LESSON 19

Overview | Write and Identify Equivalent Expressions

STANDARDS FOR MATHEMATICAL PRACTICE (SMP)

SMP 1, 2, 3, 4, 5, and 6 are integrated into the Try-Discuss-Connect routine.*

2 Reason abstractly and quantitatively.

5 Use appropriate tools strategically.

* See page 1q to learn how every lesson includes these SMP.

Objectives

Content Objectives

- Apply the distributive property to a sum of two whole numbers.
- Apply the distributive property to algebraic expressions.
- Combine like terms.
- Recognize and generate equivalent expressions.

Language Objectives

- Read and understand a sequence of instructions demonstrating how to apply the distributive property to a sum of two whole numbers.
- Justify strategies to find equivalent expressions when applying the distributive property to algebraic expressions.
- Understand the phrase *like terms* and use it to discuss the properties of operations.
- Compare forms of an expression using the term *equivalent expressions*.

Prior Knowledge

- Find the greatest common factor of two numbers.
- Use the distributive property.
- Identify parts of an expression.

Vocabulary

Math Vocabulary

equivalent expressions two or more expressions in different forms that always name the same value.

like terms two or more terms that have the same variable factors.

Review the following key terms.

coefficient a number that is multiplied by a variable.

distributive property multiplying each term in a sum or difference by a common factor does not change the value of the expression. For any numbers a, b, and c, $a(b + c) = ab + ac$.

expression a group of numbers, variables, and/or operation symbols that represents a mathematical relationship. An expression without variables, such as $3 + 4$, is called a *numerical expression*. An expression with variables, such as $5b^2$, is called an *algebraic expression*.

greatest common factor (GCF) the greatest factor two or more numbers have in common.

term a number, a variable, or a product of numbers, variables, and/or expressions. A term may include an exponent.

Academic Vocabulary

algebraic involving variables or the rules of algebra.

value the number or quantity that a variable or expression represents.

Learning Progression

In Grade 5, students used the commutative, associative, and distributive properties to write and evaluate numerical expressions involving the order of operations.

Earlier in Grade 6, students evaluated algebraic expressions for given values of the variables, including expressions that involve exponents. They identified the parts of an expression using mathematical terms and found the greatest common factor of two numbers.

In this lesson, students will build on prior understandings of numerical and algebraic expressions. They recognize equivalent expressions through models, manipulatives, diagrams, or real-world contexts and identify and combine like terms in order to generate equivalent expressions, including by using the distributive property "in reverse." Students recognize that there are many forms in which expressions can be written.

Later in Grade 6, students will solve one-variable equations of the form $x + p = q$ and $px = q$, where x, p, and q are nonnegative. They will apply their understanding of writing equivalent expressions when they combine like terms on one side of an equation in order to produce an equation in one of those forms.

In Grade 7, students will add, subtract, factor, and expand linear expressions with both positive and negative coefficients.

Pacing Guide

Items marked with 🖱 are available on the **Teacher Toolbox**.

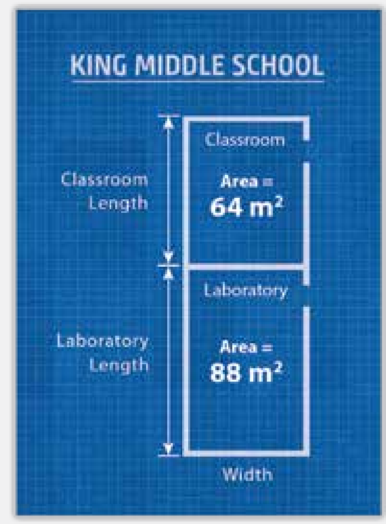
	MATERIALS	DIFFERENTIATION
SESSION 1 **Explore** Equivalent Expressions (35–50 min)		
• **Start** (5 min) • **Try It** (5–10 min) • **Discuss It** (10–15 min) • **Connect It** (10–15 min) • **Close: Exit Ticket** (5 min) **Additional Practice** (pages 439–440)	**Math Toolkit** grid paper, sticky notes, unit tiles Presentation Slides 🖱	**PREPARE** Interactive Tutorial 🖱 **RETEACH or REINFORCE** Hands-On Activity **Materials** For each group: 24 two-color counters
SESSION 2 **Develop** Using the Distributive Property to Write Equivalent Expressions (45–60 min)		
• **Start** (5 min) • **Try It** (10–15 min) • **Discuss It** (10–15 min) • **Connect It** (15–20 min) • **Close: Exit Ticket** (5 min) **Additional Practice** (pages 445–446)	**Math Toolkit** algebra tiles, grid paper Presentation Slides 🖱	**RETEACH or REINFORCE** Hands-On Activity **Materials** For each group: algebra tiles (6 x-tiles, 18 1-tiles) **REINFORCE** Fluency & Skills Practice 🖱 **EXTEND** Deepen Understanding
SESSION 3 **Develop** Combining Like Terms (45–60 min)		
• **Start** (5 min) • **Try It** (10–15 min) • **Discuss It** (10–15 min) • **Connect It** (15–20 min) • **Close: Exit Ticket** (5 min) **Additional Practice** (pages 451–452)	**Math Toolkit** algebra tiles, grid paper Presentation Slides 🖱	**RETEACH or REINFORCE** Hands-On Activity **Materials** For each group: algebra tiles (10 x-tiles, 10 1-tiles) **REINFORCE** Fluency & Skills Practice 🖱 **EXTEND** Deepen Understanding
SESSION 4 **Develop** Identifying Equivalent Expressions (45–60 min)		
• **Start** (5 min) • **Try It** (10–15 min) • **Discuss It** (10–15 min) • **Connect It** (15–20 min) • **Close: Exit Ticket** (5 min) **Additional Practice** (pages 457–458)	**Math Toolkit** algebra tiles, grid paper Presentation Slides 🖱	**RETEACH or REINFORCE** Hands-On Activity **Materials** For each group: algebra tiles (10 x-tiles, 10 1-tiles) **REINFORCE** Fluency & Skills Practice 🖱 **EXTEND** Deepen Understanding
SESSION 5 **Refine** Writing and Identifying Equivalent Expressions (45–60 min)		
• **Start** (5 min) • **Monitor & Guide** (15–20 min) • **Group & Differentiate** (20–30 min) • **Close: Exit Ticket** (5 min)	**Math Toolkit** Have items from previous sessions available for students. Presentation Slides 🖱	**RETEACH** Hands-On Activity **Materials** For each student: algebra tiles (10 x-tiles, 10 1-tiles) **REINFORCE** Problems 4–9 **EXTEND** Challenge **PERSONALIZE** i-Ready
Lesson 19 Quiz 🖱 or **Digital Comprehension Check**		**RETEACH** Tools for Instruction 🖱 **REINFORCE** Math Center Activity 🖱 **EXTEND** Enrichment Activity 🖱

Connect to Culture

➤ **Use these activities to connect with and leverage the diverse backgrounds and experiences of all students. Engage students in sharing what they know about contexts before you add the information given here.**

SESSION 1 ■ □ □ □ □

Try It Ask students to raise their hands if they know anyone who has designed or constructed a building, such as a school. When an architect is designing a school, one important factor to consider is the estimated number of students in the school and the estimated number of students in each classroom. Lighting is another important factor, so the number and size of windows and the number and type of lighting fixtures must be considered. A science laboratory must be large enough to include tools and equipment for labs, as well as additional space for equipment. The architect works together with a large team of people to finalize the design for the school. To promote student discussion, ask: *If you could redesign or create a new part of our school, what would it be and why?*

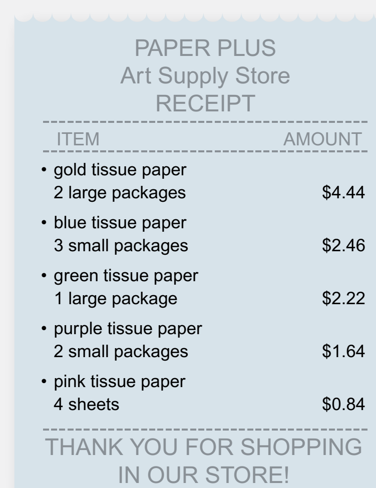


SESSION 2 ■ ■ □ □ □

Try It Ask students to stand up if they like to watch movies. Select a few students that are standing and ask what their favorite movie genre is. Streaming movie services have become more popular and are beginning to replace other formats in which people view movies. While in the past movie stores were popular, it is more common to have kiosk at a different store where you can rent a movie. Even the largest movie companies are starting their own streaming movie service because the Internet is becoming the most popular way to watch movies. Survey students to find out how they watch movies most often: at the theater, on a disc, or streaming over the Internet.

SESSION 3 ■ ■ ■ □ □

Try It Ask: *Has anyone ever used papel picado for decorations before? Can you describe it for the class? Papel picado* refers to tissue paper flags used for decoration during many celebrations in Mexico. In the 1500s, one of the towns that the Spaniards colonized started using the thin paper left there by the Spaniards to cut and create designs. In the 1900s, they started taking it to other towns and cities to sell, and it has since become a tradition all around Mexico and the world. Have students discuss other celebrations they are familiar with that include flags or other special decorations.

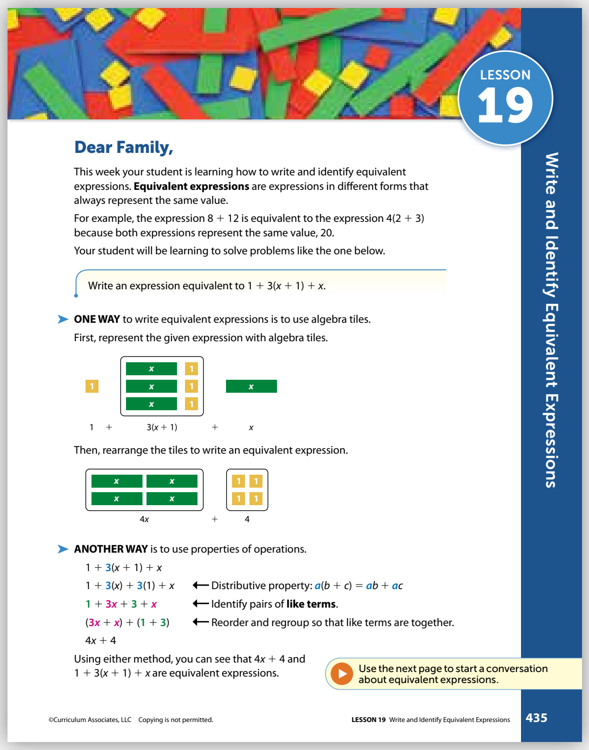


SESSION 5 ■ ■ ■ ■ ■

Apply It **Problem 8** Swimming pools have a long history. The oldest known pool is over 5,000 years old and was built in Pakistan. The Ancient Greeks and Romans used pools for swimming, bathing, socializing, and training their soldiers for war. Some of these pools were heated by giant fires in a basement underneath. In the 1800s, pools were first used for swimming competitions in Britain. After World War II, people started purchasing their own swimming pools, and now there are millions of pools found in community parks, hotels, and private residences. Ask students whether they like to swim or if they prefer to participate in other activities.

Connect to Family and Community

After the Explore session, have students use the Family Letter to let their families know what they are learning and to encourage family involvement.



LESSON 19

Write and Identify Equivalent Expressions

Dear Family,

This week your student is learning how to write and identify equivalent expressions. **Equivalent expressions** are expressions in different forms that always represent the same value.

For example, the expression $8 + 12$ is equivalent to the expression $4(2 + 3)$ because both expressions represent the same value, 20.

Your student will be learning to solve problems like the one below.

Write an expression equivalent to $1 + 3(x + 1) + x$.

ONE WAY to write equivalent expressions is to use algebra tiles.

First, represent the given expression with algebra tiles.

$1 + 3(x + 1) + x$

Then, rearrange the tiles to write an equivalent expression.

$4x + 4$

ANOTHER WAY is to use properties of operations.

$1 + 3(x + 1) + x$

$1 + 3(x) + 3(1) + x$ ← Distributive property: $a(b + c) = ab + ac$

$1 + 3x + 3 + x$ ← Identify pairs of **like terms**.

$(3x + x) + (1 + 3)$ ← Reorder and regroup so that like terms are together.

$4x + 4$

Using either method, you can see that $4x + 4$ and $1 + 3(x + 1) + x$ are equivalent expressions.

Use the next page to start a conversation about equivalent expressions.

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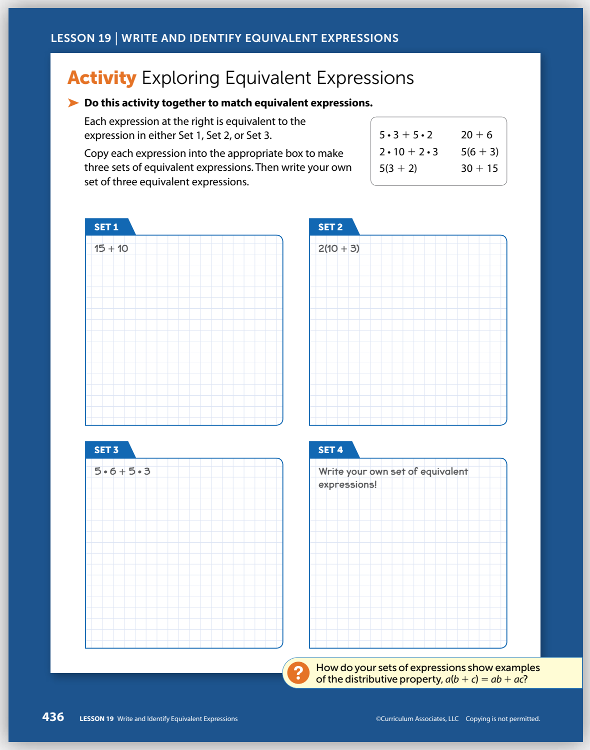

LESSON 19 | WRITE AND IDENTIFY EQUIVALENT EXPRESSIONS

Activity Exploring Equivalent Expressions

Do this activity together to match equivalent expressions.

Each expression at the right is equivalent to the expression in either Set 1, Set 2, or Set 3.

Copy each expression into the appropriate box to make three sets of equivalent expressions. Then write your own set of three equivalent expressions.

$5 \cdot 3 + 5 \cdot 2$	$20 + 6$
$2 \cdot 10 + 2 \cdot 3$	$5(6 + 3)$
$5(3 + 2)$	$30 + 15$

SET 1 $15 + 10$

SET 2 $2(10 + 3)$

SET 3 $5 \cdot 6 + 5 \cdot 3$

SET 4 Write your own set of equivalent expressions!

How do your sets of expressions show examples of the distributive property, $a(b + c) = ab + ac$?

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Connect to Language

For English language learners, use the Differentiation chart to scaffold the language in each session. Use the Academic Vocabulary routine for academic terms before Session 1.

DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Use with **Session 1** **Connect It**

ACADEMIC VOCABULARY

To *rewrite* is to write again in a different way. In math, simplifying and expanding are two ways to *rewrite* an expression.

Levels 1–3: Reading/Speaking

Support students as they make sense of Connect It problem 2. Read the problem aloud. Display important terms as you encounter them in context, such as *common factor, equivalent expressions, distributive property, rewrite*, and *greatest common factor*. Have students rate their knowledge of each term:

- *1–I know the term and can use it in speaking and writing.*
- *2–I've heard the term and think I know what it means.*
- *3–I don't know the term.*

Clarify unfamiliar terms and rephrase ideas as needed.

Levels 2–4: Reading/Speaking

Support students as they make sense of and discuss Connect It problem 2. Read the problem with students. Call on volunteers to identify new and review math terms, such as *common factor, equivalent expressions, distributive property,* and *greatest common factor.* Compile the list of terms and ask students to rate their knowledge of the terms (1-2-3).

Encourage students to use the Interactive Glossary or a dictionary to check meanings of terms rated 2 or 3. Have partners use **Say It Another Way** to paraphrase the problem and confirm understanding.

Levels 3–5: Reading/Speaking

Support students as they make sense of and discuss Connect It problem 2. Have students read the problem independently. Ask them to circle words and phrases that are unfamiliar or might be challenging for classmates.

Ask partners to compare the terms they selected and discuss meanings. Encourage partners to use the Interactive Glossary or a dictionary to check meanings as needed. Have partners use **Say It Another Way** to paraphrase the problem and confirm understanding.

Explore Equivalent Expressions

Purpose

- **Explore** the idea that writing a numerical expression in a different form can help you make sense of a problem.
- **Understand** that the distributive property can be applied in reverse to write a sum as a product.

START CONNECT TO PRIOR KNOWLEDGE

Same and Different

A: 6(8)	B: 6(5 + 1)
C: 4(3 + 6)	D: 4(14 – 2)

Possible Solutions

All four expressions are made up of 2 factors.

A and D have a value of 48.

B and C have a value of 36.

A and B have a factor of 6.

C and D have a factor of 4.

WHY? Support students' ability to recognize expressions with the same value.

TRY IT SMP 1, 2, 4, 5, 6

Make Sense of the Problem

See ***Connect to Culture*** to support student engagement. Before students work on Try It, use **Co-Craft Questions** to help them make sense of the problem. Read the problem aloud. Ask: *What is the problem about?* Have students write questions that might be asked about this situation and have them compare questions with a partner. Have students share their questions with the class.

DISCUSS IT SMP 2, 3, 6

Support Partner Discussion

After students work on Try It, have them respond to Discuss It with a partner. Listen for understanding of:

- finding the GCF of the areas of the rooms by listing the factors of each area.
- recognizing that the GCF is the width of each room.
- finding the length of each room by dividing the area by the width, which is the GCF of the areas.

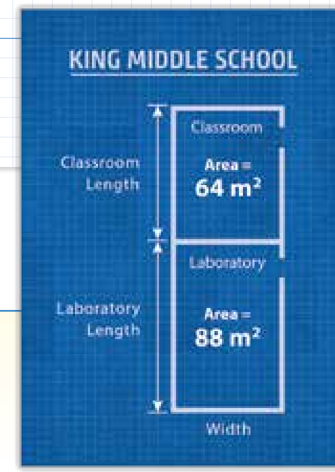
LESSON 19 | SESSION 1 ■ □ □ □ □

Explore Equivalent Expressions

Previously, you learned how to write and evaluate expressions. In this lesson, you will learn about equivalent expressions.

➤ **Use what you know to try to solve the problem below.**

In the design for a new school, a classroom needs to have the same width as a laboratory. The architect wants the width to be as great as possible. The length and width of each room should be a whole number of meters. What length and width should the architect use for each room?



TRY IT **Math Toolkit** grid paper, sticky notes, unit tiles

Possible work:

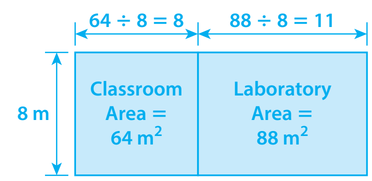
SAMPLE A

Factors of 64: 1, 2, 4, 8, 16, 32, 64

Factors of 88: 1, 2, 4, 8, 11, 22, 44, 88

The GCF is 8, so the greatest possible width is 8 m.

The width of both rooms is 8 m. The length of the classroom is 8 m, and the length of the laboratory is 11 m.



SAMPLE B

Classroom area (64): 1 × 64, 2 × 32, 4 × 16, 8 × 8

Laboratory area (88): 1 × 88, 2 × 44, 4 × 22, 8 × 11

The width could be 1 m, 2 m, 4 m, or 8 m. The greatest width is 8 m.

The classroom should be 8 m wide and 8 m long.
The laboratory should be 8 m wide and 11 m long.

DISCUSS IT

Ask: How did you determine the greatest possible width of the rooms?

Share: I started by . . . Then I . . .

Learning Targets SMP 1, SMP 2, SMP 3, SMP 4, SMP 5, SMP 6
- Apply the properties of operations to generate equivalent expressions.
- Identify when two expressions are equivalent.
- Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor.

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Common Misconception Listen for students who find a common factor that is not the greatest common factor. They may think that any common factor, such as 2 or 4, can be used to answer the question. As students share their strategies, address this misconception by asking: *Is the width you used the greatest width possible? How can you check that it is the greatest width?*

Select and Sequence Student Strategies

Select 2–3 samples that represent the range of student thinking in your classroom. Here is one possible order for class discussion:

- using a model or drawing to display the dimensions of each room
- (misconception) finding a common factor that is not the greatest common factor
- listing possible pairs of lengths and widths of each room
- finding the GCF of the areas to determine the greatest width, and dividing the area by the width to find the lengths of each room

Facilitate Whole Class Discussion

Call on students to share selected strategies. Review with students that one way to listen to understand is to see if you can paraphrase the speaker's ideas. Have students confirm their paraphrase with the original speaker.

Guide students to **Compare and Connect** the representations. Use turn and talk to help students think through their responses before sharing with the group.

> **ASK** *How are [student name]'s and [student name]'s strategies used to determine the width of the rooms?*
>
> **LISTEN FOR** The width of the rooms is determined by the GCF of the areas of the rooms. Because the rooms share the greatest common width, the GCF is the width.

CONNECT IT

SMP 2, 4, 5

1 Look Back Look for understanding that the width of the rooms is equal to the GCF of the areas of the rooms and the lengths of the rooms are equal to the areas divided by the width.

DIFFERENTIATION | RETEACH or REINFORCE

Hands-On Activity

Model expressions that have the same value.

If students are unsure about the concept of equivalent expressions, then use this activity to visualize how to form equivalent expressions.

Materials For each group: 24 two-color counters

- Have students make a group of 9 red and 15 yellow counters. Ask: *What is the GCF of 9 and 15?* [3]
- Have students organize each color into an array with 3 rows. Ask: *How many counters are in each row of the red array?* [3] *The yellow array?* [5]
- Have students combine the two arrays into one array with 3 rows. Ask: *How many counters are in each row of the large array?* [8] *How is this number related to numbers of counters in the smaller arrays?* [It is the sum.]
- Have students write a multiplication equation that represents the total number of counters in the large array and a multiplication equation that represents the total number of counters as the sum of the smaller arrays. Ask: *How does each equation represent the total number of counters in the large array?* [large array: $3 \times 8 = 24$; sum of smaller arrays: $(3 \times 3) + (3 \times 5) = 24$; Both show the total number of counters is 24.]

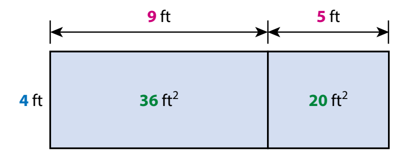
CONNECT IT

1 Look Back What length and width should the architect use for the classroom and the laboratory? Explain how you know.

Classroom: 8 m by 8 m; laboratory: 11 m by 8 m; Possible explanation: The shared width is equal to the GCF of the areas. The length of each room is equal to the area divided by the width.

2 Look Ahead When two rectangles share a common side, the areas of the rectangles have a common factor.

a. The expressions $36 + 20$ and $4(9 + 5)$ both represent the area, in square feet, of the outer rectangle. They are **equivalent expressions** because they name the same value. Show that these expressions are equivalent by finding the value of each expression.



$36 + 20 =$ __56__

$4(9 + 5) = 4 \times$ __14__

$=$ __56__

b. You can also use the distributive property to show that the sum $36 + 20$ is equivalent to the product $4(9 + 5)$. To rewrite $36 + 20$ as a product, you can use the greatest common factor (GCF) of 36 and 20 as one of the factors.

The GCF of 36 and 20 is __4__.	$36 + 20$
Rewrite each term using the GCF as a factor.	__4__ $\times$ 9 + __4__ $\times$ 5
Use the distributive property.	__4__ $\times$ (__9__ + __5__)

c. Rewrite the sum $42 + 35$ as a product. Use the GCF of 42 and 35 as one of the factors. Use a sum as the other factor.

$42 + 35 = (7 \cdot 6) + (7 \cdot 5)$

$= 7(6 + 5)$

3 Reflect How can you use the distributive property to rewrite a sum of two terms as a product of two factors?

Possible answer: Use the GCF of the terms to rewrite each term as a product. Then rewrite the sum as the product of the GCF and the sum of the other factors.

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2 Look Ahead Point out that two or more expressions in different forms that name the same value are equivalent. Students should recognize that they can use the distributive property to show that two expressions are equivalent.

Ask a volunteer to rephrase the definition of equivalent expressions, looking for understanding that these name the same value. Support student understanding by having students show that there is more than one way to factor an expression. For example, $18 + 24$ can be written as $2(9 + 12)$ and $3(6 + 8)$.

CLOSE | EXIT TICKET

3 Reflect Look for understanding that expressions can be written in different forms to show how they are still equivalent.

Common Misconception If students think the distributive property can only be used to rewrite a product as a sum, then have them look at a sum and identify a common factor of the addends. Encourage them to use the common factor to write the expression as a product with the common factor as one factor and the sum as the other factor. This shows how you can use the distributive property "in reverse."

Prepare for Writing and Identifying Equivalent Expressions

Support Vocabulary Development

Assign **Prepare for Writing and Identifying Equivalent Expressions** as extra practice in class or as homework.

If you have students complete this in class, then use the guidance below.

Ask students to consider the term *expression*. Provide support as needed, helping students use their previous knowledge of numerical expressions to guide their thinking.

Have students work individually or in pairs to complete the graphic organizer. Invite students to share their completed organizers and prompt a whole-class comparative discussion of meanings of the words in the graphic organizer and the examples.

Have students look at the expression in problem 2 and discuss with a partner how to determine the coefficient of a variable without a number in front of it. Encourage students to use *term* and *coefficient* in their explanation.

Problem Notes

1. Students should understand that expressions can include numbers, variables, and operation symbols but not an equal sign. Student responses may include numerical expressions or expressions with variable terms. Students may recognize that coefficients are parts of terms and terms are parts of expressions. Students may understand that the distributive property is a way to write a sum as a product.

2. Students should recognize that the coefficient of a variable term is the number that the variable is multiplied by, and that if there is no number shown in front of the variable, then the coefficient of the term is 1.

LESSON 19 | SESSION 1 Name:

Prepare for Writing and Identifying Equivalent Expressions

1. Think about what you know about expressions. Fill in each box. Use words, numbers, and pictures. Show as many ideas as you can.

Possible answers:

Word	In My Own Words	Examples
expression	An expression can include numbers, variables, and operation symbols, but not an equal sign.	$12 + 5$ n $2x + 7$
term	A term is a number, a variable, or a product of one or more numbers and one or more variables.	The expression $12a + 3b + 15$ has 3 terms. $12a + 3b + 15$ term term term
coefficient	In a term with a variable, the coefficient is the number that the variable is multiplied by.	12 is the coefficient of $12a$, and 3 is the coefficient of $3b$. $12a + 3b + 15$ coefficient coefficient
distributive property	You can multiply a sum by a number by multiplying each addend by the number and adding the results.	$8(60 + 3) = 8(60) + 8(3)$

2. What is the coefficient of n in the expression $n + 15$? Explain how you know.

1; Possible explanation: Any number times 1 is equal to the number. So, you can rewrite the term n as $n \cdot 1$, or $1n$. This shows that n has a coefficient of 1.

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REAL-WORLD CONNECTION

When engineers send satellites to orbit the Earth, they use equivalent expressions to keep the satellites in orbit. The gravity of the Earth is continually pulling the satellites toward the ground, so engineers must determine how to keep them from falling. An engineer creates an expression for an equivalent speed of the satellite going around the Earth, so that instead of falling straight down, the satellite moves around the planet but does not get close to the ground. Encourage a whole-class discussion about reasons why there are satellites orbiting the Earth, such as for cell phone communications, streaming radio and television shows, and predicting weather patterns. Ask students to think of other real-world examples when equivalent expressions might be useful.

Additional Practice

3 Problem 3 provides another look at equivalent expressions. This problem is similar to the problem about finding the width and lengths of a classroom and laboratory. In both problems, the areas of two rooms are given. This problem asks for students to find the dimensions of the rooms, given the parameters that the rooms have the same length and the length should be as great as possible.

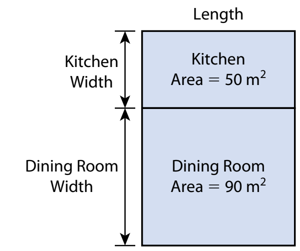
Students may want to make a list of possible dimensions or make a list of factors to find the GCF to solve.

Suggest that students use **Three Reads**, asking themselves one of the following questions each time:

- *What is this problem about?*
- *What is the question trying to answer?*
- *What information is important?*

LESSON 19 | SESSION 1

3 An architect is designing a sandwich shop, as shown in the diagram. The kitchen and dining room will be the same length. The length needs to be as great as possible. The length and width of each room should be a whole number of meters.



a. What length and width should the architect use for the kitchen and for the dining room? Show your work.

Possible work:

Factors of 50: 1, 2, 5, 10, 25, 50

Factors of 90: 1, 2, 3, 5, 6, 9, 10, 15, 18, 30, 45, 90

The GCF is 10, so the greatest possible length is 10 m.

Area = length × width, so the width of each room is area ÷ length.

Kitchen width = 50 ÷ 10 = 5   Dining room width = 90 ÷ 10 = 9

SOLUTION The length of each room is 10 m. The width of the kitchen should be 5 m. The width of the dining room should be 9 m.

b. Check your answer to problem 3a. Show your work.

Possible work:

Kitchen area (50): 1 × 50, 2 × 25, 5 × 10

Dining room area (90): 1 × 90, 2 × 45, 3 × 30, 5 × 18, 6 × 15, 9 × 10

The length could be 1 m, 2 m, 5 m, or 10 m. The greatest is 10 m.

The kitchen should be 10 m long and 5 m wide.

The dining room should be 10 m long and 9 m wide.

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DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Use with **Session 2** Connect It

Levels 1–3: Listening/Writing

Help students make sense of Connect It problem 5 and respond in writing. Read the problem and then provide sentence frames to help students rephrase the first sentence with a partner:

- *We used the* _____ *to rewrite equations with* _____.
- We *can also use the* _____ *to rewrite equations with* _____.

Allow students time to think about strategies and rewrite the expression. Before helping students with written responses, call on several volunteers to share their ideas. Display the definition of *distributive property*. Prompt students to justify their answers using the terms *distributive property* and *common factor*.

Levels 2–4: Listening/Writing

Help students make sense of Connect It problem 5 and respond in writing. Have students read the problem and discuss the definition of *distributive property*. Clarify that students should use what they learned in Analyze It to form a strategy for rewriting the equation. Use **Co-Constructed Word Bank** to develop a list of words and phrases that might be used to talk or write about the distributive property, like *terms*, *parentheses*, and *common factor*.

Allow time for students to solve the problem. Prompt them to refer to the definition of *distributive property* as they justify their strategy by explaining in writing how the property supports their ideas.

Levels 3–5: Listening/Writing

Support students as they make sense of Connect It problem 5 and justify strategies in a written response. Have students read the problem and discuss with a partner how the distributive property can help them form a strategy for rewriting the equation. Call on several volunteers to share ideas so that everyone hears the ideas more than once and in more than one way. Ask students to rephrase responses, when appropriate, using precise mathematical language.

Have students craft their written responses. Prompt students to use definitions, properties, and what they already know as they justify their strategies and solutions in writing.

Develop Using the Distributive Property to Write Equivalent Expressions

Purpose

- **Develop** strategies for applying the distributive property to write equivalent expressions.
- **Recognize** that algebraic expressions can be written in different forms to show how the quantities in a problem are related.

START CONNECT TO PRIOR KNOWLEDGE

Which One Doesn't Belong?

A: 6(4 + 3)	B: 4(6 + 14)
C: 3(8 + 6)	D: 18 + 24

Possible Solutions

A is the only expression that uses the GCF.

B is the only expression not equal to 24 + 18 (or 32).

C is the only expression that uses an odd common factor.

D is the only expression written as a sum.

WHY? Support students' understanding of how to use the distributive property to write equivalent expressions.

DEVELOP ACADEMIC LANGUAGE

WHY? Support students as they justify ideas.

HOW? Prompt students to use definitions, properties, and what they already know as they justify their strategies and solutions. Model this by displaying the definitions of *distributive property* and *common factor* and thinking aloud to justify the explanation presented in Analyze It.

Highlight times during whole-class discussion when students use definitions or properties to justify their strategies.

TRY IT

SMP 1, 2, 4, 5, 6

Make Sense of the Problem

See ***Connect to Culture*** to support student engagement. Before students work on Try It, use **Notice and Wonder** to help them make sense of the problem. Ask: *What do you notice in this situation?* Discuss the things that students noticed and wondered about that are relevant to the problem.

LESSON 19 | SESSION 2

Develop Using the Distributive Property to Write Equivalent Expressions

➤ **Read and try to solve the problem below.**

The Romano family pays for a streaming movie service that costs \$8 per month. They want to add a second movie service for *d* dollars per month. Write two expressions for the total cost of both services for 3 months. One expression should be a sum of two terms, and one should be a product of two factors.

TRY IT Math Toolkit algebra tiles, grid paper

Possible work:

SAMPLE A

24			3*d*		
8	8	8	*d*	*d*	*d*

8 + *d*		8 + *d*		8 + *d*	
8	*d*	8	*d*	8	*d*

The expressions 24 + 3*d* and 3(8 + *d*) both represent the total cost in dollars.

SAMPLE B

Sum of two terms: Add the total cost of the first service to the total cost of the second service.

$3 \cdot 8 + 3 \cdot d$

$24 + 3d$

Product of two factors: Multiply the total cost for 1 month by the number of months, 3.

$3 \cdot (8 + d)$

$3(8 + d)$

DISCUSS IT

Ask: What does each part of your two expressions represent?

Share: In my first expression, . . . In my second expression, . . .

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DISCUSS IT

SMP 2, 3, 6

Support Partner Discussion

After students work on Try It, have them respond to Discuss It with a partner. If students need support in getting started, prompt them to ask each other questions such as:

- *Why did you choose the model or strategy you used to represent the costs of the streaming movie services?*
- *How did your model help you determine the total cost?*

Common Misconception Listen for students who identify 3(8 + *d*) as a sum, not a product. Remind students that parentheses can be written to show multiplication. For example, 3(8) is the same as 3 • 8, or 3 groups of 8. So, 3(8 + *d*) is the same as 3 groups of 8 + *d*.

Select and Sequence Student Strategies

Select 2–3 samples that represent the range of student thinking in your classroom. Here is one possible order for class discussion:

- using grid paper to draw a model of the situation
- (misconception) identify $3(8 + d)$ as a sum, not a product
- model with algebra tiles
- use the distributive property to write equivalent expressions using the sum of two terms and the product of two factors

Facilitate Whole Class Discussion

Call on students to share selected strategies. Prompt students to use definitions, properties, and what they already know to justify their strategies and solutions.

Guide students to **Compare and Connect** the representations. Prompt students to connect each representation to the total cost for both services.

ASK *How do [student name]'s and [student name]'s models show that both expressions have the same value?*

LISTEN FOR One representation adds the cost of one service for 3 months to the cost of the other service for 3 months. The other representation multiplies the cost of each service for one month by 3 months.

Model It/Analyze It

If students presented these models, have students connect these models to those presented in class.

If no student presented at least one of these models, have students first analyze key features of the models, and then connect them to the models presented in class.

ASK *How does each model represent the cost of each service for 3 months?*

LISTEN FOR Each model shows the cost of three months of the first service as 3(8) and the cost of three months of the second service as $3d$.

For the algebra tiles, prompt students to identify what each tile and group of tiles represents in the situation.

- *How does the group of square tiles represent the first streaming movie service?*
- *How does the group of rectangular tiles represent the second streaming movie service?*

For the expression, prompt students to explain how they know the expressions are equivalent.

- *How can you check that $24 + 3d$ and $3(8 + d)$ are equivalent expressions?*

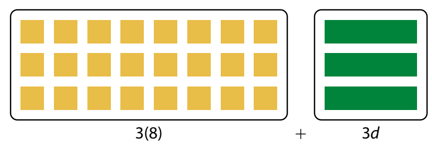
➤ **Explore different ways to understand using the distributive property to write equivalent expressions.**

The Romano family pays for a streaming movie service that costs $8 per month. They want to add a second movie service for d dollars per month. Write two expressions for the total cost of both services for 3 months. One expression should be a sum of two terms, and one should be a product of two factors.

Model It

You can use algebra tiles to help you write an algebraic expression.

Each square tile represents $1. Each rectangular tile represents d dollars.



The tiles show that the expression $24 + 3d$ represents the total cost of both services for 3 months.

Analyze It

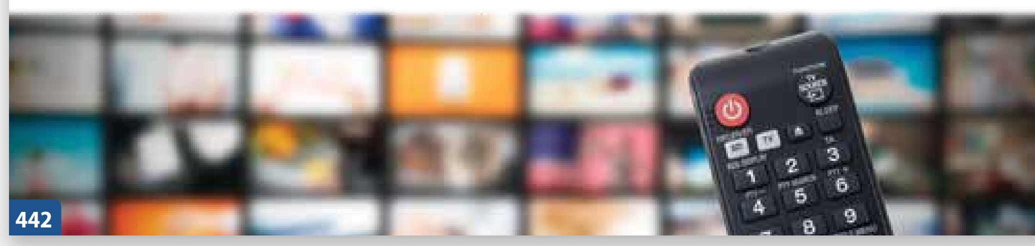
You can use the distributive property to find an equivalent expression.

Rewrite the expression $24 + 3d$ as a product of two factors. One factor is a common factor of the two terms.

$24 + 3d$

$3(8) + 3(d)$ ⟵ The GCF of 24 and $3d$ is 3.

$3(8 + d)$



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DIFFERENTIATION | EXTEND

SMP 2

Deepen Understanding
Making Sense of Quantities and the Relationships Between Them

Prompt students to think about how the expressions for the services would change if the context was changed in different ways.

ASK *How would the expression as a product change if you wanted to find the total cost for 5 months? What about the expression as a sum? How would either expression stay the same?*

LISTEN FOR Both terms would be multiplied by 5, so the expression as a product would be $5(8 + d)$. The expression as a sum would be $40 + 5d$. The expression as a product would still have $8 + d$ as a factor.

ASK *How would the expression as product change if the cost of the first streaming service was raised to $10 per month? What about the expression as a sum? How would either expression stay the same?*

LISTEN FOR There would still be a factor of 3, but the addend 8 would be replaced with 10. The expression as a product would be $3(10 + d)$. The expression as a sum would be $30 + 3d$. The expression as a sum would still have $3d$ as an addend.

Develop Using the Distributive Property to Write Equivalent Expressions

CONNECT IT

SMP 2, 4, 5, 6

Remind students that the quantities and the relationships between them are the same in each representation. Explain that they will now use those relationships to reason about equivalent expressions.

Before students begin to record and expand on their work in Model It/Analyze It, tell them that problem 4 will prepare them to provide the explanation asked for in problem 5.

Monitor and Confirm Understanding 1 – 3

- The expression $8 + d$ is within parentheses, so it is a factor.
- 24 square tiles and 3 rectangular tiles can be divided into three equal groups of 8 square tiles and 1 rectangular tile.
- There are three groups of 8 square tiles and 1 rectangular tile. This is the same as 24 square tiles and 3 rectangular tiles.

Facilitate Whole Class Discussion

4 Look for understanding that writing a product as a sum is the reverse of the process of writing a sum as a product.

> **ASK** *How do you rewrite 24 + 3d as 3(8 + d)?*
>
> **LISTEN FOR** You divide each addend by the GCF, which is 3. Then you write the expression with the factors 3 and $8 + d$.

5 Look for understanding that an equivalent expression can be written for an expression given as a product or as a sum.

> **ASK** *How can you use the distributive property to write equivalent expressions?*
>
> **LISTEN FOR** If an expression is a sum, you can use the GCF of the addends to write the expression as a product. If the expression is given as a product, you can multiply the first factor through the parentheses to write the sum of two terms.

6 Look for the idea that the distributive property applies to both differences and sums.

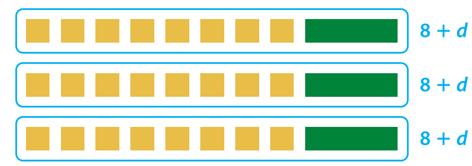
7 **Reflect** Have all students focus on the strategies used to solve the Try It. If time allows, have students discuss their ideas with a partner.

CONNECT IT

➤ **Use the problem from the previous page to help you understand how to use the distributive property to write equivalent expressions.**

1 Look at the expressions $24 + 3d$ and $3(8 + d)$. Which expression is a sum of two terms? Which expression is a product of two factors?
$24 + 3d$ is a sum of two terms; $3(8 + d)$ is a product of two factors.

2 The tiles in **Model It** are grouped to model the expression $24 + 3d$. The same set of tiles is shown at the right. Circle and label groups of tiles to show how the tiles also model the expression $3(8 + d)$. **See model.**



3 Equivalent expressions always name the same value. How do the algebra tiles show that $24 + 3d$ and $3(8 + d)$ will always have the same value?
Each expression is represented by the same number and type of tiles.

4 In **Analyze It**, the distributive property is used to rewrite the sum $24 + 3d$ as the product $3(8 + d)$. How do you use the distributive property to rewrite the product $3(8 + d)$ as the sum $24 + 3d$?
Multiply each term inside the parentheses by 3 and add the results.

5 What are two ways you can use the distributive property to write equivalent expressions?
You can rewrite a sum as a product or a product as a sum.

6 The distributive property applies to differences as well as sums. How could you use the distributive property to rewrite $6x - 6$ as a product?
The terms have a common factor of 6, so you can use the distributive property to rewrite the expression as $6(x - 1)$.

7 **Reflect** Think about all the models and strategies you have discussed today. Describe how one of them helped you better understand how to use the distributive property to write equivalent expressions.
Responses will vary. Check student responses.

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DIFFERENTIATION | RETEACH or REINFORCE

Hands-On Activity
Use the distributive property to write equivalent expressions.

If students are unsure about using the distributive property to write equivalent expressions, then use this activity to visualize the distributive property.

Materials For each group: algebra tiles (6 x-tiles, 18 1-tiles)

- Distribute 18 1-tiles and 6 x-tiles to each group. Ask: *Using x as the value of each rectangular tile, what sum do the tiles represent?* [$6x + 18$]
- Have students divide the tiles into equal groups that include both 1-tiles and x-tiles. Ask: *How can you show the total number of tiles using a multiplication expression?* [Possible answers: $6(x + 3)$; $2(3x + 9)$; $3(2x + 6)$]
- Challenge students to regroup the tiles and write the multiplication expression three different ways. Ask: *How are the expressions the same?* [They all can be written as $6x + 18$.]

Apply It

For all problems, encourage students to use a model to support their thinking. Allow some leeway in precision; drawing tiles that are the exact same size can be difficult, and precise drawings are not necessary to solve the problems.

8 **a.** Students may use the commutative property to rewrite the problem as $(2)(5x + 3)$ before using the distributive property.

b. Students may also use the associative property to regroup $3 \cdot y \cdot 2$ as $(3 \cdot 2) \cdot y$.

9 **A** **is correct.** Students may solve the problem by finding the GCF of 63 and 56, which is 7, and then dividing each addend by 7.

B is not correct. This answer subtracts 3 from 63 instead of dividing by the greatest common factor of 63 and 56.

C is not correct. This answer rewrites each number by removing the common digit of 6 from both instead of finding a common factor of 63 and 56.

D is not correct. This answer only divides the first addend in the parentheses by the GCF of 7 instead of dividing 56 by 7 as well.

LESSON 19 | SESSION 2

Apply It

➤ **Use what you learned to solve these problems.**

8 **a.** Use the distributive property to rewrite $(5x + 3)(2)$ as a sum of two terms. Show your work. Possible work:

$(5x + 3)(2)$

$(5x \cdot 2) + (3 \cdot 2)$

$(5 \cdot 2 \cdot x) + (3 \cdot 2)$

$10x + 6$

SOLUTION $10x + 6$

b. You can use the commutative and associative properties of multiplication to reorder and regroup factors. Explain how you used one or both of these properties in your work for problem 8a.

Possible answer: I reordered and regrouped the factors in the term $(5x \cdot 2)$ to get $(5 \cdot 2)x$, or $10x$.

9 Which expression is equivalent to $63 + 56$?

A $7(9 + 8)$

B $3(60 + 56)$

C $6(3 + 5)$

D $7(9 + 56)$

10 A company sells fruit cups in packs of 4. The packs currently weigh 20 oz. The company plans to reduce the weight of each cup by n oz. The expression $20 - 4n$ represents the new weight, in ounces, of a pack of fruit cups. Rewrite the expression for the new weight as a product of two factors. Show your work.

Possible work:

$20 - 4n$

$4 \cdot 5 - 4 \cdot n$

$4(5 - n)$

SOLUTION Possible answer: $4(5 - n)$

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CLOSE EXIT TICKET

10 Students may also rewrite the expression as $2(10 - 2n)$. Students' solutions should show an understanding of:

- finding a factor of two numbers.
- using the distributive property to write an equivalent expression.

Error Alert If a student writes the expression $20 - 4n$ as $4(5 - 4n)$, then emphasize that when rewriting the expression as a product, the same factor must be divided from each term in the difference. In this case, divide both 20 and $4n$ by 4 to get $4(5 - n)$.

Practice Using the Distributive Property to Write Equivalent Expressions

Problem Notes

Assign **Practice Using the Distributive Property to Write Equivalent Expressions** as extra practice in class or as homework.

1. ***Basic***
2. Students should recognize that when using the distributive property, the factor outside of the parentheses is multiplied by each term within the parentheses. ***Medium***
3. Students may write a list of factors to determine the GCF. ***Basic***

LESSON 19 | SESSION 2   Name:

Practice Using the Distributive Property to Write Equivalent Expressions

➤ **Study the Example showing how to use the distributive property to rewrite a product. Then solve problems 1–6.**

Example

Rewrite the expression $3(7a - 4b)$ as a difference.

You can use the distributive property to rewrite the product.

Multiply $7a$ and $4b$ by 3.	$3(7a - 4b)$
	$3 \cdot 7a - 3 \cdot 4b$
Use the associative property.	$(3 \cdot 7)a - (3 \cdot 4)b$
Multiply inside the parentheses.	$21a - 12b$

The difference $21a - 12b$ is equivalent to $3(7a - 4b)$.

1. You use the associative property of multiplication to change how factors are grouped. In the Example, why are the factors of the term $3 \cdot 7a$ regrouped as $(3 \cdot 7)a$?

   Possible explanation: Regrouping the factors lets you write the product with one numerical factor and one variable factor.

2. Jesse says that the expressions $7(2x + 9)$ and $14x + 9$ are equivalent. Do you agree with Jesse? Explain.

   No; Possible explanation: When you use the distributive property to multiply $(2x + 9)$ by 7, you multiply both $2x$ and 9 by 7. The expression $7(2x + 9)$ is equivalent to $14x + 63$, not $14x + 9$.

3. Use the greatest common factor of 84 and 48 to write the sum $84 + 48$ as a product. Write a whole number in each blank.

   $84 + 48$

   $\underline{\ 12\ } \times (7 + \underline{\ 4\ })$

Vocabulary

distributive property
for any numbers a, b, and c, $a(b + c) = ab + ac$.

equivalent expressions
two or more expressions in different forms that always name the same value.

greatest common factor (GCF)
the greatest factor two or more numbers have in common.

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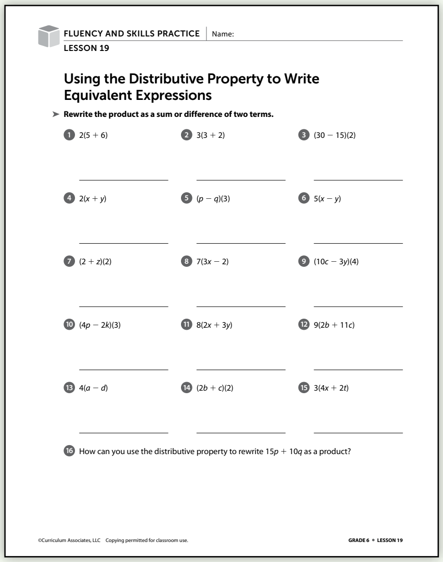
Fluency & Skills Practice

Using the Distributive Property to Write Equivalent Expressions

In this activity, students use the distributive property to rewrite the product as a sum or difference of two terms.


FLUENCY AND SKILLS PRACTICE | Name:
LESSON 19

Using the Distributive Property to Write Equivalent Expressions

➤ Rewrite the product as a sum or difference of two terms.

1. $2(5 + 6)$
2. $3(3 + 2)$
3. $(30 - 15)(2)$
4. $2(x + y)$
5. $(p - q)(3)$
6. $5(x - y)$
7. $(2 + z)(2)$
8. $7(3x - 2)$
9. $(10c - 3y)(4)$
10. $(4p - 2k)(3)$
11. $8(2x + 3y)$
12. $9(2b + 11c)$
13. $4(a - d)$
14. $(2b + c)(2)$
15. $3(4x + 2t)$
16. How can you use the distributive property to rewrite $15p + 10q$ as a product?

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Additional Practice

4 Students may use the distributive property to rewrite the product as a difference. Remind students that the commutative property does not work for subtraction, and rewriting the difference as $8k - 6$ is incorrect. ***Medium***

5 **a.** The expressions are equivalent because the distributive property was applied correctly.

b. The expressions are equivalent because the distributive property was applied correctly.

c. The expressions are not equivalent. The coefficient of each term should not be increased by 4. Rather, each term in the sum should be multiplied by 4.

d. The expressions are not equivalent. Both terms should be multiplied by 9, not just the second term.

Medium

6 **a.** Students may find the GCF of $3x$ and 18 and then divide each term of the expression by the GCF. ***Medium***

b. Students may determine which value in the expression represents the increased amount of food Kaley fed her puppy each day. The part of the expression that represents the additional amount is "$x +$," so the number 6 represents the number of ounces the puppy was fed each day before the increase. ***Challenge***

LESSON 19 | SESSION 2

4 Rewrite the expression $2(3 - 4k)$ as a difference. Show your work.

Possible work: $2(3 - 4k)$

$2 \cdot 3 - 2 \cdot 4k$

$2(3) - (2 \cdot 4)k$

$6 - 8k$

SOLUTION $6 - 8k$

5 Tell whether each pair of expressions is *Equivalent* or *Not Equivalent*.

	Equivalent	Not Equivalent
a. $5(3t - 6)$ and $15t - 30$	●	○
b. $16 + 72n$ and $(2 + 9n)(8)$	●	○
c. $4(6a + 8b)$ and $10a + 12b$	○	●
d. $7x - 9y$ and $(7x - y)(9)$	○	●

6 Kaley plans to increase the amount of food she feeds her puppy each day by x oz. The expression $3x + 18$ represents the total weight of food, in ounces, Kaley will need for her puppy for the next three days.

a. Rewrite the expression as a product of two factors. Show your work. Possible work:

x	6	$x + 6$
x	6	$x + 6$
x	6	$x + 6$

SOLUTION $3(x + 6)$

b. How many ounces of food did Kaley feed her puppy each day before she increased the amount? Explain how you know.

6 oz; Possible answer: The expression $3(x + 6)$ shows that Kaley's puppy now gets $(x + 6)$ oz each day, so it used to get 6 oz each day.

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DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Use with **Session 3** Picture It/Analyze It

MATH TERM

To *regroup* means to compose or decompose tens, hundreds, thousands, and so forth.

ACADEMIC VOCABULARY

To *reorder* means to arrange or place in a different order.

Levels 1–3: Reading/Listening

Prepare students to interpret Picture It and Analyze It. Read the problem aloud. Call on a volunteer to give examples of *like terms*.

Use **Act It Out** to help students make sense of the strategies. Guide pairs of students to use markers and note cards to replicate Picture It. Help them label the note cards. Then read each sentence of Analyze It aloud. Prompt partners to reorder and regroup the note cards as shown in the expressions. Confirm understanding of *like terms* and *pair of like terms*. Have partners work together to describe each step using everyday language.

Levels 2–4: Reading/Listening

Help students interpret Picture It and Analyze It using **Act It Out**. Ask partners to use note cards to replicate Picture It. Encourage partners to discuss how they can label the note cards. For example, the first column may have two note cards labeled y sheets or one note card labeled $2y$ sheets.

Read Analyze It aloud and ask: *What like terms are in the expression?* Prompt partners to follow steps 2 and 3 to reorder and regroup the note cards as shown in the expressions and to rewrite the y term. Have partners take turns paraphrasing each step.

Levels 3–5: Reading/Listening

To support Analyze It, use **Notice and Wonder** to help students analyze the expressions. Record statements for reference. Then read aloud each sentence in Analyze It and have students discuss how they relate to the notice and wonder statements. Encourage students to identify any statements that can be reworded using precise math or academic language.

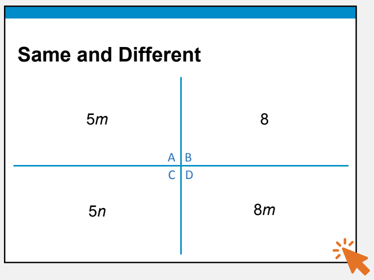
Next, ask partners to discuss how the expressions in Analyze It relate to the model in Picture It. Encourage students to listen for understanding by asking their partners clarifying questions or by paraphrasing their partners' ideas.

Develop Combining Like Terms

Purpose

- **Develop** strategies for identifying and combining like terms.
- **Recognize** that the distributive property allows you to combine like terms.

START CONNECT TO PRIOR KNOWLEDGE



Possible Solutions

All four are a single term.

A and D both have the same variable factor.

A and C have the same coefficient but different variable factors.

The number 8 appears in B and D, but only D has a variable.

WHY? Support students' understanding of identifying terms.

DEVELOP ACADEMIC LANGUAGE

WHY? Develop awareness of precision in academic language.

HOW? Explain that words and phrases that add or qualify details make ideas more precise, or exact. Display: *The commutative property of addition lets you reorder the terms.* Ask students to discuss how naming the property makes the idea more precise. Then have them name the properties of operations that are used to rewrite the expressions in Analyze It.

TRY IT SMP 1, 2, 4, 5, 6

Make Sense of the Problem

See ***Connect to Culture*** to support student engagement. Before students work on Try It, use **Co-Craft Questions** to help them make sense of the problem. If discussion lags, have students turn and talk with a partner about what each number and variable in the problem represents.

Develop Combining Like Terms

➤ **Read and try to solve the problem below.**

Ryan is making papel picado as decorations for his aunt's wedding. The receipt shows how much tissue paper he bought. Each small package holds x sheets, and each large package holds y sheets. Write an expression with exactly three terms to represent the total number of sheets of tissue paper Ryan bought.

PAPER PLUS
Art Supply Store
RECEIPT

ITEM	AMOUNT
• gold tissue paper 2 large packages	\$4.44
• blue tissue paper 3 small packages	\$2.46
• green tissue paper 1 large package	\$2.22
• purple tissue paper 2 small packages	\$1.64
• pink tissue paper 4 sheets	\$0.84

THANK YOU FOR SHOPPING IN OUR STORE!

TRY IT

Math Toolkit algebra tiles, grid paper

Possible work:

SAMPLE A

Number of large packages: $2 + 1 = 3$

Number of small packages: $3 + 2 = 5$

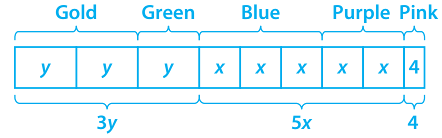
Sheets in 3 large packages: $3y$

Sheets in 5 small packages: $5x$

Total sheets: $3y + 5x + 4$

An expression for the total number of sheets is $3y + 5x + 4$.

SAMPLE B



$3y + 5x + 4$ represents the total number of sheets of tissue paper.

DISCUSS IT

Ask: What does each term in your expression represent?

Share: The first term represents . . .

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DISCUSS IT SMP 2, 3, 6

Support Partner Discussion

After students work on Try It, encourage them to respond to Discuss It with a partner. If students need support in getting started, prompt them to ask each other questions such as:

- *How would you describe your model?*
- *How does your model represent the problem situation?*

Common Misconception Listen for students who add terms that do not have the same variable, such as adding 4 sheets to $3y$ or to $5x$. As students share their strategies, ask them to explain why they think the number of packages can be added together. Discuss how the order of operations can be used to justify this. To find the value of $4 + 5x$, the order of operations state that you need to multiply 5 and the value of x before adding 4. So, $4 + 5x$ does not equal $9x$.

Select and Sequence Student Strategies

Select 2–3 samples that represent the range of student thinking in your classroom. Here is one possible order for class discussion:

- drawing a diagram to represent the total number of sheets of tissue paper
- (misconception) adding terms that do not have the same variable
- writing expressions for the number of sheets in the small and large packages and combining the expressions for the total the number of sheets

Facilitate Whole Class Discussion

Call on students to share selected strategies. Allow think time for students to process the ideas presented.

Guide students to **Compare and Connect** the representations. Call on volunteers to reword any vague or unclear statements.

ASK *How do [student name]'s and [student name]'s models show the total number of sheets?*

LISTEN FOR The number of sheets in a large package is multiplied by the number of large packages. The number of sheets in a small package is multiplied by the number of small packages. The total number of sheets is the sum of these parts and the pink sheets.

Picture It/Analyze It

If students presented these models, have students connect these models to those presented in class.

If no student presented at least one of these models, have students first analyze key features of the models, and then connect them to the models presented in class.

ASK *What term represents the total number of sheets that came in the large packages? What term represents the total number of sheets that came in the small packages?*

LISTEN FOR The number of sheets in the large packages is represented by the term $3y$. The number of sheets in the small packages is represented by the term $5x$.

For the picture, prompt students to identify how each box represents a number of sheets.

- *What do the y sheets represent?*
- *What do the x sheets represent?*
- *What does a single line represent?*

For the expression, prompt students to explain how to identify the like terms.

- *How can you determine if two terms are like terms?*

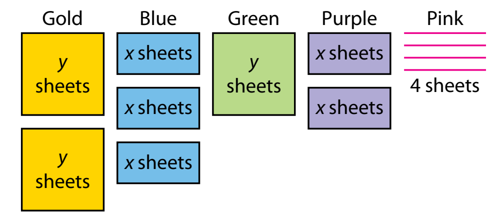
➤ **Explore different ways to combine like terms.**

Ryan is making papel picado as decorations for his aunt's wedding. The table shows how much tissue paper he bought. Each small package holds x sheets, and each large package holds y sheets. Write an expression with exactly three terms to represent the total number of sheets of tissue paper Ryan bought.

Paper Color	Amount
Gold	2 large packages
Blue	3 small packages
Green	1 large package
Purple	2 small packages
Pink	4 sheets

Picture It

You can draw a picture to help you write an algebraic expression.



$2y + 3x + y + 2x + 4$

Analyze It

You can use properties of operations to combine terms with the same variable parts.

Identify the **like terms**.	$2y + 3x + y + 2x + 4$
Reorder and regroup the terms.	$(2y + y) + (3x + 2x) + 4$
Rewrite the term y as $1y$.	$(2y + 1y) + (3x + 2x) + 4$
Use the distributive property with each pair of like terms.	$(2 + 1)y + (3 + 2)x + 4$


Papel picado

DIFFERENTIATION | EXTEND

SMP 2

Deepen Understanding
Making Sense of Quantities and the Relationships Between Them

Have students think about the relationships between the number of sheets of tissue paper in each package by thinking about the coefficients and variables. Have students use these ideas to visualize how the picture shows the relationship between the number of sheets of paper in the different packages.

ASK *How can you draw the picture to visualize combining like terms?*

LISTEN FOR Draw all boxes labeled *y sheets* together and all boxes labeled *x sheets* together.

ASK *Why can the amount of blue and purple sheets be combined? Why can the amount of gold and green sheets be combined?*

LISTEN FOR Each small package of blue and purple paper has the same number of sheets, and each package of gold and green paper has the same number of sheets.

ASK *Can you combine the number of pink sheets with another term? Why or why not?*

LISTEN FOR You cannot combine the pink sheets with the large or small packages because the number of sheets in those packages is unknown.

Develop Combining Like Terms

CONNECT IT

SMP 2, 4, 5, 6

Remind students that the quantities and the relationships between them are the same in each representation. Explain that they will now use those relationships to reason about how to combine like terms.

Before students begin to record and expand on their work in Picture It, tell them that problem 2 will prepare them to provide the explanation asked for in problem 3.

Monitor and Confirm Understanding 1 – 2

- The number of different shapes in Picture It is the same as the number of terms in Analyze It.

Facilitate Whole Class Discussion

2 Look for understanding that terms with the same variable are like terms. Look for understanding of how the commutative and distributive properties are used to write equivalent expressions and combine like terms.

> **ASK** *How can you identify like terms? How can the commutative and distributive properties help when combining like terms?*
>
> **LISTEN FOR** Like terms are terms that have the same variable factor. The commutative property allows the terms to be rearranged so that like terms are closer together. The distributive property allows terms with a common factor to be rewritten as a product of two factors.

3 Look for understanding that one term includes a variable factor to represent the number of sheets in a large package, a second term includes a variable to represent the number of sheets in a small package, and a third term represents the number of individual sheets.

> **ASK** *How do you know that the expression $3y + 5x + 4$ cannot be written with fewer terms? Why would you want to combine terms that have the same variable factor?*
>
> **LISTEN FOR** Each term in the expression contains a different variable factor, or no variable at all, and represents a different amount in a package in the context of the problem. You want to combine terms with the same variable to write an expression with the least number of terms.

4 **Reflect** Have all students focus on the strategies used to solve the Try It. If time allows, have students discuss their ideas with a partner.

CONNECT IT

➤ **Use the problem from the previous page to help you understand how to combine like terms.**

1 Look at **Picture It**. How do the shapes show that an expression for the total number of sheets of tissue paper will have at least three terms?
There are 3 different shapes, so there will be 1 term for each shape.

2 Look at the four equivalent expressions in **Analyze It**.

a. What are the like terms in the expression $2y + 3x + y + 2x + 4$? Explain.
$2y$ and y are like terms because they both include the variable y. $3x$ and $2x$ are like terms because they both include the variable x.

b. The commutative property of addition lets you reorder terms. How is this property used to rewrite the expression?
The commutative property is used to reorder the terms so that like terms that include x and like terms that include y are next to each other.

c. Look at the last expression. Why can you use the distributive property to combine like terms?
The first two terms have a common factor of y and the second two terms have a common factor of x. You use the distributive property to rewrite each pair of terms as a product, with one factor the common factor.

3 What expression with exactly three terms can you write to represent the total number of sheets of tissue paper Ryan bought? How are the coefficients of the variable terms related to the coefficients of the variable terms of the original expression?
$3y + 5x + 4$; The coefficient of each variable term is the sum of the coefficients of the corresponding variable terms of the original expression.

4 **Reflect** Think about all the models and strategies you have discussed today. Describe how one of them helped you better understand how to combine like terms of an algebraic expression.
Responses will vary. Check student responses.

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DIFFERENTIATION | RETEACH or REINFORCE

Hands-On Activity
Use algebra tiles to combine like terms.

If students are unsure about combining like terms, then use this activity to visualize and work through the process.

Materials For each group: algebra tiles (10 x-tiles, 10 1-tiles)

- Give each group a set of 20 algebra tiles. Have students create the following groups: 3 square tiles, 5 rectangular tiles, 3 square tiles, and 3 rectangular tiles.
- Have students identify the groups that represent like terms, using x to represent the square tiles and y to represent the rectangular tiles. Have them combine the groups of like terms to create the fewest number of groups. Ask: *How can you use the variable x to write an equation to describe the number of squares?* [$3x + 3x = 6x$] *Use the variable y to do the same for the number of rectangles.* [$5y + 3y = 8y$]
- Have students create different groups of two sets of square tiles and two sets of rectangular tiles and write equations to represent the groups. Then ask them to write an expression using the fewest number of terms. Have students describe how they can combine like terms.

Apply It

For all problems, encourage students to use a model to support their thinking.

5 Students may use the commutative property to reorder the addends.

6 **D and E are correct.** Students may use the distributive property to rewrite $6a - a$ as a product of two factors.

A is not correct. This answer implies that $6a - a$ equals 6, and then adds 6 to $4b$.

B is not correct. The answer combines all the coefficients, and then combines the variables.

C is not correct. This answer implies that $6a - a$ equals 6 rather than $5a$.

F is not correct. This answer multiplies the coefficients of $6a$ and a rather than subtracting the coefficients.

LESSON 19 | SESSION 3

Apply It

➤ **Use what you learned to solve these problems.**

5 Write an expression equivalent to $12g - 3g + 7$ with exactly two terms. Show your work.

Possible work:

$12g - 3g + 7$

$(12 - 3)g + 7$

$9g + 7$

SOLUTION $9g + 7$

6 Which expressions are equivalent to $6a - a + 4b$? Select all that apply.

A $10b$

B $9ab$

C $6 + 4b$

D $5a + 4b$

E $(6 - 1)a + 4b$

F $(6 \cdot 1)a + 4b$

7 An athletic store receives an order for 8 blue jerseys, 12 pairs of blue shorts, 10 gold jerseys, and 5 pairs of gold shorts. Each jersey weighs j oz, and each pair of shorts weighs s oz. They are packed in a box that weighs 16 oz when empty. Write an expression with exactly three terms for the total weight, in ounces, of the box. Show your work.

Item	Quantity
blue jersey	8
blue shorts	12
gold jersey	10
gold shorts	5

Possible work:

$8j + 12s + 10j + 5s + 16$

$(8j + 10j) + (12s + 5s) + 16$

$(8 + 10)j + (12 + 5)s + 16$

$18j + 17s + 16$

SOLUTION $18j + 17s + 16$

CLOSE EXIT TICKET

7 Students' solutions should show an understanding of:

- identifying like terms by recognizing the terms that contain the same variable.
- applying the commutative property to rearrange the order of the terms, and also applying the distributive property to combine like terms.

Error Alert If students combine all terms with a variable and produce the expression $35js + 16$, then remind students to consider how the order of operations can be used to evaluate an expression. To find the value of $5s + 16$, the order of operations state that you need to multiply 5 and the value of s before adding 16.

Practice Combining Like Terms

Problem Notes

Assign **Practice Combining Like Terms** as extra practice in class or as homework.

1. Students should rewrite the expression from the Example to replace the cost of the frames, $11f$, with the new cost of the frames, $13f$. ***Basic***

2. **B is correct.** Students add $3a$ and $9a$ to find $12a$ and subtract $1b$ from $7b$ to find $6b$.

   A is not correct. This answer removes the b variable when subtracting instead of using a coefficient of 1 for b and combining like terms to write $6b$.

   C is not correct. This answer adds all of the coefficients and combines the variables instead of recognizing that terms with a variable of a are like terms and terms with a variable of b are like terms.

   D is not correct. This answer adds the coefficients 3, 9, and 7, and removes the variable b entirely, instead of identifying and combining like terms.

   Medium

LESSON 19 | SESSION 3 Name:

Practice Combining Like Terms

➤ **Study the Example showing how to combine like terms. Then solve problems 1–6.**

Example

The Woodworking Club is selling picture frames at the school craft fair. The frames sell for \$11 each. Materials for each frame cost \$6, and renting a booth costs \$36. The expression $11f - 6f - 36$ represents the amount of money the club will make for selling f frames. Rewrite the expression with exactly two terms.

You can use the distributive property to combine like terms.

The terms $11f$ and $6f$ are like terms because both have the variable f.

$11f - 6f - 36$

$(11 - 6)f - 36$

$5f - 36$

The terms of $5f - 36$ are not like terms, so they cannot be combined.

The equivalent expression is $5f - 36$.

1. Look at the Example. Suppose the club increases the selling price of a frame to \$13. Write an expression with exactly two terms for the amount of money the club will make for selling f frames. Show your work.

   Possible work:

   $13f - 6f - 36$

   $(13 - 6)f - 36$

   $7f - 36$

   SOLUTION $7f - 36$

2. Which expression is equivalent to $3a + 9a + 7b - b$?

   A $12a + 7$

   B $12a + 6b$

   C $18ab$

   D $19a$

Vocabulary

equivalent expressions
two or more expressions in different forms that always name the same value.

like terms
two or more terms that have the same variable factors.

perimeter
the distance around a two-dimensional shape.

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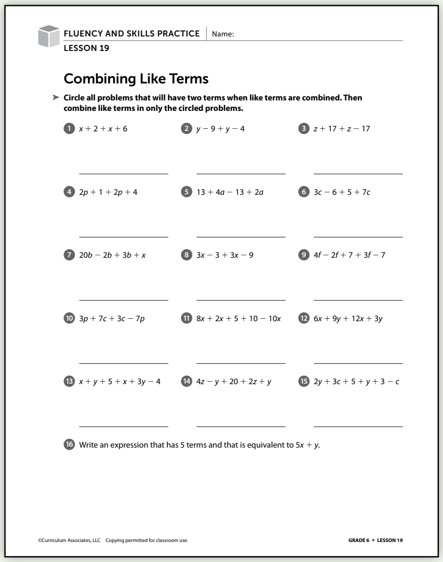
Fluency & Skills Practice

Combining Like Terms

In this activity, students circle all expressions that can be written as the sum or difference of two terms, and then they combine like terms in only the circled expressions.


FLUENCY AND SKILLS PRACTICE Name:
LESSON 19

Combining Like Terms

➤ **Circle all problems that will have two terms when like terms are combined. Then combine like terms in only the circled problems.**

1. $x + 2 + x + 6$
2. $y - 9 + y - 4$
3. $z + 17 + z - 17$
4. $2p + 1 + 2p + 4$
5. $13 + 4a - 13 + 2a$
6. $3c - 6 + 5 + 7c$
7. $20b - 2b + 3b + x$
8. $3x - 3 + 3x - 9$
9. $4f - 2f + 7 + 3f - 7$
10. $3p + 7c + 3c - 7p$
11. $8x + 2x + 5 + 10 - 10x$
12. $6x + 9y + 12x + 3y$
13. $x + y + 5 + x + 3y - 4$
14. $4z - y + 20 + 2z + y$
15. $2y + 3c + 5 + y + 3 - c$
16. Write an expression that has 5 terms and that is equivalent to $5x + y$.

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Additional Practice

3 Students may use the commutative property to reorder the terms. ***Medium***

4 ***Basic***

5 Students should recognize that if all terms are like terms, they can all be combined into 1 term. ***Medium***

6 **A, B, C, and E are correct.** Each of these expressions can be simplified by combining like terms and using the commutative and distributive properties. Each expression equals $6w$.

D is not correct. This answer does not include the 2 sides of the rectangle that are width w.

Challenge

3 Neena and Carissa collect trading cards. Neena has 4 packs of castle cards and 5 packs of hero cards. Carissa has 6 packs of castle cards and 4 packs of hero cards. Each castle pack holds c cards, and each hero pack holds h cards. Write an expression with exactly two terms for the total number of cards Neena and Carissa have. Show your work.

Possible work:

$4c + 5h + 6c + 4h$

$(4c + 6c) + (5h + 4h)$

$10c + 9h$

SOLUTION $10c + 9h$

4 Write a whole number in each blank to show an expression that is equivalent to $15x + 10$.

$20x - \underline{\ 5\ } \cdot x + \underline{\ 10\ }$

5 Isaiah writes an expression with 5 terms. All 5 terms are like terms. How many terms are in the equivalent expression with the least number of terms? Explain.

1 term; Like terms can be combined into a single term. All 5 terms are like terms, so there will be only 1 term once they are combined.

6 The length of a rectangle is twice the width w. Which expressions represent the perimeter of the rectangle? Select all that apply.

Ⓐ $2w + w + 2w + w$

Ⓑ $2w + 2(2w)$

Ⓒ $2(w + 2w)$

D $2(2w)$

Ⓔ $6w$

DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Use with **Session 4** **Connect It**

Levels 1–3: Speaking/Writing

Prepare students for Connect It problem 3 by reviewing how the properties of operations are used in Analyze It. Ask students to number the steps from 1 to 4, and to label the steps with *distributive property, commutative property, associative property, rewrite as a sum, group like terms,* and *combine like terms.*

Ask students to name the properties used in each step and tell how the properties are used. Call on volunteers to repeat and rephrase key ideas. Reword as needed using math language, such as *rewrite, sum, group, combine,* and *like terms.*

Next, help students write responses using:

- *Step _____: The _____ property is used to _____.*

Levels 2–4: Speaking/Writing

Prepare students for Connect It problem 3 by reviewing how the properties of operations are used in Analyze It. Ask students to number steps 1–4 and to label them with *distributive property, commutative property,* or *associative property.* Create a **Co-Constructed Word Bank** of terms to describe the steps. Ensure students include *rewrite, sum, group, combine,* and *like terms.*

Encourage students to use the word bank to discuss how the properties are used in the steps. Ask students to use the sequence words *first, next,* and *then* in their responses. Emphasize key ideas by calling on volunteers to rephrase information.

Levels 3–5: Speaking/Writing

Have students prepare for Connect It problem 3 by reviewing how the properties of operations are used in Analyze It. Ask students to work with partners to create a **Co-Constructed Word Bank** of terms they can use in their written responses.

Have students pre-write their responses to the problem. Then use **Stronger and Clearer Each Time** to help students clarify and revise their responses. Ask partners to discuss how responses can be strengthened using precise language, such as the name of the properties or sequence words. Then have students work independently to revise their written responses.

Develop Identifying Equivalent Expressions

Purpose

- **Develop** strategies for determining whether algebraic expressions are equivalent.
- **Recognize** that algebraic expressions are equivalent if they can be rewritten in exactly the same form.

START CONNECT TO PRIOR KNOWLEDGE

Same and Different

A: $4(n + 3)$	B: $5(b - 3)$
C: $7(b + 5)$	D: $8(n - 2)$

Possible Solutions

All four expressions can rewritten using the distributive property.

A and D contain the variable *n*.

B and C contain the variable *b*.

A and C have addition within the parentheses.

B and D have subtraction within the parentheses.

WHY? Support students' understanding of using the distributive property.

DEVELOP ACADEMIC LANGUAGE

WHY? Reinforce understanding of *reorder, rearrange, rewrite,* and *regroup* through prefixes.

HOW? Students may be familiar with *back* or *again* as the meaning of the prefix *re-*. Explain that *re-* can also mean *again in a different way*. Have students discuss how the prefix influences the meaning of *reorder, rearrange, rewrite,* and *regroup*. Read Analyze It. Have students choose expressions and use words with *re-* to describe the steps to simplify them.

TRY IT

SMP 1, 2, 4, 5, 6

Make Sense of the Problem

Before students work on Try It, use **Notice and Wonder** to help them make sense of the problem. Ask students: *What do you notice in this problem?* Discuss the things that students noticed and wondered about that are relevant to the problem.

LESSON 19 | SESSION 4

Develop Identifying Equivalent Expressions

➤ **Read and try to solve the problem below.**

Which of these three expressions are equivalent?

$3(x + 2) + 2x$ $\quad$ $2 + 4(x + 1) + x$ $\quad$ $2(3 + 3x) - 2x$

TRY IT

Math Toolkit algebra tiles, grid paper

Possible work:

SAMPLE A

x	2	x	2	x	2	x	x

$3(x + 2) + 2x$ is equivalent to $5x + 6$.

2	x	1	x	1	x	1	x	1	x

$2 + 4(x + 1) + x$ is equivalent to $5x + 6$.

3	x	x	x	3	x	$\cancel{x}$	$\cancel{x}$

$2(3 + 3x) - 2x$ is equivalent to $4x + 6$.

Only the expressions $3(x + 2) + 2x$ and $2 + 4(x + 1) + x$ are equivalent.

SAMPLE B

x	$3(x + 2) + 2x$	$2 + 4(x + 1) + x$	$2(3 + 3x) - 2x$
0	$3(2) + 0 = 6$	$2 + 4(1) + 0 = 6$	$2(3 + 0) - 0 = 6$
1	$3(3) + 2 = 11$	$2 + 4(2) + 1 = 11$	$2(3 + 3) - 2 = 10$

When $x = 1$, $2(3 + 3x) - 2x$ does not have the same value as the other two expressions, so it is not equivalent to either of them.

$3(x + 2) + 2x$	$2 + 4(x + 1) + x$
$3x + 6 + 2x$	$2 + 4x + 4 + x$
$3x + 2x + 6$	$2 + 4 + 4x + x$
$5x + 6$	$6 + 5x$, or $5x + 6$

The expressions $3(x + 2) + 2x$ and $2 + 4(x + 1) + x$ are equivalent because they both are equivalent to $5x + 6$.

DISCUSS IT

Ask: What is another way you could show which expressions are equivalent?

Share: I could also . . .

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DISCUSS IT

SMP 2, 3, 6

Support Partner Discussion

After students work on Try It, encourage them to respond to Discuss It with a partner. If students need support in getting started, prompt them to ask each other questions such as:

- *Why did you choose the strategy you used?*
- *How did your strategy help you make sense of the problem?*

Common Misconception Listen for students who assume that expressions are equivalent when they are equivalent for one value of the variable. For example, $2x + 3$ and $3x + 3$ have the same value when $x = 0$ but different values when $x = 1$. As students share their strategies, explain that the variables must have the same value for any value assigned to the variable in order for the expressions to be equivalent. For example, $2x + 3$ and $2(x + 1) + 1$ have the same value for any value of x. Encourage students to create a table of values containing several values for x to help them verify that two expressions are equivalent. Explain that this alone does not show that two expressions are equivalent, but if a value of x results in different values for the expressions, it can be confirmed that the expressions are not equivalent.

Select and Sequence Student Strategies

Select 2–3 samples that represent the range of student thinking in your classroom. Here is one possible order for class discussion:

- creating a table with the values of the expressions for different values of the variables
- (misconception) using only one value for the variable instead of multiple values
- drawing a picture of each expression to see whether they are equivalent
- modeling each expression using algebra tiles

Facilitate Whole Class Discussion

Call on students to share selected strategies. Prompt students to ask for more information as needed during partner and whole-class discussion.

Guide students to **Compare and Connect** the representations. To emphasize a key mathematical idea, call on two or three others to rephrase so that students hear the idea in more than one way.

> **ASK** *How do [student name]'s and [student name]'s models show which expressions are equivalent?*
>
> **LISTEN FOR** It shows that $3(x + 2) + 2x$ and $2 + 4(x + 1) + x$ have the same number of x's and 1s, and that $2(3 + 3x) - 2x$ has a different number of x's than the others.

Model It/Analyze It

If students presented these models, have students connect these models to those presented in class.

If no student presented at least one of these models, have students first analyze key features of the models, and then connect them to the models presented in class.

> **ASK** *How are algebra tiles used to represent the multiplication of the values in parentheses by a number?*
>
> **LISTEN FOR** The tiles that represent the expression in the parentheses are repeated the same number of times as the coefficients of the expressions.

For the algebra tiles, prompt students to identify how each part of the algebra tile model represents each part of the expression.

- *Which parts of the models represent the expressions in parentheses?*

For the expressions, prompt students to explain the steps used to rewrite the expressions.

- *How does each expression change from one step to another? Why is each step necessary?*

➤ **Explore different ways to identify equivalent expressions.**

Which of these three expressions are equivalent?

$3(x + 2) + 2x$ $\quad$ $2 + 4(x + 1) + x$ $\quad$ $2(3 + 3x) - 2x$

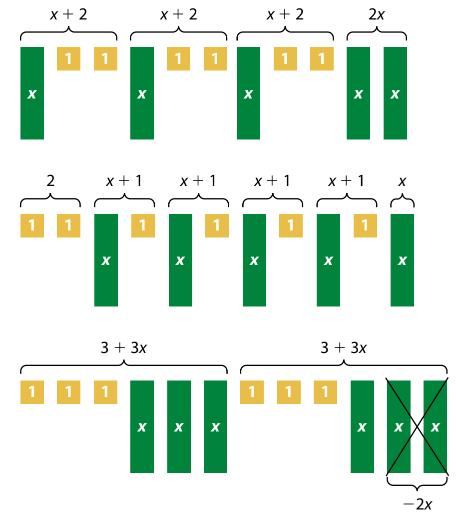
Model It

You can use algebra tiles to model each expression.

$3(x + 2) + 2x$

$2 + 4(x + 1) + x$

$2(3 + 3x) - 2x$



Analyze It

You can use properties of operations to write each expression without parentheses and with the fewest number of terms possible.

$3(x + 2) + 2x$	$2 + 4(x + 1) + x$	$2(3 + 3x) - 2x$
$3 \cdot x + 3 \cdot 2 + 2x$	$2 + 4 \cdot x + 4 \cdot 1 + x$	$2 \cdot 3 + 2 \cdot 3x - 2x$
$3x + 6 + 2x$	$2 + 4x + 4 + x$	$6 + 6x - 2x$
$(3x + 2x) + 6$	$(2 + 4) + (4x + x)$	$6 + (6x - 2x)$
$5x + 6$	$6 + 5x$	$6 + 4x$

DIFFERENTIATION | EXTEND $\quad$ **SMP 5**

Deepen Understanding
Using Models Strategically

Prompt students to compare the advantages and disadvantages of using algebra tiles and using properties of operations to determine whether expressions are equivalent.

> **ASK** *What are some advantages of using algebra tiles to identify equivalent expressions?*
>
> **LISTEN FOR** Algebra tiles can be moved around to combine and count like terms in order to create equivalent expressions.

> **ASK** *What are some advantages of using properties of operations to identify equivalent expressions?*
>
> **LISTEN FOR** You would not want to use algebra tiles if the coefficients and/or constants were numbers like 200 or 2.7. A property of operations, such as the distributive property $a(b + c) = ab + ac$, is true for all values of its variables. This means you can use the property with an expression that includes decimals or fractions and know that you have written an equivalent expression.

Develop Identifying Equivalent Expressions

CONNECT IT

SMP 2, 4, 5, 6

Remind students that the quantities and the relationships between them are the same in each representation. Explain that they will now use those relationships to reason about identifying equivalent expressions.

Before students begin to record and expand on their work in Model It, tell them that problem 3 and problem 4 will prepare them to provide the explanation asked for in problem 5.

Monitor and Confirm Understanding 1 – 2

- When modeling with algebra tiles, use the coefficients of the expressions to determine the number of x-tiles and 1-tiles to use.
- Move and combine groups of algebra tiles to show the commutative, associative, and distributive property.

Facilitate Whole Class Discussion

3 Look for understanding that the distributive property may be used more than once when writing an equivalent expression.

4 Look for understanding that you can use a model or an algebraic expression to show that expressions are equivalent.

> **ASK** *How do you know whether two expressions are equivalent?*
>
> **LISTEN FOR** After combining like terms, the expressions have the same value of the x-term and the same value of the term without a variable.

5 Look for the idea that the commutative, associative, and distributive properties can be used to rewrite expressions.

> **ASK** *What properties are useful when rewriting expressions?*
>
> **LISTEN FOR** The commutative, associative, and distributive properties allow you to write equivalent expressions and then rearrange terms in order to combine them. Once like terms have been combined, two expressions can be identified as equivalent or not equivalent.

6 **Reflect** Have all students focus on the strategies used to solve the Try It. If time allows, have students discuss their ideas with a partner.

CONNECT IT

➤ **Use the problem from the previous page to help you understand how to identify equivalent expressions.**

1 Look at **Model It**. How do you use tiles to model $3(x + 2) + 2x$?

To show $3(x + 2)$, show 3 groups of one x-tile and two 1-tiles. To show $2x$, show two more x-tiles.

2 How could rearranging the algebra tiles help you determine which expressions are equivalent?

For each expression, you could group all the x-tiles and all the 1-tiles to make the expressions easier to compare.

3 Look at the first group of expressions in **Analyze It**. List the properties of operations that are used to rewrite the expression $3(x + 2) + 2x$.

Possible answer: The distributive property is used to rewrite $3(x + 2)$ as a sum. Then, the commutative and associative properties are used to group like terms. Next, the distributive property is used to combine like terms.

4 Which of the three expressions are equivalent? Explain how you know.

Possible answer: The expressions $3(x + 2) + 2x$ and $2 + 4(x + 1) + x$ are equivalent because both can be rewritten in the form $5x + 6$. The expression $2(3 + 3x) - 2x$ cannot be written in this form.

5 How are properties of operations useful for identifying equivalent expressions?

Properties of operations let you rewrite expressions in different forms to make them easier to compare. If you can write the expressions in exactly the same form, then they are equivalent.

6 **Reflect** Think about all the models and strategies you have discussed today. Describe how one of them helped you better understand how to solve the **Try It** problem.

Responses will vary. Check student responses.

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DIFFERENTIATION | RETEACH or REINFORCE

Hands-On Activity

Use algebra tiles to determine whether expressions are equivalent.

If students are unsure about identifying equivalent expressions, then use this activity to reinforce modeling expressions with algebra tiles.

Materials For each group: algebra tiles (10 x-tiles, 10 1-tiles)

- Have students model $2(x + 3) + 4x + 1$ using their algebra tiles. Ask: *What property did you use to find the value of $2(x + 3)$ as a sum of two terms? Explain.* [The distributive property; Because $(x + 3)$ is being multiplied by 2, you multiply each addend by 2.]
- Have students combine the tiles. Ask: *What expression is represented?* [$6x + 7$]
- Next, have students model $3(2x + 1) + x + 4$ using their algebra tiles.
- Have students combine the x-tiles and then combine the 1-tiles. Ask: *What expression is represented?* [$6x + 7$]
- Have students compare the original expressions. Ask: *What do you know about the expressions $2(x + 3) + 4x + 1$ and $3(2x + 1) + x + 4$?* [They are equivalent because the combined tiles for each expression result in $6x + 7$.]
- Challenge students to write at least one more expression using the distributive property that is equivalent to $6x + 7$. [Possible answer: $5(x + 1) + x + 2$]

Apply It

For all problems, encourage students to use a model to support their thinking. Allow some leeway in precision; drawing models that are the exact same size can be difficult, and models that are the exact same size are not necessary to solve the problem.

7 Students may use the commutative property to write the expressions as $7 + 6w$.

8 **B and C are correct.** Students may use the distributive property and then combine like terms to show that $2a + 6(a - 1)$ and $4a + 2(2a - 3)$ are equivalent to $8a - 6$.

A is not correct. This answer is equivalent to $2a + 6$. Students may incorrectly think that the commutative property can be applied to subtraction.

D is not correct. This answer is equivalent to $6a - 4$. Students may have applied the order of operations incorrectly.

E is not correct. This answer is equivalent to $12a - 6$. Students may have applied the order of operations incorrectly.

Apply It

➤ **Use what you learned to solve these problems.**

Size	Group 1	Group 2
small	1	2
medium	3	1
large	2	3

7 Two groups of campers carry their water in reusable packs that come in three sizes. The table shows how many packs each group carries. A medium water pack holds 1 liter more than a small pack holds. A large pack holds 2 liters more than a small pack. Do the two groups carry the same amount of water? If not, which group carries more? Use w to represent the number of liters of water a small pack can hold. Show your work. Possible work:

First group:	Second group:
$w + 3(w + 1) + 2(w + 2)$	$2w + w + 1 + 3(w + 2)$
$w + 3w + 3 + 2w + 4$	$2w + w + 1 + 3w + 6$
$w + 3w + 2w + 3 + 4$	$2w + w + 3w + 1 + 6$
$6w + 7$	$6w + 7$

SOLUTION Yes, they carry the same amount.

8 Which expressions are equivalent to $8a - 6$? Select all that apply.

A $5a + 6 - 3a$

B $2a + 6(a - 1)$

C $4a + 2(2a - 3)$

D $2 + 3a + 3(a - 2)$

E $11a - a + 2(a - 3)$

9 Are the expressions $3(x + y) + 2y + 10$ and $x + 5y + 2(x + 5)$ equivalent? Show your work. Possible work:

$3(x + y) + 2y + 10$	$x + 5y + 2(x + 5)$
$3 \cdot x + 3 \cdot y + 2y + 10$	$x + 5y + 2 \cdot x + 2 \cdot 5$
$3x + 3y + 2y + 10$	$x + 5y + 2x + 10$
$3x + (3y + 2y) + 10$	$(x + 2x) + 5y + 10$
$3x + 5y + 10$	$3x + 5y + 10$

SOLUTION Yes; the expressions are equivalent.

CLOSE EXIT TICKET

9 Students' solutions should show an understanding of:

- using properties of operations to rewrite expressions.
- identifying expressions as equivalent.

Error Alert If students only multiply the factor outside of the parentheses by the first number within the parentheses, then remind students that the factor on the outside must be multiplied by each term within the parentheses. Provide an example using a variable, such as $3(x + 7)$, and substitute in a value for x to show students that the expression $3x + 7$ is not equivalent to $3x + 21$.

Practice Identifying Equivalent Expressions

Problem Notes

Assign **Practice Identifying Equivalent Expressions** as extra practice in class or as homework.

1 Students may use the commutative property before using the distributive property to show that the expressions are equivalent. ***Medium***

2 **a.** This expression is not equivalent to $48a - 36b$. Students may have added 30 to each term in parentheses instead of multiplying.

b. This expression is equivalent to $48a - 36b$.

c. This expression is equivalent to $48a - 36b$.

d. This expression is equivalent to $48a - 36b$.

Medium

Practice Identifying Equivalent Expressions

➤ **Study the Example showing how to determine whether expressions are equivalent. Then solve problems 1–5.**

Example

Are the expressions $4(x + 1) - 1$ and $2(x + 1) + 2x$ equivalent?

You can use properties of operations to rewrite the expressions.

$4(x + 1) - 1$	$2(x + 1) + 2x$
$4 \cdot x + 4 \cdot 1 - 1$	$2 \cdot x + 2 \cdot 1 + 2x$
$4x + 4 - 1$	$2x + 2 + 2x$
$4x + (4 - 1)$	$(2x + 2x) + 2$
$4x + 3$	$4x + 2$

No matter what the value of x is, $4x + 3$ will always be 1 more than $4x + 2$. The expressions $4x + 3$ and $4x + 2$ never name the same value.

The expressions $4(x + 1) - 1$ and $2x + 2(x + 1)$ are not equivalent.

1 Explain how the distributive property and the commutative property of addition are used in the Example to show that $2(x + 1) + 2x$ is equivalent to $4x + 2$.

Possible explanation: The distributive property is used to rewrite $2(x + 1) + 2x$ as $2 \cdot x + 2 \cdot 1 + 2x$. The commutative property of addition is used to reorder the terms of $2x + 2 + 2x$ as $(2x + 2x) + 2$.

2 Is each expression equivalent to the expression $48a - 36b$? Select *Yes* or *No* for each expression.

	Yes	No
a. $30(18a - 6b)$	○	●
b. $12a + 36(a - b)$	●	○
c. $12(3a + a - 3b)$	●	○
d. $4(10a + 2a - 9b)$	●	○

Vocabulary

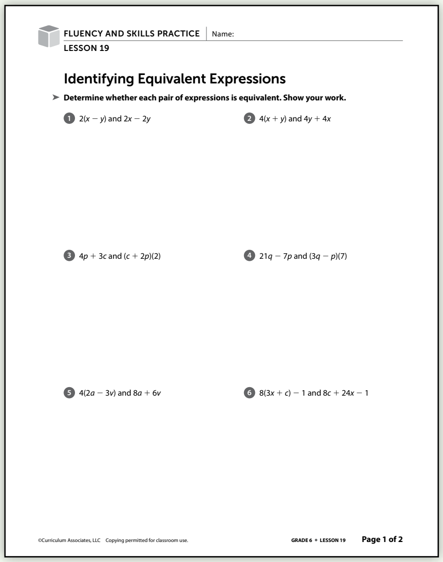
equivalent expressions
two or more expressions in different forms that always name the same value.

term
a number, a variable, or a product of numbers, variables, and/or expressions.

Fluency & Skills Practice

Identifying Equivalent Expressions

In this activity, students determine whether each given pair of expressions is equivalent.


FLUENCY AND SKILLS PRACTICE | Name:
LESSON 19

Identifying Equivalent Expressions

➤ **Determine whether each pair of expressions is equivalent. Show your work.**

1 $2(x - y)$ and $2x - 2y$

2 $4(x + y)$ and $4y + 4x$

3 $4p + 3c$ and $(c + 2p)(2)$

4 $21q - 7p$ and $(3q - p)(7)$

5 $4(2a - 3v)$ and $8a + 6v$

6 $8(3x + c) - 1$ and $8c + 24x - 1$

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Additional Practice

3 Students should recognize that the cost of an adult ticket is $c + 4$ and the cost of a senior ticket is $c + 3$. Students may create a picture to model this situation. ***Medium***

4 Students should recognize that for two expressions to be equivalent, they have the same variables with the same coefficients. Students may substitute a value in for the variable and determine whether the expressions have the same numerical value. ***Medium***

5 Students may use the commutative, associative, and distributive properties to write equivalent expressions with the least number of terms possible for each given expression. ***Basic***


Corn maze

3 An adult ticket to a corn maze costs \$4 more than a child ticket. A senior ticket costs \$3 more than a child ticket. Amelia's family has 3 children and 2 adults. Manuel's family has 2 children, 1 adult, and 2 seniors. Do the two families pay the same amount for tickets to the maze? If not, who pays more? Use c to represent the cost of a child ticket. Show your work.

Possible work:

Amelia's family:	Manuel's family:
$3c + 2(c + 4)$	$2c + (c + 4) + 2(c + 3)$
$3c + 2c + 8$	$2c + c + 4 + 2c + 6$
$(3c + 2c) + 8$	$(2c + c + 2c) + (4 + 6)$
$5c + 8$	$5c + 10$

The expression $5c + 10$ is 2 more than $5c + 8$.

SOLUTION No; Manuel's family pays more.

4 You can use the commutative property to reorder the terms of an expression. James says that you can use the commutative property to rewrite $5m + 10$ as $10m + 5$. Is James correct? Explain.

No; Possible explanation: The commutative property lets you change the order of terms. The terms of $5m + 10$ are $5m$ and 10. You can use the commutative property to rewrite $5m + 10$ as $10 + 5m$, but not $10m + 5$.

5 Which of these three expressions are equivalent? Show your work.

$7(2 + 3x) - 3x$ $\quad$ $2(6x + 7) + 10x$ $\quad$ $4(3 + 3x) + 2(1 + 3x)$

Possible work:

$7(2 + 3x) - 3x$	$2(6x + 7) + 10x$	$4(3 + 3x) + 2(1 + 3x)$
$7 \cdot 2 + 7 \cdot 3x - 3x$	$2 \cdot 6x + 2 \cdot 7 + 10x$	$4 \cdot 3 + 4 \cdot 3x + 2 \cdot 1 + 2 \cdot 3x$
$14 + 21x - 3x$	$12x + 14 + 10x$	$12 + 12x + 2 + 6x$
$14 + (21x - 3x)$	$14 + (12x + 10x)$	$(12 + 2) + (12x + 6x)$
$14 + 18x$	$14 + 22x$	$14 + 18x$

SOLUTION Only $7(2 + 3x) - 3x$ and $4(3 + 3x) + 2(1 + 3x)$ are equivalent.

DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Use with **Session 5** **Apply It**

Levels 1–3: Reading/Speaking

Support students as they make sense of Apply It problem 5 and justify their answers in partner discussion. Read the problem chorally. Ask students to underline *distributive property* and the phrase *product of a number and a sum*. Display the term *distributive property* and its definition. Have partners identify the part of the definition that shows an expression with a *product of a number and a sum*. [$a(b + c)$]

Allow students to work together to find values for the variables in the expression $72 = a(b + c)$. Encourage them to use the phrase *product of a number and a sum* as they justify their answers.

Levels 2–4: Reading/Speaking

Help students make sense of Apply It problem 5 and justify their answers in partner discussion. Read the problem with students. Ask them to circle the term *distributive property*. Have students work in pairs to define the term and discuss how the definition applies to the problem.

After students solve the problem individually, have them meet with a different partner to share expressions. Encourage students to refer to the definition of *distributive property* as they justify their answers using the sentence stem:

- *My expression shows the distributive property because* ____.

Levels 3–5: Reading/Speaking

Support students as they make sense of Apply It problem 5 and justify their answers in small-group discussion. Have students read and discuss the problem with a partner before solving individually. Encourage them to underline important words and phrases that will help them solve the problem.

Combine two sets of partners to form a small group. Have students take turns sharing and justifying their expressions. Review that one way to justify a solution is to use definitions, properties, and what you already know to support your thinking.

Refine Writing and Identifying Equivalent Expressions

Purpose

- **Refine** strategies for writing and identifying equivalent expressions.
- **Refine** understanding of applying properties of operations to expressions.

START CHECK FOR UNDERSTANDING

Rewrite $3(2m + 4n) + 2m$ as an expression with two terms.

Solution

$8m + 12n$

WHY? Confirm students' understanding of writing and identifying equivalent expressions, identifying common errors to address as needed.

MONITOR & GUIDE

Before students begin to work, use their responses to the **Start** to determine those who will benefit from additional support. Use the **Error Analysis** table below to guide remediation.

Have all students complete the Example and problems 1–3, using Consider This and Pair/Share as appropriate. Observe and monitor their reasoning and guide or redirect students as needed.

LESSON 19 | SESSION 5

Refine Writing and Identifying Equivalent Expressions

➤ **Complete the Example below. Then solve problems 1–10.**

Example

Are the expressions $(3x)(x) + 6$ and $x + x + 6 + x$ equivalent?

Look at how you could rewrite the expressions to compare them.

$(3x)(x) + 6$	$x + x + 6 + x$
$3(x \cdot x) + 6$	$(x + x + x) + 6$
$3x^2 + 6$	$3x + 6$

SOLUTION The expressions are not equivalent.

CONSIDER THIS . . . Are the terms $3x^2$ and $3x$ like terms? Why or why not?

PAIR/SHARE Explain how you know whether the expressions are equivalent.

Apply It

1. The fine for an overdue library book is \$0.75 for the first day and \$0.50 for each additional day. Rewrite the expression $0.75 + 0.5(d - 1)$ for the fine as a sum of two terms where d represents the number of days overdue. Show your work.

Possible work:

$0.75 + 0.5(d - 1)$

$0.75 + 0.5d - 0.5$

$0.5d + 0.75 - 0.5$

$0.5d + 0.25$

SOLUTION $0.5d + 0.25$

CONSIDER THIS . . . How could you use the distributive property to rewrite the expression?

PAIR/SHARE What would the fine be for a book that is 4 days overdue?

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START ERROR ANALYSIS

If the error is . . .	Students may . . .	To support understanding . . .
$8m + 4n$	have only distributed the 3 to the first term in parentheses.	Have students use algebra tiles or draw a picture to model the distributive property to show that 3 groups of $2m + 4n$ means that 3 should be multiplied by both $2m$ and $4n$.
$18n + 2m$	have added $2m$ and $4n$ and written the sum as $6n$.	Have students use algebra tiles to represent $2m$ and $4n$. Use 1-tiles for m and x-tiles for n. Have them explain why the 1-tiles and x-tiles combined and therefore m and n do not represent like terms.
$7m + 7n$	have added 3 to $2m$ and $4n$.	Have students draw an arrow with a multiplication sign from 3 to each term in parentheses to reinforce the idea of multiplication. Then ask students to rewrite the expression showing the two multiplication expressions involved.

Example

Guide students in understanding the Example. Ask:

- *What properties of operations can you use to show that the two expressions are equivalent?*
- *How do you know whether two expressions are equivalent?*
- *What are like terms?*

Help all students focus on the Example and responses to the questions by asking them to explain the reasons why their answers make sense.

Look for understanding that expressions can be rewritten to show whether they are equivalent.

Apply It

1 Students may also use the commutative property to write this expression as $0.25 + 0.5d$. *DOK 1*

2 Students may use the distributive property to write $(2.4r + 1.8r)$ as $(2.4 + 1.8)r$ and $(0.8c + 1.3c)$ as $(0.8 + 1.3)c$. ***DOK 2***

3 **D is correct.** $6(x + 2y) - y$ is equivalent to $3(2x + 4y) - y$ because both expressions can be rewritten as $6x + 2y$.

A is not correct. This answer is found by adding all the coefficients and combining the variables instead of combining like terms.

B is not correct. This answer is found by thinking that subtracting y removes y from the term $12y$.

C is not correct. This answer is found by adding the coefficients and combining the variables in the parentheses.

DOK 3

2 Ana and Katrina are buying fruit to bring to their book club meeting. Ana buys 2.4 lb of oranges and 0.8 lb of cherries. Katrina buys 1.8 lb of oranges and 1.3 lb of cherries. Oranges cost r dollars per pound, and cherries cost c dollars per pound. Write an expression with exactly two terms for the total cost of the fruit Ana and Katrina buy. Show your work.

Possible work:

$2.4r + 0.8c + 1.8r + 1.3c$

$(2.4r + 1.8r) + (0.8c + 1.3c)$

$4.2r + 2.1c$

SOLUTION $4.2r + 2.1c$

CONSIDER THIS . . . The cost of Ana's oranges is equal to the weight of her oranges times the cost per pound.

PAIR/SHARE What does each term in your expression represent?

3 Which expression is equivalent to $3(2x + 4y) - y$?

A $17xy$

B $6x + 12$

C $3(6xy) - y$

D $6(x + 2y) - y$

Kimani chose C as the correct answer. How might she have gotten that answer?

Possible answer: She tried to combine the terms $2x$ and $4y$ inside the parentheses to get $6xy$. These terms are not like terms because they have different variables, so they cannot be combined.

CONSIDER THIS . . . How can you use properties of operations to rewrite $3(2x + 4y) - y$ in a different way?

PAIR/SHARE How could you use $x = 1$ and $y = 2$ to check your answer?

GROUP & DIFFERENTIATE

Identify groupings for differentiation based on the **Start** and problems 1–3. A recommended sequence of activities for each group is suggested below. Use the resources on the next page to differentiate and close the lesson.

Approaching Proficiency
- **RETEACH** Hands-On Activity
- **REINFORCE** Problems 7, 9

Meeting Proficiency
- **REINFORCE** Problems 4–9

Extending Beyond Proficiency
- **REINFORCE** Problems 4–9
- **EXTEND** Challenge

Have all students complete the **Close: Exit Ticket**.

Resources for Differentiation are found on the next page.

Refine Writing and Identifying Equivalent Expressions

Apply It

4 **D and E are correct.** The area shown in the diagram can be represented by the expression $15.5(20 + x)$. Using the distributive property, this expression can be written as $15.5x + 310$.

A is not correct. This answer has a different coefficient for the x term.

B is not correct. This answer uses multiplication for the expression in parentheses rather than addition.

C is not correct. This answer uses multiplication for the expression of length rather than addition and then adds the side lengths rather than multiplying.

F is not correct. This answer adds the numbers in the diagram and then adds the variable.

DOK 2

5 There are many other possible answers, such as $9(5 + 3)$ and $3(17 + 7)$. ***DOK 2***

6 **a.** $f + f + f = 3f$

b. $(x + x) + y \cdot y \cdot y = 2x + y^3$

c. $2.5(2n - 4) = 5n - 10$

DOK 1

7 Students may use factor trees to determine the GCF of 84 and 72. ***DOK 2***

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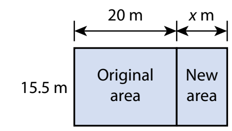
4 A park meadow is planted with wildflowers. The Parks Department plans to extend the length of the rectangular meadow by x meters. Which expressions represent the total area, in square meters, after the meadow's length is increased? Select all that apply.



A $310 + x$

B $15.5(20x)$

C $20x + 15.5$

D $15.5x + 310$

E $15.5(20 + x)$

F $35.5 + x$

5 Use the distributive property to write two different expressions equal to 72. Each expression should be the product of a number and a sum. Show your work.

Possible work:

$72 = 70 + 2$
$= 2 \cdot 35 + 2 \cdot 1$
$= 2(35 + 1)$

$72 = 30 + 42$
$= 6 \cdot 5 + 6 \cdot 7$
$= 6(5 + 7)$

SOLUTION Possible answers: $2(35 + 1)$ and $6(5 + 7)$

6 Look at each pair of expressions. Select *Equivalent* or *Not Equivalent* for each pair.

	Equivalent	Not Equivalent
a. $f + f + f$ and $3f$	●	○
b. $x^2 + 3y$ and $(x + x) + y \cdot y \cdot y$	○	●
c. $2.5(2n - 4)$ and $5n - 4$	○	●

7 Show how to use the greatest common factor of 84 and 72 to rewrite $84m - 72n$ as a product of two factors. Label the step that shows the distributive property.

Possible work:

84: 1, 2, 3, 4, 6, 7, 12, 14, 21, 48, 84

72: 1, 2, 3, 6, 8, 9, 12, 24, 36, 72

$84m - 72n$

$12 \cdot 7m - 12 \cdot 3n$

$12(7m - 6n)$ ⟵ distributive property

DIFFERENTIATION

RETEACH

Hands-On Activity

Make a concrete model to visualize equivalent expressions.

Students approaching proficiency with writing equivalent expressions will benefit from using algebra tiles to create equivalent expressions.

Materials For each student: algebra tiles (10 x-tiles, 10 1-tiles)

- Distribute algebra tiles to students.
- Have them model $3(x + 2) + 5x + 1$ using the algebra tiles.
- To discuss the model, ask: *How is $3(x + 2)$ represented in your model?* [There are 3 groups of one x-tile and two 1-tiles.]
- Ask: *How can you demonstrate using the distributive property with your tiles?* [Rearrange the 3 groups of one x-tile and two 1-tiles to be a group of three x-tiles and a group of six 1-tiles.]
- Have students demonstrate combining like terms with the algebra tiles. Ask: *What is an expression with two terms that is equivalent to $3(x + 2) + 5x + 1$?* [$8x + 7$]
- Have each student remove three 1-tiles from their model. Ask: *What expression do the algebra tiles now represent?* [$8x + 4$]
- Discuss with students how to rewrite the original expression based on removing three 1-tiles. Ask: *How can you rewrite the expression $3(x + 2) + 5x + 1$ to show that it is equivalent to $8x + 4$?* [Possible answer: Change the 2 in the parentheses to 1, so there are three groups of 1 instead of three groups of 2. The expression would be $3(x + 1) + 5x + 1$.]

Refine

8 See ***Connect to Culture*** to support student engagement. **B and E are correct.**

A is not correct. This answer is the volume of the swimming pool only.

C is not correct. This answer does not take into account that the swimming pool is 15 times the volume of the wading pool.

D is not correct. This answer is found by adding 15 to twice the volume of the wading pool.

DOK 2

9 Students may first use the distributive property to eliminate the parentheses, then use the commutative property to rearrange the order of the terms, and finally combine like terms. *DOK 1*

CLOSE EXIT TICKET

10 **Math Journal** Look for understanding that expressions are equivalent only if they are equal for all values of x.

Error Alert If students think the expressions are equivalent, then have them create a table of values for $x = 0$, 1, and 2. Have them note that they are not equivalent when $x = 1$ or 2.

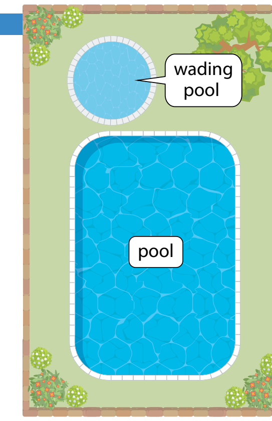
End of Lesson Checklist

INTERACTIVE GLOSSARY Support students by having them draw squares for a and rectangles for b.

SELF CHECK Have students review and check off any new skills on the Unit 5 Opener.

LESSON 19 | SESSION 5

8 A wading pool holds g gallons of water. A swimming pool holds 15 times as much water as the wading pool. Which expressions represent the total number of gallons of water in both pools? Select all that apply.



A $15g$

B $16g$

C $15 + g$

D $2g + 15$

E $g + 15g$

9 Are the expressions $\frac{1}{2}(4c + 8) + 3c - 2$ and $4c + 2(0.5c + 1)$ equivalent? Show your work.

Possible work:

$\frac{1}{2}(4c + 8) + 3c - 2$	$4c + 2(0.5c + 1)$
$\frac{1}{2} \cdot 4c + \frac{1}{2} \cdot 8 + 3c - 2$	$4c + 2 \cdot 0.5c + 2 \cdot 1$
$2c + 4 + 3c - 2$	$4c + 1c + 2$
$(2c + 3c) + (4 - 2)$	$(4c + 1c) + 2$
$5c + 2$	$5c + 2$

SOLUTION Yes, the expressions are equivalent.

10 **Math Journal** Brian says that the expressions $2(x + 2)$ and $3(x + 1) + 1$ are equivalent because they name the same value when $x = 0$. Is Brian's reasoning correct? Explain.

No; Possible explanation: When $x = 1$, the expression $2(x + 2)$ has a value of 6, and the expression $3(x + 1) + 1$ has a value of 7. The expressions do not always name the same value, so they are not equivalent.

End of Lesson Checklist

☐ **INTERACTIVE GLOSSARY** Find the entry for *like terms*. Explain why the terms $5a$ and $5b$ in the expression $5a + 5b + 8$ are not like terms. Label your explanation as a non-example.

☐ **SELF CHECK** Go back to the Unit 5 Opener and see what you can check off.

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REINFORCE

Problems 4–9

Identify equivalent expressions.

Students meeting proficiency will benefit from additional work with writing and identifying equivalent expressions by solving problems in a variety of formats.

- Have students work on their own or with a partner to solve the problems.
- Encourage students to show their work.

EXTEND

Challenge

Write equivalent expressions with 3 variables.

Students extending beyond proficiency will benefit from writing equivalent expressions with 3 variables.

- Display the expression $4(x + 2y - z) + 2x - y + 5z$.
- Explain that the distributive property can be applied to expressions with more than two terms within parentheses.
- Have students determine whether this expression is equivalent to $6x + 7y + 5z$. [No; The expression is equivalent to $6x + 7y + z$. If students answer yes, they may have forgotten to multiply 4 by the z within the parentheses and then subtract that term from $5z$.]

PERSONALIZE

i-Ready

Provide students with opportunities to work on their personalized instruction path with *i-Ready* Online Instruction to:

- fill prerequisite gaps.
- build up grade-level skills.

Overview | Write and Solve One-Variable Equations

STANDARDS FOR MATHEMATICAL PRACTICE (SMP)

SMP 1, 2, 3, 4, 5, and 6 are integrated into the Try-Discuss-Connect routine.*

This lesson provides additional support for:

2 Reason abstractly and quantitatively.

7 Look for and make use of structure.

* See page 1q to learn how every lesson includes these SMP.

Objectives

Content Objectives

- Write one-step equations to model real-world and mathematical problems.
- Solve one-step equations by using algebraic reasoning.
- Interpret the solution to an algebraic equation in context.

Language Objectives

- Interpret word problems by identifying important quantities and determining the relationship between quantities.
- Explain solution strategies for one-step equations using algebraic reasoning and the lesson vocabulary.
- Demonstrate understanding of the solution to an equation by explaining how a variable represents a problem situation in class discussion.
- Connect to problems by explaining how they are similar to and/or different from other problems they have solved.

Prior Knowledge

- Write algebraic expressions to model real-world situations.
- Understand what it means to solve an equation.
- Determine whether a given number is a solution of an equation.
- Represent multi-step word problems using equations with variables representing unknown quantities.

Vocabulary

Math Vocabulary

There is no new vocabulary. Review the following key terms.

equation a mathematical statement that uses an equal sign (=) to show that two expressions have the same value.

inverse operations operations that undo each other. For example, addition and subtraction are inverse operations, and multiplication and division are inverse operations.

reciprocal for any nonzero number a, the reciprocal is $\frac{1}{a}$. The reciprocal of any fraction $\frac{a}{b}$ is $\frac{b}{a}$. Zero does not have a reciprocal. The reciprocal of a number is also called the multiplicative inverse of that number.

solution of an equation a value that can be substituted for a variable to make an equation true.

Academic Vocabulary

verify to prove, show, find out, or state that something is true or correct.

Learning Progression

In earlier grades, students used letters, question marks, or other symbols to represent unknown quantities when writing expressions to model real-world situations.

Earlier in Grade 6, students understood what it means for a value to be a solution of an equation. They have experience writing equations with an unknown factor or unknown addend and solving by using reasoning about inverse operations.

In this lesson, students will understand that an equation is balanced and that it remains balanced as long as the same operation is performed on both sides of the equation. They will write a one-step equation to represent a problem and state what the variable represents. Students will know how to rewrite one side of an equation as necessary before solving algebraically.

Later in Grade 6, students will use variables to represent two quantities that change in relation to one another. They will write equations that express one quantity (the dependent variable) in terms of the other quantity (the independent variable). Students will also write and graph one-variable inequalities.

In Grade 7, students will write and solve two-step equations that represent real-world situations.

Pacing Guide

Items marked with are available on the **Teacher Toolbox**.

Session	Materials	Differentiation
SESSION 1 **Explore** One-Variable Equations (35–50 min)		
• **Start** (5 min) • **Try It** (5–10 min) • **Discuss It** (10–15 min) • **Connect It** (10–15 min) • **Close: Exit Ticket** (5 min) **Additional Practice** (pages 479–480)	**Math Toolkit** algebra tiles, counters, grid paper Presentation Slides	**PREPARE** Interactive Tutorial **RETEACH or REINFORCE** Hands-On Activity **Materials** For each pair: algebra tiles (3 *x*-tiles, 10 1-tiles)
SESSION 2 **Develop** Solving One-Variable Addition Equations (45–60 min)		
• **Start** (5 min) • **Try It** (10–15 min) • **Discuss It** (10–15 min) • **Connect It** (15–20 min) • **Close: Exit Ticket** (5 min) **Additional Practice** (pages 485–486)	**Math Toolkit** algebra tiles, counters, grid paper Presentation Slides	**RETEACH or REINFORCE** Hands-On Activity **Materials** For each pair: algebra tiles (1 *x*-tile, 20 1-tiles), 1 index card **REINFORCE** Fluency & Skills Practice **EXTEND** Deepen Understanding
SESSION 3 **Develop** Solving One-Variable Multiplication Equations (45–60 min)		
• **Start** (5 min) • **Try It** (10–15 min) • **Discuss It** (10–15 min) • **Connect It** (15–20 min) • **Close: Exit Ticket** (5 min) **Additional Practice** (pages 491–492)	**Math Toolkit** algebra tiles, counters, grid paper Presentation Slides	**RETEACH or REINFORCE** Hands-On Activity **Materials** For each pair: 15 two-color counters, 4 sticky notes **REINFORCE** Fluency & Skills Practice **EXTEND** Deepen Understanding
SESSION 4 **Develop** Writing and Solving One-Variable Equations (45–60 min)		
• **Start** (5 min) • **Try It** (10–15 min) • **Discuss It** (10–15 min) • **Connect It** (15–20 min) • **Close: Exit Ticket** (5 min) **Additional Practice** (pages 497–498)	**Math Toolkit** algebra tiles, grid paper, number line Presentation Slides	**RETEACH or REINFORCE** Hands-On Activity **Materials** For each pair: 2 index cards, algebra tiles (10 *x*-tiles, 25 1-tiles) **REINFORCE** Fluency & Skills Practice **EXTEND** Deepen Understanding
SESSION 5 **Refine** Writing and Solving One-Variable Equations (45–60 min)		
• **Start** (5 min) • **Monitor & Guide** (15–20 min) • **Group & Differentiate** (20–30 min) • **Close: Exit Ticket** (5 min)	**Math Toolkit** Have items from previous sessions available for students. Presentation Slides	**RETEACH** Hands-On Activity **Materials** For each pair: 4 index cards, 30 two-color counters **REINFORCE** Problems 4-8 **EXTEND** Challenge **PERSONALIZE** i-Ready
Lesson 21 Quiz or **Digital Comprehension Check**		**RETEACH** Tools for Instruction **REINFORCE** Math Center Activity **EXTEND** Enrichment Activity

Overview | Write and Solve One-Variable Equations

Connect to Culture

➤ **Use these activities to connect with and leverage the diverse backgrounds and experiences of all students. Engage students in sharing what they know about contexts before you add the information given here.**

SESSION 2 ■ ■ □ □ □

Try It Pixie frogs are large African bullfrogs that can live over 20 years. They have strong hind legs for digging, sharp teeth, and fragile skin. They can grow to be up to 10 inches long, but they do not move around very much. They can be kept in glass fish tanks, or terrariums, with heating pads underneath to keep them warm. Pixie frogs like to burrow under the surface of their habitat, which may include coconut fiber or moss. They also like to soak in shallow dishes of water. They eat big bugs, fish, and mice. Ask students to share other facts they know about frogs.

SESSION 3 ■ ■ ■ □ □

Try It Before erasers were invented, people erased with wet, balled-up pieces of bread. In 1770, Joseph Priestly, who also discovered oxygen, discovered that rubber could be used to erase pencil marks. Erasers work because the polymers they are made of are stickier than the paper, so the pencil-marking particles get stuck in the eraser as it is rubbed across the paper. Most erasers today are made of vinyl or plastic because these materials are more durable and flexible than rubber. Some erasers also contain volcanic ash. In Japan, kawaii (cute) erasers gained popularity in the 1990s. Today, these colorful puzzle-style erasers are designed to look like anything from animals to food. Ask students about the types of materials they like to write or draw with.

SESSION 4 ■ ■ ■ ■ □

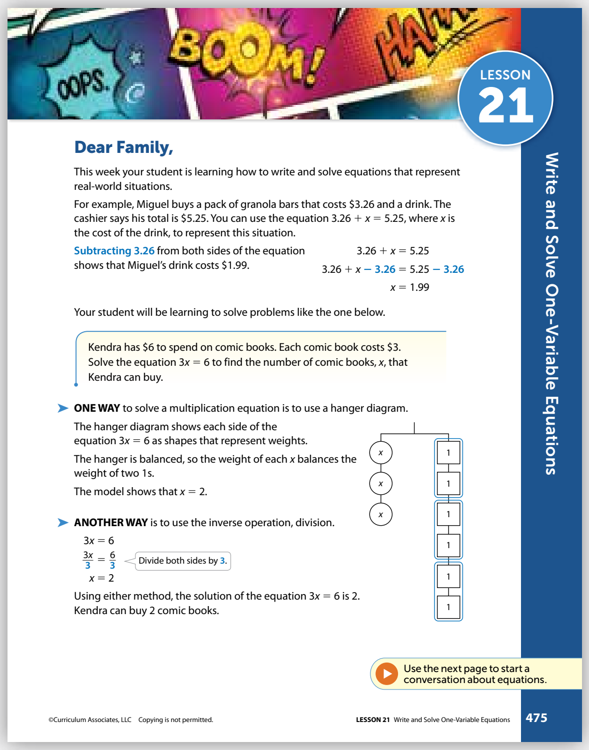
Try It Ask students for a show of hands if they like hip hop dancing. Hip hop has its roots in African rhythms and dance movements. Hip hop dancing is a full-body workout that promotes cardiovascular health, strengthens muscles, and helps relieve stress. Dance classes are a great way to meet new people and gain confidence. Dancers learn basic dance moves but also add their own moves that reflect their creativity and personal style. Ask students who take dance classes, such as hip hop or ballet, to share some of their dance moves with the class.

SESSION 5 ■ ■ ■ ■ ■

Apply It **Problem 3** Ask students if they have ever donated to a food drive. Food drives are a great way to collect packaged and canned foods for people in need. Canned foods are popular items for food drives because the cans stay on shelves longer and are easier to store than fresh or frozen products. Canned fruits and vegetables provide essential nutrition for a healthy diet. You can help your community by setting up a canned food drive in your school. If time allows, have students research how they could organize a food drive in their school.

Connect to Family and Community

➤ **After the Explore session, have students use the Family Letter to let their families know what they are learning and to encourage family involvement.**



LESSON 21

Write and Solve One-Variable Equations

Dear Family,

This week your student is learning how to write and solve equations that represent real-world situations.

For example, Miguel buys a pack of granola bars that costs \$3.26 and a drink. The cashier says his total is \$5.25. You can use the equation $3.26 + x = 5.25$, where x is the cost of the drink, to represent this situation.

Subtracting 3.26 from both sides of the equation shows that Miguel's drink costs \$1.99.

$$3.26 + x = 5.25$$
$$3.26 + x - 3.26 = 5.25 - 3.26$$
$$x = 1.99$$

Your student will be learning to solve problems like the one below.

> Kendra has \$6 to spend on comic books. Each comic book costs \$3. Solve the equation $3x = 6$ to find the number of comic books, x, that Kendra can buy.

➤ **ONE WAY** to solve a multiplication equation is to use a hanger diagram.

The hanger diagram shows each side of the equation $3x = 6$ as shapes that represent weights.

The hanger is balanced, so the weight of each x balances the weight of two 1s.

The model shows that $x = 2$.

➤ **ANOTHER WAY** is to use the inverse operation, division.

$$3x = 6$$
$$\frac{3x}{3} = \frac{6}{3} \quad \text{Divide both sides by 3.}$$
$$x = 2$$

Using either method, the solution of the equation $3x = 6$ is 2. Kendra can buy 2 comic books.

Use the next page to start a conversation about equations.

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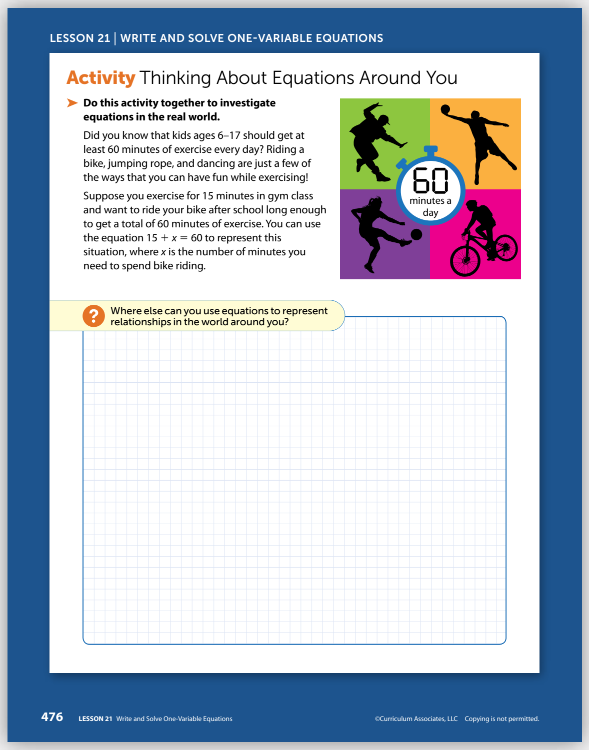

LESSON 21 | WRITE AND SOLVE ONE-VARIABLE EQUATIONS

Activity Thinking About Equations Around You

➤ **Do this activity together to investigate equations in the real world.**

Did you know that kids ages 6–17 should get at least 60 minutes of exercise every day? Riding a bike, jumping rope, and dancing are just a few of the ways that you can have fun while exercising!

Suppose you exercise for 15 minutes in gym class and want to ride your bike after school long enough to get a total of 60 minutes of exercise. You can use the equation $15 + x = 60$ to represent this situation, where x is the number of minutes you need to spend bike riding.

60 minutes a day

Where else can you use equations to represent relationships in the world around you?

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Connect to Language

➤ **For English language learners, use the Differentiation chart to scaffold the language in each session. Use the Academic Vocabulary routine for academic terms before Session 1.**

DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS Use with **Session 1** **Connect It**

Levels 1–3: Speaking/Writing

Prepare students to write responses to Connect It problem 3. Display a word bank with the terms *balanced, add, remove, the same amount, different amount,* and *both sides.* Ask: *How do we keep a hanger balanced when we add amounts? What do we need to do to both sides?*

- *We need to add ____ amount to both sides.*

Record responses for reference. Then, repeat the questions to ask about removing amounts. Ask students to explain their reasoning orally before writing. Allow students to use **Act It Out** with a clothes hanger and paper clips to form and explain their ideas. Reword student responses, as needed, using terms from the word bank.

Levels 2–4: Speaking/Writing

Help students prepare written responses for Connect It problem 3. Create a **Co-Constructed Word Bank** to identify words and phrases from the problem that students can use in their written responses. Ensure students include *balance, add, remove, same, different, amount,* and *both sides.* Allow partners to use a clothes hanger with paper clips as they talk about the meanings of the terms and discuss their responses to the problem.

Have students use **Stronger and Clearer Each Time** to draft, discuss, and revise responses. Encourage students to ask clarifying questions during partner discussions.

Levels 3–5: Speaking/Writing

Support students as they prepare written responses for Connect It problem 3. First, have partners read and discuss the problem. Remind students that they will need to give reasons in their written response. Help students list words and phrases they can use to give reasons, such as *because, so, one reason is,* and *this means that.* Have partners create a **Co-Constructed Word Bank** with terms they can use in written response, including math terms and words that show reasons.

Next, have students use **Stronger and Clearer Each Time** to draft, discuss, and revise responses. Encourage students to ask clarifying questions during partner discussions.

Explore One-Variable Equations

Purpose

- **Explore** the idea that equations can be balanced.
- **Understand** that an equation remains balanced if you perform the same operation on both sides of the equal sign.

START CONNECT TO PRIOR KNOWLEDGE

Same and Different

A $2 + 9$	B $6 - 4$
C $17 - 6$	D $1 + 1$

Possible Solutions

All can be evaluated with one operation.

A and D are addition expressions.

B and C are subtraction expressions.

A and C have a value of 11.

B and D have a value of 2.

WHY? Support students' facility with identifying operations and equivalent expressions.

TRY IT

SMP 1, 2, 4, 5, 6

Make Sense of the Problem

Before students work on Try It, use **Notice and Wonder** to help them make sense of the problem. After students explain what they notice and wonder about the problem, point out the hanger diagram. Review what students know about this type of diagram.

DISCUSS IT

SMP 2, 3, 6

Support Partner Discussion

After students work on Try It, have them explain their work and then respond to Discuss It with a partner. Listen for understanding that:

- the hanger diagram is balanced, so the value of the left side is equal to the value of the right side.
- the value of the right side is 7, and the value of the left side is $x + 3$.

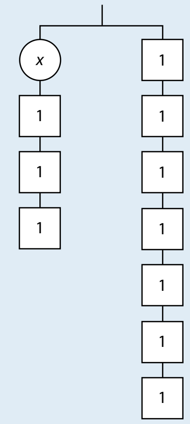
LESSON 21 | SESSION 1

Explore One-Variable Equations

Previously, you learned about solutions of equations. In this lesson, you will learn how to solve addition and multiplication equations in one variable.

➤ **Use what you know to try to solve the problem below.**

What is the value of x in the hanger diagram? How do you know?

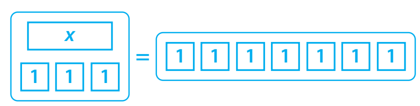


TRY IT

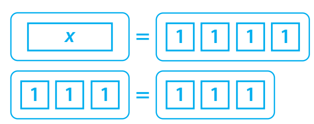
Math Toolkit algebra tiles, counters, grid paper

Possible work:

SAMPLE A



The weight for x has to be the same as the weight for four 1s.



The value of x is 4.

SAMPLE B

The left side of the hanger shows $x + 3$.

The right side of the hanger shows 7.

Since the hanger is balanced, $x + 3 = 7$.

Subtract to find the unknown addend: $x = 7 - 3$.

So, the value of x is 4.

DISCUSS IT

Ask: How can you verify that you found the correct value of x?

Share: I know my answer makes sense because . . .

Learning Target SMP 1, SMP 2, SMP 3, SMP 4, SMP 5, SMP 6

Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers.

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Common Misconception Listen for students who add all of the values in the hanger diagram together. They may say the hanger diagram represents the equation $x = 10$ instead of $x + 3 = 7$. As students share their strategies, have them use substitution to see if the hanger diagram remains balanced.

Select and Sequence Student Strategies

Select 2–3 samples that represent the range of student thinking in your classroom. Here is one possible order for class discussion:

- drawings that show the problem using algebra tiles
- (misconception) equations that show the sum of the known values equal to the variable without considering the placement of the values in the diagram
- an equation that represents each side of the hanger diagram separated by an equal sign

Facilitate Whole Class Discussion

Call on students to share selected strategies. Ask students to provide reasons to explain how they know the solutions are correct.

Guide students to **Compare and Connect** the representations. Ask students to take individual think time and then turn and talk to answer the question, *How does this representation show the value of x?*

ASK *How do [student name]'s and [student name]'s representations show the information from the hanger diagram?*

LISTEN FOR Both show that the value of the left side is $x + 3$ and the value of the right side is 7. They also show that the value of x is 4 because that is the only value that would keep the diagram balanced.

CONNECT IT

SMP 2, 4, 5

1 Look Back Look for understanding that the hanger diagram is a tool used to show that the two sides of an equation have the same value. The hanger is balanced, meaning that the value of each side is the same. To solve, find the value of x that makes the left side equal to the right side.

DIFFERENTIATION | RETEACH or REINFORCE

Hands-On Activity

Use algebra tiles to explore the meaning of *equals*.

If students are unsure about what the equal sign represents, then use this activity to show the meaning of an equal sign.

Materials For each pair: algebra tiles (3 x-tiles, 10 1-tiles)

- Have one student in each pair use all the 1-tiles to show a balanced hanger diagram (by placing the tiles in two columns).
- Have the other student in each pair remove 3 of the tiles from one column and replace them with an x-tile.
- Ask: *What is the value of x? How do you know?* [3; If you substitute 3 for x, the hanger is balanced.] Have students write an equation for their model using the variable x. [$x + 2 = 5$]
- Ask: *How can you say the equation using the term equals? What does equals mean?* [x plus 2 equals 5; Equals means that both sides of the equation are balanced.]

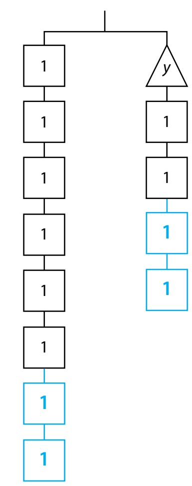
LESSON 21 | SESSION 1

CONNECT IT

1 Look Back What is the value of x in the hanger diagram? Explain how you could find the value of x.

4; Possible explanation: Write the equation $x + 3 = 7$ to show that the weights on the two sides are the same. Then x must be 4 in order for $x + 3$ to be 7.

2 Look Ahead Use the hanger diagram.



a. What expression does the left side of the hanger represent? What expression does the right side of the hanger represent?

$1 + 1 + 1 + 1 + 1 + 1$ (or 6); $y + 1 + 1$ (or $y + 2$)

b. The hanger is balanced. What does that tell you about the two expressions you wrote in problem 2a? What equation does the hanger diagram represent?

The expressions have the same value; $6 = y + 2$

c. Draw two more 1s on each side of the hanger. Why is the hanger still balanced? What equation does the hanger represent now?

See diagram. The hanger is still balanced because the same weight was added to each side; $8 = y + 4$.

d. Suppose you remove three 1s from the right side of the hanger. What should you do to keep the hanger balanced?

Remove three 1s from the left side.

3 Reflect To keep a hanger diagram balanced, what must you do when adding and removing amounts on the sides of the hanger? Why?

You must add or remove the same amount on both sides of the hanger. If you add or remove different amounts, the sides will no longer represent the same weight. The hanger will become unbalanced.

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2 Look Ahead Point out that the hanger remains balanced when the same amount is added or taken away from each side. Ask: *What would happen if 17 were added to both sides? If 1 was moved from one side to the other?* [The hanger diagram would remain balanced if 17 were added to both sides because the same operation is applied to each side. It would be unbalanced if 1 was added to one side and taken away from the other.]

CLOSE **EXIT TICKET**

3 Reflect Look for understanding of how a hanger diagram is kept in balance. The same amount must be added to or removed from each side.

Common Misconception If students indicate that they must remove the amount they add to one side from the other side, then use a different example to explain why this is false. Have students use counters or draw a hanger diagram with 5 ones on each side and write the equation $5 = 5$. Add 2 to both sides and determine if the equation is true or false: $7 = 7$, so the equation is true. Have students make another model to show $5 = 5$. Then, subtract 2 from one side and add 2 to the other side. Determine if the equation is true or false: $7 = 3$, so the equation is false.

Prepare for Writing and Solving One-Variable Equations

Support Vocabulary Development

Assign **Prepare for Writing and Solving One-Variable Equations** as extra practice in class or as homework.

If you have students complete this in class, then use the guidance below.

Ask students to consider the term *solution of an equation*. Be aware of the fact that some students may have difficulty understanding the new uses of the term *solution*. Prior to the last lesson, students used that term to describe a final answer to a problem. When talking about a solution of an equation, students need to understand that a solution is a value that makes an equation true.

Have students work in pairs to complete the graphic organizer. Invite pairs to share their completed organizers and prompt a whole-class comparative discussion of the definition, prior knowledge, examples, and non-examples.

Have students look at the equations in problem 2 and discuss with a partner how they will decide which equations to circle. Students may realize that they need to check every equation to determine whether 6 is a solution because the number of correct answers is not specified.

Problem Notes

1. Students should understand that a solution of an equation is a value of a variable that makes the equation true. Student responses may include examples and non-examples of solutions of equations with a variable. Students should recognize that a value is a solution of an equation if both sides of the equation are true after the value is substituted for the variable.

2. Students should understand that the equations they will circle will be true when 6 is substituted for the variable used in the equation. Students should understand that 6 can be substituted into any of the equations because it is not explicitly defined as being equal to only one of the variables used in the equations.

 Name:

Prepare for Writing and Solving One-Variable Equations

1. Think about what you know about solutions of equations. Fill in each box. Use words, numbers, and pictures. Show as many ideas as you can.
Possible answers:

solution of an equation

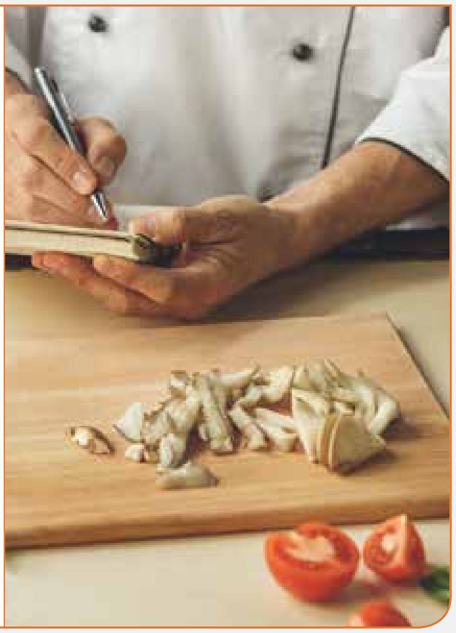
What Is It?	What I Know About It
A solution of an equation is a value of a variable in an equation that makes the equation true.	To check if a value is a solution of an equation, substitute the value for the variable and see if both sides of the equation have the same value.
Examples	**Non-Examples**
5 is a solution of $6 + x = 11$ because $6 + 5 = 11$ is a true statement. 2 is a solution of $7y = 14$ because $7(2) = 14$ is a true statement.	1 is not a solution of $6 + x = 11$ because $6 + 1 = 11$ is a not true statement. 8 is not a solution of $7y = 14$ because $7(8) = 14$ is not a true statement.

2. Circle the equations for which 6 is a solution of the equation.

$4 = x + 2$	$y - 2 = 4$	$6z = 36$	$12 \div w = 6$
$x + 2 = 6$	$56 = 5y$	$8 + z = 14$	$2 = \frac{w}{12}$

REAL-WORLD CONNECTION

A chef at a restaurant may be tasked with creating daily or weekly specials. The chef may even be responsible for ordering the ingredients for any special meals he or she prepares. It is important that the chef buys an adequate amount of each ingredient. The chef should be aware of which days or times of day that most customers come to the restaurant and what percent of those customers are likely to order the special. The chef can use variables when planning and purchasing ingredients. If a chef knows how much of each ingredient is needed for one person, he or she can multiply each ingredient by *n* people to determine how much of each ingredient to purchase based on the number of guests expected to order the special. Ask students to think of other real-world examples when using variables would be useful.



Additional Practice

3 Problem 3 provides another look at the idea of a balanced equation. This problem is similar to the problem about solving for *x* using a hanger diagram. In both problems, a hanger diagram is used to indicate that both sides of the hanger are equal or balanced. This problem asks for students to find the value of *z* given a hanger diagram.

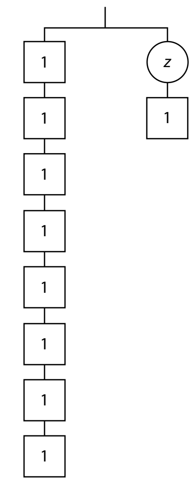
Students may want to use algebra tiles, counters, equations, or grid paper to solve.

Suggest that students use **Notice and Wonder,** asking themselves the following questions:

- What do you notice about the problem?
- What are you wondering that mathematics can answer?

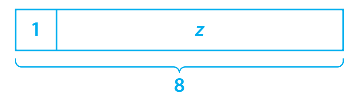
LESSON 21 | SESSION 1

3 Look at the hanger diagram.



a. What is the value of z in the hanger diagram? Show your work.

Possible work:



The bar model shows that $z = 8 - 1$.

The value of z is 7.

SOLUTION 7

b. Check your answer to problem 3a. Show your work.

Possible work:

The left side of the hanger diagram is 8.

The right side of the hanger diagram is $z + 1$.

Since the diagram is balanced, $8 = z + 1$.

Subtract to find the unknown addend: $z = 8 - 1$.

So, $z = 7$.

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DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Use with **Session 2** Apply It

Levels 1–3: Reading/Writing

Prepare students to write about solving the equation in Apply It problem 6. Read the problem aloud. Ask students to underline *downloads, deletes,* and *ends up with;* help them discuss the meaning of each term and identify any related math language and symbols. Then, ask students to highlight the equation. Help them identify the parts of the equation and explain their solutions using the following sentence frames:

- *____ represents the photos that Teresa ____.*
- *First, ____ from both sides.*
- *Then, ____ from both sides.*
- *Teresa has ____ photos on Monday.*

Levels 2–4: Reading/Writing

Prepare students to write about solving an equation in Apply It problem 6. Have students read the problem and discuss the meanings of *downloads* and *deletes*. Have students model with counters to show how the number of photos changes. Ask students to use sequence words, like *first, next,* and *then,* and math terms like *add* and *subtract,* as they describe the actions.

Display questions to encourage mathematical thinking*: What does the variable p represent? How can you solve the equation? What is the first step?* Support writing with sentence frames:

- *____ represents ____.*
- *First, I ____. Then, I ____. The result is ____.*

Levels 3–5: Reading/Writing

Support students as they write about solving an equation in Apply It problem 6. Have students read the problem individually. Encourage students to draw models or use counters to represent the problem. Then have students craft responses using **Stronger and Clearer Each Time**. Allow think time for students to draft written responses. During partner discussion, encourage them to explain the math operation related to the words *downloads* and *deletes*. Adapt the routine by having partners read each other's drafts and make suggestions about how to rephrase the response with precise language. Ask students to think about the feedback as they revise their writing.

Develop Solving One-Variable Addition Equations

Purpose

- **Develop** strategies for solving a one-step addition equation with a variable.
- **Recognize** that you can solve a one-step addition equation with a variable by performing subtraction on both sides of the equation to undo the addition performed on the variable.

START CONNECT TO PRIOR KNOWLEDGE

Which One Doesn't Belong?

A: $13 - 9 = 4$	B: $9 + 4 = 10 + 3$
C: $6 + 5 = 11$	D: $13 = 4 + 9$

Possible Solutions

A is the only subtraction equation.

B is the only equation with an operation on both sides of the equal sign.

C does not include the value 9.

D is the only equation with no operation on the left side of the equation.

WHY? Support students' recognizing similarities and relationships among equations.

DEVELOP ACADEMIC LANGUAGE

WHY? Support understanding of phrases that define variables in a sentence.

HOW? Present conventions for defining variables, such as the phrase from Try It: *. . . how many more terrariums, t, the rescue center needs.* Explain that another way to define a variable is to use the phrase: *Let t be . . .* Have students practice using the phrases to define the variable in Apply It problem 6.

TRY IT

SMP 1, 2, 4, 5, 6

Make Sense of the Problem

See ***Connect to Culture*** to support student engagement. Before students work on Try It, use **Three Reads** to help them make sense of the problem. After the first read, have students use the ***Connect to Culture*** information to determine the meanings of pixie frog and terrarium. Once students understand the context of the problem, continue with the **Three Reads** routine.

LESSON 21 | SESSION 2

Develop Solving One-Variable Addition Equations

➤ **Read and try to solve the problem below.**

An animal rescue group in Africa is taking care of 5 injured baby pixie frogs. They plan to release them back into the wild when they are healthy. Each pixie frog needs its own terrarium. One of the rescue volunteers checks and sees that there are only 3 terrariums. Solve the equation $t + 3 = 5$ to find how many more terrariums, t, the rescue center needs.

TRY IT **Math Toolkit** algebra tiles, counters, grid paper

Possible work:

SAMPLE A

t	3

$\underbrace{\qquad\qquad}_{5}$

$t = 5 - 3$

$= 2$

The rescue center needs 2 more terrariums.

SAMPLE B

$\underbrace{\boxed{t}\ \boxed{1}\ \boxed{1}\ \boxed{1}}_{t+3} = \underbrace{\boxed{1}\ \boxed{1}\ \boxed{1}\ \boxed{1}\ \boxed{1}}_{5}$

$t = 2$

The rescue center needs 2 more terrariums.

DISCUSS IT

Ask: What did you do first to solve the problem?

Share: The first thing I did was . . .

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DISCUSS IT

SMP 2, 3, 6

Support Partner Discussion

After students work on Try It, have them explain their work and then respond to Discuss It with a partner. To support students in extending the conversation, prompt them to discuss these questions:

- *What would it be like if the quantities in this problem were greater? For example, what if there are 25 pixie frogs and the rescue center has 18 terrariums? How would the steps you take to solve this problem compare with the steps you took to solve the previous problem?*
- *Where does your solution show the number of terrariums the rescue center needs to buy?*

Common Misconception If students say that the rescue center needs to buy 8 terrariums, then discuss how the values 3 and 5 are used in the problem. As students share their strategies, encourage students to check their solutions to make sure that they make the equation true and that their solution makes sense for the context of the problem. Have students who arrived at the solution 8 substitute 8 for t in the equation $t + 3 = 5$. Guide them to see that while $3 + 5 = 8$, this is not the relationship between 3 and 5 in this problem.

Select and Sequence Student Strategies

Select 2–3 samples that represent the range of student thinking in your classroom. Here is one possible order for class discussion:

- a bar diagram that shows the relationship between the quantities and the variable in the equation
- (misconception) drawings or equations that add 3 and 5 to arrive at a solution of 8
- a drawing with an equation that uses algebra tiles to show the relationships in the problem

Facilitate Whole Class Discussion

Call on students to share selected strategies. Prompt students to explain what they noticed or assumed about the problem, what they decided to do as a result, and why.

Guide students to **Compare and Connect** the representations. Allow time for students to think by themselves before starting the discussion.

> **ASK** *Where in [student name]'s and [student name]'s models is the number of pixie frogs shown? The number of terrariums the rescue center has?*
>
> **LISTEN FOR** Representations should show a total of 5 pixie frogs with 3 terrariums. They may also show the number of additional terrariums the rescue center needs to buy, which is 2.

Model It & Analyze It

If students presented these models, have students connect these models to those presented in class.

If no student presented at least one of these models, have students first analyze key features of the models, and then connect them to the models presented in class.

> **ASK** *How is the number of terrariums the rescue center already has and the number of pixie frogs shown in each representation?*
>
> **LISTEN FOR** Each representation shows 3 terrariums along with an unknown number of additional terrariums and 5 pixie frogs.

For the Model It, prompt students to think about each part of the hanger diagram.

- *How are the number of terrariums the rescue center already has represented? Pixie frogs?*
- *What does the variable t represent?*
- *How is the value of t found?*

For the Analyze It, prompt students to consider how an equation is used to find the solution.

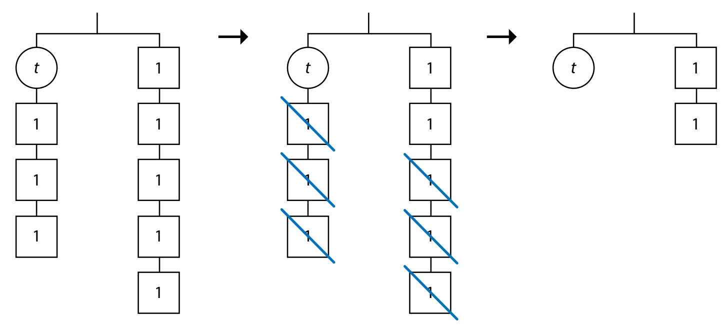
- *Why is 3 subtracted from both sides of the equation?*

➤ **Explore different ways to solve a one-variable addition equation.**

An animal rescue group in Africa is taking care of 5 injured baby pixie frogs. They plan to release them back into the wild when they are healthy. Each pixie frog needs its own terrarium. One of the rescue volunteers checks and sees that there are only 3 terrariums. Solve the equation $t + 3 = 5$ to find how many more terrariums, t, the rescue center needs.

Model It

You can use a hanger diagram to model and solve an addition equation.



Analyze It

You can use subtraction to solve for a variable in an addition equation.

$$t + 3 = 5$$

$$t + 3 - 3 = 5 - 3 \quad \longleftarrow \text{Subtract 3 from both sides of the equation.}$$

$$t = 2$$

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DIFFERENTIATION | EXTEND

SMP 2

Deepen Understanding
Using Models to Reason Abstractly and Quantitatively

Prompt students to think about why subtraction is used to solve addition equations involving variables by decontextualizing the situation. Relate the equation to the model.

> **ASK** *If you know an addend and the sum in an equation, what operation can you use to find the other addend? Why?*
>
> **LISTEN FOR** Subtraction can be used because subtracting involves taking away an addend from a sum to find the addend that remains.

> **ASK** *Could you use any addition strategies to solve the equation $t + 3 = 5$?*
>
> **LISTEN FOR** You could use basic facts and think, *What number do I add to 3 to have a sum of 5?*

> **ASK** *How can you check to make sure your answer is correct by using a hanger diagram?*
>
> **LISTEN FOR** You could replace t with 2 squares and check to see if the hanger diagram is balanced.

Develop Solving One-Variable Addition Equations

CONNECT IT

SMP 2, 4, 5, 6

Remind students that the quantities and the relationships between them are the same in each representation. Explain that they will now use those relationships to reason about how an addition equation can be solved using subtraction.

Before students begin to record and expand on their work in Model It & Analyze It, tell them that problem 2 will prepare them to provide the description asked for in problem 4. Ask students to take individual think time, and then turn and talk to answer the question, *How are all the equations in problem 4 alike?*

Monitor and Confirm Understanding 1 – 3

- Hanger diagrams can demonstrate how to remove the same number from each side in order to keep things balanced.
- Once a solution is determined, substitute it into the original equation to make sure that the equation is true.

Facilitate Whole Class Discussion

2 Look for understanding that both solution methods start with the equation. Then 3 is removed from each side of the hanger, as well as from each side of the equation.

ASK *What would happen to the hanger diagram or the equation if you did not take 3 away from both sides?*

LISTEN FOR The equation and the hanger would be unbalanced.

4 Look for the idea that all of the equations involve the addition of a number and a variable, with a single number on the other side of the equation, so they can all be solved using subtraction.

ASK *What would a hanger diagram for each equation look like? How would you solve using a hanger diagram?*

LISTEN FOR All of the hanger diagrams would have the sum on one side of the hanger, and the addends on the other side. To find the value of the variable, subtract the known addend from each side of the hanger. The quantity that remains is the value of the variable.

5 **Reflect** Have all students focus on the strategies used to solve the Try It. If time allows, have students discuss their ideas with a partner.

CONNECT IT

➤ **Use the problem from the previous page to help you understand how to solve a one-variable addition equation.**

1 Look at **Model It**. Why are three 1s removed from each side of the hanger diagram?

Removing three 1s from the left side of the hanger leaves the variable t by itself on the left side. Then you remove three 1s from the right side of the hanger to keep the hanger balanced.

2 Look at **Analyze It**. How are the steps for solving the equation similar to the steps for solving the equation with the hanger diagram?

Start with the given equation. Then, subtract 3 from both sides of the equation. This is like removing three 1s from each side of the hanger. The final equation and the final hanger diagram both show the value of t.

3 How many more terrariums does the rescue center need? How can you use substitution to verify that your answer is correct?

2 terrariums; Substitute 2 for t in the equation $t + 3 = 5$ and check that the equation is a true statement: $2 + 3 = 5$.

4 How are all the equations below similar to $t + 3 = 5$? Why could you solve each equation by subtracting a number from both sides of the equation?

$8 + y = 92$ $\quad$ $50 = a + 17$ $\quad$ $n + 2.2 = 5$ $\quad$ $12.5 = 7.4 + z$

Each equation has a number on one side and a sum of a variable and a number on the other. Subtracting the number that is added to the variable from both sides keeps the equation balanced, with the variable on one side and the solution on the other.

5 **Reflect** Think about all the models and strategies you have discussed today. Describe how one of them helped you better understand how to solve one-variable addition equations.

Responses will vary. Check student responses.

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DIFFERENTIATION | RETEACH or REINFORCE

Hands-On Activity

Make a model to show solving an addition equation.

If students are unsure about solving an addition equation with a variable, then use this activity to show the relationship among the terms.

Materials For each pair: algebra tiles (1 x-tile, 20 1-tiles), 1 index card

- Have pairs model $8 + x = 12$ using algebra tiles and an index card with an equal sign written on it. Have pairs work together so that each student models one side of the equation.
- Ask: *How can you find a solution for the equation by using subtraction? What is the value of x?* [You can take away the same number of square tiles from each side until one of the sides has only the variable; $x = 5$]
- Ask: *How can you use addition to check your answer?* [You can substitute 5 for x to see if it makes a true equation.]

Apply It

For all problems, encourage students to use a model to support their thinking. Allow some leeway in precision; for example, if using a hanger diagram, the amounts can be represented by shapes of different sizes to represent different values. Students may use whatever solution method makes the most sense to them. Remind students to be precise when defining a variable.

6 Students may wonder where the variable is mentioned in the word problem. Have students reread the problem and emphasize that the number of photos Teresa has on her computer on Monday is not given, and that is what the solution to the equation will reveal.

7 **B** **is correct.** Students may solve the problem by finding the sum of 7 and 5 and then subtracting 4 from each side to find the value of *y*.

A is not correct. This answer is the result of finding that the difference of 7 and 5 is 2 and assuming that the numbers on the right side of the equation should also have a difference of 2 because $6 - 4 = 2$.

C is not correct. This answer is the result of finding only the sum of the left side of the equation.

D is not correct. This answer is the result of finding the sum of the numbers in the problem without using the relationship between the numbers to find the value of *y*.

Apply It

➤ **Use what you learned to solve these problems.**

6 Teresa has some photos on her computer on Monday. On Tuesday, she downloads 24 new photos. Then she deletes 9 photos. She ends up with 33 photos. Explain how the equation $p + 24 - 9 = 33$ represents the situation. Then solve the equation for the variable *p* and interpret the solution.

Possible explanation: The variable *p* is the number of photos Teresa has on her computer on Monday. She downloads 24 photos, so add 24 to *p*. Then she deletes 9 photos, so subtract 9 from $p + 24$. The result is the number of photos she ends up with, 33.

$$p + 24 - 9 = 33$$
$$p + 15 = 33$$
$$p + 15 - 15 = 33 - 15$$
$$p = 18$$

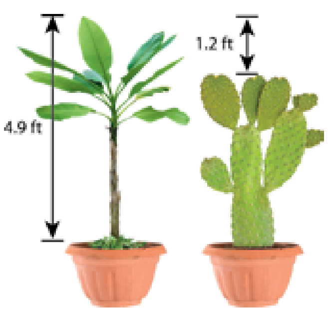
Teresa has 18 photos on Monday.

7 What is the solution of the equation $7 + 5 = 4 + y$?

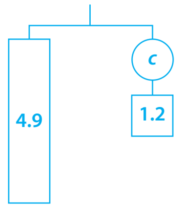
A 6 **B** 8

C 12 **D** 16

8 At a local gardening store, a palm tree is 1.2 ft taller than a cactus. The palm tree is 4.9 ft tall. Use the equation $4.9 = c + 1.2$ to find the height in feet, *c*, of the cactus. Show your work.



Possible work:



$$c = 4.9 - 1.2$$
$$c = 3.7$$

SOLUTION The cactus is 3.7 ft tall.

CLOSE **EXIT TICKET**

8 Students' solutions should show an understanding of:

- how models and/or equations can be used to show a problem situation
- how to find the solution to an addition equation with one variable

Error Alert If students' solution is 6.1 ft, then review how subtraction can be used to solve addition equations. Have students reread the problem and use the information in the picture. Guide students to see that adding 1.2 to the height of the palm tree would match a problem situation where the cactus is 1.2 ft taller than the palm tree, but it does not match the given problem situation.

Practice Solving One-Variable Addition Equations

Problem Notes

Assign **Practice Solving One-Variable Addition Equations** as extra practice in class or as homework.

1 **a.** Students may understand that the total cost of the notebooks must be subtracted from the amount Cameron spent to find the cost of the stapler. ***Basic***

b. Students may recognize that adding and then subtracting the same value will not change the value of an expression. ***Medium***

2 Students may also subtract 43 from both sides of the equation. ***Medium***

LESSON 21 | SESSION 2 Name:

Practice Solving One-Variable Addition Equations

➤ **Study the Example showing how to solve a one-variable addition equation. Then solve problems 1–5.**

Example

Cameron buys a stapler. Then he buys 5 notebooks that cost \$1.50 each. He spends \$13.50 in all. Use the equation $d + 5(1.50) = 13.50$ to find the number of dollars, d, that Cameron spends on the stapler.

$d + 5(1.50) = 13.50$

Multiply 1.50 by 5. $d + 7.50 = 13.50$

Subtract 7.50 from both sides. $d + 7.50 - 7.50 = 13.50 - 7.50$

$d = 6$

Cameron spends \$6 on the stapler.

1 **a.** In the Example, why is 7.50 subtracted from both sides of the equation?

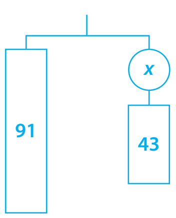
Subtracting 7.50 from both sides keeps the equation balanced and leaves the variable by itself on one side with the solution on the other side.

b. Why can you replace the expression $d + 7.50 - 7.50$ with just the variable d in the last step of solving the equation?

You know that $7.50 - 7.50 = 0$ and $d + 0 = d$.

2 Solve the equation $91 = 43 + x$. Show your work.

Possible work:



$x = 91 - 43$

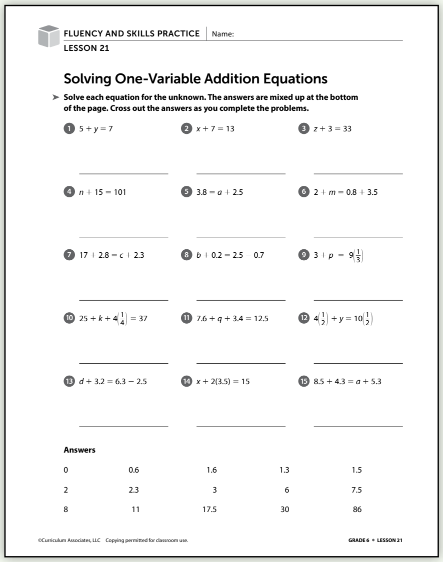
$x = 48$

SOLUTION 48

Fluency & Skills Practice

Solving One-Variable Addition Equations

In this activity, students practice solving one-variable addition equations and check their work by crossing out answers provided at the bottom of the page.



FLUENCY AND SKILLS PRACTICE | Name:
LESSON 21

Solving One-Variable Addition Equations

➤ **Solve each equation for the unknown. The answers are mixed up at the bottom of the page. Cross out the answers as you complete the problems.**

1 $5 + y = 7$
2 $x + 7 = 13$
3 $z + 3 = 33$
4 $n + 15 = 101$
5 $3.8 = a + 2.5$
6 $2 + m = 0.8 + 3.5$
7 $17 + 2.8 = c + 2.3$
8 $b + 0.2 = 2.5 - 0.7$
9 $3 + p = 9\left(\frac{1}{3}\right)$
10 $25 + k + 4\left(\frac{1}{4}\right) = 37$
11 $7.6 + q + 3.4 = 12.5$
12 $4\left(\frac{1}{2}\right) + y = 10\left(\frac{1}{2}\right)$
13 $d + 3.2 = 6.3 - 2.5$
14 $x + 2(3.5) = 15$
15 $8.5 + 4.3 = a + 5.3$

Answers

0	0.6	1.6	1.3	1.5
2	2.3	3	6	7.5
8	11	17.5	30	86

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Additional Practice

3 Students may solve the problem using a hanger diagram, even though the distance Jessica runs is shown as a fraction. ***Challenge***

4 Students may understand that this is an addition equation, so it can be solved the same way as any other addition equation. They may initially notice that the sum and one addend are both 100, so the variable must be equal to 0. ***Basic***

5 Students also may benefit from making a simple drawing to show why some values are added and some are subtracted. This may help them see why this is still an addition equation, which can be solved by subtraction. The amount of juice that Dylan pours out is less than the total amount of orangeade, so subtraction can be used to find the amount of sparkling water Dylan adds. ***Medium***

LESSON 21 | SESSION 2

3 On Monday, Jessica runs on a track at her school. Each day from Tuesday through Friday, she runs $1\frac{1}{2}$ mi in a park. The total distance she runs for the week is 8 mi. Solve the equation $m + 4\left(1\frac{1}{2}\right) = 8$ to find the number of miles, m, Jessica runs on Monday. Show your work.

Possible work:

$m + 4\left(1\frac{1}{2}\right) = 8$

$m + 6 = 8$

$m = 8 - 6$

$m = 2$

m	$1\frac{1}{2}$	$1\frac{1}{2}$	$1\frac{1}{2}$	$1\frac{1}{2}$

$\underbrace{\qquad\qquad\qquad\qquad}_{8}$

SOLUTION Jessica runs 2 mi on Monday.

4 What operation can you use on both sides of the equation $100 = 100 + y$ to solve the equation for y? Solve the equation for y. Then explain how to check the solution.

Possible explanation: Since 100 is added to the unknown, y, subtract 100 from both sides to get y by itself.

$100 = 100 + y$

$100 - 100 = 100 - 100 + y$

$0 = y$

You can check that $y = 0$ by substituting 0 for y in the equation $100 = 100 + y$ and showing that this is a true statement: $100 = 100 + 0$.

5 Dylan has a pitcher with 1.65 L of orange juice. He pours out 0.2 L of the juice. Then he adds some sparkling water to the pitcher to make orangeade. He ends up with 1.9 L of orangeade. Solve the equation $1.65 - 0.2 + x = 1.9$ to find the amount of sparkling water, x, Dylan adds to the pitcher. Show your work.

Possible work:

$1.65 - 0.2 + x = 1.9$

$1.45 + x = 1.9$

$1.45 - 1.45 + x = 1.9 - 1.45$

$x = 0.45$

SOLUTION Dylan adds 0.45 L of sparkling water.

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DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Use with **Session 3** **Connect It**

Levels 1–3: Speaking/Writing

To prepare students for Connect It problem 5, familiarize them with language they might use to communicate their ideas before they engage in discussion.

Read the problem aloud. Review comparison words, like *similar, different,* and *both*. Ask students to solve each equation side-by-side. Have them turn and talk with a partner about the steps they used. Ask volunteers to share the steps. Record their statements for reference. Provide sentence frames to support writing:

- *In both equations, you need to ____.*
- *To solve $2x = 8$, you ____ both sides by ____.*
- *To solve $2 + x = 8$, you ____ both sides.*

Levels 2–4: Speaking/Writing

To help students compare and contrast equations in Connect It problem 5, ask questions that students can consider and build on during partner discussion.

Read the problem with students. Point to each equation as you ask: *What do we do to isolate the variable?* Then have partners discuss how to compare the equations and answer the question. Adapt **Co-Constructed Word Bank** by having partners list terms they can use in their written responses. Remind them to include math terms and comparison words, such as *similar, different,* and *both*. Encourage students to refer to the word bank as they craft their written responses.

Levels 3–5: Speaking/Writing

To help students compare and contrast equations in Connect It problem 5, encourage them to think about math terms and comparison words they might use to talk about similarities and differences.

Have students read the problem individually and solve each equation. Then have students compare and contrast the equations with a partner. Ask partners to create a **Co-Constructed Word Bank** with terms to support written responses, such as *both sides, divide,* and *subtract.*

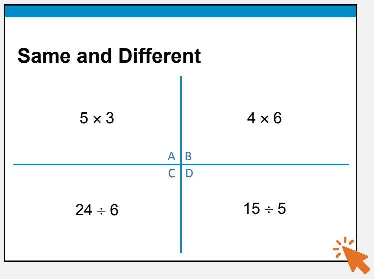
After discussion, allow students to draft responses independently. Then ask partners to compare and discuss responses by providing reasons they agree or disagree.

Develop Solving One-Variable Multiplication Equations

Purpose

- **Develop** strategies for solving a one-variable multiplication equation.
- **Recognize** that you can solve a one-variable multiplication equation by performing division on both sides of the equation to undo the multiplication performed on the variable.

START CONNECT TO PRIOR KNOWLEDGE



Possible Solutions

All are expressions shown with one operation.

A and B are multiplication expressions.

C and D are division expressions.

A and D are in the same fact family.

B and C are in the same fact family.

WHY? Support students' fluency with thinking about how multiplication and division are related.

DEVELOP ACADEMIC LANGUAGE

WHY? Understand questions with *similar to*.

HOW? Discuss with students when they have used the phrase *similar to*. Then have students find the phrase in Discuss It and in Connect It problems 3 and 5. Call on volunteers to convert the questions into statements by telling what they need to compare, for example: *I need to compare the steps to solve the equation to the steps to solve with a diagram.*

TRY IT

SMP 1, 2, 4, 5, 6

Make Sense of the Problem

See ***Connect to Culture*** to support student engagement. Before students work on Try It, use **Notice and Wonder** to help them make sense of the problem. If no one mentions the equation and variable, lead a discussion about how the equation matches the problem situation and make sure students understand that the variable is defined as the number of erasers in each package.

LESSON 21 | SESSION 3

Develop Solving One-Variable Multiplication Equations

➤ **Read and try to solve the problem below.**

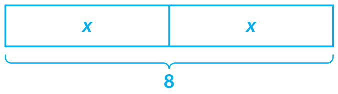
Tamera is buying party favors. She chooses pencil erasers that are shaped like pieces of sushi. Each package she buys contains the same number of erasers. She buys 2 packages and gets a total of 8 erasers. Solve the equation $2x = 8$ to find the number of erasers, x, in each package.

TRY IT **Math Toolkit** algebra tiles, counters, grid paper

Possible work:

SAMPLE A

$2x = 8$

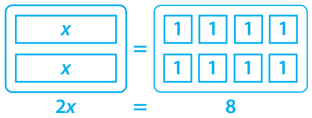


$x = 8 \div 2$

$= 4$

There are 4 erasers in each package.

SAMPLE B



$x = 4$

There are 4 erasers in each package.

DISCUSS IT

Ask: How is your strategy similar to mine? How is it different?

Share: My strategy is similar because . . . It is different because . . .

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DISCUSS IT

SMP 2, 3, 6

Support Partner Discussion

After students work on Try It, have them respond to Discuss It with a partner. Remind students that:

- a good explanation describes what you did and justifies why you decided to do it.
- good listeners rephrase a partner's ideas and add details or examples to refine these ideas.

Common Misconception Listen for students who misread the equation as *2 times 8*. As students share their strategies, review that when a number and a variable are placed side-by-side with no space or operator, the operation is multiplication. Read the equation *2 times x equals 8* aloud to reinforce the relationship between the numbers and the variable in the equation. Have students consider whether their answer makes sense in the context of the problem.

Select and Sequence Student Strategies

Select 2–3 samples that represent the range of student thinking in your classroom. Here is one possible order for class discussion:

- a bar diagram showing 2 equal sections, each labeled x and a total of 8
- (misconception) a drawing or equation that solves the problem 2×8, instead of $2x = 8$
- algebra tiles that represent $2x$ and 8, relating the value of both sides of the equation

Facilitate Whole Class Discussion

Call on students to share selected strategies. Prompt students to connect strategies by showing how each represents an important quantity in the problem.

Guide students to **Compare and Connect** the representations. Ask students to use *variable* and *solution* in their explanations. Allow think time before asking them to respond.

ASK *Where do [student name]'s and [student name]'s model show 2 packages? 8 erasers? The value of x?*

LISTEN FOR Representations should show 2 equal groups of x and a total of 8. The representation should also show that x has a value of 4.

Model It & Analyze It

If students presented these models, have students connect these models to those presented in class.

If no student presented at least one of these models, have students first analyze key features of the models, and then connect them to the models presented in class.

ASK *Why can you divide both sides of the equation by 2?*

LISTEN FOR You can divide both sides by any nonzero number as long as you do the same thing to each side of the equation.

For the Model It, prompt students to identify the differences in the hanger diagram as you move from one step to the next.

- *How does the diagram change from the first diagram to the second diagram?*
- *Why does the third diagram show only one x?*

For the Analyze It, prompt students to notice that division is used to undo multiplication.

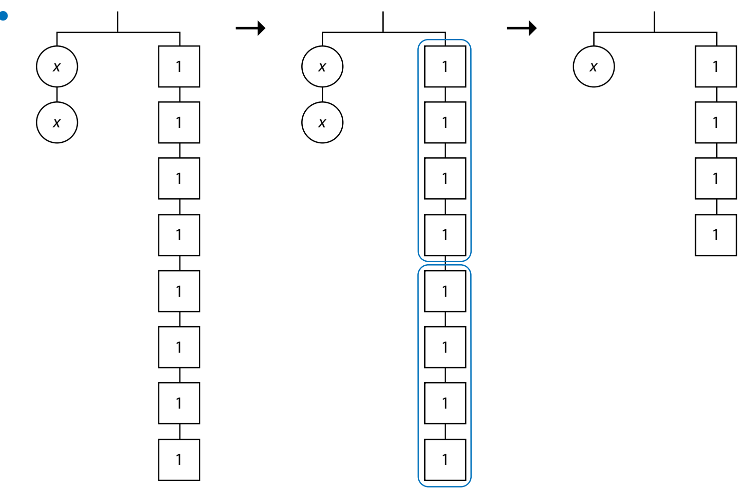
- *Why can you use division to find the value of x?*
- *Why is 2 the divisor?*
- *What is the value of $8 \div 2$?*

➤ Explore different ways to solve a one-variable multiplication equation.

Tamera is buying party favors. She chooses pencil erasers that are shaped like pieces of sushi. Each package she buys contains the same number of erasers. She buys 2 packages and gets a total of 8 erasers. Solve the equation $2x = 8$ to find find the number of erasers, x, in each package

Model It

You can use a hanger diagram to model and solve a multiplication equation.



Analyze It

You can use division to solve for a variable in a multiplication equation.

$2x = 8$

$\frac{2x}{2} = \frac{8}{2}$ ⟵ Divide both sides of the equation by 2.

$x = 4$

DIFFERENTIATION | EXTEND

SMP 7

Deepen Understanding

Make Use of Structure in a Model to Write an Equation

Prompt students to compare differences between a hanger diagram for an addition equation and one for a multiplication equation.

ASK *What addition equation could you write for the first hanger diagram? How is this related to the equation $2x = 8$?*

LISTEN FOR You could write the addition equation $x + x = 8$. It is related to the equation $2x = 8$ because $x + x = 2x$.

ASK *In what situations could a hanger diagram be used to represent addition or multiplication? When could it only represent addition?*

LISTEN FOR Multiplication is repeated addition. If the variable addend repeats, then you can use a multiplication equation. If not, then you have to use an addition equation.

ASK *How does each hanger diagram help you know what operation to use to solve?*

LISTEN FOR A multiplication diagram shows equal groups. Division is used to find the number of groups or the number in each group. An addition diagram shows the joining of addends. Subtraction is used to find an addend when the sum and the other addend or addends are known.

Develop Solving One-Variable Multiplication Equations

CONNECT IT

SMP 2, 4, 5, 6

Remind students that the quantities and the relationships between them are the same in each representation. Explain that they will now use those relationships to reason about why division can be used to solve an equation when a number and a variable are multiplied.

Before students begin to record and expand on their work in Model It & Analyze It, tell them that problem 3 will prepare them to provide the description asked for in problem 5. To confirm understanding, ask several students to rephrase, or paraphrase, the ideas.

Monitor and Confirm Understanding

1 – 2, 4

- Since there are two *x*s on the left side and you are finding the value of *x*, you can divide the 1s on the right into two equal groups.
- The fraction bar is another division symbol. It indicates that the numerator is divided by the denominator.
- Remember to check your work by substituting the solution into the given equation to see if it makes the equation true.

Facilitate Whole Class Discussion

3 Look for the understanding that the solution process is similar for each representation.

ASK *How would you represent dividing both sides of the equation by 2 with a hanger diagram?*

LISTEN FOR Dividing by 2 is the same as putting the objects in 2 equal groups, so you would split the sides into 2 equal groups.

5 Look for the idea that one-step equations are solved by performing the same operation on each side to get the variable by itself.

ASK *How do you solve any one-step addition or multiplication equation involving a variable?*

LISTEN FOR To solve, you need to get the variable by itself. In an addition equation, you can subtract the number added to the variable from both sides. In a multiplication equation, you can divide by the number multiplied by the variable on both sides.

6 **Reflect** Have all students focus on the strategies used to solve the Try It. If time allows, have students discuss their ideas with a partner.

CONNECT IT

➤ **Use the problem from the previous page to help you understand how to solve a one-variable multiplication equation.**

1 Look at the second hanger diagram in **Model It**. Why are the 1s separated into two equal groups?

There are two *x*'s on the left side and you want to know the value of *x*, so you separate the 1s on the right side into two equal groups.

2 Look at **Analyze It**. Why can you use the fractions $\frac{2x}{2}$ and $\frac{8}{2}$ to show dividing both sides of the equation $2x = 8$ by 2?

You can use a fraction to show division. Dividing by a number is the same as multiplying by its reciprocal. $\frac{1}{2} \cdot 2x = \frac{2x}{2}$ and $\frac{1}{2} \cdot 8 = \frac{8}{2}$.

3 How are the steps for solving the equation similar to the steps for solving the problem with the hanger diagram?

Start with the given equation. Then, divide both sides of the equation by 2. This is like separating the 1s into two equal groups on the hanger diagram. The final equation and the final hanger diagram both show the value of one *x*.

4 How many erasers are in each package? How you can use substitution to check your answer?

4 erasers; Substitute 4 for *x* in the equation $2x = 8$ and check that the equation is a true statement: $2 \cdot 4 = 8$.

5 How is solving $2x = 8$ similar to solving $2 + x = 8$? How is it different?

Solving each equation involves getting the variable by itself on one side by using the same operation on both sides of the equation. To solve $2x = 8$, you divide both sides by 2. To solve $2 + x = 8$, you subtract 2 from both sides.

6 **Reflect** Think about all the models and strategies you have discussed today. Describe how one of them helped you better understand solving one-variable multiplication equations.

Responses will vary. Check student responses.

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DIFFERENTIATION | RETEACH or REINFORCE

Hands-On Activity

Make a model to explore solving one-step equations.

If students are unsure about how to solve a one-step equation with a variable, then use this activity to give them concrete experience.

Materials For each pair: 15 two-color counters, 4 sticky notes

- Display the equations $3y = 9$ and $3 + y = 9$. Have one student in each pair model the addition equation and the other model the multiplication equation. Have students use the same color to model 9 and write an equal sign on a sticky note for their model.
- Ask: *How could you use sticky notes to model 3y?* [Write one *y* on 3 different sticky notes.] *How could you find the value of each y?* [Divide the 9 counters into 3 equal groups.] *What is the value of y?* [3] Ask: *How could you use sticky notes and counters to model 3 + y?* [Show 3 counters of the other color and one sticky note with *y*.] *How do you find the value of y?* [Subtract 3 from each side.] *What is the value of y?* [6]
- Have pairs compare models. Ask: *How are the models similar and different?* [In both equations, you do one step to find the value of *y*. In the addition equation, you subtract. In the multiplication equation, you divide.]

Apply It

For all problems, encourage students to use a model to support their thinking.

7 **a.** Students may recognize that a variable represents a quantity. The other quantities in the problem are stated (4 campers, \$27.68 on food, and \$42 for the campsite), so the only quantity left for the variable to represent is the cost per person.

b. Students may recognize that they should first add the amounts spent on food and for the campsite and then they can divide this by 4 campers.

8 **B is correct.** Students may solve the problem by finding $12\left(\frac{1}{2}\right) = 6$ and then dividing both sides of the equation by 6.

A is not correct. This answer is the result of dividing 2 by 12 without taking $\frac{1}{2}$ into consideration.

C is not correct. This answer is the result of dividing 6 by 2, instead of dividing 2 by 6.

D is not correct. This answer is the result of finding $12\left(\frac{1}{2}\right) = 6$ and then not taking the next step of dividing both sides by 6.

Apply It

➤ **Use what you learned to solve these problems.**

7 Four friends go on a camping trip. They decide to share the cost equally. They spend \$27.68 on food and \$42 to reserve the campsite. You can use the equation $4c = 42 + 27.68$ to find each person's share of the cost.

a. What does the variable c represent? Explain how the equation $4c = 42 + 27.68$ represents the situation.

c is each person's share and 4*c* is the cost for 4 people. Add 42 and 27.68 to represent the total cost of the trip for all 4 friends.

b. Solve the equation for c and interpret the solution. Show your work.

Possible work:

$27.68 + 42 = 69.68$

$$\underbrace{\boxed{c}\boxed{c}\boxed{c}\boxed{c}}_{69.68}$$

$c = 69.68 \div 4$

$c = 17.42$

SOLUTION Each person's share of the total cost is \$17.42.

8 What is the solution of the equation $12\left(\frac{1}{2}\right)y = 2$?

A $\frac{1}{6}$

B $\frac{1}{3}$

C 3

D 6

9 Benjamin ran a total of 21 miles last month. This is 75% of the number of miles he wants to run this month. Solve the equation $0.75m = 21$ to find out how many miles, m, Benjamin wants to run this month. Show your work. Possible work:

$0.75m = 21$

$\frac{0.75m}{0.75} = \frac{21}{0.75}$

$m = 28$

SOLUTION Benjamin wants to run 28 miles this month.

CLOSE **EXIT TICKET**

9 Students' solutions should show an understanding that division is used to solve for the variable

Error Alert If students' response is 20.25 miles, then have them consider what operation should be used to find the value of the variable. Because the coefficient and the variable are shown as one term, they are being multiplied. So, division should be used to find the value of the variable. Remind students to check the reasonableness of their answer. The problem states that 21 miles is 75% of the number of miles that Benjamin wants to run this month, so the solution should be a number greater than 21.

Practice Solving One-Variable Multiplication Equations

Problem Notes

Assign **Practice Solving One-Variable Multiplication Equations** as extra practice in class or as homework.

1. Students may recognize that the reciprocal of $\frac{1}{4}$ is $\frac{4}{1}$, which is equal to 4. They should also remember that numbers can be multiplied in any order and the product remains the same. ***Basic***

2. Students may recognize that the mixed number $5\frac{1}{2}$ needs to be rewritten as a fraction before dividing by $\frac{1}{2}$. Also, students may question the reasonableness of the answer, since 11 is greater than either value in the equation. Remind students that the quotient tells the number of shorter pieces of string that can be cut from $5\frac{1}{2}$ yd, and that number is greater than either number in the equation. ***Medium***

LESSON 21 | SESSION 3 Name:

Practice Solving One-Variable Multiplication Equations

➤ **Study the Example showing how to solve a one-variable multiplication equation. Then solve problems 1–5.**

Example

Mariko is making potato pancakes. She has $4\frac{3}{4}$ lb of shredded potatoes. She uses $\frac{1}{4}$ lb to make each pancake. Solve the equation $4\frac{3}{4} = \frac{1}{4}p$ to find the number of potato pancakes, p, Mariko can make.

You can divide by the coefficient of the variable to solve the equation for p.

$$4\frac{3}{4} = \frac{1}{4}p$$

$$4\frac{3}{4} \cdot 4 = \frac{1}{4}p \cdot 4 \quad \longleftarrow \text{To divide by } \tfrac{1}{4}\text{, multiply by the reciprocal, 4.}$$

$$\frac{19}{4} \cdot \frac{4}{1} = p$$

$$19 = p$$

Mariko can make 19 potato pancakes.

1. In the Example, why can you replace the expression $\frac{1}{4}p \cdot 4$ on the right side of the equation with just the variable p?

   Possible explanation: You can reorder the factors: $\frac{1}{4}p \cdot 4 = \frac{1}{4} \cdot 4 \cdot p$. You know that $\frac{1}{4} \cdot 4 = 1$ and that $1 \cdot p = p$.

2. Mindy has a piece of string that is $5\frac{1}{2}$ yd long. She cuts it into pieces that are each $\frac{1}{2}$ yd long. Solve the equation $5\frac{1}{2} = \frac{1}{2}x$ to find out how many pieces of string, x, she gets. Show your work. Possible work:

$$5\frac{1}{2} = \frac{1}{2}x$$

$$5\frac{1}{2} \div \frac{1}{2} = \frac{1}{2}x \div \frac{1}{2}$$

$$\frac{11}{2} \div \frac{1}{2} = x$$

$$11 = x$$

SOLUTION Mindy gets 11 pieces of string.

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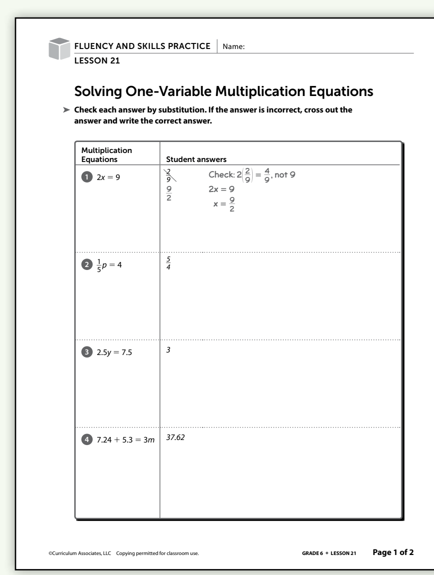
Fluency & Skills Practice

Solving One-Variable Multiplication Equations

In this activity, students practice solving one-variable multiplication equations. They use substitution to check the answers provided and correct them if they are incorrect.



Additional Practice

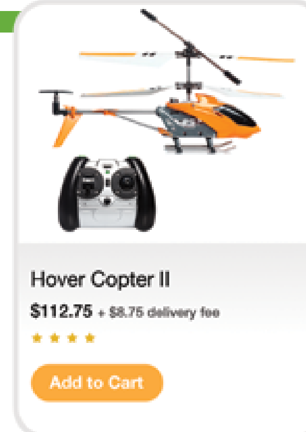
3 **a.** Students may recognize that the first step is finding the total amount Khalid needs to save. This is the value of the right side of the equation. Then, they divide that total by 6 to find the amount Khalid needs to save each month. ***Medium***

b. Students may also add just the cost of the helicopter and delivery, and then divide by 6 to find the new amount. Or, they could divide the amount of the coupon by the number of months and add that to the amount Khalid needs to save each month. ***Medium***

4 Students may recognize this as a multiplication problem, even though the variable is on the right side of the equal sign. They also should recognize that the numbers will not always evenly divide. Some equations have fractions or decimals as solutions. ***Basic***

5 Students may solve the problem by first dividing 480 by 12, and then dividing the quotient, 40, by 8 to get the variable by itself. ***Medium***

3 Khalid plans to save the same amount of money each month for 6 months to buy a remote control helicopter. The helicopter costs \$112.75 plus \$8.75 for delivery. Khalid has a coupon for \$18 off the price of the helicopter.



a. Use the equation $6m = 112.75 - 18 + 8.75$ to find out how much money, m, Khalid should save each month. Show your work.

Possible work:

$112.75 - 18 + 8.75 = 103.5$

m	m	m	m	m	m

$\underbrace{\hspace{6em}}_{103.5}$

$m = 103.5 \div 6 = 17.25$

SOLUTION Khalid should save \$17.25 each month.

b. How much money would Khalid need to save each month if he did not have the \$18 coupon? Explain your reasoning.

\$20.25; Possible explanation: The total cost would increase by \$18. $18 \div 6 = 3$, so Khalid would need to save \$3 more per month.

4 What operation would you use to solve $10 = 6y$? Explain your reasoning.

Possible answer: Division; The variable, y, is multiplied by 6. You want the variable by itself on one side. So, use the inverse operation, dividing by 6.

5 A can of tomato sauce contains 8 oz of sauce. A case contains 12 cans. Chef Hugo orders some cases of tomato sauce and gets a total of 480 oz of sauce. Use $12(8)x = 480$ to find out how many cases of tomato sauce Chef Hugo orders, x. Show your work. Possible work:

$12(8)x = 480$

$96x = 480$

$\frac{96x}{96} = \frac{480}{96}$

$x = 5$

SOLUTION Chef Hugo orders 5 cases of tomato sauce.

DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Use with **Session 4** Apply It

Levels 1–3: Reading/Listening

To help students make sense of Apply It problem 7, adapt **Three Reads** by reading the problem aloud each time as students follow along. For the first read, use **Act It Out** with pictures or realia to demonstrate the context. Guide students to circle and discuss the meanings of *pours in ____ more cups* and *removes ____ cups*. For the second read, have student make a sketch to illustrate the popcorn bin and the different amounts of kernels. For the third read, guide students to label their sketches with a variable, numbers, and mathematical operations.

Help students discuss how they can write an equation to solve for the cups of kernels in the bin when Tyler arrived.

Levels 2–4: Reading/Listening

Help students make sense of Apply It problem 7 by adapting **Three Reads**. After each read, encourage partners to look at the problem and identify the sentence that will help them answer the question for the read. Encourage them to ask questions to clarify meanings of unfamiliar terms, as needed.

Ask: *How do you know how many kernels were in the bin when Tyler arrived? How can you write an equation to solve for the variable?*

Have partners discuss how they can write an equation to solve for the cups of kernels in the bin when Tyler arrived.

Levels 3–5: Reading/Listening

Support students as they make sense of Apply It problem 7 using an adaptation of **Three Reads**. Have students review the focus question before each read. Then provide individual think time for students to consider their answer before turning to talk with partners. Call on volunteers to share ideas with the class.

Next, ask students to think about strategies to solve the problem and the take turns explaining their ideas to partners. When it is their turn to listen, prompt students to check their understanding by paraphrasing the speaker's ideas and confirming that the paraphrase is correct.

Develop Writing and Solving One-Variable Equations

Purpose

- **Develop** strategies for writing an equation to represent a problem situation.
- **Recognize** that an equation can show the relationship between quantities in a word problem, with a variable representing the unknown and the value of the variable representing the solution.

START CONNECT TO PRIOR KNOWLEDGE

Always, Sometimes, Never

A $4x = 4$ when x is 1.
B $4x = 4$ when x is a factor of 4.
C $4x = 4$ when x is a multiple of 4.

Possible Solutions

A is always true.

B is sometimes true.

C is always true.

WHY? Support students' facility with determining solutions of multiplication equations.

DEVELOP ACADEMIC LANGUAGE

WHY? Support students as they make connections to deepen understanding.

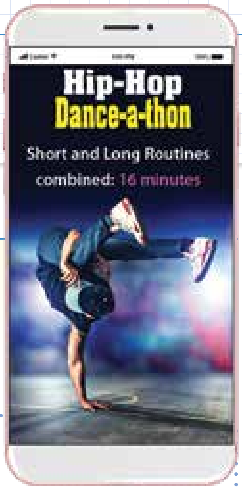
HOW? Explain that one way to understand more about a problem is by connecting it with others they have solved. Ask students to explain how the Try It problem is like or different from another problem they have solved. After students complete Apply It, ask them to choose a problem and explain how it is connected to the Try It problem. Highlight times during discussion when students intentionally make connections to previous problems.

TRY IT SMP 1, 2, 4, 5, 6

Make Sense of the Problem

See ***Connect to Culture*** to support student engagement. Before students work on Try It, use **Co-Craft Questions** to help them make sense of the problem. Have students share their questions with a partner. Then, open the discussion up to the class. Record and display the questions so students can compare their questions to the one in the book.

Develop Writing and Solving One-Variable Equations



➤ **Read and try to solve the problem below.**

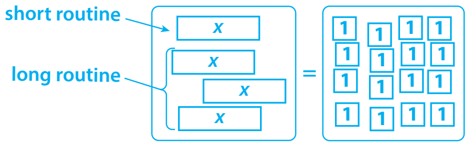
A dance crew is entering a hip-hop dance contest with a short routine and a long routine. The combined length of the two routines must be 16 min. The crew's long routine is 3 times as long as the short routine. Write and solve an equation to find the lengths of the crew's two routines.

TRY IT Math Toolkit algebra tiles, grid paper, number lines

Possible work:

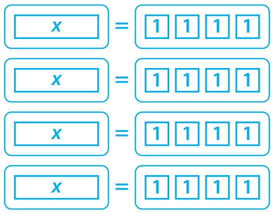
SAMPLE A

Use three times as many x-tiles for the long routine as for the short routine.



The equation is $4x = 16$.

$x = 4$, so the length of the short routine is 4 min and the length of the long routine is 12 min.



SAMPLE B

The total time is 16 minutes.

The short routine is x. The long routine is $x + x + x$.

The equation is $16 = x + x + x + x$, or $16 = 4x$.

Divide to find x: $x = 16 \div 4 = 4$

The short routine is 4 min. The long routine is 12 min.

DISCUSS IT

Ask: What does each side of your equation represent?

Share: The left side of my equation shows . . . The right side of my equation shows . . .

DISCUSS IT SMP 2, 3, 6

Support Partner Discussion

After students work on Try It, have them explain their work and then respond to Discuss It with a partner. Listen for understanding that:

- one side of the equation represents the total length of the routine, 16 minutes.
- one side of the equation represents the combined length of the short routine and long routine in minutes.
- the length of the long routine is three times the length of the short routine.
- the variable describes the length of the short routine in minutes.

Error Alert If students' response is to solve the equation $3x = 16$, where x is the length of the short routine, have them draw a model for that equation. Remind them that the long routine is 3 times the length of the short routine, so they need to also account for the length of the short routine in their equation.

Select and Sequence Student Strategies

Select 2–3 samples that represent the range of student thinking in your classroom. Here is one possible order for class discussion:

- using algebra tiles to model the problem and then write an equation from the model
- drawing a bar diagram that relates the lengths of each routine to the total length
- writing and solving an equation that relates the lengths of each routine to the total length

Facilitate Whole Class Discussion

Call on students to share selected strategies. Prompt students to understand more about the problem by connecting it with others they have solved.

Guide students to **Compare and Connect** the representations. After each strategy, allow individual think time for students to process the ideas.

ASK *How do [student name]'s and [student name]'s models show the length of the combined routines?*

LISTEN FOR Models show that the combined length of the short and long routines is 16 min. Models also show that the length of the long routine is 3 times the length of the short routine.

Model Its & Analyze It

If students presented these models, have students connect these models to those presented in class.

If no student presented at least one of these models, have students first analyze key features of the models, and then connect them to the models presented in class.

ASK *How are the lengths of the short routine, long routine, and the combined length represented in the models and equation?*

LISTEN FOR The length of the short routine is represented with x, the length of the long routine is represented with $3x$, and the combined length is represented with 16.

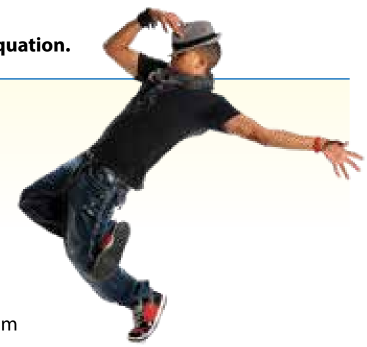
For the bar model, prompt students to identify how the model is labeled to match the problem. **For the first equation,** prompt students to connect the words to the numeric equation.

- *What does each quantity represent?*

For the second equation, prompt students to notice how the equation is solved.

- *Why can you combine x and 3x?*
- *What operation is used to solve for x?*

➤ **Explore different ways to write and solve a one-variable equation.**



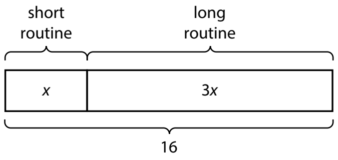
A dance crew is entering a hip-hop dance contest with a short routine and a long routine. The combined length of the two routines must be 16 min. The crew's long routine is 3 times as long as the short routine. Write and solve an equation to find the lengths of the crew's two routines.

Model It

You can use a bar model to show how the quantities in the problem are related.

Let x be the length of the short routine in minutes.

Then the length of the long routine in minutes is $3x$.



Model It

You can use words to help you write an equation with a variable.

The combined length of the two routines is 16 min.

length of short routine	+	length of long routine	=	total length of routines
↓		↓		↓
x	+	$3x$	=	16

Analyze It

You can use equivalent expressions and an inverse operation to solve an equation.

$$x + 3x = 16$$
$$4x = 16$$
$$\frac{4x}{4} = \frac{16}{4}$$
$$x = ?$$

DIFFERENTIATION | EXTEND

SMP 2

Deepen Understanding
Represent Quantities with Expressions

Prompt students to consider the contexts in word problems that demonstrate the relationship between quantities and can be used write expressions.

ASK *How is the relationship between the lengths of the routines described in the problem?*

LISTEN FOR The long routine is *3 times as long* as the short routine.

ASK *How can you show the lengths of both the short and long routines using a single term? Why is this possible?*

LISTEN FOR Since the long routine is compared to the short routine, the same variable can be used to write expressions for both the long and the short routine. This means that they are like terms. The term $4x$ represents the combined length.

ASK *How could you apply this understanding to other problems, for example, "Javier read 4 times as many books as Alita. How many books did they read in all?"*

LISTEN FOR To find the total number of books read, the term $5b$ could be used. It represents the number of books, b, Alita read plus the number of books Javier read, $4b$. The coefficient of the term that combines both quantities will be $1 + 4$ because they are like terms.

Develop Writing and Solving One-Variable Equations

CONNECT IT

SMP 2, 4, 5, 6

Remind students that the quantities and the relationships between them are the same in each representation. Explain that they will now use those relationships to reason about how equations can be used to represent problem situations.

Before students begin to record and expand on their work in Model Its & Analyze It, tell them that problem 2 will prepare them to provide the description asked for in problem 5. To help students collect their ideas for problem 5, ask them to turn and talk about how writing an equation can help solve a real-world problem.

Monitor and Confirm Understanding

1, 3 – 4

- Both the bar model and the word form of the equation show the same relationship between the quantities in the problem.
- The models show 3*x* plus 1 more *x* is the same as 4*x*.
- Like terms can be combined in order to solve an equation.
- Check a solution by using substitution to make sure the equation is true.

Facilitate Whole Class Discussion

2 Look for the understanding that the terms from the first Model It can be used to write an equation.

> **ASK** *What are two ways you can write the same quantity in the Try It, using a variable in one but not the other?*
>
> **LISTEN FOR** The two ways are $x + 3x$ (or $4x$) and 16. You can use an equal sign to compare the two ways.

5 Look for the idea that equations show how the quantities in the problem are related and can be used to solve for an unknown quantity.

> **ASK** *What strategies can you use to write an equation to solve a real-world problem?*
>
> **LISTEN FOR** Look at how the quantities are described in the problem and consider how they are related.

6 **Reflect** Have all students focus on the strategies used to solve the Try It. If time allows, have students discuss their ideas with a partner.

CONNECT IT

➤ **Use the problem from the previous page to help you understand how to write and solve a one-variable equation.**

1 Look at the two **Model Its**. How are they similar?

Possible answer: The bar model shows that the lengths of the bars for the short routine and long routine together equal the total length. This is the same relationship as represented with the words.

2 How do the two **Model Its** help you write an equation?

Possible answer: Both models show that one expression for the total length of the routines is $x + 3x$ and another expression for the total length of the routine is 16. So, these two expressions are equal.

3 Look at the first two equations in **Analyze It**. Why can you replace expression $x + 3x$ with the expression $4x$? Why is this step necessary?

x and 3*x* are like terms. You need to combine the like terms before you can divide both sides by the same quantity.

4 What are the lengths of the crew's routines? How you can use substitution to check your answer?

The short routine is 4 min and the long routine is 12 min. You can substitute 4 for *x* in the equation $x + 3x = 16$ and see if the equation is true.

5 How can writing and solving an equation help you solve a real-world problem?

Possible answer: Writing an equation shows the relationships between the different quantities in the problem. You can use a variable to represent an unknown quantity. Then you can solve for the value of the variable.

6 **Reflect** Think about all the models and strategies you have discussed today. Describe how one of them helped you better understand how to solve the **Try It** problem.

Responses will vary. Check student responses.

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DIFFERENTIATION | RETEACH or REINFORCE

Hands-On Activity

Make a model to represent real-world problems with an equation.

If students are unsure about whether any real-world problem can be solved with an equation, then use this activity to show how problems can be represented.

Materials For each pair: 2 index cards, algebra tiles (10 *x*-tiles, 25 1-tiles)

- Display this problem: *Reth plants 5 times as many trees as Ava. If they plant 24 trees in all, how many trees does each person plant?*
- Direct students to use rectangular tiles and an index card with + written on it to model the number of trees Reth and Ava each plant. Ask: *How many rectangular tiles do you use to model the number of trees Reth plants? Ava?* [5; 1] *What does one rectangular tile represent?* [The number of trees Ava planted] *What expression shows the combined number of trees planted?* [Let t = the number of trees Ava planted; so, $t + 5t$ represents the combined number of trees.]
- Have students use 1-tiles and an index card with = on it to model the equation. Ask: *What equation can you write using equivalent expressions?* [$t + 5t = 24$] *How many trees did Ava plant? Reth?* [4; 20]

Apply It

For all problems, encourage students to use a model to support their thinking.

7 Students should understand that this problem requires both addition and subtraction in the equation. Some may choose to draw the action in the problem. Representations should show 4 added to the initial amount, then 1.5 taken from this total, with the result of 6.

8 **A, B, and C are correct.** Students may solve the problem by subtracting the 17 fewer cards Colin has from the number of cards Demi has, or by adding the number of cards Colin has to the 17 fewer cards he has, or by subtracting the number of cards Colin has from the number of cards Demi has. Since the number of cards Colin has is unknown, a variable is used to represent that quantity.

- **D** is not correct. This answer is the sum of the number of cards Demi has and the 17 fewer cards Colin has.
- **E** is not correct. This answer is the result of subtracting the 17 fewer cards Colin has from the number of cards Colin has.
- **F** is not correct. This answer is the result of subtracting the number of cards Demi has from the number of cards Colin has.

Apply It

➤ **Use what you learned to solve these problems.**

7 Tyler sells popcorn at the movies. He arrives at work and sees there are some kernels in the popcorn bin. He pours in 4 more cups of kernels to fill the bin. Then he removes 1.5 cups of kernels. Now there are 6 cups of kernels in the bin. Write and solve an equation to find out how many cups of kernels were in the bin when Tyler arrived. Show your work. Possible work:

x = number of cups of kernels when Tyler arrived

arrive	add 4 cups	remove 1.5 cups	=	now
x	$+ 4$	$- 1.5$	=	6

$x + 2.5 = 6$

$x = 6 - 2.5$

$x = 3.5$

SOLUTION There were 3.5 cups of kernels when Tyler arrived.

8 Colin has 17 fewer baseball cards than Demi. Demi has 28 cards. Which equations can you use to find n, the number of baseball cards Colin has? Select all that apply.

- **A** $n = 28 - 17$
- **B** $28 = n + 17$
- **C** $17 = 28 - n$
- **D** $n = 17 + 28$
- **E** $n - 17 = 28$
- **F** $17 = n - 28$

9 Gabe mails two packages. One package weighs $\frac{3}{4}$ as much as the other package. The total weight of the packages is 7 lb. Write and solve an equation to find the weights of the two packages. Show your work. Possible work:

x = weight of one package

$\frac{3}{4}x$ = weight of the other package

sum of weights = 7 lb

$x + \frac{3}{4}x = 7$

$1\frac{3}{4}x = 7$

$\frac{7}{4}x = 7$

$\frac{7}{4}x \div \frac{7}{4} = 7 \div \frac{7}{4}$

$x = 4$ $\quad \frac{3}{4}x = \frac{3}{4} \cdot 4 = 3$

SOLUTION The packages weigh 4 lb and 3 lb.

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CLOSE EXIT TICKET

9 Students' solutions should show an understanding that:

- an equation can be used to show how the quantities in a problem are related to each other
- a variable can be used to represent the unknown quantity in an equation
- you can solve the equation to determine the value of the variable

Error Alert If students' response is $x = 9\frac{1}{3}$, then have students reread the problem and define the variable. Students may have solved the equation $\frac{3}{4}x = 7$ without understanding the relationship between the quantities in the problem. Have students draw two packages to represent those Gabe mailed. Students should label one package x, and then work through the problem together to decide what the label on the second package should be. $\left[\frac{3}{4}x\right]$ Then, have students explain how the weights of the packages are related to 7 lb and decide which operation to use to represent this relationship. [The sum of the weights is 7 lb; use addition to show the relationship.]

Practice Writing and Solving One-Variable Equations

Problem Notes

Assign **Practice Writing and Solving One-Variable Equations** as extra practice in class or as homework.

1. Students should realize that the 12 mystery books on the shelf represent only 40% of all the books on the bookshelf; so the total number of books on the shelf must be greater than 12. ***Basic***

2. Students may benefit from making a list with each friend's name and then the number of points for each friend.

   Jamila: $4\frac{1}{2}$ more points than Carter; 26 points

   Carter: $7\frac{1}{2}$ more points than Aisha

   Aisha: *a* points

   They can work backward to write the equation. Since Jamila has 26 points, the equation will have another expression equivalent to 26. Write an expression to represent the number of points Carter has, $a + 7\frac{1}{2}$, and then add $4\frac{1}{2}$ to this to show the number of points Jamila has. Write the addends as an expression equivalent to 26: $a + 7\frac{1}{2} + 4\frac{1}{2} = 26$. Add the fractions and then subtract that sum from both sides to solve for *a*. ***Medium***

LESSON 21 | SESSION 4 Name:

Practice Writing and Solving One-Variable Equations

➤ **Study the Example showing how to write and solve a one-variable equation. Then solve problems 1–5.**

Example

At the grocery store, Samuel spends $9 on fruits and vegetables. This is 80% of the money he spends in all. How much does Samuel spend?

Let m = the amount in dollars that Samuel spends.

Write an equation. 80% of the money Sam spends is $9.

$$0.8 \cdot m = 9$$

Solve the equation for *m*.

$$0.8m = 9$$

$$\frac{0.8m}{0.8} = \frac{9}{0.8}$$

$$m = 11.25$$

Samuel spends $11.25.

1. There are 12 paperback mysteries on a shelf. This is 40% of the books on the shelf. Write and solve an equation to find the number of books on the shelf. Show your work.

   Possible work:

   b = the number of books on the shelf

   $40\% = 0.4$

   $0.4b = 12$

   $\frac{0.4b}{0.4} = \frac{12}{0.4}$

   $b = 30$

   SOLUTION There are 30 books.

2. Three friends play a game. Jamila has $4\frac{1}{2}$ more points than Carter. Carter has $7\frac{1}{2}$ more points than Aisha. Jamila has 26 points. Write and solve an equation to find the number of points Aisha has. Show your work.

   Possible work:

   a = number of points Aisha has

   $a + 7\frac{1}{2} + 4\frac{1}{2} = 26$

   $a + 12 = 26$

   $a + 12 - 12 = 26 - 12$

   $a = 14$

   SOLUTION Aisha has 14 points.

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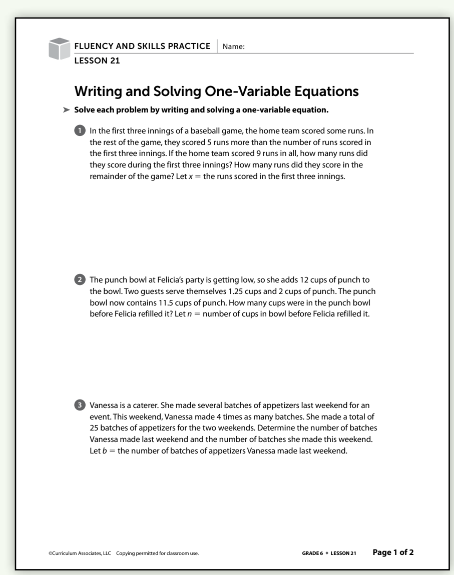
Fluency & Skills Practice

Writing and Solving One-Variable Equations

In this activity, students practice solving real-world problems by writing and solving one-variable equations.



FLUENCY AND SKILLS PRACTICE | Name:
LESSON 21

Writing and Solving One-Variable Equations

➤ **Solve each problem by writing and solving a one-variable equation.**

1. In the first three innings of a baseball game, the home team scored some runs. In the rest of the game, they scored 5 runs more than the number of runs scored in the first three innings. If the home team scored 9 runs in all, how many runs did they score during the first three innings? How many runs did they score in the remainder of the game? Let x = the runs scored in the first three innings.

2. The punch bowl at Felicia's party is getting low, so she adds 12 cups of punch to the bowl. Two guests serve themselves 1.25 cups and 2 cups of punch. The punch bowl now contains 11.5 cups of punch. How many cups were in the punch bowl before Felicia refilled it? Let n = number of cups in bowl before Felicia refilled it.

3. Vanessa is a caterer. She made several batches of appetizers last weekend for an event. This weekend, Vanessa made 4 times as many batches. She made a total of 25 batches of appetizers for the two weekends. Determine the number of batches Vanessa made last weekend and the number of batches she made this weekend. Let b = the number of batches of appetizers Vanessa made last weekend.

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Additional Practice

3 Students may choose to draw a bar diagram to show the relationship between the quantities. Some may also write the equation $4b = 30$, recognizing that the waiting time for the bumper cars plus the waiting time for the roller coaster can be represented by the expression $b + 3b$, which is equal to $4b$. ***Medium***

4 Students should understand that they need to use the perimeter formula for a rectangle, $p = 2\ell + 2w$, to find the length and width of the rectangle. They can substitute the given information into the perimeter formula and then solve for the width. Then, they can multiply the width by 2 to solve for the length. ***Challenge***

5 Students may recognize that this problem is similar to the one they solved in Try It, except this involves a decimal. The distance Neva runs is the unknown, so it will be represented by a variable in the equation. The distance Neva bikes is 5.5 times the distance she runs, so the variable is multiplied by 5.5. The sum of the two distances is 26 miles, so the expression showing the distance Neva runs plus the distance Neva bikes is set equal to 26. ***Medium***

LESSON 21 | SESSION 4

3 At a theme park, the waiting time for the roller coaster is usually 3 times as long as the waiting time for the bumper cars. The park's website says that visitors who go on both rides should expect to wait a total of 30 min. What is the usual waiting time for the bumper cars? Show your work. **Possible work:**

b = the waiting time in minutes for the bumper cars;
$3b$ = the waiting time for the roller coaster

$$b + 3b = 30$$
$$4b = 30$$
$$\frac{4b}{4} = \frac{30}{4}$$
$$b = 7\tfrac{1}{2}$$

SOLUTION The usual waiting time is $7\frac{1}{2}$ min.

4 The length of a rectangle is twice its width. The perimeter of the rectangle is 36 ft. What are the length and width of the rectangle? Show your work. **Possible work:**

w = the width of the rectangle in feet; $2w$ = the length of the rectangle

$$w + 2w + w + 2w = 36$$
$$6w = 36$$
$$\frac{6w}{6} = \frac{36}{6}$$
$$w = 6 \qquad 6 \times 2 = 12$$

SOLUTION The width is 6 ft. The length is 12 ft.

5 Neva is training for a race. This week, she bikes 5.5 times as far as she runs. Her total distance running and biking this week is 26 mi. How far does Neva run this week? Show your work. **Possible work:**

r = the distance in miles that Neva runs; $5.5r$ = the distance she bikes

$$r + 5.5r = 26$$
$$6.5r = 26$$
$$6.5r \div 6.5 = 26 \div 6.5$$
$$r = 4$$

SOLUTION Neva runs 4 mi this week.

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DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Use with **Session 5** **Apply It**

Levels 1–3: Reading/Speaking

Help students make sense of Apply It problem 2. Read the problem as students follow along. Then review the terms *checking account, checks,* and *balance.* Read Consider This and help students discuss how to use subtraction to show that the balance decreases.

After students solve the problem, read Pair/Share and review the meaning of *estimation* and *reasonable*. Have partners work together to use estimation to find the approximate starting balance. Provide frames to help students express ideas about estimation and reasonableness:

- *My estimated answer is ____. This helps me know my answer ____ reasonable.*

Levels 2–4: Reading/Speaking

Have students read Apply It problem 2 and use **Say It Another Way** to confirm understanding.

Next, have partners read Consider This. Ask them to discuss what mathematical operation they will use to show that the account balance is decreasing with each check that is written.

After students solve the problem, have them read Pair/Share. Support discussion with sentence frames:

- *I can estimate the balance at the start of the day by ____. My estimate is ____.*
- *My estimate shows that my answer ____ reasonable because ____.*

Levels 3–5: Reading/Speaking

Ask partners to read Apply It problem 2 and confirm understanding using **Say It Another Way**. Then have partners use Consider This and to deepen their understanding of the problem.

After students solve the problem, ask partners to read Pair/Share and discuss how to use estimation to check their solution. Provide these sentence starters to help students build on to their partner's ideas and/or explain their own ideas:

- *I think you are right because ____.*
- *I have a different idea because ____.*

Refine Writing and Solving One-Variable Equations

Purpose

- **Refine** strategies for writing and solving one-variable equations.
- **Refine** understanding of how to use inverse operations to solve an equation.

START CHECK FOR UNDERSTANDING

Solve the equation for n.

$14n + n = 52.5$

$n = 3.5$

WHY? Confirm students' understanding of how applying an operation to both sides of an equation keeps the equation in balance, identifying common errors to address as needed.

MONITOR & GUIDE

Before students begin to work, use their responses to the **Start** to determine those who will benefit from additional support. Use the **Error Analysis** table below to guide remediation.

Have all students complete the Example and problems 1–3, using Consider This and Pair/Share as appropriate. Observe and monitor their reasoning and guide or redirect students as needed.

LESSON 21 | SESSION 5

Refine Writing and Solving One-Variable Equations

➤ **Complete the Example below. Then solve problems 1–9.**

Example

At Valley Middle School, $\frac{3}{5}$ of the students study Spanish. There are 96 students who study Spanish. Solve the equation $\frac{3}{5}n = 96$ to find the number of students, n, at the school.

Look at how you could multiply by the reciprocal to solve a multiplication equation.

$\frac{3}{5}n = 96$

$\frac{5}{3} \cdot \frac{3}{5}n = \frac{5}{3} \cdot 96$

$1 \cdot n = \frac{5 \cdot 96}{3}$

$n = 5 \cdot 32$

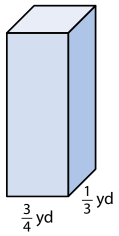
SOLUTION There are 160 students at the school.

CONSIDER THIS . . . The product of a number and its reciprocal is 1.

PAIR/SHARE Why is $\frac{5 \cdot 96}{3}$ equivalent to $5 \cdot 32$?

Apply It

1 The volume of the right rectangular prism is $\frac{1}{2}$ yd^3. What is the height of the prism? Show your work.



Possible work:

$V = \ell wh$

$\frac{1}{2} = \left(\frac{3}{4}\right)\left(\frac{1}{3}\right)h$

$\frac{1}{2} = \frac{1}{4}h$

$\frac{1}{2} \div \frac{1}{4} = \frac{1}{4}h \div \frac{1}{4}$

$\frac{1}{2} \cdot 4 = h$

$2 = h$

SOLUTION The height of the prism is 2 yd.

CONSIDER THIS . . . The formulas for the volume of a right rectangular prism are $V = \ell wh$ and $V = Bh$.

PAIR/SHARE How can you verify that your answer is correct?

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START ERROR ANALYSIS

If the error is . . .	Students may . . .	To support understanding . . .
$n = 787.5$	have multiplied both sides by 15 instead of dividing by 15.	Ask students to write a division equation with the same values and the same relationship between the values with n alone on one side of the equal sign: $52.5 \div 15 = n$. Guide them to see that the value of n will be less than 52.5.
$n = 3.75$	have divided both sides by 14 instead of dividing by 15.	Have students write each expression in a different color and then simplify each side as much as they can before using inverse operations to find the value of the variable.
$n = 37.5$	have subtracted 15 instead of dividing by 15.	Ask students to draw a bar model to show that the expression $15n$ is equivalent to 52.5. Elicit from students that 52.5 is divided into 15 different groups, and 1 group represents the value of n.

Example

Guide students in understanding the Example. Ask:

- *What do $\frac{3}{5}$ and 96 represent in the problem? How are the numbers related?*
- *What operation is indicated by $\frac{3}{5}n$? How would you get the variable by itself?*
- *How do inverse operations help you solve this problem? How else might you solve it?*

Help all students focus on the Example and responses to the questions by reminding students that they can ask questions about ideas that are not clear.

Look for understanding that division can be used to undo multiplication. In the case where a fraction is multiplied by a variable, multiply by the reciprocal as you would to divide by any other fraction.

Apply It

1. Students may also choose to find the area of the base first, and then substitute that into the equation $V = Bh$. ***DOK 1***

2. Students may recognize that this problem situation involves subtraction, since they are given the amounts of 3 checks and the ending balance. Also, students may solve the equation $b = 3(65) + 330.25$ to find the starting balance. ***DOK 2***

3. **B is correct.** Students may solve the problem by letting x represent the number of cans of food Erin collects. $5x$ represents the number of cans of food Carmen collects, so set the expression $x + 5x$ equal to the total number of cans of food, 60. Combine like terms and solve for x.

   A is not correct. This answer is the result of only setting the number of cans of food Carmen collects to the total number of cans.

   C is not correct. This answer is the result of misreading "5 times as" as indicating division instead of multiplication. It also does not include the cans Erin collected.

   D is not correct. This answer is the result of multiplying the quantities in the problem without relating them correctly.

   DOK 3

2. Yukio has a checking account. On Monday, he writes 3 checks for \$65 each. His balance at the end of the day is \$330.25. Use an equation with a variable to find Yukio's balance at the start of the day. Show your work.

CONSIDER THIS . . . When writing checks, your account balance decreases.

Possible work:

b = starting balance

start balance − 3 checks = final balance

$b - 3(65) = 330.25$

$b - 195 = 330.25$

$b - 195 + 195 = 330.25 + 195$

$b = 525.25$

PAIR/SHARE How can you use estimation to make sure your answer is reasonable?

SOLUTION Yukio starts with \$525.25.

3. Carmen and Erin collect cans of food for a food drive. Carmen collects 5 times as many cans of food as Erin. Together, they collect 60 cans of food. Which equation can you solve to find the number of cans of food Erin collects?

CONSIDER THIS . . . You can write an equation where one side is an algebraic expression for the total number of cans of food Carmen and Erin collect.

A $5x = 60$

B $6x = 60$

C $\frac{1}{5}x = 60$

D $x = 60 \cdot 5$

Jelani chose B as the correct answer. How might he have gotten that answer?

Possible answer: Jelani let x represent the number of cans of food Erin collects. Then $5x$ is the number of cans of food Carmen collects. He wrote the equation $x + 5x = 60$, which is the same as $6x = 60$.

PAIR/SHARE How can you find the number of cans of food Carmen collects?

GROUP & DIFFERENTIATE

Identify groupings for differentiation based on the **Start** and problems 1–3. A recommended sequence of activities for each group is suggested below. Use the resources on the next page to differentiate and close the lesson.

Approaching Proficiency
- **RETEACH** Hands-On Activity
- **REINFORCE** Problems 5, 6, 8

Meeting Proficiency
- **REINFORCE** Problems 4–8

Extending Beyond Proficiency
- **REINFORCE** Problems 4–8
- **EXTEND** Challenge

Have all students complete the **Close: Exit Ticket**.

Resources for Differentiation are found on the next page.

Refine Writing and Solving One-Variable Equations

Apply It

4 Students may recognize that they must rewrite the percent as a fraction or decimal before solving. The words "This is 24% of the fish" can be used to help students write their equation, where "this" indicates 6, "is" indicates the equal sign, "of" indicates multiplication, and a variable is used to represent the total number of fish. *DOK 2*

5 Students may be able to solve this problem mentally if they recognize that when the number added to the variable is equal to the number on the other side of the equation, then the variable has a value of 0. *DOK 1*

6 **a.** Students should recognize that multiplying both sides by the reciprocal will change the coefficient before y to 1.

b. Students should recognize that multiplying by the reciprocal is the same as dividing by the fractional coefficient.

c. Students should recognize that $\frac{5}{2} \neq \frac{2}{5}$. These equations will not have the same solution.

d. Students should recognize that both sides of the equation were multiplied by 2, so the relationship between the quantities remains the same.

DOK 2

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4 Ju-long has 6 guppies in his aquarium. This is 24% of the fish in the aquarium. How many fish are in Ju-long's aquarium? Use an equation with a variable. Show your work.

Possible work:

f = the number of fish in the aquarium

24% means $\frac{24}{100}$, or 0.24.

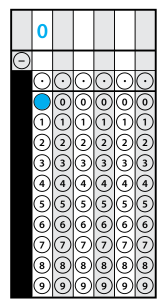
$0.24f = 6$

$0.24f \div 0.24 = 6 \div 0.24$

$f = 25$

SOLUTION There are 25 fish in Ju-long's aquarium.

5 What is the solution of the equation $9\frac{1}{3} = x + 9\frac{1}{3}$?



6 Consider the equation $\frac{2}{5}y = 20$. Tell whether each statement is *True* or *False*.

	True	False
a. You can solve the equation by multiplying both sides by $\frac{5}{2}$.	●	○
b. You can solve the equation by dividing both sides by $\frac{2}{5}$.	●	○
c. The equation has the same solution as $\frac{5}{2}y = 20$.	○	●
d. The equation has the same solution as $\frac{4}{5}y = 40$.	●	○

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DIFFERENTIATION

RETEACH

Hands-On Activity

Write and solve one-variable equations.

Students approaching proficiency with writing and solving one-variable equations will benefit from using manipulatives to write equations for real-world situations.

Materials For each pair: 4 index cards, 30 two-color counters

- Display the problem: *Dara has 4 times as many white golf balls as yellow golf balls. He has 20 golf balls in all. How many golf balls does Dara have of each color?*
- Ask: *What quantity will you represent with a variable and why?* [The number of yellow golf balls; The number of white golf balls is given in terms of the number of yellow golf balls.]
- Have pairs use two-color counters to represent the number of white golf balls for every yellow golf ball. Ask: *If g represents the number of yellow golf balls, what represents the number of white golf balls and why?* [$4g$; There are 4 times as many white golf balls as yellow.] *What operation will you use to show how the number of each color relates to the total number of golf balls and why?* [Addition, because you are looking for a total amount.]
- Have pairs write the quantities and operations on index cards: $4g$, g, 20, $=$, and $+$. Then have them order the cards to write an equation. Students should note that like terms can be combined and replace the $4g$, g, and $+$ cards with a single card that reads $5g$. Pairs should use the cards to solve for g. [$g = 4$; 4 yellow and 16 white golf balls]
- Have pairs illustrate the equation $4g + g = 20$ using the counters to check their work. They should find that a total of 20 golf balls will have 16 white (4 groups of 4) and 4 yellow golf balls.

7 Students should understand that there are 5 days in a work week. Students may use a bar model or other drawing to help visualize how many trips Enrico makes each week in order to identify the relationship between the quantities and write an equation. ***DOK 2***

8 Students should recognize that the value of an equation will not change as long as the same operation with the same quantity is applied to both sides of an equation. ***DOK 3***

CLOSE EXIT TICKET

9 **Math Journal** Look for understanding that equations can be written with one variable and solved using inverse operations. Students can check solutions by substituting the value of the variable into the original equation.

Error Alert If students write an equation such as $x - 3.2 = 5$, then remediate by discussing how subtraction can be used to solve for the variable, not represent an addition problem.

End of Lesson Checklist

INTERACTIVE GLOSSARY Support students by suggesting they write an example of an equation and note how one-variable equations can be solved.

SELF CHECK Have students review and check off any new skills on the Unit 5 Opener.

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7 Each day from Monday to Friday, Enrico uses a rideshare scooter to take the same route to and from work. According to the rideshare app, the total distance he rides each week is 13 mi. Explain how to write an equation that Enrico can use to find the distance of one trip to or from work. Show how to solve the equation.

Possible work: Let d represent the distance in miles to or from work. There are two trips each day. So, each day's total distance is $2d$. There are 5 days in the week. So, each week's total distance is $5 \times 2d$ or $10d$. This is equal to the total distance, 13 mi.

So, the equation is $10d = 13$.

$$\frac{10d}{10} = \frac{13}{10}$$

$$d = 1.3$$

The distance of one trip to or from work is 1.3 mi.

8 Is the value of w the same in both equations shown below? Explain how you can decide without solving the equations.

$w + 16 = 43$ $\qquad$ $w + 16 - 7 = 43 - 7$

Yes; Possible explanation: The second equation is the same as the first equation, except that 7 is subtracted from both sides. Subtracting 7 from both sides keeps the equation balanced and does not change the solution.

9 **Math Journal** Write an equation that you can solve by subtracting 3.2 from both sides of the equation. Then show how to solve the equation and check your solution.

Possible answer:

$$x + 3.2 = 8$$
$$x + 3.2 - 3.2 = 8 - 3.2$$
$$x = 4.8$$

To check, substitute 4.8 for x in the original equation.

$4.8 + 3.2 = 8$; This is a true statement, so the solution checks.

End of Lesson Checklist

- ☐ **INTERACTIVE GLOSSARY** Find the entry for *equation*. Add two important things you learned about equations in this lesson.
- ☐ **SELF CHECK** Go back to the Unit 5 Opener and see what you can check off.

REINFORCE

Problems 4–8
Solve one-variable equations.

Students meeting proficiency will benefit from additional work with writing and solving one-variable equations by solving problems in a variety of formats.

- Have students work on their own or with a partner to solve the problems.
- Encourage students to show their work.

EXTEND

Challenge
Write and solve two-step equations.

Students extending beyond proficiency will benefit from writing and solving two-step equations.

- Have partners work to solve this problem: *Mora has 5 more than twice the number of pens Kennedy has. Mora has 11 pens. How many does Kennedy have?*
- Some students may work backward to write the equation: $11 = 5 + 2p$. Others may subtract the number of Mora's additional pens and write the equation $11 - 5 = 2p$. All equations should show $p = 3$, so Kennedy has 3 pens. Encourage students to check their work.
- Challenge students to solve this extension of the problem: *Mrs. McClary has 3 fewer than 3 times the number of pens as Mora. How many pens does she have?* [30 pens]

PERSONALIZE

i-Ready

Provide students with opportunities to work on their personalized instruction path with *i-Ready* Online Instruction to:

- fill prerequisite gaps.
- build up grade-level skills.