Precision to Work with Data Sets with an Odd Number of Values

Prompt students to understand how to determine the median, lower quartile, and upper quartile for a data set with an odd number of values.
Use the data set 70, 100, 140, 250, 320, 375, 425.

ASK *What is the median of the given data set?*

LISTEN FOR The value that is in the middle of the list of values is the median. The median is 250 because it is the fourth number out of a list of seven numbers.

ASK *When calculating the lower and upper quartiles, why do you not include the median?*

LISTEN FOR The median does not belong to either half of the data. Each half has three values, so the halves have an equal number of values.

ASK *What is the lower quartile and upper quartile for the data set? How do you know?*

LISTEN FOR The lower quartile is 100 and the upper quartile is 375. Each half has three values, so the middle value of each half is the median.

Develop Finding the Median and Quartiles

CONNECT IT

SMP 2, 4, 5, 6

Remind students that the quantities and the relationships between them are the same in each representation. Explain that they will now use those relationships to reason about how the upper and lower quartiles and the median can provide information about a data set.

Before students begin to record and expand on their work in Model It & Analyze It, tell them that problem 5 will prepare them to provide the description asked for in problem 6. To engage all students, ask them to turn and talk to answer the question, *What is the upper quartile, the lower quartile, and the median of the data set?*

Monitor and Confirm Understanding 1 – 3

- The median is the middle value of a data set. Half of the data values are greater and half are less than the median.
- The first quartile, Q1, would describe a typical amount of rain that falls during a dry month.
- The third quartile, Q3, would describe a value for the amount of rain that falls during a wet month.

Facilitate Whole Class Discussion

4 Look for the idea that the median divides an ordered data set into two halves, and the quartiles divide the data set into four quarters.

> **ASK** *How can you use fractions and percents to describe groups of data values?*
>
> **LISTEN FOR** The median marks the value greater than half and less than half of the other values, and $\frac{1}{2}$ represents 50%. Quartiles divide each half of the values in half. This forms four equal groups, which can be referred to by the fraction $\frac{1}{4}$ or the percent 25%. Half of the data are between Q1 and Q3, and this represents the middle 50% of the data.

5 Look for the idea that lower/upper quartiles and the median can be used to describe data distributions.

6 **Reflect** Have all students focus on the strategies used to solve the Try It. If time allows, have students discuss their ideas with a partner.

CONNECT IT

➤ **Use the problem from the previous page to help you understand how to use medians to describe a data distribution.**

1 Look at **Model It**. What does the median, 85, tell you about the amount of rainfall during the year in Darwin, Australia?
Possible answer: Half the months get less than 85 mm rain and half the months get more than 85 mm rain.

2 Look at **Analyze It**. What is the value of the lower quartile (Q1) for the monthly rainfall data? Use Q1 to describe a typical amount of rainfall per month during the drier 6 months of the year.
10; A drier month typically gets about 10 mm of rain.

3 What is the value of the upper quartile (Q3) for the monthly rainfall data? What does this value tell you about rainfall in Darwin, Australia?
285; Possible answer: A wetter month typically gets about 285 mm of rain.

4 You can use a median or quartile to help you identify groups of data values.

a. What fraction of the monthly rainfall data is greater than 85 mm? $\frac{1}{2}$

b. What percent of the data is less than 285 mm? 75%

c. Which data values are in the middle 50% of the monthly rainfall data set?
15, 20, 70, 100, 140, and 250

5 Why are the median, lower quartile, and upper quartile useful for describing data distributions?
Possible answer: The median can describe a typical value for a data set. Half the data are greater than the median and half are less than the median. The lower and upper quartiles can describe a typical value for each half.

6 **Reflect** Think about all the models and strategies you have discussed today. Describe how one of them helped you better understand how to use medians to describe data distributions.
Responses will vary. Check student responses.

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DIFFERENTIATION | RETEACH or REINFORCE

Hands-On Activity

Find the Q1, median, and Q3 of a data set.

If students are unsure about how to find the first and third quartiles, then use this activity to show how the values are determined.

Materials For each pair: scissors, Activity Sheet *1-Inch Grid Paper*

- Have pairs write the values from the Try It data set on the Activity Sheet, with one value in each box. Cut out the boxes. Order the values in a row from least to greatest.
- Have one partner start on the left side and the other partner start on the right side, pointing to a value as they move toward each other. As they reach the sixth and seventh values, they should realize that they cannot go further.
- Ask: *How do you find the median?* [It is halfway between 70 and 100, which is 85.]
- Direct students' attention to the lower half of the data. Ask: *What numbers do these values represent?* [the drier months] *What could you use as a typical value to describe the drier months?* [Find the median of the numbers.] Have pairs repeat the process of moving toward each other to identify Q1. [10] Then have them use the same process with the upper half of the data to find Q3. [285]

Apply It

For all problems, encourage students to use a model to support their thinking. Allow some leeway in precision; if students choose to draw a number line or make a dot plot to help solve the problems, number lines with precisely sized sections are not required. Any model used helps students keep track of the order of the values.

7 **a.** Students should recognize that they have to put the data from the table in order from least to greatest before they can identify the median. There are 15 values, so the median will be the eighth number with no additional calculations needed.

b. Students should understand that the median represents a typical value in the data set. Half of the values will be greater and half will be less than the median.

8 **A is correct.** Students may solve the problem by correctly interpreting the median.

B is not correct. This answer is the result of finding Q3 but not realizing that 75% of students sold fewer than 3 coupon books, not more than 3 coupon books.

C is not correct. This answer is the result of finding Q1 but not realizing that one fourth, or 25%, of students sold less than 1 coupon book, not more than 1 coupon book.

D is not correct. This answer is the result of misinterpreting Q1 and the median. Q1 and the median represent values that are typical of the data set, but they may not represent all of the values in the data set.

Apply It

➤ **Use what you learned to solve these problems.**

7 A gym has rowing machines with digital screens that display time, distance, and speed. The data show the numbers of miles that members row on Tuesday.

a. Find the median, the lower quartile, and the upper quartile of the data set. (*Note:* When the number of data values is odd, do not include the median in either half of the data as you find Q1 and Q3.) Show your work. Possible work:

0.1, 0.9, 1.0, 1.4, 1.8, 2.0, 2.2, 2.3, 2.4, 2.4, 3.5, 3.9, 4.3, 6.8, 10.0

SOLUTION The median is 2.3 mi, Q1 is 1.4 mi, and Q3 is 3.9 mi.

b. What does the median tell you about the number of miles rowed?

Possible explanation: About half the members rowed less than 2.3 mi, and half the members rowed more than 2.3 mi.

8 Students sell coupon books for a fundraiser and report the number sold. The median number of coupon books sold is 2 and the lower and upper quartiles for the data are Q1 = 1 and Q3 = 3.5. Which statement is true?

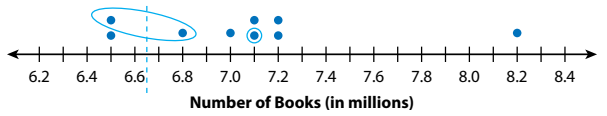
A About half of the students sold 2 or more coupon books.

B About 75% of the students sold more than 3 coupon books.

C About one fourth of the students sold more than 1 coupon book.

D About 50% of the students sold either 1 or 2 coupon books.

9 The dot plot shows the number of books in some large libraries in the U.S. What is the median and the lower quartile of the data set? Show your work. Possible work:



6.8 − 6.5 = 0.3 and half of 0.3 = 0.15.

6.5 + 0.15 = 6.65

SOLUTION The median is 7.1 million books and Q1 is 6.65 million books.

CLOSE EXIT TICKET

9 Students' solutions should show an understanding of: how to find the median, lower quartile, and upper quartile of a data set.

Error Alert If students identify Q1 as 6.8, then have students list the values from the dot plot from least to greatest. Students may have included dots greater than 7.1 when determining Q1, without recognizing that this is a data set with an odd number of values. The lower half of the data set is 6.5, 6.5, 6.8, and 7.0, so Q1 will be between 6.5 and 6.8.

Practice Finding the Median and Quartiles

Problem Notes

Assign **Practice Finding the Median and Quartiles** as extra practice in class or as homework.

1 **a.** Students should understand that they should find the value halfway between 0.99 and 1.14 to find Q1 and the value halfway between 1.30 and 1.84 to find Q3. ***Basic***

b. Students may recognize that they can use Q1 and Q3 to make a generalization about the typical cost. The numbers from Q1 to Q3 give a typical range of costs for granola bars. ***Basic***

2 Students may recognize that this value must be added to the list of values in numerical order. The value 2.75 becomes the new greatest value, so Q1, the median, and Q3 all increase. ***Medium***

Practice Finding the Median and Quartiles

➤ **Study the Example showing how to summarize a data set with a single number. Then solve problems 1–5.**

Example

Abran recorded the price of his favorite granola bar at 9 different stores. What is the median cost of the granola bar at these stores?

$0.85, $0.99, $1.15, $1.27, $1.28, $1.30, $1.30, $1.84, $1.89

Order the values from least to greatest. Find the middle value.

0.85, 0.99, 1.15, 1.27, (1.28,) 1.30, 1.30, 1.84, 1.89

The median cost for the granola bar is $1.28.

1 **a.** Look at the Example. What is the lower quartile (Q1) and upper quartile (Q3) of the granola bar prices? Show your work. **Possible work:**

0.85, [0.99,] 1.15, 1.27, (1.28,) 1.30, [1.30, 1.84,] 1.89

1.15 − 0.99 = 0.16 and half of 0.16 = 0.08. So, Q1 = 0.99 + 0.08 = 1.07.

1.84 − 1.30 = 0.54 and half of 0.54 = 0.27. So, Q3 = 1.30 + 0.37 = 1.57.

SOLUTION Q1 = $1.07 and Q3 = $1.57

b. What do these values tell you about the cost of the granola bars?

Possible answer: Half of the granola bars cost from about $1.07 to $1.57.

2 Abran sees his favorite granola bar from the Example in a vending machine at an airport. The cost is $2.75. What are the new values of the median, lower quartile, and upper quartile? Show your work.

Possible work:

0.85, 0.99, [1.15,] 1.27, (1.28, 1.30,) 1.30, [1.84,] 1.89, 2.75

Q1 Q2 = 1.29 Q3

SOLUTION Q1 = $1.15, median = $1.29, and Q3 = $1.84

Vocabulary

median
the middle number, or halfway point between the two middle numbers, in an ordered set of numbers.

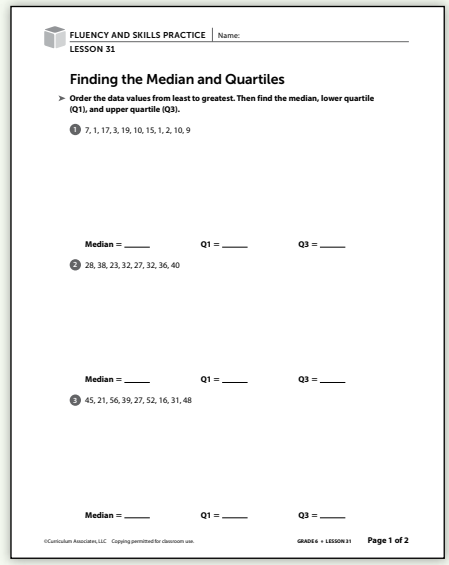
lower quartile
the middle number between the minimum and the median in an ordered set of numbers.

upper quartile
the middle number between the median and the maximum in an ordered set of numbers.

Fluency & Skills Practice

Finding the Median and Quartiles

In this activity, students order data values from least to greatest to find the median, lower quartile (Q1), and upper quartile (Q3).



FLUENCY AND SKILLS PRACTICE | Name:
LESSON 31

Finding the Median and Quartiles

➤ Order the data values from least to greatest. Then find the median, lower quartile (Q1), and upper quartile (Q3).

1 7, 1, 17, 3, 19, 10, 15, 1, 2, 10, 9

Median = ____ Q1 = ____ Q3 = ____

2 28, 38, 23, 32, 27, 32, 36, 40

Median = ____ Q1 = ____ Q3 = ____

3 45, 21, 56, 39, 27, 52, 16, 31, 48

Median = ____ Q1 = ____ Q3 = ____

3 **a.** Students should recognize that the numbers on the table are not in order, so they need to order the numbers before identifying the quartiles. ***Medium***

b. Students should recognize that the values between Q1 and Q3 represent 50%, or half, of the data. ***Medium***

c. Students should recognize that removing an outlier from the greatest end of the values will decrease the Q1, the median, and Q3. ***Challenge***

4 **a.** The median of the data set is 9, so more than 50% of the data values are 9 or greater.

b. The lower quartile is 8, so 25% of the data values are 8 or less.

c. The upper quartile is 11, so he worked 10 or more hours about 25% of the time.

Medium

5 Students should recognize that they first need to order the data: 0, 0, 1, 2, 3, 4, 5, 9, 12. The median is 3. The upper quartile is 7, which is halfway between 5 and 9. ***Challenge***

3 The table shows the lengths of various musicals in hours.

Musical Lengths (hours)								
2.8	2.8	2.5	2.5	2.3	2.9	2.5	2.6	2.3
2.5	2.5	2.5	2.3	2.6	2.3	5.3	2.5	

a. What are the lower quartile and upper quartile? Show your work.

Possible work:
2.3, 2.3, 2.3, 2.3, 2.5, 2.5, 2.5, 2.5, 2.5, 2.5, 2.6, 2.6, 2.8, 2.8, 2.9, 5.3

SOLUTION Q1 = 2.4 h and Q3 = 2.7 h

b. What do the lower and upper quartiles tell you about the middle 50% of the data?

Possible answer: Half the musicals last from about 2.4 h to 2.7 h.

c. Suppose Elias removes the outlier of 5.3 hours. How do the median, Q1, and Q3 change?

2.3, 2.3, 2.3, 2.3, 2.5, 2.5, 2.5, 2.5, 2.5, 2.5, 2.6, 2.6, 2.8, 2.8, 2.9

The median is still 2.5 h, the Q1 is still 2.4 h, and Q3 is now 2.6 h.

4 The data show the number of hours a part-time waiter works each week.

7, 11, 8, 10, 11, 8, 13, 9, 10, 9, 9

Tell whether each statement about the data is *True* or *False*.

	True	False
a. He works more than 9 hours about 50% of the time.	●	○
b. He works 8 or fewer hours about 25% of the time.	●	○
c. He works 10 or more hours about 75% of the time.	○	●

5 Each day for 9 days, a school principal records the number of 6th graders who are absent. Hai says the upper quartile for the data below is 5. Is Hai correct? Explain.

0, 1, 0, 2, 4, 3, 5, 12, 9

No; Possible explanation: Hai included the median in the upper half of the data when he found Q3. The upper quartile is 7, the number that is halfway between 5 and 9.

DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Use with **Session 3** Connect it

Levels 1–3: Reading/Writing

Support students as they interpret and prepare a written response to Connect It problem 4. Review the terms *center* and *variability.* Ask: *In this situation comparing the battery life of two companies' batteries, what does the variability mean?* Guide students to understand the context as needed.

Read the problem. Make a **Co-Constructed Word Bank** of terms that students might use in their writing, including comparison words such as *more* or *less.* Then ask students to explain their thinking in their own words, and help them rephrase, as needed, to write responses in complete sentences.

Levels 2–4: Reading/Writing

Support students as they interpret and prepare a written response to Connect It problem 4. Discuss the terms *center* and *variability.* Ask: *What does variability mean in terms of the battery life of the two batteries? Would a company want its battery life to be more or less variable?*

Read the problem with students, and have them use **Co-Constructed Word Bank** to list words or phrases for their responses. Suggest math terms such as *range, IQR,* and *box plot,* and comparison words such as *more* and *less.* Then ask student to use **Stronger and Clearer Each Time** to draft responses and get partner feedback for revision.

Levels 3–5: Reading/Writing

Support students as they interpret and prepare a written response to Connect It problem 4. Discuss the terms *center* and *variability.* Have partners define each term and explain the significance in terms of the problem's context.

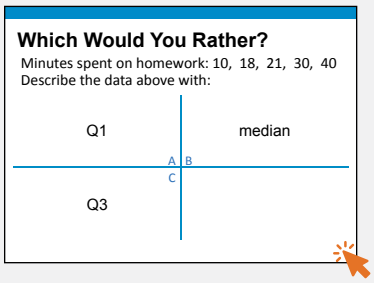
Ask students to read the problem and draft a written response. Have them use **Stronger and Clearer Each Time** to draft responses and get partner feedback. Reinforce that partners may ask questions to clarify meanings of terms or to ask for more information. After discussions, have students revise responses independently.

Develop Using Box Plots and Interquartile Range to Describe Variability

Purpose

- **Develop** strategies for answering statistical questions using a box plot and the interquartile range (IQR).
- **Recognize** that box plots and the IQR provide information about the center and symmetry of a data set, and provide more than a verbal description of the shape.

START CONNECT TO PRIOR KNOWLEDGE



Possible Solutions

A because 14 indicates most days require more than 14 minutes for homework.

B because 21 tells the typical number of minutes.

C because 35 indicates most days require less than 35 minutes for homework.

WHY? Support students' ability to understand and describe data using quartiles.

DEVELOP ACADEMIC LANGUAGE

WHY? Develop understanding of the phrase *summary of a data set*.

HOW? Discuss the meaning of *summary*. Have students tell about summarizing information in other contexts, such as in another content area or when telling about a book or movie. Then read Model It and have students find the term. Ask: *In what way do these five values provide a summary of the data set?*

TRY IT

SMP 1, 2, 4, 5, 6

Make Sense of the Problem

See ***Connect to Culture*** to support student engagement. Before students work on Try It, use **Say It Another Way** to help them make sense of the problem. Allow students to read the problem with a partner and paraphrase the problem together. Then lead a class discussion about the term *typically* and what it means in the context of this problem.

LESSON 31 | SESSION 3 ■ ■ ■ □

Develop Using Box Plots and IQR to Describe Variability

➤ **Read and try to solve the problem below.**

Ziplife company claims that its external battery pack typically adds 9 hours of battery life to a smartphone. Researchers tested a group of the battery packs and recorded the number of extra hours of battery life. Their data are shown in the table. Do you agree with the company's claim? Use data to support your reasoning.

Extra Hours of Battery Life

9.5	3.5	5.5	6.5	6.0
12.5	11.5	13.5	7.5	8.0
8.5	9.5	10.0	10.5	9.0

TRY IT

Math Toolkit graph paper, number lines, sticky notes

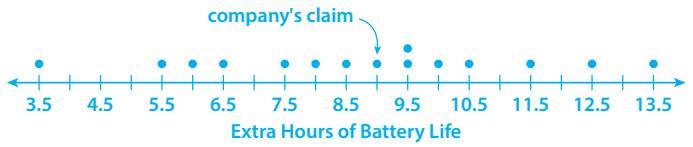
Possible work:

SAMPLE A

3.5, 5.5, 6.0, 6.5, 7.5, 8.0, 8.5, (9.0), 9.5, 9.5, 10.0, 10.5, 11.5, 12.5, 13.5

The median is 9, which means that a battery pack typically adds 9 hours of battery life. I agree with the company's claim.

SAMPLE B



There is a cluster of data around 9, but the data are very spread out from 3.5 to 13.5. Because much of the data is so spread out and not close to 9, I do not agree with the company's claim.

DISCUSS IT

Ask: How did you get started?

Share: I started by . . .

DISCUSS IT

SMP 2, 3, 6

Support Partner Discussion

After students work on Try It, have them respond to Discuss It with a partner. Listen for understanding that:

- the data values must be ordered from least to greatest.
- the median can be used to find the typical number of hours of additional battery life.
- the median can be compared with the company's claim to answer the question presented in the problem.

Error Alert If students say the median is 8.5, then they may have incorrectly used a dot plot to determine the median. They may have found the midpoint between 3.5 and 13.5 without considering the frequency of the data. Have students list all of the data from least to greatest to compare the data with the number line.

Select and Sequence Student Strategies

Select 2–3 samples that represent the range of student thinking in your classroom. Here is one possible order for class discussion:

- an organization of the data set in numerical order is used to find the median and compare the median with the claim
- a dot plot is used to organize the data set and compare the data with the claim

Facilitate Whole Class Discussion

Call on students to share selected strategies. Ask students to use precise language, such as *median*, *Q1*, or *Q3* in their explanations.

Guide students to **Compare and Connect** the representations. Use turn and talk to help students think through their responses before sharing with the group.

ASK *How does this model show the data? How does this model compare the data with the claim?*

LISTEN FOR Representations should show the data ordered from least to greatest. The claim of 9 hours should be noted in relation to the data, with other data used to support or refute the claim.

Model It

If students presented these models, have students connect these models to those presented in class.

If no student presented at least one of these models, have students first analyze key features of the models and then connect them to the models presented in class.

ASK *How are the representations similar? How are they different?*

LISTEN FOR Both show the spread of the data, Q1, and Q3. The box plot uses a visual to show the minimum, maximum, and quartiles. The dot plot shows the same information using words and numbers.

For the first Model It, prompt students to note how the dot plot was constructed.

- *What numbers are included in the five-number summary? Where are they located on the dot plot?*

For the second Model It, prompt students to discuss the box plot.

- *How do measures of center compare with measures of variability?*
- *What is the difference between the range and the IQR? When might you use each calculation?*

➤ **Explore different ways to describe the variability of a data set.**

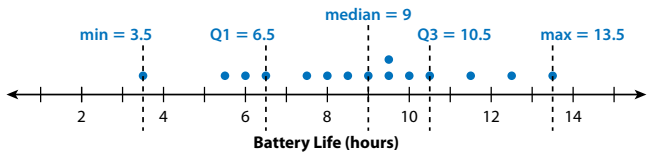
ZipLife company claims that its external battery pack typically adds 9 hours of battery life to a smartphone. Researchers tested a group of the battery packs and recorded the number of extra hours of battery life. Their data are shown in the table. Do you agree with the company's claim? Use data to support your reasoning.

Extra Hours of Battery Life				
9.5	3.5	5.5	6.5	6.0
12.5	11.5	13.5	7.5	8.0
8.5	9.5	10.0	10.5	9.0

Model It

You can use a dot plot to show the variability of a data set.

The minimum (min) and maximum (max) data values determine the range of the data. Locate the median, the lower quartile (Q1), and the upper quartile (Q3).

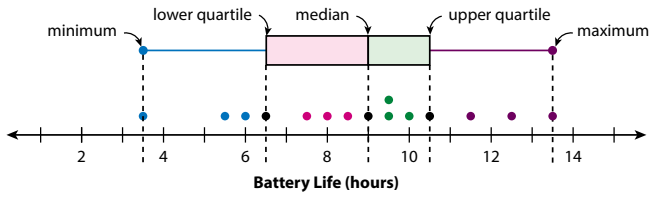


Together, these values are called the *five-number summary* of the data. They can be used to construct a data display called a **box plot**.

Model It

You can use a box plot to analyze the variability of a data set.

A box plot shows how the data in each quarter of the data set are spread out.



The range and the **interquartile range (IQR)** are both **measures of variability** of a data set.

Range = max − min

Interquartile range = Q3 − Q1

DIFFERENTIATION | EXTEND

SMP 5

Deepen Understanding

Using Box Plots to Strategically Analyze Data Sets

Prompt students to consider when and how box plots are useful representations of data.

ASK *What are some advantages to using a box plot to display a data set?*

LISTEN FOR A box plot shows the five-number summary visually. This means you can locate each quarter of the data and easily identify the range, IQR, and median.

ASK *Why is the median not directly in the middle of the box plot?*

LISTEN FOR The median represents the middle value of the data set. This is why there are seven data values on either side of the median. The median does not mean the middle value between the minimum and maximum.

ASK *What are some disadvantages to using a box plot to display a data set?*

LISTEN FOR The box plot does not show individual data values like a dot plot. So, for example, while you can see where the lowest 25% of the data is located between 3.5 and 6.5, you do not know the exact value of those data values.

Generalize Encourage students to tell when they might use a box plot instead of a dot plot to represent a data set.

Develop Using Box Plots and IQR to Describe Variability

CONNECT IT

SMP 2, 4, 5, 6

Remind students that the quantities and the relationships between them are the same in each representation. Explain that they will now use those relationships to reason about how box plots and IQR are used to describe a data set.

Before students begin to record and expand on their work in Model It, tell them that problem 4 will prepare them to provide the description asked for in problem 5. Allow time for students to think by themselves before starting the discussion.

Monitor and Confirm Understanding 1 – 2

- The total length of the box represents the IQR, which is 50% of the data.
- The size of each box in a box plot tells the relationship between Q1 and Q2, and between Q2 and Q3. A short box indicates that the values are close together, while a long box indicates that they are far apart.

Facilitate Whole Class Discussion

3 Look for understanding of how to read a box plot and use IQR to justify an answer.

4 Look for understanding that a double box plot can be used to compare data sets.

> **ASK** *How can you compare data presented in box plots?*
>
> **LISTEN FOR** You can compare all the numbers from the five-number summary using a double box plot. You can see which data set has a greater range by comparing the width of the boxes and the length of the line indicating range.

5 Look for the idea that box plots, range, and IQR can be used to compare data sets, as they represent the middle 50% of the data.

> **ASK** *What information can you gather from a box plot?*
>
> **LISTEN FOR** Box plots display the range of the data. You can view the data as two halves or four quarters. Longer parts mean there is more variability, and shorter parts mean less variability. A high IQR or range means more variability, and a low IQR or range means less variability.

6 **Reflect** Have all students focus on the strategies used to solve the Try It. If time allows, have students discuss their ideas with a partner.

CONNECT IT

➤ **Use the problem from the previous page to help you understand how to describe the variability of a data set.**

1 Look at the second **Model It**. What are the range and the IQR for the Ziplife battery data? How is the IQR represented in the box plot?

Range = 10; IQR = 4; The IQR is the width of the box between Q1 and Q3.

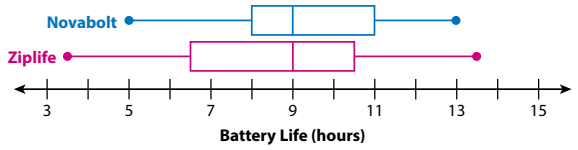
2 Why is the left side of the box wider than the right side of the box?

Possible answer: The data values from the lower quartile to the median are more spread out that the data values from the median to the upper quartile.

3 Could Ziplife use the box plot to support its claim? Explain.

Possible answer: Yes; the box plot shows that half the data are between 7.5 and 10, with a median of 9. So, 9 h is a reasonable amount of time to expect.

4 The researchers test a similar external battery made by the company Novabolt. The double box plot shows the distribution of the data for each company. What can you tell about the center and variability of the two distributions?



Possible answer: They have the same median. The data for Ziplife are more varied than the data for Novabolt because they have a greater range and IQR.

5 How do box plots, range, and IQR help you to analyze variability?

Possible answer: A box plot shows the spread of the data. You can see the data as four quarters. Wider parts show greater variability. A high IQR or range means more variability and a low IQR or range means less variability.

6 **Reflect** Think about all the models and strategies you have discussed today. Describe how one of them helped you better understand how to describe the variability of a data set.

Responses will vary. Check student responses.

DIFFERENTIATION | RETEACH or REINFORCE

Hands-On Activity

Construct a box plot for a data set.

If students are unsure about what each component of a box plot represents, then use this activity to show how different box plots represent different sets of data.

Materials For each student: Activity Sheet *Number Lines*

- Have each student construct any box plot as long as it has a median of 10. Have partner pairs compare the similarities and differences between their box plots.
- Have each student construct any box plot as long as it has a median of 10 and an IQR of 5. Again, have partner pairs compare the similarities and differences between their box plots. Listen for conversation that the size of the boxes are the same, but the location on the number line is different.
- Continue having students individually construct box plots and then compare similarities and differences with a partner. The third box plot will have a median of 10, 8 as Q1, and 13 as Q3. Listen for conversation that the size and location of the box is now the same.
- The fourth box plot will have a median of 10, 8 as Q1, 13 as Q3, and a range of 9. Listen for conversation that the range is 9, but the location of the maximum and minimum vary.

Apply It

For all problems, encourage students to use a model to support their thinking. Allow some leeway in precision; if students choose to construct a box plot to compare data sets, remind them to precisely label the number line in order to correctly compare data sets.

7 Students may recognize that the minimum and Q1 are the same for both data sets. Encourage them to find the other parts of the five-number summary so they can more accurately compare the variability. Some students may choose to make box plots to compare the data as well.

8 Students may recognize that the data set for the advanced class does not include a line to represent the median. This is because the median is equal to either Q1 or Q3. In order to compare the medians for the data set, compare the median for the beginner class with Q3 of the advanced class. Whether the median is equal to Q1 or Q3, the median of the beginner class will still be greater than the median of the advanced class.

Apply It

➤ **Use what you learned to solve these problems.**

7 Two food truck owners record the number of people who visit their trucks each day for one week. Which data set has more variability? Explain.

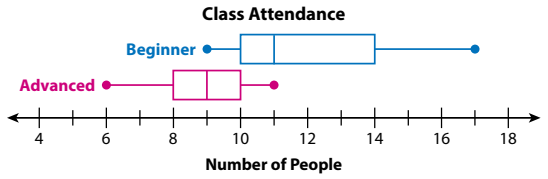
Possible work:

Truck 1: 15, [16,] 32, (36,) 45, [50,] 60 IQR: 50 − 16 = 34

Truck 2: 15, [16,] 22, (25,) 26, [65,] 72 IQR: 65 − 16 = 49

Truck 2 has more variability because it has a greater IQR than Truck 1.

8 A yoga teacher leads a beginner class and an advanced class. He records the number of people who attend each class for several months. The box plots show the data. Compare the measures of center and variability.



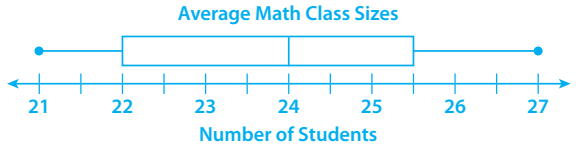
Possible answer: The beginner class has a greater median, which means more people typically attended each class. The advanced class has a lesser IQR, which means it has a more consistent number of attendees.

9 Ignacio researched the average number of students per math class in different countries. The table shows the data. Display the data in a box plot.

Number of Students								
21.0	21.5	25.0	21.5	22.0	27.0	22.5	22.0	23.0
23.5	24.0	25.0	25.0	25.0	26.0	26.0	27.0	

Possible work:

21, 21.5, 21.5, [22, 22,] 22.5, 23, 23.5, (24,) 25, 25, 25, [25, 26,] 26, 27, 27



CLOSE EXIT TICKET

9 Students' solutions should show an understanding of:

- how to organize data to determine the five-number summary.
- how to use the five-number summary to construct a box plot.

Error Alert If students forget to order the values when making the box plot, then have them write each value on a small piece of paper. They can order the pieces of paper by the values written on each one, and then work from that list to find the five-number summary and make the box plot.

Practice Using Box Plots and IQR to Describe Variability

Problem Notes

Assign **Practice Using Box Plots and IQR to Describe Variability** as extra practice in class or as homework.

1 **a.** Students may wish to use a ruler to help them draw the number line or box for their box plot. ***Basic***

b. Students may remember that the range is provided in the Example. The question asks them to interpret the range. ***Medium***

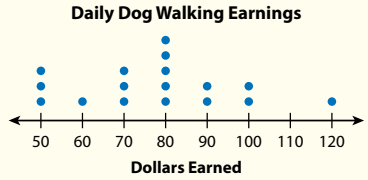
c. Students may use the definition of IQR to help them answer the question. Students may realize that the IQR is a value that is not in the data set. The IQR is the difference between Q1 and Q3, not a value listed in the data set. ***Medium***

Practice Using Box Plots and IQR to Describe Variability

➤ **Study the Example showing how to find measures of variability of a data set. Then solve problems 1–4.**

Example

Julio earns money as a dog walker. He charges $10 per dog. He records how much money he earns each day in a dot plot. Find the range and IQR of his data.



Identify the maximum and minimum data values, the median, and the upper and lower quartiles.

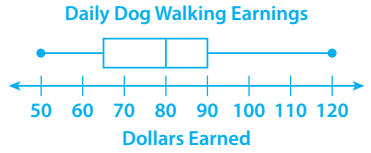
50, 50, 50, 60, 70, 70, 70, 80, 80, 80, 80, 90, 90, 100, 100, 120

minimum; Q1 = 65; median; Q3 = 90; maximum

Range: maximum − minimum = 120 − 50 = 70

IQR: Q3 − Q1 = 90 − 65 = 25

1 **a.** Use a box plot to display the data from the Example.



b. What does the range tell you about the data?

The difference between the least and greatest amount of money earned per day is $70.

c. What does the IQR tell you about the data?

The IQR tells you 50% of the money earned per day was within a $25 range.

Vocabulary

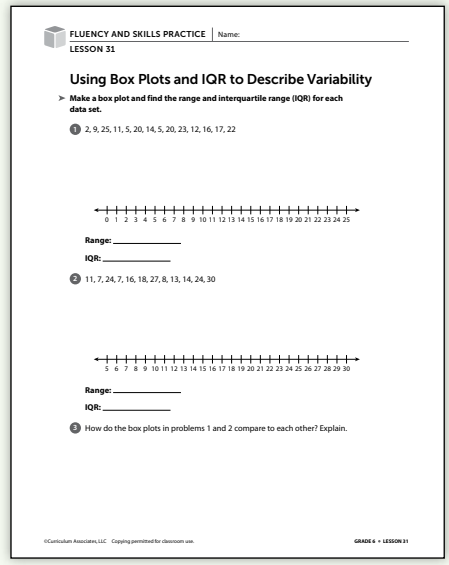
box plot
a visual display of a data set that shows the minimum, the lower quartile, the median, the upper quartile, and the maximum.

interquartile range (IQR)
the difference between the upper quartile and the lower quartile.

Fluency & Skills Practice

Using Box Plots and IQR to Describe Variability

In this activity, students construct box plots from data sets containing even and odd number values. Then they state the IQR and range of each data set.



FLUENCY AND SKILLS PRACTICE | Name:
LESSON 31

Using Box Plots and IQR to Describe Variability

➤ Make a box plot and find the range and interquartile range (IQR) for each data set.

1 2, 9, 25, 11, 5, 20, 14, 5, 20, 23, 12, 16, 17, 22

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25

Range: _____

IQR: _____

2 11, 7, 24, 7, 16, 18, 27, 8, 13, 14, 24, 30

5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30

Range: _____

IQR: _____

3 How do the box plots in problems 1 and 2 compare to each other? Explain.

GRADE 6 • LESSON 31

2 **a.** Students may think the data is already ordered, since the first value listed is 0 and the last is 11. Remind them to verify that all numbers are in order before using a data set to draw a box plot. ***Medium***

b. Students may think that the data sets cannot be compared because the problem only gives the IQR. IQRs can be compared, without having more specific data, because the IQR describes the variability of the middle 50% of the data values. Support students by defining the word *consistent* to ensure they understand its meaning in the context of the problem. ***Medium***

3 **A, B, and D are correct.** Students may solve the problem by finding the five-number summary for the data set and then comparing it with the answer choices.

C is not correct. This answer is the result of confusing Q3 with the maximum number.

E is not correct. This answer is the result of confusing the IQR with the number of values in a data set. Box plots do not show the number of values in a data set.

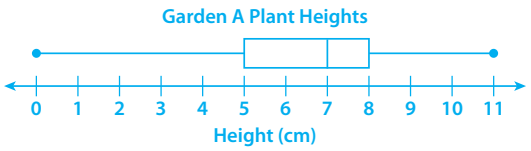
Basic

4 Students may also sketch out generic box plots for data sets with the given IQRs. While they will not be able to complete the box plot because they do not have enough information, they will be able to compare the IQRs visually to answer the question. ***Medium***

2 Students measure the heights, in centimeters, of the plants in two different gardens.

Garden A Plant Heights (cm)								
0	5	6	6	5	5	5	7	8
10	8	7	8	8	8	8	11	7

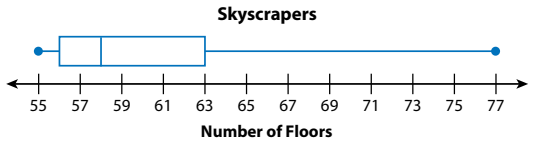
a. The table shows the data for Garden A. Display the data in a box plot.



b. Garden B has plant heights with an IQR of 1.5 cm. Which garden has less variability in its plant heights? Explain.

Garden B; Possible explanation: Garden A has an IQR of 3, which is greater than the IQR of Garden B.

3 The box plot shows the number of floors of some skyscrapers in the U.S. Which statements about the box plot are true?



(A) The range of the data is 22.

(B) The median number of floors is 58.

C The greatest number of floors is 63.

(D) About half of the buildings have 56 to 63 floors.

E There are 15 buildings in the data set.

4 Two airlines report their number of delayed flights each month for one year. Airline A has an IQR of 83.5 and Airline B has an IQR of 22. Which airline is the most consistent in not having delays? Explain.

Airline B; Since it has a lower IQR, it has lower variability and is more consistent.

DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Use with **Session 4** Apply It

Levels 1–3: Reading/Speaking

Use **Notice and Wonder** to prepare students to discuss the box plots in Apply It problem 3. Have students meet with partners to tell what they notice about the box plots. Then ask them to tell what they wonder or want to find out. Next, read the question. Have partners take turns saying and comparing the 5-number summaries for the classes:

- *The ____ for Class 1 is ____ Class 2.*

Read the statements, pausing to restate or explain. Ask questions to check comprehension. (Ex. *Does the IQR increase if you remove . . .?*) Support partners as they discuss the answer choices to determine whether each statement is true.

Levels 2–4: Reading/Speaking

Use **Notice and Wonder** to prepare students to discuss the box plots in Apply It problem 3. Have students meet with partners to tell what they notice about the box plots. Then ask them to tell what they wonder.

Next, read the question and have partners take turns saying and comparing the 5-number summaries for Class 1 and Class 2. Read the statements with students, pausing to restate or explain as needed. Invite students to tell if any of the statements describe what they noticed. Ask students to turn to partners and use **Say It Another Way** to justify their answers using words such as *since, if, then, because, true, not true,* and *false*.

Levels 3–5: Reading/Speaking

Use **Notice and Wonder** to prepare students to discuss the box plots in Apply It problem 3. Have students tell what they notice and wonder about the box plots. Reinforce that students can listen and build on the discussion with their own related ideas.

Next, have students read the problem and then turn to partners to discuss whether the statements are true or false. Encourage the use of words that connect ideas, such as *since, if/then,* and *because*. Invite volunteers to reword the false statements to make them true. For example, students can use "not" to make the sentence true: *If you combined the data from both classes into one box plot, the range would not be 105*.

Refine Interpreting Median and Interquartile Range in Box Plots

Purpose

- **Refine** strategies for finding and interpreting what the median and interquartile range show in the data.
- **Refine** understanding of how data sets can be described using measures of center and variability to answer statistical questions.

START CHECK FOR UNDERSTANDING

The 10-day forecast shows these high temperatures in °F. What is the five-number summary for this data set?

74, 78, 71, 77, 73, 72

Min: 71

Q1: 72

Q2: 73.5

Q3: 77

Max: 78

WHY? Confirm students' understanding of finding the five-number summary for a data set, identifying common errors to address as needed.

MONITOR & GUIDE

Before students begin to work, use their responses to the **Start** to determine those who will benefit from additional support. Use the **Error Analysis** table below to guide remediation.

Have all students complete the Example and problems 1–3, using Consider This and Pair/Share as appropriate. Observe and monitor their reasoning and guide or redirect students as needed.

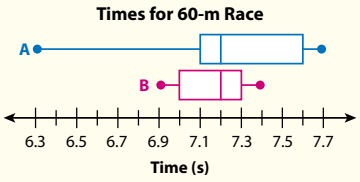
LESSON 31 | SESSION 4

Refine Interpreting Median and Interquartile Range in Box Plots

➤ **Complete the Example below. Then solve problems 1–9.**

Example

A track team needs another runner. The team analyzes the times for the 60-m dash for Runner A and Runner B. What information can you learn from the box plots?



Look how you could analyze the distributions.

A: median = 7.2; IQR = 7.6 − 7.1 = 0.5; range = 7.7 − 6.3 = 1.4

B: median = 7.2; IQR = 7.3 − 7.0 = 0.3; range = 7.4 − 6.9 = 0.5

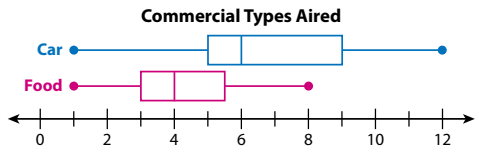
SOLUTION Both runners typically run the 60-m race in 7.2 s. However, Runner B has more consistent times because her IQR and range are less than the IQR and range for Runner A.

CONSIDER THIS . . . Which runner is more consistent? How do you know?

PAIR/SHARE How do you think the data for the box plots were collected and measured?

Apply It

1. Two types of commercials during awards shows are tracked over a 25-year period. Which type of commercial was more consistently aired? Explain.



Food commercials; Possible explanation: The shorter box for food commercials shows that the data for food commercials has less variability.

CONSIDER THIS . . . Without calculating, how do you know that the data for car commercials have a greater range?

PAIR/SHARE What are the medians and what do they tell you about the data?

START ERROR ANALYSIS

If the error is . . .	Students may . . .	To support understanding . . .
7	have found the range in place of one of the values in the five-number summary.	Ask students to refer back to Model It in Session 3 to review the five-number summary. Prompt students to organize and label this data set in a similar way to identify each value.
Q2: 73 (or 74)	have missed including a data value and found the median of a data set with 9 values.	Ask students to copy the data set as presented first, and then cross off each data value as they order the set from least to greatest to make sure they do not miss any values.
Q1: 72; Q2: 73; Q3: 76	not have included the repeated number 78 when determining the five-number summary.	Ask students to identify if the Q2 or the Q1/Q3 are found by identifying the number halfway between two values. Describe how the calculation changes based on whether the set has an odd or even number of data values.

Example

Guide students in understanding the Example. Ask:

- *What do A and B represent on the box plot?*
- *What is the scale for the number line?*
- *How do median and interquartile range help you solve this problem? How else might you solve it?*

Help all students focus on the Example and responses to the questions by reminding students to suggest other reasons that they know the solution is correct or reasonable.

Look for understanding that the IQRs and ranges must be compared because the medians are the same in each data set.

Apply It

1. Students may solve the problem by describing which value from the five-number summaries on each box plot that they compared in order to analyze the data sets. ***DOK 3***

2. See ***Connect to Culture*** to support student engagement. Students may recognize that the box plot does not have a left whisker, which means that the minimum and lower quartile are the same number. ***DOK 3***

3. **A is correct.** Students should recognize that moving students from the upper quartile of Class 1 would increase the median of Class 2.

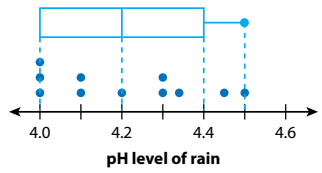
   B is not correct. This answer is the result of assuming that removing lower values would increase the IQR. Removing the lower values would decrease the IQR.

   C is not correct. This answer is the result of adding the ranges.

   D is not correct. This answer is the result of adding the medians.

   DOK 3

2. Acid rain damages a town's trees. During each rainfall, a scientist collects data on the rain's pH (a measure of acidity). The dot plot shows her data. Draw a box plot above the dot plot to display the distribution. About what percent of the data does the box represent? Explain how the box plot shows this.

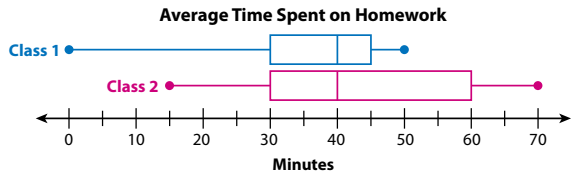


See graph; about 75%; The lower quartile is equal to the minimum, so there is no left whisker.

CONSIDER THIS . . .
In a box plot, the lines that connect the box to the maximum and minimum are sometimes called *whiskers*.

PAIR/SHARE
Suppose a box plot does not show a line for the median inside the box. What would this tell you about Q1, Q2, and Q3?

3. Which statement about the box plots is true?



A Moving the students from the top 25% of Class 1 into Class 2 would increase the Class 2 median.

B Removing the data for students in Class 1 who did not homework would increase the Class 1 IQR.

C Combining the data from both classes into one box plot would result in a box plot with a range of 105.

D Combining the data from both classes into one box plot would double the current median for Class 1.

Noor chose B as the correct answer. How might she have gotten that answer?
Possible answer: Noor thinks that the IQR is affected by outliers.

CONSIDER THIS . . .
Think about the 5-number summary for each class and then compare them.

PAIR/SHARE
How would Noor have gotten C as an answer?

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GROUP & DIFFERENTIATE

Identify groupings for differentiation based on the **Start** and problems 1–3. A recommended sequence of activities for each group is suggested below. Use the resources on the next page to differentiate and close the lesson.

Approaching Proficiency
- **RETEACH** Hands-On Activity
- **REINFORCE** Problems 4, 5, 6

Meeting Proficiency
- **REINFORCE** Problems 4–8

Extending Beyond Proficiency
- **REINFORCE** Problems 4–8
- **EXTEND** Challenge

Have all students complete the **Close: Exit Ticket**.

Resources for Differentiation are found on the next page.

Refine Interpreting Median and Interquartile Range in Box Plots

Apply It

4 Students may recognize that there are two parts to this answer. They must compare the medians, which are the measures of center, and the range or IQR, which are measures of variability. ***DOK 2***

5 **B, D, and E are correct.** Students may recognize that any value in the range could be in the data set. They should recognize that the range is given in the problem and understand that 50% of the values must be less than the median, which is 5.

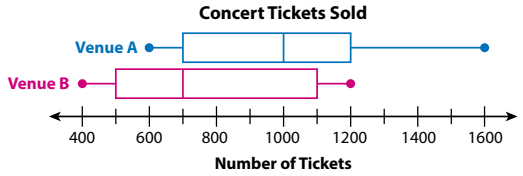
A is not correct. This answer is the result of assuming that all values in a range must be included in a data set.

C is not correct. This answer is the result of finding the sum of the numbers in the problem, $14 + 1 + 5 = 20$. The range, 14, is given in the problem. The problem also states that each pig has at least 1 spot, which means 1 is the minimum. The maximum must be 15, because $1 + 14 = 15$.

DOK 2

6 Students may find it helpful to draw box plots for the data sets in order to compare the medians. ***DOK 2***

4 Two concert venues record the number of tickets they sell at each show for a year. The box plots show the distribution of the data. Compare the measures of center and variability and explain what they mean in terms of the problem.



Possible answer: Venue A typically sells more tickets, because the median of A is 1,000 tickets and the median of B is 700 tickets. Venue B has a greater IQR (600 > 500), while Venue A has a greater range (1,000 > 800).

5 A farm has 14 spotted pigs. Each spotted pig has at least 1 spot. The median number of spots is 5. The difference between the maximum number of spots and the minimum number of spots is 14.

Which statements are true? Select all that apply.

A There must be at least 1 pig with 5 spots.

B One of the pigs could have 3 spots.

C A pig could have more than 20 spots.

D All pigs must have 15 or fewer spots.

E At least 1 pig has fewer than 5 spots.

6 Researchers test the miles per gallon for two cars. Which car has the greater median? Explain what this means in terms of the situation.

Car 1: 15, 16, 17, 18, 20, 22, (25), 27, 28, 29, 29, 29, 30

Car 2: 19, 19, 19, 19, 19, 20, (22), 22, 22, 24, 24, 25, 25

Car 1 has a greater median (25 > 22). This means that Car 1 typically gets more miles per gallon.

Car 1					Car 2				
30	15	16	17	18	25	24	22	22	19
20	22	29	29	25	20	19	22	19	24
27	28	29			25	19	19		

DIFFERENTIATION

RETEACH

Hands-On Activity

Construct and interpret a box plot for a self-generated number set.

Students approaching proficiency with interpreting median and IQR in box plots will benefit from generating data in order to construct box plots and find the median and IQR.

Materials For each pair: 2 number cubes, Activity Sheet *Number Lines*

- Display directions for students to generate data: *Work with a partner to roll a number cube 10 times, and record the numbers rolled as Set A. Then roll 2 number cubes together a total of 10 times, and record the results as Set B. Compare the data sets. Write at least one question that could be answered by your data.*
- Ask: *What is the five-number summary for each data set?* [Answers will vary.] Direct students to use the Activity Sheet to make a box plot for each data set.
- Ask: *How do the measures of center and variability for your data sets compare?* [Possible answers: The median for Set B is greater than it is for Set A because the possible values for Set B are from 2 to 12. The range is less for Set A because the possible values are from 1 to 6.]
- Have pairs write at least one question that could be answered by using their data sets. They may write a context for the data sets if that is helpful. If pairs need help thinking of questions, have them look back to the lesson for guidance.
- Have pairs share their questions. If no one offers these questions, discuss the following: *Do either of your data sets have one or more outliers? Based on your data sets, what number would you expect to roll on your 11th roll and why? How will the range change if you roll more than 10 times? Would you expect the IQR to change? Why?*

7 Students should recognize that they can make a box plot even though the complete data set is not given. Only the five-number summary is used to make a box plot, so they have enough information. ***DOK 3***

8 Students may recognize that more information is provided than they need to solve this problem. They only need to find the IQR for Team A and compare it with the IQR for Team B. ***DOK 2***

CLOSE EXIT TICKET

9 **Math Journal** Look for understanding that 10 will be the median and 7 will be the IQR of the data set.

Error Alert If students construct a data set with a range or IQR of 10 and a median of 7, then review the definitions of *measure of center* and *measure of variability*. Have students explain each definition and identify these in other problems in the lesson.

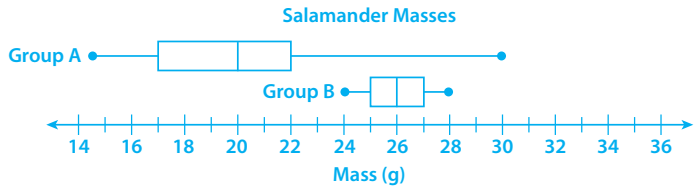
End of Lesson Checklist

INTERACTIVE GLOSSARY Support students by suggesting they include a small data set with the calculation for IQR as part of their entry.

SELF CHECK Have students review and check off any new skills on the Unit 7 Opener.

7 A scientist reports the the information shown about the masses, in grams, of two groups of salamanders. Construct a box plot for each data set. Which group of salamanders shows greater variability in mass? Explain how you know.

	Min	Q1	Median	Q3	Max
Group A	14.5	17.0	20.0	22.0	30.0
Group B	24.0	25.0	26.0	27.0	28.0



Group A; Possible explanation: The IQR for Group A is 22.0 − 17.0 = 5.0, and the IQR for Group B is 27.0 − 25.0 = 2.0. Group A shows more variability in mass because it has the greater IQR.

8 Hockey Team A records its final score for every game in a season. The minimum is 0, Q1 is 1, the median is 2, Q3 is 3, and the maximum is 5. Hockey Team B's final scores for the season have an IQR of 6. Which team is more consistent in its number of points scored per game? Explain your reasoning.

Team A; Possible explanation: Team A's IQR is 3 − 1 = 2. Team B's IQR, 6, is greater than Team A's. This means the middle 50% of Team B's scores is much more spread out than the middle 50% of Team A's scores.

9 **Math Journal** Construct a data set that has 10 as a measure of center and 7 as a measure of variability. Explain what these measures mean in terms of the data.

Possible answer: Dollars spent on lunch at a food truck:

4, 4, [8], 9, 9, (10), 12, 12, [15], 18, 18

People typically spent about $10 on lunch. The middle 50% of the people spent from $8 to $15.

End of Lesson Checklist

- ☐ **INTERACTIVE GLOSSARY** Write a new entry for *interquartile range (IQR)*. Tell how the interquartile range (IQR) of a data set varies with the situation.
- ☐ **SELF CHECK** Go back to the Unit 7 Opener and see what you can check off.

714

REINFORCE

Problems 4–8

Solve problems interpreting median and IQR.

Students meeting proficiency will benefit from additional work with interpreting median and IQR in box plots by solving problems in a variety of formats.

- Have students work on their own or with a partner to solve the problems.
- Encourage students to show their work.

EXTEND

Challenge

Solve median and IQR problems with fractions.

Students extending beyond proficiency will benefit from solving median and IQR problems with fractional values.

- Have partners work to solve this problem: *Becky weighs muffins sold at her bakery on Saturday (in ounces): $1\frac{1}{4}$, $1\frac{1}{2}$, $1\frac{1}{3}$, $1\frac{2}{3}$, $1\frac{7}{8}$, $1\frac{4}{5}$, $1\frac{1}{6}$. The muffins Becky sold on Friday had a median of $1\frac{2}{5}$ and an IQR of $\frac{1}{2}$. How do the data sets compare?*
- Some pairs may change all to have common denominators. [Saturday data: median: $1\frac{1}{2}$; IQR: $\frac{11}{20}$] Pairs should find that the median and IQR for Friday are less than Saturday's median and IQR.

PERSONALIZE

i-Ready

Provide students with opportunities to work on their personalized instruction path with *i-Ready* Online Instruction to:

- fill prerequisite gaps.
- build up grade-level skills.

Overview | Interpret Mean and Mean Absolute Deviation

STANDARDS FOR MATHEMATICAL PRACTICE (SMP)

SMP 1, 2, 3, 4, 5, and 6 are integrated into the Try-Discuss-Connect routine.*

This lesson provides additional support for:

2 Reason abstractly and quantitatively.

* See page 1q to learn how every lesson includes these SMP.

Objectives

Content Objectives

- Calculate the mean of a data set.
- Calculate the MAD of a data set.
- Interpret the mean and MAD of data sets in different contexts.
- Determine the effect of outliers on the mean and MAD of data sets.

Language Objectives

- Compare strategies to calculate the mean of a data set by explaining how the strategies are the same and different.
- Understand the terms *mean absolute deviation (MAD)* and *generally* and use them to describe how much data values generally vary from the mean.
- Ask and answer questions about mean and MAD during partner and class discussion.
- Explain the effect of outliers on measures of center orally and in writing, using the lesson vocabulary.

Prior Knowledge

- Understand measures of center and variability of a data set.
- Find outliers in a data set.
- Construct a dot plot to visualize a data distribution.

Vocabulary

Math Vocabulary

mean the sum of a set of values divided by the number of values. This is often called the *average*.

mean absolute deviation (MAD) the sum of the distances of each data point from the mean of the data set divided by the number of data points. It is always positive.

Review the following key terms.

measure of center a single number that summarizes what is typical for all the values in a data set. Mean and median are measures of center.

measure of variability a single number that summarizes how much the values in a data set vary. Mean absolute deviation and interquartile range are measures of variability.

median the middle number, or the halfway point between the two middle numbers, in an ordered set of values.

outlier a data value that is much greater or much less than most of the other values in the data set. An outlier seems to not quite fit with the rest of the data points.

variability how spread out or close together values in a data set are.

Academic Vocabulary

distributed equally shared or divided so that each gets the same amount.

generally most of the time or in most cases.

Learning Progression

In earlier grades, students constructed line plots and interpreted data. They worked with measurement techniques, which helped with interpreting data sets. They used addition and division to find equal groups of a total.

Earlier in Grade 6, students analyzed the distribution of data using dot plots and histograms. They found the median and IQR as measures of center and variability. They identified outliers and asked meaningful questions about data.

➤

In this lesson, students learn another way to find the center of a data set, the mean. They also learn to find the mean absolute deviation, or MAD. They understand how outliers affect these measures. They interpret these measures in different contexts and visualize the way they represent different distributions of data.

➤

In Grade 6, students will determine which measure of center and variability should be used in various contexts. They will summarize data using the different measures.

In Grade 7, students will compare populations after reasoning about random samples of data. They will use measures of center and variability to interpret the given data for a population.

Pacing Guide

Items marked with ↖ are available on **Teacher Toolbox**.

	MATERIALS	DIFFERENTIATION
SESSION 1 **Explore** The Mean of a Data Set (35–50 min)		
• **Start** (5 min) • **Try It** (5–10 min) • **Discuss It** (10–15 min) • **Connect It** (10–15 min) • **Close: Exit Ticket** (5 min) **Additional Practice** (pages 719–720)	**Math Toolkit** counters, graph paper, number lines, sticky notes Presentation Slides ↖	**PREPARE** Interactive Tutorial ↖ **RETEACH or REINFORCE** Hands-On Activity **Materials** For each group: 36 paper clips
SESSION 2 **Develop** Finding the Mean (45–60 min)		
• **Start** (5 min) • **Try It** (10–15 min) • **Discuss It** (10–15 min) • **Connect It** (15–20 min) • **Close: Exit Ticket** (5 min) **Additional Practice** (pages 725–726)	**Math Toolkit** connecting cubes, counters, graph paper, number lines Presentation Slides ↖	**RETEACH or REINFORCE** Hands-On Activity **Materials** For each student: 10 counters, ruler **REINFORCE** Fluency & Skills Practice ↖ **EXTEND** Deepen Understanding
SESSION 3 **Develop** Finding and Interpreting Mean Absolute Deviation (45–60 min)		
• **Start** (5 min) • **Try It** (10–15 min) • **Discuss It** (10–15 min) • **Connect It** (15–20 min) • **Close: Exit Ticket** (5 min) **Additional Practice** (pages 731–732)	**Math Toolkit** graph paper, number lines, sticky notes Presentation Slides ↖	**RETEACH or REINFORCE** Hands-On Activity **Materials** For each student: 11 counters, ruler, Activity Sheet *Number Lines* ↖ **REINFORCE** Fluency & Skills Practice ↖ **EXTEND** Deepen Understanding
SESSION 4 **Refine** Interpreting Mean and Mean Absolute Deviation (45–60 min)		
• **Start** (5 min) • **Monitor & Guide** (15–20 min) • **Group & Differentiate** (20–30 min) • **Close: Exit Ticket** (5 min)	**Math Toolkit** Have items from previous sessions available for students. Presentation Slides ↖	**RETEACH** Visual Model **REINFORCE** Problems 4–7 **EXTEND** Challenge **PERSONALIZE** i-Ready
Lesson 32 Quiz ↖ or **Digital Comprehension Check**		**RETEACH** Tools for Instruction ↖ **REINFORCE** Math Center Activity ↖ **EXTEND** Enrichment Activity ↖

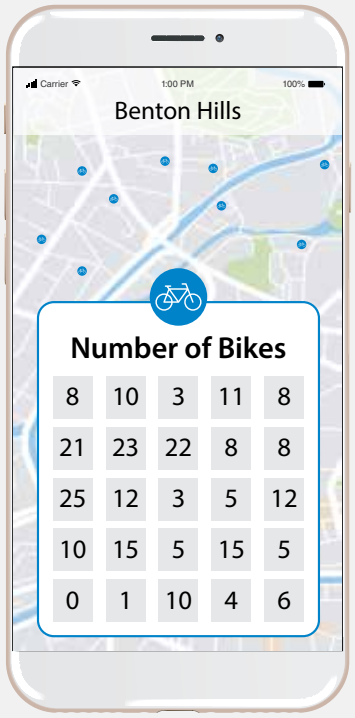
Overview | Interpret Mean and Mean Absolute Deviation

Connect to Culture

➤ **Use these activities to connect with and leverage the diverse backgrounds and experiences of all students. Engage students in sharing what they know about contexts before you add the information given here.**

SESSION 1 ■ □ □ □

Try It Ask students if they have ever ridden a bike or would ever like to ride a bike. Some cities have become so crowded that they are looking for any opportunity to reduce the number of cars on the streets and people in the subway. One solution is bike-sharing. Bike-sharing allows a person to pick up a bike from a certain location and then drop it off later, either at this same location or at another designated spot. People who use the program do not have to worry about storing or locking their own bikes, and they can ride a bike instead of getting stuck in car traffic or taking a crowded bus or subway train. Some bike-sharing programs have as many as 90,000 bicycles. Discuss with students the advantages and disadvantages of participating in a bike-sharing program.



SESSION 2 ■ ■ □ □

Try It Survey students to see if they have ever been involved in Earth Day activities. The roots of Earth Day go back to the 1960s in the United States, when people were starting to realize the harmful effects of pollution on the environment. A senator from Wisconsin decided to start teaching groups of college students about these harmful effects. In 1970, Earth Day became an official holiday. Earth Day has been effective at raising awareness of the harm that humans can cause to the planet. Ask students to generate a list of possible Earth Day activities that could be done in their neighborhoods or towns.

SESSION 3 ■ ■ ■ □

Try It Ask students to share examples of science experiments they have performed themselves or ones they have seen performed. One popular science experiment that has grown into a competition is balloon-powered cars. Directions for how to make these cars can be found online, including on the NASA website. Each group builds a car that is propelled by an inflated balloon. Sometimes these cars can travel as far as 70 feet! Encourage a discussion about other popular science experiments that could be made into a competition.

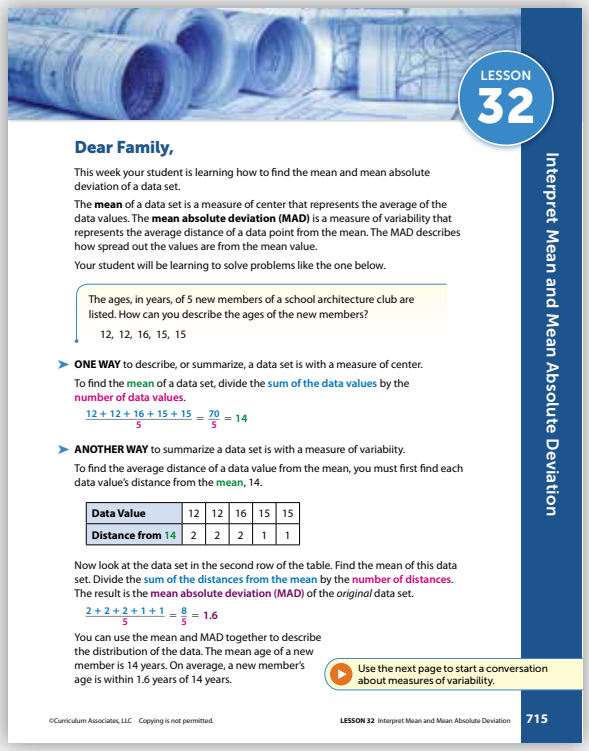
SESSION 4 ■ ■ ■ ■

Apply It **Problem 7** One popular Olympic sport is speed skating. It first appeared in 1924 at the Olympic Winter Games, although the sport itself had been around a long time before that. In the 1300s, the Dutch used skates to travel over frozen canals to different towns. The first competition involving skating may date back to 1676 in the Netherlands, but officially, the first recorded competition was in 1863 in Norway. Now, the sport features sleek, colorful suits and long, sharp blades on the skates that allow skaters to travel up to 40 miles per hour. Ask students what their favorite Olympic sports are and why they enjoy watching them.

Lap Times (seconds)			
26.6	26.5	25.1	29.3
28.5	27.0	28.5	27.7

Connect to Family and Community

➤ **After the Explore session, have students use the Family Letter to let their families know what they are learning and to encourage family involvement.**



LESSON 32

Interpret Mean and Mean Absolute Deviation

Dear Family,

This week your student is learning how to find the mean and mean absolute deviation of a data set.

The **mean** of a data set is a measure of center that represents the average of the data values. The **mean absolute deviation (MAD)** is a measure of variability that represents the average distance of a data point from the mean. The MAD describes how spread out the values are from the mean value.

Your student will be learning to solve problems like the one below.

> The ages, in years, of 5 new members of a school architecture club are listed. How can you describe the ages of the new members?
>
> 12, 12, 16, 15, 15

➤ **ONE WAY** to describe, or summarize, a data set is with a measure of center.

To find the mean of a data set, divide the sum of the data values by the number of data values.

$$\frac{12+12+16+15+15}{5} = \frac{70}{5} = 14$$

➤ **ANOTHER WAY** to summarize a data set is with a measure of variability.

To find the average distance of a data value from the mean, you must first find each data value's distance from the mean, 14.

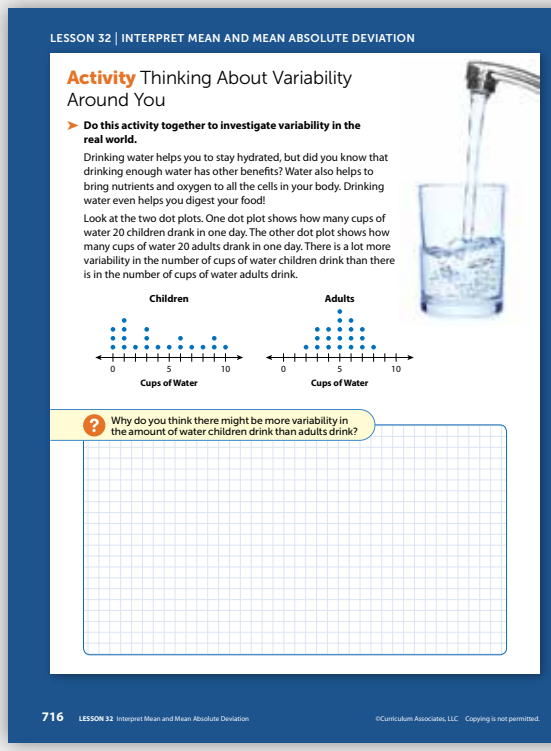
Data Value	12	12	16	15	15
Distance from 14	2	2	2	1	1

Now look at the data set in the second row of the table. Find the mean of this data set. Divide the sum of the distances from the mean by the number of distances. The result is the mean absolute deviation (MAD) of the *original* data set.

$$\frac{2+2+2+1+1}{5} = \frac{8}{5} = 1.6$$

You can use the mean and MAD together to describe the distribution of the data. The mean age of a new member is 14 years. On average, a new member's age is within 1.6 years of 14 years.

Use the next page to start a conversation about measures of variability.



Activity Thinking About Variability Around You

➤ **Do this activity together to investigate variability in the real world.**

Drinking water helps you to stay hydrated, but did you know that drinking enough water has other benefits? Water also helps to bring nutrients and oxygen to all the cells in your body. Drinking water even helps you digest your food!

Look at the two dot plots. One dot plot shows how many cups of water 20 children drank in one day. The other dot plot shows how many cups of water 20 adults drank in one day. There is a lot more variability in the number of cups of water children drink than there is in the number of cups of water adults drink.

? Why do you think there might be more variability in the amount of water children drink than adults drink?

Connect to Language

➤ **For English language learners, use the Differentiation chart to scaffold the language in each session. Use the Academic Vocabulary routine for academic terms before Session 1.**

DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS Use with **Session 1** **Connect It**

Levels 1–3: Reading/Speaking

Support students as they interpret and discuss Connect It problem 2. First, point to the graph and ask students to label the bar graph with the number of bikes at each station. Then ask: *Do the stations have equal numbers of bikes?* Use **Act It Out** to model the problem with unit cubes. Provide sentence frames for students to reference as they work in pairs to distribute the cubes so that all stations have equal numbers of bikes:

- *Station ____ has ____. Station ____ has ____.*
- *The numbers ____ equal.*
- *We can ____. Then we can ____.*
- *Now stations ____ have equal numbers.*

Levels 2–4: Reading/Speaking

Support students as they interpret and discuss Connect It problem 2. Read problems 2a and 2b with students and have them discuss the term *distributed equally.* Then have students use **Say It Another Way** to confirm their understanding of the tasks. Encourage students to refer to the first bar graph to support their paraphrasing.

Ask students to use **Act It Out** to model the problem with unit cubes and then work with partners to describe the number of bikes at each station and discuss how to distribute the bikes equally. Suggest that students use terms like *add, subtract, change,* and *distribute equally* as they discuss.

Levels 3–5: Reading/Speaking

Support students as they interpret and discuss Connect It problem 2. Ask students to read the first paragraph and make a prediction about how the bar graph can help them explore measures of center. Remind students to listen as their classmates share their ideas and to build on to the discussion by adding their own ideas.

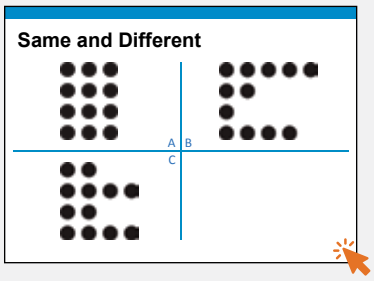
Next, ask students to read problems 2a and 2b with partners and discuss the meaning of *distributed equally.* Have students complete the problems individually. When students have finished, have them turn and talk with their partner to compare and discuss their results.

Explore The Mean of a Data Set

Purpose

- **Explore** the idea that the center of a data set can be summarized with a single number called the mean.
- **Understand** that mean is a measure of center that can be calculated and represents the balance point or fair share of the data set.

START CONNECT TO PRIOR KNOWLEDGE



Possible Solutions

All have 12 dots.

All are organized into four rows.

A has equal rows.

B has all different amounts in each row.

C has two sets of equal rows.

WHY? Support students' facility with divisibility.

TRY IT SMP 1, 2, 4, 5, 6

Make Sense of the Problem

See ***Connect to Culture*** to support student engagement. Before students work on Try It, use **Co-Craft Questions** to help them make sense of the problem. Have students write questions that might be asked about the situation and that they think are answerable by performing calculations with the numbers given in the problem.

DISCUSS IT SMP 2, 3, 6

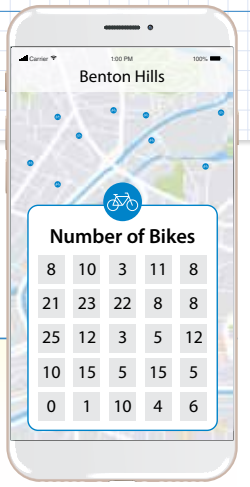
Support Partner Discussion

After students work on Try It, have them explain their work and then respond to Discuss It with a partner. Listen for understanding of:

- data listed in numerical order.
- finding the data value in the middle.

LESSON 32 | SESSION 1 ■ □ □ □

Explore The Mean of a Data Set



Previously, you learned to use the median and interquartile range (IQR) to describe the center and variability of a data set. In this lesson, you will learn about another measure of center, the mean, and another measure of variability, the mean absolute deviation.

➤ **Use what you know to try to solve the problem below.**

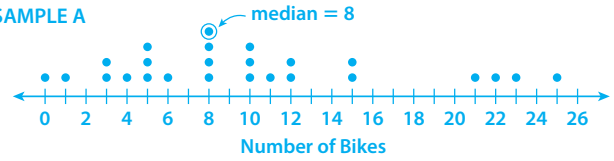
A city has a bike-sharing program. A manager of the program wants to know whether there are enough bikes at the bike stations in the neighborhood of Benton Hills. The data show the number of bikes currently at the neighborhood's 25 bike stations. Based on the data set, what is a typical number of bikes at a station?

TRY IT

Math Toolkit counters, graph paper, number lines, sticky notes

Possible work:

SAMPLE A



The typical number of bikes at a station is about 8.

SAMPLE B

There are 25 data values. Order the data and find the 13th number.

0	1	3	3	4	5	5	5	6	8	8	8	(8)
10	10	10	11	12	12	15	15	21	22	23	25	

The median is 8. The typical number of bikes at a station is 8.

DISCUSS IT

Ask: What did you do first to identify a typical value for the data set? Why did you do this step?

Share: First, I . . . because . . .

Learning Targets SMP 1, SMP 2, SMP 3, SMP 4, SMP 5, SMP 6

- Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.
- Summarize numerical data sets in relation to their context by giving quantitative measures of center and variability, as well as describing any overall pattern and any striking deviations from the overall pattern.

717

Common Misconception Listen for students who believe that the data value with the highest frequency always represents the typical number of bikes for the data set. As students share their strategies, facilitate discussion about whether the median always has the highest frequency. Discuss with students that although the median corresponds to the highest frequency of bikes in this example, this is not true of every data set.

Select and Sequence Student Strategies

Select 2–3 samples that represent the range of student thinking in your classroom. Here is one possible order for class discussion:

- data listed in order from least to greatest or from greatest to least
- **(misconception)** description of the highest frequency as the only typical number
- dot plots that show the median
- histograms showing groupings of values

Facilitate Whole Class Discussion

Call on students to share selected strategies. Prompt students to justify their solutions by providing reasons that they are correct in the situation.

Guide students to **Compare and Connect** the representations. Use turn and talk to help students think through their responses before sharing with the group.

ASK *How do [student name]'s and [student name]'s strategies show the typical number for the data set?*

LISTEN FOR All the data was put in order before finding the middle value.

CONNECT IT

SMP 2, 4, 5

1 **Look Back** Look for understanding that the median can describe the distribution of data as a measure of center.

DIFFERENTIATION | RETEACH or REINFORCE

Hands-On Activity

Model finding a typical number by distributing paper clips to a group.

If students are unsure about what typical numbers represent in a data set, then use this activity to demonstrate equal distribution.

Materials For each group: 36 paper clips

- Place students into groups of 3 or 4. Give each group 36 paper clips, divided unevenly among students.
- Ask: *How can you estimate a typical amount to determine about how many paper clips each person has?* [Students may use various estimation strategies.] Ask: *How can you determine a typical number?* [Students may suggest ways to divide all paper clips into equal groups, listing all values, finding median, or making a dot plot.]
- Have students put all the paper clips into a pile and then take turns removing one paper clip from the pile until all are distributed. Ask: *How many paper clips do you have now?* [12 or 9, depending on the size of the group] Compare estimates to the number found by dividing the paper clips evenly. Ask: *What would happen if you each took different amounts of paper clips?* [There would not be an even distribution that would result in a typical number.] *Is there a more efficient way to find a typical number?* [Divide the number of paper clips by the number of students in the group.]

CONNECT IT

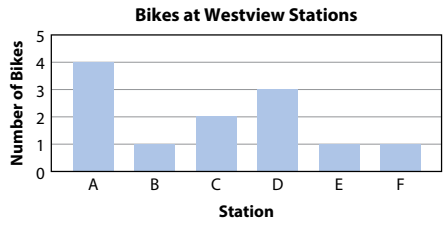
1 **Look Back** Based on the data set, what is a typical number of bikes at a bike station in the Benton Hills neighborhood? Explain.

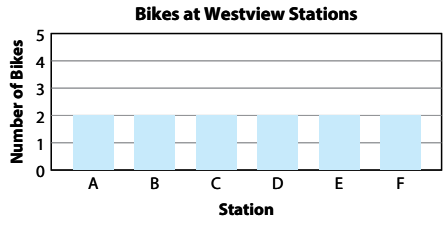
8 bikes; The median of the data set is 8. The median is the middle value in a data set, so it is a good measure of the typical value.

2 **Look Ahead** The median is not the only way to use a single number to summarize a data set. Use the data set shown in the bar graph to explore another measure of center.

a. The first bar graph shows the number of bikes currently at bike stations A–F in the neighborhood of Westview. Complete the second bar graph to show how many bikes would be at stations A–F if all the bikes are distributed equally among the stations.

See graph.





b. How does the number of bikes at each station change when the bikes are distributed equally among the stations?

Station A loses 2 bikes. Stations B, E, and F gain 1 bike. Station C has no change. Station D loses 1 bike.

c. When the bikes are distributed equally, the number of bikes at a station is the **mean**, or average, of the original data set. Like the median of a data set, the mean is a measure of center. What is the mean number of bikes at a station?

2 bikes

3 **Reflect** Why do you think the mean of a data set is sometimes described as a *fair share* of the data set?

The mean gives the amount of each share if the total amount in the data set were shared equally and the number of data values stayed the same.

2 **Look Ahead** Point out that the mean is another measure of center that can answer statistical questions in a data set. Students may recognize that data sets can be described with more than one measure.

Ask a volunteer to rephrase the definition of *mean*. Support student understanding by encouraging students to speak clearly and loudly and by asking others to repeat what is said.

CLOSE EXIT TICKET

3 **Reflect** Look for understanding of the mean as a measure of center that can be calculated and that it represents the balance point or fair share of the data set.

Common Misconception If students fail to describe a *fair share* of a data set as one of the amounts when the total amount in the data set is divided into equal amounts, then direct students to count the number of bikes represented by each bar graph and then compare the numbers. Students should see that the totals are the same because they are the same data, but only the second graph illustrates fair shares.

Prepare for Interpreting Mean and Mean Absolute Deviation

Support Vocabulary Development

Assign **Prepare for Interpreting Mean and Mean Absolute Deviation** as extra practice in class or as homework.

If you have students complete this in class, then use the guidance below.

Ask students to consider the term *variability* by asking several students to rephrase the definition. Clarify that examples of variability can include how variability is interpreted in different data sets.

Have students work in pairs to complete the graphic organizer. Invite pairs to share their completed organizers, and prompt a whole-class comparative discussion of different examples of variability.

Have students look at the question presented in problem 2 and discuss with a partner whether they need to know actual ages of sixth-grade teachers in order to answer the question.

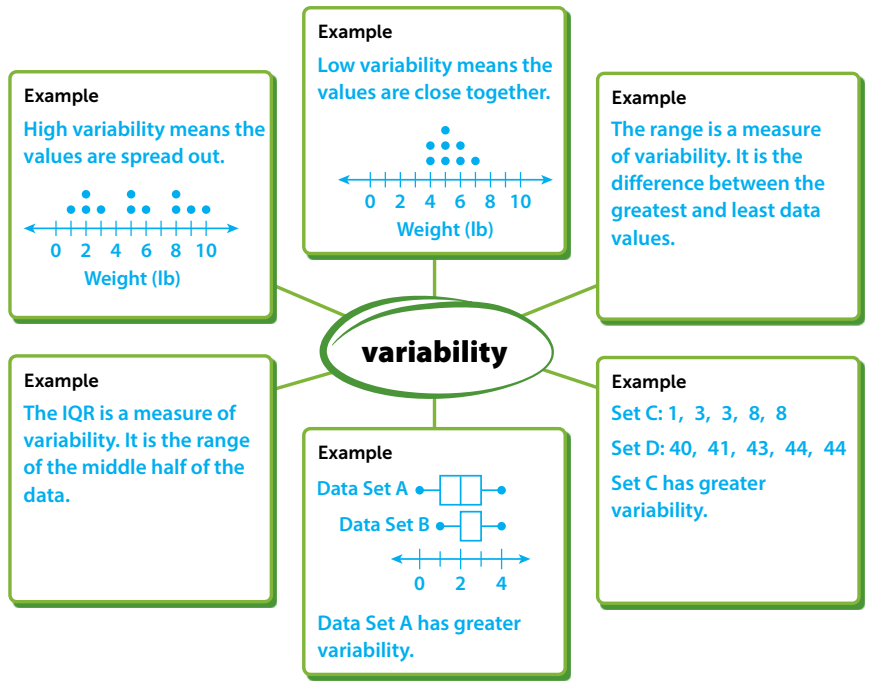
Problem Notes

1. Students may understand that variability refers to how far the data is spread out. Student responses might include examples of high and low variability or that outliers affect variability. Students may recognize that a larger range has more variability.

2. Students may recognize that a large range of ages is possible in the workforce, but students start and progress through school at approximately the same age.

Prepare for Interpreting Mean and Mean Absolute Deviation

1. Think about what you know about data and variability. Fill in each box. Use words, numbers, and pictures. Show as many ideas as you can. **Possible answers:**



2. Which data set would you expect to have more variability: the ages of the 6th grade students at a school or the ages of the 6th grade teachers at the school? Explain.
There is likely to be more variability in the teacher ages. Most 6th grade students are 11 or 12, but 6th grade teachers can be almost any adult age.

REAL-WORLD CONNECTION

In almost every sport, statisticians use numbers to describe facts about both the sport and the athletes. The mean is often used to discuss statistics relating to points, speed, and distance. For example, the sum of the points a team scores in a given season divided by the number of games results in the mean number of points per game. In football, when a running back has a total number of carries in a season, the mean number of yards per carry can be calculated. In baseball, a player's batting average is calculated by the mean number of hits divided by the number of times the player is up to bat. Even miles per hour for runners, bikers, and drivers can be described with a mean. Ask students to think of other real-world examples in sports when calculating the mean is useful for describing performance.

Additional Practice

3 Problem 3 provides another look at describing data quantitatively to find the measure of center. This problem is similar to the problem about the bike-sharing program. In both problems, students must find the center of the data set. This problem asks for students to solve the problem in two ways.

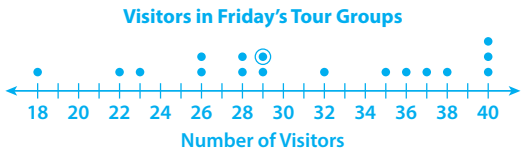
Students may choose to use dot plots and ordering data values to solve.

Suggest that students use **Co-Craft Questions** by reading the problem aloud and thinking about questions that are answerable by doing math.

3 Visitors to a state capitol building can take a free tour. The manager in charge of the tours wants to know a typical number of visitors in a tour group. The list shows the number of visitors in the 17 tour groups on Friday.

38, 23, 40, 35, 36, 28, 40, 26, 40, 37, 22, 32, 18, 28, 29, 26, 29

a. Based on the data set, what is a typical number of visitors in a tour group on Friday? Show your work.



A dot plot of the data shows that the middle value is 29.

SOLUTION A typical number of visitors in a Friday tour group is 29.

b. Check your answer to problem 3a. Show your work.

18, 22, 23, 26, 26, 28, 28, 29, 29, 32, 35, 36, 37, 38, 40, 40, 40

The median of the data values is 29.

A typical number of visitors in Friday's tour groups is 29.

720

DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Use with **Session 2** Apply It

Levels 1–3: Reading/Writing

Prepare students to write about the effect of outliers in Apply It problem 7. Read the problem aloud and discuss the context of archery, as needed. Review *outlier, median,* and *mean*, and ask students to identify each for the data set.

Rephrase problem 7a for students: *The outlier is 0. Remove the outlier. What is the new data set? What is the new mean? What is the new median?* Have students work with a partner to find the mean and median for the data set without an outlier. Then support problem 7b by asking why the median does not change. Encourage students to answer in their own words. Reword responses to model the use of academic language.

Levels 2–4: Reading/Writing

Prepare students to write about the effect of outliers in Apply It problem 7. Use a **Co-Constructed Word Bank** to identify important words and phrases. If needed, suggest that students include terms such as *mean, median, outlier, change,* and *effect*.

Have students identify the mean and median of the data set with all 6 scores. Then ask students to rewrite the data set without the outlier, and to find the new mean and median. For problem 7a, ask students to write responses using *would change* and *would not change*. For problem 7b, provide sentence frames to support writing:

- *All data values affect ____, including ____.*
- *Only the middle data values affect ____.*

Levels 3–5: Reading/Writing

Have students prepare to write about the effect of outliers in Apply It problem 7. Adapt a **Co-Constructed Word Bank** by having students work in pairs to identify key words and phrases. Compile responses into a class word bank. For problem 7a, suggest words that describe how a number might change, such as *increase* and *decrease*. For problem 7b, clarify the phrase *have an effect on* and introduce the phrase *be affected by*.

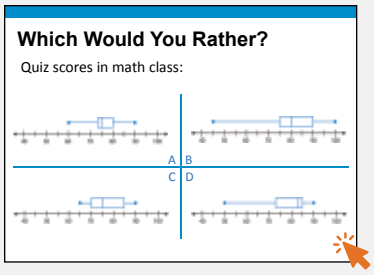
Have students read the problem again and solve independently. Encourage them to write responses using words from the word bank. Then ask partners to compare responses and discuss their use of academic vocabulary.

Develop Finding the Mean

Purpose

- **Develop** strategies for finding the mean of a data set and finding the deviation from the mean.
- **Recognize** that mean is a measure of center of a data set that includes all values in the data set.

START CONNECT TO PRIOR KNOWLEDGE



Possible Solutions

A has the most consistent scores.

B includes a score of 100.

C has the smallest range.

D has the highest median score.

WHY? Support students' facility with comparing values.

DEVELOP ACADEMIC LANGUAGE

WHY? Use context to make sense of the academic phrase *balance point*.

HOW? Read Picture It with students and ask them to circle the phrase *balance point*. Call on volunteers to define *balance* in their own words. Have students look at the diagram with a partner and paraphrase the steps for finding the balance point. After discussing Analyze It, ask students to make connections between mean and balance point.

TRY IT

SMP 1, 2, 4, 5, 6

Make Sense of the Problem

See ***Connect to Culture*** to support student engagement. Before students work on Try It, use **Three Reads** to help them make sense of the problem. Have students focus on identifying quantities and relationships needed to answer the question.

LESSON 32 | SESSION 2 ■ ■ □ □

Develop Finding the Mean

Read and try to solve the problem below.

For Earth Day, volunteers are cleaning up the shore of a lake. A team of 9 students is collecting trash. Their goal is to collect 5 lb of trash per person. The list shows the weight, in pounds, of trash each student collects.

What is a typical amount of trash that a student on the team collects? How does this amount compare to the team's goal?

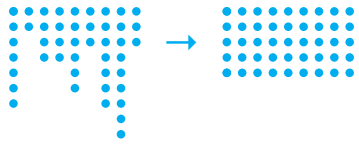
TRY IT

Math Toolkit connecting cubes, counters, graph paper, number lines

Possible work:

SAMPLE A

Each dot represents a pound of trash. Redistribute the dots into 9 equal groups.



Each group has 5 dots, so a typical amount of trash that a student collects is 5 lb. The typical amount is the same as the team's goal.

SAMPLE B

Find the median amount of trash collected.

2, 3, 3, 4, (4), 6, 7, 7, 9

4 lb is a typical amount of trash collected by a student. This is less than the team's goal, but close.

DISCUSS IT

Ask: How is your strategy for finding the typical value similar to mine? How is it different?

Share: My strategy is similar to yours because . . . It is different because . . .

721

DISCUSS IT

SMP 2, 3, 6

Support Partner Discussion

After students work on Try It, encourage them to respond to Discuss It with a partner. If students need support in getting started, prompt them to ask each other questions such as:

- *What is the problem asking?*
- *What do you need to do first?*

Error Alert Listen for students who did not copy all the data values into an organized list. They may have skipped values that appear more than once, or did not carefully check to make sure that all of the data values are accounted for. Have students count the number of data values in the info bubble and the number of values in their organized list. If the numbers of values are not equal, have students cross out each value in the info bubble that appears on their organized list. Facilitate discussion on the importance of including all data values, even if data values repeat.

Select and Sequence Student Strategies

Select 2–3 samples that represent the range of student thinking in your classroom. Here is one possible order for class discussion:

- organizing cubes, counters, or dots to find the mean
- **(misconception)** including repeated values only once
- using graphs to evenly divide the total weight of the trash among 9 students

Facilitate Whole Class Discussion

Call on students to share selected strategies. Prompt students to build on ideas they agree with by showing why the solution is reasonable.

Guide students to **Compare and Connect** the representations. Have students reword ideas by using the sentence starter: *I know [student name]'s approach works because ____.*

ASK *How does this model represent the typical amount of trash?*

LISTEN FOR The total amount of trash is distributed evenly among the total number of students.

Picture It & Analyze It

If students presented these models, have students connect these models to those presented in class.

If no student presented at least one of these models, have students first analyze key features of the models, and then connect them to the models presented in class.

ASK *How is the total weight of the trash represented on the dot plot and explained in the analysis?*

LISTEN FOR The total weight is represented by the location of the dots along the dot plot. The analysis shows the sum of all data values, which is the total weight.

For the dot plot, prompt students to describe how the model changes each time.

- *Why are the dots at different values in the dot plot?*
- *Does the total weight of the trash ever change?*
- *How is the weight represented in the final model?*

For the analysis, prompt students to connect the calculations to the models.

- *How do you know the total number of data values? How is the frequency of the data values represented?*
- *How is the sum of data values used?*
- *Why is division used to find the mean?*

➤ **Explore different ways to find the mean of a data set.**

For Earth Day, volunteers are cleaning up the shore of a lake. A team of 9 students is collecting trash. Their goal is to collect 5 lb of trash per person. The list shows the weight, in pounds, of trash each student collects.

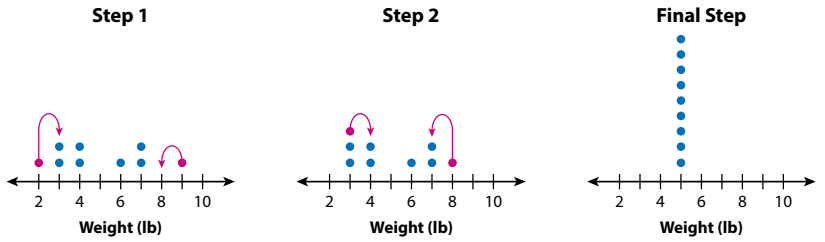
7, 2, 4, 4, 6, 3, 7, 9, 3

What is a typical amount of trash that a student on the team collects? How does this amount compare to the team's goal?

Picture It

You can think of the mean of a data set as the balance point of the data.

To find the balance point, move the left-most and right-most points 1 unit each toward the middle. Repeat this step until all data points stack above a single value.



Analyze It

You can calculate the mean by finding the average of the data.

The average of a data set is the sum of the data values divided by the number of data values.

Sum of data values: $7 + 2 + 4 + 4 + 6 + 3 + 7 + 9 + 3 = 45$

Sum ÷ number of data values: $\frac{45}{9}$

Mean = ?

DIFFERENTIATION | EXTEND

SMP 2

Deepen Understanding

Using Repeated Calculations with Dot Plots and Divisibility to Find the Mean

Prompt students to think about the relationship between the repeated movements of dots along a number line and division.

ASK *Do you always have to move 2 dots in each step? What would happen if you only moved 1 dot? How would an outlier affect the mean?*

LISTEN FOR If you only move 1 dot, you would lose the balance between data values. Data values are distributed evenly among the number of values in the data set when finding mean. An outlier skews data to the right or left.

ASK *If the dots do not line up exactly above a particular value, does that signal that no mean exists? Explain.*

LISTEN FOR The mean does not need to be a whole number. The dots can fall anywhere on the dot plot, including between tick marks. If finding the mean with division, a mean may have a decimal remainder.

Develop Finding the Mean

CONNECT IT

SMP 2, 4, 5, 6

Remind students that the quantities and the relationships between them are the same in each representation. Explain that they will now use those quantities to reason about finding the mean.

Before students begin to record and expand on their work in Picture It & Analyze It, tell them that problem 4 will prepare them to provide the analysis asked for in problem 5.

Monitor and Confirm Understanding 1 – 3

- The data values change in equal amounts.
- The mean represents the balance of values less than the mean with values greater than the mean.
- The mean is a measure of center.

Facilitate Whole Class Discussion

4 Look for understanding that the mean is a measure that can summarize all values in a data set with a single number.

> **ASK** *What is the relationship between the mean and the sum of the values in the data set?*
>
> **LISTEN FOR** The mean is calculated by the sum of the values and the number of values. The values that comprise the sum can change, but if the sum and the total number of values remain the same, the mean is unaffected.

5 Look for the idea that the mean and the median are both measures used to describe a data set, but one can be a better measure than the other in some cases.

> **ASK** *What are advantages to using the mean or the median?*
>
> **LISTEN FOR** The mean is the balance point of the data. It is one of the better measures of center when the range of the data is small. The median can be used to find the center value of the data and is a good measure to use when there are outliers in the data.

> **ASK** *Why is the mean sometimes called a balance point?*
>
> **LISTEN FOR** The mean is a balance point because distances from data points to the mean on either side of the mean are equal.

6 **Reflect** Have all students focus on the strategies used to solve the Try It. If time allows, have students discuss their ideas with a partner.

CONNECT IT

➤ **Use the problem from the previous page to help you understand how to find the mean of a data set.**

1 Look at **Picture It**. What does moving a pair of points represent about the pounds of trash two students collect? Does the total amount the team collects change?

Moving a point 1 unit to the right means one person collects 1 more pound of trash. Moving a point 1 unit to the left means one person collects 1 less pound of trash. The total amount collected does not change.

2 How does the final dot plot show that the team met its goal?

It shows that if everyone collects the same amount of trash, each person would collect 5 lb. So, the team collected 5 lb per person.

3 Look at **Analyze It**. What is the mean weight of trash a student on the team collects? Why could you call this a typical amount of trash collected by a student?

5 lb; The mean tells the average amount collected per person.

4 The dot plots represent three different data sets. Why do the three data sets have the same mean?

Possible answer: Moving the points in pairs does not change the sum of the data values or the number of data values, so the mean does not change.

5 Why is the mean considered a measure of center of a data set? How is the mean different from the median?

Possible answer: The mean is a way to summarize the values in a data set with a single number. It describes a fair share or balance point of the data. The median gives the middle number in a data set.

6 **Reflect** Think about all the models and strategies you have discussed today. Describe how one of them helped you better understand how to find the mean of a data set.

Responses will vary. Check student responses.

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DIFFERENTIATION | RETEACH or REINFORCE

Hands-On Activity

Model the mean as a balance point.

If students are unsure about mean, then use this activity to demonstrate why the mean can be thought of as a balance point.

Materials For each student: 10 counters, ruler

- Display the following data set: 3, 4, 8, 8, 1, 3, 4, 2, 1, 6. On a sheet of paper, have students use a ruler to label a number line with 8 tick marks 1 inch apart and put counters above each value to represent the data.
- Ask: *What is the mean of this data?* [4] Have students move each counter to above 4, keeping track of how many units they move each counter.
- Ask: *What do you notice about the total units the counters to the left of 4 had to move and the counters to the right of 4 had to move to get to 4?* [The totals are the same.]
- Ask: *Why can the mean be described as the balance point of a data set?* [The total units the data points from the left of the mean are moved to get to the mean equals the total units the data points from the right of the mean are moved to get to the mean.]

Apply It

For all problems, encourage students to use a model to support their thinking. Allow some leeway in precision; tick marks on dot plots do not need to be perfectly spaced.

7 **a.** Students may also solve the problem using division to find the mean. Students should understand that they must put the data values in order before finding the median, with the 0 as the first value.

b. Students may understand that mean and median are both measures of center but can differ drastically. The mean represents the average and is affected more by an outlier, and the median represents the middle data point, which is typically not significantly affected by an outlier.

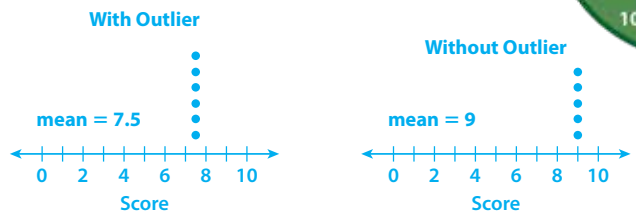
Apply It

➤ **Use what you learned to solve these problems.**

7 Lola is in an archery club. She shoots 6 arrows and earns the scores shown in the list.

a. How does the mean and median change if the outlier of 0 is left out of the data set? Show your work.

Possible work:



Median with outlier:
0, 7, (9, 9,) 10, 10

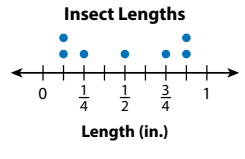
Median without outlier:
7, 9, (9,) 10, 10

SOLUTION The mean increases from 7.5 to 9. The median does not change.

b. Why does the outlier have a greater effect on the mean than on the median?

The median is only affected by the middle values in the data set, but the mean is affected by all the data values, including the outlier.

8 There are several rare insects on display at the insect exhibit at a science museum. The dot plot shows the lengths of the insects to the nearest $\frac{1}{8}$ in. What is the mean length of the insects? Show your work.



Possible work:

sum of data values $= \frac{1}{8} + \frac{1}{8} + \frac{2}{8} + \frac{4}{8} + \frac{6}{8} + \frac{7}{8} + \frac{7}{8} = \frac{28}{8}$

mean $= \frac{28}{8} \div 7 = \frac{4}{8}$

SOLUTION The mean length is $\frac{4}{8}$ in., or $\frac{1}{2}$ in.

CLOSE EXIT TICKET

8 Students' solutions should show an understanding of:

- including all the data values when calculating the mean.
- adding and dividing fractions.

Common Misconception If students find the mean of the frequencies and not the data values, then have them evaluate their solution in the context of the problem.

Practice Finding the Mean

Problem Notes

Assign **Practice Finding the Mean** as extra practice in class or as homework.

1 **a.** Students may understand that they need to adjust the number of values when dividing if more data values are added to the set, such as Lian's 13 commercials. Also, they need to ensure that the data value of 13 is included in the dividend. ***Medium***

b. ***Basic***

Practice Finding the Mean

➤ **Study the Example showing how to find the mean of a data set. Then solve problems 1–4.**

Example

Students in Fiona's class each listen to a different radio station for 20 min one night and count the number of commercials. The list shows their data.

6, 3, 4, 2, 3, 1, 3, 3, 7, 1, 0, 0, 0, 2, 4

What is the mean number of commercials in 20 min?

You can find the mean by adding the data values and then dividing the sum by the number of values. You can use multiplication to group the values that are the same before adding.

$$\text{Mean} = \frac{(3 \cdot 0) + (2 \cdot 1) + (2 \cdot 2) + (3 \cdot 4) + (2 \cdot 4) + 6 + 7}{15}$$

$$= \frac{39}{15} = 2.6$$

The mean is 2.6, so the mean number of commercials in 20 min is 2.6.

1 Lian is absent from class the day the students in the Example combine their data. He counted 13 commercials when he listened for 20 min.

a. How does the mean change when Lian includes his data in the class data set? Show your work.

Possible work:

$$\text{Mean} = \frac{39 + 13}{15 + 1}$$

$$= \frac{52}{16} = 3.25$$

SOLUTION The mean increases from 2.6 to 3.25.

b. Does Lian's value also change the median of the data set? Why or why not?

No; His value is an outlier. An outlier is not near the middle of a data set. The middle number will be close to what it was before.

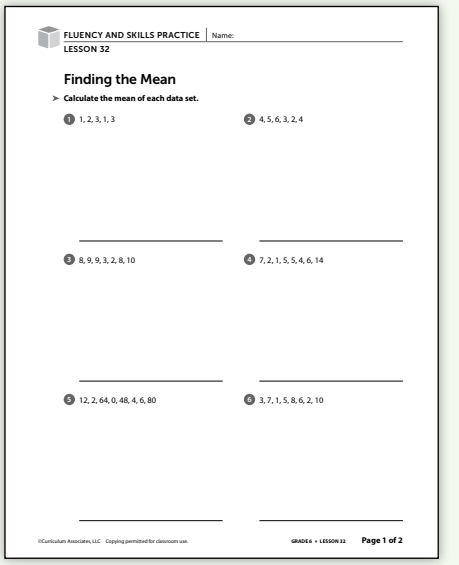
Median without 13: 0, 0, 0, 1, 1, 2, 2, 3, 3, 3, 3, 4, 4, 6, 7

Median with 13: 0, 0, 0, 1, 1, 2, 2, 3, 3, 3, 4, 4, 6, 7, 13

Fluency & Skills Practice

Finding the Mean

In this activity, students calculate the mean of data sets given either as a list or in a dot plot.



FLUENCY AND SKILLS PRACTICE | Name:
LESSON 32

Finding the Mean

➤ **Calculate the mean of each data set.**

1. 1, 2, 3, 1, 3
2. 4, 5, 6, 3, 2, 4
3. 8, 9, 9, 3, 2, 8, 10
4. 7, 2, 1, 5, 5, 4, 6, 14
5. 12, 2, 64, 0, 48, 4, 6, 80
6. 3, 7, 1, 5, 8, 6, 2, 10

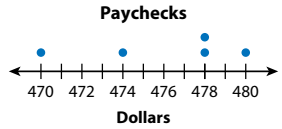
2 Students may also solve by moving pairs of numbers on the dot plot toward the center so that all values are above the same number. The solution should include a dollar sign. ***Basic***

3 **a.** ***Basic***

b. Students may recognize that the mean distributes the data values evenly among the number of values in the data set. The data values are the number of students in each classroom. The number of values in the data set is 10, the total number of classrooms. ***Medium***

4 Students may also solve by finding the mean of both data sets. Students may need to calculate the mean of the original data values and then calculate the mean if 2 people changed teams in order to understand that the mean is not affected if the total number of players and total number of teams remains the same. ***Challenge***

2 The dot plot shows the amounts of Michael's last 5 paychecks. What is the mean of the amounts? Show your work.



Possible work:

sum of data values = 470 + 474 + 478 + 478 + 480
= 2,380

Mean = 2,380 ÷ 5
= 476

SOLUTION The mean is $476.

3 The lists show the numbers of students in each math class at two different schools.

East Middle School: 24, 26, 28, 27, 24, 24, 22, 26, 27, 22

Grove Middle School: 18, 25, 25, 29, 28, 26, 28, 28, 27, 26

a. Which school has the greater mean number of students per math class? Show your work.

Possible work:

East: $\frac{24 + 26 + 28 + 27 + 24 + 24 + 22 + 26 + 27 + 22}{10} = \frac{250}{10} = 25$

Grove: $\frac{18 + 25 + 25 + 29 + 28 + 26 + 28 + 28 + 27 + 26}{10} = \frac{260}{10} = 26$

SOLUTION The mean is greater at Grove Middle School.

b. What do the means of the data sets represent in this situation?

They represent the number of students that would be in each class if each had the same number of students.

4 The table shows the number of players on each team in a softball league. Suppose two people on the team with the most players move to the team with the fewest players. How would the mean number of players per team change? Explain how you know.

Number of Players				
12	11	12	12	11
9	13	15	13	12

When two players change teams, the total number of players in the league does not change and the number of teams does not change. The mean number of players per team stays the same.

DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Use with **Session 3** Apply It

Levels 1–3: Reading/Writing

Help students make sense of Apply It problem 9 with an adaptation of **Three Reads**. After Read 1, use **Act It Out** by asking volunteers to role-play waiting in a waiting room at a doctor's office. For Read 2, review the term *unusual* and rephrase the question: *How much time do patients usually wait? Is 25 minutes usual?* For Read 3, have students identify the mean and MAD and then use a table or number line to solve the problem.

Help students write responses, using:

- *The MAD shows that usual wait times are ____ to ____ minutes.*
- *So, 25 minutes is ____ wait time.*

Levels 2–4: Reading/Writing

Have students make sense of Apply It problem 9 using **Three Reads**. After Read 1, invite students to share their experiences with wait times and waiting rooms. After Read 2, help students discuss words to interpret MAD. Review *usual* and *unusual,* and review that *un-* means *not.* After Read 3, have students discuss how MAD can help them interpret the information.

Encourage students to write responses using *usual* and/or *unusual.* Provide sentence starters for support:

- *The MAD shows that ____.*
- *I know that ____. This means ____.*

Levels 3–5: Reading/Writing

Have students work in pairs to make sense of Apply It problem 9 using **Three Reads**. Ask them to discuss the meaning of *unusual.* Explain that *usual* and *unusual* are two descriptive words that can be used when interpreting MAD. Ask students to list other antonym pairs with similar meanings, like *typical/atypical, expected/unexpected,* and *predictable/unpredictable.*

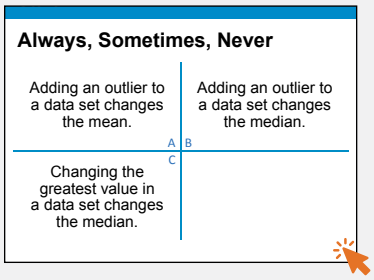
Ask students to solve the problem and write responses independently. Encourage them to use descriptive words as they explain how the MAD helps them interpret the information.

Develop Finding and Interpreting Mean Absolute Deviation

Purpose

- **Develop** strategies for interpreting the amount of variance in a data set.
- **Recognize** that the mean absolute deviation is a single number that summarizes how much the data varies from the mean and that it offers more precision than a verbal description.

START CONNECT TO PRIOR KNOWLEDGE



Possible Solutions

A is always true.

B is sometimes true.

C is never true.

WHY? Support students' understanding of how data values affect mean and median.

DEVELOP ACADEMIC LANGUAGE

WHY? Promote effective mathematical discourse.

HOW? Before students share methods in Discuss It, review that effective conversations include clear explanations and reasons to justify thinking. Encourage listeners to paraphrase the speaker's message to check understanding and then agree and build on or disagree and explain why. Remind students to make connections to prior learning or real-world contexts as they share their ideas. After partners share, call on volunteers to model effective discourse.

TRY IT

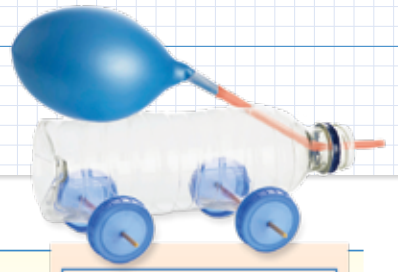
SMP 1, 2, 4, 5, 6

Make Sense of the Problem

See ***Connect to Culture*** to support student engagement. Before students work on Try It, use **Say It Another Way** to help them make sense of the problem. Listen for understanding that the question asks for the difference between the data values and the mean.

LESSON 32 | SESSION 3

Develop Finding and Interpreting Mean Absolute Deviation



Read and try to solve the problem below.

Teams in a science competition make balloon-powered cars. The list shows the distance, in feet, each car travels. The mean distance traveled is 19 ft. How much do the data values generally vary from the mean distance?

Distance Traveled (ft)					
11	12	14	15	16	17
18	21	22	23	28	31

TRY IT

Math Toolkit graph paper, number lines, sticky notes

Possible work:

SAMPLE A

Find the distance from each data value to the mean.

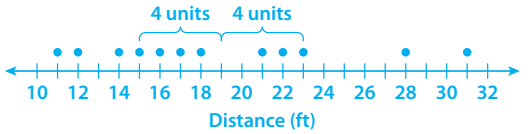
Data value	11	12	14	15	16	17	18	21	22	23	28	31
Distance from 19	8	7	5	4	3	2	1	2	3	4	9	12

Find the median of the distances from the mean:

1, 2, 2, 3, 3, (4, 4), 5, 7, 8, 9, 12; median = 4

A typical data value is about 4 away from the mean.

SAMPLE B



The dot plot shows that about half the data points are within 4 units of the mean, so the data values generally vary from the mean distance by about 4 ft.

DISCUSS IT

Ask: How did you determine the amount that the data values generally vary from the mean distance?

Share: The method I used was . . .

DISCUSS IT

SMP 2, 3, 6

Support Partner Discussion

After students work on Try It, encourage them to respond to Discuss It with a partner. If students need support in getting started, prompt them to ask each other questions such as:

- *What is a typical value of a data set?*
- *What steps can you take for finding how far each data value is from the typical distance?*

Error Alert Listen for students who answer only by listing all the differences from the mean and do not take it further and find the average of these distances. As students share their strategies, redirect their attention to the problem. Ask: *Do these values answer the question? What does "generally vary" refer to in this situation?*

Select and Sequence Student Strategies

Select 2–3 samples that represent the range of student thinking in your classroom. Here is one possible order for class discussion:

- finding differences, and then finding the median of the differences
- making a dot plot with the mean labeled and also the distances labeled from the mean
- creating a box plot with deviations from the mean and reporting an IQR of 5

Facilitate Whole Class Discussion

Call on students to share selected strategies. Reinforce that good listeners ask questions to clarify ideas or ask for more information during math discussions.

Guide students to **Compare and Connect** the representations. Ask a student to reword any unclear statements so that others understand. Confirm with the speaker that the rewording is accurate.

ASK *How does each model show the difference from the mean?*

LISTEN FOR Models may compare each data value to the mean or models may compare the majority of data points to the mean.

Model It & Analyze It

If students presented these models, have students connect these models to those presented in class.

If no student presented at least one of these models, have students first analyze key features of the models, and then connect them to the models presented in class.

ASK *How do the number line and the MAD display the distance traveled by each car?*

LISTEN FOR All the distances traveled are plotted on the number line; the MAD shows the distances from the mean.

For the number line, prompt students to identify how the number line is labeled.

- *What do the seven values to the left of the mean line represent? What do the five values to the right of the mean line represent?*
- *How does distance from the mean connect to absolute value?*

For the MAD, prompt students to compare the values in the table with the expression.

- *How is each part of the MAD represented in the expression?*
- *What does the value 12 represent in the denominator of the expression?*

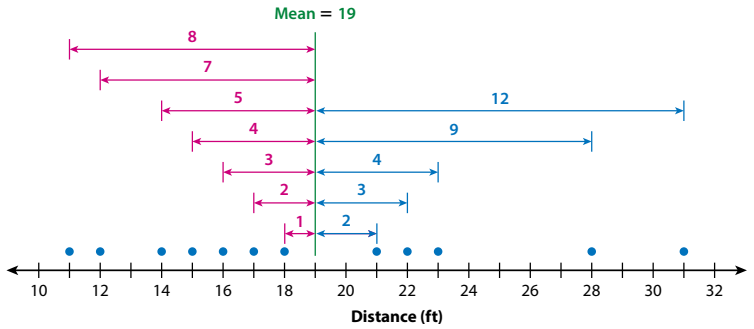
➤ **Explore different ways to understand variability in a data set.**

Teams in a science competition make balloon-powered cars. The list shows the distance, in feet, each car travels. The mean distance traveled is 19 ft. How much do the data values generally vary from the mean distance?

11, 12, 14, 15, 16, 17, 18, 21, 22, 23, 28, 31

Model It

You can use a number line to show how far each data value is from the mean.



Analyze It

You can use the average distance of data values from the mean as a measure of variability. This average distance is called the **mean absolute deviation (MAD)**.

Data Value	Distance from Mean	Data Value	Distance from Mean
11	19 − 11 = 8	21	21 − 19 = 2
12	19 − 12 = 7	22	22 − 19 = 3
14	19 − 14 = 5	23	23 − 19 = 4
15	19 − 15 = 4	28	28 − 19 = 9
16	19 − 16 = 3	31	31 − 19 = 12
17	19 − 17 = 2		
18	19 − 18 = 1		

MAD

$$= \frac{\text{sum of distances from mean}}{\text{number of data values}}$$

$$= \frac{8+7+5+4+3+2+1+2+3+4+9+12}{12}$$

$$= \frac{60}{12}$$

$$= 5$$

DIFFERENTIATION | EXTEND

SMP 2

Deepen Understanding

Using the MAD as a Numerical Descriptor of Variability in Data

Prompt students to think about the importance of MAD as a measure used to describe the distribution of data by having them identify and define the words that compose the acronym MAD.

ASK *How do you define each word in "mean absolute deviation?"*

LISTEN FOR *Mean* is the average. *Absolute* represents the absolute value or distance of each value from the mean. *Deviation* means change or difference. Together, *MAD* represents the average or mean of all the differences from the mean of a data set.

ASK *Why is the MAD an important measure to describe the distribution of values in a data set?*

LISTEN FOR The MAD describes the variability of the values in a data set. More specifically, it describes the spread of the data values in terms of the mean of the data set. The MAD also helps in understanding if a data set has values that are clustered or spread apart.

Develop Finding and Interpreting Mean Absolute Deviation

CONNECT IT

SMP 2, 4, 5, 6

Remind students that the quantities and the relationships between them are the same in each representation. Explain that they will now use those quantities to reason about mean absolute deviation (MAD).

Before students begin to record and expand on their work in Model It & Analyze It, tell them that problem 4 will prepare them to provide the explanation asked for in problem 5.

Monitor and Confirm Understanding 1 – 3

- The difference between the mean and the value of the data point is the distance from each value to the mean.
- The mean is a way to describe the data distribution with a single value.
- Distance is always positive.

Facilitate Whole Class Discussion

4 Confirm understanding that MAD is another way to describe the data quantitatively.

> **ASK** *What is the relationship between the MAD and the mean?*
>
> **LISTEN FOR** MAD is based on the mean. It describes the relationship between the data values and the mean.

5 Look for the idea that MAD shows the spread of the values in the data set.

> **ASK** *How can MAD help you understand the spread of the data?*
>
> **LISTEN FOR** It uses the deviations from the mean and describes how spread out the data set is in compared to the mean of 19. On average, the cars vary 5 feet from the typical distance of 19 feet.

6 Confirm understanding that MAD describes the typical distance from the mean.

> **ASK** *Are the distances that all the cars traveled within the MAD?*
>
> **LISTEN FOR** No, some of the cars traveled more than 5 feet from the mean, but most of them are within that distance.

7 **Reflect** Have all students focus on the strategies used to solve the Try It. If time allows, have students discuss their ideas with a partner.

CONNECT IT

➤ **Use the problem from the previous page to help you understand variability in a data set.**

1 Look at **Model It**. Find the point for the car that traveled 28 ft. How much farther than the mean distance did this car travel? How is this shown in the model?
9 ft; The distance labeled 9 shows the distance from 28 to the mean, 19.

2 Describe what the distance labeled **5** tells you about one of the cars.
The car that traveled 14 ft traveled 5 ft less than the mean distance, 19 ft.

3 Look at **Analyze It**. What do the data values in the left table have in common? What do the data values in the right table have in common? How are the subtraction equations in the tables related to the number line model?
The data values in the left table are less than the mean. The data values in the right table are greater than the mean. Each subtraction equation shows finding one of the distances shown on the number line model.

4 Look at the fractions used to find the MAD. How is the process of finding a MAD like the process of finding a mean? What data set are you finding the mean of?
The fractions show finding the sum of a set of data values and then dividing by the number of values. The data set is the set of distances from the mean.

5 The MAD of the distances the cars traveled is 5. What does a MAD of 5 tell you in this situation?
On average, the distance a car traveled varied from the mean distance by 5 ft.

6 How many cars traveled distances that are within 5 ft of the mean? How does this help you understand MAD as a measure of variability in a data set?
8 cars; Possible explanation: The cars traveled many different distances, but it is typical for the distances to be within 5 ft of the mean distance traveled.

7 **Reflect** Think about all the models and strategies you have discussed today. Describe how one of them helped you better understand variability in a data set.
Responses will vary. Check student responses.

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DIFFERENTIATION | RETEACH or REINFORCE

Hands-On Activity

Model MAD in relation to mean as a balance point.

If students are unsure about the MAD, then use this activity to demonstrate the relationship between the mean and the MAD.

Materials For each student: 11 counters, ruler, Activity Sheet *Number Lines*

- Display the following data set: 6, 3, 10, 14, 14, 12, 2, 4, 6, 15, 2. On a sheet of paper, have students use a ruler to label a number line with tick marks 1 inch apart and put counters above each value to represent the data. Have students move the counters until they balance on 8. Ask: *What does 8 represent?* [the mean]
- Have students draw a representation of their original dot plot on the Activity Sheet, draw arrows from each dot to the mean, and label these distances. [2, 5, 2, 6, 6, 4, 6, 4, 2, 7, 6] Ask: *How can you find the mean of all the distances?* [Add all of the distances and divide by 11, which is the total number of values.]
- Have students find the MAD by placing counters representing the distances between the data values and the mean on another number line. Have them move counters until they reach a balance point. [4.5]

Apply It

For all problems, encourage students to use a model to support their thinking. Allow some leeway in precision; if students draw a dot plot, precise measures between values are not necessary because the model is being used to think through a problem.

8 Students should recognize that values of 0 must be included when calculating MAD.

9 Students may recognize that values greater than the mean and the MAD are possible in a data set.

Apply It

➤ **Use what you learned to solve these problems.**

8 Imani asks 10 students about the number of hours they slept last night. She finds that the mean sleeping time is 9 h. Find the MAD of Imani's data set and describe what it means in this situation. Show your work.

Possible work:

Data Value	7	8	8	9	9	9	10	10	10	10
Distance from Mean	2	1	1	0	0	0	1	1	1	1

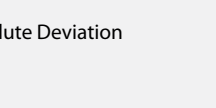
$$\text{MAD} = \frac{2+1+1+0+0+0+1+1+1+1}{10} = \frac{8}{10} = 0.8$$

SOLUTION The MAD is 0.8 h; Possible explanation: The number of hours slept is generally within 0.8 h (or 48 min) of the mean of 9 h.

9 At a doctor's office, the mean amount of time patients spend in the waiting room is 18 min. The MAD of the wait times is 7 min. Based on this information, would it be unusual for a patient to wait for 25 min in the doctor's waiting room? Explain.

No; The MAD shows that the typical distance of the data values from the mean of 18 is 7, and 25 is 7 units from 18 on a number line. So, a waiting time of 25 min would not be unusual.

10 Students in Naomi's class record the number of hours they spend volunteering in May. The dot plot shows their data. The mean time is 3 hours. Find the MAD of the data set. Show your work.



Possible work:

Data Value	0	0	1	2	2	3	5	5	6	6
Distance from Mean	3	3	2	1	1	0	2	2	3	3

$$\text{MAD} = \frac{3+3+2+1+1+0+2+2+3+3}{10} = \frac{20}{10} = 2$$

SOLUTION The MAD is 2 hours.

CLOSE EXIT TICKET

10 Students' solutions should show an understanding of:

- including a value of 0 when calculating measures of center.
- finding the distance from the mean for all data values and finding the mean of the distances to determine the MAD.

Error Alert If students forget or confuse steps when calculating MAD, such as forgetting to divide the deviations by the number of data points, then encourage students to ask themselves whether their answer makes sense. Ask: *Does a MAD of 20 make sense? If the mean is 3 hours, does it make sense that the typical deviation from the mean is 20 hours? What should you do after adding the distances from the mean?*

Practice Finding and Interpreting Mean Absolute Deviation

Problem Notes

Assign **Practice Finding and Interpreting Mean Absolute Deviation** as extra practice in class or as homework.

1 **a.** Students may recognize that MAD shows the average distance of the values in the data set from the mean. The mean of 77 is the balancing point of the data, and most values in the data set are 2.8 inches greater than or less than the mean. ***Basic***

b. Students may understand that they can compare MAD in order to interpret the data distribution and respond to statistical questions. Students compare the means of 77 and 78 that summarize the data. Then students compare MAD to see how much the heights vary from each mean. ***Medium***

Practice Finding and Interpreting Mean Absolute Deviation

➤ **Study the Example showing how to find the mean absolute deviation of a data set. Then solve problems 1–4.**

Example

The list shows the heights, in inches, of the five starting players on a men's college basketball team.

74, 74, 76, 80, 81

The mean height is 77 in. What is the MAD of the heights?

You can find the MAD (mean absolute deviation) of the data by finding the **distance** between each data value and the **mean**. Then find the average of the distances from the mean.

$$\text{MAD} = \frac{\text{sum of distances from mean}}{\text{number of data values}}$$

$$= \frac{3 + 3 + 1 + 3 + 4}{5}$$

$$= \frac{14}{5} = 2.8$$

The MAD of the heights is 2.8 in.

Data Value	Distance from Mean
74	77 − 74 = 3
74	77 − 74 = 3
76	77 − 76 = 1
80	80 − 77 = 3
81	81 − 77 = 4

1 **a.** What does the MAD of the heights in the Example tell you?

On average, the heights of the starting players differ from the mean height by 2.8 in.

b. The heights of the starting players on a different team have a mean of 78 in. and a MAD of 3.6 in. How do the heights of the players on the two teams compare?

On average, the players on the second team are taller than the players on the first team, but the heights of the players on the second team are more spread out from the mean.

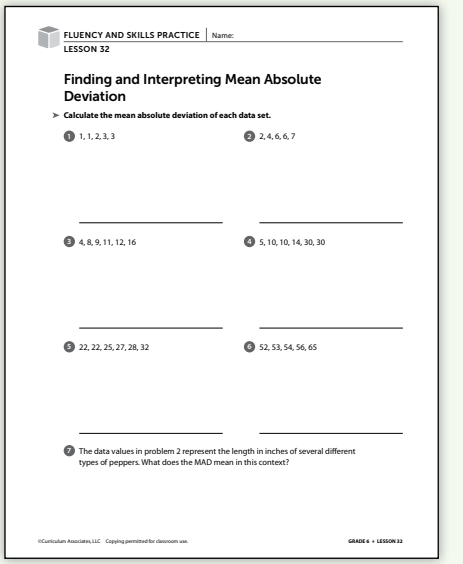
Vocabulary

mean absolute deviation (MAD)
the sum of the distances of each data point from the mean of the data set divided by the number of data points.

Fluency & Skills Practice

Finding and Interpreting Mean Absolute Deviation

In this activity, students calculate the mean absolute deviation of data sets.



FLUENCY AND SKILLS PRACTICE | Name:
LESSON 32

Finding and Interpreting Mean Absolute Deviation

➤ **Calculate the mean absolute deviation of each data set.**

1. 1, 1, 2, 3, 3
2. 2, 4, 6, 6, 7
3. 4, 8, 9, 11, 12, 16
4. 5, 10, 10, 14, 30, 30
5. 22, 22, 25, 27, 28, 32
6. 52, 53, 54, 56, 65
7. The data values in problem 2 represent the length in inches of several different types of peppers. What does the MAD mean in this context?

GRADE 6 • LESSON 33

2 Students may recognize that the number of data values remains the same, so the divisor does not change. ***Medium***

3 Students may understand that MAD uses the deviations from the mean and describes the spread of the data set in terms of the mean. By looking at each number line, students may recognize that Ravi's photos are distributed throughout the number line and Inés's photos are clustered in one area. ***Basic***

4 Students may solve by adding 23 + 6, which is the mean plus the MAD, to get a sum of 29. Then students compare that sum to 29 snacks, which is within the typical number of snacks per bag. ***Challenge***

2 The list shows the weights of the bobcats at a nature reserve. What are the mean and MAD of the weights? Show your work.

Possible work:

$$\text{Mean} = \frac{17 + 20 + 18 + 14 + 26 + 13 + 23 + 22 + 27}{9}$$

$$= \frac{180}{9} = 20$$

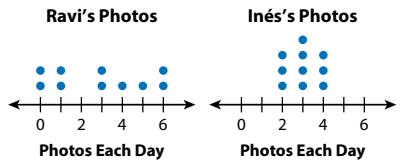
Data Value	17	20	18	14	26	13	23	22	27
Distance from Mean	3	0	2	6	6	7	3	2	7

$$\text{MAD} = \frac{3 + 0 + 2 + 6 + 6 + 7 + 3 + 2 + 7}{9}$$

$$= \frac{36}{9} = 4$$

SOLUTION The mean is 20 lb and the MAD is 4 lb.

3 The dot plots show the number of photos Ravi and Inés took each day for 10 days. How can you tell, without calculating, which data set has a greater MAD?



The dot plots show that Ravi's data set is more spread out than Inés's. The MAD is a measure of variability, or spread, so Ravi's data set has a greater MAD.

4 Paulo counts the number of cherry fruit snacks in 7 bags of mixed fruit snacks. The list shows his data. The mean is 23 cherry snacks per bag.

14, 23, 29, 31, 15, 19, 30

Based on the data, would it be unusual to get a bag with 17 cherry snacks? Use the MAD of the data to support your answer.

No; The MAD is $\frac{9 + 8 + 4 + 0 + 6 + 7 + 8}{7} = \frac{42}{7}$, or 6. A MAD of 6 means that the typical distance of the data values from the mean of 23 is 6, and 17 is 6 units from 23.

DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Use with **Session 4** Example

Levels 1–3: Listening/Speaking

Guide students as they make sense of the Example problem. Support comprehension of the word problem with **Three Reads**.

Explain that each question in the problem corresponds to a paragraph in the sample response. Read the sample responses aloud, pausing to unpack the following sentences and confirm understanding:

- *The mean is a measure of center, so it can represent a typical value in a data set.*
- *The less variability a data set has, the more consistent the data values are.*

Work with students to co-craft a solution using a compound sentence.

Levels 2–4: Listening/Speaking

Guide students as they make sense of the Example problem. Support comprehension of the word problem with **Three Reads**.

Explain that each question in the problem corresponds to a paragraph in the sample response. Read the first question and the first paragraph together. Point out similarities in wording, such as *typically* and *typical,* and *make more money* and *greater than the mean sales.* For the second question and paragraph, help students make connections between the phrase *generally more consistent* and the terms *variability* and *MAD.*

Have students work in pairs to co-craft a solution using a compound sentence.

Levels 3–5: Listening/Speaking

Support students as they interpret the Example problem. Have partners work together to make sense of the word problem using **Three Reads**.

Explain that each question in the problem corresponds to a paragraph in the sample response. Allow time for students to read the paragraphs individually. Have students use **Say It Another Way** with partners to confirm their understanding of the interpretation. Then ask students to write their solutions independently, using the interpretation to support their responses.

Refine Interpreting Mean and Mean Absolute Deviation

Purpose

- **Refine** strategies for finding and using measures of center and measures of variability.
- **Refine** understanding of the utility of MAD and mean.

START CHECK FOR UNDERSTANDING

Find the mean and the MAD.
0, 2, 3, 3, 3, 5, 8, 16

Solution

Mean = 5

MAD = 3.5

WHY? Confirm students' understanding of finding mean and mean absolute deviation (MAD), identifying common errors to address as needed.

MONITOR & GUIDE

Before students begin to work, use their responses to the **Start** to determine those who will benefit from additional support. Use the **Error Analysis** table below to guide remediation.

Have all students complete the Example and problems 1–3, using Consider This and Pair/Share as appropriate. Observe and monitor their reasoning and guide or redirect students as needed.

LESSON 32 | SESSION 4

Refine Interpreting Mean and Mean Absolute Deviation

➤ **Complete the Example below. Then solve problems 1–8.**

Example

On Fridays, the mean amount a smoothie shop makes in sales is $2,287, with a MAD of $314. On Saturdays, the mean amount the shop makes in sales is $1,934, with a MAD of $152. On which of the two days does the shop typically make more money? On which day are the sales generally more consistent?

Look at how you could interpret the means and MADs.

The mean is a measure of center, so it can represent a typical value in a data set. The mean sales for Fridays are greater than the mean sales for Saturdays.

The MAD is a measure of variability. The less variability a data set has, the more consistent the data values are. The MAD for Saturdays is less than the MAD for Fridays.

SOLUTION The shop typically makes more money on Fridays, but its sales on Saturdays are generally more consistent.

CONSIDER THIS . . .
The more consistent the values in a data set are, the closer the values are to each other.

PAIR/SHARE
Would it be unusual for the shop to make $2,500 in sales on a Friday? Explain.

Apply It

1 Roberto sells lemonade to raise money for charity. He collects data on the cost of lemonade at other lemonade stands. He uses the mean of his data as the price of lemonade at his stand. How much does lemonade cost at Roberto's stand? Show your work.

Cost of Lemonade ($)		
2.00	1.00	1.25
1.50	0.50	1.25
1.00	0.50	3.00
1.00	1.25	1.50
1.25	1.25	1.25

Possible work:

$\frac{2 \cdot 0.5 + 3 \cdot 1 + 6 \cdot 1.25 + 2 \cdot 1.5 + 2 + 3}{15} = \frac{19.5}{15} = 1.3$

SOLUTION Lemonade costs $1.30 at Roberto's stand.

CONSIDER THIS . . .
You can find the mean of a set of decimals the same way you find the mean of a set of whole numbers.

PAIR/SHARE
How does the cost of lemonade at Roberto's stand compare to the median cost of lemonade at the other stands?

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START ERROR ANALYSIS

If the error is . . .	Students may . . .	To support understanding . . .
Mean = 40 or MAD = 28	have forgotten to divide when finding the mean of the given data values and instead used the total sum of the data values.	Encourage students to check whether their answers are reasonable. Facilitate a discussion about whether mean or MAD can ever be larger than all the data values. Remind students to show the computation of the sum of the values divided by the number of values in order to ensure they complete all steps.
Mean ≈ 5.7 or MAD = 4	have neglected to use the value of 0 in their calculations.	Have students think about counters above a number line. Facilitate a discussion about whether values would balance if a counter above 0 was ignored. Prompt students for examples of real-world data sets in which 0 is a value in the set.
Mean ≈ 5.6 or MAD = 4.2	have included the repeated value of 3 only once in the calculations.	Have students make a dot plot for the data set. Have students compare data values in the dot plot to values used in calculations to show that each value must be used, even if it is repeated. Remind students that values can repeat in data sets, and discuss real-world examples.

Example

Guide students in understanding the Example. Ask:

- *What does the mean represent in the context of the problem?*
- *What does the MAD tell you about the likely data values for Fridays? For Saturdays?*
- *How might the smoothie shop owner use the information from the mean and the MAD when deciding how many employees should work on Fridays and Saturdays?*

Help all students focus on the Example and responses to the questions by reminding students to be respectful when they disagree with another's ideas.

Look for understanding that MAD measures the consistency of the data.

Apply It

1. Students may also solve by drawing a dot plot and finding the balance point. ***DOK 1***

2. Students should recognize that they must first find the mean before calculating MAD. This is the first problem where mean is not provided or referenced for students when asked to determine MAD. ***DOK 1***

3. **B is correct.** Students may solve by creating a dot plot to display the values and interpreting the mean as a balance point of the data set.

   A is not correct. This answer is interpreting median as affected by any increase in a data value.

   C is not correct. This answer is the reversal of the measures of center for mean and median.

   D is not correct. This answer is assuming that neither the mean nor median change, since the number of shelves is still the same.

   DOK 3

2 The list shows the lengths, in seconds, of the routines in a dance competition.

162, 140, 160, 159, 141, 163, 159, 164

Zara wants to know how much the times for the dance routines vary. What is the MAD of the dance times? Show your work.

Possible work:

$$\text{Mean} = \frac{162 + 140 + 160 + 159 + 141 + 163 + 159 + 164}{8}$$

$$= \frac{1{,}248}{8} = 156$$

Data Value	162	140	160	159	141	163	159	164
Distance from Mean	6	16	4	3	15	7	3	8

$$\text{MAD} = \frac{6 + 16 + 4 + 3 + 15 + 7 + 3 + 8}{8}$$

$$= \frac{62}{8} = 7.75$$

SOLUTION The MAD of the dance times is 7.75 s.

CONSIDER THIS . . .
To find the MAD of a data set, you first need to know the mean of the data set.

PAIR/SHARE
How do you know that your answer is reasonable?

3 The table shows the number of books on the shelves in the science fiction section of a library. A librarian adds 32 more books to the shelf with only 6 books. How does this change affect the mean and median number of books per shelf?

Books on a Shelf			
62	56	63	52
56	48	57	6

A It increases both the mean and the median.

B It increases the mean, but not the median.

C It increases the median, but not the mean.

D It does not increase the mean or the median.

Uma chose A as the correct answer. How might she have gotten that answer?

Possible answer: She may have thought that increasing a value in a data set will always cause the median to increase, even if the value that increases is not near the middle of the data set.

CONSIDER THIS . . .
How do outliers affect measures of center?

PAIR/SHARE
How would the mean and median be affected if the librarian added 32 more books to the shelf with 63 books, instead of the shelf with 6 books?

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GROUP & DIFFERENTIATE

Identify groupings for differentiation based on the **Start** and problems 1–3. A recommended sequence of activities for each group is suggested below. Use the resources on the next page to differentiate and close the lesson.

Approaching Proficiency
- **RETEACH** Visual Model
- **REINFORCE** Problems 4, 6, 7

Meeting Proficiency
- **REINFORCE** Problems 4–7

Extending Beyond Proficiency
- **REINFORCE** Problems 4–7
- **EXTEND** Challenge

Have all students complete the **Close: Exit Ticket**.

Resources for Differentiation are found on the next page.

Refine Interpreting Mean and Mean Absolute Deviation

Apply It

4 **a.** Students should understand that the mean is the total sum of the data set divided by the number of data values in the set.

b. Students should recognize that the data set increased by 9,924 steps and 1 extra day. Students may solve by showing an additional column in the Number of Steps table. ***DOK 3***

5 Students may also respond that there is less variability in San Diego. Students may recognize that they can interpret mean and MAD without calculating it themselves. ***DOK 2***

6 **A, D, and E are correct.** Students may understand that individual data values are needed to find MAD, and a dot plot, a frequency table, and a list of values will display each data value in a set.

B is not correct. This answer is a 5-point data summary, and the display of the box plot does not include each data value.

C is not correct. This answer only displays data in intervals and does not show each individual value.

DOK 1

4 Elijah tracks the number of steps he takes each day for 7 days.

Number of Steps						
10,740	12,168	13,760	12,468	11,541	12,847	11,904

a. What is Elijah's mean number of steps per day? Show your work.

Possible work:

$$\text{Mean} = \frac{10{,}740 + 12{,}168 + 13{,}760 + 12{,}468 + 11{,}541 + 12{,}847 + 11{,}904}{7}$$

$$= \frac{85{,}428}{7} = 12{,}204$$

SOLUTION Elijah's mean number of steps is 12,204.

b. On Day 8, Elijah walks 9,924 steps. He says the expression $\frac{85{,}428 + 9{,}924}{8}$ represents the mean with the value for Day 8 included. Is Elijah correct? Explain.

Yes; The sum of the first 7 data values is 85,428. So, the new total is equal to 85,428 + 9,924. There are now 8 data values, so divide the new total by 8 to find the mean.

5 During a 2-week time period, Denver has a mean high temperature of 72°F with a MAD of 5.66°F. During the same 2-week period, San Diego has a mean high temperature of 70°F with a MAD of 1.65°F. Compare the means and MADs and tell what they indicate about the high temperatures of the two cities.

Possible answer: The mean is greater for Denver, which shows that Denver's high temperatures are typically warmer than San Diego's. The MAD is also greater for Denver, which shows that Denver's high temperatures generally vary more from the mean than San Diego's do.

6 Which of the following could be used to calculate the MAD of a data distribution? Select all that apply.

A Dot plot
B Frequency table
C Box plot
D List of data values
E Histogram

DIFFERENTIATION

RETEACH

Visual Model

Interpret mean and MAD.

Students approaching proficiency with interpreting mean and MAD will benefit from small-group discussion of mean and MAD.

- Display a table showing two groups of students who tracked their running times for a week. *Group 1 ran 13, 8, 1, 3, and 15 hours. Group 2 ran 18, 14, 20, 22, and 16 hours.* Have students find the mean for each group. [Group 1: mean = 8; Group 2: mean = 18] Display number lines with the means marked. Show the distances between the mean and each data value. Write an equation to find MAD for Group 1. $\left[\frac{5 + 0 + 7 + 5 + 7}{5}\right]$ Have students find MAD for each group. [Group 1: MAD = 4.8; Group 2: MAD = 2.4]
- Have students individually think about the measures for each group and record at least two observations comparing the groups. Have students share their observations with a partner. Students may notice that Group 2 has a higher mean and less variability than Group 1. Ask: *How do you know which group has more variability?* [A higher MAD means more variability.] *What are some possible factors for the variability in the running times?* [Some runners in the group are faster than others. A runner might have been ill for most of the week. A runner might have been too busy to run a lot.]
- Ask: *Why do you think the mean of Group 2 is higher than the mean of Group 1?* [The two groups may have run different distances; one group may consist of high school students, and the other group may consist of middle school students, which might mean the high school students can run longer.]

7 **a.** See ***Connect to Culture*** to support student engagement. Students may solve this by drawing a dot plot to see how far each data value is from the mean. ***DOK 2***

b. Students should understand that less variability in a data set is interpreted as that data having more consistency. Brianna's MAD is 1 second less than Layla's MAD. Brianna's times are more clustered, so she is more consistent. ***DOK 1***

CLOSE EXIT TICKET

8 **Math Journal** Look for an understanding of the difference between mean and median.

Error Alert If students confuse steps, suggest that they start with 5 vertical blank lines representing the 5 data values. Have students place the median on the appropriate line and discuss if the values to the right and left of the median are greater or less than the median. Discuss ways to ensure that the average of the values chosen is 10.

End of Lesson Checklist

INTERACTIVE GLOSSARY Support students by suggesting that they show an example of a data set with five values and discuss finding the mean and median in their own words.

SELF CHECK Have students review and check off any new skills on the Unit 7 Opener.

Lap Times (seconds)			
26.6	26.5	25.1	29.3
28.5	27.0	28.5	27.7

7 Brianna's speed-skating coach times her as she skates laps during practice. The table shows her times, to the nearest tenth of a second, for 8 laps.

a. Brianna's mean lap time is 27.4 seconds. What is the MAD of her lap times? Show your work.

Possible work:

Data Value	26.6	26.5	25.1	29.3	28.5	27.0	28.5	27.7
Distance from Mean	0.8	0.9	2.3	1.9	1.1	0.4	1.1	0.3

$$\text{MAD} = \frac{0.8 + 0.9 + 2.3 + 1.9 + 1.1 + 0.4 + 1.1 + 0.3}{8} = \frac{8.8}{8} = 1.1$$

SOLUTION The MAD of Brianna's lap times is 1.1 s.

b. During the same practice, Layla's mean time for skating a lap is 26.6 seconds with a MAD of 2.1 seconds. Which skater, Brianna or Layla, had more consistent times during practice? Explain how you know.

Brianna; A data set that is more consistent has less variability. So, Brianna's times are more consistent because the MAD of her times is less than the MAD of Layla's times.

8 **Math Journal** Make up a data set of five values with a mean of 10 and a median of 12. None of the values in your data set can be repeated. Explain how you determined the values in your set.

Possible answer: 3, 8, 12, 13, 14; The middle value must be 12 because the median is 12 and the number of values is odd. I chose 13 and 14 to be the values above the median and 8 as one of the values below the median. For five values to have a mean of 10, their sum must be 50. To find the fifth value, I subtracted the other 4 values from 50: $50 - 12 - 13 - 14 - 8 = 3$.

End of Lesson Checklist

- ☐ **INTERACTIVE GLOSSARY** Find the entry for *mean*. Tell how the mean and median of a data set are alike.
- ☐ **SELF CHECK** Go back to the Unit 7 Opener and see what you can check off.

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REINFORCE

Problems 4–7

Solve problems with mean and MAD.

Students meeting proficiency will benefit from additional work with interpreting mean and MAD by solving problems in a variety of formats.

- Have students work on their own or with a partner to solve the problems.
- Encourage students to show their work.

EXTEND

Challenge

Use logic with mean and median.

Students extending beyond proficiency will benefit from applying knowledge of measures of center and using these measures to find unknown values in a data set.

- Generate a data set of 6 values with a mean of 7.5, a median of 7.5, and 14 as the greatest value.
- Students may use the median value to recognize that 7 and 8 are the middle values of the data set. Students may use $\frac{29 + x}{6} = 7.5$ to solve for the sum of the remaining 3 values and discover that these values must equal 16. Two of these values must be less than 7, and one value must be less than 14 but greater than 8. [Possible answers: 2, 3, 7, 8, 11, 14 or 1, 5, 7, 8, 10, 14]

PERSONALIZE

i-Ready

Provide students with opportunities to work on their personalized instruction path with *i-Ready* Online Instruction to:

- fill prerequisite gaps.
- build up grade-level skills.