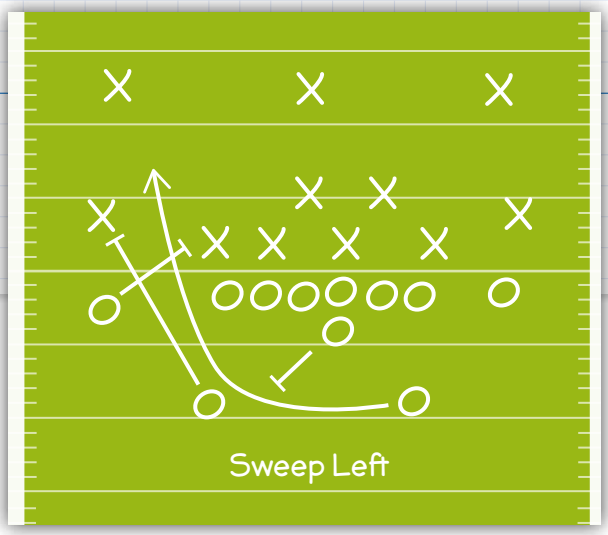
Explore Ordering Positive and Negative Numbers



A diagram of a football play

Previously, you learned about positive and negative numbers. In this lesson, you will learn about ordering and comparing positive and negative numbers.

➤ **Use what you know to try to solve the problem below.**

A youth football team tries several different plays. The goal of each play is to gain yards. The coach records the result of each play. List the plays from worst to best.

Name of Play	Wedge	Hook	Flag	Draw	Sweep	Toss
Result: Yards Gained (+) or Lost (−)	−3	+4	−5	+2	0	−4

TRY IT

Math Toolkit algebra tiles, number lines, two-color counters

DISCUSS IT

Ask: What did you do first to decide which play is the worst?

Share: The first thing I did was . . .

Learning Targets SMP 1, SMP 2, SMP 3, SMP 4, SMP 5, SMP 6

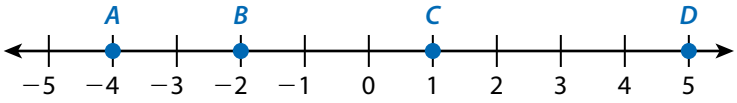
- Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram.
- Write, interpret, and explain statements of order for rational numbers in real-world contexts.

CONNECT IT

1 Look Back List the plays from worst to best. Explain how you know.

2 Look Ahead The goal of a football play is to gain yards. The more yards gained or the fewer yards lost, the better the play is. Number lines can be used to help make these types of comparisons with positive and negative numbers.

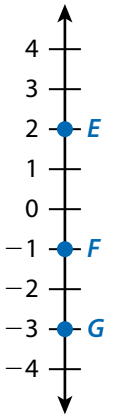
a. Look at the horizontal number line. Point *D* is farther to the right from 0 than point *C*. Which point represents a greater number?



b. Point *A* is farther to the left from 0 than point *B*. Which point represents a greater number?

c. Look at the vertical number line. Point *G* is farther down from 0 than Point *F*. Which point represents a greater number?

d. Point *E* is above Point *F*. Which point represents a lesser number? What is always true when comparing a negative number and a positive number?



3 Reflect How do the values change on a horizontal number line as you move left? How do the values change on a vertical number line as you move up?

 Name:

Prepare for Ordering Positive and Negative Numbers

1 Think about what you know about positive and negative numbers. Fill in each box. Use words, numbers, and pictures. Show as many ideas as you can.

Word	In My Own Words	Example
positive numbers		
negative numbers		
rational numbers		
inequality		

2 Choose a negative rational number. Write an inequality using the symbol $>$ to compare your number to 0. Explain your thinking.

3 Some friends play history trivia. Players gain 1 point for a correct answer. Players lose 1 point for an incorrect answer. The player with the greatest score wins. The players' scores are shown in the table.

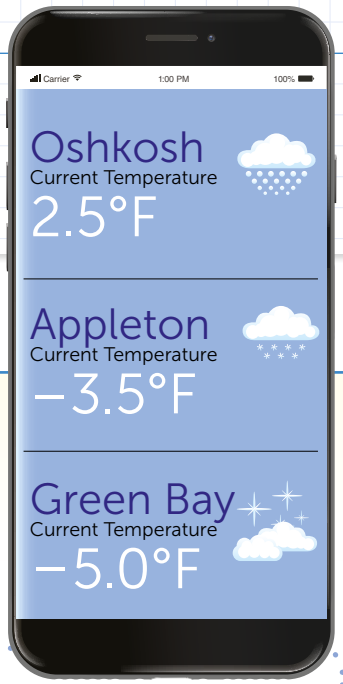
Player	Score
Brett	−7
Ellema	−1
Felipe	+3
Jennifer	0
Kamal	+2
Riley	−5

a. List the players from worst score to best score. Show your work.

SOLUTION

b. Check your answer to problem 3a. Show your work.

Develop Comparing Positive and Negative Numbers



➤ **Read and try to solve the problem below.**

On a winter day, Adnan looks up the current temperatures in three nearby cities. Adnan chooses two of the temperatures and writes a comparison. What are all the possible comparisons he can write? You can use words and/or symbols.

TRY IT

Math Toolkit graph paper, number lines

DISCUSS IT

Ask: How can you determine that you have found all the possible comparisons?

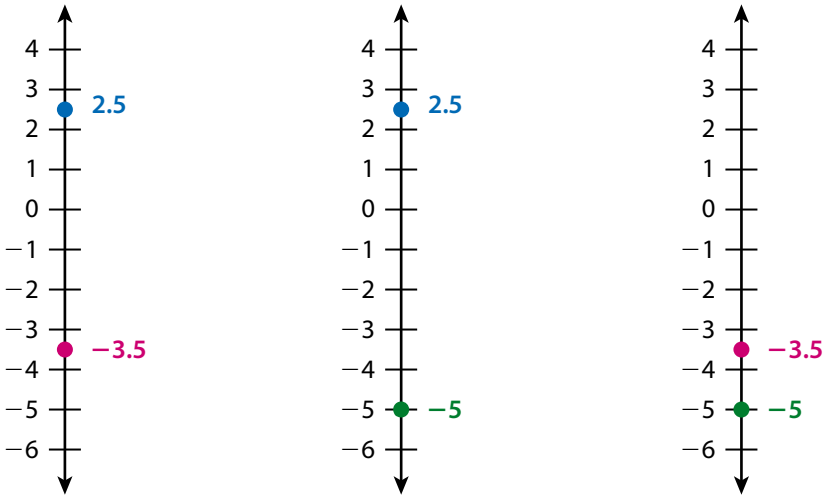
Share: I know I found all the comparisons because . . .

➤ **Explore different ways to compare positive and negative numbers.**

On a winter day, Adnan looks up the current temperatures in three nearby cities. The temperatures are 2.5°F, −3.5°F, and −5°F. Adnan chooses two of the temperatures and writes a comparison. What are all the possible comparisons he can write? You can use words and/or symbols.

Model It

You can use a number line to compare positive and negative numbers.



Model It

You can write an inequality to compare positive and negative numbers.

2.5 > −3.5 2.5 > −5 −3.5 > −5

Analyze It

You can use words to interpret the meaning of an inequality in a real-world situation.

2.5°F is warmer than −3.5°F.

2.5°F is warmer than −5°F.

−3.5°F is warmer than −5°F.

CONNECT IT

➤ **Use the problem from the previous page to help you understand how to compare positive and negative numbers.**

1. Look at the two **Model Its**. How can you use a number line to help you write an inequality?

2. Look at **Analyze It**. How can the inequalities help to determine which of two temperatures is warmer?

3. Use the symbol $<$ to rewrite the comparison between -3.5°F and 2.5°F. Then interpret the meaning of the inequality using the words *colder than*.

4. What are all the possible inequality statements Adnan might write? Use $<$ and $>$.

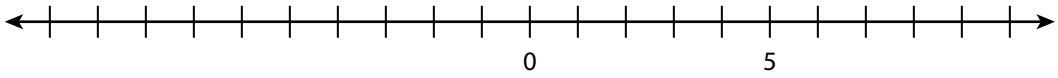
5. When given a pair of numbers in a real-world situation, how can you compare the numbers using $<$ and $>$? How can an inequality help you interpret the comparison in the real-world situation?

6. **Reflect** Think about all the models and strategies you have discussed today. Describe how one of them helped you better understand how to compare positive and negative numbers.

Apply It

Use what you learned to solve these problems.

7 Plot and label the numbers −6.5 and −8.5 on the number line. Then write an inequality using the symbol > to compare the two numbers.



SOLUTION _____

8 Which of the following statements are true? Select all that apply.

A 4 > −17 because 4 is to the right of −17 on a horizontal number line.

B 4 > −17 because 4 is to the left of −17 on a horizontal number line.

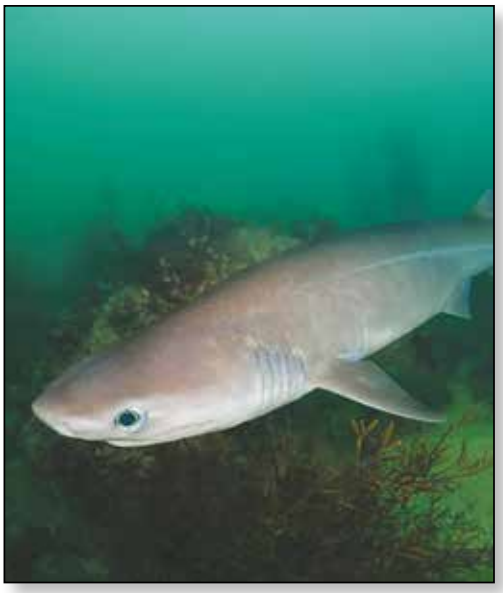
C 4 > −17 because −17 is to the right of 4 on a horizontal number line.

D 4 > −17 because 4 is above −17 on a vertical number line.

E 4 > −17 because 4 is below −17 on a vertical number line.

F 4 > −17 because −17 is below 4 on a vertical number line.

9 Notah is studying ocean animals. He learns that the sixgill shark can dive to an elevation of about −8,200 ft relative to sea level and the elephant seal can dive to an elevation of about −7,800 ft. Write an inequality to compare these elevations. Which animal can dive to a lower elevation? Show your work.



A sixgill shark

SOLUTION _____

 Name:

Practice Comparing Positive and Negative Numbers

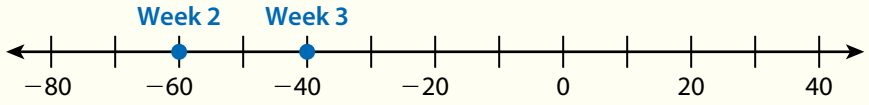
➤ **Study the Example showing how to compare two negative numbers. Then solve problems 1–5.**

Example

The table shows the amount of money Savanna either withdraws (−) or deposits (+) into her bank account over 5 weeks. Write an inequality to compare the withdrawals for Week 2 and Week 3.

Week	Week 1	Week 2	Week 3	Week 4	Week 5
Amount	+$40	−$40	−$60	+$100	−$80

Plot the amounts on a number line.

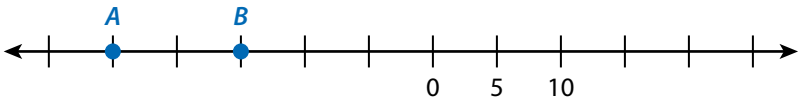


−60 is to the left of −40. So, $-60 < -40$.

1 **a.** Compare the two amounts in the Example using the symbol >.

b. Does using > for the inequality change which amount represents withdrawing more money? Explain.

2 Write an inequality that compares the value of point *A* and the value of point *B*. Show your work.

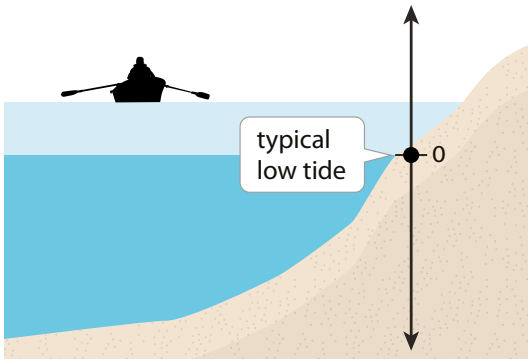


SOLUTION _____

Vocabulary

inequality
a mathematical statement that uses an inequality symbol to show the relationship between values of expressions.

3 The typical level of a low tide at a beach is the 0 point on a number line. Each day's high and low tides are measured relative to the typical low tide. On Monday morning, low tide is at -0.8 ft. On Tuesday morning, low tide is at -0.4 ft.



a. Write an inequality to compare the low tides on Monday and Tuesday mornings. Show your work.

SOLUTION

b. Which day has a higher low tide? Explain.

4 Consider the inequality $-3 < -2\frac{1}{2}$. What does the inequality tell you about the location of -3 compared to the location of $-2\frac{1}{2}$ on a horizontal number line? Use *to the right* and *to the left* in your answer.

5 In golf, the winner is the person with the lowest score. At the end of a round of golf, Jada's score is positive. Isabel's score is negative. Can you determine who wins? If so, tell who wins and why. If not, explain why not.

Refine Ordering Positive and Negative Numbers

➤ **Complete the Example below. Then solve problems 1–9.**

Example

Order the following rational numbers from least to greatest.

$\frac{1}{4}, -1.25, -\frac{3}{4}, 0.5, 1, -\frac{3}{2}$

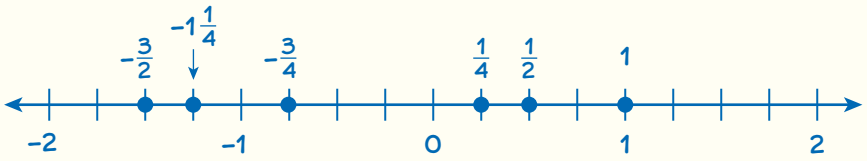
Look at how you could use a number line to order rational numbers.

Write the decimals as fractions.

$-1.25 = -1\frac{1}{4}$

$0.5 = \frac{1}{2}$

Plot the numbers on a number line.



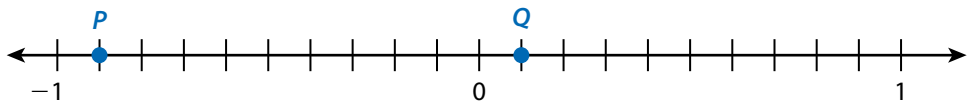
SOLUTION _____

CONSIDER THIS . . .
You can write all the rational numbers as fractions or write them all as decimals.

PAIR/SHARE
How would the order change if you changed $-\frac{3}{2}$ to $\frac{3}{2}$?

Apply It

1 Write two inequalities that compare the value of point *P* and the value of point *Q*. Show your work.



SOLUTION _____

CONSIDER THIS . . .
What does each tick mark on the number line represent?

PAIR/SHARE
What inequality can you write to compare the value of point *P* to −1?

2 A vending machine in a cafeteria sells sandwiches. The machine is restocked once during the day. At the end of each day, a cafeteria worker records how many more (+) or fewer (−) sandwiches are in the machine than there were at the start of the day. The table shows the changes for one week.

Day	Change in Number of Sandwiches
Monday	−3
Tuesday	+4
Wednesday	−5
Thursday	−2
Friday	0

Write an inequality to compare the changes for Monday and Thursday. Tell what your inequality means in terms of the situation. Show your work.

CONSIDER THIS . . .
The number of sandwiches at the end of a day is the result of some sandwiches being sold and the machine being restocked with more sandwiches.

SOLUTION

PAIR/SHARE
What does the 0 in the row for Friday mean in this situation?

3 An elevation of −4 m is higher than an elevation of −8 m. An elevation of −8 m is lower than an elevation of −6 m. Which set of inequalities correctly expresses these relationships?

A $-4 < -8$ and $-8 < -6$

B $-4 < -8$ and $-8 > -6$

C $-4 > -8$ and $-8 < -6$

D $-4 > -8$ and $-8 > -6$

Anders chose B as the correct answer. How might he have gotten that answer?

CONSIDER THIS . . .
How can you plot the elevations on a vertical number line or a horizontal number line to help write the inequalities?

PAIR/SHARE
What is a different way you can write the correct pair of inequalities?

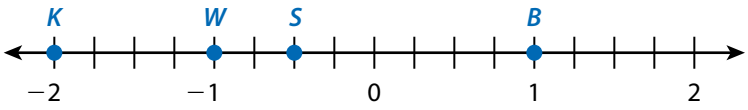
Lake Eyre, Australia

4 Sea level has an elevation of 0 ft. Lake Eyre is the lowest point in Australia. It has an elevation of −15 m relative to sea level. Which of the following U.S. locations, if any, have a lower elevation than Lake Eyre? Explain.

Location	Elevation (m)
Death Valley, California	−86
New Orleans, Louisiana	−2.4
Imperial, California	−18
Ouachita River, Arkansas	17

5 Doug says that $-7 > -5$ because $7 > 5$. Do you agree? Explain.

6 Tell whether each statement about the points on the number line is *True* or *False*.



	True	False
a. The value of point *K* is greater than −1.	○	○
b. The value of point *B* is greater than the value of point *W*.	○	○
c. The value of point *S* is less than 1.	○	○
d. The value of point *W* is less than −0.5.	○	○

7 Order the following rational numbers from least to greatest. Show your work.

$-1.5, \frac{3}{4}, -\frac{1}{4}, -1.75, -1, 1.5$

SOLUTION

8 Lilia wants to replace both question marks with the same number so that the inequalities correctly compare the numbers.

$? > -5$ and $? < 2$

Which of these numbers could Lilia use? Select all that apply.

A -7

B -4

C -2

D 0

E 1

F 5

9 **Math Journal** Choose two of the rational numbers shown below. Write two inequalities to compare the numbers, using $<$ and $>$. Then describe the location of one number compared to the other on a vertical number line. Use *above* and/or *below* in the description.

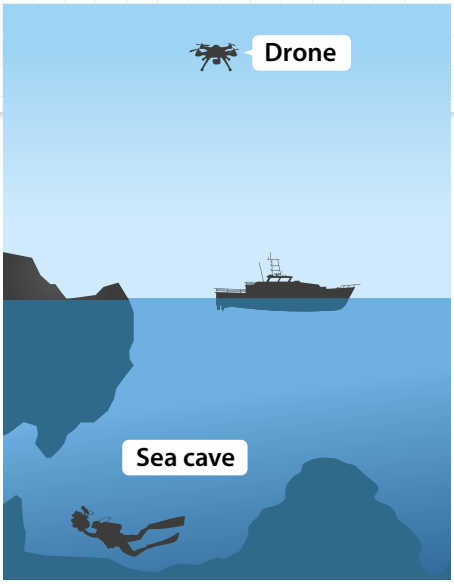
$-\frac{3}{4}$ -1.5 $\frac{1}{4}$ $-1\frac{1}{4}$ -0.5

End of Lesson Checklist

- ☐ **INTERACTIVE GLOSSARY** Write a new entry for *interpret*. Write at least one synonym for *interpret*.
- ☐ **SELF CHECK** Go back to the Unit 6 Opener and see what you can check off.

UNDERSTAND: What is the absolute value of a number?

Explore Absolute Value



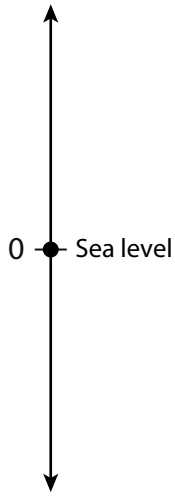
Model It

➤ **Complete the problems about distance from 0.**

1 A scientist standing on the deck of a boat uses a drone, and a scuba diver uses a camera to explore a sea cave. The table shows the elevations of four objects relative to sea level.

Object	Camera	Cave floor	Drone	Boat deck
Elevation	−20 ft	−30 ft	20 ft	5 ft

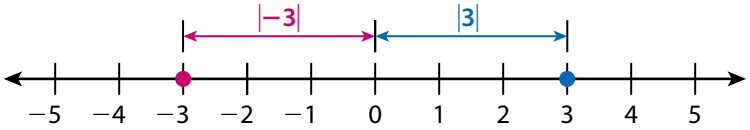
a. Use the number line to show the elevations of the objects from the table. Label each object at its elevation.



b. Are any of the objects the same distance from sea level? If so, how far from sea level are they?

c. Another object is 3 ft from sea level. Is the object's elevation *positive*, *negative*, or could it be *either*? Explain.

2 The **absolute value** of a number is its distance from 0. The notation $|-3|$ is read as *the absolute value of* -3 and represents the distance of -3 from 0.



$|3|$ = _____ because the distance from 0 to 3 is _____ units.

$|-3|$ = _____ because the distance from 0 to -3 is _____ units.

DISCUSS IT

Ask: How is absolute value related to zero on the number line?

Share: I think two numbers will have the same absolute value when . . .

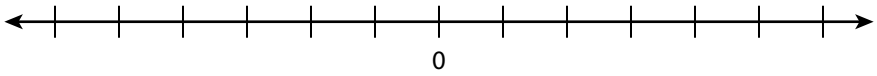
Learning Targets SMP 2, SMP 3, SMP 7

- Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation.
- Distinguish comparisons of absolute value from statements about order.

Model It

➤ **Complete the problems about absolute value.**

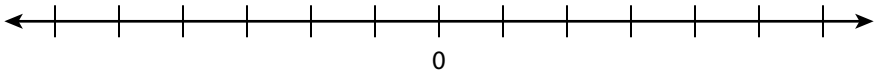
3 **a.** Plot and label the numbers 3, 4, 5, and 6 on the number line. Do the values of the numbers *increase* or *decrease* as the numbers go from 3 to 6?



b. Write the absolute value of each number. Do the absolute values of the numbers *increase* or *decrease* the numbers go from 3 to 6?

$|3| =$ _____ $|4| =$ _____ $|5| =$ _____ $|6| =$ _____

4 **a.** Plot and label the numbers -3, -4, -5, and -6 on the number line. Do the values of the numbers *increase* or *decrease* as the numbers go from -3 to -6?



b. Write the absolute value of each number. Do the absolute values of the numbers *increase* or *decrease* as the numbers go from -3 to -6?

$|-3| =$ _____ $|-4| =$ _____ $|-5| =$ _____ $|-6| =$ _____

5 Write *lesser* or *greater* to complete each statement.

a. The farther a number is from 0, the _____ the number's absolute value.

b. The closer a number is to 0, the _____ the number's absolute value.

DISCUSS IT

Ask: How are distance and absolute value related?

Share: I think the absolute value of 0 is . . . because . . .

6 **Reflect** Is the absolute value of a number ever negative? Explain your reasoning.

 Name:

Prepare for Understanding Absolute Value

1 Think about what you know about opposite numbers. Fill in each box. Use words, numbers, and pictures. Show as many ideas as you can.

In My Own Words

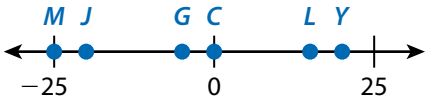
My Illustrations

opposite numbers

Examples

Non-Examples

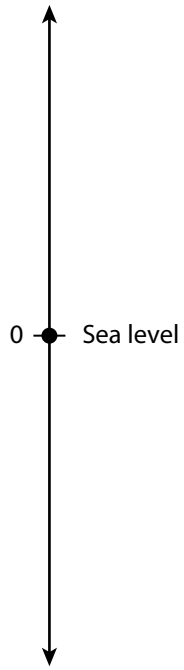
2 Look at the number line. Which pair of points appears to show a pair of opposite numbers? Explain your reasoning.



➤ **Complete problems 3–5.**

3 The table shows the elevations of four objects relative to sea level.

Object	Elevation (km)
Mountain cabin	2
Submarine	−10
Sunken ship	−6
Airplane	10



a. Use the number line to show the elevations of the four objects. Label each object at its elevation.

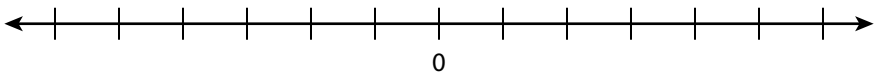
b. Circle the two objects on your number line that are the same distance from 0.

4 The notation $|40|$ means *the absolute value of 40*.

a. $|40| =$ _____ because the distance from 0 to 40 is _____.

b. $|-40| =$ _____ because the distance from 0 to −40 is _____.

5 **a.** Plot and label the numbers −2, −4, −6, and −8 on the number line. Do the values of the numbers *increase* or *decrease* as the numbers go from −2 to −8?



b. Find the absolute value of each number. Do the absolute values of the numbers *increase* or *decrease* as the numbers go from −2 to −8?

$|-2| =$ _____ $|-4| =$ _____ $|-6| =$ _____ $|-8| =$ _____

Vocabulary

absolute value

a number's distance from 0 on the number line. Absolute value is never negative.

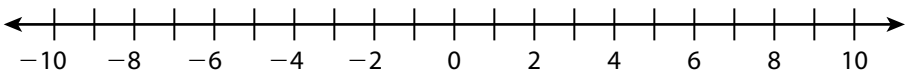
UNDERSTAND: What is the absolute value of a number?

Develop Understanding of Absolute Value

Model It: Compare Absolute Values

➤ **Try these two problems about comparing absolute values.**

1 Use the number line to help you compare the numbers and compare their absolute values. Write $<$, $>$, or $=$ in each circle to make a true statement. Explain how you know.

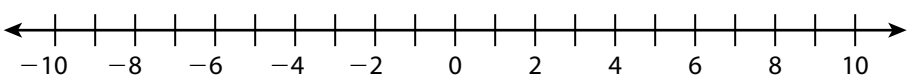


a. $-9 \bigcirc 5$ $\quad |-9| \bigcirc |5|$

b. $-1 \bigcirc 2$ $\quad |-1| \bigcirc |2|$

c. $-8 \bigcirc 8$ $\quad |-8| \bigcirc |8|$

2 Plot and label points for two numbers a and b so that $a < b$ and $|a| > |b|$. Explain your thinking.



DISCUSS IT

Ask: How does a number line help you determine which absolute value is greater?

Share: I think that when you compare two numbers and then compare their absolute values, the inequality symbols can be different because . . .

Model It: Interpret Absolute Value

➤ **Try these two problems about interpreting absolute value.**

3 The absolute value of a number may be used to describe the size, or magnitude, of a real-world quantity. Complete each equation and sentence.

a. $|-20| =$ _____ $-\$20$ means you owe \$_____.

b. $|+10| =$ _____ A score of $+10$ points means you win _____ points.

c. $|-10| =$ _____ A score of -10 points means you _____ points.

4 In each turn of a game, a player either wins or loses points. After the first turn, Jacob's score is -250 points and Indira's score is -300 points. Circle the inequality that makes a correct comparison. Then write a sentence to tell what the inequality means in this situation.

a. $-300 > -250$ $-300 < -250$

b. $|-300| > |-250|$ $|-300| < |-250|$

DISCUSS IT

Ask: How would you interpret the absolute value of a negative temperature?

Share: I think you can use positive numbers to describe negative quantities because . . .

CONNECT IT

➤ **Complete the problems below.**

5 A whale starts at an elevation of -200 ft relative to sea level and then swims to an elevation of -150 ft. Write an inequality using absolute value notation to compare the distances below sea level. Explain your reasoning.

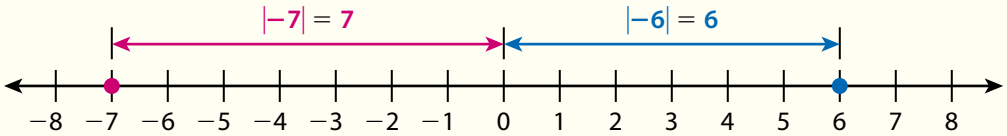
6 Luis says $|4|$ is greater than $|-5|$ because 4 is positive, -5 is negative, and any positive number is greater than any negative number. Do you agree? Explain.

Practice Comparing Absolute Values

Study how the Example shows comparing two numbers and their absolute values. Then solve problems 1–5.

Example

Use the numbers −7 and 6. Which number has the greater value? Which number has the greater absolute value?



6 is to the right of −7 on the number line, so 6 is greater than −7.

−7 is 7 units from 0.

6 is 6 units from 0.

So, −7 has the greater absolute value.

$-7 < 6$ and $|-7| > |6|$.

1. Choose a number less than −2 that is on the number line in the Example. Is your number's absolute value *greater than 2* or *less than 2*? Explain how you know.

2. Use the number line from the Example to help you compare the numbers and compare their absolute values. Write <, >, or = in each circle to make a true statement. Explain how you know.

a. −3 ◯ 5 $\quad |-3| \bigcirc |5|$

b. 4 ◯ −4 $\quad |4| \bigcirc |-4|$

Vocabulary

absolute value

a number's distance from 0 on the number line. Absolute value is never negative.

3 Sophia, Malcolm, and Oren are playing a money game. Their bank balances are shown in the table. Complete the table by writing the absolute value of each bank balance to show how much each player owes. Who owes the greatest amount?

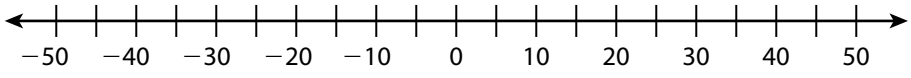
Player	Bank Balance	Amount Owed
Sophia	−$150	
Malcolm	−$325	
Oren	−$275	

4 The temperature on Monday is −24°C. The temperature on Tuesday is −21°C. Circle the inequality that makes a correct comparison. Then write a sentence to tell what the inequality means in this situation.

a. $-24 < -21$ $-24 > -21$

b. $|-24| < |-21|$ $|-24| > |-21|$

5 Plot and label points for two numbers c and d so that $c < d$ and $|c| > |d|$. Explain your thinking.



UNDERSTAND: What is the absolute value of a number?

Refine Ideas About Absolute Value

Apply It

 **Math Toolkit** number lines

➤ **Complete problems 1–5.**

1 **Deduce** Jia is thinking of a number. She gives three clues about the number: the number is even, the number is less than -12, and the absolute value of the number is between 9 and 15. What is Jia's number? Explain how you know.

2 **Analyze** Ian says that if $x < y$, then $|x| < |y|$. Is Ian's statement *always true, sometimes true,* or *never true?* Use a model to help explain your thinking.

3 **Apply** Mrs. Shen writes the expression $|-5| + |3|$ on the board. Show or explain why the sum $|-5| + |3|$ is the distance between -5 and 3 on a number line.

4 A tour group is going sea diving. The ocean floor is at −18 ft relative to sea level. One diver is already at −11 ft. The tour guide is keeping watch on a platform 5 ft above sea level, directly above the diver.

PART A Draw a model of the situation.

PART B Write an absolute value inequality comparing the distances of the tour guide and the diver to sea level. Who is closer to sea level? Explain how you know.

5 **Math Journal** Order the numbers 5, −7, −9, and −2 from least to greatest. Then order the absolute values $|5|$, $|-7|$, $|-9|$, and $|-2|$ from least to greatest. Explain how absolute value affects which values are lesser and which values are greater.

End of Lesson Checklist

☐ **INTERACTIVE GLOSSARY** Find the entry for *absolute value*. Explain why the absolute value of −4 is greater than the absolute value of 3.