Explore Fractions as Division

You know that division is used for equal sharing and that fractions represent a number of equal parts of a whole. In this lesson, you will learn how division and fractions are related. Use what you know to try to solve the problem below.

Mrs. Tatum needs to share 4 fluid ounces of red paint equally among 5 art students. How many ounces of red paint will each student get?

Learning Target

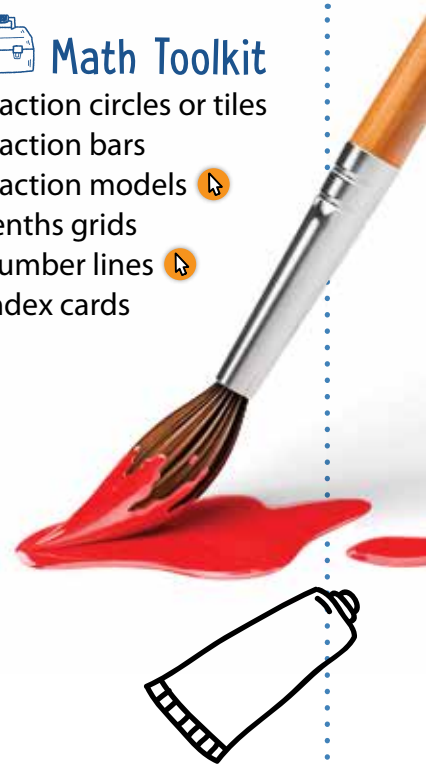
- Interpret a fraction as division of the numerator by the denominator $\left(\frac{a}{b} = a \div b\right)$. Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

SMP 1, 2, 3, 4, 5, 6

TRY IT

Math Toolkit

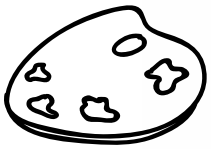
- fraction circles or tiles
- fraction bars
- fraction models
- tenths grids
- number lines
- index cards



DISCUSS IT

Ask your partner: Why did you choose that strategy?

Tell your partner: At first, I thought . . .



CONNECT IT

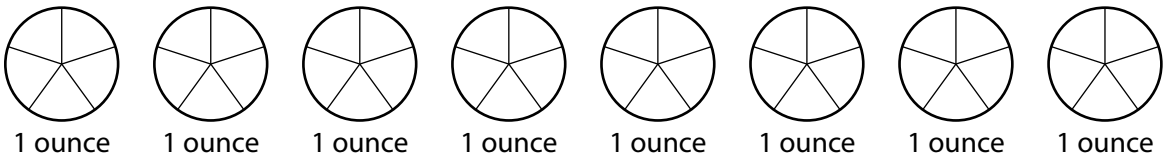
1 LOOK BACK

Explain how to find the amount of paint each student gets.

2 LOOK AHEAD

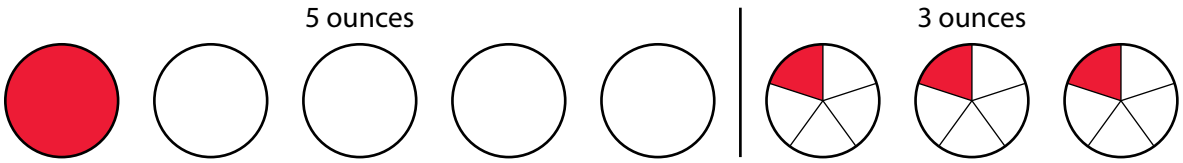
Suppose Mrs. Tatum wants to share 8 fluid ounces of paint equally among the 5 students. You can think about this quotient in two ways.

a. Think of each student getting $\frac{1}{5}$ of each ounce. Shade $\frac{1}{5}$ of each whole in the model below to show one student's share.



8 ounces ÷ 5 = 8 × _____ = _____ ounces

b. Think of 8 ounces as 5 ounces + 3 ounces. Explain how the shaded part of the model below shows one student's share.



c. Write the quotient 8 ÷ 5 as a fraction and as a mixed number. _____

3 REFLECT

How would you write the fraction $\frac{2}{5}$ as a division expression? Write a word problem that can be represented by your expression and by the fraction $\frac{2}{5}$.

_____

_____

Prepare for Fractions as Division

1 Think about what you know about division. Fill in each box. Use words, numbers, and pictures. Show as many ideas as you can.

Word	In My Own Words	Example
fraction		
division expression		
quotient		
remainder		

2 Write the fraction $\frac{3}{4}$ as a division expression. .....

How could you use multiplication to check your answer?

3 Solve the problem. Show your work.

Mrs. Tatum needs to share 3 grams of glitter equally among 8 art students. How many grams of glitter will each student get?

Solution

4 Check your answer. Show your work.

Develop Fractions as Division

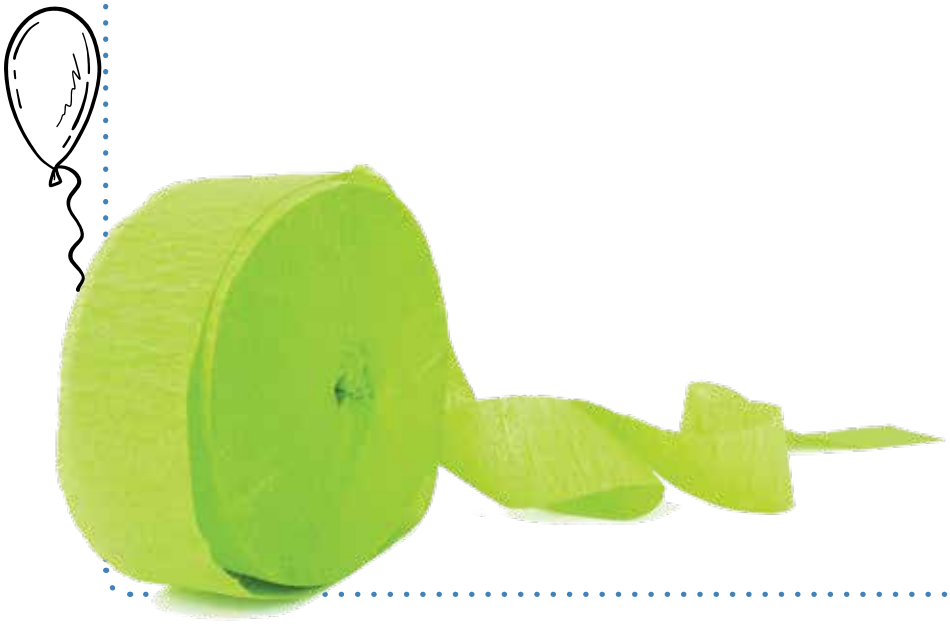
Read and try to solve the problem below.

Jared, Monica, and Heather have 5 hallways to decorate for the student council. If they share the work equally, how much will each student decorate?

TRY IT

Math Toolkit

- fraction circles or tiles
- fraction bars
- fraction models
- tenths grids
- number lines
- index cards



DISCUSS IT

Ask your partner: Do you agree with me? Why or why not?

Tell your partner: I disagree with this part because . . .

Explore different ways to understand fractions as quotients.

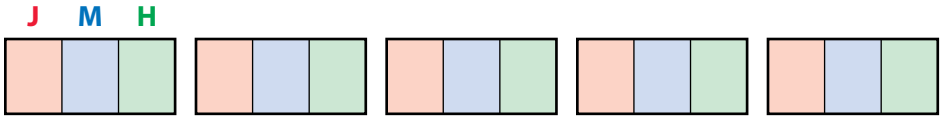
Jared, Monica, and Heather have 5 hallways to decorate for the student council. If they share the work equally, how much will each student decorate?

PICTURE IT

You can use a fraction model to picture how the students divide up the work.

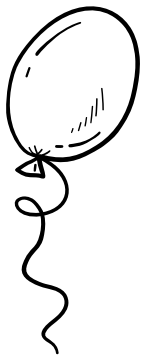
There are 5 hallways for 3 students to decorate, which is $5 \div 3$.

If they share the work equally, each student can decorate $\frac{1}{3}$ of each hallway.

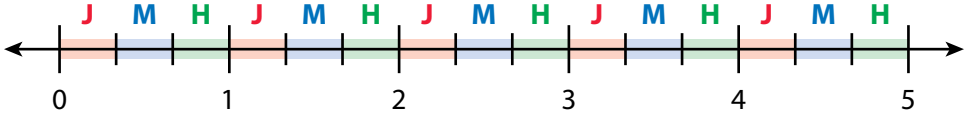


MODEL IT

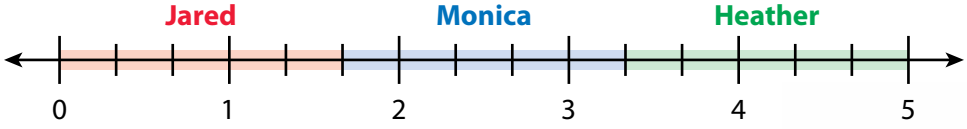
You can use a number line to model each student's share of the work.



The number line is numbered from 0 to 5 because there are 5 hallways. It is divided into thirds because each student can decorate one third of each hallway.



The thirds can be rearranged to show each student's share of the work.



CONNECT IT

Now you will use the problem from the previous page to help you understand fractions as quotients.

1. How many thirds of a hallway are there to decorate in 5 hallways? _____ thirds

2. How many thirds of a hallway will each student decorate? _____ thirds

   Write this as a fraction. _____ of a hallway

3. Write a division equation that shows the quotient as a fraction. _____

   Write a multiplication equation to check this equation. _____

4. How many whole hallways can each student decorate? _____

   How many hallways remain after those are done? _____

   How much of the 2 remaining hallways will each student decorate? _____

   Write a mixed number to show how many hallways each student will decorate.

   _____ hallways

5. Calculate using remainder notation: 5 ÷ 3 = _____ R _____

   Compare this answer to the mixed number. How are they alike?

6. How does the bar in a fraction represent division?

7 REFLECT

Look back at your **Try It**, strategies by classmates, and **Picture It** and **Model It**. Which models or strategies do you like best for finding fraction quotients? Explain.

_____

_____

_____

APPLY IT

Use what you just learned to solve these problems.

8 Five friends are equally sharing 3 packs of football cards. How many packs of cards will each friend get? Use a visual model to support your answer.

Solution

9 Elena made 10 ounces of apple chips. She puts the same amount of apple chips into each of 4 containers. How many ounces of apple chips are in 1 container?
Write a division expression to represent the problem and solve.
Write the solution in remainder form and as a mixed number.
Use a visual model to support your answer.

Solution

Does the remainder form or the mixed number form best answer the question? Explain.

10 Which expression is equivalent to $\frac{12}{7}$?

Ⓐ $12 - 7$

Ⓑ $7 - 12$

Ⓒ $12 \div 7$

Ⓓ $7 \div 12$

Practice Fractions as Division

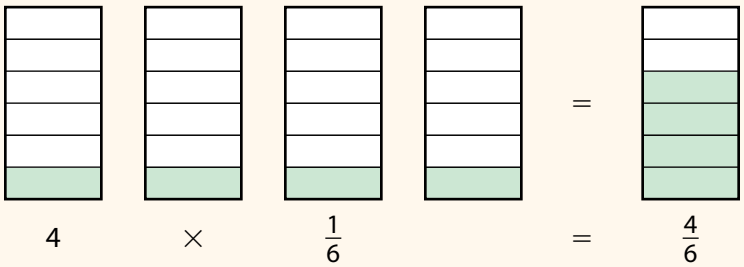
Study the Example showing whole-number division with a fraction quotient. Then solve problems 1–5.

EXAMPLE

There are 4 packages of printer paper to be divided equally among 6 classrooms. How much paper will each classroom get?

There are 4 packages for 6 classrooms to share, which is $4 \div 6$.

If you divide each package into sixths, each classroom would get one sixth of each package. So, $\frac{1}{6}$ of each package from 4 packages is the same as $\frac{4}{6}$ of a package.

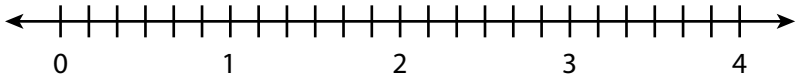


$4 \div 6 = \frac{4}{6}$

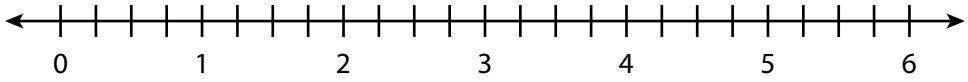
Each classroom gets $\frac{4}{6}$ of a package.

1 Circle the number line you would use to solve the problem in the Example.

Number Line A



Number Line B



2 Look at the Example. Suppose only 5 classrooms share 4 packages. How would the model in the Example change? How would the answer change?

3 Trish is taking care of the Han family's dogs. The Hans leave 7 cans of dog food for the 3 days they will be away. How much food will the dogs get each day if Trish feeds them an equal amount each day? Show your work. Write the answer in remainder form and as a mixed number.

Solution

Which best answers the question, the remainder form or the mixed number? Explain.

4 Raul plans to run 30 miles this week. He wants to run the same number of miles each day of the week. He says he will run $\frac{7}{30}$ mile each day. Is he correct? Explain.

5 Gus makes 48 fluid ounces of spiced cider. If he serves an equal amount to each of 7 people, will each person get more than 1 cup of cider or less than 1 cup? (1 cup = 8 fluid ounces) Show your work.

Solution

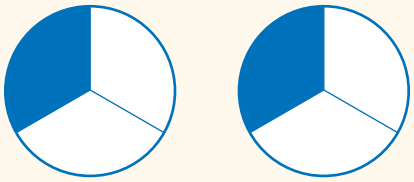
Refine Fractions as Division

Complete the Example below. Then solve problems 1–9.

EXAMPLE

Luke, Carter, and Ava have 2 quarts of juice. They want to share it equally. How many quarts of juice will each of them get?

Look at how you could show your work using a model and equations.



$$2 \div 3 = 2 \times \frac{1}{3}$$
$$= \frac{2}{3}$$

Solution

2 quarts are shared equally by 3 friends, so I know that each friend will have less than 1 quart of juice. That means the quotient is a fraction.

PAIR/SHARE
Model the problem for 3 quarts of juice divided equally among Luke, Carter, Ava, and Ava's little brother.

APPLY IT

1 Erica has 7 square feet of space in her rectangular garden to plant carrots, beans, peppers, and lettuce. Suppose she gives each vegetable an equal amount of space. How much space will each vegetable get?
Show your work.

Solution

Each vegetable will get at least 1 square foot of garden space. How will the rest of the space be divided up?

PAIR/SHARE
What are some ways you can check your solution?

2 Deon needs to make 36 pizza crusts. He has 120 ounces of dough and wants to use the same amount of dough for each crust. He weighs a portion of dough for 1 crust on a scale. The weight, in ounces, should fall between what two whole numbers? Show your work.

How many whole ounces of dough will each crust get? What will happen with the remaining ounces?

PAIR/SHARE

Create a different division story to represent $\frac{120}{36}$.

Solution

3 Jonas is doing a science experiment with his class. The teacher has 21 fluid ounces of pond water to share equally among 10 pairs of students. How much pond water will Jonas and his science partner receive?

Ⓐ $\frac{10}{21}$ fluid ounce

Ⓑ $1\frac{1}{10}$ fluid ounces

Ⓒ 2 fluid ounces

Ⓓ $\frac{21}{10}$ fluid ounces

Olivia chose Ⓐ as the correct answer. How did she get that answer?

About how much water will each pair of students receive? Will it be more or less than 2 fluid ounces?

PAIR/SHARE

Does Olivia's answer make sense?

4 Teddy makes 32 fluid ounces of hot cocoa. He pours equal amounts of cocoa into 5 cups. The amount of hot cocoa in each cup will fall between which two amounts?

Ⓐ 3 and 4 fluid ounces

Ⓑ 4 and 5 fluid ounces

Ⓒ 5 and 6 fluid ounces

Ⓓ 6 and 7 fluid ounces

5 Pierce swims 10 laps in a pool in 8 minutes. He spends the same amount of time on each lap. How much time does each lap take him?

Ⓐ $\frac{2}{10}$ minute

Ⓑ $\frac{8}{10}$ minute

Ⓒ $\frac{10}{8}$ minutes

Ⓓ $1\frac{2}{8}$ minutes

6 Dani needs 8 equal sections from a board that is 13 feet long. Does the expression represent the largest possible length of 1 section of the board, in feet?

	Yes	No
$1\frac{5}{8}$	Ⓐ	Ⓑ
$\frac{8}{13}$	Ⓒ	Ⓓ
$\frac{13}{8}$	Ⓔ	Ⓕ
$8 \div 13$	Ⓖ	Ⓗ
$13 \times \frac{1}{8}$	Ⓘ	Ⓙ

7 Which situations can be represented by $\frac{25}{9}$?

Ⓐ Melanie equally shares 25 yards of paper to make 9 banners.

Ⓑ Quill gives away 9 baseball cards from a pack of 25 cards.

Ⓒ George invites 25 kids and 9 adults to his birthday party.

Ⓓ Becca makes 9 rows with 25 buttons each.

Ⓔ Joe makes 9 equal servings from a 25-ounce bag of peanuts.

8 Paco is trying to explain to his friend that $7 \div 2 = \frac{7}{2}$.

Part A Draw a model or number line showing $7 \div 2 = \frac{7}{2}$.

Part B Explain the equivalence of $7 \div 2$ and $\frac{7}{2}$ using words.

9 MATH JOURNAL

Write a division word problem that can be represented by the expression $12 \div 5$. Then explain how to solve your problem.

SELF CHECK Go back to the Unit 3 Opener and see what you can check off.

Explore Multiplying Fractions in Word Problems

Learning Target

- Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

SMP 1, 2, 3, 4, 5, 6

Now that you have learned how to multiply fractions, you will use what you know in problem situations. Use what you know to try to solve the problem below.

Grayson lives $\frac{4}{5}$ mile from the park. He has already walked $\frac{3}{4}$ of the way to the park. How far has Grayson walked? Use a visual fraction model to show your thinking.

TRY IT

Math Toolkit

- fraction tiles or circles
- fraction bars
- fraction models
- grid paper
- number lines
- index cards
- multiplication models

DISCUSS IT

Ask your partner: Can you explain that again?

Tell your partner: A model I used was . . . It helped me . . .

CONNECT IT

1 LOOK BACK

Explain how you can use a visual model to show how far Grayson has already walked.

2 LOOK AHEAD

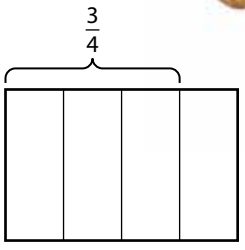
You can use what you know about multiplying fractions to think through and solve word problems involving fractions. Consider this word problem:

Ehrin spills $\frac{1}{2}$ of a $\frac{3}{4}$-pound box of cereal. How many pounds did she spill?

a. Finding $\frac{1}{2}$ of a quantity is the same as multiplying by $\frac{1}{2}$. What equation could you write for the cereal problem? Use p for the unknown amount in the problem.

b. Estimate the product. Is the amount of cereal Ehrin spills on the floor more than $\frac{3}{4}$ pound or less than $\frac{3}{4}$ pound? Why?

c. Complete the area model to show the problem.



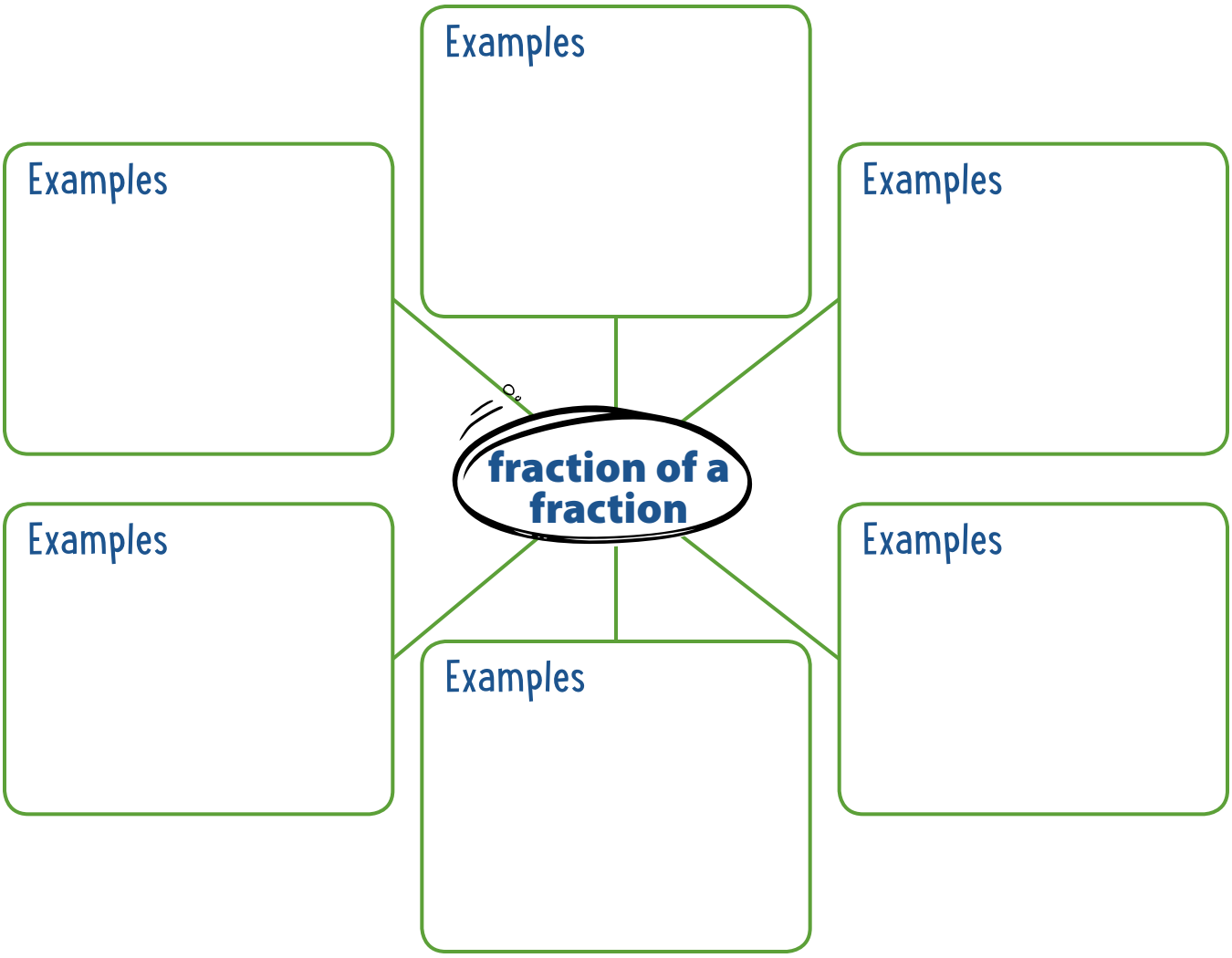
How many pounds of cereal did Ehrin spill on the floor? _____ pound

3 REFLECT

How does writing an equation, making an estimate, and drawing a model help you think through the problem?

Prepare for Multiplying Fractions in Word Problems

1 Think about what you know about fractions. Fill in each box. Use words, numbers, and pictures. Show as many ideas as you can.



2 Write a multiplication expression that can be used to find $\frac{1}{5}$ of $\frac{3}{8}$.

Why is the product less than $\frac{3}{8}$?

3 Solve the problem. Show your work.

Lola lives $\frac{3}{4}$ mile from the basketball court. She has already walked $\frac{2}{3}$ of the way to the basketball court. How far has Lola walked? Use a visual fraction model to show your thinking.

Solution

4 Check your answer. Show your work.

Develop Multiplying Fractions in Word Problems

Read and try to solve the problem below.

Brandon's mother left $\frac{3}{4}$ of a pizza on the counter. If Brandon eats $\frac{2}{3}$ of the leftover pizza, how much of the whole pizza did Brandon eat?

TRY IT

Math Toolkit

- fraction tiles or circles
- fraction bars
- fraction models
- grid paper
- number lines
- index cards
- multiplication models

DISCUSS IT

Ask your partner: How did you get started?

Tell your partner: I am not sure how to find the answer because . . .

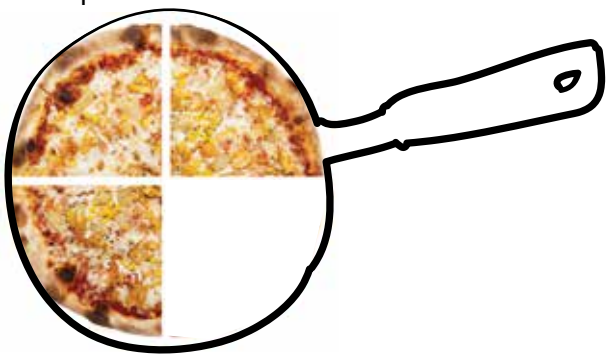
Explore different ways to understand strategies for solving word problems that involve finding a fraction of a fraction.

Brandon's mother left $\frac{3}{4}$ of a pizza on the counter. If Brandon eats $\frac{2}{3}$ of the leftover pizza, how much of the whole pizza did Brandon eat?

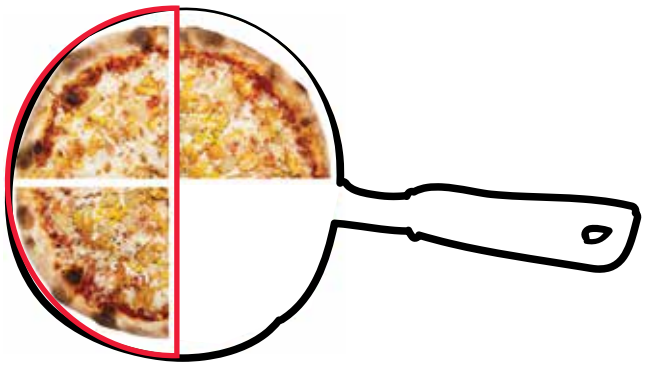
PICTURE IT

You can draw a picture to help you understand the problem.

Show $\frac{3}{4}$ of a pizza.



Since Brandon eats $\frac{2}{3}$ of what is left, outline 2 of the 3 pieces that are left. You can see from the outlined parts how much of the whole pizza Brandon ate.



MODEL IT

You can write an equation to help you understand the problem.

You need to find a fraction of a fraction: $\frac{2}{3}$ of $\frac{3}{4}$ of a pizza.

$\frac{2}{3}$ of $\frac{3}{4}$ means $\frac{2}{3} \times \frac{3}{4}$.

$$\frac{2}{3} \times \frac{3}{4} = \frac{2 \times 3}{3 \times 4}$$

CONNECT IT

Now you will use the problem from the previous page to help you understand strategies for solving word problems that involve finding a fraction of a fraction.

1. Look at **Picture It**. Why do you outline 2 of the 3 parts of the pizza?

2. How much of the whole pizza did Brandon eat? Explain your reasoning.

3. Look at **Model It**. How do you know that you should multiply $\frac{2}{3} \times \frac{3}{4}$?

4. What is $\frac{2 \times 3}{3 \times 4}$?

   Is this product the same as your answer to problem 2? Explain.

5. What strategies can you use to solve a word problem that involves finding a fraction of a fraction?

6 REFLECT

Look back at your **Try It**, strategies by classmates, and **Picture It** and **Model It**. Which models or strategies do you like best for solving word problems that involve finding a fraction of a fraction? Explain.

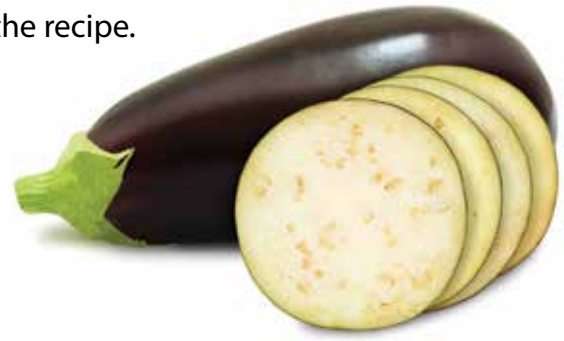
APPLY IT

Use what you just learned to solve these problems.

7 Lewis walked $\frac{8}{10}$ of a mile. Todd walked $\frac{3}{4}$ of the way with Lewis. How many miles did Todd walk with Lewis? Show your work.

Solution

8 Stan has a recipe for vegetable lasagna that calls for $\frac{9}{16}$ pound of eggplant. He wants to make a batch of lasagna that is $\frac{2}{3}$ of the amount of the recipe. How much eggplant will Stan need? Show your work.



Solution

9 Jamie worked $\frac{5}{6}$ hour filing papers for her mother. She listened to music for $\frac{4}{5}$ of the time she spent filing. How much time did Jamie spend listening to music? Show your work.

Solution

Practice Multiplying Fractions in Word Problems

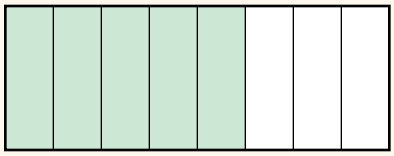
Study the Example showing one way to solve a word problem with fractions. Then solve problems 1–5.

EXAMPLE

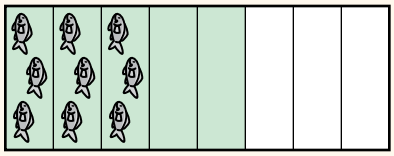
Vicky's favorite beach towel is green and white and has a fish design. The green part covers $\frac{5}{8}$ of the towel. A fish design is drawn on $\frac{3}{5}$ of that part. What part of the towel has a fish design?

You can draw a picture.

Show a towel with $\frac{5}{8}$ shaded green.



Draw fish on $\frac{3}{5}$ of the green part.



Because 3 of the 8 parts of the towel have fish drawn on them, $\frac{3}{8}$ of the towel has a fish design.

1 You can also write an equation to solve the Example. Write the numbers to complete the equation showing what part of the towel has the fish design.

$\frac{3}{5}$ of $\frac{5}{8}$ means $\frac{3}{5} \times \frac{5}{8}$.

$$\frac{3}{5} \times \frac{\square}{\square} = \frac{\square \times 5}{\square \times 8} = \frac{\square}{\square}$$

2 Is your answer to problem 1 the same as the answer of $\frac{3}{8}$ shown in the Example? Explain.

3 Suppose that the green part of Vicky's towel covers $\frac{4}{5}$ of the towel and the fish design is drawn on $\frac{3}{4}$ of that part. Draw a picture to find the part of the towel that has the fish design. Then write the answer.

Solution

4 Write an equation to show the answer to problem 3.

Solution

5 Write a word problem that can be solved by finding the product $\frac{1}{6} \times \frac{3}{8}$. Then solve your problem.

Problem

Show your work.

Solution

Develop Multiplying with Mixed Numbers in Word Problems

Read and try to solve the problem below.

Janie has a rectangular garden that is $2\frac{3}{4}$ yards in length and 1 yard in width. She grows roses in $\frac{1}{2}$ of her garden. How many square yards in Janie's garden has roses?

TRY IT

Math Toolkit

- fraction tiles or circles
- fraction bars
- fraction models
- grid paper
- number lines
- index cards
- multiplication models

DISCUSS IT

Ask your partner: Do you agree with me? Why or why not?

Tell your partner: At first, I thought . . .

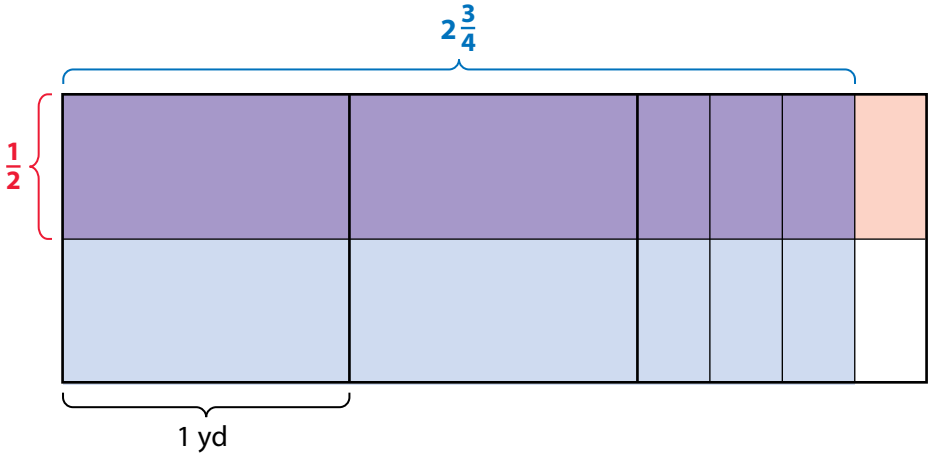
Explore different ways to understand multiplying fractions and mixed numbers.

Janie has a rectangular garden that is $2\frac{3}{4}$ yards in length and 1 yard in width. She grows roses in $\frac{1}{2}$ of her garden. How many square yards in Janie's garden has roses?

PICTURE IT

You can use an area model to help you understand the problem.

The purple shaded region of the area model shows **half** of $2\frac{3}{4}$.



MODEL IT

You can write equations to model the problem.

You can write $2\frac{3}{4}$ as a fraction.

$$2\frac{3}{4} = 2 + \frac{3}{4}$$
$$= \frac{8}{4} + \frac{3}{4}$$
$$= \frac{11}{4}$$

You need to find a fraction of a fraction: $\frac{1}{2}$ of $\frac{11}{4}$ square yards.

$\frac{1}{2}$ of $\frac{11}{4}$ means $\frac{1}{2} \times \frac{11}{4}$.

$$\frac{1}{2} \times \frac{11}{4} = \frac{1 \times 11}{2 \times 4}$$

CONNECT IT

Now you will use the problem from the previous page to understand how to multiply fractions and mixed numbers.

1. Use the last equation in **Model It** to find the area of Janie's garden that has roses.

   Janie's garden has ..... square yards of roses.

   Explain how you can use the area model in **Picture It** to find the area of Janie's garden that has roses.

2. Look at the first equation in **Model It**. Why is the mixed number rewritten as a fraction?

3. How can you multiply $\frac{1}{2} \times 2\frac{3}{4}$ without changing $2\frac{3}{4}$ to a fraction?

   What is $\frac{1}{2} \times 2$? ..... What is $\frac{1}{2} \times \frac{3}{4}$? .....

   Add the two products. ..... + ... = ...

   Is this result the same as your answer to problem 1? .....

4. How can you multiply a mixed number by a fraction?

5 REFLECT

Look back at your **Try It**, strategies by classmates, and **Picture It** and **Model It**. Which models or strategies do you like best for multiplying fractions and mixed numbers? Explain.

.....

.....

.....

APPLY IT

Use what you just learned to solve these problems.

6 Izzy has $3\frac{1}{2}$ yards of rope. She uses $\frac{3}{5}$ of the rope to attach a tire swing to a tree in her yard. How many yards of rope does Izzy use for the tire swing? Show your work.

Solution

7 Colin has a chain that is $\frac{5}{6}$ foot long. He adds links to his chain so that it is $4\frac{1}{2}$ times as long as the original chain. How many feet long is his chain now? Show your work.

Solution

8 George has $1\frac{5}{9}$ yards of fabric. He plans to use $\frac{3}{4}$ of the fabric to make a pillow. How many yards of fabric will George use for the pillow?

Ⓐ $1\frac{6}{36}$

Ⓑ $1\frac{8}{13}$

Ⓒ $1\frac{17}{36}$

Ⓓ $1\frac{5}{12}$

Practice Multiplying with Mixed Numbers

Study the Example showing one way to solve a word problem with a mixed number. Then solve problems 1–5.

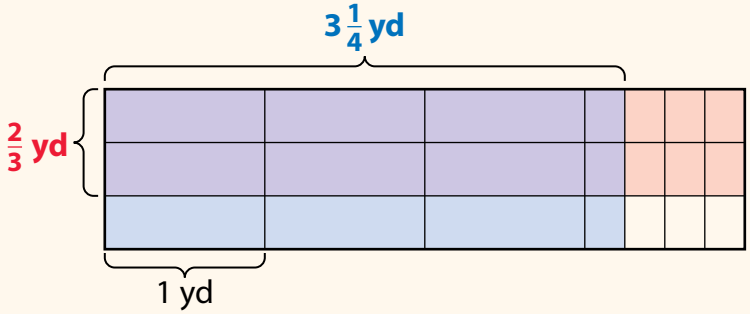
EXAMPLE

Mr. Urrego is painting his deck for the summer. He has painted a rectangular area that is $3\frac{1}{4}$ yards long and $\frac{2}{3}$ yard wide. How many square yards of deck are painted?

You can use an area model.

The larger sections of the area model are $\frac{1}{3} \times 1 = \frac{1}{3}$ square yard.

The smaller sections of the area model are $\frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$ square yard.



The model shows the number of square yards painted is:

$\frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{12} = \frac{6}{3} + \frac{2}{12} = 2 + \frac{2}{12} = 2\frac{2}{12}$

1 Write the missing numbers to complete the multiplication equation showing how much of the deck is painted.

Multiply the length and width of the painted area:

$$3\frac{1}{4} \times \frac{\square}{\square} = \left(\square \times \frac{2}{3}\right) + \left(\frac{\square}{\square} \times \frac{2}{3}\right) = \frac{\square}{3} + \frac{2}{\square} = \square\frac{2}{12}$$

_____ square yards

2 To multiply by a mixed number, you can also write the mixed number as a fraction and then multiply. Use this method to find the product $3\frac{1}{4} \times \frac{2}{3}$ in order to find how many square yards of the deck are painted. Show your work.

Solution _____

3 On Saturday, Kira ran $\frac{3}{4}$ mile. On Sunday, she ran $2\frac{1}{2}$ times as far as on Saturday. Use a multiplication equation to find how far Kira ran on Sunday. Show your work.

Solution

4 Use a visual model to show another way to find the distance Kira ran on Sunday.

5 The multipurpose room at the Cortez School is being set up for the annual book sale. Graphic novels will be displayed in a rectangular area $1\frac{1}{4}$ yards long and $\frac{3}{4}$ yard wide. Will the graphic novels be displayed in an area greater than or less than 1 square yard? Show your work.

Solution

Refine Multiplying Fractions in Word Problems

Complete the Example below. Then solve problems 1–8.

EXAMPLE

Chris uses $4\frac{1}{4}$ tubes of paint. Nico uses $1\frac{1}{2}$ times as much paint as Chris. How much paint did Nico use?

Look at how you can solve this problem using equations.

$$4\frac{1}{4} \times 1 = 4\frac{1}{4}$$

$$4\frac{1}{4} \times \frac{1}{2} = \left(4 \times \frac{1}{2}\right) + \left(\frac{1}{4} \times \frac{1}{2}\right) = 2 + \frac{1}{8}$$

$$4\frac{1}{4} + 2 + \frac{1}{8} = 6\frac{1}{4} + \frac{1}{8} = 6\frac{2}{8} + \frac{1}{8} = 6\frac{3}{8}$$

Solution

Breaking apart a mixed number happens twice in this problem.

PAIR/SHARE

How does the product compare to $4\frac{1}{4}$?

APPLY IT

1 Josh exercises at the gym $3\frac{3}{4}$ hours a week. He spends $\frac{2}{5}$ of his time at the gym lifting weights. How many hours a week does Josh spend lifting weights at the gym? Show your work.

Solution

How do I know what operation to use to solve this problem?

PAIR/SHARE

What is a reasonable estimate for the number of hours Josh lifts weights each week?

2 A field is in the shape of a rectangle $\frac{5}{6}$ mile long and $\frac{3}{4}$ mile wide. What is the area of the field? Show your work.

What model can I use to help understand this problem?

PAIR/SHARE
Can you solve this problem in another way?

Solution

3 Ari had $\frac{3}{4}$ of a bag of popcorn. His friends ate $\frac{1}{2}$ of his popcorn. What fraction of the whole bag of popcorn did Ari's friends eat?

What equation can I write to solve this problem?

Ⓐ $\frac{1}{4}$

Ⓑ $\frac{3}{8}$

Ⓒ $\frac{5}{4}$

Ⓓ $\frac{3}{2}$

Kayla chose Ⓐ as the correct answer. How did she get that answer?

PAIR/SHARE
Does Kayla's answer make sense?

4 On Sunday, Kristen bought a carton of 24 bottles of water.

- On Monday, Kristen drank $\frac{1}{6}$ of the bottles in the carton.
- On Tuesday, Kristen drank $\frac{1}{4}$ of the bottles that remained in the carton after Monday.

Which picture represents the number of bottles of water remaining in the carton after Kristen drank her water on Tuesday?

Ⓐ 

Ⓑ 

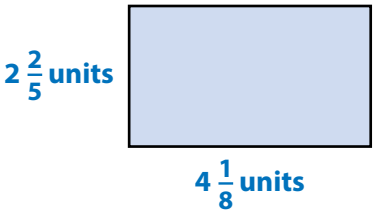
Ⓒ 

Ⓓ 

5 Milo's pancake recipe makes 9 servings. It calls for $\frac{3}{4}$ cup milk. Milo wants to make 6 servings. How much milk will he need?

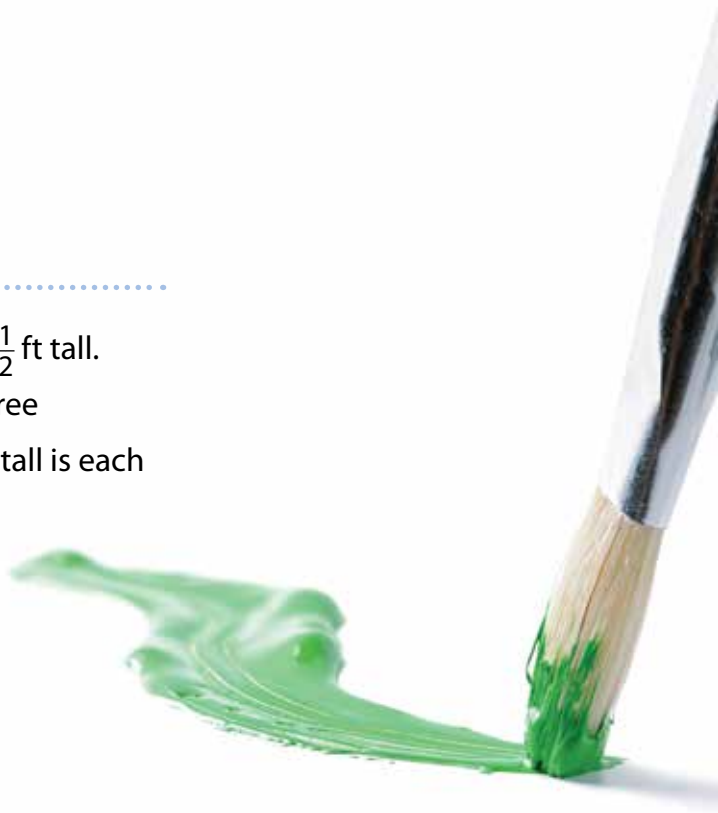
..... cup

6 Jillian draws a rectangle with the dimensions shown below. What is the area of Jillian's rectangle?



Solution

7 Lily paints 3 trees for a wall mural. The middle tree is $2\frac{1}{2}$ ft tall. The tree on the left is $\frac{3}{4}$ as tall as the middle tree. The tree on the right is $1\frac{3}{4}$ times as tall as the middle tree. How tall is each tree? Show your work.



Solution

8 MATH JOURNAL

Write a word problem for the expression $3\frac{1}{2} \times \frac{1}{2}$. Use a visual model or an equation to show how to solve your problem.

SELF CHECK Go back to the Unit 3 Opener and see what you can check off.

Explore Dividing Unit Fractions in Word Problems

Learning Target

- Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem.

SMP 1, 2, 3, 4, 5, 6, 8

Previously, you learned about what it means to divide with unit fractions. Use what you know to try to solve the problem below.

Micah is running a 6-mile race. There are water stops every $\frac{1}{2}$ mile and at the 6-mile finish line. How many water stops are there in all? Use a visual model to show your solution.

TRY IT

Math Toolkit

- fraction bars
- fraction models
- number lines
- grid paper
- index cards
- sticky notes

DISCUSS IT

Ask your partner: Why did you choose that strategy?

Tell your partner: A model I used was . . . It helped me . . .

CONNECT IT

1 LOOK BACK

How many water stops are there in all? Explain how you can use a number line to support your answer.

2 LOOK AHEAD

On the previous page, you used a visual model to solve a word problem involving dividing a whole number by a unit fraction. You can also use equations to represent and solve these types of problems. Consider this word problem.

Micah now runs in a 5-mile race. There are water stops every $\frac{1}{3}$ mile and at the 5-mile finish line in this new race. How many water stops are there in all?

a. Complete the division equation below.

miles in race ..... ÷ ..... = w number of water stops

fraction of a mile for each water stop

b. Complete the multiplication equation below.

miles in race ..... × ..... = w number of water stops

number of water stops in each mile

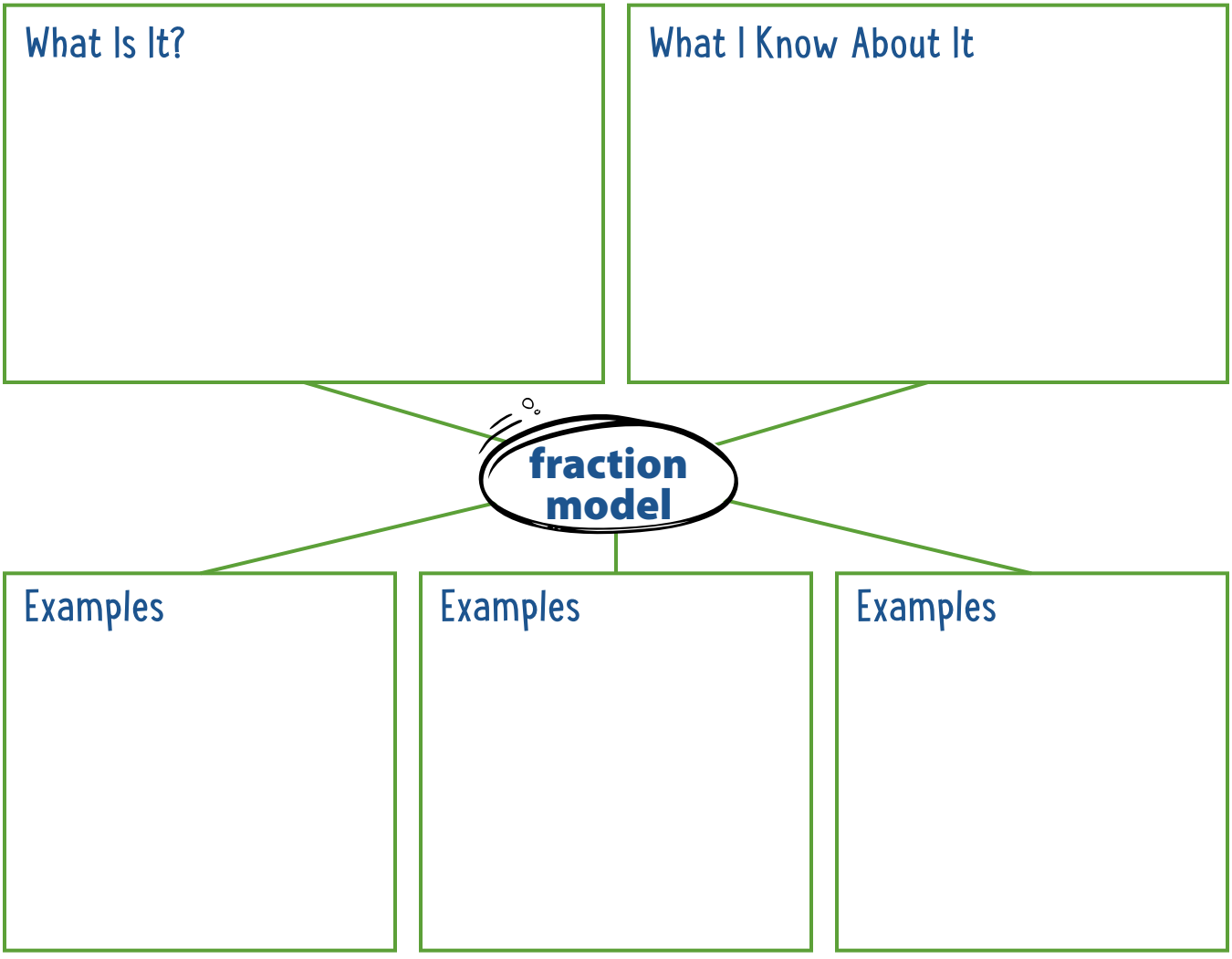
c. How many water stops are in this race? Explain how you know.

3 REFLECT

Explain what it means to divide 5 by $\frac{1}{3}$, or $5 \div \frac{1}{3}$.

Prepare for Dividing Unit Fractions in Word Problems

1 Think about what you know about fraction models. Fill in each box. Use words, numbers, and pictures. Show as many ideas as you can.



2 Draw a fraction model to show the expression $4 \div \frac{1}{2}$.

3 Solve the problem. Show your work.

Adela has a ribbon that is 2 yards long. She cuts the ribbon into pieces that are $\frac{1}{4}$ yard long. How many pieces of ribbon are there in all? Use a visual model to show your solution.

Solution

4 Check your answer. Show your work.

Develop Dividing a Unit Fraction by a Whole Number

Read and try to solve the problem below.

Piper uses $\frac{1}{6}$ yard of ribbon to make a border around an equilateral triangle. How long is the piece of ribbon that Piper uses for each side?

TRY IT

Math Toolkit

- fraction bars
- fraction models
- number lines
- grid paper
- ribbon or yarn
- index cards

DISCUSS IT

Ask your partner: How did you get started?

Tell your partner: I knew . . . so I . . .

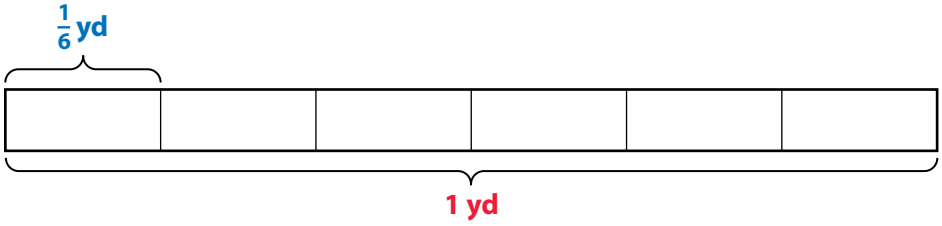
Explore different ways to understand dividing a unit fraction by a whole number to solve word problems.

Piper uses $\frac{1}{6}$ yard of ribbon to make a border around an equilateral triangle. How long is the piece of ribbon that Piper uses for each side?

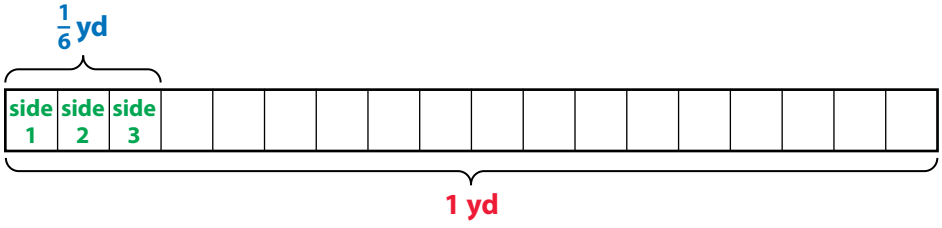
PICTURE IT

You can draw a picture to help understand the problem.

Draw a **1-yard** length of ribbon and then draw and label a **$\frac{1}{6}$-yard** length.



Divide the **$\frac{1}{6}$-yard** length into **3 equal parts**, one for each side of the equilateral triangle.



MODEL IT

You can use equations to model the problem.

Write a division equation.

$\frac{1}{6} \div 3 = s$ — length of each part

length of ribbon — $\frac{1}{6}$

number of equal parts — 3

Write a multiplication equation.

$\frac{1}{3} \times \frac{1}{6} = s$ — length of each part

fraction of the ribbon — $\frac{1}{3}$

length of ribbon — $\frac{1}{6}$

CONNECT IT

Now you will use the problem from the previous page to help you understand how to divide a unit fraction by a whole number.

1 Look at **Picture It**. What does the first diagram show? What whole is being divided?

Why does the second diagram show each $\frac{1}{6}$-yard section divided into 3 equal parts?

2 Look at **Model It**. How does each equation relate to the second diagram in **Picture It**?

3 How long is the piece of ribbon Piper uses for each side of the triangle?

4 What is $\frac{1}{6} \div 3$? How can you use a multiplication equation different from the one shown in **Model It** to check that your answer is correct?

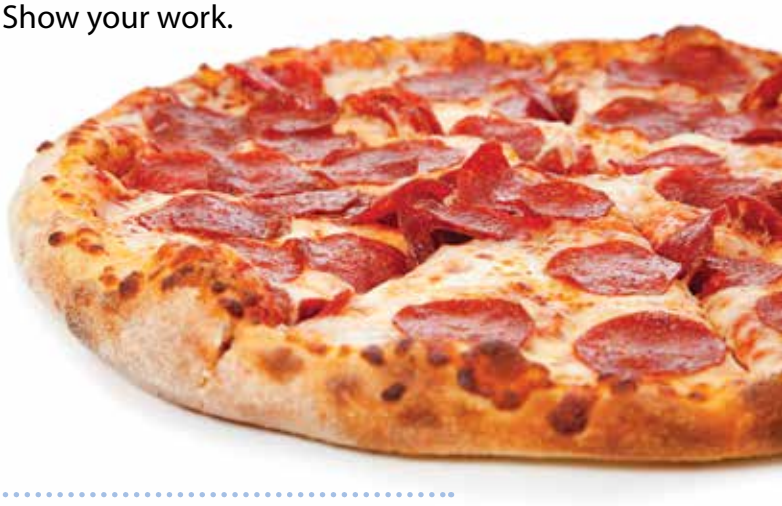
5 REFLECT

Look back at your **Try It**, strategies by classmates, and **Picture It** and **Model It**. Which models or strategies do you like best for dividing a unit fraction by a whole number to solve word problems? Explain.

APPLY IT

Use what you just learned to solve these problems.

6 Felipe has $\frac{1}{4}$ of a pizza. He wants to share it equally with a friend. How much of the original whole pizza will each of them get? Show your work.



Solution

7 Angela uses $\frac{1}{3}$ of her rectangular flower garden for roses. She plants equal rectangular areas of red, white, pink, and orange roses in this part of the garden. What fraction of the whole garden has red roses? Draw a model and write a division equation to represent and solve the problem.

Solution

8 Look at problem 7. Which multiplication expressions can be used to represent the situation or check the division equation?

Ⓐ $\frac{1}{4} \times \frac{1}{3}$

Ⓑ $4 \times \frac{1}{3}$

Ⓒ $\frac{1}{12} \times 4$

Ⓓ 3×4

Ⓔ $3 \times \frac{1}{4}$

Practice Dividing a Unit Fraction by a Whole Number

Study the Example showing one way to solve a word problem involving dividing a fraction by a whole number. Then solve problems 1–5.

EXAMPLE

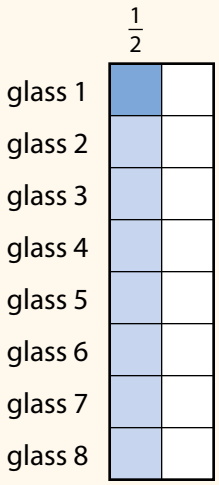
Felicia makes $\frac{1}{2}$ gallon of fruit punch. She pours an equal amount into 8 glasses. What fraction of a gallon of fruit punch is in each glass?

Find $\frac{1}{2} \div 8$.

The model shows a rectangle divided into halves and then divided into 8 equal parts. There are a total of 16 parts, and one part is the amount of fruit punch in 1 glass.

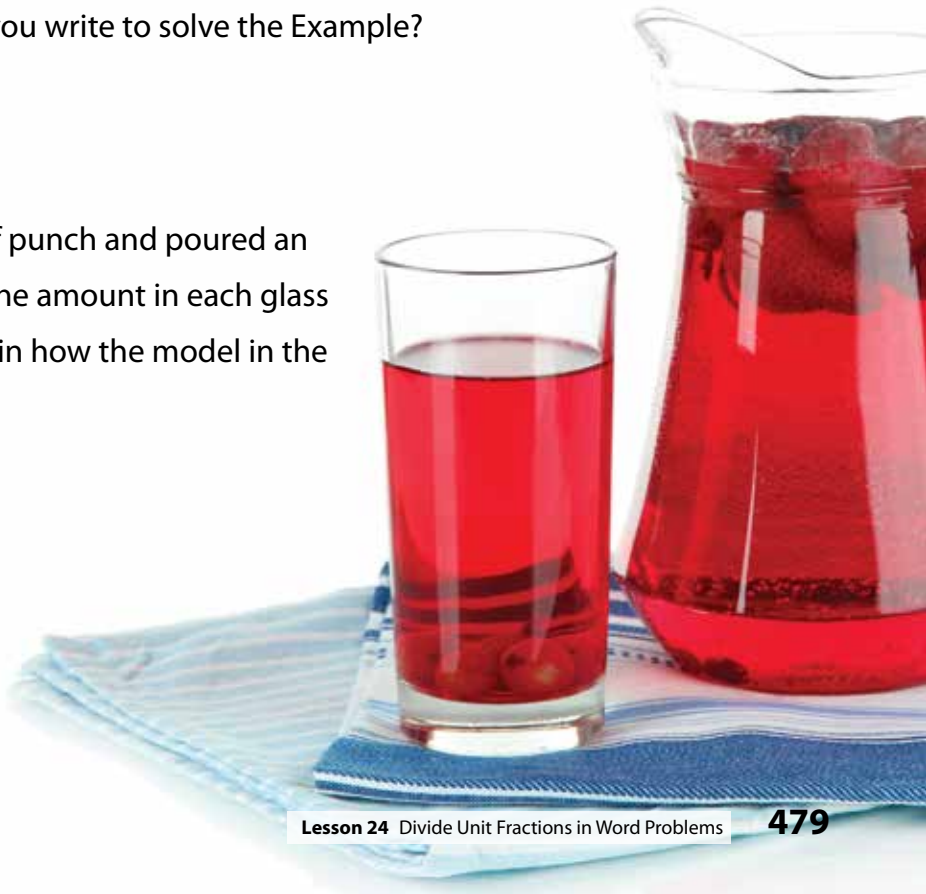
$$\frac{1}{2} \div 8 = \frac{1}{16}$$

The amount in 1 glass is $\frac{1}{16}$ gallon.



1 What multiplication equation could you write to solve the Example?

2 Suppose Felicia had made $\frac{1}{4}$ gallon of punch and poured an equal amount into 8 glasses. Would the amount in each glass be more or less than $\frac{1}{16}$ gallon? Explain how the model in the Example would change to show this.



3 Donal buys a $\frac{1}{4}$-pound package of cheese. There are 8 slices of cheese in the package. Each slice has the same weight. What fraction of a pound is each slice? Draw a model and write a division equation to represent and solve the problem.

Solution

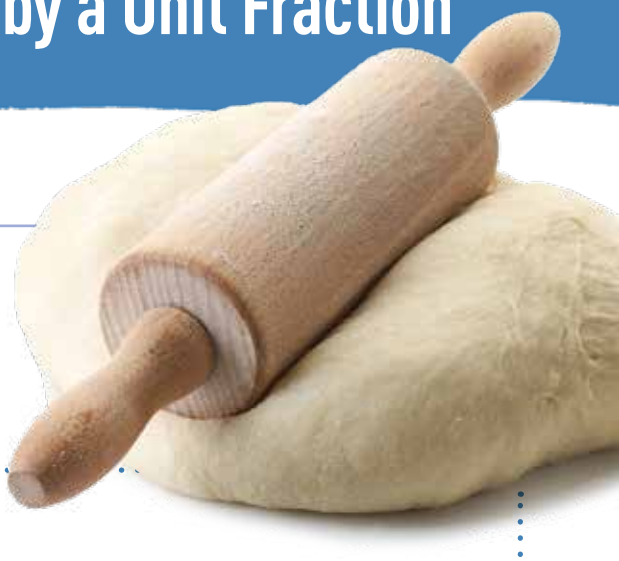
4 Student volunteers are getting ready to hand out programs at a talent show. Leah and Tomas are each given $\frac{1}{2}$ of a stack of programs to hand out. Leah divides her $\frac{1}{2}$ equally among herself and 2 friends. What fraction of the original stack of programs do Leah and her 2 friends each have? Show your work.

Solution

5 Look at problem 4. If Tomas divides his stack of programs between himself and his 3 friends, what fraction of the original stack will each of his friends have? Write a division equation to represent and solve the problem.

Solution

Develop Dividing a Whole Number by a Unit Fraction



Read and try to solve the problem below.

Alex makes 2 pounds of bread dough. He splits the dough into $\frac{1}{4}$-pound loaves before baking them in the oven. How many loaves does he make?

TRY IT

Math Toolkit

- fraction tiles
- fraction bars
- fraction models
- number lines
- grid paper
- index cards

DISCUSS IT

Ask your partner: Can you explain that again?

Tell your partner: The strategy I used to find the answer was . . .

Explore different ways to understand how to divide a whole number by a unit fraction in order to solve word problems.

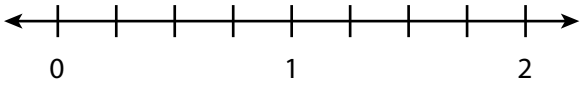
Alex makes 2 pounds of bread dough. He splits the dough into $\frac{1}{4}$-pound loaves before baking them in the oven. How many loaves does he make?

MODEL IT

You can use a number line to help understand the problem.

Draw a number line and label it to show the 2 pounds of bread dough.

Mark the number line to divide each whole into fourths.



MODEL IT

You can use what you know about equations, equivalent fractions, and common denominators to solve the problem.

The equation $2 \div \frac{1}{4} = n$ models the problem with n being the number of loaves Alex makes.

Write the numbers in the equation with a common denominator.

$$\frac{8}{4} \div \frac{1}{4} = n$$

Now you can divide $\frac{8}{4}$ into equal groups of $\frac{1}{4}$.

CONNECT IT

Now you will use the problem from the previous page to help you understand how to divide a whole number by a unit fraction.

1. In the first **Model It** number line, how are the 2 pounds of bread dough represented?

2. How are the $\frac{1}{4}$-pound loaves represented on the number line?

3. How many fourths are in one whole? In two wholes?

4. Look at the second **Model It**. How was the equation $2 \div \frac{1}{4} = n$ changed to an equation involving fractions with common denominators?

5. How many groups of $\frac{1}{4}$ are in $\frac{8}{4}$? What is $\frac{8}{4} \div \frac{1}{4}$? Explain.

6. How many loaves does Alex make? How are the first **Model It** and second **Model It** alike in showing how to find the solution?

7. What multiplication equation can you write to check your answer to $2 \div \frac{1}{4}$? Explain.

8 REFLECT

Look back at your **Try It**, strategies by classmates, and **Model Its**. Which models or strategies do you like best for dividing a whole number by a unit fraction? Explain.

APPLY IT

Use what you just learned to solve these problems.

9 Stacy has 4 sheets of paper to make cards. Each card requires $\frac{1}{2}$ sheet of paper. How many cards can Stacy make? Draw a model and write a division equation to represent and solve the problem.

Solution

10 Look at problem 9 above. Which multiplication expressions can be used to represent the situation or check the division equation?

Ⓐ 8×2

Ⓑ $4 \times \frac{1}{2}$

Ⓒ $16 \times \frac{1}{2}$

Ⓓ $8 \times \frac{1}{2}$

Ⓔ 4×2

11 Dylan makes 3 submarine sandwiches. He cuts each sandwich into sixths to share. He stacks all the sandwich pieces on a plate. How many sandwich pieces does Dylan stack on the plate? Show your work.

Solution

Practice Dividing a Whole Number by a Unit Fraction

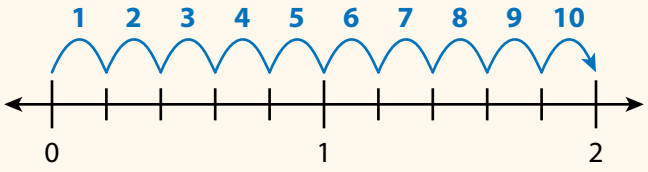
Study the Example showing one way to solve a word problem involving dividing a whole number by a fraction. Then solve problems 1–6.

EXAMPLE

Darius walks dogs at an animal shelter. He walks each dog for $\frac{1}{5}$ hour. He walks the dogs one at a time. How many dogs can Darius walk in 2 hours?

Find $2 \div \frac{1}{5}$.

The number line shows two hours. Each hour is divided into fifths.



There are 10 fifths in 2.

$$2 \div \frac{1}{5} = 10$$

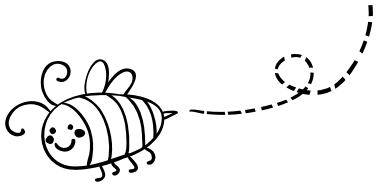
Darius can walk 10 dogs in 2 hours.

1. What multiplication equation could you write to solve the Example?

2. Use the information from the Example. In one month, Darius spends 9 hours walking dogs. How many times does he walk a dog in one month?

3. Explain how you got your answer to problem 2.

4 Mrs. Wing will tape up posters made by her students on the wall. She cuts tape into $\frac{1}{4}$-foot pieces. How many $\frac{1}{4}$-foot pieces can she cut from 5 feet of tape? Show your work.



Solution

5 Taylor is helping decorate tables with flowers for a graduation celebration. She has 7 bunches of tulips. She will put $\frac{1}{2}$ of each bunch in a vase. How many vases does she need? Draw a model and write a division equation to represent and solve the problem.

Solution

6 Look at how you solved problem 5. Use a different way to solve the problem and show how a multiplication equation can be used to check the answer.

Solution

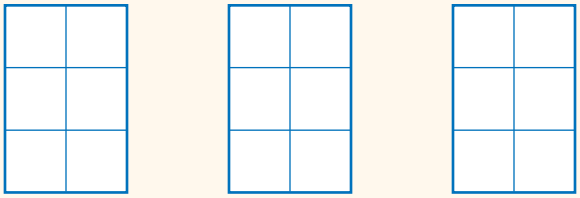
Refine Dividing Unit Fractions in Word Problems

Complete the Example below. Then solve problems 1–9.

EXAMPLE

Sierra has a photo album with 3 empty pages. Each photo uses $\frac{1}{6}$ of an album page. How many photos can Sierra put on the empty pages?

Look at how you could show your work using rectangles.



6 photos will fit on each of the 3 pages.

Solution

The student used a model to visualize the problem.

PAIR/SHARE

What related equations can you write to represent the problem?

APPLY IT

1 Corrine picked $\frac{1}{4}$ gallon of blackberries. She poured equal amounts of berries into 4 containers. What fraction of a gallon is in each container? Show your work.

Can you draw a model to help understand the problem?

PAIR/SHARE

How will the answer compare to $\frac{1}{4}$ gallon?

Solution

2 Cooper's USB drive is $\frac{1}{2}$ full with 5 video files. Each video file is the same size. What fraction of the USB drive does 1 video file use? Show your work.

How could I represent this problem using an equation?

PAIR/SHARE
How can you check your answer?

Solution

3 Devonte is studying for a history test. He uses $\frac{1}{8}$ of a side of one sheet of paper to write notes for each historical event. He fills 2 full sides of one sheet of paper. Which expression could be used to find how many events Devonte makes notes for?

Ⓐ $2 \times \frac{1}{8}$

Ⓑ $2 \div \frac{1}{8}$

Ⓒ $\frac{1}{8} \times 2$

Ⓓ $\frac{1}{8} \div 2$

Barry chose Ⓓ as the correct answer. How did he get that answer?

Is this problem like one you have seen before?

PAIR/SHARE
Does Barry's answer make sense?

4 Elise picks 6 pounds of apples. She uses $\frac{1}{2}$ pound of apples to make 1 container of applesauce. How many containers of applesauce can Elise make with all the apples?

Ⓐ 12 containers

Ⓑ $6\frac{1}{2}$ containers

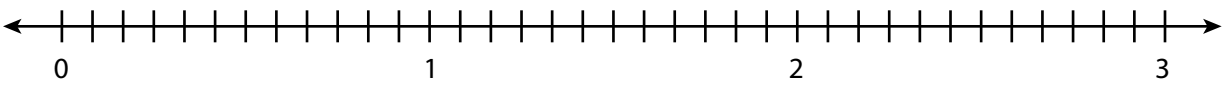
Ⓒ $5\frac{1}{2}$ containers

Ⓓ 3 containers

5 Students are running in a relay race. Each team will run a total of 3 miles. Each member of a team will run $\frac{1}{3}$ mile. How many students will a team need to complete the race? Circle the correct number below.

$\frac{1}{9}$ 3 9 12 36

You may use the number line to help find your answer.



6 Tanya has $\frac{1}{3}$ of a cake left over from a party. She cuts the leftover cake into 6 equal pieces to store in the freezer. What fraction of the original cake is each piece? Show your work.

Solution

7 Marina has a pattern to make bows that requires $\frac{1}{4}$ yard of ribbon for each bow. Fill in the table to show how many bows she can make from a given length of ribbon.

Ribbon Length (yards)	Number of Bows
1	
2	
3	
4	

8 **Part A** Ted serves $\frac{1}{6}$ gallon of ice cream. He puts an equal amount of ice cream in each of 4 bowls. How many gallons of ice cream does Ted put in each bowl? Use a visual model to support your answer.

Solution

Part B Write a division equation to represent this situation. Then write a multiplication equation you can use to check your answer.

Solution

9 MATH JOURNAL

Write a word problem represented by $\frac{1}{5} \div 4$. Explain or show how to find the answer.

SELF CHECK Go back to the Unit 3 Opener and see what you can check off.