Explore The Median of a Data Set

Grams of Sugar

Serving Size: 5 ounces
Servings per Container: about 6

7	9	15	10	9
5	13	9	8	15

Previously, you learned about displaying data distributions with dot plots and histograms. In this lesson, you will learn about displaying data distributions with box plots.

➤ **Use what you know to try to solve the problem below.**

The data show the number of grams of sugar in one serving of different types of yogurt. Suppose you are given a serving of yogurt. About how many grams of sugar would you expect it to have?

TRY IT

Math Toolkit graph paper, number lines, sticky notes

DISCUSS IT

Ask: How is your strategy similar to mine? How is it different?

Share: My strategy shows . . .

Learning Targets SMP 1, SMP 2, SMP 3, SMP 4, SMP 5, SMP 6

- Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.
- Display numerical data in plots on a number line, including dot plots, histograms, and box plots.
- Summarize numerical data sets in relation to their context by giving quantitative measures of center and variability.

CONNECT IT

1 Look Back About how many grams of sugar would you expect there to be in one serving of yogurt? Explain how you can answer this question.

2 Look Ahead In the **Try It** problem, you chose one number to represent a typical amount of sugar in a serving of yogurt. When you use a single value to summarize a data set, you are using a **measure of center**. One measure of center is the **median**, or middle value when the data values are listed in order.

a. The table shows the number of grams of sugar in one serving of different drinks. List the values in order from least to greatest. What is the middle value of the data set? Describe how you found it.

Grams of Sugar				
14	15	5	30	13
6	8	9	12	7
12	8	13	16	8
24	5			

b. What is the median number of grams of sugar in a drink? How do you know?

c. The median splits the data into two halves. Complete the statements.

About half of the drinks have less than _____ grams of sugar per serving.

About 50% of the drinks have more than _____ grams of sugar per serving.

d. When a data set has an even number of values, there are two middle values. The number halfway between these two values is the median. What is the median of the data set shown below?

4, 4, 5, (5, 7,) 8, 10, 11

3 Reflect Explain why the median could be a good value to use to summarize all the values of a data set, or to represent a typical value.

 Name:

Prepare for Interpreting Median and IQR in Box Plots

1 Think about what you know about data distributions. Fill in each box. Use words, numbers, and pictures. Show as many ideas as you can.

What Is It?

What I Know About It

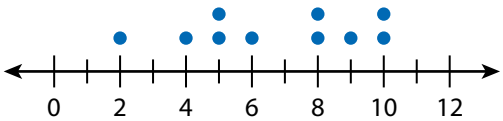
range

Examples

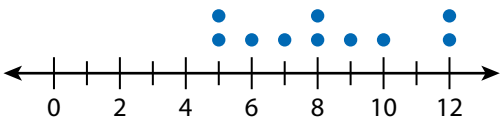
Examples

2 Explain why the data in Dot Plot A have a greater range than the data in Dot Plot B.

Dot Plot A



Dot Plot B



3 The list shows how many grams of protein there are in one serving of different brands of yogurt.

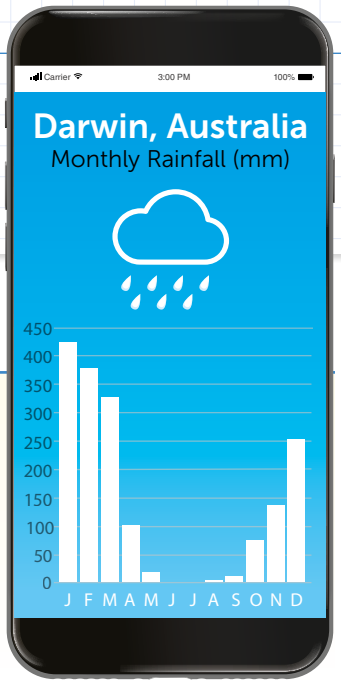
8, 10, 6, 12, 14, 6, 10, 12, 13, 6

a. About how many grams of protein would you expect a typical serving of yogurt to have? Show your work.

SOLUTION _____

b. Justify why your answer is reasonable. Show your work.

Develop Finding the Median and Quartiles



➤ **Read and try to solve the problem below.**

The town of Darwin, Australia, has a rainy season and a dry season. The data show the monthly amount of rainfall in one year. Each amount is rounded to the nearest 5 millimeters.

425, 375, 320, 100, 20, 0, 0, 5, 15, 70, 140, 250

Show that the median rainfall per month is 85 mm. How much rain does Darwin typically get per month during the drier 6 months of the year?

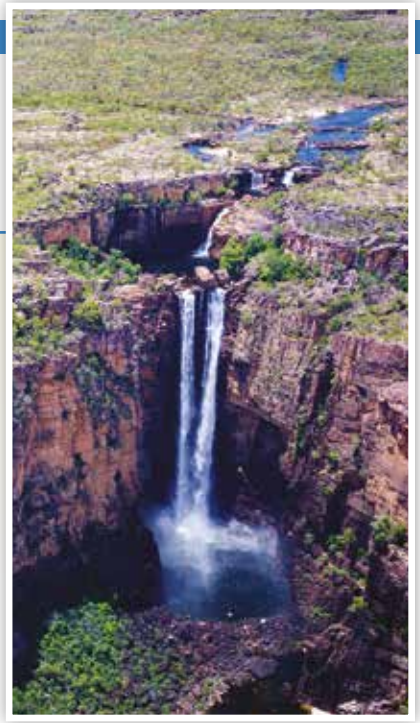
TRY IT

Math Toolkit graph paper, number lines, sticky notes

DISCUSS IT

Ask: How did you use your model to find a typical value?

Share: In my model, . . .



➤ **Explore different ways to use medians to describe a data distribution.**

The town of Darwin, Australia, has a rainy season and a dry season. The data show the monthly amount of rainfall in one year. Each amount is rounded to the nearest 5 millimeters.

425, 375, 320, 100, 20, 0, 0, 5, 15, 70, 140, 250

Show that the median rainfall per month is 85 mm. How much rain does Darwin typically get per month during the drier 6 months of the year?

Model It

You can use the median to describe the center of a data distribution.

To find the median, first list the values in order from least to greatest. Then find the middle value, or the two middle values.

0, 0, 5, 15, 20, 70, 100, 140, 250, 320, 375, 425

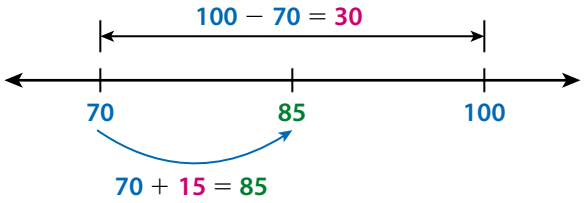
The median is halfway between the 6th and 7th values.

You can use a number line to find the halfway point between 70 and 100.

The distance from 70 to 100 is 30 and half of this distance is 15.

Add half of the distance from 70 to 100 to 70.

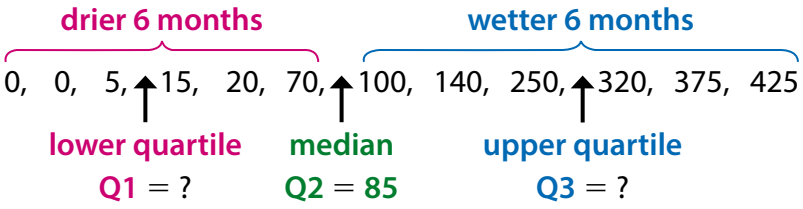
The median is 85.



Analyze It

You can find the median of each half of a data distribution.

The median, 85, separates the rainfall data into two halves. The median of the lower half of the data is the **lower quartile (Q1)**. The median of the upper half of the data is the **upper quartile (Q3)**. The median of the whole data set is Q2.



CONNECT IT

Use the problem from the previous page to help you understand how to use medians to describe a data distribution.

1. Look at **Model It**. What does the median, 85, tell you about the amount of rainfall during the year in Darwin, Australia?

2. Look at **Analyze It**. What is the value of the lower quartile (Q1) for the monthly rainfall data? Use Q1 to describe a typical amount of rainfall per month during the drier 6 months of the year.

3. What is the value of the upper quartile (Q3) for the monthly rainfall data? What does this value tell you about rainfall in Darwin, Australia?

4. You can use a median or quartile to help you identify groups of data values.

   a. What fraction of the monthly rainfall data is greater than 85 mm?

   b. What percent of the data is less than 285 mm?

   c. Which data values are in the middle 50% of the monthly rainfall data set?

5. Why are the median, lower quartile, and upper quartile useful for describing data distributions?

6. **Reflect** Think about all the models and strategies you have discussed today. Describe how one of them helped you better understand how to use medians to describe data distributions.

Apply It

Use what you learned to solve these problems.

7 A gym has rowing machines with digital screens that display time, distance, and speed. The data show the numbers of miles that members row on Tuesday.

a. Find the median, the lower quartile, and the upper quartile of the data set. (*Note:* When the number of data values is odd, do not include the median in either half of the data as you find Q1 and Q3.) Show your work.

SOLUTION _____

b. What does the median tell you about the number of miles rowed?

8 Students sell coupon books for a fundraiser and report the number sold. The median number of coupon books sold is 2 and the lower and upper quartiles for the data are Q1 = 1 and Q3 = 3.5. Which statement is true?

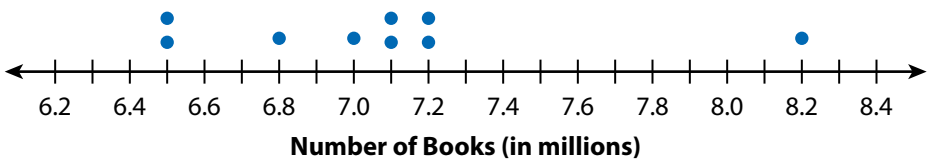
A About half of the students sold 2 or more coupon books.

B About 75% of the students sold more than 3 coupon books.

C About one fourth of the students sold more than 1 coupon book.

D About 50% of the students sold either 1 or 2 coupon books.

9 The dot plot shows the number of books in some large libraries in the U.S. What is the median and the lower quartile of the data set? Show your work.



SOLUTION _____

 Name:

Practice Finding the Median and Quartiles

➤ **Study the Example showing how to summarize a data set with a single number. Then solve problems 1–5.**

Example

Abran recorded the price of his favorite granola bar at 9 different stores. What is the median cost of the granola bar at these stores?

$0.85, $0.99, $1.15, $1.27, $1.28, $1.30, $1.30, $1.84, $1.89

Order the values from least to greatest. Find the middle value.

0.85, 0.99, 1.15, 1.27, (1.28,) 1.30, 1.30, 1.84, 1.89

The median cost for the granola bar is $1.28.

1 **a.** Look at the Example. What is the lower quartile (Q1) and upper quartile (Q3) of the granola bar prices? Show your work.

SOLUTION _____

b. What do these values tell you about the cost of the granola bars?

2 Abran sees his favorite granola bar from the Example in a vending machine at an airport. The cost is $2.75. What are the new values of the median, lower quartile, and upper quartile? Show your work.

SOLUTION _____

Vocabulary

median
the middle number, or halfway point between the two middle numbers, in an ordered set of numbers.

lower quartile
the middle number between the minimum and the median in an ordered set of numbers.

upper quartile
the middle number between the median and the maximum in an ordered set of numbers.

3 The table shows the lengths of various musicals in hours.

Musical Lengths (hours)								
2.8	2.8	2.5	2.5	2.3	2.9	2.5	2.6	2.3
2.5	2.5	2.5	2.3	2.6	2.3	5.3	2.5	

a. What are the lower quartile and upper quartile? Show your work.

SOLUTION

b. What do the lower and upper quartiles tell you about the middle 50% of the data?

c. Suppose Elias removes the outlier of 5.3 hours. How do the median, Q1, and Q3 change?

4 The data show the number of hours a part-time waiter works each week.

7, 11, 8, 10, 11, 8, 13, 9, 10, 9, 9

Tell whether each statement about the data is *True* or *False*.

	True	False
a. He works more than 9 hours about 50% of the time.	○	○
b. He works 8 or fewer hours about 25% of the time.	○	○
c. He works 10 or more hours about 75% of the time.	○	○

5 Each day for 9 days, a school principal records the number of 6th graders who are absent. Hai says the upper quartile for the data below is 5. Is Hai correct? Explain.

0, 1, 0, 2, 4, 3, 5, 12, 9

Develop Using Box Plots and IQR to Describe Variability

➤ **Read and try to solve the problem below.**

Ziplife company claims that its external battery pack typically adds 9 hours of battery life to a smartphone. Researchers tested a group of the battery packs and recorded the number of extra hours of battery life. Their data are shown in the table. Do you agree with the company's claim? Use data to support your reasoning.

Extra Hours of Battery Life

9.5	3.5	5.5	6.5	6.0
12.5	11.5	13.5	7.5	8.0
8.5	9.5	10.0	10.5	9.0

TRY IT

Math Toolkit graph paper, number lines, sticky notes

DISCUSS IT

Ask: How did you get started?

Share: I started by . . .

➤ **Explore different ways to describe the variability of a data set.**

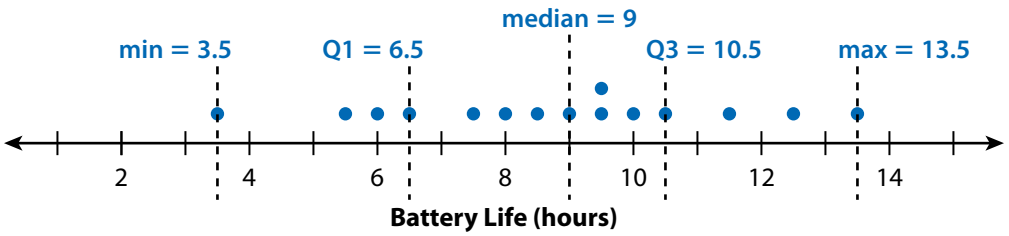
ZipLife company claims that its external battery pack typically adds 9 hours of battery life to a smartphone. Researchers tested a group of the battery packs and recorded the number of extra hours of battery life. Their data are shown in the table. Do you agree with the company's claim? Use data to support your reasoning.

Extra Hours of Battery Life				
9.5	3.5	5.5	6.5	6.0
12.5	11.5	13.5	7.5	8.0
8.5	9.5	10.0	10.5	9.0

Model It

You can use a dot plot to show the variability of a data set.

The minimum (min) and maximum (max) data values determine the range of the data. Locate the median, the lower quartile (Q1), and the upper quartile (Q3).

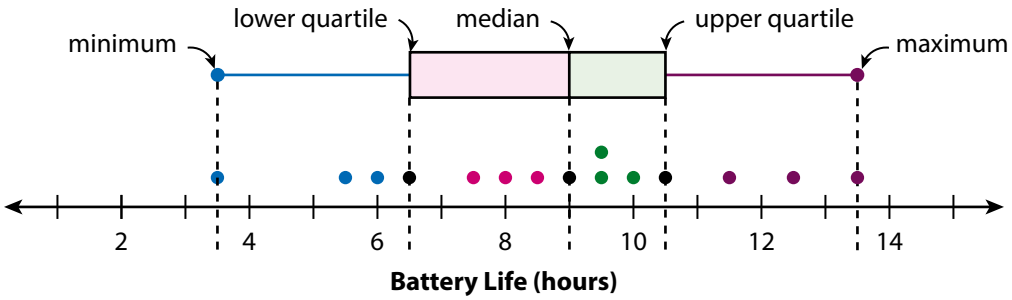


Together, these values are called the *five-number summary* of the data. They can be used to construct a data display called a **box plot**.

Model It

You can use a box plot to analyze the variability of a data set.

A box plot shows how the data in each quarter of the data set are spread out.



The range and the **interquartile range (IQR)** are both **measures of variability** of a data set.

Range = max − min

Interquartile range = Q3 − Q1

CONNECT IT

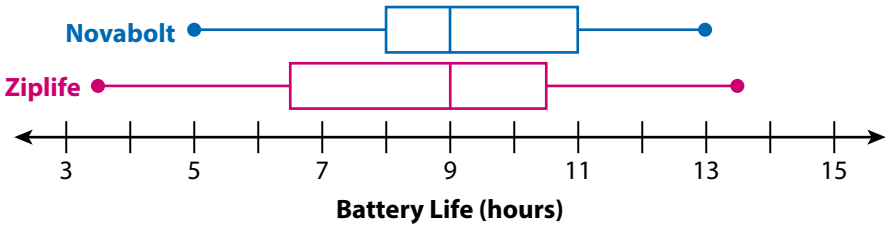
Use the problem from the previous page to help you understand how to describe the variability of a data set.

1. Look at the second **Model It**. What are the range and the IQR for the Ziplife battery data? How is the IQR represented in the box plot?

2. Why is the left side of the box wider than the right side of the box?

3. Could Ziplife use the box plot to support its claim? Explain.

4. The researchers test a similar external battery made by the company Novabolt. The double box plot shows the distribution of the data for each company. What can you tell about the center and variability of the two distributions?



5. How do box plots, range, and IQR help you to analyze variability?

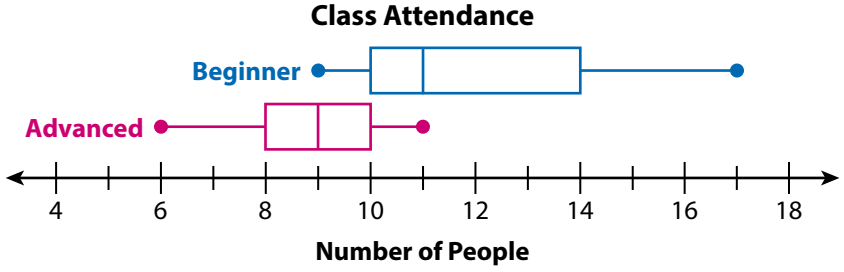
6. **Reflect** Think about all the models and strategies you have discussed today. Describe how one of them helped you better understand how to describe the variability of a data set.

Apply It

Use what you learned to solve these problems.

7 Two food truck owners record the number of people who visit their trucks each day for one week. Which data set has more variability? Explain.

8 A yoga teacher leads a beginner class and an advanced class. He records the number of people who attend each class for several months. The box plots show the data. Compare the measures of center and variability.



9 Ignacio researched the average number of students per math class in different countries. The table shows the data. Display the data in a box plot.

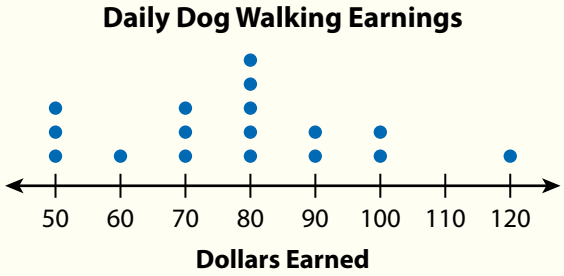
Number of Students								
21.0	21.5	25.0	21.5	22.0	27.0	22.5	22.0	23.0
23.5	24.0	25.0	25.0	25.0	26.0	26.0	27.0	

Practice Using Box Plots and IQR to Describe Variability

➤ **Study the Example showing how to find measures of variability of a data set. Then solve problems 1–4.**

Example

Julio earns money as a dog walker. He charges $10 per dog. He records how much money he earns each day in a dot plot. Find the range and IQR of his data.



Identify the maximum and minimum data values, the median, and the upper and lower quartiles.

50, 50, 50, 60, 70, 70, 70, 80, 80, 80, 80, 90, 90, 100, 100, 120

minimum = 50; Q1 = 65; median = 80; Q3 = 90; maximum = 120

Range: maximum − minimum = 120 − 50 = 70

IQR: Q3 − Q1 = 90 − 65 = 25

1 a. Use a box plot to display the data from the Example.

b. What does the range tell you about the data?

c. What does the IQR tell you about the data?

Vocabulary

box plot
a visual display of a data set that shows the minimum, the lower quartile, the median, the upper quartile, and the maximum.

interquartile range (IQR)
the difference between the upper quartile and the lower quartile.

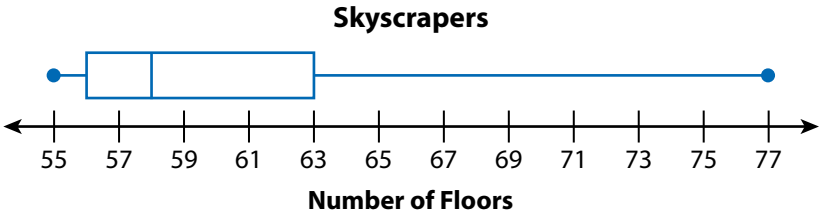
2 Students measure the heights, in centimeters, of the plants in two different gardens.

Garden A Plant Heights (cm)								
0	5	6	6	5	5	5	7	8
10	8	7	8	8	8	8	11	7

a. The table shows the data for Garden A. Display the data in a box plot.

b. Garden B has plant heights with an IQR of 1.5 cm. Which garden has less variability in its plant heights? Explain.

3 The box plot shows the number of floors of some skyscrapers in the U.S. Which statements about the box plot are true?



A The range of the data is 22.

B The median number of floors is 58.

C The greatest number of floors is 63.

D About half of the buildings have 56 to 63 floors.

E There are 15 buildings in the data set.

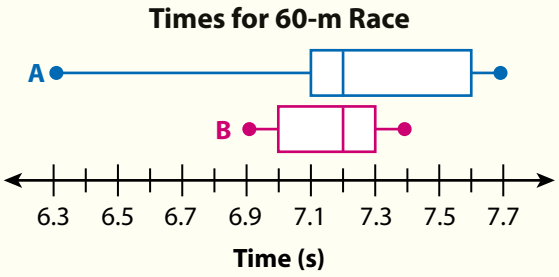
4 Two airlines report their number of delayed flights each month for one year. Airline A has an IQR of 83.5 and Airline B has an IQR of 22. Which airline is the most consistent in not having delays? Explain.

Refine Interpreting Median and Interquartile Range in Box Plots

➤ **Complete the Example below. Then solve problems 1–9.**

Example

A track team needs another runner. The team analyzes the times for the 60-m dash for Runner A and Runner B. What information can you learn from the box plots?



Look how you could analyze the distributions.

A: median = 7.2; IQR = 7.6 − 7.1 = 0.5; range = 7.7 − 6.3 = 1.4

B: median = 7.2; IQR = 7.3 − 7.0 = 0.3; range = 7.4 − 6.9 = 0.5

SOLUTION _____

_____

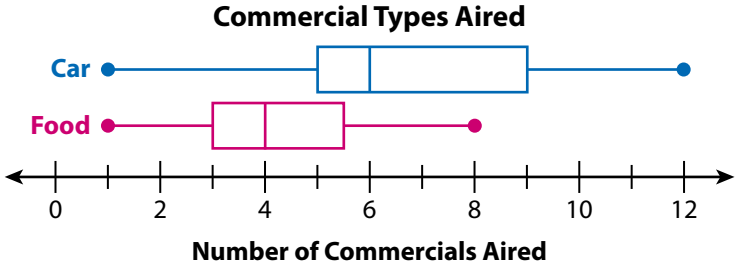
_____

CONSIDER THIS . . .
Which runner is more consistent? How do you know?

PAIR/SHARE
How do you think the data for the box plots were collected and measured?

Apply It

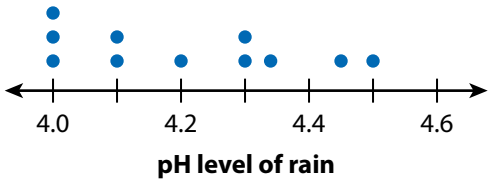
1 Two types of commercials during awards shows are tracked over a 25-year period. Which type of commercial was more consistently aired? Explain.



CONSIDER THIS . . .
Without calculating, how do you know that the data for car commercials have a greater range?

PAIR/SHARE
What are the medians and what do they tell you about the data?

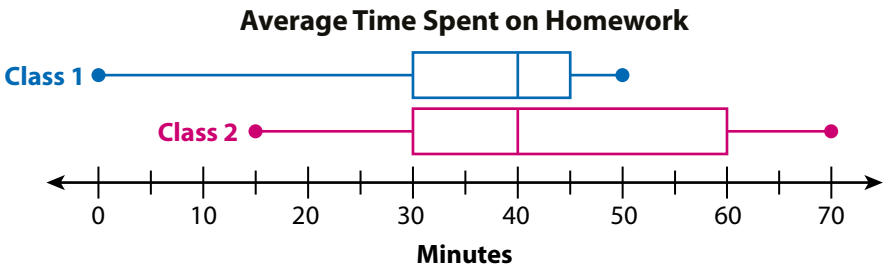
2 Acid rain damages a town's trees. During each rainfall, a scientist collects data on the rain's pH (a measure of acidity). The dot plot shows her data. Draw a box plot above the dot plot to display the distribution. About what percent of the data does the box represent? Explain how the box plot shows this.



CONSIDER THIS . . .
In a box plot, the lines that connect the box to the maximum and minimum are sometimes called *whiskers*.

PAIR/SHARE
Suppose a box plot does not show a line for the median inside the box. What would this tell you about Q1, Q2, and Q3?

3 Which statement about the box plots is true?



A Moving the students from the top 25% of Class 1 into Class 2 would increase the Class 2 median.

B Removing the data for students in Class 1 who did not homework would increase the Class 1 IQR.

C Combining the data from both classes into one box plot would result in a box plot with a range of 105.

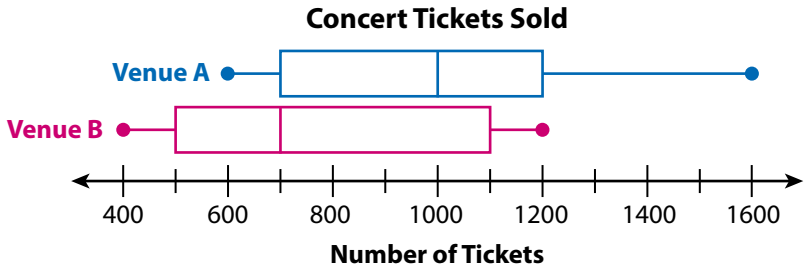
D Combining the data from both classes into one box plot would double the current median for Class 1.

Noor chose B as the correct answer. How might she have gotten that answer?

CONSIDER THIS . . .
Think about the 5-number summary for each class and then compare them.

PAIR/SHARE
How would Noor have gotten C as an answer?

4 Two concert venues record the number of tickets they sell at each show for a year. The box plots show the distribution of the data. Compare the measures of center and variability and explain what they mean in terms of the problem.



5 A farm has 14 spotted pigs. Each spotted pig has at least 1 spot. The median number of spots is 5. The difference between the maximum number of spots and the minimum number of spots is 14.

Which statements are true? Select all that apply.

A There must be at least 1 pig with 5 spots.

B One of the pigs could have 3 spots.

C A pig could have more than 20 spots.

D All pigs must have 15 or fewer spots.

E At least 1 pig has fewer than 5 spots.

6 Researchers test the miles per gallon for two cars. Which car has the greater median? Explain what this means in terms of the situation.

Miles per Gallon

Car 1					Car 2				
30	15	16	17	18	25	24	22	22	19
20	22	29	29	25	20	19	22	19	24
27	28	29			25	19	19		

7 A scientist reports the the information shown about the masses, in grams, of two groups of salamanders. Construct a box plot for each data set. Which group of salamanders shows greater variability in mass? Explain how you know.

	Min	Q1	Median	Q3	Max
Group A	14.5	17.0	20.0	22.0	30.0
Group B	24.0	25.0	26.0	27.0	28.0

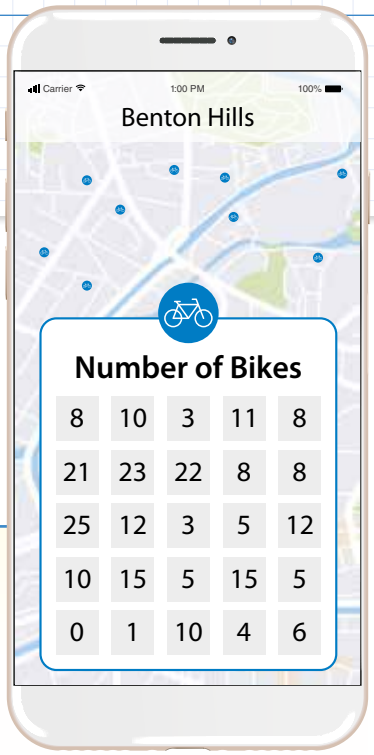
8 Hockey Team A records its final score for every game in a season. The minimum is 0, Q1 is 1, the median is 2, Q3 is 3, and the maximum is 5. Hockey Team B's final scores for the season have an IQR of 6. Which team is more consistent in its number of points scored per game? Explain your reasoning.

9 **Math Journal** Construct a data set that has 10 as a measure of center and 7 as a measure of variability. Explain what these measures mean in terms of the data.

End of Lesson Checklist

- ☐ **INTERACTIVE GLOSSARY** Write a new entry for *interquartile range (IQR)*. Tell how the interquartile range (IQR) of a data set varies with the situation.
- ☐ **SELF CHECK** Go back to the Unit 7 Opener and see what you can check off.

Explore The Mean of a Data Set



Previously, you learned to use the median and interquartile range (IQR) to describe the center and variability of a data set. In this lesson, you will learn about another measure of center, the mean, and another measure of variability, the mean absolute deviation.

➤ **Use what you know to try to solve the problem below.**

A city has a bike-sharing program. A manager of the program wants to know whether there are enough bikes at the bike stations in the neighborhood of Benton Hills. The data show the number of bikes currently at the neighborhood's 25 bike stations. Based on the data set, what is a typical number of bikes at a station?

TRY IT

Math Toolkit counters, graph paper, number lines, sticky notes

DISCUSS IT

Ask: What did you do first to identify a typical value for the data set? Why did you do this step?

Share: First, I . . . because . . .

Learning Targets SMP 1, SMP 2, SMP 3, SMP 4, SMP 5, SMP 6

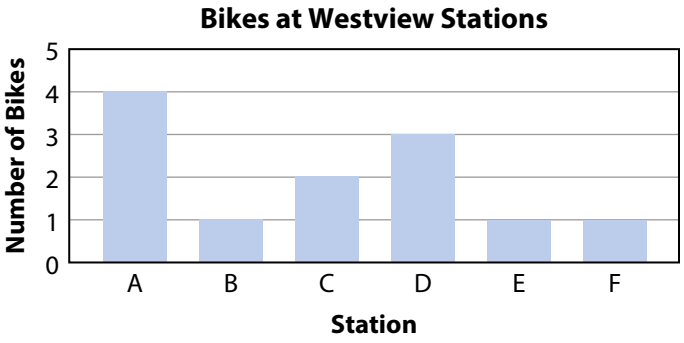
- Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.
- Summarize numerical data sets in relation to their context by giving quantitative measures of center and variability, as well as describing any overall pattern and any striking deviations from the overall pattern.

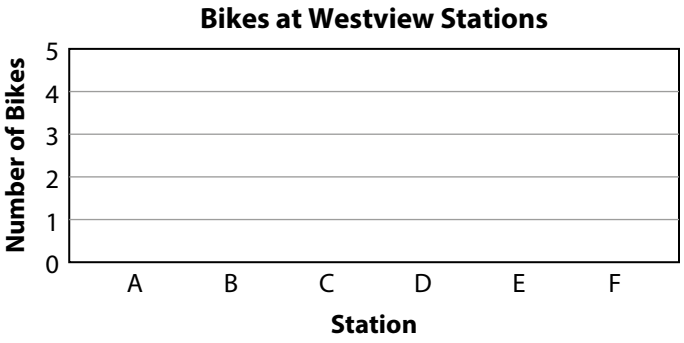
CONNECT IT

1 Look Back Based on the data set, what is a typical number of bikes at a bike station in the Benton Hills neighborhood? Explain.

2 Look Ahead The median is not the only way to use a single number to summarize a data set. Use the data set shown in the bar graph to explore another measure of center.

a. The first bar graph shows the number of bikes currently at bike stations A–F in the neighborhood of Westview. Complete the second bar graph to show how many bikes would be at stations A–F if all the bikes are distributed equally among the stations.





b. How does the number of bikes at each station change when the bikes are distributed equally among the stations?

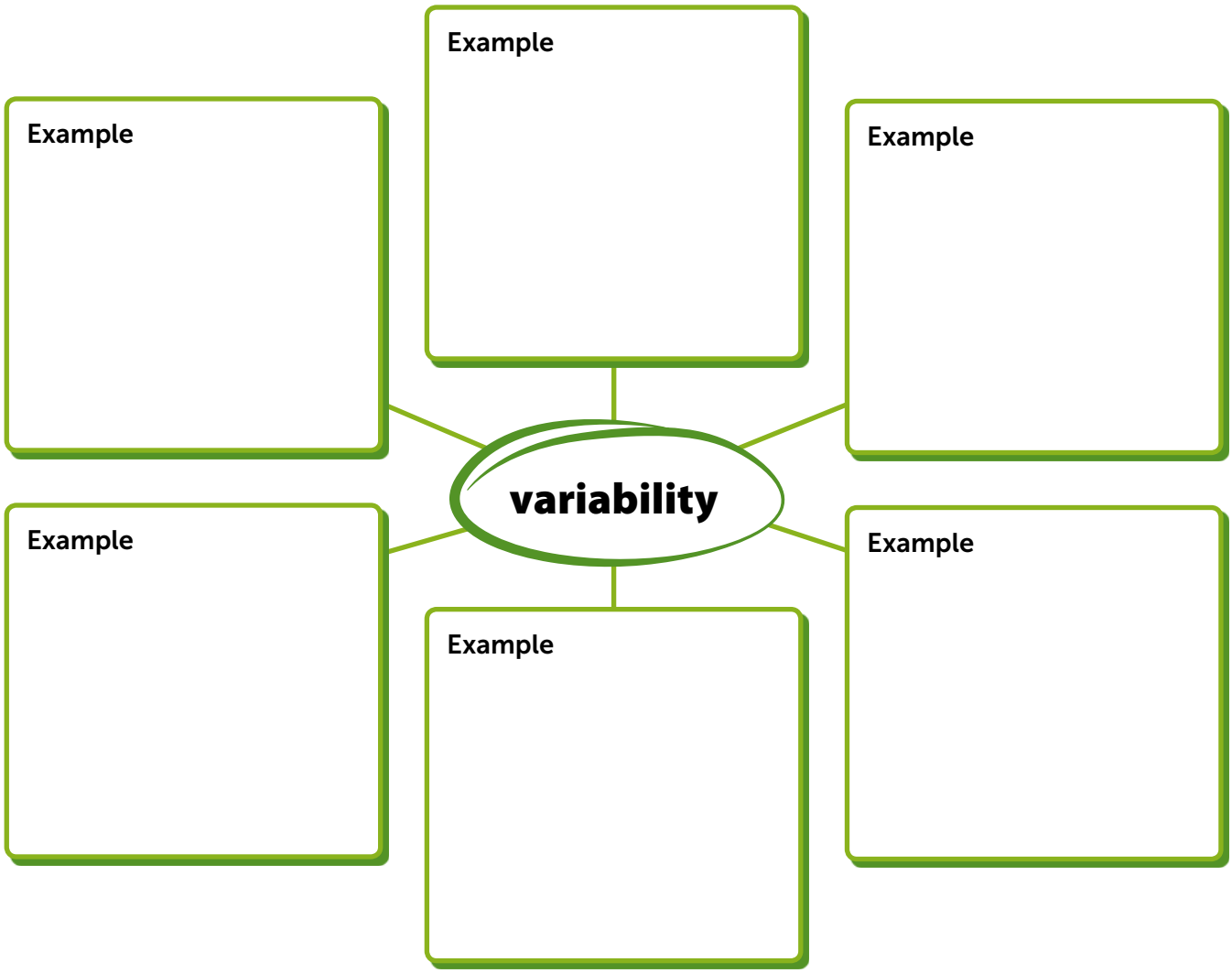
c. When the bikes are distributed equally, the number of bikes at a station is the **mean**, or average, of the original data set. Like the median of a data set, the mean is a measure of center. What is the mean number of bikes at a station?

3 Reflect Why do you think the mean of a data set is sometimes described as a *fair share* of the data set?

 Name:

Prepare for Interpreting Mean and Mean Absolute Deviation

1 Think about what you know about data and variability. Fill in each box. Use words, numbers, and pictures. Show as many ideas as you can.



2 Which data set would you expect to have more variability: the ages of the 6th grade students at a school or the ages of the 6th grade teachers at the school? Explain.

3 Visitors to a state capitol building can take a free tour. The manager in charge of the tours wants to know a typical number of visitors in a tour group. The list shows the number of visitors in the 17 tour groups on Friday.

38, 23, 40, 35, 36, 28, 40, 26, 40, 37, 22, 32, 18, 28, 29, 26, 29

a. Based on the data set, what is a typical number of visitors in a tour group on Friday? Show your work.

SOLUTION _____

b. Check your answer to problem 3a. Show your work.

Develop Finding the Mean

Read and try to solve the problem below.

For Earth Day, volunteers are cleaning up the shore of a lake. A team of 9 students is collecting trash. Their goal is to collect 5 lb of trash per person. The list shows the weight, in pounds, of trash each student collects.

What is a typical amount of trash that a student on the team collects? How does this amount compare to the team's goal?

TRY IT

Math Toolkit connecting cubes, counters, graph paper, number lines

DISCUSS IT

Ask: How is your strategy for finding the typical value similar to mine? How is it different?

Share: My strategy is similar to yours because . . . It is different because . . .

➤ **Explore different ways to find the mean of a data set.**

For Earth Day, volunteers are cleaning up the shore of a lake. A team of 9 students is collecting trash. Their goal is to collect 5 lb of trash per person. The list shows the weight, in pounds, of trash each student collects.

7, 2, 4, 4, 6, 3, 7, 9, 3

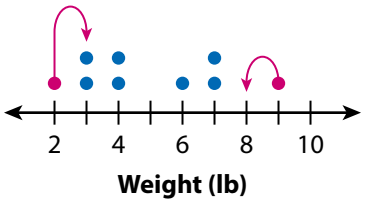
What is a typical amount of trash that a student on the team collects? How does this amount compare to the team's goal?

Picture It

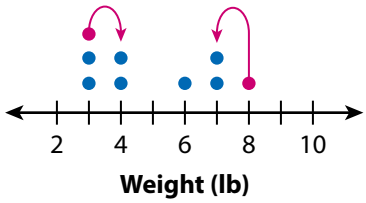
You can think of the mean of a data set as the balance point of the data.

To find the balance point, move the left-most and right-most points 1 unit each toward the middle. Repeat this step until all data points stack above a single value.

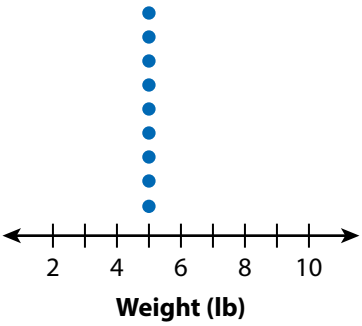
Step 1



Step 2



Final Step



Analyze It

You can calculate the mean by finding the average of the data.

The average of a data set is the sum of the data values divided by the number of data values.

Sum of data values: $7 + 2 + 4 + 4 + 6 + 3 + 7 + 9 + 3 = 45$

Sum ÷ number of data values: $\frac{45}{9}$

Mean = ?

CONNECT IT

Use the problem from the previous page to help you understand how to find the mean of a data set.

1. Look at **Picture It**. What does moving a pair of points represent about the pounds of trash two students collect? Does the total amount the team collects change?

2. How does the final dot plot show that the team met its goal?

3. Look at **Analyze It**. What is the mean weight of trash a student on the team collects? Why could you call this a typical amount of trash collected by a student?

4. The dot plots represent three different data sets. Why do the three data sets have the same mean?

5. Why is the mean considered a measure of center of a data set? How is the mean different from the median?

6. **Reflect** Think about all the models and strategies you have discussed today. Describe how one of them helped you better understand how to find the mean of a data set.

Apply It

Use what you learned to solve these problems.

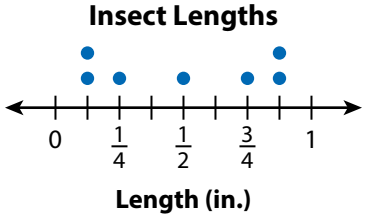
7 Lola is in an archery club. She shoots 6 arrows and earns the scores shown in the list.

a. How does the mean and median change if the outlier of 0 is left out of the data set? Show your work.

SOLUTION

b. Why does the outlier have a greater effect on the mean than on the median?

8 There are several rare insects on display at the insect exhibit at a science museum. The dot plot shows the lengths of the insects to the nearest $\frac{1}{8}$ in. What is the mean length of the insects? Show your work.



SOLUTION

Practice Finding the Mean

➤ **Study the Example showing how to find the mean of a data set. Then solve problems 1–4.**

Example

Students in Fiona's class each listen to a different radio station for 20 min one night and count the number of commercials. The list shows their data.

6, 3, 4, 2, 3, 1, 3, 3, 7, 1, 0, 0, 0, 2, 4

What is the mean number of commercials in 20 min?

You can find the mean by adding the data values and then dividing the sum by the number of values. You can use multiplication to group the values that are the same before adding.

$$\text{Mean} = \frac{(3 \cdot 0) + (2 \cdot 1) + (2 \cdot 2) + (3 \cdot 4) + (2 \cdot 4) + 6 + 7}{15}$$

$$= \frac{39}{15} = 2.6$$

The mean is 2.6, so the mean number of commercials in 20 min is 2.6.

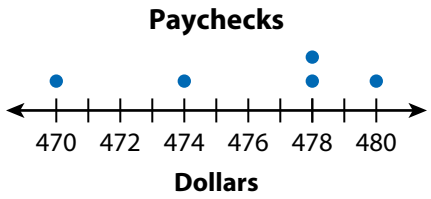
1 Lian is absent from class the day the students in the Example combine their data. He counted 13 commercials when he listened for 20 min.

a. How does the mean change when Lian includes his data in the class data set? Show your work.

SOLUTION _____

b. Does Lian's value also change the median of the data set? Why or why not?

2 The dot plot shows the amounts of Michael's last 5 paychecks. What is the mean of the amounts? Show your work.



SOLUTION

3 The lists show the numbers of students in each math class at two different schools.

East Middle School: 24, 26, 28, 27, 24, 24, 22, 26, 27, 22

Grove Middle School: 18, 25, 25, 29, 28, 26, 28, 28, 27, 26

a. Which school has the greater mean number of students per math class? Show your work.

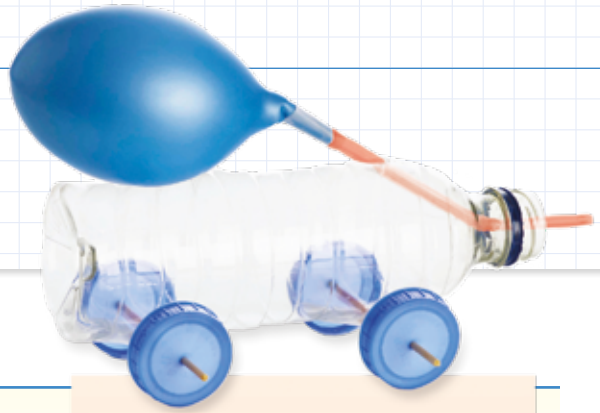
SOLUTION

b. What do the means of the data sets represent in this situation?

4 The table shows the number of players on each team in a softball league. Suppose two people on the team with the most players move to the team with the fewest players. How would the mean number of players per team change? Explain how you know.

Number of Players				
12	11	12	12	11
9	13	15	13	12

Develop Finding and Interpreting Mean Absolute Deviation



➤ **Read and try to solve the problem below.**

Teams in a science competition make balloon-powered cars. The list shows the distance, in feet, each car travels. The mean distance traveled is 19 ft. How much do the data values generally vary from the mean distance?

Distance Traveled (ft)					
11	12	14	15	16	17
18	21	22	23	28	31

TRY IT

Math Toolkit graph paper, number lines, sticky notes

DISCUSS IT

Ask: How did you determine the amount that the data values generally vary from the mean distance?

Share: The method I used was . . .

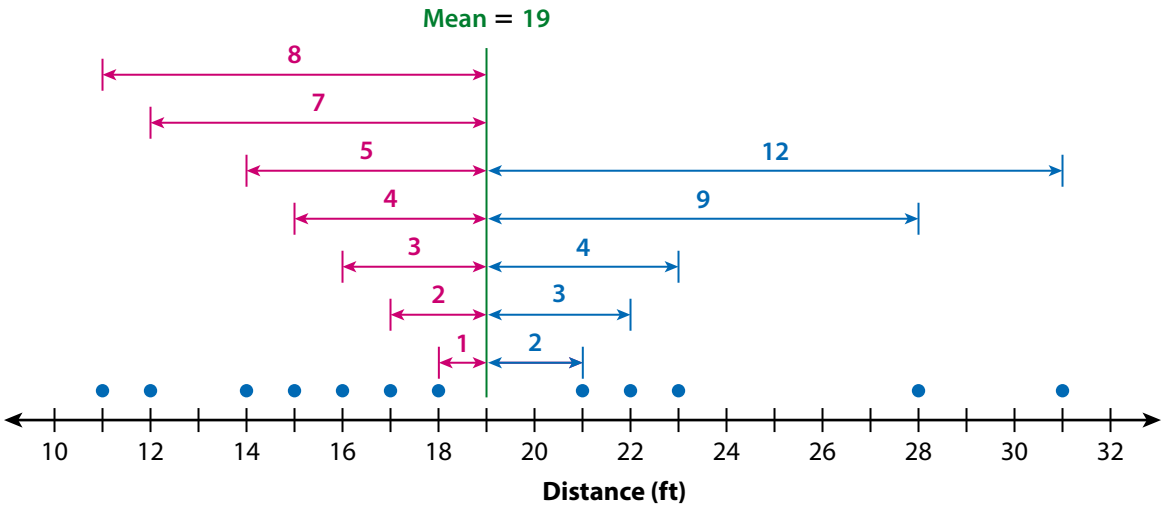
➤ **Explore different ways to understand variability in a data set.**

Teams in a science competition make balloon-powered cars. The list shows the distance, in feet, each car travels. The mean distance traveled is 19 ft. How much do the data values generally vary from the mean distance?

11, 12, 14, 15, 16, 17, 18, 21, 22, 23, 28, 31

Model It

You can use a number line to show how far each data value is from the mean.



Analyze It

You can use the average distance of data values from the mean as a measure of variability. This average distance is called the **mean absolute deviation (MAD)**.

Data Value	Distance from Mean
11	19 − 11 = 8
12	19 − 12 = 7
14	19 − 14 = 5
15	19 − 15 = 4
16	19 − 16 = 3
17	19 − 17 = 2
18	19 − 18 = 1
21	21 − 19 = 2
22	22 − 19 = 3
23	23 − 19 = 4
28	28 − 19 = 9
31	31 − 19 = 12

MAD

$= \frac{\text{sum of distances from mean}}{\text{number of data values}}$

$= \frac{8 + 7 + 5 + 4 + 3 + 2 + 1 + 2 + 3 + 4 + 9 + 12}{12}$

$= \frac{60}{12}$

$= 5$

CONNECT IT

➤ **Use the problem from the previous page to help you understand variability in a data set.**

1. Look at **Model It**. Find the point for the car that traveled 28 ft. How much farther than the mean distance did this car travel? How is this shown in the model?

2. Describe what the distance labeled **5** tells you about one of the cars.

3. Look at **Analyze It**. What do the data values in the left table have in common? What do the data values in the right table have in common? How are the subtraction equations in the tables related to the number line model?

4. Look at the fractions used to find the MAD. How is the process of finding a MAD like the process of finding a mean? What data set are you finding the mean of?

5. The MAD of the distances the cars traveled is 5. What does a MAD of 5 tell you in this situation?

6. How many cars traveled distances that are within 5 ft of the mean? How does this help you understand MAD as a measure of variability in a data set?

7. **Reflect** Think about all the models and strategies you have discussed today. Describe how one of them helped you better understand variability in a data set.

Apply It

Use what you learned to solve these problems.

Number of Hours Slept				
7	8	8	9	9
9	10	10	10	10

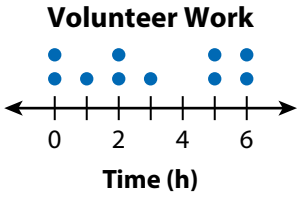
8 Imani asks 10 students about the number of hours they slept last night. She finds that the mean sleeping time is 9 h. Find the MAD of Imani's data set and describe what it means in this situation. Show your work.

SOLUTION _____

_____

9 At a doctor's office, the mean amount of time patients spend in the waiting room is 18 min. The MAD of the wait times is 7 min. Based on this information, would it be unusual for a patient to wait for 25 min in the doctor's waiting room? Explain.

10 Students in Naomi's class record the number of hours they spend volunteering in May. The dot plot shows their data. The mean time is 3 hours. Find the MAD of the data set. Show your work.



SOLUTION _____

Practice Finding and Interpreting Mean Absolute Deviation

➤ **Study the Example showing how to find the mean absolute deviation of a data set. Then solve problems 1–4.**

Example

The list shows the heights, in inches, of the five starting players on a men's college basketball team.

74, 74, 76, 80, 81

The mean height is 77 in. What is the MAD of the heights?

You can find the MAD (mean absolute deviation) of the data by finding the **distance** between each data value and the **mean**. Then find the average of the distances from the mean.

$$\text{MAD} = \frac{\text{sum of distances from mean}}{\text{number of data values}}$$

$$= \frac{3 + 3 + 1 + 3 + 4}{5}$$

$$= \frac{14}{5} = 2.8$$

The MAD of the heights is 2.8 in.

Data Value	Distance from Mean
74	77 − 74 = 3
74	77 − 74 = 3
76	77 − 76 = 1
80	80 − 77 = 3
81	81 − 77 = 4

1 a. What does the MAD of the heights in the Example tell you?

b. The heights of the starting players on a different team have a mean of 78 in. and a MAD of 3.6 in. How do the heights of the players on the two teams compare?

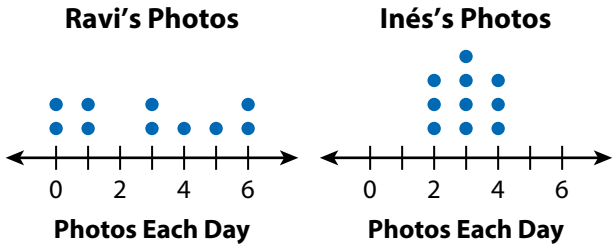
Vocabulary

mean absolute deviation (MAD)
the sum of the distances of each data point from the mean of the data set divided by the number of data points.

2 The list shows the weights of the bobcats at a nature reserve. What are the mean and MAD of the weights? Show your work.

SOLUTION

3 The dot plots show the number of photos Ravi and Inés took each day for 10 days. How can you tell, without calculating, which data set has a greater MAD?



4 Paulo counts the number of cherry fruit snacks in 7 bags of mixed fruit snacks. The list shows his data. The mean is 23 cherry snacks per bag.

14, 23, 29, 31, 15, 19, 30

Based on the data, would it be unusual to get a bag with 17 cherry snacks? Use the MAD of the data to support your answer.

Refine Interpreting Mean and Mean Absolute Deviation

➤ **Complete the Example below. Then solve problems 1–8.**

Example

On Fridays, the mean amount a smoothie shop makes in sales is $2,287, with a MAD of $314. On Saturdays, the mean amount the shop makes in sales is $1,934, with a MAD of $152. On which of the two days does the shop typically make more money? On which day are the sales generally more consistent?

Look at how you could interpret the means and MADs.

The mean is a measure of center, so it can represent a typical value in a data set. The mean sales for Fridays are greater than the mean sales for Saturdays.

The MAD is a measure of variability. The less variability a data set has, the more consistent the data values are. The MAD for Saturdays is less than the MAD for Fridays.

SOLUTION _____

_____

CONSIDER THIS . . .
The more consistent the values in a data set are, the closer the values are to each other.

PAIR/SHARE
Would it be unusual for the shop to make $2,500 in sales on a Friday? Explain.

Apply It

1 Roberto sells lemonade to raise money for charity. He collects data on the cost of lemonade at other lemonade stands. He uses the mean of his data as the price of lemonade at his stand. How much does lemonade cost at Roberto's stand? Show your work.

Cost of Lemonade ($)		
2.00	1.00	1.25
1.50	0.50	1.25
1.00	0.50	3.00
1.00	1.25	1.50
1.25	1.25	1.25

SOLUTION _____

CONSIDER THIS . . .
You can find the mean of a set of decimals the same way you find the mean of a set of whole numbers.

PAIR/SHARE
How does the cost of lemonade at Roberto's stand compare to the median cost of lemonade at the other stands?

2 The list shows the lengths, in seconds, of the routines in a dance competition.

162, 140, 160, 159, 141, 163, 159, 164

Zara wants to know how much the times for the dance routines vary. What is the MAD of the dance times? Show your work.

CONSIDER THIS . . .
To find the MAD of a data set, you first need to know the mean of the data set.

PAIR/SHARE
How do you know that your answer is reasonable?

SOLUTION

3 The table shows the number of books on the shelves in the science fiction section of a library. A librarian adds 32 more books to the shelf with only 6 books. How does this change affect the mean and median number of books per shelf?

Books on a Shelf			
62	56	63	52
56	48	57	6

A It increases both the mean and the median.

B It increases the mean, but not the median.

C It increases the median, but not the mean.

D It does not increase the mean or the median.

Uma chose A as the correct answer. How might she have gotten that answer?

CONSIDER THIS . . .
How do outliers affect measures of center?

PAIR/SHARE
How would the mean and median be affected if the librarian added 32 more books to the shelf with 63 books, instead of the shelf with 6 books?

4 Elijah tracks the number of steps he takes each day for 7 days.

Number of Steps						
10,740	12,168	13,760	12,468	11,541	12,847	11,904

a. What is Elijah's mean number of steps per day? Show your work.

SOLUTION

b. On Day 8, Elijah walks 9,924 steps. He says the expression $\frac{85{,}428 + 9{,}924}{8}$ represents the mean with the value for Day 8 included. Is Elijah correct? Explain.

5 During a 2-week time period, Denver has a mean high temperature of 72°F with a MAD of 5.66°F. During the same 2-week period, San Diego has a mean high temperature of 70°F with a MAD of 1.65°F. Compare the means and MADs and tell what they indicate about the high temperatures of the two cities.

6 Which of the following could be used to calculate the MAD of a data distribution? Select all that apply.

A Dot plot

B Frequency table

C Box plot

D List of data values

E Histogram

7 Brianna's speed-skating coach times her as she skates laps during practice. The table shows her times, to the nearest tenth of a second, for 8 laps.

Lap Times (seconds)			
26.6	26.5	25.1	29.3
28.5	27.0	28.5	27.7

a. Brianna's mean lap time is 27.4 seconds. What is the MAD of her lap times? Show your work.

SOLUTION

b. During the same practice, Layla's mean time for skating a lap is 26.6 seconds with a MAD of 2.1 seconds. Which skater, Brianna or Layla, had more consistent times during practice? Explain how you know.

8 **Math Journal** Make up a data set of five values with a mean of 10 and a median of 12. None of the values in your data set can be repeated. Explain how you determined the values in your set.

End of Lesson Checklist

- ☐ **INTERACTIVE GLOSSARY** Find the entry for *mean*. Tell how the mean and median of a data set are alike.
- ☐ **SELF CHECK** Go back to the Unit 7 Opener and see what you can check off.