Overview | Divide Fractions

STANDARDS FOR MATHEMATICAL PRACTICE (SMP)

SMP 1, 2, 3, 4, 5, and 6 are integrated into the Try-Discuss-Connect routine.*

This lesson provides additional support for:

1 Make sense of problems and persevere in solving them.

7 Look for and make use of structure.

* See page 1q to learn how every lesson includes these SMP.

Objectives

Content Objectives

- Divide fractions by fractions using visual models and equations.
- Use visual models and equations to represent word problems involving fraction division.
- Interpret the quotient of two fractions.
- Solve fraction division problems, including those that have quotients that are fractions.
- Write a word problem to match a given expression involving division of fractions.

Language Objectives

- Discuss connections between visual models and equations in word problems involving fraction division with a partner.
- Read and interpret word problems to determine which quantity is the dividend and which quantity is the divisor.
- Justify the size of the quotient in relationship to the dividend and divisor using reasoning and understanding of fraction division in class discussion.
- Apply understanding of division with fractions to write a word problem to match a given expression using complete sentences.
- Understand and use lesson vocabulary to accurately explain division of fractions.

Prior Knowledge

- Understand that a divisor can represent the size of a group or the number of groups.
- Add, subtract, and multiply fractions.
- Interpret a fraction as division.
- Use visual models to represent division with fractions.
- Write a mixed number as a fraction greater than 1.

Vocabulary

Math Vocabulary

reciprocal for any nonzero number a, the reciprocal is $\frac{1}{a}$. The reciprocal of any fraction $\frac{a}{b}$ is $\frac{b}{a}$. Zero does not have a reciprocal.

Review the following key terms.

common denominator a number that is a common multiple of the denominators of two or more fractions.

fraction a number that names equal parts of a whole. A fraction names a point on the number line and can also represent the division of two numbers.

Academic Vocabulary

nonzero not equal to zero.

Learning Progression

In Grade 5, students multiplied fractions by whole numbers and other fractions. They also divided whole numbers by unit fractions and unit fractions by whole numbers.

In the previous lesson, students used visual models, including bar models and number lines, to find a quotient of two fractions when the fractions were compatible and when the quotient was a whole number.

➤

In this lesson, students use models and strategies to divide fractions, including mixed numbers. They look at situations in which the quotient is not a whole number, including when the divisor is a whole number or a fraction. When there is a remainder, they understand how to write the remainder as a fraction of the divisor. Students explore an algorithm for dividing fractions—multiplying by the reciprocal of the divisor—and they understand why this algorithm works. They solve a variety of word problems involving division of fractions.

➤

In the next lesson, students build on their skills and understandings of multiplying and dividing fractions as they solve volume problems for prisms with fractional edge lengths.

In Grade 7, students will extend their work with fractions to perform operations with positive and negative rational numbers.

Overview

Pacing Guide

Items marked with ↖ are available on the **Teacher Toolbox**.

	MATERIALS	DIFFERENTIATION
SESSION 1 **Explore** Dividing Fractions (35–50 min)		
• **Start** (5 min) • **Try It** (5–10 min) • **Discuss It** (10–15 min) • **Connect It** (10–15 min) • **Close: Exit Ticket** (5 min) **Additional Practice** (pages 211–212)	**Math Toolkit** fraction bars, fraction circles, grid paper, number lines Presentation Slides ↖	**PREPARE** Interactive Tutorial ↖ **RETEACH or REINFORCE** Hands-On Activity **Materials** For each pair: 1 set of fraction tiles
SESSION 2 **Develop** Interpreting a Remainder When the Divisor Is a Fraction (45–60 min)		
• **Start** (5 min) • **Try It** (10–15 min) • **Discuss It** (10–15 min) • **Connect It** (15–20 min) • **Close: Exit Ticket** (5 min) **Additional Practice** (pages 217–218)	**Math Toolkit** fraction bars, fraction circles, grid paper, number lines Presentation Slides ↖	**RETEACH or REINFORCE** Hands-On Activity **Materials** For each pair: 13 connecting cubes **REINFORCE** Fluency & Skills Practice ↖ **EXTEND** Deepen Understanding
SESSION 3 **Develop** Dividing by a Fraction (45–60 min)		
• **Start** (5 min) • **Try It** (10–15 min) • **Discuss It** (10–15 min) • **Connect It** (15–20 min) • **Close: Exit Ticket** (5 min) **Additional Practice** (pages 223–224)	**Math Toolkit** fraction bars, fraction circles, grid paper, number lines Presentation Slides ↖	**RETEACH or REINFORCE** Hands-On Activity **Materials** For each pair: 2 sets of fraction circles **REINFORCE** Fluency & Skills Practice ↖ **EXTEND** Deepen Understanding
SESSION 4 **Refine** Dividing Fractions (45–60 min)		
• **Start** (5 min) • **Monitor & Guide** (15–20 min) • **Group & Differentiate** (20–30 min) • **Close: Exit Ticket** (5 min)	**Math Toolkit** Have items from previous sessions available for students. Presentation Slides ↖	**RETEACH** Hands-On Activity **Materials** For each group: 7 sets of fraction circles **REINFORCE** Problems 4–8 **EXTEND** Challenge **PERSONALIZE** i-Ready
Lesson 10 Quiz ↖ or **Digital Comprehension Check**		**RETEACH** Tools for Instruction ↖ **REINFORCE** Math Center Activity ↖ **EXTEND** Enrichment Activity ↖

Connect to Culture

➤ **Use these activities to connect with and leverage the diverse backgrounds and experiences of all students. Engage students in sharing what they know about contexts before you add the information given here.**

SESSION 1 ■ □ □ □

Try It Ask students if they have ever made anything out of ceramic pottery clay or polymer modeling clay. Clay can be used to make a variety of things, such as bricks, tiles, dishware, pots, sculptures, and even toys. Once a figure is formed, depending on the type of clay, it can be air dried, baked, or fired in a kiln at very high temperatures to become hard. Ask students to share objects they have seen or used that are made from clay.

SESSION 2 ■ ■ □ □

Try It Juneteenth is a celebration of the ending of slavery in the United States. On June 19th, 1865, news of Lincoln's Emancipation Proclamation finally reached Texas, two and a half years after slaves were officially freed. Now, some people make a journey back to Galveston, Texas to celebrate Juneteenth. Sporting events, barbecues, and guest speakers have become important parts of the tradition. The number of cities and states that celebrate Juneteenth continues to grow to this day. Survey students to determine if they know of or have participated in any other celebrations of freedom.

SESSION 3 ■ ■ ■ □

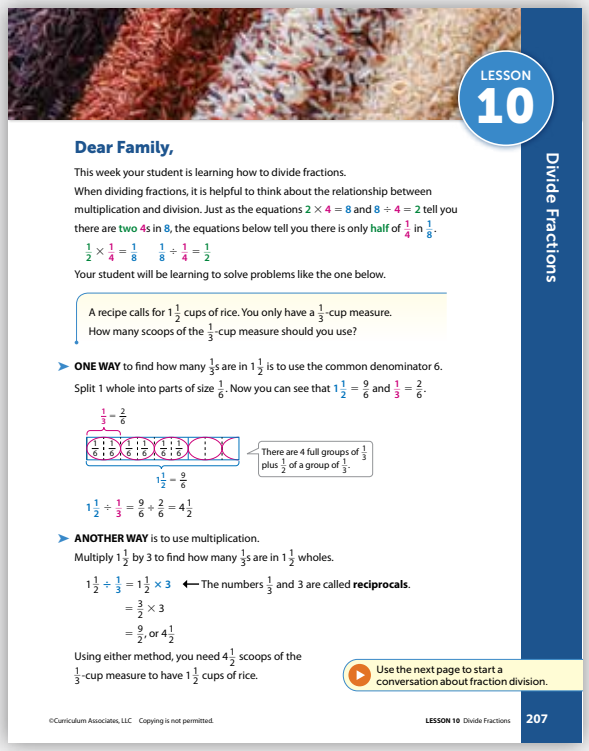
Try It *Chicken feed* is the food given to chickens. Different feed is used for younger chickens versus older ones: starter feed for babies, grower feed for young chickens, and various types of feed for adults. There are different forms and different ingredients of chicken feed. Depending on the role of the chicken—for example, laying eggs—there should be a different mix of mash, pellets, shell grit, and chicken scratch for a good meal. Have students describe different varieties of food they have seen or know of for other animals.

SESSION 4 ■ ■ ■ ■

Apply It **Problem 5** Tomatoes are a popular plant for many gardeners. There are many different types of tomatoes. Tomatoes are commonly red, but there are also orange, yellow, purple, and black tomatoes. Tomatoes can be small like grapes or as large as melons. They are typically grown in soil, but some farmers choose to use only water and nutrients instead. Have students discuss different dishes that include tomatoes as an ingredient.

Connect to Family and Community

➤ **After the Explore session, have students use the Family Letter to let their families know what they are learning and to encourage family involvement.**



LESSON 10

Divide Fractions

Dear Family,

This week your student is learning how to divide fractions.

When dividing fractions, it is helpful to think about the relationship between multiplication and division. Just as the equations $2 \times 4 = 8$ and $8 \div 4 = 2$ tell you there are two 4s in 8, the equations below tell you there is only half of $\frac{1}{4}$ in $\frac{1}{8}$.

$\frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$ $\frac{1}{8} \div \frac{1}{4} = \frac{1}{2}$

Your student will be learning to solve problems like the one below.

A recipe calls for $1\frac{1}{2}$ cups of rice. You only have a $\frac{1}{3}$-cup measure. How many scoops of the $\frac{1}{3}$-cup measure should you use?

➤ **ONE WAY** to find how many $\frac{1}{3}$s are in $1\frac{1}{2}$ is to use the common denominator 6.

Split 1 whole into parts of size $\frac{1}{6}$. Now you can see that $1\frac{1}{2} = \frac{9}{6}$ and $\frac{1}{3} = \frac{2}{6}$.

$1\frac{1}{2} \div \frac{1}{3} = \frac{9}{6} \div \frac{2}{6} = 4\frac{1}{2}$

➤ **ANOTHER WAY** is to use multiplication.

Multiply $1\frac{1}{2}$ by 3 to find how many $\frac{1}{3}$s are in $1\frac{1}{2}$ wholes.

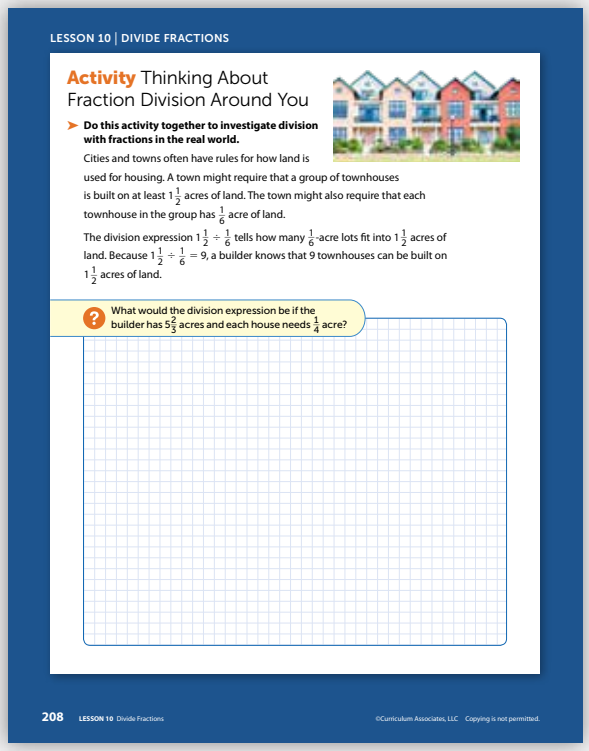
$1\frac{1}{2} \div \frac{1}{3} = 1\frac{1}{2} \times 3$ ← The numbers $\frac{1}{3}$ and 3 are called **reciprocals**.

$= \frac{3}{2} \times 3$

$= \frac{9}{2}$, or $4\frac{1}{2}$

Using either method, you need $4\frac{1}{2}$ scoops of the $\frac{1}{3}$-cup measure to have $1\frac{1}{2}$ cups of rice.

Use the next page to start a conversation about fraction division.



Activity Thinking About Fraction Division Around You

➤ **Do this activity together to investigate division with fractions in the real world.**

Cities and towns often have rules for how land is used for housing. A town might require that a group of townhouses is built on at least $1\frac{1}{2}$ acres of land. The town might also require that each townhouse in the group has $\frac{1}{6}$ acre of land.

The division expression $1\frac{1}{2} \div \frac{1}{6}$ tells how many $\frac{1}{6}$-acre lots fit into $1\frac{1}{2}$ acres of land. Because $1\frac{1}{2} \div \frac{1}{6} = 9$, a builder knows that 9 townhouses can be built on $1\frac{1}{2}$ acres of land.

What would the division expression be if the builder has $5\frac{2}{3}$ acres and each house needs $\frac{1}{4}$ acre?

Connect to Language

➤ **For English language learners, use the Differentiation chart to scaffold the language in each session. Use the Academic Vocabulary routine for academic terms before Session 1.**

DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS Use with **Session 1** **Connect It**

MATH TERM

To *estimate* means to give an approximate number or answer based on mathematical thinking.

ACADEMIC VOCABULARY

A *portion* is a part of a larger amount.

Levels 1–3: Reading/Listening

Support students as they interpret Connect It problem 2. Read the problem aloud. Display and clarify the Math Term and Academic Vocabulary.

Guide students to make sense of the problem by sketching the examples provided in parts a and b and asking students to make observations about the sketches. Call on volunteers to help you label the quotient and dividend in each sketch. Have students turn and talk about each example using the phrases *greater than* or *less than*.

Call on volunteers to share answers and explain their thinking. Reword as needed to model precise language and clarify ideas.

Levels 2–4: Reading/Listening

Support students as they interpret Connect It problem 2. Read the problem with students. Have students turn and talk to define *estimate*. Display the definition provided and have students revise and improve their definitions based on the one provided. Repeat for *portion*.

Allow time for partners to make sense of parts a and b by sketching the examples provided in each part. Review the sentence frames and ask partners to decide which to use for parts a and b:

- *The quotient ____ is greater than the dividend ____.*
- *The quotient ____ is less than the dividend ____.*

Levels 3–5: Reading/Listening

Support students as they interpret Connect It problem 2. Read the problem with students. Have them talk with a partner to develop a definition for *estimate*. Display the definition provided. Adapt **Say It Another Way** by recording different ways that partners state the meaning of the term. Repeat for *portion*.

Begin a **Co-Constructed Word Bank** of terms that might be used to talk and write about the problem. Students are likely to suggest *quotient, dividend, greater than,* and *less than.* You might add *estimate* and *portion.* Encourage students to use the terms as they respond to the problem.

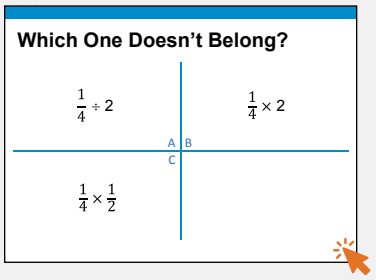
Explore Dividing Fractions

Purpose

- **Explore** the idea that dividing a fraction by a whole number can be thought of as decomposing the fraction into equal-size parts.
- **Understand** that when dividing by a fraction, the quotient is sometimes greater than the dividend.

START CONNECT TO PRIOR KNOWLEDGE

Which One Doesn't Belong?



Possible Solutions

A is the only expression written as a division expression.

B is the only expression that is not equivalent to 3.

C is the only expression that does not include a whole number.

WHY? Support students' facility with recognizing the connection between division and multiplication with fractions.

TRY IT — SMP 1, 2, 4, 5, 6

Make Sense of the Problem

See ***Connect to Culture*** to support student engagement. Before students work on Try It, use **Say It Another Way** to help them make sense of the problem. If students identify 3 as an important quantity in the problem, have them reread the third sentence carefully and turn and talk to discuss what this means in terms of what the problem is asking.

DISCUSS IT — SMP 2, 3, 6

Support Partner Discussion

After students work on Try It, have them respond to Discuss It with a partner. Listen for understanding of:

- drawing a model that represents $\frac{3}{4}$ lb.
- dividing $\frac{3}{4}$ into 4 equal parts.
- finding $\frac{1}{4}$ of $\frac{3}{4}$.

LESSON 10 | SESSION 1 ■ □ □ □

Explore Dividing Fractions

Previously, you learned what it means to divide with fractions. In this lesson, you will learn more about dividing fractions.

➤ **Use what you know to try to solve the problem below.**

Ramona is making clay animals with her friends. She has $\frac{3}{4}$ lb of clay. She shares the clay equally among herself and 3 friends. How much clay does each person have?

TRY IT

Math Toolkit fraction bars, fractions circles, grid paper, number lines

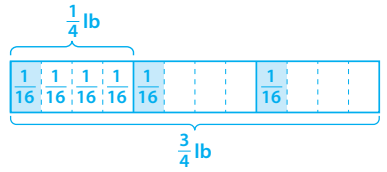
Possible work:

SAMPLE A

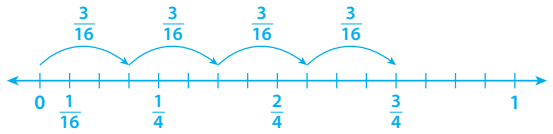
There are 4 people, so each person gets $\frac{1}{4}$ of each $\frac{1}{4}$ lb of clay.

$\frac{1}{4}$ of $\frac{1}{4} = \frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$

Each person has $\frac{3}{16}$ lb of clay.



SAMPLE B



$\frac{3}{4} = \frac{12}{16}$. When you divide $\frac{12}{16}$ into 4 equal parts, each part is $\frac{3}{16}$.

Each person has $\frac{3}{16}$ lb of clay.

DISCUSS IT

Ask: How does your model show that Ramona shares the clay equally among herself and 3 friends?

Share: My model shows . . .

Learning Target SMP 1, SMP 2, SMP 3, SMP 4, SMP 5, SMP 6, SMP 7

Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions.

Common Misconception Listen for students who divide the denominator of 4 in $\frac{3}{4}$ by 4, giving a result of $\frac{3}{1}$ instead of $\frac{3}{16}$. As students share their strategies, ask them to think about how they can use multiplication to check their answer. For example, have students write $\frac{3}{1} \times 4$ to understand that the product would be 12, not $\frac{3}{4}$.

Select and Sequence Student Strategies

Select 2–3 samples that represent the range of student thinking in your classroom. Here is one possible order for class discussion:

- **(misconception)** expressions that divide only the denominator by 4 instead of $\frac{3}{4} \div 4$
- bar models that show 3 fourths each divided into 4 equal parts
- area models that show $\frac{3}{4}$ divided into 4 equal parts
- number lines that show 4 equal jumps of $\frac{3}{16}$ from 0 to $\frac{3}{4}$
- equations for conversion of $\frac{3}{4}$ pound to 12 ounces and division by 4

Facilitate Whole Class Discussion

Call on students to share selected strategies. Use turn and talk to help students reason through their models before sharing with the group.

After each strategy, allow individual think time for students to process the ideas. Encourage students to identify ideas they agree with and add details to build onto those ideas.

ASK *How does [student name]'s model show division?*

LISTEN FOR Division is splitting a quantity into equal groups to find the number or size of the groups. Representations should show a quantity, $\frac{3}{4}$, divided into 4 equal groups.

CONNECT IT

SMP 2, 4, 5

1 **Look Back** Look for understanding that the models show the dividend as 12 equal parts that are each $\frac{1}{16}$, the divisor as 4, and the quotient as $\frac{3}{16}$, or three $\frac{1}{16}$s.

DIFFERENTIATION | RETEACH or REINFORCE

Hands-On Activity

Use fraction tiles to model a fraction divided by a whole number.

If students are unsure about dividing a fraction by a whole number, then use this activity to help them model division of a fraction by a whole number.

Materials For each pair: 1 set of fraction tiles

- Explain to students that they will be modeling $\frac{2}{3} \div 4$. Have students start with 2 one-third tiles. Ask: *How does this model represent* $\frac{2}{3}$*?* [The total length of 2 one-third tiles is $\frac{2}{3}$.]
- Ask: *In terms of fraction tiles, what does it mean to divide* $\frac{2}{3}$ *by 4?* [It means you have to find 4 equal-size tiles that have the same total length as the 2 one-third tiles set end-to-end.]
- Allow time for students to determine which size fraction tile they can use 4 of to have the same length as the 2 one-third tiles.
- Ask: *Which fraction tile can you use 4 of to equal the length of the 2 one-third tiles?* $\left[\frac{1}{6}\right]$
- Ask: *What is the quotient* $\frac{2}{3} \div 4$*?* $\left[\frac{1}{6}\right]$
- Have students repeat the activity for other quotients of fractions and whole numbers, such as $\frac{2}{5} \div 4$ and $\frac{1}{2} \div 6$. $\left[\frac{1}{10}; \frac{1}{12}\right]$

CONNECT IT

1 **Look Back** How much clay does each person have when Ramona shares her clay? Explain how you know.

$\frac{3}{16}$ lb; Possible explanation: You can divide $\frac{3}{4}$ lb of clay into 4 equal parts by making 12 parts that are each $\frac{1}{16}$ lb. $12 \div 4 = 3$, so each person has $\frac{3}{16}$ lb.

2 **Look Ahead** Dividing $\frac{3}{4}$ lb of clay equally among several people is a division situation that involves fractions. As when you divide with whole numbers, it can be helpful to estimate a quotient before you divide.

a. Estimate whether $\frac{3}{4} \div 6$ is *greater than* or *less than* the dividend, $\frac{3}{4}$. Use an example of dividing $\frac{3}{4}$ lb of clay into 6 equal portions to explain your thinking.

Less than $\frac{3}{4}$; Possible explanation: Each portion will weigh less than $\frac{3}{4}$ lb because you divide the $\frac{3}{4}$ lb into 6 smaller pieces.

b. Estimate whether $2\frac{1}{2} \div \frac{1}{4}$ is *greater than* or *less than* the dividend, $2\frac{1}{2}$. Use an example of dividing $2\frac{1}{2}$ lb of clay into $\frac{1}{4}$-lb portions to explain your thinking.

Greater than $2\frac{1}{2}$; Possible explanation: Each pound of clay is made up of 4 parts of size $\frac{1}{4}$ lb, so the number of $\frac{1}{4}$-lb portions is at least $2 \cdot 4$, or 8.

c. Yolanda estimates that $3\frac{1}{4} \div \frac{1}{8}$ is about 24. Show how to use multiplication to check whether Yolanda's estimate is reasonable.

$24 \cdot \frac{1}{8} = \frac{24}{1} \cdot \frac{1}{8}$

$= \frac{24}{8} = 3$

3 is close to $3\frac{1}{4}$, so the estimate of 24 is reasonable.

3 **Reflect** What division equation can you write to show that when $\frac{3}{4}$ lb of clay is divided equally among 6 people, each person gets $\frac{1}{8}$ lb of clay? How would you use multiplication to check that the quotient is correct?

$\frac{3}{4} \div 6 = \frac{1}{8}$; Possible explanation: Check that the product $6 \cdot \frac{1}{8}$ is $\frac{3}{4}$.

2 **Look Ahead** Point out that the estimates in parts a and b help students recognize whether the quotient is greater than or less than the dividend. Students should recognize the multiplicative relationship between the quotient, the divisor, and the dividend.

CLOSE EXIT TICKET

3 **Reflect** Look for understanding of writing an equation that correctly shows the dividend, divisor, and quotient and that multiplying the quotient by the divisor can be used to check the equation.

Common Misconception If students are unclear about where to write the dividend, divisor, and quotient in an equation, then have them write a sentence to describe the situation and then substitute symbols for words. For example, students may write "$\frac{3}{4}$ **divided equally among** 6 people **is** $\frac{1}{8}$ lb each" and translate this to the equation $\frac{3}{4} \div 6 = \frac{1}{8}$.

Prepare for Dividing Fractions

Support Vocabulary Development

Assign **Prepare for Dividing Fractions** as extra practice in class or as homework.

If you have students complete this in class, then use the guidance below.

Ask students to consider the term *fraction* and to consider the parts of a fraction. Review that the numerator tells the number of equal parts that are described and the denominator tells the number of equal parts that are in the whole.

Have students work in pairs to complete the graphic organizer. Invite pairs to share their completed organizers, and prompt a whole-class comparative discussion of the previous knowledge and examples given.

Have students look at the model in problem 2 and discuss with a partner ways to remember the meaning of each term. Encourage students to use the terms *numerator* and *denominator* as they discuss their thinking.

Problem Notes

1. Students likely understand that fractions are numbers that express parts of a whole. Student responses may include a fraction with labeled parts; a shaded model with labeled parts; or a fraction that represents a division expression.

2. Students should recognize that the shaded part of the model represents the numerator and the number of equal parts in the model represents the denominator.

Prepare for Dividing Fractions

1. Think about what you know about fractions. Fill in each box. Use words, numbers, and pictures. Show as many ideas as you can.

Possible answers:

What Is It?

a number that names equal parts of a whole

What I Know About It

The numerator tells you the number of equal parts there are.

The denominator tells you the number of equal parts in the whole.

A fraction can represent the division of two numbers.

fraction

Examples

$\frac{7}{10}$ ← 7 equal parts
← 10 equal parts in the whole

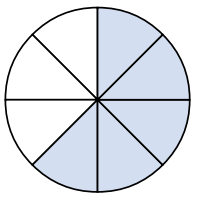
$\frac{7}{10} = 7 \div 10$

Examples

$\frac{5}{6}$ of the figure is shaded.

2. A fraction of the model is shaded. What are the numerator and the denominator of the fraction? Explain how you know.

The numerator is 5 because 5 parts of the whole are shaded. The denominator is 8 because the whole circle is divided into 8 equal parts.



REAL-WORLD CONNECTION

Fashion designers work with fractions when they design and sew clothes. Figuring out how to place buttonholes on the front of a shirt may require dividing with fractions. Suppose a designer wants to place 7 buttons on a shirt so that the distance between the top and bottom buttons is $13\frac{1}{2}$ inches. The space between the buttons can be found by calculating $13\frac{1}{2} \div 6$. Or, suppose a designer wants to put buttons $3\frac{1}{4}$ inches apart so that the distance between the top and bottom buttons is $19\frac{1}{2}$ inches. The number of buttons needed can be found by calculating $19\frac{1}{2} \div 3\frac{1}{4}$, and then adding 1 to the quotient. Ask students to think of other real-word examples where dividing fractions might be useful.

Additional Practice

3 Problem 3 provides another look at dividing a fraction by a whole number. This problem is similar to the problem about clay animals. In both problems, a fraction of a whole is divided into smaller parts. This problem asks for the weight of each layer of sand when a fraction of one pound of sand is divided into smaller equal parts.

Students may want to use fraction bars, fraction circles, grid paper, or number lines to solve.

Suggest that students use **Three Reads**, asking themselves one of the following questions each time:

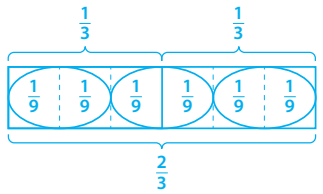
- *What is this problem about?*
- *What is the question I am trying to answer?*
- *What information is important?*

3 Patrick is using layers of colored sand in an art project. He divides $\frac{2}{3}$ lb of blue sand into 3 equal parts in order to make 3 layers.

a. What is the weight of the sand in each layer of blue sand? Show your work.

Possible work:

$\frac{1}{3}$ of $\frac{1}{3}$ is $\frac{1}{3} \cdot \frac{1}{3}$, or $\frac{1}{9}$.



SOLUTION The weight of the sand in each layer of blue sand is $\frac{2}{9}$ lb.

b. Check your answer to problem 3a. Show your work.

Possible work:

If each layer of blue sand weighs $\frac{2}{9}$ lb, then the total weight of 3 layers is 3 times $\frac{2}{9}$. Check by multiplying.

$$3 \cdot \frac{2}{9} = \frac{3}{1} \cdot \frac{2}{9}$$
$$= \frac{3 \cdot 2}{1 \cdot 9}$$
$$= \frac{6}{9}$$
$$= \frac{2}{3}$$

So, $\frac{2}{9}$ lb is correct since 3 times $\frac{2}{9}$ equals the starting amount.

DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Use with **Session 2** Apply It

Levels 1–3: Reading/Speaking

Support students as they make sense of Apply It problem 8. Read the problem aloud. Use photos or realia to illustrate the meaning of *pitcher*. Read the first two sentences again and think aloud as you sketch a pitcher labeled $1\frac{2}{5}$ L and a larger container labeled $4\frac{1}{5}$ L. Read the question. Ask students to locate and underline the two expressions. Call on a volunteer to interpret the expressions using the sketches and everyday language.

Display the expressions and have students turn and talk about which expression can be used to find the number of pitchers Sofia can fill. Allow time for students to solve individually.

Levels 2–4: Reading/Speaking

Support students as they make sense of Apply It problem 8 with **Three Reads**. Adapt the routine by displaying sentence starters to help students respond in complete sentences to the questions that correspond with each reading:

- *The problem is about . . .*
- *We are trying to find out . . .*
- *The important quantities in the problem are . . .*
- *They are related because . . .*

After each reading, present the question and allow think time for students to formulate a response before talking with a partner. Have students sketch to show their understanding and solve individually.

Levels 3–5: Reading/Speaking

Support students as they make sense of Apply It problem 8 with **Three Reads**. Adapt the routine by having students work in pairs to answer the questions and understand the problem. Encourage students to speak in complete sentences when responding to the questions after each reading. Review with students that making a sketch or model can help them justify their thinking.

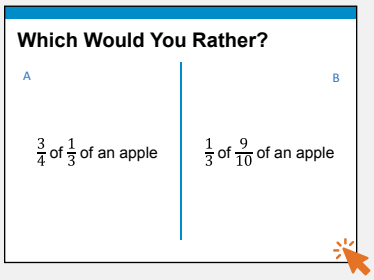
Allow time for students to solve the problem individually, then have them compare and justify their answers by explaining why it makes sense given the situation.

Develop Dividing Fractions

Purpose

- **Develop** strategies for dividing fractions (including mixed numbers) by fractions.
- **Recognize** that when dividing by a fraction, any remainder can be expressed as a fraction of the divisor.

START CONNECT TO PRIOR KNOWLEDGE



Possible Solutions

A because it is the lesser amount, $\frac{1}{4}$.

B because it is the greater amount, $\frac{3}{10}$.

WHY? Support students' facility with determining a fraction of a fraction and prepare students to think about a remainder as a fraction of a fractional divisor.

DEVELOP ACADEMIC LANGUAGE

WHY? Support students as they make connections between ideas.

HOW? Explain that making connections is an effective way to deepen understanding of an idea. Display the bar model from Model It and the division equation from Analyze It. Invite volunteers to circle parts of the model that are also represented in the equation and explain the connection. Provide a sentence frame: ____ *connects to* ____ *because* ____.

TRY IT

SMP 1, 2, 4, 5, 6

Make Sense of the Problem

See ***Connect to Culture*** to support student engagement. Before students work on Try It, use **Notice and Wonder** to help them make sense of the problem. Listen for understanding that the number of time slots depends on how many $\frac{3}{4}$-hour time slots fit into $3\frac{1}{2}$ hours.

LESSON 10 | SESSION 2

Develop Dividing Fractions

Read and try to solve the problem below.

Imani is planning her city's Juneteenth festival. There will be $3\frac{1}{2}$ hours of performances on the main stage. Each performer's time slot lasts $\frac{3}{4}$ hour. How many time slots can Imani plan to have?

TRY IT

Math Toolkit fraction bars, fraction circles, grid paper, number lines

Possible work:

SAMPLE A

1 hour = 60 minutes, and $\frac{1}{2}$ hour = 30 minutes.

$3 \cdot 60 + 30 = 210$, so there are 210 minutes in $3\frac{1}{2}$ hours.

$\frac{1}{4}$ hour = 15 min and $3 \cdot 15 = 45$. So, $\frac{3}{4}$ of an hour is 45 min.

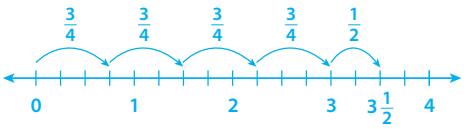
$4 \cdot 45 = 180$, so at least four 45-min time slots will fit in 210 min.

$210 - 180 = 30$, so there are not enough minutes for another time slot.

Imani can plan on having 4 time slots that are each $\frac{3}{4}$ hour long.

SAMPLE B

$3\frac{1}{2} \div \frac{3}{4} = ?$



You can fit $\frac{3}{4}$ hour into $3\frac{1}{2}$ hours 4 times, with $\frac{1}{2}$ hour left over.

The remaining $\frac{1}{2}$ hour is not long enough to make a fifth time slot.

Imani can plan to have four $\frac{3}{4}$-hour time slots.

DISCUSS IT

Ask: How is your strategy similar to mine? How is it different?

Share: My strategy is similar to yours because . . . It is different because . . .

DISCUSS IT

SMP 2, 3, 6

Support Partner Discussion

After students work on Try It, encourage them to respond to Discuss It with a partner. If students need support in getting started, prompt them to ask each other questions such as:

- *Why did you choose the model or strategy you used?*
- *How does your model show $3\frac{1}{2}$ and $\frac{3}{4}$?*

Common Misconception Listen for students who think the quotient is always less than the dividend. As students share their strategies, ask them to explain how to determine the reasonableness of the quotient in this situation.

Select and Sequence Student Strategies

Select 2–3 samples that represent the range of student thinking in your classroom. Here is one possible order for class discussion:

- fraction circles to visually partition three and a half circles into equal parts
- **(misconception)** strategies that result in a quotient that is less than the dividend
- number lines showing how many $\frac{3}{4}$s are in $3\frac{1}{2}$.

Facilitate Whole Class Discussion

Call on students to share selected strategies. If ideas are unclear, ask the speaker to repeat and rephrase.

Guide students to **Compare and Connect** the representations. Prompt students to connect strategies by showing how each represents the important quantities in the problem.

> **ASK** *How does [student name]'s model show the total time for performances? the time for each performer?*
>
> **LISTEN FOR** Representations should show the total performance time as $3\frac{1}{2}$ hours and the time for each performer as $\frac{3}{4}$ hour.

Model Its & Analyze It

If students presented these models, have students connect these models to those presented in class.

If no student presented at least one of these models, have students first analyze key features of the models, and then connect them to the models presented in class.

> **ASK** *How does each model represent the problem? How does the multiplication equation in the first Model It relate to the bar model?*
>
> **LISTEN FOR** Each model shows a way to find how many $\frac{3}{4}$s are in $3\frac{1}{2}$. The total of $3\frac{1}{2}$ hours is shown by the length of the bar. The number of groups of 3 fourths multiplied by the size of each group, $\frac{3}{4}$, is equal to the total.

For the equations in Model It, have students make connections between multiplication and division.

- *How are the multiplication and division equations related?*

For the bar model, prompt students to describe how the model is labeled to represent the problem.

- *What do the last two fourths circled represent?*

For the common denominator strategy, encourage students to justify using common denominators.

- *What does 14 mean in this situation?*

➤ **Explore different ways to divide fractions, including when the quotient is not a whole number.**

Imani is planning her city's Juneteenth festival. There will be $3\frac{1}{2}$ hours of performances on the main stage. Each performer's time slot lasts $\frac{3}{4}$ hour. How many time slots can Imani plan to have?

Dance performers at a Juneteenth festival

Model It

You can use the relationship between multiplication and division to represent a division situation with equations.

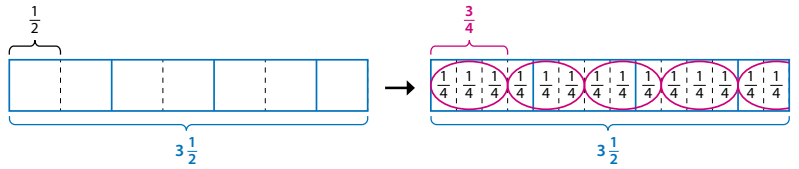
How many $\frac{3}{4}$-hour time slots are in $3\frac{1}{2}$ hours? $? \times \frac{3}{4} = 3\frac{1}{2}$

Use division to find the unknown factor. $3\frac{1}{2} \div \frac{3}{4} = ?$

Model It

You can use a bar model to divide fractions.

Show 3 wholes and $\frac{1}{2}$ of another whole. Split each whole into 2 halves. Then divide each half into 2 fourths and separate the fourths into groups of 3.



Analyze It

You can use a common denominator to divide fractions.

$$3\tfrac{1}{2} \div \tfrac{3}{4} = \tfrac{7}{2} \div \tfrac{3}{4}$$
$$= \tfrac{14}{4} \div \tfrac{3}{4}$$

Now you can think of dividing 14 fourths by 3 fourths, which is the same as $14 \div 3$.

DIFFERENTIATION | EXTEND

SMP 1

Deepen Understanding

Using Multiple Models to Understand and Solve a Problem

Prompt students to share how the models help them understand the problem and the solution process.

> **ASK** *How does the bar model help you see the relationships between the dividend, divisor, and quotient?*
>
> **LISTEN FOR** The bar model shows the dividend as the entire length of the model, the divisor as sections of three unit fractions, and the quotient as the number of times the divisor fits into the length of the model.
>
> **ASK** *How can you look at the bar model in a different way to make sense of using a common denominator?*
>
> **LISTEN FOR** The bar model shows 14 quarter hours. The total, or dividend, in the problem can be thought of as 14 quarter hours, and the divisor is equal groups of 3 quarter hours. So 14 can be divided by 3 to find the number of 3 quarter hours in 14 quarter hours.

Develop Dividing Fractions

CONNECT IT

SMP 2, 4, 5, 6

Remind students that the quantities and the relationships between them are the same in each representation. Explain that they will now use those relationships to reason about different strategies for interpreting a remainder.

Before students begin to record and expand on their work in the Model Its and Analyze It, tell them that problems 2 and 3 will prepare them to provide the answer asked for in problem 4.

Monitor and Confirm Understanding 1 – 2

- Dividing $3\frac{1}{2}$ hours into $\frac{3}{4}$-hour parts tells how many time slots are available for performers.
- A group that has fewer than 3 parts of size $\frac{1}{4}$ represents a partial time slot.

Facilitate Whole Class Discussion

3 Students learn that dividing two fractions with the same denominator gives the same result as dividing the two numerators.

> **ASK** *How is $\frac{14}{4} \div \frac{3}{4}$ similar to $14 \div 3$? How is it different?*
>
> **LISTEN FOR** Each expression represents 14 equal parts divided into groups of 3 parts. In the first expression, each part is $\frac{1}{4}$. In the second expression, each part is 1 whole.

4 Students should recognize that the remainder represents a fraction of the divisor, regardless of whether the divisor is a whole number or a fraction.

> **ASK** *How is finding the remainder for $14 \div 3$ similar to finding the remainder for $\frac{14}{4} \div \frac{3}{4}$?*
>
> **LISTEN FOR** $14 \div 3$ has 2 remaining whole parts, and $\frac{14}{4} \div \frac{3}{4}$ has 2 remaining fractional parts. In each case, the remaining parts are $\frac{2}{3}$ of the divisor.

5 Look for understanding that $3\frac{1}{2} \div \frac{3}{4} = 4\frac{2}{3}$. Have students compare approaches to finding the product $4\frac{2}{3} \cdot \frac{3}{4}$.

6 **Reflect** Have all students focus on the strategies used to solve the Try It. If time allows, have students discuss their ideas with a partner.

CONNECT IT

➤ **Use the problem from the previous page to help you understand how to divide fractions when the quotient is not a whole number.**

1 Look at the second **Model It**. How many full $\frac{3}{4}$-hour time slots does Imani have for performances? How does the second bar model show this?

4 full $\frac{3}{4}$-hour time slots; Possible explanation: The bar model shows that you can make 4 groups that each have 3 parts of size $\frac{1}{4}$.

2 The second bar model shows one group that has only 2 parts of size $\frac{1}{4}$. Explain why this group is $\frac{2}{3}$ of a full time slot.

This group has 2 out of the 3 parts that you need for a full time slot.

3 Look at **Analyze It** and the second **Model It**. How does the bar model show that you can use the quotient $14 \div 3$ to find the quotient $\frac{14}{4} \div \frac{3}{4}$?

The second bar model shows 14 parts of size $\frac{1}{4}$ separated into groups of 3 parts. To find how many groups there are, divide 14 by 3.

4 When you divide 14 by 3, the result is 4 with a remainder of 2. When you divide $\frac{14}{4}$ by $\frac{3}{4}$, the remainder is $\frac{2}{4}$. Where do you see this remainder in the bar model? What fraction of the divisor, $\frac{3}{4}$, does this remainder represent?

The remainder is the last group that shows only 2 parts of size $\frac{1}{4}$. It represents $\frac{2}{3}$ of the divisor.

5 How many times does $\frac{3}{4}$ fit into $3\frac{1}{2}$? Use multiplication to check your answer.

$4\frac{2}{3}$; Possible work: $4\frac{2}{3} \cdot \frac{3}{4} = \frac{14}{3} \cdot \frac{3}{4}$

$= \frac{14 \cdot 3}{3 \cdot 4} = \frac{14}{4} = \frac{7}{2} = 3\frac{1}{2}$

6 **Reflect** Think about all the models and strategies you have discussed today. Describe how one of them helped you better understand how to divide fractions when the quotient is not a whole number.

Responses will vary. Check student responses.

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DIFFERENTIATION | RETEACH or REINFORCE

Hands-On Activity

Use connecting cubes to interpret a remainder.

If students are unsure about interpreting a remainder when the divisor is a fraction, then use this activity to help them physically explore the remainder.

Materials For each pair: 13 connecting cubes

- Display the division problem $4\frac{1}{3} \div \frac{2}{3}$. Have students connect their 13 cubes in one row to model the dividend. Ask: *What unit fraction does each cube represent?* $\left[\frac{1}{3}\right]$ *Why does your model show $4\frac{1}{3}$?* [13 cubes show $\frac{13}{3}$, or $4\frac{1}{3}$.]
- Have partners model dividing the cubes into equal groups of $\frac{2}{3}$, the divisor. Ask: *How many full groups of $\frac{2}{3}$ are there?* [6] *How many cubes remain?* [1]
- Ask: *What fraction of a full group does the remainder represent?* $\left[\frac{1}{2}\right]$ *How do the cubes show this?* [Each full group of $\frac{2}{3}$ is shown by 2 cubes and the 1 remaining cube is half of that.] *What is the quotient $4\frac{1}{3} \div \frac{2}{3}$?* $\left[6\frac{1}{2}\right]$

Apply It

For all problems, encourage students to use a model to support their thinking. Allow some leeway in precision; for example, drawing equal-size sections in fraction circles, bar models, or number lines can be challenging, and precise drawings are not necessary as the model is being used to think through a problem.

7 Students may also use a number line or connecting cubes to reason through the problem. Students should recognize that the quotient is not a whole number and partial batches are reasonable in this situation.

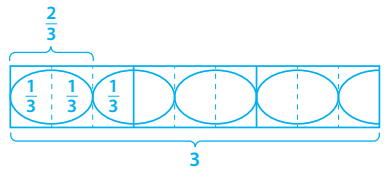
8 Students should recognize the amount each pitcher can hold as the divisor and the amount of iced tea as the dividend. Each pitcher represents an equal group.

Apply It

➤ **Use what you learned to solve these problems.**

7 Mr. Lincoln is making slime for his kindergarten class. He has 3 cups of glue. Each batch of slime uses $\frac{2}{3}$ cup of glue. How many batches of slime can Mr. Lincoln make? Show your work.

Possible work:



SOLUTION Mr. Lincoln can make $4\frac{1}{2}$ batches of slime.

8 Sofia has pitchers that each hold $1\frac{2}{5}$ L. She has $4\frac{1}{5}$ L of iced tea. Which division expression can you use to find the number of pitchers Sofia can fill with iced tea: $1\frac{2}{5} \div 4\frac{1}{5}$ or $4\frac{1}{5} \div 1\frac{2}{5}$? How many pitchers can Sofia fill? Show your work.

Possible work:

$$4\tfrac{1}{5} \div 1\tfrac{2}{5} = \tfrac{21}{5} \div \tfrac{7}{5}$$
$$= 21 \div 7$$
$$= 3$$

SOLUTION $4\frac{1}{5} \div 1\frac{2}{5}$; Sofia can fill 3 pitchers.

9 What is $2\frac{3}{4} \div \frac{3}{8}$? Show your work. Possible work:

$$2\tfrac{3}{4} \div \tfrac{3}{8} = \tfrac{11}{4} \div \tfrac{3}{8}$$
$$= \tfrac{22}{8} \div \tfrac{3}{8}$$
$$= 22 \div 3$$
$$= \tfrac{22}{3} = 7\tfrac{1}{3}$$

SOLUTION $7\frac{1}{3}$

CLOSE EXIT TICKET

9 Students' solutions should show an understanding of:

- interpreting and determining dividends and divisors.
- modeling and strategizing division of a mixed number by a fraction.

Error Alert If students think that the quotient is $7\frac{1}{8}$, they may not understand how to interpret the remainder. Encourage students to use a bar model to visualize that the remainder is $\frac{1}{3}$ of the divisor.

Practice Dividing Fractions

Problem Notes

Assign **Practice Dividing Fractions** as extra practice in class or as homework.

1 **a.** Students should recognize that the arrow from $2\frac{2}{3}$ to $3\frac{1}{3}$ shows that after fitting $1\frac{1}{3}$ into $3\frac{1}{3}$ two times, there are 2 remaining pieces of size $\frac{1}{3}$. ***Basic***

b. Students should recognize the 2 remaining thirds as half of the 4 thirds that make up 1 whole jogging loop. ***Medium***

2 Students may demonstrate through a model or equation that the quotient is 4. Students should understand that when the quotient is a whole number, there is no remainder. ***Medium***

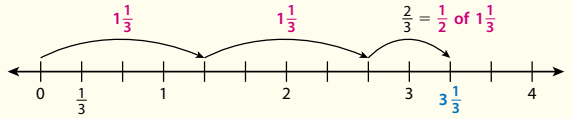
Practice Dividing Fractions

➤ **Study the Example showing how to divide fractions when the quotient is not a whole number. Then solve problems 1–5.**

Example

The jogging loop at Lake Park is $1\frac{1}{3}$ mi long. Brianna wants to jog $3\frac{1}{3}$ mi. How many times should she jog around the loop?

You can use a number line to show dividing $3\frac{1}{3}$ mi into lengths of $1\frac{1}{3}$ mi.



There are $2\frac{1}{2}$ lengths of $1\frac{1}{3}$ in $3\frac{1}{3}$. So, $3\frac{1}{3} \div 1\frac{1}{3} = 2\frac{1}{2}$.

Brianna should jog $2\frac{1}{2}$ times around the loop.

1 The number line model in the Example shows that $1\frac{1}{3}$ fits into $3\frac{1}{3}$ two times, with a remainder of $\frac{2}{3}$.

a. How is the remainder $\frac{2}{3}$ shown in the number line model?

The remainder $\frac{2}{3}$ is shown by the arrow from $2\frac{2}{3}$ to $3\frac{1}{3}$.

b. How do you know that the remainder represents $\frac{1}{2}$ of the jogging loop?

The loop is $1\frac{1}{3}$ mi. There are 4 thirds in $1\frac{1}{3}$ and 2 thirds is $\frac{1}{2}$ of 4 thirds.

2 Show that there is no remainder when you divide $2\frac{1}{2}$ by $\frac{5}{8}$.

Possible work: $2\frac{1}{2} \div \frac{5}{8} = \frac{5}{2} \div \frac{5}{8}$

$= \frac{20}{8} \div \frac{5}{8}$

$= 4$

$\frac{5}{8}$ fits into $2\frac{1}{2}$ exactly 4 times, with no remainder.

Vocabulary

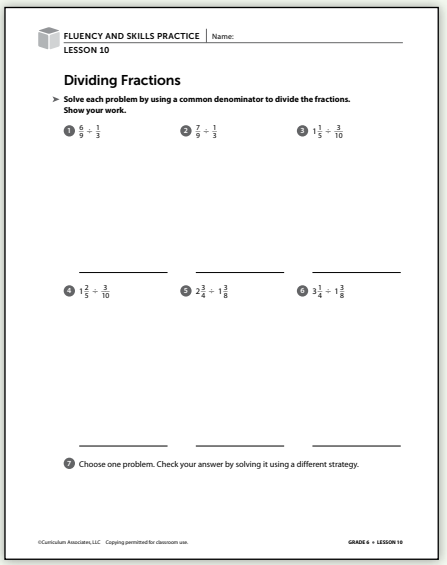
remainder
the amount left over when one number does not divide another number a whole number of times.

quotient
the result of division.

Fluency & Skills Practice

Dividing Fractions

In this activity, students solve fraction division problems by using a common denominator.


FLUENCY AND SKILLS PRACTICE | Name:
LESSON 10

Dividing Fractions

➤ Solve each problem by using a common denominator to divide the fractions. Show your work.

1 $\frac{6}{9} \div \frac{1}{3}$ 2 $\frac{7}{9} \div \frac{1}{3}$ 3 $1\frac{1}{5} \div \frac{3}{10}$

4 $1\frac{2}{5} \div \frac{3}{10}$ 5 $2\frac{3}{4} \div 1\frac{3}{8}$ 6 $3\frac{1}{4} \div 1\frac{3}{8}$

7 Choose one problem. Check your answer by solving it using a different strategy.

GRADE 6 • LESSON 10

3 Students may also use bar models or number lines to solve. They may recognize that the weight of the kitten is the dividend when representing this problem as division. Students may also solve by recognizing that if you multiply $\frac{1}{4}$ by $\frac{7}{2}$, the result is $\frac{7}{8}$. ***Challenge***

4 Students may also use number lines or find common denominators to divide the fractions. ***Medium***

5 Students may also draw a model or use multiplication to find the quotient to support their interpretation of the dividend and divisor in this situation. ***Medium***

3 Andre is comparing the weights of his pets. His gerbil weighs $\frac{1}{4}$ lb. His kitten weighs $\frac{7}{8}$ lb. How many times the gerbil's weight is the kitten's weight? Show your work.

Possible work:

$? \times \frac{1}{4} = \frac{7}{8}$

$\frac{7}{8} \div \frac{1}{4} = \frac{7}{8} \div \frac{2}{8}$

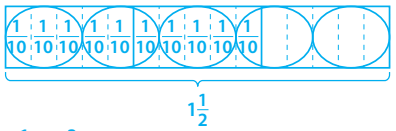
$= 7 \div 2$

$= 3\frac{1}{2}$

SOLUTION The kitten's weight is $3\frac{1}{2}$ times the gerbil's weight.

4 An ant walks along a stick. The stick is $1\frac{1}{2}$ ft long. The ant travels $\frac{3}{10}$ ft every second. How long does it take the ant to to walk the whole length of the stick? Show your work.

Possible work:



$1\frac{1}{2} \div \frac{3}{10} = 5$

SOLUTION It takes the ant 5 seconds.

5 A serving of dried fruit is $\frac{1}{5}$ cup. A bag contains $\frac{9}{10}$ cup of dried fruit. Which division expression can you use to find the number of servings in the bag: $\frac{1}{5} \div \frac{9}{10}$ or $\frac{9}{10} \div \frac{1}{5}$? Explain your reasoning.

$\frac{9}{10} \div \frac{1}{5}$; Divide the total amount by the amount in each serving to find the number of servings.

DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Use with **Session 3** Analyze It

Levels 1–3: Listening/Speaking

Help students understand the steps in Analyze It. Read the first two sentences aloud. Display the expression $\frac{3}{4} \div \frac{3}{2}$. Point to the divisor. Point out the use of red, blue and green font in the divisor and within each step.

Read the first multiplication step aloud. Rephrase as needed. For example, you might say: *Multiply $\frac{3}{4}$ by 2 to find the number of $\frac{1}{2}$s.* Point out the connection to Aimee's model in Model It. Repeat for the second step.

Then have students turn and talk to explain Analyze It in their own words.

Levels 2–4: Listening/Speaking

Guide students to interpret the steps in Analyze It. Read each step aloud, pausing between each step to allow think time for students to process each statement.

Have students turn and talk with a partner to paraphrase the steps. If students have trouble getting started, ask them to identify the font color used for each part of the fraction and make connections between corresponding parts of the expression in each step.

Call on volunteers to share their statements with the class.

Levels 3–5: Listening/Speaking

Support students as they make sense of Analyze It. Read the steps aloud, then have students turn and talk with a partner to paraphrase the steps.

Have students meet with a different partner. Ask them to cover the text on the left side of the page, so that only the expression and equations for each step are showing. Ask partners to take turns summarizing the steps using complete sentences, sequencing words, and precise mathematical language.

Develop Using Multiplication to Divide by a Fraction

Purpose

- **Develop** strategies for dividing by a fraction.
- **Recognize** that dividing by a fraction gives the same result as multiplying by that fraction's reciprocal.

START CONNECT TO PRIOR KNOWLEDGE

Always, Sometimes, Never

A When two numbers are divided, the quotient is greater than the dividend.

B When the divisor is less than the dividend, the quotient is less than 1.

C When the divisor is the same as the dividend, the quotient is equal to 1.

Solutions

A is sometimes true.

B is never true.

C is always true.

WHY? Support students' understanding of how the size of a quotient is related to the sizes of the dividend and divisor.

DEVELOP ACADEMIC LANGUAGE

WHY? Understand questions by restating them as statements.

HOW? Tell students that one way to start to understand *how* and *why* questions is to restate all or part of them as statements. Demonstrate with Connect It problem 2: *Why does multiplying by 2 tell you how many $\frac{1}{2}$s are in a number?* ➡ *Multiplying by 2 tells you how many $\frac{1}{2}$s are in a number. Why?* Encourage students to restate questions they find confusing.

TRY IT

SMP 1, 2, 4, 5, 6

Make Sense of the Problem

See ***Connect to Culture*** to support student engagement. Before students work on Try It, use **Notice and Wonder** to help them make sense of the problem. Record student responses as time and interest allow and then lead a discussion about what students noticed and wondered that may be relevant to the problem.

LESSON 10 | SESSION 3

Develop Using Multiplication to Divide by a Fraction

➤ **Read and try to solve the problem below.**

Paulo and Aimee each have $\frac{3}{4}$ cup of feed left in their bags of chicken feed. Paulo uses $\frac{1}{2}$ cup of feed each time he gives his chickens a meal. Aimee uses $\frac{3}{2}$ cups of feed each time she gives her chickens a meal. How many meals can Paulo give his chickens? How many meals can Aimee give her chickens?

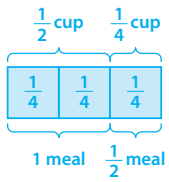
TRY IT

Math Toolkit fraction bars, fraction circles, grid paper, number lines

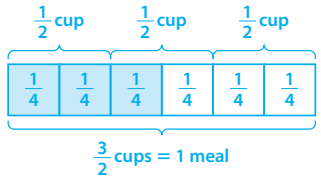
Possible work:

SAMPLE A

Paulo



Aimee



Paulo can give his chickens 1 full meal plus $\frac{1}{2}$ of a second meal.

Aimee can give her chickens only $\frac{1}{2}$ of a full meal.

SAMPLE B

Paulo: How many $\frac{1}{2}$s are in $\frac{3}{4}$?

$? \times \frac{1}{2} = \frac{3}{4}$

$\frac{3}{4} \div \frac{1}{2} = \frac{3}{4} \div \frac{2}{4}$

$= 3 \div 2 = 1\frac{1}{2}$

Aimee: How many $\frac{3}{2}$s are in $\frac{3}{4}$?

$? \times \frac{3}{2} = \frac{3}{4}$

$\frac{3}{4} \div \frac{3}{2} = \frac{3}{4} \div \frac{6}{4}$

$= 3 \div 6 = \frac{1}{2}$

Paulo can give his chickens $1\frac{1}{2}$ meals, and Aimee can give hers $\frac{1}{2}$ meal.

DISCUSS IT

Ask: How does your model show the relationship between $\frac{3}{4}$ and $\frac{1}{2}$? Between $\frac{3}{4}$ and $\frac{3}{2}$?

Share: In my model, . . . represents . . .

DISCUSS IT

SMP 2, 3, 6

Support Partner Discussion

After students work on Try It, encourage them to respond to Discuss It with a partner. If students need support getting started, prompt them to ask each other questions such as:

- *How would you describe your model?*
- *How does your model show the amount of feed Paulo and Aimee have?*

Common Misconception Listen for students who confuse the dividend and the divisor, giving answers of $\frac{2}{3}$ of a meal and 2 meals. As students share their strategies, ask them to think about whether each person has enough feed to make one full meal. Have them discuss whether this means each quotient should be greater than 1 or less than 1.

Select and Sequence Student Strategies

Select 2–3 samples that represent the range of student thinking in your classroom. Here is one possible order for class discussion:

- drawings such as bar models to show the fraction of a meal that three fourths represents
- (misconception) equations or drawings that confuse the dividend and divisor
- equations, including equations that use a common denominator strategy

Facilitate Whole Class Discussion

Call on students to share selected strategies. After each strategy, allow individual think time for students to process the ideas.

Guide students to **Compare and Connect** the representations. Encourage students to build on each other's ideas by offering reasons or observations about why each model makes sense.

> **ASK** *How do these models show whether Aimee can make more than 1 meal or less than 1 meal?*
>
> **LISTEN FOR** Each model shows that the amount of feed Aimee needs for 1 meal is greater than the amount of feed she has, so she can make less than 1 meal.

Model It & Analyze It

If students presented these models, have students connect these models to those presented in class.

If no student presented at least one of these models, have students first analyze key features of the models and then connect them to the models presented in class.

> **ASK** *How does each model show the dividend and divisor?*
>
> **LISTEN FOR** The diagrams show the dividend as 3 shaded fourths and the divisor as groups of $\frac{1}{2}$ with brackets. The equations show dividing the amount of feed each person has (the dividend) by the amount needed for 1 meal (the divisor).

For the diagrams, prompt students to relate the words and equations to the diagrams.

- *How do the diagrams and words help you write the division equations?*
- *How can the diagrams help you find the quotients?*

For the multiplication strategy, prompt students to reason about the two multiplication steps.

- *How does multiplying by the denominator of the divisor relate to division?*
- *Why do you divide by the numerator of the divisor? Why is this the same as multiplying by $\frac{1}{3}$?*

➤ **Explore different ways to solve problems that involve dividing by a fraction.**

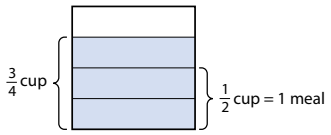
Paulo and Aimee each have $\frac{3}{4}$ cup of feed left in their bags of chicken feed. Paulo uses $\frac{1}{2}$ cup of feed each time he gives his chickens a meal. Aimee uses $\frac{3}{2}$ cups of feed each time she gives her chickens a meal. How many meals can Paulo give his chickens? How many meals can Aimee give her chickens?

Model It

You can use a diagram and words to help you represent a division situation.

Shade $\frac{3}{4}$ of a square to represent the amount of feed Paulo and Aimee have.

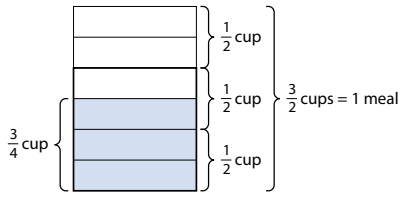
Paulo



How many $\frac{1}{2}$ cups are in $\frac{3}{4}$ cup of feed?

$\frac{3}{4} \div \frac{1}{2} = ?$

Aimee



How many $\frac{3}{2}$ cups are in $\frac{3}{4}$ cup of feed?

$\frac{3}{4} \div \frac{3}{2} = ?$

Analyze It

You can use multiplication to divide by a fraction.

To find how many $\frac{3}{2}$s are in a number, you can use two multiplication steps.

First find how many $\frac{1}{2}$s are in the number.

To do this, **multiply by 2**.

Then separate the **number of $\frac{1}{2}$s** into 3 equal parts.

To divide by 3, **multiply by $\frac{1}{3}$**.

$\frac{3}{4} \div \frac{3}{2}$

$\frac{3}{4} \times 2 = \frac{6}{4}$

$\frac{6}{4} \times \frac{1}{3} = \frac{6}{12}$, or $\frac{1}{2}$

DIFFERENTIATION | EXTEND

SMP 7

Deepen Understanding

Making Use of Structure to Understand Division of Fractions

Prompt students to relate the steps in Analyze It to the diagrams in Model It. Point out that the first step, multiplying by 2 to find how many halves are in $\frac{3}{4}$, tells how many meals Paulo can feed his chickens. You can write $\frac{6}{4}$ meals as $1\frac{1}{2}$ meals.

> **ASK** *How does the diagram for Paulo show that $\frac{1}{2}$ fits into $\frac{3}{4}$ one and a half times?*
>
> **LISTEN FOR** The diagram shows that $\frac{3}{4}$ is made up of 1 half plus $\frac{1}{2}$ of another half.

Then use the diagram for Aimee to explore the step of dividing by 3 (or multiplying by $\frac{1}{3}$).

> **ASK** *How does the diagram for Aimee show that she needs 3 times as much feed for 1 meal as Paulo needs? Why can you multiply Paulo's number of meals by $\frac{1}{3}$ to find Aimee's number of meals?*
>
> **LISTEN FOR** You can see 1 meal for Paulo in the diagram for Aimee by looking at one of the groups labeled as $\frac{1}{2}$ cup. You can see that Aimee needs 3 times this amount by looking at the 3 groups that are each labeled as $\frac{1}{2}$ cup. Since Aimee needs 3 times as much as Paulo, she has only $\frac{1}{3}$ of the number of meals Paulo has.

Develop Using Multiplication to Divide by a Fraction

CONNECT IT

SMP 2, 4, 5, 6

Remind students that the quantities and the relationships between them are the same in each representation. Explain that they will now use those relationships to reason about how to use multiplication to divide by a fraction.

Before students begin to record and expand on their work in Analyze It, tell them that problem 3 will prepare them to provide the explanation asked for in problem 4.

Monitor and Confirm Understanding 1 – 2

- Finding the number of $\frac{1}{2}$ cups in $\frac{3}{4}$ cup and the number of $\frac{3}{2}$ cups in $\frac{3}{4}$ cup tells the number of meals each person can give.
- Multiplying by 2 tells how many $\frac{1}{2}$s are in a number.
- Multiplying by 2 and by $\frac{1}{3}$ tells how many $\frac{3}{2}$s are in a number.

Facilitate Whole Class Discussion

3 Support students in understanding that the *reciprocal* of any fraction $\frac{a}{b}$ is $\frac{b}{a}$.

> **ASK** *What is the reciprocal of 3? Why?*
>
> **LISTEN FOR** The reciprocal of 3 is $\frac{1}{3}$. You can write 3 as $\frac{3}{1}$, so the reciprocal is $\frac{1}{3}$.

4 Look for the understanding that dividing by a fraction gives the same result as multiplying by the reciprocal of that fraction.

> **ASK** *If the divisor is a fraction, why can you use multiplication to find the quotient?*
>
> **LISTEN FOR** Multiplying by the denominator of the divisor tells how many fractional parts of that size are in the dividend. Then multiplying by the reciprocal of the numerator divides those parts into groups of the size of the numerator.

5 Before students complete the generalization of the algorithm stated with letters, suggest that they first record the process symbolically with numbers using the quotient $\frac{3}{4} \div \frac{3}{2}$ from the Try It problem and the quotient $4 \div \frac{2}{5}$ from problem 4.

5 **Reflect** Have all students focus on the strategies used to solve the Try It. If time allows, have students discuss their ideas with a partner.

CONNECT IT

➤ **Use the problem from the previous page to help you understand how to use multiplication to divide by a fraction.**

1 How many meals can Paulo give his chickens? How many meals can Aimee give her chickens? Explain how you know.

$1\frac{1}{2}$ meals; $\frac{1}{2}$ meal; Possible explanation: For Paulo, $\frac{2}{4}$ is 1 meal and $\frac{1}{4}$ is $\frac{1}{2}$ meal. Aimee has 3 of 6 fourths, which is half of what she needs for 1 meal.

2 Look at **Analyze It**. Why does multiplying by 2 tell you how many $\frac{1}{2}$s are in a number? Why does multiplying the number of $\frac{1}{2}$s by $\frac{1}{3}$ tell you how many $\frac{3}{2}$s are in the number?

There are 2 halves in each whole; there are only $\frac{1}{3}$ as many $\frac{3}{2}$s as $\frac{1}{2}$s.

3 The fraction $\frac{2}{3}$ is called the **reciprocal** of $\frac{3}{2}$. Explain why dividing by $\frac{3}{2}$ gives the same result as multiplying by its reciprocal, $\frac{2}{3}$.

$\frac{2}{3} = 2 \times \frac{1}{3}$, so multiplying by $\frac{2}{3}$ means multiplying by 2 and by $\frac{1}{3}$. Multiplying by 2 and then by $\frac{1}{3}$ is how you divide by $\frac{3}{2}$.

4 Explain why dividing by any fraction gives the same result as multiplying by that fraction's reciprocal. Use the example $4 \div \frac{2}{5}$.

You multiply 4 by 5 to find how many $\frac{1}{5}$s are in 4. Then you find half of that number of fifths to see how many $\frac{2}{5}$s are in 4. You are multiplying by 5 and by $\frac{1}{2}$, so you are multiplying by $5 \times \frac{1}{2}$, or $\frac{5}{2}$. The fraction $\frac{5}{2}$ is the reciprocal of $\frac{2}{5}$.

5 Complete the equations to show how to divide any fraction $\frac{a}{b}$ by any fraction $\frac{c}{d}$.

$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \boxed{d} \times \frac{1}{\boxed{c}} \rightarrow \frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{\boxed{d}}{\boxed{c}}$

6 **Reflect** Think about all the models and strategies you have discussed today. Describe how one of them helped you better understand how solve problems that involve dividing by a fraction.

Responses will vary. Check student responses.

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DIFFERENTIATION | RETEACH or REINFORCE

Hands-On Activity

Use fraction circles to understand an algorithm for fraction division.

If students are unsure about the connection between dividing by a fraction and multiplying by that fraction's reciprocal, then have them use fraction circles to look at the relationships.

Materials For each pair: 2 sets of fraction circles

- Display $\frac{3}{2} \div \frac{1}{8}$. Have students use fraction circles to model $\frac{3}{2}$ using one-eighth pieces. Ask: *How many eighths are in* $\frac{3}{2}$*?* [12] *What is* $\frac{3}{2} \div \frac{1}{8}$*?* [12]
- Have students connect dividing by $\frac{1}{8}$ to multiplying by the denominator of $\frac{1}{8}$. Ask: *What is the product* $\frac{3}{2} \times 8$*?* $\left[\frac{24}{2} = 12\right]$ *Why is the number of eighths in* $\frac{3}{2}$ *equivalent to* $\frac{3}{2} \times 8$*?* [To find how many eighths are in a number, you multiply the number by 8.]
- Now have students model $\frac{3}{2} \div \frac{2}{8}$. Ask: *How does the number of* $\frac{2}{8}$*s in* $\frac{3}{2}$ *compare to the number of* $\frac{1}{8}$*s in* $\frac{3}{2}$*?* [There are $\frac{1}{2}$ as many.] Connect dividing by 2 to multiplying by $\frac{1}{2}$.
- Have students repeat using $\frac{3}{2} \div \frac{3}{8}$ to see that there are $\frac{1}{3}$ as many $\frac{3}{8}$s as $\frac{1}{8}$s in $\frac{3}{2}$.

Apply It

For all problems, encourage students to use a model to support their thinking. Allow some leeway in precision; students who draw fraction models may not precisely show fractional parts that are equal in size.

7 Students also may use bar models, number lines, or a common denominator to find $\frac{2}{3} \div \frac{2}{5}$.

8 Students may recognize the length of the base and the height as factors and the area as the product.

Apply It

➤ **Use what you learned to solve these problems.**

7 Find the quotient $\frac{2}{3} \div \frac{2}{5}$. Show your work.

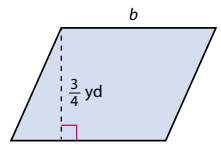
Possible work:

$\frac{2}{3} \div \frac{2}{5} = \frac{2}{3} \times \frac{5}{2}$

$= \frac{5}{3}$

SOLUTION $\frac{5}{3}$

8 The parallelogram has an area of $1\frac{1}{8}$ yd^2. Use the formula $A = bh$ to find the length of the base, b. Show your work.



Possible work: $1\frac{1}{8} = b \cdot \frac{3}{4}$

$b = 1\frac{1}{8} \div \frac{3}{4}$

$= \frac{9}{8} \cdot \frac{4}{3}$

$= \frac{36}{24}$, or $\frac{3}{2}$

SOLUTION The length of the base is $\frac{3}{2}$ yd, or $1\frac{1}{2}$ yd.

9 It takes Francisco $\frac{5}{6}$ minute to upload a video to his blog. How much of one video can he upload in $\frac{1}{2}$ minute? Show your work.

Possible work:

How many $\frac{5}{6}$s are in $\frac{1}{2}$?

$\frac{1}{2} \div \frac{5}{6} = \frac{1}{2} \cdot \frac{6}{5}$

$= \frac{6}{10}$

$= \frac{3}{5}$

SOLUTION Francisco can upload $\frac{3}{5}$ of one video in $\frac{1}{2}$ minute.

CLOSE EXIT TICKET

9 Students' solutions should show an understanding of:

- $\frac{5}{6}$ minute as the time to upload 1 video and $\frac{1}{2}$ minute as the time available for an upload.
- the time available to upload the video is less than the time needed to upload 1 video.

Error Alert If students divide $\frac{5}{6}$ by $\frac{1}{2}$ because of the order given in the problem, then have them check the solution for reasonableness. Prompt students to recognize that $\frac{1}{2}$ minute is less than $\frac{5}{6}$ minute, so in $\frac{1}{2}$ minute an entire video cannot be fully uploaded. Ask: *Should the quotient be greater than 1 or less than 1?* [less than 1] Generate open discussion about different ways to think about what makes sense for determining the dividend and divisor in problems involving fraction division.

Practice Using Multiplication to Divide by a Fraction

Problem Notes

Assign **Practice Using Multiplication to Divide by a Fraction** as extra practice in class or as homework.

1 **a.** Students should understand that $\frac{8}{3}$ and $8 \times \frac{1}{3}$ are equivalent expressions. ***Basic***

b. Students may recognize that multiplying by 8 tells the number of $\frac{1}{8}$s in $1\frac{1}{2}$, and then multiplying by $\frac{1}{3}$ tells the number of groups of $\frac{3}{8}$ in $1\frac{1}{2}$. ***Medium***

Practice Using Multiplication to Divide by a Fraction

➤ **Study the Example showing how to use multiplication to divide by a fraction. Then solve problems 1–4.**

Example

Tyrone has $1\frac{1}{2}$ quarts of honey. He is pouring the honey into jars that each hold $\frac{3}{8}$ quart. How many jars can Tyrone fill?

You can divide the total quarts of honey, $1\frac{1}{2}$, by the number of quarts each jar can hold, $\frac{3}{8}$.

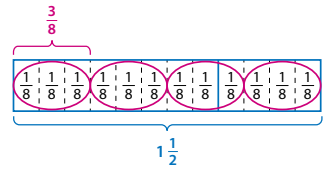
$1\frac{1}{2} \div \frac{3}{8} = 1\frac{1}{2} \times \frac{8}{3}$ ← To divide by $\frac{3}{8}$, multiply by its reciprocal, $\frac{8}{3}$.

$= \frac{3}{2} \times \frac{8}{3}$

$= \frac{24}{6} = 4$

Tyrone can fill 4 jars of honey.

1 The Example shows finding the quotient $1\frac{1}{2} \div \frac{3}{8}$ by multiplying $1\frac{1}{2}$ by the reciprocal of $\frac{3}{8}$, or $\frac{8}{3}$. You can relate multiplying by the reciprocal to a bar model that represents the division equation $1\frac{1}{2} \div \frac{3}{8} = 4$.



a. Explain why multiplying by $\frac{8}{3}$ gives the same result as first multiplying by 8 and then multiplying by $\frac{1}{3}$.

You can break apart $\frac{8}{3}$ into the product of two factors: $\frac{8}{3} = 8 \times \frac{1}{3}$.

b. What is the value of the expression $\left(1\frac{1}{2} \times 8\right) \times \frac{1}{3}$? Explain how finding the value of the expression is related to the bar model.

4; Possible explanation: You first find $1\frac{1}{2} \times 8 = 12$, which is the same as the number of parts of size $\frac{1}{8}$ in the bar model. Then you multiply 12 by $\frac{1}{3}$, which is $\frac{12}{3}$, or $12 \div 3 = 4$. This is like dividing the 12 eighths in the bar model into 4 groups of $\frac{3}{8}$s.

Vocabulary

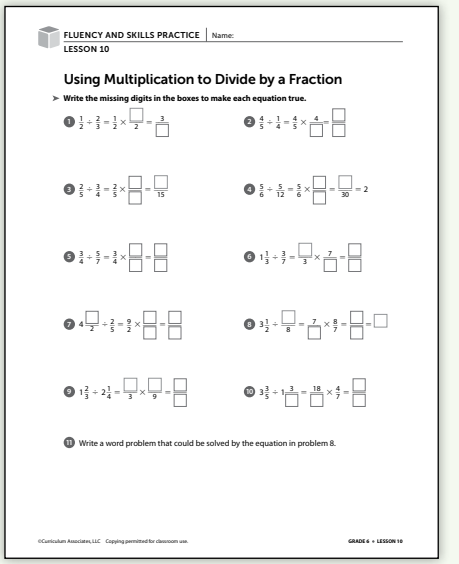
reciprocal

for any nonzero number a, the reciprocal is $\frac{1}{a}$. The reciprocal of any fraction $\frac{a}{b}$ is $\frac{b}{a}$.

Fluency & Skills Practice

Using Multiplication to Divide by a Fraction

In this activity, students complete division equations by filling in missing numbers.



FLUENCY AND SKILLS PRACTICE | Name:
LESSON 10

Using Multiplication to Divide by a Fraction

➤ Write the missing digits in the boxes to make each equation true.

1. $\frac{1}{2} \div \frac{2}{3} = \frac{1}{2} \times \frac{\square}{2} = \frac{3}{\square}$
2. $\frac{4}{5} \div \frac{1}{4} = \frac{4}{5} \times \frac{4}{\square} = \frac{\square}{\square}$
3. $\frac{2}{5} \div \frac{3}{4} = \frac{2}{5} \times \frac{\square}{\square} = \frac{\square}{15}$
4. $\frac{5}{6} \div \frac{5}{12} = \frac{5}{6} \times \frac{\square}{\square} = \frac{\square}{30} = 2$
5. $\frac{3}{4} \div \frac{5}{7} = \frac{3}{4} \times \frac{\square}{\square} = \frac{\square}{\square}$
6. $1\frac{1}{3} \div \frac{3}{7} = \frac{\square}{3} \times \frac{7}{\square} = \frac{\square}{\square}$
7. $4\frac{\square}{2} \div \frac{2}{5} = \frac{9}{2} \times \frac{\square}{\square} = \frac{\square}{\square}$
8. $3\frac{1}{2} \div \frac{\square}{8} = \frac{7}{\square} \times \frac{8}{7} = \frac{\square}{\square} = \square$
9. $1\frac{2}{3} \div 2\frac{1}{4} = \frac{\square}{3} \times \frac{\square}{9} = \frac{\square}{\square}$
10. $3\frac{3}{5} \div 1\frac{3}{\square} = \frac{18}{\square} \times \frac{4}{7} = \frac{\square}{\square}$
11. Write a word problem that could be solved by the equation in problem 8.

GRADE 6 • LESSON 10

Additional Practice

2 Students also may use an area model to visually represent the area and length of the park to find the width. ***Medium***

3 Students also may use a bar model to explore $6 \div 3\frac{3}{4}$. ***Challenge***

4 Students may also use a bar model to label and represent the situation and visualize the quotient. ***Medium***

2 A rectangular city park is $\frac{6}{8}$ mi long. The park has an area of $\frac{1}{4}$ mi^2. What is the width of the park? Show your work.

Possible work:

$A = \ell w$, so $w = A \div \ell$.

$w = \frac{1}{4} \div \frac{6}{8}$

$= \frac{1}{4} \cdot \frac{8}{6}$

$= \frac{8}{24}$, or $\frac{1}{3}$

SOLUTION The width of the park is $\frac{1}{3}$ mi.

3 Find the value of $6 \div 3\frac{3}{4}$. Show your work.

Possible work:

$6 \div 3\frac{3}{4} = 6 \div \frac{15}{4}$

$= 6 \cdot \frac{4}{15}$

$= \frac{24}{15} = \frac{8}{5}$

SOLUTION $\frac{8}{5}$, or $1\frac{3}{5}$

4 During a community service day, 6 teams of students clean a beach by picking up trash. The beach is $1\frac{4}{5}$ mi long. Each team cleans the same length of beach. What is the length of beach that each team cleans? Show your work.

Possible work:

$1\frac{4}{5} \div 6 = \frac{9}{5} \times \frac{1}{6}$

$= \frac{9}{30}$, or $\frac{3}{10}$

SOLUTION Each team cleans $\frac{3}{10}$ mi of the beach.

DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Use with **Session 4** Apply It

Levels 1–3: Listening/Speaking

Support students as they engage in partner discussion about Apply It problem 6. Read the problem aloud. Paraphrase the problem using *If*. Draw a number line to review *less than*, *greater than,* and *equal*. Have students point to a number. Ask them to tell if the number is *less than, greater than,* or *equal to 1*.

Have students talk to a partner to complete the chart. Encourage students to refer to their models or equations to support their answers. Provide a sentence starter:

- *The quotient is _____.*

Levels 2–4: Listening/Speaking

Support students as they engage in partner discussion about Apply It problem 6. Call on a volunteer to read the problem aloud. Review *less than, greater than,* and *equal to* as needed. Have students work individually to complete the chart. Then have them turn to partners to explain their reasoning. Encourage them to refer to their models or equations to support their explanations.

Have students complete the sentence frame:

- *The quotient _____ is _____.*

Levels 3–5: Listening/Speaking

Support students as they engage in partner discussion about Apply It problem 6. Call on a volunteer to read the problem aloud. Have students work individually to complete the chart. Organize students into pairs and ask them to take turns explaining their reasoning using their models or equations.

Then have students meet with a different partner and take turns asking each other questions about their charts. Suggest students use the following sentence frames:

- *Why is _____ less/greater than 1?*
- *Why is _____ equal to 1?*
- *How did you decide that _____ is _____?*

LESSON 10 | SESSION 4

Refine Dividing Fractions

Purpose

- **Refine** strategies for solving problems involving division with fractions.
- **Refine** understanding of why and how to multiply by the reciprocal when dividing by a fraction.

START CHECK FOR UNDERSTANDING

What is $\frac{3}{8} \div \frac{2}{3}$? Show your work.

Solution

$\frac{9}{16}$;

Possible work:

$\frac{3}{8} \times \frac{3}{2} = \frac{9}{16}$

WHY? Confirm students' understanding of dividing fractions, identifying common errors to address as needed.

MONITOR & GUIDE

Before students begin to work, use their responses to the **Start** to determine those who will benefit from additional support. Use the **Error Analysis** table below to guide remediation.

Have all students complete the Example and problems 1–3, using Consider This and Pair/Share as appropriate. Observe and monitor their reasoning and guide or redirect students as needed.

LESSON 10 | SESSION 4

Refine Dividing Fractions

➤ **Complete the Example below. Then solve problems 1–9.**

Example

Sarah uses this recipe to make trail mix. She puts the trail mix in small bags. Each bag holds $1\frac{1}{4}$ cups. How many bags does Sarah fill?

Look at how you could show your work using operations with fractions.

Trail Mix

- $2\frac{1}{2}$ cups almonds
- $\frac{3}{4}$ cup dried cherries
- $2\frac{1}{4}$ cups walnuts
- $\frac{3}{4}$ cup raisins

First, find the total amount of trail mix.

$$2\frac{1}{2} + \frac{3}{4} + 2\frac{1}{4} + \frac{3}{4} = 2\frac{1}{2} + 3 + \frac{3}{4}$$
$$= 5\frac{1}{2} + \frac{3}{4} = 6\frac{1}{4}$$

Then divide the total amount by the amount for 1 bag.

$$6\frac{1}{4} \div 1\frac{1}{4} = \frac{25}{4} \div \frac{5}{4} = 25 \div 5$$

SOLUTION Sarah fills 5 bags.

CONSIDER THIS . . .
The total amount of trail mix is the sum of the amounts of each ingredient.

PAIR/SHARE
What is another way you could find the value of $6\frac{1}{4} \div 1\frac{1}{4}$?

Apply It

1 Hiroaki buys $3\frac{1}{2}$ gal of paint. He uses $1\frac{1}{2}$ gal of paint. What fraction of the paint that Hiroaki buys does he use? Show your work.

Possible work: What fraction of $3\frac{1}{2}$ is $1\frac{1}{2}$?

$$? \times 3\frac{1}{2} = 1\frac{1}{2}$$
$$? = \frac{3}{2} \div \frac{7}{2}$$
$$= 3 \div 7$$
$$= \frac{3}{7}$$

SOLUTION Hiroaki uses $\frac{3}{7}$ of the paint.

CONSIDER THIS . . .
How could you use a multiplication equation with an unknown factor to represent the question?

PAIR/SHARE
How could you use estimation to help you check your answer?

START ERROR ANALYSIS

If the error is . . .	Students may . . .	To support understanding . . .
$\frac{1}{4}$	have multiplied the fractions.	Ask students to review the given expression and their expression to explain how $\frac{3}{8} \div \frac{2}{3}$ is not the same as $\frac{3}{8} \times \frac{2}{3}$. Remind students that they must multiply by the reciprocal of the divisor.
$\frac{7}{24}$	have subtracted the fractions after writing them with a common denominator.	Have students rewrite the division expression using a common denominator of 24. Have them describe the expression $\frac{9}{24} \div \frac{16}{24}$ using words to recognize that 9 twenty-fourths divided by 16 twenty-fourths is the same as 9 divided by 16.
$\frac{16}{9}$	have multiplied by the reciprocal of the dividend rather than the divisor.	Prompt students to name the dividend, divisor, and quotient and to use a related multiplication equation to check the accuracy of their answer. Prompt students to recognize that multiplying the quotient by the divisor should result in the dividend.

Example

Guide students in understanding the Example. Ask:

- *How do you know the amount of trail mix that Sarah has?*
- *What represents the dividend in this situation? What represents the divisor?*
- *What are other ways you could solve the problem?*

Help all students focus on the Example and responses to the questions by asking them to agree, disagree, or add on to classmates' responses.

Look for understanding that the sum of the amounts in the recipe is the total amount of trail mix, and this total must be divided into servings of $1\frac{1}{4}$ cups each.

Apply It

1. Students may also draw a model to see the relationship between $\frac{3}{2}$ and $\frac{7}{2}$. Students may shade 3 of the 7 equal parts of a bar model in which each part represents $\frac{1}{2}$ to see that $\frac{3}{2}$ is $\frac{3}{7}$ of $\frac{7}{2}$. ***DOK 2***

2. Students also may write a multiplication equation with an unknown factor to model the situation. ***DOK 1***

3. **B is correct.** Students may solve the problem by using a model or writing the related multiplication equation.

   A is not correct. This answer is the product of the dividend and the divisor.

   C is not correct. This answer is the product of the divisor and the reciprocal of the dividend.

   D is not correct. This answer is the product of the reciprocal of the divisor and the reciprocal of the dividend.

   DOK 3

2. Tiana uses $1\frac{3}{5}$ oz of detergent for each full load of laundry. How many full loads of laundry can she do with 100 oz of detergent? Show your work.

   Possible work:

   $$100 \div 1\frac{3}{5} = 100 \div \frac{8}{5}$$
   $$= 100 \cdot \frac{5}{8}$$
   $$= \frac{500}{8}, \text{ or } 62\frac{1}{2}$$

   SOLUTION Tiana can do 62 full loads of laundry.

CONSIDER THIS . . .
What would a remainder represent in this division situation?

PAIR/SHARE
How do you know that your answer is reasonable?

3. Which expression can be used to determine the number of $\frac{3}{4}$-cup servings in $\frac{1}{2}$ cup of pasta salad?

   A $\frac{1}{2} \times \frac{3}{4}$

   B $\frac{1}{2} \times \frac{4}{3}$

   C $\frac{3}{4} \times \frac{2}{1}$

   D $\frac{4}{3} \times \frac{2}{1}$

   Lillie chose C as the correct answer. How might she have gotten that answer?

   Possible answer: Instead of dividing $\frac{1}{2}$ by $\frac{3}{4}$, Lillie divided $\frac{3}{4}$ by $\frac{1}{2}$ and then rewrote the division as a multiplication problem.

CONSIDER THIS . . .
How can you rewrite a division expression as a multiplication expression?

PAIR/SHARE
Is there *more than* one full serving of pasta salad or *less than* one full serving? How do you know?

GROUP & DIFFERENTIATE

Identify groupings for differentiation based on the **Start** and problems 1–3. A recommended sequence of activities for each group is suggested below. Use the resources on the next page to differentiate and close the lesson.

Approaching Proficiency
- **RETEACH** Hands-On Activity
- **REINFORCE** Problems 4, 6, 8

Meeting Proficiency
- **REINFORCE** Problems 4–8

Extending Beyond Proficiency
- **REINFORCE** Problems 4–8
- **EXTEND** Challenge

Have all students complete the **Close: Exit Ticket.**

Resources for Differentiation are found on the next page.

Refine Dividing Fractions

Apply It

4 Students also may use multiplication to confirm that $\frac{2}{10} \times 4 = \frac{8}{10}$ or use reasoning to recognize that the answer should be less than $\frac{8}{10}$ because $\frac{8}{10}$ is divided into 4 sections. ***DOK 1***

5 See ***Connect to Culture*** to support student engagement. Students should recognize that this is a multi-step problem that involves subtracting the used soil first and then dividing the remaining amount into groups of $\frac{3}{4}$ ft^3. ***DOK 3***

6 **a.** The dividend is greater than the divisor, so the quotient is greater than 1.

b. The dividend is less than the divisor, so the quotient is less than 1.

c. The dividend is greater than the divisor, so the quotient is greater than 1.

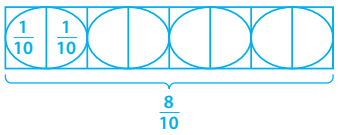
d. The dividend is equivalent to the divisor, so the quotient is equal to 1.

DOK 1

7 Students may use reasoning to realize that the amount of seeds Querida has is less than the amount needed to cover the entire lawn, so the quotient will be less than 1. ***DOK 2***

4 A new nature trail is $\frac{8}{10}$ mi long. A park ranger divides the trail into 4 equal sections. How long is each section of the trail? Show your work.

Possible work:



SOLUTION Each section is $\frac{2}{10}$ mi, or $\frac{1}{5}$ mi.

5 Estela has 10 ft^3 of soil. She uses $3\frac{1}{2}$ ft^3 in her garden. She uses the rest of the soil for tomato plants. She needs $\frac{3}{4}$ ft^3 of the soil for each tomato plant. How many tomato plants can she plant? Show your work.

Possible work: $10 - 3\frac{1}{2} = 6\frac{1}{2}$

$\frac{13}{2} \div \frac{3}{4} = \frac{13}{2} \times \frac{4}{3}$

$= \frac{52}{6}$ or $8\frac{4}{6}$

SOLUTION Estela can plant 8 tomato plants.

6 Without dividing, tell whether each quotient is *less than 1*, *greater than 1*, or *equal to 1*.

	Less Than 1	Greater Than 1	Equal to 1
a. $\frac{2}{9} \div \frac{1}{27}$	○	●	○
b. $\frac{1}{2} \div \frac{3}{4}$	●	○	○
c. $\frac{4}{3} \div \frac{3}{5}$	○	●	○
d. $\frac{20}{8} \div 2\frac{1}{2}$	○	○	●

DIFFERENTIATION

RETEACH

Hands-On Activity

Use fraction circles to model fraction division as repeated subtraction.

Students approaching proficiency with dividing fractions will benefit from relating a model to a context involving division of fractions to understand division as repeated subtraction.

Materials For each group: 7 sets of fraction circles

- Use the division problem from the Example, $6\frac{1}{4} \div 1\frac{1}{4}$. Tell students they can use fraction circles to model dividing the total amount of trail mix into equal portions.
- Ask: *Which number in the division expression $6\frac{1}{4} \div 1\frac{1}{4}$ represents the total amount of trail mix? Which number represents the size of each portion?* $\left[6\frac{1}{4}; 1\frac{1}{4}\right]$ Have students model $6\frac{1}{4}$ cups of trail mix with fraction circles.
- Elicit from students that just as multiplication can be thought of as repeated addition, division can be thought of as repeated subtraction. Have students model dividing the total amount of trail mix into equal portions by repeatedly removing groups of pieces that represent $1\frac{1}{4}$ cups from their model for $6\frac{1}{4}$ cups, keeping each group separate.
- Ask: *How many times can you subtract $1\frac{1}{4}$ from $6\frac{1}{4}$?* [5 times] *How can you tell whether there is a remainder?* There are no fraction pieces left, so the remainder is 0. *What is $6\frac{1}{4} \div 1\frac{1}{4}$?* [5]
- Have students record the division symbolically by multiplying $6\frac{1}{4}$ by the reciprocal of $1\frac{1}{4}$ and confirm the quotient is 5. $\left[6\frac{1}{4} \div 1\frac{1}{4} = \frac{25}{4} \div \frac{5}{4} = \frac{25}{4} \times \frac{4}{5} = \frac{100}{20} = 5\right]$

8 **C is correct.** Students may solve the problem by rewriting the dividend and divisor as fractions greater than 1 and then multiplying the dividend by the reciprocal of the divisor.

A is not correct. This answer is $\frac{1}{7} \times \frac{1}{14}$.

B is not correct. This answer is the product of $2\frac{1}{14}$ and the reciprocal of $4\frac{1}{7}$.

D is not correct. This answer is $\frac{14}{1} \times \frac{7}{1}$.

DOK 1

CLOSE EXIT TICKET

9 **Math Journal** Look for a story context that involves dividing the dividend into equal parts or determining the fraction of the divisor that is the dividend.

Error Alert If students confuse the dividend and divisor in the context, then have them select a problem from the Student Worktext that they solved successfully and use that as a guide when they rewrite their problem.

End of Lesson Checklist

INTERACTIVE GLOSSARY Support students by suggesting they work with a partner and have one student write a number and the other student explain how to write the reciprocal.

SELF CHECK Have students review and check off any new skills on the Unit 2 Opener.

7 Querida spreads grass seed on her lawn. She needs $\frac{5}{6}$ lb of grass seed to cover her whole lawn. She has $\frac{1}{3}$ lb of grass seed. How much of her lawn can she cover? Show your work.

Possible work:

Use a rectangle to represent 1 lb of grass seed.

Querida has 2 of the 5 parts she needs.

SOLUTION Querida can cover $\frac{2}{5}$ of her lawn.

8 What is $4\frac{1}{7} \div 2\frac{1}{14}$?

A $\frac{1}{98}$ **B** $\frac{1}{2}$

C 2 **D** 98

9 **Math Journal** Write a word problem that you can use the division expression $\frac{3}{4} \div \frac{3}{8}$ to solve. Then solve the problem.

Possible answer: Ryan has $\frac{3}{4}$ gal of water. He pours the water into bottles that each hold $\frac{3}{8}$ gal. How many bottles can Ryan fill?

$\frac{3}{4} \div \frac{3}{8} = \frac{6}{8} \div \frac{3}{8}$

$= 6 \div 3$

$= 2$

Ryan can fill 2 bottles.

End of Lesson Checklist

- ☐ **INTERACTIVE GLOSSARY** Find the entry for *reciprocal*. Give examples of three numbers and their reciprocals.
- ☐ **SELF CHECK** Go back to the Unit 2 Opener and see what you can check off.

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REINFORCE

Problems 4–8

Solve problems by dividing fractions.

Students meeting proficiency will benefit from additional work with dividing fractions by solving problems in a variety of formats.

- Have students work on their own or with a partner to solve the problems.
- Encourage students to show their work.

EXTEND

Challenge

Write and solve fraction divison problems.

Students extending beyond proficiency will benefit from generating a variety of quotients using a given set of digits.

- Prompt pairs of students to use the digits 2, 4, 6, and 8 to create a fractional dividend and a fractional divisor, using each digit only once. Then have them find the quotient for the expression that they write.
- Encourage a few rounds of writing division expressions, motivating pairs to aim for the greatest and least quotients possible. [The greatest is 6 and least is $\frac{1}{6}$.]
- Ask students to share their expressions and quotients with their peers to generate a list of all possible quotients. [9 unique quotients: $\frac{1}{6}$, $\frac{3}{16}$, $\frac{3}{8}$, $\frac{2}{3}$, $\frac{3}{2}$, 2, $2\frac{2}{3}$, 3, 6]

PERSONALIZE

i-Ready

Provide students with opportunities to work on their personalized instruction path with *i-Ready* Online Instruction to:

- fill prerequisite gaps.
- build up grade-level skills.