Lesson Overview

LESSON 12

Add Fractions

Lesson Objectives

Content Objectives

- Given two fractions with unlike denominators, write equivalent fractions with a common denominator.
- Use visual models to represent adding fractions with unlike denominators.
- Use equivalent fractions to add fractions and mixed numbers with unlike denominators.

Language Objectives

- Define *common denominator* and use the term in a discussion with a partner.
- Draw fraction models or number lines to show a sum of fractions with unlike denominators.
- Rewrite sums of fractions with unlike denominators as sums with like denominators using equivalent fractions.

Prerequisite Skills

- Find multiples of whole numbers.
- Understand the meaning of adding fractions.
- Add fractions and mixed numbers with like denominators.
- Find fractions equivalent to a given fraction.
- Regroup fractions greater than 1 as mixed numbers.

Standards for Mathematical Practice (SMP)

SMPs 1, 2, 3, 4, 5, and 6 are integrated in every lesson through the *Try-Discuss-Connect* routine.*

In addition, this lesson particularly emphasizes the following SMPs:

2 Reason abstractly and quantitatively.

5 Use appropriate tools strategically.

7 Look for and make use of structure.

*See page 1i to see how every lesson includes these SMPs.

Lesson Vocabulary

There is no new vocabulary. Review the following key terms.

- **common denominator** a number that is a common multiple of the denominators of two or more fractions.
- **denominator** the number below the line in a fraction that tells the number of equal parts in the whole.
- **equivalent fractions** two or more different fractions that name the same part of a whole or the same point on a number line.
- **mixed number** a number with a whole number part and a fractional part.
- **multiple** the product of a given number and any other whole number.
- **numerator** the number above the line in a fraction that tells the number of equal parts that are being described.

Learning Progression

In Grade 4 students added fractions (including mixed numbers) with like denominators. They understood the concept of fraction addition as similar to addition with whole numbers. Students also learned to find equivalent fractions by multiplying the numerator and denominator of a fraction by the same number.

In this lesson students extend the concept of fraction addition to adding fractions (including mixed numbers) with unlike denominators. They replace fractions with equivalent fractions to show equivalent sums of fractions and mixed numbers with like denominators. Then they add the fractions using the methods they used in Grade 4.

In the next lesson students will subtract fractions and mixed numbers with unlike denominators. Students will do further work with estimating sums and differences of decimals in word problem contexts in a later lesson.

Lesson Pacing Guide

Whole Class Instruction

SESSION 1 Explore *45–60 min*	**Adding Fractions** • Start *5 min* • Try It *10 min* • Discuss It *10 min* • Connect It *15 min* • Close: Exit Ticket *5 min*	**Additional Practice** Lesson pages 229–230
SESSION 2 Develop *45–60 min*	**Adding Fractions with Unlike Denominators** • Start *5 min* • Try It *10 min* • Discuss It *10 min* • Picture It & Model It *5 min* • Connect It *10 min* • Close: Exit Ticket *5 min*	**Additional Practice** Lesson pages 235–236 **Fluency** Adding Fractions with Unlike Denominators
SESSION 3 Develop *45–60 min*	**Adding with Mixed Numbers** • Start *5 min* • Try It *10 min* • Discuss It *10 min* • Picture It & Model It *5 min* • Connect It *10 min* • Close: Exit Ticket *5 min*	**Additional Practice** Lesson pages 241–242 **Fluency** Adding with Mixed Numbers
SESSION 4 Refine *45–60 min*	**Adding Fractions** • Start *5 min* • Example & Problems 1–3 *15 min* • Practice & Small Group Differentiation *20 min* • Close: Exit Ticket *5 min*	**Lesson Quiz** or **Digital Comprehension Check**

Lesson Materials

Lesson *(Required)*	*Per pair:* 1 set of fraction tiles, 2 sets of fraction circles *Activity Sheet:* 1-Centimeter Grid Paper
Activities	*Per pair:* 1 set of fraction circles, 1 set of fraction tiles *Activity Sheet:* Number Lines
Math Toolkit	fraction tiles, fraction circles, fraction bars, grid paper, number lines
Digital Math Tools	Fraction Models, Number Line

Teacher Toolbox

Small Group Differentiation

PREPARE

Ready Prerequisite Lessons

Grade 4
- Lesson 17 Understand Equivalent Fractions
- Lesson 20 Add and Subtract Fractions
- Lesson 21 Add and Subtract Mixed Numbers

RETEACH

Tools for Instruction

Grade 4
- Lesson 17 Equivalent Fractions
- Lesson 20 Add and Subtract Fractions
- Lesson 21 Add and Subtract Mixed Numbers

Grade 5
- Lesson 12 Add Fractions with Unlike Denominators

REINFORCE

Math Center Activity

Grade 5
- Lesson 12 Fraction Addition: True or False!

EXTEND

Enrichment Activity

Grade 5
- Lesson 12 Addition Grids

Connect to Family, Community, and Language Development

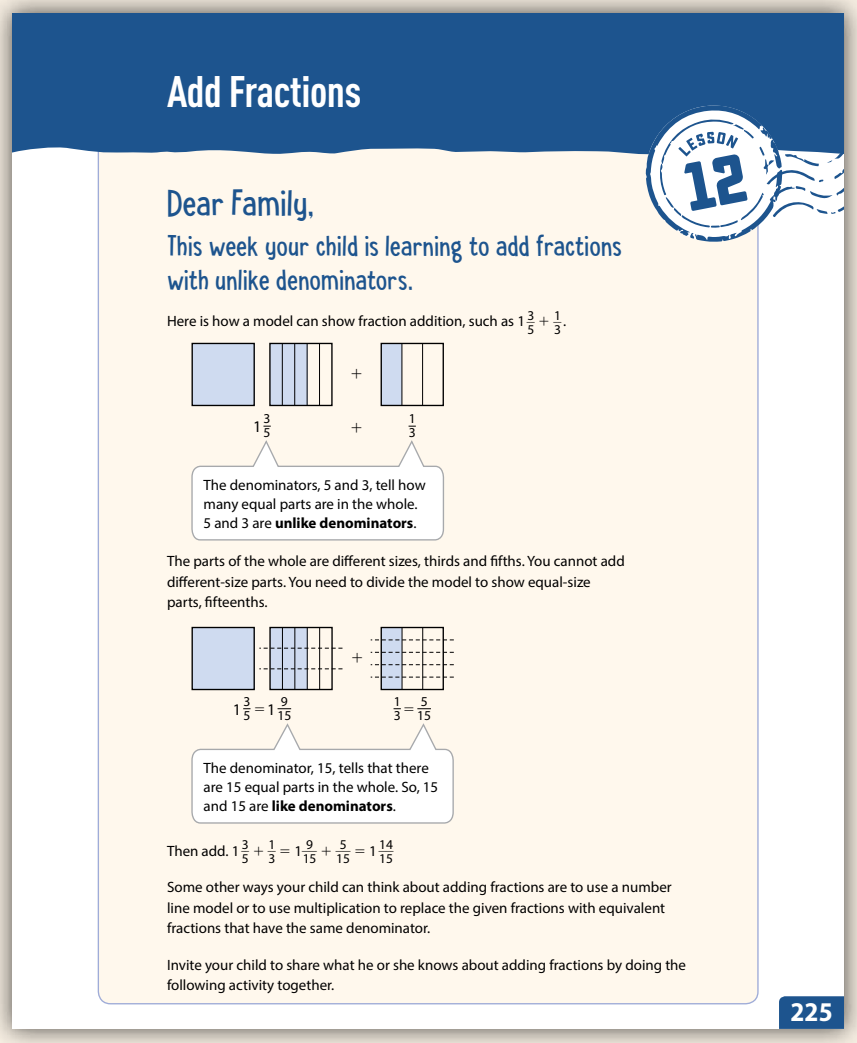
The following activities and instructional supports provide opportunities to foster school, family, and community involvement and partnerships.

Connect to Family

Use the **Family Letter**—which provides background information, math vocabulary, and an activity—to keep families apprised of what their child is learning and to encourage family involvement.

Available in Spanish

Teacher Toolbox



Add Fractions

LESSON 12

Dear Family,

This week your child is learning to add fractions with unlike denominators.

Here is how a model can show fraction addition, such as $1\frac{3}{5} + \frac{1}{3}$.

The denominators, 5 and 3, tell how many equal parts are in the whole. 5 and 3 are **unlike denominators**.

The parts of the whole are different sizes, thirds and fifths. You cannot add different-size parts. You need to divide the model to show equal-size parts, fifteenths.

The denominator, 15, tells that there are 15 equal parts in the whole. So, 15 and 15 are **like denominators**.

Then add. $1\frac{3}{5} + \frac{1}{3} = 1\frac{9}{15} + \frac{5}{15} = 1\frac{14}{15}$

Some other ways your child can think about adding fractions are to use a number line model or to use multiplication to replace the given fractions with equivalent fractions that have the same denominator.

Invite your child to share what he or she knows about adding fractions by doing the following activity together.

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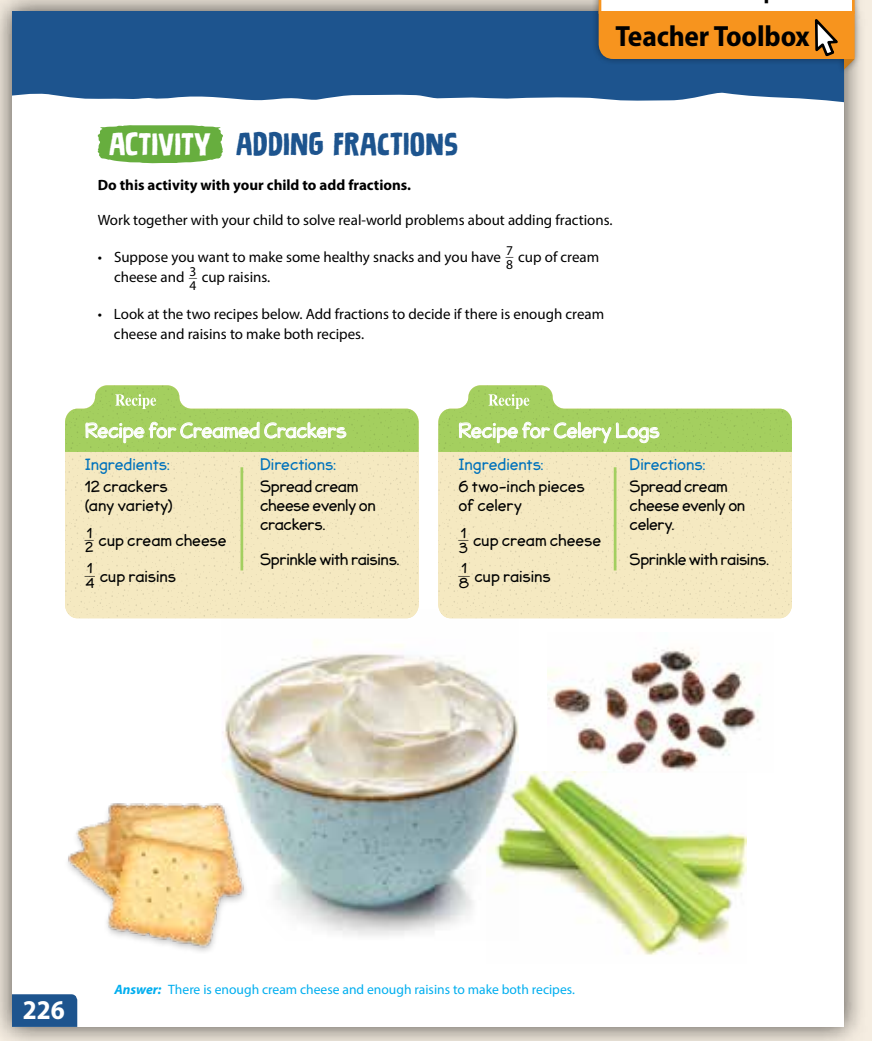

ACTIVITY ADDING FRACTIONS

Do this activity with your child to add fractions.

Work together with your child to solve real-world problems about adding fractions.

- Suppose you want to make some healthy snacks and you have $\frac{7}{8}$ cup of cream cheese and $\frac{3}{4}$ cup raisins.
- Look at the two recipes below. Add fractions to decide if there is enough cream cheese and raisins to make both recipes.

Recipe

Recipe for Creamed Crackers

Ingredients:
12 crackers (any variety)
$\frac{1}{2}$ cup cream cheese
$\frac{1}{4}$ cup raisins

Directions:
Spread cream cheese evenly on crackers.
Sprinkle with raisins.

Recipe

Recipe for Celery Logs

Ingredients:
6 two-inch pieces of celery
$\frac{1}{3}$ cup cream cheese
$\frac{1}{8}$ cup raisins

Directions:
Spread cream cheese evenly on celery.
Sprinkle with raisins.

Answer: There is enough cream cheese and enough raisins to make both recipes.

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Goal

The goal of the Family Letter is to enlist the family in helping students learn how to add fractions with unlike denominators.

- Students are encouraged to discuss adding fractions using different strategies, such as using an area model, a number line, or multiplication to find equivalent fractions with like denominators.

Activity

Students and family members review two recipes with fractions with unlike denominators to determine if there are enough ingredients to make both dishes. Look at the *Adding Fractions* activity and adjust if needed to connect with students.

Math Talk at Home

Encourage students to discuss situations in which they have to add fractions at home. This could be when using recipes to prepare food dishes or when mixing soil and fertilizer in a garden.

Conversation Starters Below are additional conversation starters students can write in their Family Letter or math journal to engage family members:

- *How do you add fractions in recipes?*
- *Have you had to adjust a recipe for more people? How do you calculate the amounts?*
- *Do all the cooking tools use only fractions? Why?*

Connect to Community and Cultural Responsiveness

Use these activities to connect with and leverage the diverse backgrounds and experiences of all students.

Sessions 1 and 2 **Use anytime during these sessions.**

- Adding fractions is very common when using recipes. Ask students to bring recipes that use fractions from their homes. Have students work in small groups and search online or in cookbooks for recipes that use fractions to specify the amounts of some ingredients. Encourage students to make different scenarios in which they must double or triple the amounts called for in a recipe to feed a larger number of people, such as their fellow students in class.

Sessions 3 and 4 **Use anytime during these sessions.**

- Have students form small groups and assign one of the following countries to each group: India, China, France, Italy, or Russia. Have students research a traditional dessert from each of the countries and make a chart that lists the ingredients for each recipe. Ask students to compare the ingredients for the different recipes. If there is a common ingredient in all the recipes, have students find which recipe uses the most of that ingredient and which recipe uses the least.

Connect to Language Development

For ELLs, use the Differentiated Instruction chart to plan and prepare for specific activities in every session.

ELL English Language Learners: Differentiated Instruction

Prepare for Session 1
Use with *Try It*.

Levels 1–3	Levels 2–4	Levels 3–5
Reading/Speaking Before reading the ***Try It*** problem, have students underline the terms *like denominators* and *unlike denominators* in the directions. Read the ***Try It*** problem and ask students to identify the unlike denominators. Prompt students to restate the problem to a partner, using the picture of the stick of butter to support their retellings. Allow time for students to solve the problem independently, then call on volunteers to share their solution strategies. Validate understanding and revoice their answers as needed to model fluent speaking.	**Reading/Speaking** Read the ***Try It*** problem with students. Organize them into small groups to share ideas about how to add fractions with unlike denominators. Have each group member choose a tool from the *Math Toolkit* to model the problem and explain why they think it is the best tool for the problem. After each group member has shared, give students the opportunity to change their mind if they discovered a better tool during the discussion. Allow time for students to engage in productive struggle to solve the problem independently, then have them compare answers with the group.	**Reading/Speaking** Have students read the ***Try It*** problem. Allow time for students to apply what they know about adding like denominators and choose a strategy for adding unlike denominators. Have them form pairs to compare answers and solutions strategies. If possible, have students find a partner who solved the problem using a different method. Have partners who solved the problem differently compare and contrast their solution strategies. Ask partners who used the same strategy to think of a different way to solve or model the problem together.

LESSON 12

SESSION 1 Explore

Purpose In this session students draw on their understanding of adding fractions with like denominators. They explore various solution methods based on replacing one fraction in a sum with an equivalent fraction to produce fractions with like denominators. They will look ahead to think about sums in which both addends need to be replaced with equivalent fractions.

Start

Connect to Prior Knowledge

Materials For each pair: 1 set of fraction tiles

Why Support students' facility with finding equivalent fractions for $\frac{1}{2}$, foreshadowing the work they will do with equivalent fractions to solve the *Try It* problem.

How Have students use sets of fraction tiles or fraction circles to build 5 possible equivalent fractions for $\frac{1}{2}$.

Use fraction tiles to make five fractions equivalent to $\frac{1}{2}$.

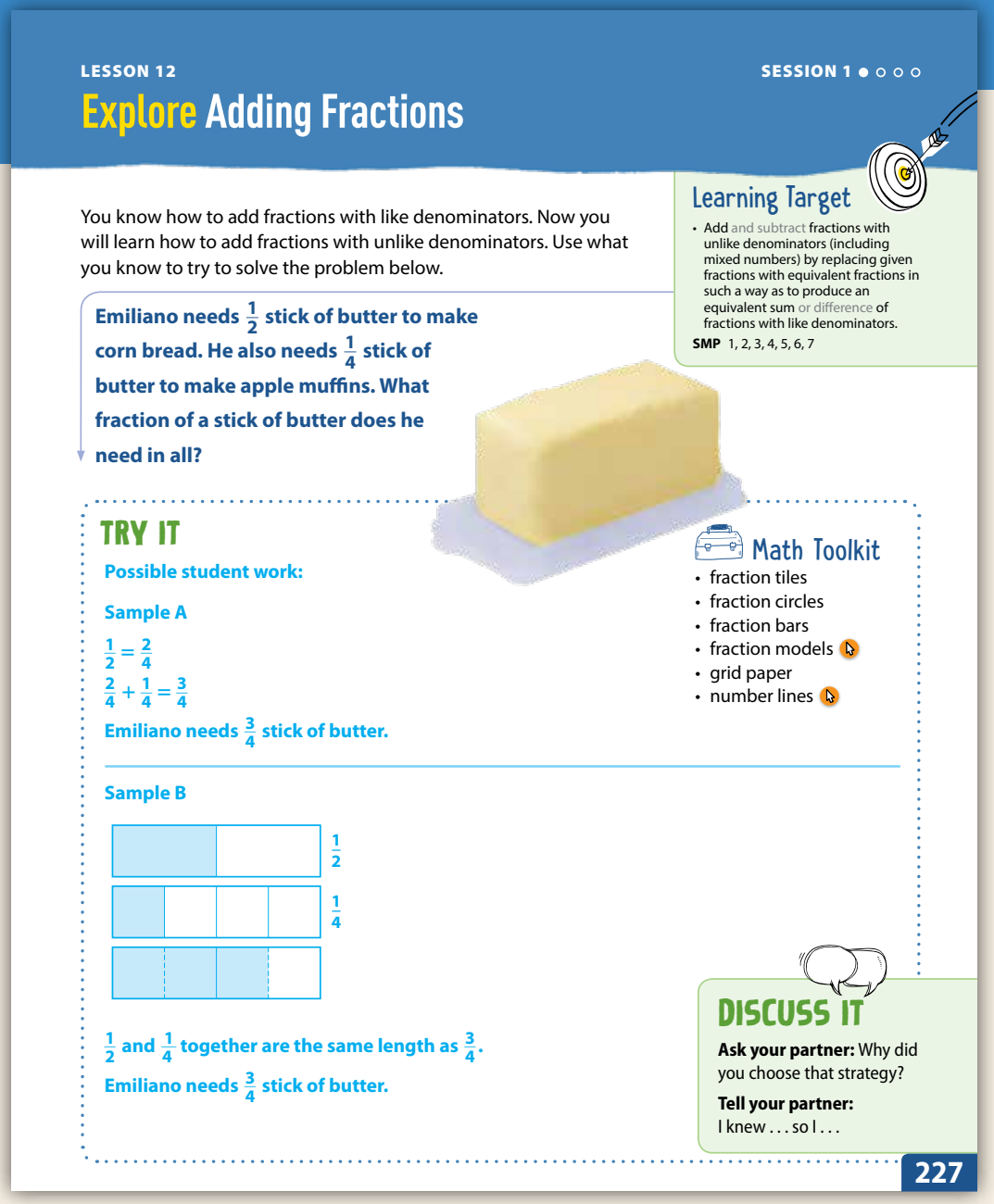
$\frac{1}{2} =$ ..... $\frac{1}{2} =$ .....

$\frac{1}{2} =$ ..... $\frac{1}{2} =$ .....

$\frac{1}{2} =$ .....

Possible Solutions

$\frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \frac{5}{10}, \frac{6}{12}$



LESSON 12

SESSION 1 ● ○ ○ ○

Explore Adding Fractions

You know how to add fractions with like denominators. Now you will learn how to add fractions with unlike denominators. Use what you know to try to solve the problem below.

Learning Target

- Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators.

SMP 1, 2, 3, 4, 5, 6, 7

Emiliano needs $\frac{1}{2}$ stick of butter to make corn bread. He also needs $\frac{1}{4}$ stick of butter to make apple muffins. What fraction of a stick of butter does he need in all?

TRY IT

Possible student work:

Sample A

$\frac{1}{2} = \frac{2}{4}$

$\frac{2}{4} + \frac{1}{4} = \frac{3}{4}$

Emiliano needs $\frac{3}{4}$ stick of butter.

Math Toolkit

- fraction tiles
- fraction circles
- fraction bars
- fraction models
- grid paper
- number lines

Sample B

$\frac{1}{2}$

$\frac{1}{4}$

$\frac{1}{2}$ and $\frac{1}{4}$ together are the same length as $\frac{3}{4}$.

Emiliano needs $\frac{3}{4}$ stick of butter.

DISCUSS IT

Ask your partner: Why did you choose that strategy?

Tell your partner: I knew . . . so I . . .

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TRY IT

Make Sense of the Problem

To support students in making sense of the problem, have them identify the amount of butter Emiliano needs for corn bread and for apple muffins.

DISCUSS IT

Support Partner Discussion

Encourage students to share what did not work for them as well as what did as they talk to each other.

Look for, and prompt as necessary for, understanding of:

- only same-size parts can be added
- an equivalent amount for $\frac{1}{2}$ can be shown using fourths

Common Misconception Look for students who count parts without concern that each whole is divided into same-size parts. As students present solutions, have them specify why they used 4 as the denominator.

Select and Sequence Student Solutions

One possible order for whole class discussion:

- physical parts showing halves and fourths
- drawings to represent the problem
- number lines marked in fourths
- equations showing $\frac{1}{2}$ replaced with the equivalent fraction $\frac{2}{4}$

Support Whole Class Discussion

Prompt students to note the relationship between the numbers in each model and the numbers in the problem.

Ask *How do [student name]'s and [student name]'s models show the number of fourths that is the same amount as $\frac{1}{2}$ and the amount of butter needed in all?*

Listen for $\frac{2}{4}$ is the same amount of the whole as $\frac{1}{2}$. The parts $\frac{2}{4}$ and $\frac{1}{4}$ combined are $\frac{3}{4}$ of a stick of butter.

CONNECT IT

1 LOOK BACK

Look for understanding that an equivalent amount in fourths had to be shown for $\frac{1}{2}$ in order to find how much butter is needed in all.

Hands-On Activity

Use fraction circles to add fractions.

If . . . *students are unsure about the concept of finding an equivalent fraction to add fractions,*

Then . . . *use this activity to have them model an equivalent fraction when one denominator is a multiple of the other denominator.*

Materials For each pair: 1 set of fraction circles

- Have students model $\frac{1}{2}$ and $\frac{1}{4}$.
- Point out that the sizes of the pieces are not the same. Ask: *Can you name this sum?* [no]
- Have students find the number of fourths it takes to cover the half and replace the half with those pieces. [2 fourths] Ask: *Can you name this sum?* $\left[\frac{3}{4}\right]$
- Repeat activity for other sums, such as $\frac{2}{3} + \frac{1}{6}$ and $\frac{1}{4} + \frac{3}{8}$. $\left[\frac{5}{6}; \frac{5}{8}\right]$

2 LOOK AHEAD

Point out that when the fractions in a sum have unlike denominators and one denominator is not a multiple of the other denominator, each addend must be replaced with an equivalent fraction.

Students should be able to explain the terms *common denominator, multiple,* and *equivalent fractions* and use this language to discuss the process of writing two fractions with denominators that show same-sized parts of the whole.

Ask *The first pair of fraction models in problem 2b shows that when you change the number of parts in the whole from 2 to 6, you change the number of parts in $\frac{1}{2}$ from 1 to 3. What operation changes 2 into 6 and 1 into 3? What equation can you write to show changing $\frac{1}{2}$ into $\frac{3}{6}$?*

Listen for You can change 2 into 6 and 1 into 3 by multiplying by 3. So you can change $\frac{1}{2}$ into $\frac{3}{6}$ by multiplying the numerator and denominator by 3: $\frac{1}{2} = \frac{3 \times 1}{3 \times 2} = \frac{3}{6}$.

CONNECT IT

1 LOOK BACK

Explain how you found how much butter Emiliano needs.

Possible answer: I found a common denominator for $\frac{1}{2}$ and $\frac{1}{4}$, fourths. Then I added to get $\frac{3}{4}$ of a stick of butter.

2 LOOK AHEAD

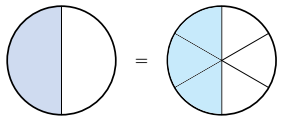
Before adding fractions, the fractions must have the same-size parts of a whole. You can use what you know about multiples and equivalent fractions to find a common denominator for fractions that have unlike denominators.

a. Use the fractions $\frac{1}{2}$ and $\frac{1}{3}$. Write the next four multiples of each denominator. Then circle the multiple that 2 and 3 have in common.

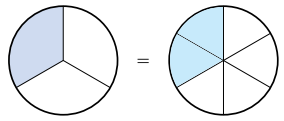
Multiples of 2: 2, 4, (6), 8, 10

Multiples of 3: 3, (6), 9, 12, 15

b. Complete the models and equations to show equivalent fractions for $\frac{1}{2}$ and $\frac{1}{3}$ using the common multiple as the common denominator. **Possible models shown.**



$\frac{1}{2} = \frac{3}{6}$



$\frac{1}{3} = \frac{2}{6}$

c. Use the equivalent fractions to add. $\frac{1}{2} + \frac{1}{3} = \frac{3}{6} + \frac{2}{6} = \frac{5}{6}$

3 REFLECT

What equivalent fractions could you write to add $\frac{1}{2}$ and $\frac{2}{3}$? Explain.

Possible answer: The number 6 is a common multiple of 2 and 3. So, you can use the equivalent fractions $\frac{3}{6}$ and $\frac{4}{6}$ to add $\frac{1}{2}$ and $\frac{2}{3}$.

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Close: Exit Ticket

3 REFLECT

Look for understanding that to be added, fractions must have same-size parts of a whole. Student responses should include the common multiple of 2 and 3 to be used as the common denominator, as well as the equivalent fractions for $\frac{1}{2}$ and $\frac{2}{3}$.

Common Misconception **If** students correctly name a multiple to use as the common denominator in each equivalent fraction but neglect to make a corresponding change in the numerators, **then** provide students with fraction tiles. Have students find the number of $\frac{1}{6}$s that equal $\frac{1}{2}$ and the number of $\frac{1}{6}$s that equal $\frac{2}{3}$ and write the corresponding fractions. Discuss what students notice about the numerators and denominators.

Real-World Connection

Encourage students to think about everyday places or situations where people might need to add fractions with unlike denominators. Have volunteers share ideas. Examples include combining measurements given in fractions of a pound, fractions of an hour, or fractions of a mile.

Solutions

Support Vocabulary Development

1 Have students repeat *common denominator* after you. Have students form pairs and discuss when they have used the term *common* before. Clarify the meaning as needed and provide an example sentence, such as: *People from Spanish-speaking countries speak a common language: Spanish*.

Point out that in mathematics, a common denominator is a denominator that is the same for two or more fractions.

Have students work in pairs to complete the graphic organizer. Provide examples as needed to stimulate student thinking.

2 Read the problem. Provide the following vocabulary terms for the discussion: *multiply*, *multiple*, *common*, and *same*. Have students work with a partner to discuss how they can find the common denominator.

Supplemental Math Vocabulary

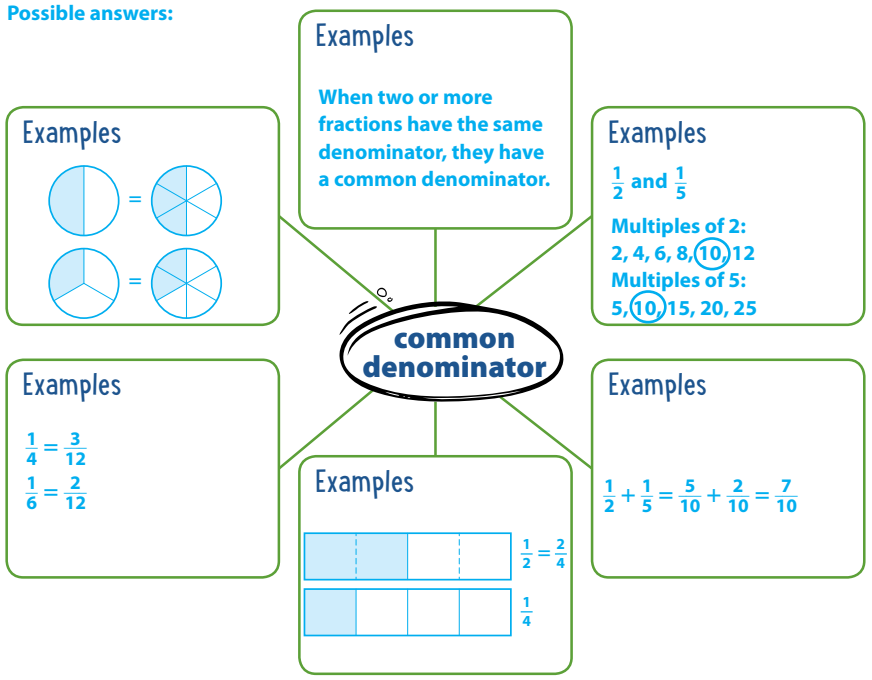
- *denominator*
- *fraction*
- *multiple*
- *multiply*
- *same*

Name: _____ LESSON 12 SESSION 1

Prepare for Adding Fractions

1 Think about what you know about equivalent fractions. Fill in each box. Use words, numbers, and pictures. Show as many ideas as you can.

Possible answers:



2 How can you find a common denominator for $\frac{1}{3}$ and $\frac{1}{5}$?

Possible answer: Find a number that is a multiple of both 3 and 5.
Multiples of 3: 3, 6, 9, 12, (15), 18
Multiples of 5: 5, 10, (15), 20
15 is a common multiple.

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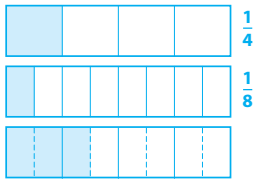
3 Solve the problem. Show your work.

Naeem needs $\frac{1}{4}$ cup of milk to make a carrot cake. He also needs $\frac{1}{8}$ cup of milk to make the icing for the cake. What fraction of a cup of milk does Naeem need in all?

Possible student work using pictures:



$\frac{1}{4}$ and $\frac{1}{8}$ together are the same length as $\frac{3}{8}$.

Solution **Naeem needs $\frac{3}{8}$ cup of milk.**

4 Check your answer. Show your work.

Possible student work:

$\frac{1}{4} = \frac{2}{8}$
$\frac{2}{8} + \frac{1}{8} = \frac{3}{8}$
Naeem needs $\frac{3}{8}$ cup of milk.

3 Assign problem 3 to provide another look at solving a problem by adding fractions with unlike denominators.

This problem is very similar to the problem about how much butter Emiliano needs to make corn bread and apple muffins. In both problems, students are asked to find the sum of two fractions with unlike denominators. The question asks students to find how much milk Naeem needs to make both the carrot cake and the icing.

Students may want to use fraction tiles, fraction circles, or fraction bars.

Suggest that students read the problem three times, asking themselves one of the following questions each time:

- *What is this problem about?*
- *What is the question I am trying to answer?*
- *What information is important?*

Solution: $\frac{1}{4} + \frac{1}{8} = \frac{2}{8} + \frac{1}{8} = \frac{3}{8}$; Naeem needs $\frac{3}{8}$ cup of milk.

Medium

4 Have students solve the problem another way to check their answer.

ELL English Language Learners: Differentiated Instruction

Prepare for Session 2
Use with *Try It*.

Levels 1–3

Reading/Speaking Read the ***Try It*** problem aloud. Ask students to circle the fractions in the problem. Have them form pairs and use gestures, words and pictures to restate the problem. Display the terms *common denominator* and *multiples*. Have partners explore the multiples of 2 and 5 to identify a common multiple. Support students as they generate equivalent fractions with like denominators. After partners solve the problem, have them complete the sentence frame orally and in writing:

- *I can use what I know about* ___*multiples*___ *to find a* ___*common denominator*___.

Levels 2–4

Reading/Speaking Read the ***Try It*** problem with students. Provide the sentence frames:

- *I can use what I know about* ___*multiples*___ *to find a* ___*common denominator*___.
- *I can use multiplication to find* ___*equivalent fractions*___.

Have students form pairs and talk about how they can use what they already know to solve the problem. Call on volunteers to share their solution strategies with the class. Encourage them to use terms from the sentence frames in their explanations.

Levels 3–5

Reading/Speaking Have students read the ***Try It*** problem and form pairs to discuss possible solutions strategies. Provide a bank of terms to support discussion, such as: *unlike denominators, common denominator, equivalent fractions, multiples, common multiple.*

After students solve the problem independently, have them explain their solution in writing using terms from the word bank.

LESSON 12

SESSION 2 Develop

Purpose In this session students solve a problem that requires finding the sum of $\frac{1}{2}$ and $\frac{4}{5}$. Neither denominator is a multiple of the other, so students need to replace both addends with equivalent fractions. Students model the fractions in the word problem either on paper or with manipulatives to develop strategies for adding fractions with unlike denominators.

Start

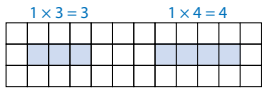
Connect to Prior Knowledge

Materials For each student: Activity Sheet *1-Centimeter Grid Paper*

Why Support students' understanding of finding common multiples.

How Have students draw arrays to show multiples of 3 and 4 and identify a common multiple.

Show four multiples of 3 and 4 as arrays of squares.



Is there a common multiple?

Solutions

Multiples of 3, shown with 1–4 rows of three squares: 3, 6, 9, 12

Multiples of 4, shown with 1–4 rows of four squares: 4, 8, 12, 16

Common multiple: 12

Develop Language

Why Explore the antonyms *agree* and *disagree*.

How Ask students to circle the words *agree* and *disagree* in *Discuss It*. Ask students to share the meaning of *agree* and address any misconceptions. Tell students that the prefix *dis-* means *apart* or *not*. Ask students to provide examples of words with the prefix *dis-*, such as *disappear*, *disassemble*, and *disease*. Explain that *to agree* means to have the same opinion about something. Guide students to tell the meaning of *disagree*.

TRY IT

Make Sense of the Problem

To support students in making sense of the problem, have them describe Maggie's kayaking trip.

Ask *How can you describe the two parts of Maggie's trip? Does Maggie go the same distance in each part?*

Develop Adding Fractions with Unlike Denominators

Read and try to solve the problem below.

Maggie paddles her kayak $\frac{1}{2}$ mile to an island. Then she paddles $\frac{4}{5}$ mile to a beach. How far does Maggie paddle her kayak in all?

TRY IT

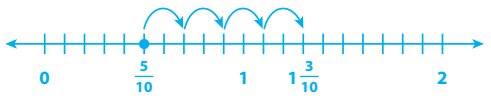
Possible student work:

Sample A

Multiples of 2: 2, 4, 6, 8, (10)
Multiples of 5: 5, (10)

$\frac{1}{2} = \frac{5}{10}$

$\frac{4}{5}$ is 4 jumps of $\frac{2}{10}$ each.



She paddles $1\frac{3}{10}$ miles.

Sample B

Multiples of 2: 2, 4, 6, 8, (10)
Multiples of 5: 5, (10)

$\frac{1}{2} + \frac{4}{5} = \frac{5}{10} + \frac{8}{10} = \frac{13}{10}$

$\frac{13}{10}$ miles

Math Toolkit

- fraction tiles
- fraction circles
- fractions bars
- fraction models
- grid paper
- number lines

DISCUSS IT

Ask your partner: Do you agree with me? Why or why not?

Tell your partner: I disagree with this part because . . .

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DISCUSS IT

Support Partner Discussion

Encourage students to use the terms *common multiple* and *common denominator.*

Support as needed with questions such as:

- *How did you get started?*
- *How are the distances to the island and to the beach shown in your model?*

Common Misconception Look for students who add numerators and denominators separately for a solution of $\frac{5}{7}$ mile. As students present solutions, have them explain how they chose a common denominator.

Select and Sequence Student Solutions

One possible order for whole class discussion:

- physical parts showing halves, fifths, and tenths
- drawings to represent the problem
- number lines marked in tenths
- equations showing equivalent fractions with like denominators for $\frac{1}{2}$ and $\frac{4}{5}$

Support Whole Class Discussion

Compare and connect the different representations and have students identify how they are related.

Ask *Where does your model show the two distances as same-size parts of a mile? the total distance?*

Listen for Students should recognize that accurate responses show the use of a common multiple of 2 and 5 as a common denominator. Responses may include $\frac{5}{10}$ mile as the distance to the island, $\frac{8}{10}$ mile as the distance from the island to the beach, and $\frac{13}{10}$ mile, or $1\frac{3}{10}$ miles, as the total distance.

PICTURE IT & MODEL IT

If no student presented these models, connect them to the student models by pointing out the ways they each represent:

- the parts to be added (distance to the island and distance to the beach)
- a distance equivalent to $\frac{1}{2}$ mile
- a distance equivalent to $\frac{4}{5}$ mile
- the total distance

Ask *How is tenths shown as the common denominator for halves and fifths in the picture? in the equations?*

Listen for In the picture, both the halves fraction bar and the fifths fraction bar are divided into 10 equal-size parts. In the equations, both denominators, 2 and 5, are multiplied by a number that results in a product of 10.

For the fraction bars, prompt students to interpret how the bars are drawn to represent the problem.

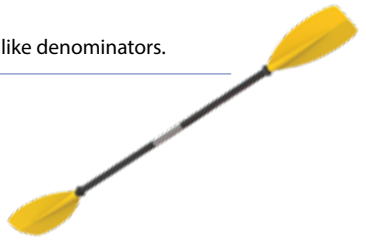
- *Why are there two fraction bars labeled Island?*
- *How are the two fraction bars divided into same-size parts by the red dashed lines alike? How are they different?*

For the equation model, prompt students to identify how it relates to the fraction bar drawing.

- *Which fractions represent the two fraction bars labeled Island?*
- *What operation is used to show the halves as tenths?*

Explore different ways to understand adding fractions with unlike denominators.

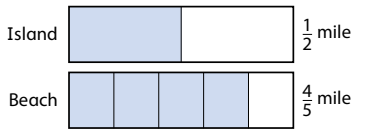
Maggie paddles her kayak $\frac{1}{2}$ mile to an island. Then she paddles $\frac{4}{5}$ mile to a beach. How far does Maggie paddle her kayak in all?



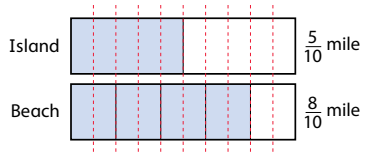
PICTURE IT

You can picture the fractions in the problem using fraction bars.

The fraction bars are divided into halves and fifths.



Both fraction bars need to be divided into same-size parts.



Now you can add equivalent fractions.

MODEL IT

You can model the problem with an equation.

Replace the fractions $\frac{1}{2}$ and $\frac{4}{5}$ with equivalent fractions with a common denominator.

$\frac{1}{2} = \frac{1 \times 5}{2 \times 5} = \frac{5}{10}$ and
$\frac{4}{5} = \frac{4 \times 2}{5 \times 2} = \frac{8}{10}$

So, $\frac{1}{2} + \frac{4}{5} = \frac{5}{10} + \frac{8}{10}$.

Deepen Understanding

Equation Model

SMP 7 Look for structure.

When discussing the equation model, prompt students to recognize that the product of the denominators is always one choice for a common denominator.

Ask *What is 2 multiplied by to make the common denominator 10? What is 5 multiplied by to make 10? What do you notice?*

Listen for Each denominator was multiplied by the other denominator to make the common denominator.

Ask *Why is 10 a multiple of both 2 and 5?*

Listen for 2 and 5 are the factors of 10. A multiple of a number is the product of that number and another whole number.

Generalize *Do you think that multiplying one denominator by the other will always work as a way to find a common denominator?* Look for understanding that you are looking for a common multiple of the denominators, so by definition, if you multiply one denominator by the other, the product is a multiple of both.

CONNECT IT

- Remind students that one thing that is alike about all the representations is that they show how to replace fractions that do not have same-size parts with equivalent fractions that do.
- Explain that on this page, students will look more closely at processes that can be used to find a common denominator.

Monitor and Confirm

1 – 3 Check for understanding that:

- 10 is a multiple of both 2 and 5, so tenths allow you to show halves, fifths, and tenths
- Maggie paddles $\frac{13}{10}$, or $1\frac{3}{10}$, miles
- multiplying the denominators, 2 and 5, gives you a common denominator, 10

Support Whole Class Discussion

4 Tell students that this problem will prepare them to provide the explanation in problem 5.

Ask *Suppose 20 is used as the common denominator. How would the model in* Picture It *change?*

Listen for The second set of fraction bars would each be divided into 20 equal parts; 10 of the 20 parts would be shaded for the distance to the island; 16 of the 20 parts would be shaded for the distance from the island to the beach.

Ask *When 10 is used as the common denominator, the sum is $\frac{13}{10}$. When 20 is used as the common denominator, the sum is $\frac{26}{20}$. Do $\frac{13}{10}$ miles and $\frac{26}{20}$ miles represent the same distance? How do you know?*

Listen for The distances added together do not change; they are just renamed. The total distance does not change, whether it is described as *tenths* or *twentieths*.

5 Look for the idea that you replace the fractions with equivalent fractions that have a common denominator and then add the numerators to show the number of same-sized parts in the whole. Students may note that one way to find a common denominator is to multiply the denominators of the fractions.

6 REFLECT

Have all students focus on the strategies used to solve this problem. If time allows, have students share their preferences with a partner.

CONNECT IT

Now you will use the problem from the previous page to help you understand how to add any two fractions with unlike denominators.

1 Explain why both fraction bars in **Picture It** are divided into 10 equal pieces.

Possible answer: 10 is a multiple of both 2 and 5, so dividing each whole into 10 equal pieces allows you to group the pieces into halves for the island bar and fifths for the beach bar.

2 Write the total distance Maggie paddles as a fraction. $\frac{13}{10}$ miles

Write the total distance Maggie paddles as a mixed number. $1\frac{3}{10}$ miles

3 Look at the denominators in **Model It**. What do you notice about the relationship between the original denominators, 2 and 5, and the common denominator, 10?

Possible answer: The common denominator is the product of the original denominators, 2 and 5.

4 Is 10 the only common denominator for 2 and 5? Justify your answer.

No; Possible answer: All common multiples of 2 and 5 could be common denominators for 2 and 5. For example, 10, 20, or 30 could be used as common denominators.

5 Explain how to add two fractions with unlike denominators.

Possible answer: Find a common denominator of the two fractions and use it to write equivalent fractions. Then add the equivalent fractions. One way to find a common denominator for two fractions is to multiply their denominators.

6 REFLECT

Look back at your **Try It**, strategies by classmates, and **Picture It** and **Model It**. Which models or strategies do you like best for adding fractions with unlike denominators? Explain.

Some students may prefer using fraction bars because the bars help them see how to rewrite the fractions with a common denominator; other students may like using multiplication to rewrite the given fractions as equivalent fractions that have the same denominator.

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Hands-On Activity

Model equivalent sums.

If . . . *students are unsure about writing equivalent expressions,*

Then . . . *use this activity to connect equivalent sums.*

Materials For each pair: 1 set of fraction tiles

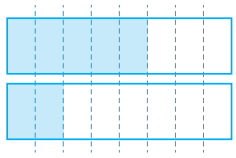
- Have students write $\frac{1}{2} + \frac{1}{3}$ at the top of a sheet of paper. Right below this expression have them model each addend using fraction tiles.
- Together, list six multiples of 2 and 3. [2, 4, 6, 8, 10, 12; 3, 6, 9, 12, 15, 18] Ask: *What are two common multiples for 2 and 3?* [6, 12]
- Keeping the half and third tiles in place, have students find the number of $\frac{1}{6}$ tiles it takes to cover each. Below the tiles have them write the equivalent sum now shown. $\left[\frac{3}{6} + \frac{2}{6}\right]$ Discuss how the part of the total represented by each addend and the total itself did not change.
- Repeat the steps to show an equivalent sum using twelfths as the common denominator. Discuss how the sums $\frac{1}{2} + \frac{1}{3}$, $\frac{3}{6} + \frac{2}{6}$, and $\frac{6}{12} + \frac{4}{12}$ are equivalent.

APPLY IT

Use what you just learned to solve these problems.

7 Hank practices $\frac{5}{8}$ of the words on his spelling list on Monday. He practices another $\frac{1}{4}$ of his list on Tuesday. What fraction of his spelling list has Hank practiced so far? Show your work.

Possible student work:



$\frac{5}{8} + \frac{2}{8} = \frac{7}{8}$

Solution $\frac{7}{8}$ **or** $\frac{28}{32}$**; Accept any equivalent fraction.**

8 What is the sum of $\frac{7}{6}$ and $\frac{5}{8}$? Show your work.

Possible student work:

$\frac{7 \times 4}{6 \times 4} = \frac{28}{24}$ and $\frac{5 \times 3}{8 \times 3} = \frac{15}{24}$

$\frac{28}{24} + \frac{15}{24} = \frac{43}{24} = 1\frac{19}{24}$

Solution $\frac{43}{24}$ **or** $1\frac{19}{24}$**; Accept any equivalent fraction.**

9 What is the value of the expression $\frac{2}{3} + \frac{5}{12}$?

Ⓐ $\frac{7}{36}$

Ⓑ $\frac{7}{15}$

Ⓒ $\frac{13}{12}$

Ⓓ $\frac{38}{36}$

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APPLY IT

For all problems, encourage students to draw some kind of model to support their thinking. Allow some leeway in precision; drawing same-size parts in a whole can be difficult.

7 $\frac{7}{8}$; Accept any equivalent fraction. See possible work on the Student Worktext page. Students may also show $\frac{5}{8}$ on a number line divided into eighths and count 2 marks to the right. They may also write the equations $\frac{2 \times 1}{2 \times 4} = \frac{2}{8}$ and $\frac{5}{8} + \frac{2}{8} = \frac{7}{8}$. Students who choose to use a common denominator that is a product of the two denominators will show models that divide the wholes into 32 parts. Students may also use a common denominator of 16.

8 $\frac{43}{24}$ or $1\frac{19}{24}$; Accept any equivalent fraction. Students may also write $\frac{86}{48}$ or $1\frac{38}{48}$, using the product of the denominators as the common denominator; See possible work on the Student Worktext page. Students might also use models, number lines, or some other method to write equivalent fractions to find the sum.

Close: Exit Ticket

9 **C**; 12 is a multiple of both denominators. Multiply both the numerator and the denominator of $\frac{2}{3}$ by 4 to write the equivalent fraction $\frac{8}{12}$. Add $\frac{8}{12}$ and $\frac{5}{12}$.

Error Alert **If** students choose A, B, or D, **then** they may have found a correct common denominator but not adjusted the numerators (A), added the numerators and added the denominators (B), or made a calculation error (D). Have them use fraction tiles to find that $\frac{8}{12}$ is equivalent to $\frac{2}{3}$. Review how both the numerator and denominator in the fraction $\frac{2}{3}$ are multiplied by the same factor (4) to show the equivalent fraction. Have students replace $\frac{2}{3}$ with $\frac{8}{12}$ in the expression $\frac{2}{3} + \frac{5}{12}$ and find the sum.

Solutions

1 **a.** Possible answer: 24

b. Possible answer: There would be twice as many equal parts in each model, and each part would be smaller. The areas shaded would be the same.

c. $\frac{18}{24}$; $\frac{4}{24}$

Medium

Name: _____ LESSON 12 SESSION 2

Practice Adding Fractions with Unlike Denominators

Study the Example showing one way to add fractions with unlike denominators. Then solve problems 1–4.

EXAMPLE

What is $\frac{3}{4} + \frac{1}{6}$?

To add fractions, the size of the parts must be the same. Write each addend as an equivalent fraction with a common denominator.

Identify 12 as a common multiple of the denominators 4 and 6. Divide the models into 12 equal parts.

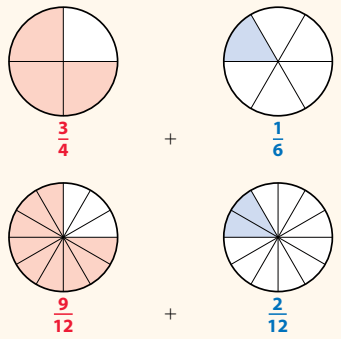
Write the equivalent fractions.

$\frac{3}{4} = \frac{9}{12}$ and $\frac{1}{6} = \frac{2}{12}$

Find the sum.

$\frac{3}{4} + \frac{1}{6} = \frac{9}{12} + \frac{2}{12}$

$= \frac{11}{12}$



1 The Example uses 12 as the common multiple of 4 and 6.

a. Name a different common multiple of 4 and 6.

Possible answer: 24

b. If you used the common multiple from part a as the common denominator, how would the models in the Example be different? How would they be the same?

Possible answer: There would be twice as many equal parts in each model, and each part would be smaller. The areas shaded would be the same.

c. Use the common multiple from part a as the common denominator to write equivalent fractions for $\frac{3}{4}$ and $\frac{1}{6}$.

$\frac{3}{4} =$ **18/24** $\frac{1}{6} =$ **4/24**

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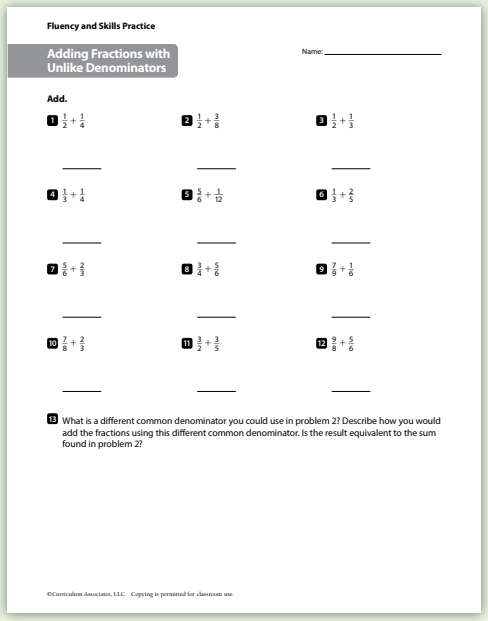
Fluency & Skills Practice

Teacher Toolbox

Assign *Adding Fractions with Unlike Denominators*

In this activity students add fractions with unlike denominators. Students may apply their understanding of adding fractions with unlike denominators in real-world situations. For example, students may want to find the total amount of flour they need if one recipe uses $\frac{3}{4}$ cup of flour and a different recipe uses $1\frac{1}{2}$ cups of flour



Fluency and Skills Practice

Adding Fractions with Unlike Denominators

Name: _____

Add.

1 $\frac{1}{2} + \frac{1}{4}$ 2 $\frac{1}{2} + \frac{3}{8}$ 3 $\frac{1}{2} + \frac{1}{3}$

4 $\frac{1}{3} + \frac{1}{4}$ 5 $\frac{5}{6} + \frac{1}{12}$ 6 $\frac{1}{3} + \frac{2}{5}$

7 $\frac{5}{6} + \frac{2}{3}$ 8 $\frac{3}{4} + \frac{5}{6}$ 9 $\frac{7}{9} + \frac{1}{6}$

10 $\frac{7}{8} + \frac{2}{3}$ 11 $\frac{3}{2} + \frac{3}{5}$ 12 $\frac{9}{8} + \frac{5}{6}$

13 What is a different common denominator you could use in problem 2? Describe how you would add the fractions using this different common denominator. Is the result equivalent to the sum found in problem 2?

2 Find a common denominator for each pair of fractions. Then use multiplication to write each fraction as an equivalent fraction with the common denominator.
Possible answers are shown.

a. $\frac{1}{8}$ and $\frac{1}{2}$ common denominator **8**

$\frac{1 \times 1}{8 \times 1} = \frac{1}{8}$ $\frac{1 \times 4}{2 \times 4} = \frac{4}{8}$

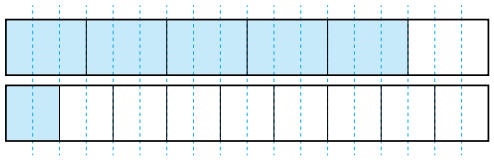
b. $\frac{1}{8}$ and $\frac{9}{5}$ common denominator **40**

$\frac{1 \times 5}{8 \times 5} = \frac{5}{40}$ $\frac{9 \times 8}{5 \times 8} = \frac{72}{40}$

c. $\frac{1}{8}$ and $\frac{11}{6}$ common denominator **24**

$\frac{1 \times 3}{8 \times 3} = \frac{3}{24}$ $\frac{11 \times 4}{6 \times 4} = \frac{44}{24}$

3 Show how to find the sum of $\frac{5}{6}$ and $\frac{1}{9}$ using the fraction bars below.



Write an equation for the sum.

Possible answer: $\frac{5}{6} + \frac{1}{9} = \frac{15}{18} + \frac{2}{18} = \frac{17}{18}$

4 Glenn swims $\frac{2}{3}$ mile on Monday, $\frac{3}{4}$ mile on Wednesday, and $\frac{5}{6}$ mile on Friday. What is the total distance Glenn swims on those three days? Show your work.

Students might use models, number lines, equations, or some other method to use equivalent fractions to find $\frac{2}{3} + \frac{3}{4} + \frac{5}{6}$.

Solution **$2\frac{1}{4}$ miles; Accept any equivalent fraction.**

2 **a.** Possible answer: common denominator 8; See completed equations on the student page. Students may also use 8×2, or 16, as the common denominator. Accept any equivalent fraction.

b. Possible answer: common denominator 40; See completed equations on the student page. Accept any equivalent fraction.

c. Possible answer: common denominator 24; See completed equations on the student page. Students may also use 8×6, or 48, as the common denominator. Accept any equivalent fraction.

Medium

3 See completed fraction bar models on the student page; $\frac{5}{6} + \frac{1}{9} = \frac{15}{18} + \frac{2}{18} = \frac{17}{18}$.

Medium

4 $2\frac{1}{4}$ miles; Accept any equivalent fraction. Students might use models, number lines, equations, or some other method to use equivalent fractions to find the sum.

Challenge

ELL English Language Learners: Differentiated Instruction

Prepare for Session 3
Use with *Connect It*.

Levels 1–3

Speaking Read ***Connect It*** problem 5. Ask students to look at ***Picture It*** and point to the mixed number, $1\frac{2}{3}$. Remind students that a mixed number includes both a whole number and a fraction. Provide the following sentence frames to help students explain one way to add with mixed numbers.

- *First, I rewrite the fractional parts using a* ___*common denominator*___.
- *Second, I add the* ___*fractional parts*___.
- *Then I* ___*regroup*___ *if the fractional part is greater than 1.*

Provide a word bank as needed: *common denominator, fractional parts*, and *regroup*.

Levels 2–4

Speaking Read ***Connect It*** problem 5. Have students form pairs and look at ***Picture It***. Provide the following sentence frames to explain one way to add with mixed numbers.

- *First, I rewrite the fractional parts by using a* ___*common denominator*___.
- *Second, I add the* ___*fractional parts*___.
- *Then I* ___*regroup*___ *if the fractional part is greater than 1.*

Have pairs take turns saying the process using the completed sentences.

Levels 3–5

Writing Read ***Connect It*** problem 5 and have students form pairs. Provide the terms *common denominator*, *fractional parts*, and *regroup* and the sequence words *first*, *second,* and *then*. Ask pairs to look at ***Picture It*** and work together to write an explanation for how to add with mixed numbers using the terms and sequence words provided. Ask students to use complete sentences. After completing their explanations, have pairs share them with other partner groups.

LESSON 12

SESSION 3 Develop

Purpose In this session, students solve a problem that requires finding the sum of the mixed number $1\frac{2}{3}$ and the fraction $\frac{3}{4}$. Students model the mixed number and fraction in the word problem on paper or with manipulatives to develop strategies for adding with mixed numbers.

Start

Connect to Prior Knowledge

Materials For each pair: 2 sets of fraction circles

Why Support students' facility with regrouping fractions greater than 1 as mixed numbers.

How Have students use fraction circles to show a whole-number part and a fractional part for three fractions greater than 1.

Model each fraction. Then write the fraction as a mixed number.

$\frac{7}{6} =$ ..... $\frac{8}{6} =$ .....

$\frac{11}{6} =$ .....

Solutions

$\frac{7}{6} = 1\frac{1}{6}$

$\frac{8}{6} = 1\frac{2}{6}$

$\frac{11}{6} = 1\frac{5}{6}$

Develop Language

Why Explore the multiple meanings of the word *spend*.

How Tell students that they can use the verb *to spend* to talk about money they use to pay for something. They can also use the verb *to spend* to describe using a period of time for an activity, for example: *I am going to spend three hours doing my homework*. Guide students to identify the past tense of the verb in the *Try It* problem.

TRY IT

Make Sense of the Problem

To support students in making sense of the problem, have them identify $1\frac{2}{3}$ hours as the time spent mowing one part of the lawn and $\frac{3}{4}$ hour as the time spent mowing the other part.

Ask *What is the problem asking us to find out about Jenna?*

Develop Adding with Mixed Numbers

Read and try to solve the problem below.

Jenna spent $1\frac{2}{3}$ hours mowing the back yard. After taking a break, she spent $\frac{3}{4}$ hour mowing the front yard. How many hours did she spend mowing the whole yard?

TRY IT

Math Toolkit
- fraction tiles
- fraction circles
- fractions bars
- fraction models
- grid paper
- number lines

Possible student work:

Sample A

$1\frac{2}{3} + \frac{3}{4}$

$\frac{2 \times 4}{3 \times 4} = \frac{8}{12}$

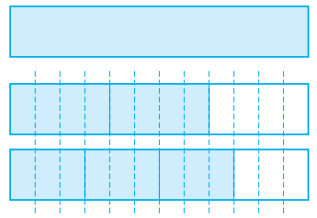
$\frac{3 \times 3}{4 \times 3} = \frac{9}{12}$

$1\frac{8}{12} + \frac{9}{12} = 1\frac{17}{12}$

$1\frac{17}{12} = 2\frac{5}{12}$

Jenna spent $2\frac{5}{12}$ hours mowing.

Sample B

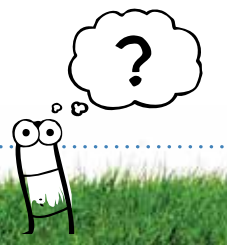


$1\frac{2}{3} = 1\frac{8}{12}$

$\frac{3}{4} = \frac{9}{12}$

$1\frac{8}{12} + \frac{9}{12} = 1\frac{17}{12}$ or $2\frac{5}{12}$

She mowed for $2\frac{5}{12}$ hours.



DISCUSS IT

Ask your partner: Can you explain that again?

Tell your partner: I do not understand how . . .

DISCUSS IT

Support Partner Discussion

Encourage students to use the terms *mixed number* and *fraction* in their discussion.

Support as needed with questions such as:

- *How did you get started?*
- *What strategy did you use?*

Common Misconception Look for students who add the whole number 1 to the numerators 8 and 9 to get a sum of $\frac{18}{12}$. Have them show that $\frac{18}{12} = 1\frac{6}{12}$, not $1\frac{17}{12}$.

Select and Sequence Student Solutions

One possible order for whole class discussion:

- physical parts showing thirds, fourths, and twelfths
- drawings to represent the problem
- number lines marked in twelfths
- equations showing equivalent fractions with like denominators for $1\frac{2}{3}$ and $\frac{3}{4}$, or for $\frac{5}{3}$ and $\frac{1}{4}$

Support Whole Class Discussion

Compare and connect the different representations and have students identify how they relate.

Ask *Where does your model show the time spent on the front yard and the time spent on the back yard as same-size parts of an hour? the total amount of time Jenna spent mowing?*

Listen for Students should recognize that accurate responses show the use of a common multiple for 3 and 4 as a common denominator. Responses may include $1\frac{8}{12}$ or $\frac{20}{12}$ as the number of hours spent mowing the front yard, $\frac{9}{12}$ as the number of hours spent mowing the back yard, and $1\frac{17}{12}$, or $2\frac{5}{12}$ as the total time.

PICTURE IT & MODEL IT

If no student presented these models, connect them to the student models by pointing out the ways they each represent:

- the time spent mowing the front yard and the time spent mowing the back yard
- a common multiple of 3 and 4
- the total time Jenna spent mowing

Ask *In order to add the two times, how many equal parts is an hour divided into in the square fraction model? on the number line?*

Listen for The fraction model and the number line both show an hour divided into 12 equal parts.

For the fraction models, prompt students to explain how the models represent the problem.

- *What does each square represent? Why are 2 squares needed to represent $1\frac{2}{3}$ hours?*
- *In the second row of models, the model for $\frac{8}{12}$ shows 4 rows of 3 pieces and the model for $\frac{9}{12}$ shows 3 rows of 4 pieces. Are the pieces the same size? How do you know?*

For the number line model, prompt students to explain how the number line represents the problem.

- *What do the red tick marks show? How does the number line show that $1\frac{2}{3}$ is equivalent to $1\frac{8}{12}$?*
- *How do you know where to start on the number line? Could you start at a different point instead?*

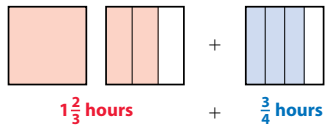
Explore different ways to understand adding with mixed numbers.

Jenna spent $1\frac{2}{3}$ hours mowing the back yard. After taking a break, she spent $\frac{3}{4}$ hour mowing the front yard. How many hours did she spend mowing the whole yard?

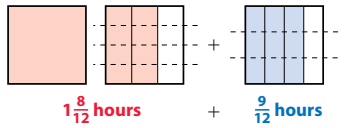
PICTURE IT

You can picture the fractions in the problem using models.

The shaded parts represent time spent on the back yard, $1\frac{2}{3}$ **hours**, and the front yard, $\frac{3}{4}$ **hour**.



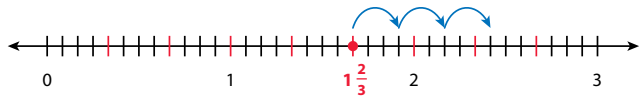
The sections need to be divided into same-size parts to add. Use dashed lines to divide the fraction models into 12 equal parts.



MODEL IT

You can use a number line to add fractions.

The number line is divided first into thirds and then into twelfths, with a point at $1\frac{2}{3}$.



$\frac{1}{4} = \frac{3}{12}$, so $\frac{3}{4} = \frac{3}{12} + \frac{3}{12} + \frac{3}{12}$.

$\frac{3}{12} + \frac{3}{12} + \frac{3}{12} = \frac{9}{12}$

Start at $1\frac{2}{3}$ and jump right a total of $\frac{9}{12}$.

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Deepen Understanding

Fraction Model

SMP 2 Reason abstractly and quantitatively.

When discussing the fraction models, prompt students to consider whether representing an amount using an equivalent fraction in place of a mixed number changes the problem.

Direct students' attention to the first row of fraction models in *Picture It*, representing $1\frac{2}{3}$ hours and $\frac{3}{4}$ hour with squares.

Discuss how you could represent $1\frac{2}{3}$ as a fraction if you divided the 1 whole square into thirds and counted the total number of thirds. Have students explain that this results in the fraction $\frac{5}{3}$.

Ask *Does this make the problem a different problem?*

Listen for The amount represented (the time spent mowing the back yard) does not change. $1\frac{2}{3}$ hours and $\frac{5}{3}$ hours are the same quantity represented in different ways.

CONNECT IT

- Remind students that one thing that is alike about all the representations is that they show how to replace fractions that do not have same-size parts with equivalent fractions that do.
- Explain that on this page, students will write equations to show replacing fractions with equivalent fractions in order to add with mixed numbers.

Monitor and Confirm

1 – 3 Check for understanding that:

- multiplying the denominators 3 and 4 gives you a common denominator, 12
- $\frac{8}{12}$ is an equivalent fraction for the fractional part of the mixed number $1\frac{2}{3}$
- $\frac{9}{12}$ is an equivalent fraction for $\frac{3}{4}$
- Jenna spent $1\frac{17}{12}$ hours mowing the whole yard

Support Whole Class Discussion

4 Tell students that this problem will prepare them to provide the explanation required in problem 5.

Make sure students understand this problem is asking them to write the sum they found in problem 3, $1\frac{17}{12}$, another way.

> **Ask** *How do you know the fractional part of the mixed number is greater than 1?*
>
> ***Listen for*** The numerator, 17, is greater than the denominator, 12. 12 twelfths is 1 whole. 17 twelfths is greater than 12 twelfths, so it is greater than 1 whole.

5 Look for the idea that you rewrite fractional parts of the addends as equivalent fractions with a common denominator, add the fractional parts, and then add the whole number(s). If the sum has a fractional part greater than 1, regroup.

6 REFLECT

Have all students focus on the strategies used to solve this problem. If time allows, have students share their preferences with a partner.

CONNECT IT

Now you will use the problem from the previous page to help you understand how to add with mixed numbers.

1 Look at the models on the previous page. What is a common denominator of $1\frac{2}{3}$ and $\frac{3}{4}$? 12

2 You can find this common denominator without a model. Write a multiplication equation that shows how the denominators 3 and 4 are related to 12.

$3 \times 4 = 12$

3 Use this common denominator to find equivalent fractions for $1\frac{2}{3}$ and $\frac{3}{4}$.

$1\frac{2}{3} + \frac{3}{4} = 1\frac{8}{12} + \frac{9}{12}$

Then write the sum as a mixed number.

$= 1\frac{17}{12}$

4 The fractional part of the mixed number in problem 3 is more than 1. How could you rewrite the mixed number so that its fractional part is less than 1?

Possible answer: You can write $\frac{17}{12}$ as $1\frac{5}{12}$, so $1\frac{17}{12} = 1 + 1\frac{5}{12} = 2\frac{5}{12}$.

How many hours did Jenna spend mowing the whole yard? **$2\frac{5}{12}$ hours**

5 Explain how to add with mixed numbers.

Possible answer: Rewrite the fractional parts with a common denominator, add the fractional parts, add the whole number(s), and regroup if the fractional part of the sum is greater than 1.

6 REFLECT

Look back at your **Try It**, strategies by classmates, and **Picture It** and **Model It**. Which models or strategies do you like best for adding with mixed numbers? Explain.

Possible answer: I like using an area model because it helps me see how to rewrite the fractions with a common denominator.

Visual Model

Regroup fractions greater than 1 as mixed numbers.

If . . . *students are unsure about regrouping fractional parts as wholes,*

Then . . . *use this activity to connect fractions greater than 1 and mixed numbers.*

Materials For each student: Activity Sheet *Number Lines*

- Have students mark a number line from 0 to 3, with each whole divided into twelfths. Have them label the appropriate tick marks with the numerals 1, 2, and 3 and with the equivalent fractions in twelfths. $\left[\frac{12}{12}, \frac{24}{12}, \frac{36}{12}\right]$
- Direct them to mark and label $\frac{17}{12}$ on the number line and then write the mixed number it shows. $\left[1\frac{5}{12}\right]$ If necessary, prompt students with questions such as: *How many wholes are there? How many more twelfths?*
- Repeat for two other fractions greater than 1 with 12 as the denominator, such as $\frac{20}{12}$ and $\frac{27}{12}$. Ask students to show the fraction on a number line and then write it as a mixed number. $\left[1\frac{8}{12}, 2\frac{3}{12}\right]$

APPLY IT

Use what you just learned to solve these problems.

7 What is the sum $1\frac{1}{6} + 2\frac{3}{8}$? Show your work.

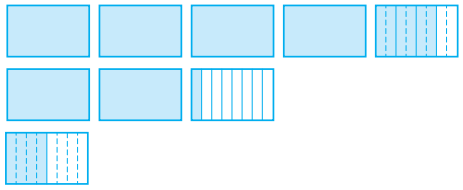
Possible student work:

$\frac{1}{6} = \frac{4}{24}$ and $\frac{3}{8} = \frac{9}{24}$

$1\frac{1}{6} + 2\frac{3}{8} = 1\frac{4}{24} + 2\frac{9}{24} = 3\frac{13}{24}$

Solution **$3\frac{13}{24}$; Accept any equivalent fraction.**

8 Cameron has $4\frac{3}{4}$ pounds of raspberries, $2\frac{1}{8}$ pounds of blueberries, and $\frac{1}{2}$ pound of blackberries to make a fruit salad. How many pounds of fruit does Cameron have in all? Show your work. **Possible student work:**



$4\frac{6}{8} + 2\frac{1}{8} + \frac{4}{8} = 6\frac{11}{8} = 7\frac{3}{8}$

Solution **$7\frac{3}{8}$ pounds of fruit; Accept any equivalent fraction.**

9 What is the value of the expression $5\frac{3}{5} + 4\frac{1}{15}$? Show your work.

Possible student work:

$\frac{3}{5} = \frac{9}{15}$

$5\frac{3}{5} + 4\frac{1}{15} = 5\frac{9}{15} + 4\frac{1}{15} = 9\frac{10}{15}$, or $9\frac{2}{3}$

Solution **$9\frac{2}{3}$; Accept any equivalent fraction.**

APPLY IT

For all problems, encourage students to draw a model to support their thinking. Allow some leeway in precision; drawing same-size parts in a whole can be difficult.

7 $3\frac{13}{24}$; Accept any equivalent fraction. See possible work on the Student Worktext page. Students may also use 48, the product of the denominators 6 and 8, or some other common multiple as the common denominator.

8 $7\frac{3}{8}$ pounds of fruit; Accept any equivalent fraction. See possible work on the Student Worktext page. Students may also show a number line divided into eighths, or use a different common multiple of 4, 8, and 2 as the common denominator.

Close: Exit Ticket

9 $9\frac{2}{3}$; Accept any equivalent fraction. See possible work on the Student Worktext page. Students may also show models for the mixed numbers with the fractional parts divided into 15ths. Students may also use a different common multiple of 5 and 15 as the common denominator.

Students' solutions should indicate understanding of:

- replacing fractional parts of mixed numbers with equivalent fractions with a common denominator before adding
- accurate use of visual fraction models or equations to represent the problem

Error Alert **If** students' solution is $9\frac{4}{15}$, **then** review finding equivalent fractions to remind them that the numerator must be multiplied by the same factor as the denominator. Have them draw models to show $\frac{3}{5} = \frac{9}{15}$ and identify the factor that both 3 and 5 are multiplied by.

Solutions

1 $5\frac{5}{12}$; Accept any equivalent fraction. Students may use a different common denominator for 6 and 4; their models should reflect the common denominator chosen.

Medium

Name: _____ LESSON 12 SESSION 3

Practice Adding with Mixed Numbers

Study the Example showing how to add with mixed numbers. Then solve problems 1–4.

EXAMPLE

What is $1\frac{2}{3} + 1\frac{1}{2}$?

To add mixed numbers, the fractional parts must be the same size.

Replace the given fractions with equivalent fractions that have the denominator 6.

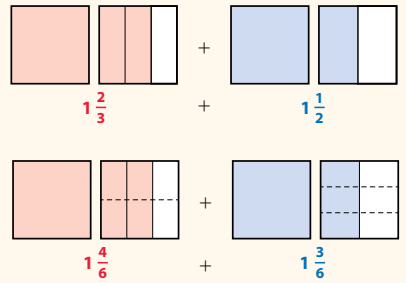
Find the sum. $1\frac{2}{3} + 1\frac{1}{2} = 1\frac{4}{6} + 1\frac{3}{6}$

$= 2\frac{7}{6}$

Rewrite the mixed number so that the fractional part is less than 1.

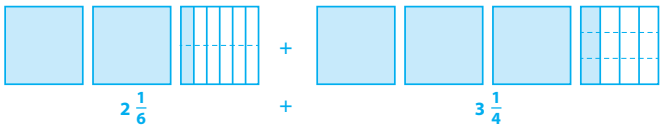
$2\frac{7}{6} = 2 + \frac{7}{6} = 2 + 1\frac{1}{6}$

$= 3\frac{1}{6}$



1 Draw a model to show how you can use equivalent fractions to find the sum $2\frac{1}{6} + 3\frac{1}{4}$. Show your work.

Possible student work:



$2\frac{1}{6} + 3\frac{1}{4} = 2\frac{2}{12} + 3\frac{3}{12} = 5\frac{5}{12}$

Solution $5\frac{5}{12}$; Accept any equivalent fraction.

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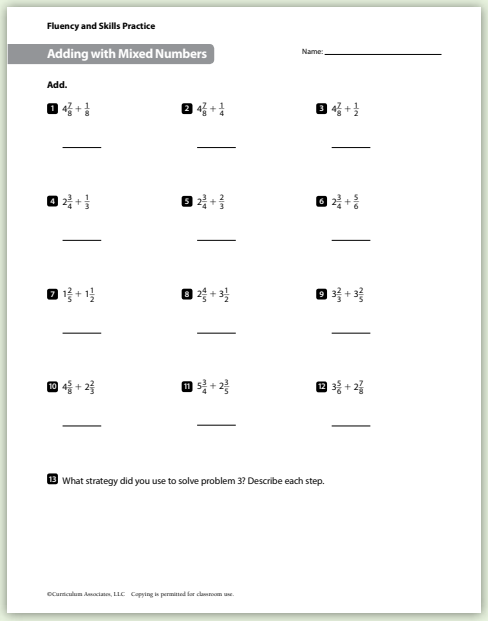
Fluency & Skills Practice Teacher Toolbox

Assign *Adding with Mixed Numbers*

In this activity students add mixed numbers, some with like denominators and others with unlike denominators. Students may add mixed numbers in different real-world contexts, such as finding the total cups of vegetables and pasta needed for a pasta salad or finding the total length of lumber needed to build a deck or a dock. Students may use a variety of strategies to solve these problems, such as drawing fraction bars or making number lines.



Fluency and Skills Practice

Adding with Mixed Numbers

Name: _____

Add.

1 $4\frac{2}{8} + \frac{1}{8}$ _____

2 $4\frac{2}{8} + \frac{1}{4}$ _____

3 $4\frac{7}{8} + \frac{1}{2}$ _____

4 $2\frac{3}{4} + \frac{1}{3}$ _____

5 $2\frac{3}{4} + \frac{2}{3}$ _____

6 $2\frac{3}{4} + \frac{5}{6}$ _____

7 $1\frac{2}{5} + 1\frac{1}{2}$ _____

8 $2\frac{4}{5} + 3\frac{1}{2}$ _____

9 $3\frac{2}{3} + 3\frac{2}{5}$ _____

10 $4\frac{5}{8} + 2\frac{2}{3}$ _____

11 $5\frac{3}{4} + 2\frac{3}{5}$ _____

12 $3\frac{5}{6} + 2\frac{7}{8}$ _____

13 What strategy did you use to solve problem 3? Describe each step.

2 **a.** $1\frac{3}{5} = 1\frac{12}{20}$; $1\frac{3}{4} = 1\frac{15}{20}$

b. $2\frac{1}{2} = 2\frac{5}{10}$; $\frac{4}{5} = \frac{8}{10}$; Accept any equivalent fractions using a common denominator.

c. $\frac{3}{8} = \frac{18}{48}$; $\frac{1}{6} = \frac{8}{48}$; Accept any equivalent fractions using a common denominator.

Medium

3 Dot at $2\frac{5}{10}$ on number line and arrow showing 8 jumps of $\frac{1}{10}$ each; $2\frac{1}{2} + \frac{4}{5} = 2\frac{5}{10} + \frac{8}{10} = 3\frac{3}{10}$; Students may also show the sum of $2\frac{5}{10} + \frac{8}{10}$ as $2\frac{13}{10}$ and then regroup to show $3\frac{3}{10}$. Accept any equivalent fractions using a common denominator.

Medium

4 $2\frac{7}{8}$ pounds; Accept any equivalent fraction.

$\frac{1}{2} + 2\frac{1}{8} + \frac{1}{4} = \frac{4}{8} + 2\frac{1}{8} + \frac{2}{8} = 2\frac{7}{8}$; Students may use a different common denominator of 2, 4 and 8; They may also use models or number lines to find the sum.

Challenge

2 One way to find a common denominator is by multiplying the denominators of the two fractions together and using the product as the common denominator.

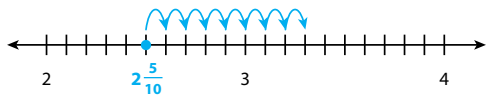
Use this method to find a common denominator for each pair of fractions. Write the equivalent fractions.

a. $1\frac{3}{5} = 1\frac{\boxed{12}}{20}$ $1\frac{3}{4} = 1\frac{\boxed{15}}{20}$

b. $2\frac{1}{2} =$ **$2\frac{5}{10}$** $\frac{4}{5} =$ **$\frac{8}{10}$**

c. $\frac{3}{8} =$ **$\frac{18}{48}$** $\frac{1}{6} =$ **$\frac{8}{48}$**

3 Show how to add $2\frac{1}{2} + \frac{4}{5}$ using the number line below. **Possible model shown.**



Write an equation to represent the problem.

Possible answer: $2\frac{1}{2} + \frac{4}{5} = 2\frac{5}{10} + \frac{8}{10} = 3\frac{3}{10}$

4 Maya is packing her backpack for a hike. In one pocket, she puts in a $\frac{1}{2}$-pound bag of trail mix, a water bottle weighing $2\frac{1}{8}$ pounds, and a flashlight weighing $\frac{1}{4}$ pound. How much weight do these three items add to her backpack? Show your work.

Students might use models, number lines, equations, or some other method to find $\frac{1}{2} + 2\frac{1}{8} + \frac{1}{4}$.

Solution **$2\frac{7}{8}$; Accept any equivalent fraction.**

ELL English Language Learners: Differentiated Instruction

Prepare for Session 4
Use with *Apply It*.

Levels 1–3	Levels 2–4	Levels 3–5
Speaking/Listening Read ***Apply It*** problem 3 aloud. Reread the first sentence. Ask students to point to the term *flour*. Tell students that both muffins and bread use flour. Read the first question to students. Ask them to say what needs to be found. Have students form pairs and work together to solve the problem. After students have answered the first question, have them tell another pair how they solved the problem. Then have pairs discuss the second question. Encourage students to use gestures, numbers and words to share ideas about how Nicola got the wrong answer.	**Reading/Speaking** Read ***Apply It*** problem 3. Have students work in pairs to restate the problem. Have pairs work together to solve the problem and then take turns saying and listening to the steps they used. Have pairs compare their answer with another partner group. Encourage students to comment on the other pair's answers by using *I agree* and/or *I disagree*. Ask pairs to look at the second part of the problem and use the sentence frame below to answer the question in a complete sentence. *Nicola chose answer A because* _____.	**Reading/Writing** Have students form pairs and read ***Apply It*** problem 3. Have pairs work together to solve the problem and then write the steps they used. Have pairs compare their answer and the process they used with another partner group. Ask pairs to look at the second part of the problem and write a sentence that explains how Nicola chose answer A. Have students take turns reading the sentence. Then call on pairs to read their sentences to the class.

LESSON 12

SESSION 4 Refine

Purpose In this session students solve word problems involving adding fractions and mixed numbers with unlike denominators and then discuss and confirm their answers with a partner.

Before students begin to work, use their responses to the *Check for Understanding* to determine those who will benefit from additional support.

As students complete the Example and problems 1–3, observe and monitor their reasoning to identify groupings for differentiated instruction.

Start

Check for Understanding

Why Confirm understanding of adding fractions with unlike denominators.

How Have students find $3\frac{5}{6} + \frac{1}{8}$ using any strategy they want.

Use any strategy to find the sum.

$3\frac{5}{6} + \frac{1}{8} = ?$

Solution

$3\frac{23}{24}$ or $3\frac{46}{48}$ or any equivalent fraction.

Refine Adding Fractions

Complete the Example below. Then solve problems 1–8.

EXAMPLE

What is $\frac{11}{8} + \frac{5}{6}$?

Look at how you could show your work using equations.

24 is a multiple of 8 and 6.

$\frac{11 \times 3}{8 \times 3} = \frac{33}{24}$ and $\frac{5 \times 4}{6 \times 4} = \frac{20}{24}$

$\frac{33}{24} + \frac{20}{24} = \frac{53}{24}$

$\frac{53}{24} = \frac{24}{24} + \frac{24}{24} + \frac{5}{24} = 1 + 1 + \frac{5}{24} = 2\frac{5}{24}$

Solution **$2\frac{5}{24}$; Accept any equivalent fraction.**

The student found a common denominator before adding.

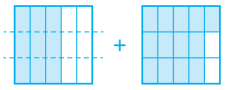
PAIR/SHARE

How would the solution method be different if you first wrote $\frac{11}{8}$ as $1\frac{3}{8}$?

APPLY IT

1 What is $\frac{3}{5} + \frac{13}{15}$? Show your work.

Possible student work using a model:



$\frac{3}{5} + \frac{13}{15} = \frac{9}{15} + \frac{13}{15} = \frac{22}{15} = 1\frac{7}{15}$

Solution **$1\frac{7}{15}$; Accept any equivalent fraction.**

How can you use an estimate to check if your answer is reasonable?

PAIR/SHARE

Explain why you chose the type of model you did to solve this problem.

Error Alert

If the error is . . .	*Students may . . .*	*To support understanding . . .*
$3\frac{6}{24}$	not have multiplied each numerator by the factor used to make the common denominator.	Have students draw models to show $\frac{1}{6}$ and $\frac{1}{8}$, make twenty-fourths in each model, and then use the models to illustrate the fractions in the problem and find the sum.
$3\frac{6}{14}$	have added the numerators and added the denominators because they may not understand what a denominator means.	Have students draw models to review finding the sum of two unit fractions such as $\frac{1}{2} + \frac{1}{4}$ or $\frac{1}{2} + \frac{1}{3}$. Reinforce the idea that only when denominators are equal are the parts the same. Only then can you name the total number of parts by adding. Then use models to represent the given sum.

EXAMPLE

$2\frac{5}{24}$; Accept any equivalent fraction. Using equations as shown is one way to solve the problem. Students could also solve the problem by drawing models or number lines divided into parts that show a common denominator for 8 and 6.

Look for First write the fractions with a common denominator. Then add the numerators.

APPLY IT

1 $1\frac{7}{15}$; Accept any equivalent fraction. Students could solve the problem by representing each fraction on a model divided into fifteenths, finding the total number of fifteenths shaded and writing the sum as a mixed number. They could also use a different common denominator for 5 and 15.

DOK 1

Look for Because $\frac{13}{15}$ is close to 1 and $\frac{3}{5} > \frac{1}{2}$, the sum should be close to $1\frac{1}{2}$.

2 $4\frac{1}{2}$ miles; Accept equivalent fraction. Students could solve the problem by drawing a number line divided into sixths, a point at $2\frac{2}{3}$, and an arrow showing jumps to the right—one jump of 1 and five jumps of $\frac{1}{6}$. They could also write the equations $2\frac{2}{3} + 1\frac{5}{6} = 2\frac{4}{6} + 1\frac{5}{6} = 3\frac{9}{6} = 4\frac{3}{6}$. They may also use a different common denominator of 3 and 6.

DOK 2

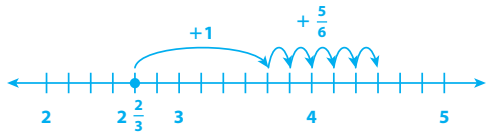
Look for The denominator 6 is a multiple of the other denominator, 3. Use 6 as the common denominator so that only one fraction needs to be replaced with an equivalent fraction.

3 **D**; Students could solve the problem by using the equations $\frac{3}{4} = \frac{9}{12}$, $\frac{2}{3} = \frac{8}{12}$, and $\frac{3}{4} + 1\frac{2}{3} = \frac{9}{12} + 1\frac{8}{12} = 1\frac{17}{12} = 2\frac{5}{12}$.

2 Michael rides his bike $2\frac{2}{3}$ miles on Saturday. He rides another $1\frac{5}{6}$ miles on Sunday. How many miles did Michael ride his bike on both days combined? Show your work.

How are the denominators 3 and 6 related?

Possible student work using a number line:



$2\frac{2}{3} + 1\frac{5}{6} = 2\frac{4}{6} + 1\frac{5}{6} = 4\frac{3}{6} = 4\frac{1}{2}$

PAIR/SHARE
How did you decide what common denominator to use to solve this problem?

Solution **$4\frac{1}{2}$ miles; Accept any equivalent fraction.**

3 Sasha needs $\frac{3}{4}$ cup of flour to make a batch of muffins and $1\frac{2}{3}$ cups of flour to make a loaf of bread. How many cups of flour does Sasha need to make both a batch of muffins and a loaf of bread?

How can you replace the given fractions with equivalent fractions that have like denominators?

Ⓐ $\frac{17}{12}$

Ⓑ $\frac{8}{7}$

Ⓒ $1\frac{5}{7}$

Ⓓ $2\frac{5}{12}$

Nicola chose Ⓐ as the correct answer. How did she get that answer?

Possible answer: She only added the fractional part of the mixed number to $\frac{3}{4}$.

PAIR/SHARE
What should be the denominator of the sum?

Explain why the other two answer choices are not correct:

B is not correct because it is the result of changing the mixed number to an improper fraction and then adding both the numerators and denominators.

C is not correct because it is the result of adding the numerators and denominators of the fractional parts.

DOK 3

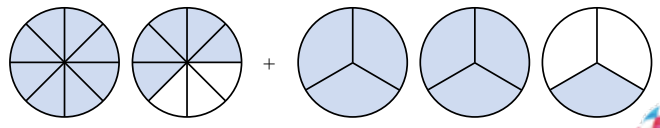
4 **A**; Identify $1 + 2 + \frac{6}{11}$ as the only choice that is not equivalent to $1\frac{5}{8} + 2\frac{1}{3}$.
DOK 1

Error Alert Students may think the circles that are entirely shaded must be represented by fractions and so choose D, the only expression that does not include whole numbers.

5 $9\frac{1}{3}$ yards; Accept equivalent fraction. Add the number of yards Sam has and the additional number he needs, finding a common denominator for 2 and 6 and writing equivalent fractions as needed. See possible equations on the Student Worktext page.
DOK 2

6 $\frac{5}{6}$ and $\frac{4}{9}$; Find the fractions with denominators that are factors of 18.
DOK 1

4 The model below represents the expression $1\frac{5}{8} + 2\frac{1}{3}$.



Which of the following could NOT be represented by the model?

Ⓐ $1 + 2 + \frac{6}{10}$

Ⓑ $1\frac{15}{24} + 2\frac{8}{24}$

Ⓒ $1 + 2 + \frac{23}{24}$

Ⓓ $\frac{39}{24} + \frac{56}{24}$

5 Sam has $5\frac{1}{2}$ yards of rope. He needs an additional $3\frac{5}{6}$ yards for a project. What is the total length of rope Sam needs for his project? Show your work.

Possible student work:

$\frac{1}{2} = \frac{3}{6}$

$5\frac{1}{2} + 3\frac{5}{6} = 5\frac{3}{6} + 3\frac{5}{6} = 8\frac{8}{6}$

$8\frac{8}{6} = 9\frac{2}{6} = 9\frac{1}{3}$

Solution $9\frac{1}{3}$ yards; Accept any equivalent fraction.

6 Which two fractions below can be added using the denominator 18?

$\frac{1}{4}$	$\frac{5}{6}$	$\frac{7}{12}$	$\frac{1}{8}$	$\frac{4}{9}$

Solution $\frac{5}{6}$ and $\frac{4}{9}$

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Differentiated Instruction

RETEACH

Hands-On Activity
Use fraction tiles to rewrite and add fractions.

Students struggling with concepts that fractions can be replaced with equivalent fractions to find a sum

Will benefit from additional work with concrete models

Materials For each pair: 1 set of fraction tiles

- Organize students in small groups and distribute fraction tiles.
- Guide students to use fraction tiles to model adding $\frac{3}{5}$ and $\frac{1}{10}$. Students can visually arrange the tiles to find an equivalent fraction for $\frac{3}{5}$ by placing six $\frac{1}{10}$ tiles on top of the three $\frac{1}{5}$ tiles to see that they name the same amount.
- Once students have found equivalent fractions with the same denominator, ask: *What is the sum of $\frac{3}{5}$ and $\frac{1}{10}$? How do you know?* $\left[\frac{7}{10}; \frac{3}{5} = \frac{6}{10} \text{ and } \frac{6}{10} + \frac{1}{10} = \frac{7}{10}\right]$
- Repeat with other pairs of fraction addends, such as $\frac{1}{4} + \frac{5}{8}$ and $\frac{1}{3} + \frac{1}{2}$. $\left[\frac{7}{8}; \frac{5}{6}\right]$

EXTEND

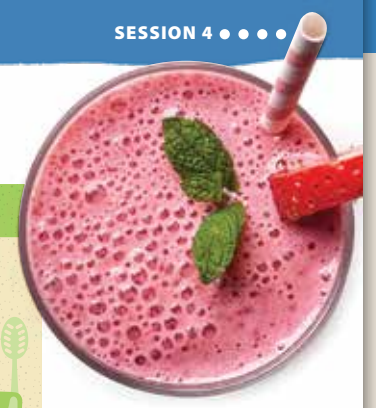
Challenge Activity
Write addition problems with unlike denominators.

Students who have achieved proficiency

Will benefit from deepening understanding of adding with mixed numbers

- Have students work in pairs to write addition problems with unlike fractions and mixed numbers. Each student writes a problem and its solution. Partners exchange problems and solve.
- Partners discuss the solutions and come to an agreement on what the correct solutions are.

7 Lucy is making a smoothie by following the recipe below.



Recipe

Sunshine Smoothie

- $1\frac{1}{3}$ cups banana
- $\frac{1}{2}$ cup yogurt
- 1 cup strawberries
- $\frac{3}{4}$ cup orange juice

Place ingredients in blender.

Blend until smooth.

Explain whether the recipe will make enough for Lucy and 3 friends to each have at least 1 cup of smoothie. If it does not make enough, explain how to change the recipe to make enough.

The recipe will not make enough. Possible explanation: I found the total amount by adding the amounts of all the ingredients. The recipe will make $3\frac{7}{12}$ or $3\frac{14}{24}$ cups. That's not enough for 4 people. Lucy needs 4 cups. Change the recipe to use $1\frac{1}{2}$ cups of strawberries. Then the recipe will make enough: $4\frac{1}{12}$ or $4\frac{2}{24}$ cups.

8 MATH JOURNAL

Lin says that $\frac{5}{7} + \frac{3}{2}$ is $\frac{31}{14}$. Carlos says the sum is $\frac{8}{9}$. Who is right? Explain your answer.

Lin; Possible explanation: Carlos added both the numerators and the denominators. Lin found the common denominator 14 and replaced each given fraction with an equivalent fraction that has the common denominator: $\frac{10}{14} + \frac{21}{14} = \frac{31}{14}$.

SELF CHECK Go back to the Unit 2 Opener and see what you can check off.

7 No, the recipe will not make enough; See possible explanation on the Student Worktext page. Add the number of cups of ingredients to find the sum is $3\frac{7}{12}$ or $3\frac{14}{24}$ cups. This is less than 4 cups. Increase amount of one or more ingredients to make the sum equal to or greater than 4 cups.

DOK 2

REINFORCE

Problems 4–8

Add fractions.

All students will benefit from additional work with adding fractions by solving problems in a variety of formats.

- Have students work on their own or with a partner to solve the problems.
- Encourage students to show their work.

PERSONALIZE

i-Ready

Provide students with opportunities to work on their personalized instruction path with *i-Ready* Online Instruction to:

- fill prerequisite gaps
- build up grade-level skills

Close: Exit Ticket

8 MATH JOURNAL

See possible explanation on the Student Worktext page; Student responses should indicate understanding that to add fractions and mixed numbers with unlike denominators, you first find a common denominator and replace given fractions with equivalent fractions with that denominator.

Error Alert **If** students add denominators as well as numerators of fractions and mixed numbers after finding a common denominator, **then** have students add using fraction tiles, with attention being paid to which part of the fraction changes (numerator).

SELF CHECK Have students consider whether they feel they are ready to check off any new skills on the Unit 2 Opener.

Lesson Overview

LESSON 13

Subtract Fractions

Lesson Objectives

Content Objectives

- Given two fractions with unlike denominators, write equivalent fractions with a common denominator.
- Use visual models to represent subtracting fractions with unlike denominators.
- Use equivalent fractions to subtract fractions and mixed numbers with unlike denominators.

Language Objectives

- Define common denominator and use the term in a discussion with a partner.
- Draw fraction models or number lines to show a difference of fractions with unlike denominators.
- Rewrite differences of fractions with unlike denominators as differences with like denominators using equivalent fractions.

Prerequisite Skills

- Find multiples of whole numbers.
- Understand the meaning of subtracting fractions.
- Subtract fractions and mixed numbers with like denominators.
- Find fractions equivalent to a given fraction.
- Regroup wholes in mixed numbers to write mixed numbers with fractions greater than 1.

Standards for Mathematical Practice (SMP)

SMPs 1, 2, 3, 4, 5, and 6 are integrated in every lesson through the *Try-Discuss-Connect* routine.*

In addition, this lesson particularly emphasizes the following SMPs:

4 Model with mathematics.

5 Use appropriate tools strategically.

7 Look for and make use of structure.

*See page 1i to see how every lesson includes these SMPs.

Lesson Vocabulary

There is no new vocabulary. Review the following key terms.

- **common denominator** a number that is a common multiple of the denominators of two or more fractions.
- **denominator** the number below the line in a fraction that tells the number of equal parts in the whole.
- **equivalent fractions** two or more different fractions that name the same part of a whole or the same point on a number line.
- **numerator** the number above the line in a fraction that tells the number of equal parts that are being described.

Learning Progression

In the previous lesson students added fractions and mixed numbers with unlike denominators by replacing fractions with equivalent fractions that have like denominators.

In this lesson students extend the methods for adding fractions and mixed numbers with unlike denominators to subtracting fractions and mixed numbers with unlike denominators. They represent the subtraction with manipulatives and visual models. When necessary to subtract, they regroup mixed numbers to show equivalent mixed numbers with fractions greater than 1.

In the next lesson students will subtract fractions with unlike denominators to solve word problems, using benchmark fractions and number sense to estimate and assess the reasonableness of their answers. In later grades students will apply and extend understandings of addition and subtraction of fractions to add and subtract rational numbers.

Lesson Pacing Guide

Whole Class Instruction

SESSION 1 Explore *45–60 min*	**Subtracting Fractions** • Start *5 min* • Try It *10 min* • Discuss It *10 min* • Connect It *15 min* • Close: Exit Ticket *5 min*	**Additional Practice** Lesson pages 251–252
SESSION 2 Develop *45–60 min*	**Subtracting Fractions with Unlike Denominators** • Start *5 min* • Try It *10 min* • Discuss It *10 min* • Picture It & Model It *5 min* • Connect It *10 min* • Close: Exit Ticket *5 min*	**Additional Practice** Lesson pages 257–258 **Fluency** Subtracting Fractions with Unlike Denominators
SESSION 3 Develop *45–60 min*	**Subtracting with Mixed Numbers** • Start *5 min* • Try It *10 min* • Discuss It *10 min* • Picture It & Model It *5 min* • Connect It *10 min* • Close: Exit Ticket *5 min*	**Additional Practice** Lesson pages 263–264 **Fluency** Subtracting with Mixed Numbers
SESSION 4 Refine *45–60 min*	**Subtracting Fractions** • Start *5 min* • Example & Problems 1–3 *15 min* • Practice & Small Group Differentiation *20 min* • Close: Exit Ticket *5 min*	**Lesson Quiz** or **Digital Comprehension Check**

Lesson Materials

Lesson *(Required)*	*Per pair:* 1 set of fraction tiles, 1 set of fraction circles *Activity Sheet:* Fraction Bars
Activities	*Per pair:* 1 set of fraction tiles *Activity Sheets:* Fraction Bars, Number Lines
Math Toolkit	fraction tiles, fraction circles, fraction bars, grid paper, number lines
Digital Math Tools	Fraction Models, Number Line

Teacher Toolbox

Small Group Differentiation

PREPARE

Ready Prerequisite Lessons

Grade 4
- Lesson 17 Understand Equivalent Fractions
- Lesson 21 Add and Subtract Mixed Numbers

RETEACH

Tools for Instruction

Grade 4
- Lesson 17 Equivalent Fractions
- Lesson 21 Add and Subtract Mixed Numbers

Grade 5
- Lesson 13 Subtract Fractions and Mixed Numbers with Unlike Denominators

REINFORCE

Math Center Activity

Grade 5
- Lesson 13 Fraction Subtraction: True or False!

EXTEND

Enrichment Activity

Grade 5
- Lesson 13 Race Training

i-Ready

Independent Learning

PERSONALIZE

i-Ready Lesson*

Grade 5
- Add and Subtract Fractions

Learning Game
- Prerequisite: *Bounce*

*We continually update the Interactive Tutorials. Check the Teacher Toolbox for the most up-to-date offerings for this lesson.

Connect to Family, Community, and Language Development

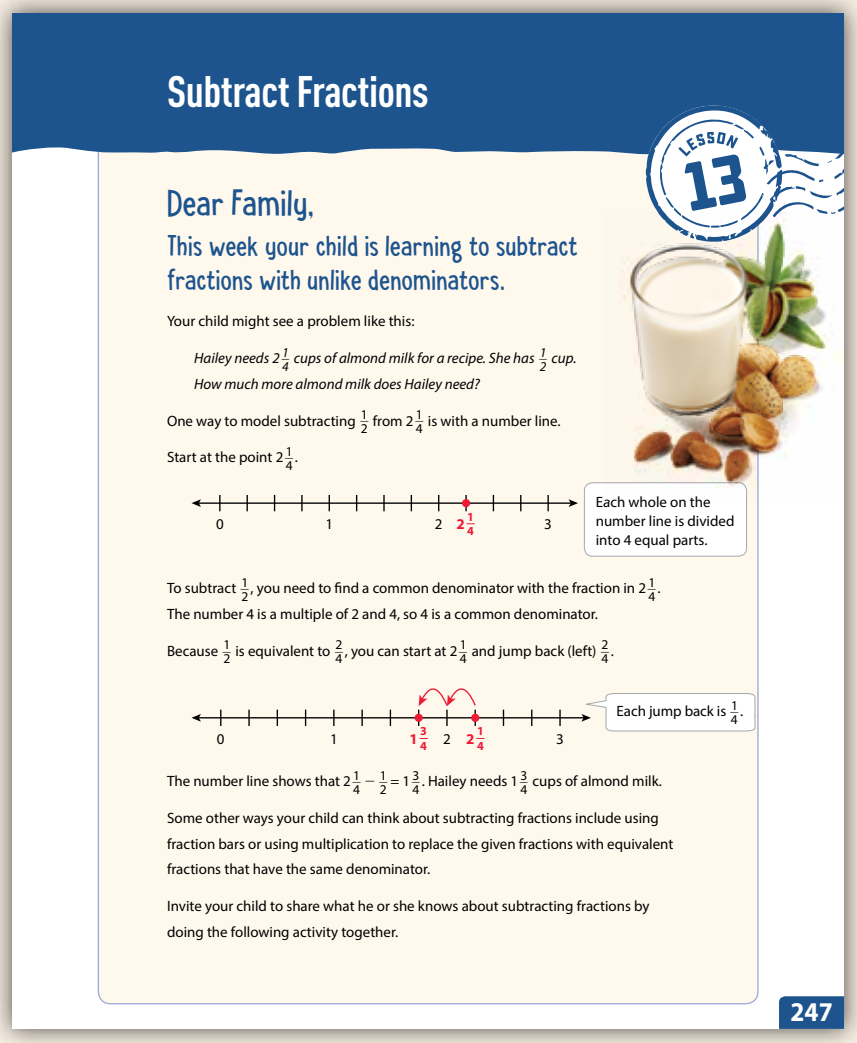
The following activities and instructional supports provide opportunities to foster school, family, and community involvement and partnerships.

Connect to Family

Use the **Family Letter**—which provides background information, math vocabulary, and an activity—to keep families apprised of what their child is learning and to encourage family involvement.

Available in Spanish

Teacher Toolbox



Subtract Fractions

LESSON 13

Dear Family,

This week your child is learning to subtract fractions with unlike denominators.

Your child might see a problem like this:

Hailey needs $2\frac{1}{4}$ cups of almond milk for a recipe. She has $\frac{1}{2}$ cup. How much more almond milk does Hailey need?

One way to model subtracting $\frac{1}{2}$ from $2\frac{1}{4}$ is with a number line.

Start at the point $2\frac{1}{4}$.

Each whole on the number line is divided into 4 equal parts.

To subtract $\frac{1}{2}$, you need to find a common denominator with the fraction in $2\frac{1}{4}$.
The number 4 is a multiple of 2 and 4, so 4 is a common denominator.

Because $\frac{1}{2}$ is equivalent to $\frac{2}{4}$, you can start at $2\frac{1}{4}$ and jump back (left) $\frac{2}{4}$.

Each jump back is $\frac{1}{4}$.

The number line shows that $2\frac{1}{4} - \frac{1}{2} = 1\frac{3}{4}$. Hailey needs $1\frac{3}{4}$ cups of almond milk.

Some other ways your child can think about subtracting fractions include using fraction bars or using multiplication to replace the given fractions with equivalent fractions that have the same denominator.

Invite your child to share what he or she knows about subtracting fractions by doing the following activity together.

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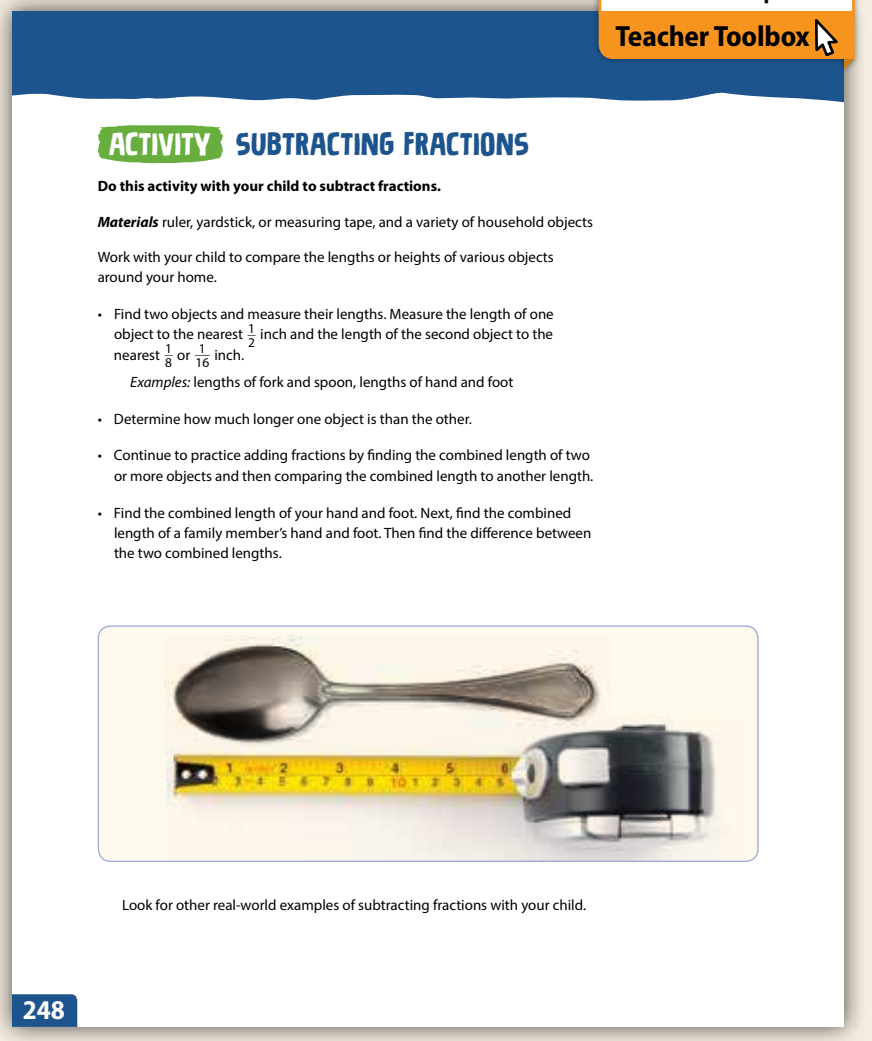

ACTIVITY SUBTRACTING FRACTIONS

Do this activity with your child to subtract fractions.

Materials ruler, yardstick, or measuring tape, and a variety of household objects

Work with your child to compare the lengths or heights of various objects around your home.

- Find two objects and measure their lengths. Measure the length of one object to the nearest $\frac{1}{2}$ inch and the length of the second object to the nearest $\frac{1}{8}$ or $\frac{1}{16}$ inch.
  Examples: lengths of fork and spoon, lengths of hand and foot
- Determine how much longer one object is than the other.
- Continue to practice adding fractions by finding the combined length of two or more objects and then comparing the combined length to another length.
- Find the combined length of your hand and foot. Next, find the combined length of a family member's hand and foot. Then find the difference between the two combined lengths.

Look for other real-world examples of subtracting fractions with your child.

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Goal

The goal of the Family Letter is to provide opportunities for students to share their learning about subtracting fractions with unlike denominators.

Activity

Look for objects of different lengths around the house to measure and then practice subtracting one length from the other. Look at the *Subtracting Fractions* activity and adjust it if necessary to connect with your students.

Math Talk at Home

Encourage students to look for objects at home that have fractions printed on them and practice subtracting fractions.

Conversation Starters Below are additional conversation starters students can write in their Family Letter or math journal to engage family members:

- *Do you use fractions at home or at work?*
- *If so, are there times when you have to add or subtract them?*

Connect to Community and Cultural Responsiveness

Use these activities to connect with and leverage the diverse backgrounds and experiences of all students.

Sessions 1 and 2 **Use anytime during these sessions.**

- Subtracting fractions may not seem as natural for students as adding them; therefore, it is important to help students make concrete connections.
- Review with students how fractions are used when telling time (e.g., a quarter of an hour and half an hour). Have students work in groups to determine the most common fractions of a clock and practice telling subtraction stories.

Sessions 3 and 4 **Use anytime during these sessions.**

- A common recommendation is to drink six to eight glasses of water a day. Have students research if this is true for all ages. In addition, students can research if different climates or weather affect the recommended amount of water per day.
- Have students work in groups to determine how to mark fractions of a cup on their own water bottles. Encourage them to write a subtraction equation each time they drink water. Students who do not have water bottles with markings can work with students who do, using the same information for their subtraction equations.

Connect to Language Development

For ELLs, use the Differentiated Instruction chart to plan and prepare for specific activities in every session.

ELL English Language Learners: Differentiated Instruction

Prepare for Session 1
Use with *Try It*.

Levels 1–3

Speaking/Writing Read the ***Try It*** problem. Have students point to the word *bolt* and the image. Ask students to circle the length of the bolt Paul has. Using the following sentence frames, have students restate the problem:

- *Paul has a bolt. It is* $\frac{3}{4}$ *inches long.*
- *Paul buys one bolt that is* $\frac{1}{8}$ *inch longer. He buys another bolt that is* $\frac{1}{8}$ *inch shorter.*
- *I need to find the* length *of each bolt that Paul buys.*

Have students form pairs to discuss how they will find the lengths of the two bolts and solve the problem.

Levels 2–4

Speaking/Writing Have students read the ***Try It*** problem with a partner. Have them label the image of the bolt. Ask students to analyze how many bolts Paul has and use the following sentence frames to restate the facts.

- *Paul has a bolt that is* $\frac{3}{4}$ *inch in length.*
- *Paul buys two more bolts. One bolt is* _____ *and the other is* _____.

Ask students to work with a partner to solve the problem and then write one or more complete sentences to state the lengths of the two bolts.

Levels 3–5

Reading/Writing Have students read the ***Try It*** problem with a partner and work together to solve it. Provide the following sentence starters for students to state what they know and to help them solve the problem.

What I know:

- *I know that Paul has* _____.
- *Paul buys* _____.
- *I need to find out* _____.

To solve the problem:

- *First, I need to* _____.
- *Then I can add* _____.
- *I also need to subtract* _____.

Ask students to present their solution to another group and justify their process.

LESSON 13

SESSION 1 Explore

Purpose In this session students draw on the similarities between adding and subtracting fractions with unlike denominators. They share models to explore how making same-size fractional parts to add can help them also to subtract. They will look ahead to think about the skills they have for adding fractions that they can apply to subtracting fractions.

Start

Connect to Prior Knowledge

Materials For each pair: 1 set of fraction tiles

Why Support students' facility with finding equivalent fractions for $\frac{3}{4}$, foreshadowing the work they will do with equivalent fractions to solve the problem.

How Have students use sets of fraction tiles or fraction circles to build the two possible equivalent fractions for $\frac{3}{4}$.

Use fraction tiles to make two fractions equivalent to $\frac{3}{4}$.

$\frac{3}{4} =$ .....

$\frac{3}{4} =$ .....

Solutions

$\frac{6}{8}$

$\frac{9}{12}$

TRY IT

Make Sense of the Problem

To support students in making sense of the problem, have them identify the number of bolts Paul must buy and what information we have about each of those bolts.

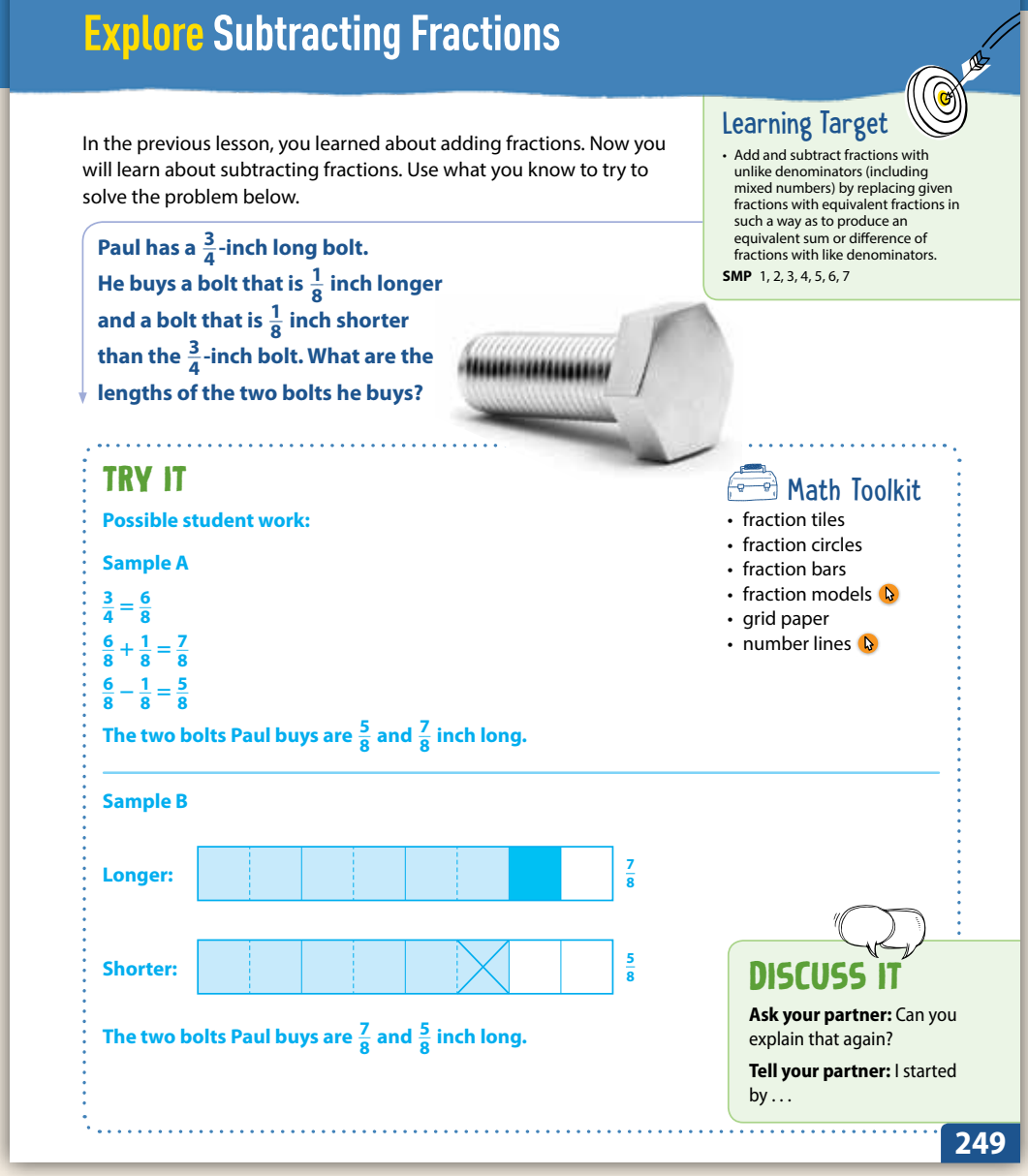
DISCUSS IT

Support Partner Discussion

Encourage students to share what did not work for them as well as what did as they talk to each other.

Look for, and prompt as necessary for, understanding of:

- a common denominator needs to be found for fourths and eighths
- the same-size parts used to show adding can also be used to show subtracting



LESSON 13 SESSION 1 ● ○ ○ ○

Explore Subtracting Fractions

In the previous lesson, you learned about adding fractions. Now you will learn about subtracting fractions. Use what you know to try to solve the problem below.

Paul has a $\frac{3}{4}$-inch long bolt. He buys a bolt that is $\frac{1}{8}$ inch longer and a bolt that is $\frac{1}{8}$ inch shorter than the $\frac{3}{4}$-inch bolt. What are the lengths of the two bolts he buys?

Learning Target

- Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators.

SMP 1, 2, 3, 4, 5, 6, 7

TRY IT

Possible student work:

Sample A

$\frac{3}{4} = \frac{6}{8}$

$\frac{6}{8} + \frac{1}{8} = \frac{7}{8}$

$\frac{6}{8} - \frac{1}{8} = \frac{5}{8}$

The two bolts Paul buys are $\frac{5}{8}$ and $\frac{7}{8}$ inch long.

Sample B

Longer: $\frac{7}{8}$

Shorter: $\frac{5}{8}$

The two bolts Paul buys are $\frac{7}{8}$ and $\frac{5}{8}$ inch long.

Math Toolkit

- fraction tiles
- fraction circles
- fraction bars
- fraction models
- grid paper
- number lines

DISCUSS IT

Ask your partner: Can you explain that again?

Tell your partner: I started by . . .

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Common Misconception Look for students who add and subtract fractions without concern that each whole is divided into same-size parts. As students present solutions, have them specify why they used the denominator they did.

Select and Sequence Student Solutions

One possible order for whole class discussion:

- physical parts showing fourths and eighths
- drawings to represent the problem
- number lines marked in eighths or another common denominator
- equations showing $\frac{3}{4}$ replaced with an equivalent fraction

Support Whole Class Discussion

Prompt students to note the relationship between the numbers in each model and the numbers in the problem.

Ask *How do [student name]'s and [student name]'s models show the length of the bolt Paul has, and the lengths of the longer and shorter bolts Paul buys?*

Listen for The lengths are shown as fractional measures of inches that have the same denominators, such as $\frac{6}{8}$ inch, $\frac{7}{8}$ inch, and $\frac{5}{8}$ inch.

CONNECT IT

1 LOOK BACK

Look for understanding that a common denominator for the measures of $\frac{3}{4}$ inch and $\frac{1}{8}$ inch had to be found, and that the same common denominator could be used to both add and subtract.

Hands-On Activity

Use fraction tiles to add and subtract fractions with unlike denominators.

If . . . *students are unsure about applying the concept of common denominators to subtracting fractions,*

Then . . . *use this activity to have them model an equivalent fraction that is then used to show both adding and subtracting.*

Materials For each pair: 1 set of fraction tiles

- Display the problems $\frac{3}{4} + \frac{1}{8}$ and $\frac{3}{4} - \frac{1}{8}$.
- Have students model $\frac{3}{4}$. [three $\frac{1}{4}$ tiles]
- Have students find the number of $\frac{1}{8}$ tiles it takes to cover $\frac{3}{4}$, replace the fourths with those pieces, and identify the equivalent fraction represented. [6 tiles; $\frac{6}{8}$]
- Have students show $\frac{6}{8} + \frac{1}{8}$ and $\frac{6}{8} - \frac{1}{8}$ by adding or removing 1 tile and record the sum and difference shown. [$\frac{7}{8}$; $\frac{5}{8}$]
- Repeat activity to show other pairs of sums and differences, such as $\frac{3}{4} + \frac{3}{12}$ and $\frac{3}{4} - \frac{3}{12}$.

2 LOOK AHEAD

Point out that the same steps and methods you use to make equal-size parts to add fractions with unlike denominators can be used to subtract fractions with unlike denominators.

Students should be able to use fraction language to discuss the steps and methods shown to find $\frac{3}{4} - \frac{1}{3}$: finding 12 as a common multiple of 4 and 3, multiplying to find the equivalent fractions $\frac{9}{12}$ and $\frac{4}{12}$, and replacing $\frac{3}{4}$ and $\frac{1}{3}$ with $\frac{9}{12}$ and $\frac{4}{12}$ in an equation to show the difference.

CONNECT IT

1 LOOK BACK

Explain how to find the lengths of the two bolts Paul buys.

Possible answer: First you need a common denominator. $\frac{3}{4} = \frac{6}{8}$ Then you can add and subtract eighths. $\frac{6}{8} + \frac{1}{8} = \frac{7}{8}$; $\frac{6}{8} - \frac{1}{8} = \frac{5}{8}$

2 LOOK AHEAD

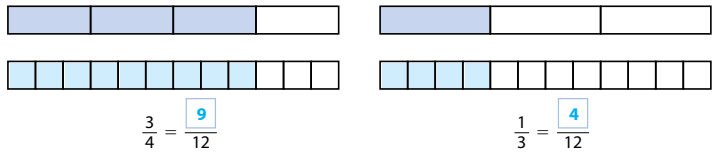
Just like with adding fractions, you must find a common denominator to subtract fractions with unlike denominators, such as $\frac{3}{4} - \frac{1}{3}$.

a. Write the first four multiples of each denominator. Then circle the common multiple to find a common denominator.

Multiples of 4: **4**, **8**, **(12)**, **16**

Multiples of 3: **3**, **6**, **9**, **(12)**

b. Complete the models and equations to show the equivalent fractions for $\frac{3}{4}$ and $\frac{1}{3}$ using the common multiple as the common denominator.



$\frac{3}{4} = \frac{\boxed{9}}{12}$ $\frac{1}{3} = \frac{\boxed{4}}{12}$

c. Use the equivalent fractions to subtract.

$\frac{3}{4} - \frac{1}{3} = \frac{\boxed{9}}{12} - \frac{\boxed{4}}{12} = \frac{\boxed{5}}{12}$

3 REFLECT

How is subtracting fractions like adding fractions?

Possible answer: The denominators must be the same. So, sometimes you need to replace one or more fractions with equivalent fractions that have common denominators.

Close: Exit Ticket

3 REFLECT

Look for understanding that just as for adding fractions, to subtract fractions the denominators of the fractions must be the same. Students' responses should include that before adding or subtracting, it may be necessary to replace one or more of the fractions with an equivalent fraction so that the fractions use a common denominator. Students may also mention that you add or subtract the numerators over the common denominator.

Common Misconception **If** students do not correctly identify the denominator to use for the difference, **then** provide students with fraction bars and have them model the problem from the Student Worktext page, $\frac{3}{4} - \frac{1}{3}$. Prompt them to find the equivalent fractions by asking: *How many shaded parts? How many parts in the whole?*

Real-World Connection

Encourage students to think about everyday places or situations where people might need to subtract fractions with unlike denominators. Have volunteers share ideas. Examples: comparing measurements given in fractions of a foot, fractions of cup, or fractions of an hour.

Solutions

Support Vocabulary Development

1 Have students say *equivalent fractions.* Assess prior knowledge by asking students to discuss what they know about fractions.

Ask students to work in small groups to complete the graphic organizer. Call on volunteers to share what they wrote for *In My Own Words*. Correct any misconceptions and ask students to revise their graphic organizers if necessary. Encourage students to look for real-world illustrations that show equivalent fractions.

To help students remember the meaning of the word *equivalent*, ask them to circle the letters *equ* and *val* in the word. Point out that the word *equivalent* has the same root words as the words *equal* and *value*.

2 Read the problem. Have students discuss with a partner what the first step should be to determine if the fractions are equivalent. As students explain their answer, ask them to justify their thinking. Provide the following sentence frame:

- *I know Jackie's answer is* incorrect *because* $\frac{3}{4} = \frac{6}{8}$.

Supplemental Math Vocabulary

- *denominator*
- *numerator*
- *multiples*

Name: _____ LESSON 13 SESSION 1

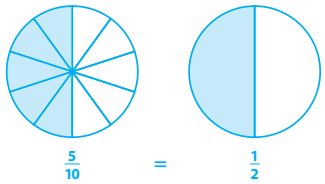
Prepare for Subtracting Fractions

1 Think about what you know about fractions. Fill in each box. Use words, numbers, and pictures. Show as many ideas as you can. **Possible answers:**

In My Own Words

Two fractions that have the same value; both fractions name the same part of a whole and the same point on a number line.

My Illustrations



$\frac{5}{10}$ = $\frac{1}{2}$

equivalent fractions

Examples

$\frac{1}{2} = \frac{2}{4}$
$\frac{1}{3} = \frac{3}{9}$
$\frac{2}{5} = \frac{4}{10}$

Non-Examples

$\frac{1}{2}$ and $\frac{1}{5}$
$\frac{2}{3}$ and $\frac{1}{3}$
$\frac{3}{5}$ and $\frac{2}{7}$

2 Jackie says that the fraction $\frac{7}{8}$ is equivalent to $\frac{3}{4}$. Is she right? Explain.

No; Possible explanation: $\frac{3}{4} = \frac{6}{8}$, so it is not equivalent to $\frac{7}{8}$.

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3 Assign problem 3 to provide another look at solving a problem that involves adding and subtracting fractions with unlike denominators.

This problem is very similar to the problem about Paul buying two new bolts. In both problems, students need to add and subtract the same fraction from a given fraction to solve the problem. The question asks for the widths of the two new nuts Solange buys.

Students may want to use fraction bars, fraction circles, or fraction tiles.

Suggest that students read the problem three times, asking themselves one of the following questions each time:

- *What is this problem about?*
- *What is the question I am trying to answer?*
- *What information is important?*

Solution:

$\frac{1}{4} = \frac{2}{8}; \frac{5}{8} + \frac{2}{8} = \frac{7}{8}; \frac{5}{8} - \frac{2}{8} = \frac{3}{8}$; The two nuts that Solange buys are $\frac{7}{8}$ inch and $\frac{3}{8}$ inch wide.

Medium

4 Have students solve the problem another way to check their answer.

3 Solve the problem. Show your work.

Solange has a nut that is $\frac{5}{8}$ inch wide. She buys a nut that is $\frac{1}{4}$ inch wider and a nut that is $\frac{1}{4}$ inch narrower than the $\frac{5}{8}$-inch nut. What are the widths of the two nuts she buys?

Possible student work using equations:

$\frac{1}{4} = \frac{2}{8}$

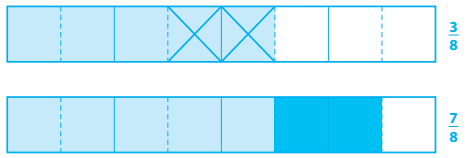
$\frac{5}{8} + \frac{2}{8} = \frac{7}{8}$

$\frac{5}{8} - \frac{2}{8} = \frac{3}{8}$

Solution **The two nuts Solange buys are $\frac{7}{8}$ and $\frac{3}{8}$ inch wide.**

4 Check your answer. Show your work.

Possible student work:



The two nuts Solange buys are $\frac{3}{8}$ and $\frac{7}{8}$ inch wide.

ELL English Language Learners: Differentiated Instruction

Prepare for Session 2
Use with *Try It*.

Levels 1–3

Listening/Speaking Read the ***Try It*** problem aloud and have students follow along. Ask students to point to how much water Gavin has in his water bottle. Provide the following sentence frames:

- *Gavin has* ___$\frac{2}{3}$___ *pint of water in his bottle.*
- *Gavin drinks* ___$\frac{1}{2}$___ *pint.*
- *I need to know* ___how much water is left___.

Ask students to work with a partner to solve the problem. Encourage them to describe the steps to solve the problem.

Levels 2–4

Listening/Speaking Read the ***Try It*** problem with students. Ask students to state the problem in their own words. Have partners discuss the steps needed to solve the problem.

Once students solve the problem, have them compare their steps with other pairs. Encourage students to justify their answers.

Levels 3–5

Writing/Speaking Have students read the ***Try It*** problem independently. Have them write the steps they took to solve the problem.

Have students trade their steps with another student. Ask them to compare the steps and diagrams that they used to solve the problem and to explain and justify their answers to their partner.

LESSON 13

SESSION 2 Develop

Purpose In this session students solve a problem that requires subtracting $\frac{1}{2}$ from $\frac{2}{3}$. Students model the fractions in the word problem either on paper or with manipulatives to show equivalent fractions with a common denominator and represent the difference. The purpose of this problem is to have students develop a strategy to subtract fractions with unlike denominators.

Start

Connect to Prior Knowledge

Materials For each pair: 1 set of fraction circles

Why Support students' facility with finding common denominators for fractions.

How Have students use fraction circles to model $\frac{2}{5}$ and $\frac{1}{2}$ as equivalent fractions with a common denominator.

Use fraction circles to show $\frac{2}{5}$ and $\frac{1}{2}$ as equivalent fractions with a common denominator.

$\frac{2}{5} =$ .....

$\frac{1}{2} =$ .....

Solutions

$\frac{2}{5} = \frac{4}{10}$

$\frac{1}{2} = \frac{5}{10}$

Develop Language

Why Reinforce students' vocabulary related to the units of measure for liquid volume, or capacity.

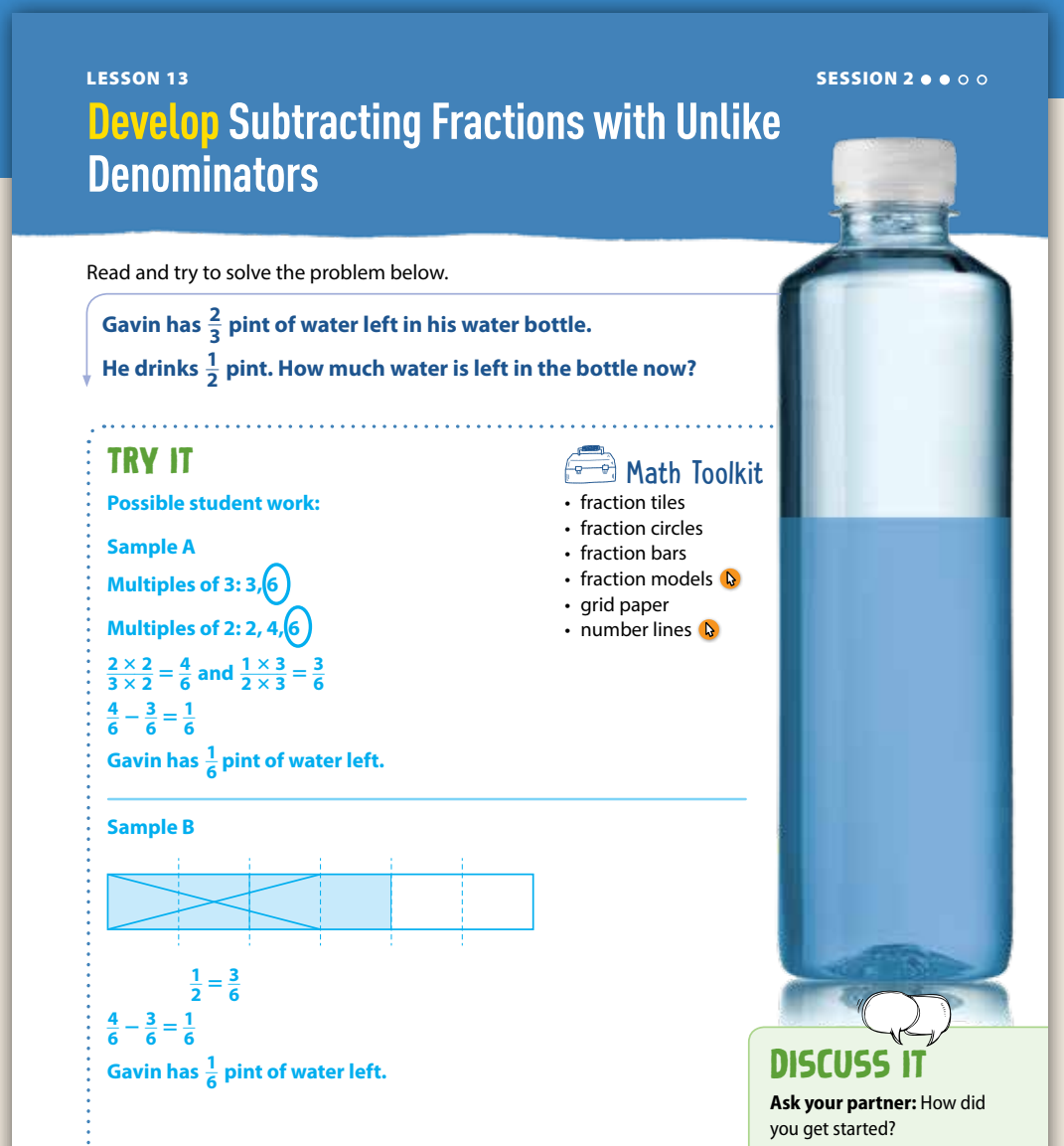
How Tell students to point to and underline the term *pint*. Remind students that a pint is a unit used to measure liquid volume in the customary system. Ask students to explore the different customary and metric units of measurement used for water bottles, milk containers, etc. They may look at containers during meals at home or during lunch at school.

TRY IT

Make Sense of the Problem

To support students in making sense of the problem, have them identify how much water Gavin has in his water bottle and how much water Gavin drinks.

Ask *Does Gavin have more or less than 1 pint of water in his bottle before he takes a drink?*



LESSON 13 SESSION 2 ● ● ○ ○

Develop Subtracting Fractions with Unlike Denominators

Read and try to solve the problem below.

Gavin has $\frac{2}{3}$ pint of water left in his water bottle.
He drinks $\frac{1}{2}$ pint. How much water is left in the bottle now?

TRY IT

Possible student work:

Sample A

Multiples of 3: 3, 6

Multiples of 2: 2, 4, 6

$\frac{2 \times 2}{3 \times 2} = \frac{4}{6}$ and $\frac{1 \times 3}{2 \times 3} = \frac{3}{6}$

$\frac{4}{6} - \frac{3}{6} = \frac{1}{6}$

Gavin has $\frac{1}{6}$ pint of water left.

Sample B

$\frac{1}{2} = \frac{3}{6}$

$\frac{4}{6} - \frac{3}{6} = \frac{1}{6}$

Gavin has $\frac{1}{6}$ pint of water left.

Math Toolkit

- fraction tiles
- fraction circles
- fraction bars
- fraction models
- grid paper
- number lines

DISCUSS IT

Ask your partner: How did you get started?

Tell your partner: I am not sure how to find the answer because . . .

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DISCUSS IT

Support Partner Discussion

Encourage students to share what did not work well for them as well as what did as they discuss their solutions.

Support as needed with questions such as:

- *How would you describe your model?*
- *What do you think is an advantage to using that model?*

Common Misconception Look for students who subtract half of the fractional part, $\frac{4}{6}$ pint, instead of half of the whole, $\frac{6}{6}$ pint, and get an answer of $\frac{2}{6}$ pint. Ask students to distinguish between $\frac{1}{2}$ pint and $\frac{1}{2}$ of 2 pints.

Select and Sequence Student Solutions

One possible order for whole class discussion:

- physical parts showing halves, thirds, and sixths
- drawings to represent the problem
- number lines marked in sixths or another common denominator of halves and thirds
- equations showing equivalent fractions with like denominators for $\frac{2}{3}$ and $\frac{1}{2}$

Support Whole Class Discussion

Compare and connect the different representations and have students identify how they are related.

Ask *Where does your model show the amount of water Gavin had to begin with? The amount he drank? The amount he has left?*

Listen for Students should recognize that accurate responses show the use of a common denominator for thirds and halves. Responses may include $\frac{4}{6}$ pint as the amount of water Gavin had to start, $\frac{3}{6}$ pint as the amount he drank, and $\frac{1}{6}$ as the amount left.

PICTURE IT & MODEL IT

If no student presented these models, connect them to the student models by pointing out the ways they each represent:

- the whole
- the amount of water equivalent to $\frac{2}{3}$ pint
- the amount of water equivalent to $\frac{1}{2}$ pint
- the amount of water Gavin has left

Ask *What common denominator for thirds and halves is shown in the picture? On the number line? Is it the same or different?*

Listen for Sixths is shown as the common denominator. Both the pint water bottle and number line are divided into 6 equal parts.

For a picture of the water bottle, prompt students to interpret how the subtraction of $\frac{3}{6}$ pint of water is represented in the second picture.

- *Why are 4 sections dark blue?*
- *Why are 3 sections enclosed in a red rectangle?*

For a number line model, prompt students to identify how the number line is divided and what the red arrows show.

- *What does each jump represent?*
- *Are the jumps to the left or the right on the number line? Why?*

Explore different ways to understand subtracting fractions with unlike denominators.

Gavin has $\frac{2}{3}$ pint of water left in his water bottle. He drinks $\frac{1}{2}$ pint. How much water is left in the bottle now?

PICTURE IT

You can use a picture to model subtracting fractions.

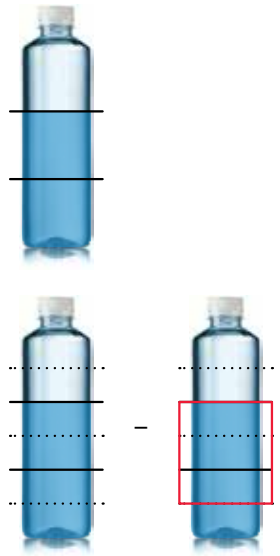
The water bottle is marked to show that it has $\frac{2}{3}$ pint of water in it. Gavin drinks $\frac{1}{2}$ pint of water.

Use equivalent fractions to find a common denominator.

$\frac{2}{3} = \frac{4}{6}$

$\frac{1}{2} = \frac{3}{6}$

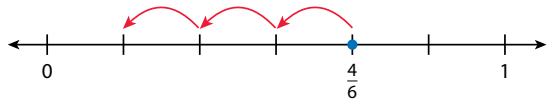
Now the water bottle is marked to show sixths. You can subtract $\frac{3}{6}$ from $\frac{4}{6}$.



MODEL IT

You can use a number line to model subtracting fractions.

The number line below is divided into sixths, the common denominator.



Start at $\frac{4}{6}$ and jump left $\frac{3}{6}$. ($\frac{3}{6}$ is three $\frac{1}{6}$ units on the number line.)

Deepen Understanding

Number Line Model

SMP 4 Model with mathematics.

After discussing how the number line represents the take-away problem given, present this comparison problem that uses the same amounts.

Problem *Gavin has $\frac{2}{3}$ of a pint of water in his water bottle. Tara has $\frac{1}{2}$ of a pint in her bottle. How much more water does Gavin have?*

Ask *Would you model this situation on a number line in the same way as shown in* Model It*, or a different way? Can you describe more than one way to show the quantities in this subtraction problem with a number line?*

Listen for You could mark the points $\frac{3}{6}$ and $\frac{4}{6}$ on the number line and count on from $\frac{3}{6}$ to $\frac{4}{6}$ to find the difference. You also can draw two segments, one from 0 to $\frac{4}{6}$ and one from 0 to $\frac{1}{2}$, and use the difference in the lengths of the segments to show the difference in the amounts of water.

Discuss what is the same and different about the ways of finding the difference. Point out that all the ways are valid ways to solve both problems.

CONNECT IT

- Remind students that one thing that is alike about all the representations is they show how to replace fractions that do not have same-size parts with equivalent fractions that do in order to subtract.
- Explain that on this page they will write subtraction equations to match representations of the problem.

Monitor and Confirm

1–4 Check for understanding that:

- to subtract, fractions must have the same-size parts—a common denominator
- 6 is a multiple of both 3 and 2, so sixths allow you to show both thirds and halves
- $\frac{2}{3} - \frac{1}{2} = \frac{4}{6} - \frac{3}{6} = \frac{1}{6}$
- $\frac{1}{6}$ pint of water is left

Support Whole Class Discussion

5 Have students share their responses to problem 5, stating other possible common denominators, such as 12 and 24. You may also want to have them write the corresponding subtraction equation.

Ask *How does the common denominator 6 used in* Picture It *and* Model It *relate to the denominators of the original fractions? How do other possible common denominators relate to 6?*

Listen for 6 is the product of the original denominators, 2 and 3. Other possible common denominators, such as 12 and 24, are multiples of 6.

6 Look for the idea that you replace the fractions with equivalent fractions that have a common denominator, then subtract. Students may also note that any common multiple of the denominators (such as their product) will allow you to write the fractions with same-size parts, so any common multiple can be used as the common denominator.

7 REFLECT Have all students focus on the strategies used to solve this problem. If time allows, have students share their preferences with a partner.

CONNECT IT

Now you will use the problem from the previous page to help you understand how to use equivalent fractions to subtract.

1 Look at **Picture It** and **Model It** from the previous page. Why is $\frac{2}{3}$ rewritten as $\frac{4}{6}$? Why is $\frac{1}{2}$ rewritten as $\frac{3}{6}$?

Possible answer: Thirds and halves are different-size parts. To subtract, the size of the parts needs to be the same.

2 Why are sixths chosen as a common denominator?

Possible answer: 6 is a multiple of both 2 and 3.

3 Use sixths as a common denominator. Write an equation to show the difference of $\frac{2}{3}$ and $\frac{1}{2}$.

$\frac{4}{6} - \frac{3}{6} = \frac{1}{6}$

4 How much water is left in the bottle? $\frac{1}{6}$ pint

5 Could you have subtracted with a different common denominator? Provide an example.

Possible answer: Yes. You could use any common multiple of 2 and 3, such as 12. You could find equivalent fractions for twelfths to subtract.

6 Explain how to subtract two fractions with unlike denominators.

Possible answer: Write the fractions as equivalent fractions that have the same denominator. Then subtract the fractions.

7 REFLECT

Look back at your **Try It**, strategies by classmates, and **Picture It** and **Model It**. Which models or strategies do you like best for subtracting fractions with unlike denominators? Explain.

Possible answer: I like using a visual model to subtract fractions with unlike denominators because I can see how to make the parts the same size before I subtract.

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Hands-On Activity

Model equivalent differences.

If . . . *students are unsure about using different possible common denominators,*

Then. . . *use this activity to model equivalent differences.*

Materials For each student: Activity Sheet *Fraction Bars* (1 whole)

- Write the problem $\frac{3}{4} - \frac{1}{2} = ?$ on the board. Ask students to show $\frac{3}{4}$ on the fraction bar. [fold into 4 one-fourths and shade 3 one-fourths]
- Together, identify 4, 8, and 16 as common multiples of 4 and 2.
- Have a volunteer rewrite the subtraction equation replacing $\frac{1}{2}$ with the equivalent fraction using fourths. $\left[\frac{3}{4} - \frac{2}{4}\right]$ Ask students to model this on the fraction bar by drawing an X over two of the three shaded fourths and then complete the subtraction equation. $\left[\frac{3}{4} - \frac{2}{4} = \frac{1}{4}\right]$
- Have students fold their fraction bars to show eighths. Ask: *What subtraction equation does your fraction bar now model? Explain.* $\left[\frac{6}{8} - \frac{4}{8} = \frac{2}{8}\right]$; The bar has 6 of 8 parts shaded and 4 of those parts are crossed out.]
- Repeat the previous step, folding to show sixteenths. $\left[\frac{12}{16} - \frac{8}{16} = \frac{4}{16}\right]$

APPLY IT

For all problems, encourage students to draw some kind of model to support their thinking. Allow some leeway in precision; drawing same-size parts in a whole can be difficult.

8 $\frac{17}{24}$ or any equivalent fraction; See possible work on the Student Worktext page. Students may also show $\frac{21}{24}$ on a number line and show 4 jumps of $\frac{1}{24}$ each to the left. They may also show a fraction model representing $\frac{21}{24}$ with $\frac{4}{24}$ crossed out.

9 Emily's shelf is $\frac{1}{12}$ foot wider than her clock; Accept equivalent answers using other denominators, and note that some students may say that the equivalent length, in inches, is 1 inch. See possible work on the Student Worktext page; Students may also use a number line model or write equations.

Close: Exit Ticket

10 $\frac{3}{10}$ or any equivalent fraction; See possible work on the Student Worktext page. Students may also show $\frac{9}{10}$ on a number line and show 3 jumps of $\frac{2}{10}$ each to the left. They may also write the equations $\frac{3}{5} = \frac{6}{10}, \frac{9}{10} - \frac{6}{10} = \frac{3}{10}$.
Students' solutions should indicate understanding of:

- making equivalent fractions so fractions operated on have a common denominator
- accurate use of visual fraction models and equations to represent the problem

Error Alert **If** students' solution is $\frac{6}{10}$, **then** review that when making equivalent fractions, both the numerator and denominator change by the same factor. Have students model $\frac{3}{5}$ on a fraction bar, then divide each section of the bar in half so the bar then shows $\frac{6}{10}$. Discuss how both the numerator and denominator doubled, and guide students to represent what they did with the equation $\frac{(3 \times 2)}{(5 \times 2)} = \frac{6}{10}$.

APPLY IT

Use what you just learned to solve these problems.

8 What is $\frac{7}{8} - \frac{1}{6}$? Show your work.

Possible student work:

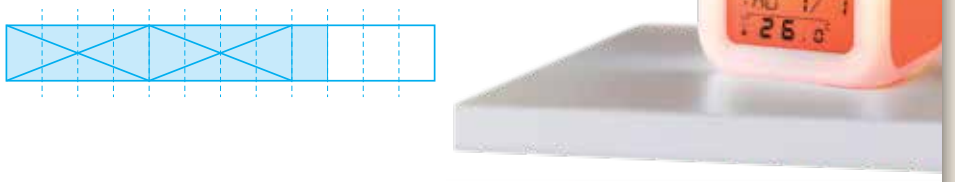
$\frac{7 \times 3}{8 \times 3} = \frac{21}{24}$ and $\frac{1 \times 4}{6 \times 4} = \frac{4}{24}$

$\frac{7}{8} - \frac{1}{6} = \frac{21}{24} - \frac{4}{24} = \frac{17}{24}$

Solution $\frac{17}{24}$; Accept any equivalent fraction.

9 Emily's shelf is $\frac{3}{4}$ foot wide. Her clock is $\frac{2}{3}$ foot wide.
How much wider is her shelf than her clock? Show your work.

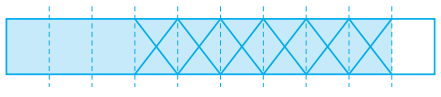
Possible student work:



Solution Emily's shelf is $\frac{1}{12}$ foot wider than her clock.

10 What is the value of the expression $\frac{9}{10} - \frac{3}{5}$? Show your work.

Possible student work:



Solution $\frac{3}{10}$; Accept any equivalent fraction.

Solutions

1. See the completed number line on the student page; The difference is $\frac{3}{10}$.
Basic

2. Possible answer: $\frac{4}{5} = \frac{8}{10}$; $\frac{5}{10} + \frac{3}{10} = \frac{8}{10}$, so $\frac{4}{5} - \frac{5}{10} = \frac{3}{10}$; Students recognize that Eric found $\frac{8}{10}$ as an equivalent fraction for $\frac{4}{5}$, then represented starting at $\frac{5}{10}$ on a number line and counting on to $\frac{8}{10}$ with the equation $\frac{5}{10} + \frac{3}{10} = \frac{8}{10}$. The related subtracted equation is $\frac{8}{10} - \frac{5}{10} = \frac{3}{10}$.
Medium

Name: _____ LESSON 13 SESSION 2

Practice Subtracting Fractions with Unlike Denominators

Study the Example showing one way to subtract fractions with unlike denominators. Then solve problems 1–5.

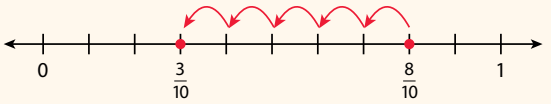
EXAMPLE

Felicia lives $\frac{4}{5}$ mile from school and $\frac{3}{10}$ mile from the soccer field. How much closer does she live to the soccer field than to school?

You can show $\frac{4}{5} - \frac{3}{10}$ using a number line. Use a common multiple to find the common denominator.

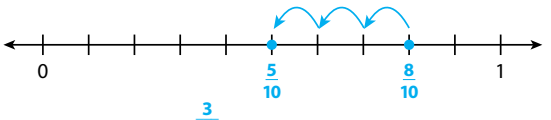
Rewrite the fractions as needed. $\frac{4}{5} = \frac{8}{10}$

Show the difference between the distance from school, $\frac{8}{10}$ mile, and the distance from the soccer field, $\frac{3}{10}$ mile, on a number line.



Felicia lives $\frac{5}{10}$, or $\frac{1}{2}$, mile closer to the soccer field than to school.

1. How could you count back on a number line to find the difference between $\frac{4}{5}$ and $\frac{5}{10}$? Show your work. **Possible answer:**



The difference is $\frac{3}{10}$.

2. Eric added up on a number line to find $\frac{4}{5} - \frac{5}{10}$. Use equivalent fractions and an addition equation to show how Eric found the difference.
Possible answer: $\frac{4}{5} = \frac{8}{10}$; $\frac{5}{10} + \frac{3}{10} = \frac{8}{10}$, so $\frac{4}{5} - \frac{5}{10} = \frac{3}{10}$.

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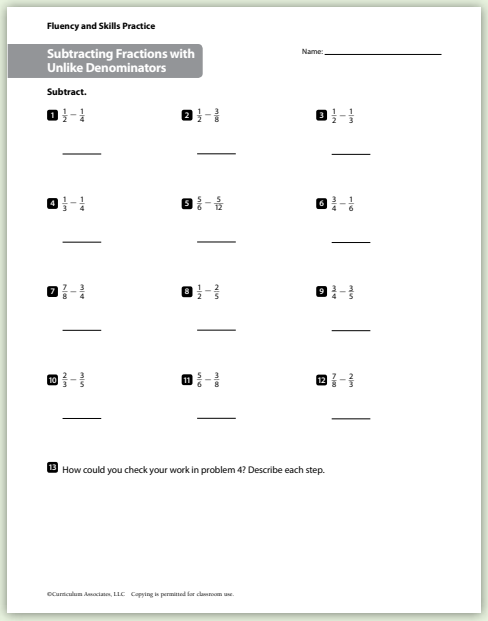
Fluency & Skills Practice Teacher Toolbox

Assign *Subtracting Fractions with Unlike Denominators*

In this activity students subtract fractions with unlike denominators. This activity reinforces students' understanding of equivalent fractions and fraction subtraction. Students may encounter real-world situations that use these skills, such as when cutting short lengths of wood, yarn, or cloth from longer lengths. Students may check their work by adding the subtrahend to the difference and making sure the sum equals the minuend.


Fluency and Skills Practice

Subtracting Fractions with Unlike Denominators

Name: _____

Subtract.

1. $\frac{1}{2} - \frac{1}{4}$ _____
2. $\frac{1}{2} - \frac{3}{8}$ _____
3. $\frac{1}{2} - \frac{1}{3}$ _____
4. $\frac{1}{3} - \frac{1}{4}$ _____
5. $\frac{5}{6} - \frac{5}{12}$ _____
6. $\frac{3}{4} - \frac{1}{6}$ _____
7. $\frac{7}{8} - \frac{3}{4}$ _____
8. $\frac{1}{2} - \frac{2}{5}$ _____
9. $\frac{3}{4} - \frac{3}{5}$ _____
10. $\frac{2}{3} - \frac{3}{5}$ _____
11. $\frac{5}{6} - \frac{3}{8}$ _____
12. $\frac{7}{8} - \frac{2}{3}$ _____

13. How could you check your work in problem 4? Describe each step.

3 $\frac{7}{12}$ or any equivalent fraction; See possible student work on the student page. Students may also show a fraction model representing $\frac{10}{12}$ with $\frac{3}{12}$ crossed out. They may also write the equations $\frac{5}{6} = \frac{10}{12}$, $\frac{1}{4} = \frac{3}{12}$, $\frac{10}{12} - \frac{3}{12} = \frac{7}{12}$.
Basic

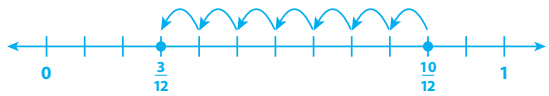
4 Students may divide the fraction bar into eight equal parts, shade six parts, and cross off five parts; $\frac{3}{4} - \frac{5}{8} = \frac{1}{8}$.
Medium

5 $\frac{7}{24}$ of a day; See possible student work on the student page. Students may also show a fraction model divided into twenty-fourths with $9 + 8$, or 17 twenty-fourths crossed out.
Challenge

3 What is the difference between $\frac{5}{6}$ and $\frac{1}{4}$? Show your work.

Possible student work:

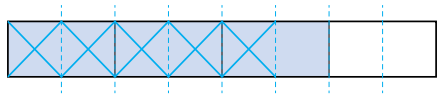
$\frac{5}{6} = \frac{10}{12}$ and $\frac{1}{4} = \frac{3}{12}$



$\frac{5}{6} - \frac{1}{4} =$ $\frac{7}{12}$; Accept any equivalent fraction.

4 Show how you can use the visual model to subtract $\frac{3}{4} - \frac{5}{8}$.

Possible student model:



$\frac{3}{4} - \frac{5}{8} =$ $\frac{1}{8}$

5 James sleeps $\frac{3}{8}$ of each day. He spends $\frac{1}{3}$ of each day at work. What fraction of his day is he not sleeping or working? Show your work.

Possible student work:

$\frac{3}{8} = \frac{3 \times 3}{8 \times 3} = \frac{9}{24}$ and $\frac{1}{3} = \frac{1 \times 8}{3 \times 8} = \frac{8}{24}$

$\frac{9}{24} + \frac{8}{24} = \frac{17}{24}$

Each day has 24 hours.

$\frac{24}{24} - \frac{17}{24} = \frac{7}{24}$

Solution $\frac{7}{24}$ of a day

ELL English Language Learners: Differentiated Instruction

Prepare for Session 3
Use with *Picture It*.

Levels 1–3

Listening/Speaking Read the explanation in ***Picture It*** aloud. Have students circle the mixed number $3\frac{1}{4}$ and the equivalent fraction $3\frac{3}{12}$. Ask them to point to the fraction bars that represent the mixed number. Then have students underline the mixed number $1\frac{2}{3}$ and the equivalent fraction $1\frac{8}{12}$. Ask them to point to the fraction bars that represent the part that is subtracted. Organize students into pairs to use gestures, numbers and drawings to explain how to use fraction bars to model subtracting with mixed numbers. Call on students to share ideas with the group. Restate their answers as needed to model fluent speaking.

Levels 2–4

Speaking Have students read the explanation in ***Picture It***. Organize students into pairs and ask them to read the ***Try It*** problem and identify which part of the model represents the amount of time Chloe spent at the park and which part represents the time she spent riding her bike. Have them underline the phrase *more twelfths* and take turns explaining how the fraction bars show *more twelfths*. [By dividing one whole fraction bar into twelve equal parts] Ask: *Why must you find a common denominator? Which part of the model shows the difference? How does the model show regrouping?*

Levels 3–5

Speaking/Reading Have students form pairs to analyze and discuss ***Picture It***. Ask them to write a short description of what they see in the model, using the following terms: *mixed number, common denominator, equivalent fraction, regroup*.

After students finish writing, have them form pairs and compare their descriptions. Then have partners work together to write a sentence that explains how fraction bars can be used to picture subtracting with mixed numbers.

LESSON 13

SESSION 3 Develop

Purpose In this session students solve a problem that requires finding the difference between $3\frac{1}{4}$ and $1\frac{2}{3}$. Students model the two mixed numbers in the word problem either on paper or with manipulatives to develop strategies for subtracting with mixed numbers.

Start

Connect to Prior Knowledge

Materials For each student: Activity Sheet *Fraction Bars* (4 wholes)

Why Support students' facility rewriting mixed numbers as fractions greater than 1 to prepare for regrouping when subtracting mixed numbers.

How Have students fold and shade fraction bars to show $1\frac{3}{4}$ and $1\frac{5}{8}$, then write the equivalent fraction.

Model each mixed number as a fraction. Write the equivalent fraction.

$1\frac{3}{4} =$ .....

$1\frac{5}{8} =$ .....

Solutions

Fold 2 fraction bars into fourths and shade 7 parts; $\frac{7}{4}$.

Fold 2 fraction bars into eighths and shade 13 parts; $\frac{13}{8}$.

Develop Language

Why Explain the word *drawback* to students.

How The Support Partner Discussion section under *Discuss It* suggests asking students if there were *drawbacks* to their strategy. Write the word *drawback* on the board and explain that a drawback is something that might be a problem or disadvantage. Encourage students to use the word as they discuss their strategies.

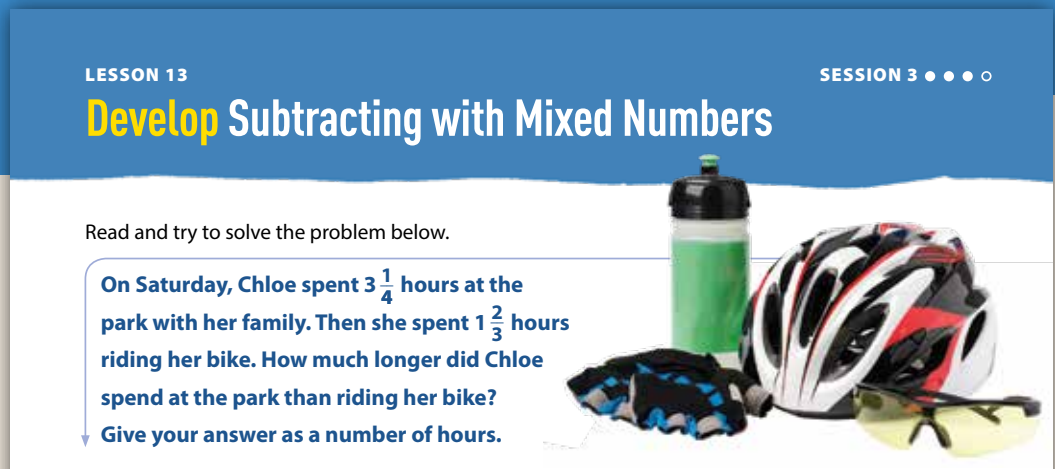
- *One drawback to my strategy is . . .*

TRY IT

Make Sense of the Problem

To support students in making sense of the problem, have them describe what the problem is asking them to find.

Ask *The times are given as mixed numbers. Which time is the longer amount of time? How do you know?*



LESSON 13 SESSION 3 ● ● ● ○

Develop Subtracting with Mixed Numbers

Read and try to solve the problem below.

On Saturday, Chloe spent $3\frac{1}{4}$ hours at the park with her family. Then she spent $1\frac{2}{3}$ hours riding her bike. How much longer did Chloe spend at the park than riding her bike? Give your answer as a number of hours.

TRY IT

Possible student work:

Sample A

$3\frac{1}{4} = \frac{13}{4}$ and $1\frac{2}{3} = \frac{5}{3}$

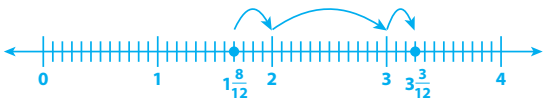
$\frac{13 \times 3}{4 \times 3} = \frac{39}{12}$ and $\frac{5 \times 4}{3 \times 4} = \frac{20}{12}$

$\frac{39}{12} - \frac{20}{12} = \frac{19}{12} = 1\frac{7}{12}$

She spent $1\frac{7}{12}$ hours longer at the park.

Sample B

$3\frac{1}{4} = 3\frac{3}{12}$ and $1\frac{2}{3} = 1\frac{8}{12}$



$\frac{4}{12} + 1 + \frac{3}{12} = 1\frac{7}{12}$

Chloe spent $1\frac{7}{12}$ hours longer at the park.

Math Toolkit

- fraction tiles
- fraction circles
- fraction bars
- fraction models
- grid paper
- number lines

DISCUSS IT

Ask your partner: Why did you choose that strategy?

Tell your partner: A model I used was . . . It helped me . . .

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DISCUSS IT

Support Partner Discussion

Encourage students to share what did not work for them as well as what did as they talk to each other.

Support as needed with questions such as:

- *Were there drawbacks to using your strategy?*
- *Would you use the same strategy the next time you solve this type of problem?*

Common Misconception Look for students who find $3\frac{3}{12} - 1\frac{8}{12}$ to be $1\frac{5}{12}$. Instead of recognizing the need to regroup, these students may subtract the lesser fraction from the greater fraction without regard to the relative sizes of the mixed numbers.

Select and Sequence Student Solutions

One possible order for whole class discussion:

- physical parts showing wholes, thirds, fourths, and twelfths
- drawings to represent the problem
- number lines marked in twelfths
- equations showing equivalent numbers for $3\frac{1}{4}$ and $1\frac{2}{3}$

Support Whole Class Discussion

Compare and connect the different representations and have students identify how they relate.

Ask *Where does your model show the time Chloe spent at the park? Riding her bike? The difference between the two times?*

Listen for Students should recognize that accurate representations show the use of a common denominator for thirds and fourths. They may include the regrouping of $3\frac{1}{4}$ into a fraction greater than 1, or a mixed number with a fraction greater than 1. If some students change the times to minutes and hours, use this as an opportunity to relate their representations to equivalent fractions and to using a common denominator of 60 $\left(\frac{1}{4} = \frac{15}{60}, \frac{2}{3} = \frac{40}{60}, \text{and } \frac{35}{60} = \frac{7}{12}\right)$.

PICTURE IT & MODEL IT

If no student presented these models, connect them to the student models by pointing out the ways they each represent:

- a common denominator for thirds and fourths
- the regrouping needed in order to subtract

Ask *To have enough twelfths to subtract, $3\frac{3}{12}$ is regrouped. How is that shown with the fraction bars? With the equations?*

Listen for Both the fraction bar model and the equations show $3\frac{3}{12}$ regrouped as $2\frac{15}{12}$.

For the fraction bars, prompt students to identify that both sets of bars represent the same amount of time.

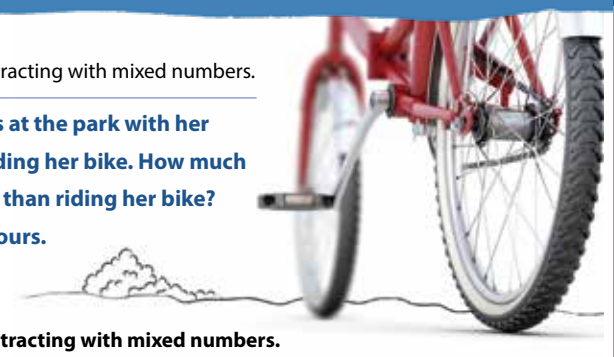
- *What mixed number is represented by the top set of fraction bars? The bottom set?*
- *Is the same total amount shaded in each set? What does that show?*

For the equation model, prompt students to relate the equations to the fraction bars.

- *How does the second sentence of* Model It *relate to the fraction bars in* Picture It*?*
- *How would you show the two equations at the end of* Model It *on the fraction bars in* Picture It*?*

Explore different ways to understand subtracting with mixed numbers.

On Saturday, Chloe spent $3\frac{1}{4}$ hours at the park with her family. Then she spent $1\frac{2}{3}$ hours riding her bike. How much longer did Chloe spend at the park than riding her bike? Give your answer as a number of hours.



PICTURE IT

You can use fraction bars to picture subtracting with mixed numbers.

Chloe spent $3\frac{1}{4}$ hours at the park. To subtract $1\frac{2}{3}$ hours, find a common denominator.

Hours spent at the park: $3\frac{1}{4} = 3\frac{3}{12}$ Hours spent riding her bike: $1\frac{2}{3} = 1\frac{8}{12}$

Model $3\frac{3}{12}$.

You need **more twelfths** to subtract $1\frac{8}{12}$.

$3\frac{3}{12} = 2\frac{15}{12}$

MODEL IT

You can use equations to subtract mixed numbers.

You can regroup one whole and break apart the mixed numbers to find $3\frac{3}{12} - 1\frac{8}{12}$.

3 wholes and $\frac{3}{12}$ is the same as 2 wholes and $\frac{15}{12}$.

$2 - 1 = 1$ and $\frac{15}{12} - \frac{8}{12} = \frac{7}{12}$

Deepen Understanding

Equation Model

SMP 7 Look for structure.

When discussing the equation method shown in *Model It*, prompt students to consider where else they have used this break-apart strategy.

Ask *How would you describe the strategy shown in* Model It*?*

Listen for You break apart each mixed number into the whole-number part and the fraction part, subtract the whole numbers, then subtract the fractions. Those two differences, together, are your answer.

Ask *Where else have you seen this strategy used? What are some examples?*

Listen for Students have used a break-apart strategy for adding and subtracting whole numbers, including one-digit, two-digit, and multi-digit numbers. Most recently, students used the strategy to subtract decimals.

Ask *How is the regrouping similar to regrouping to subtract decimals?*

Listen for If the decimal part of the number you subtract is greater than the decimal part of the total, you regroup 1 one as 10 tenths.

CONNECT IT

- Remind students that one thing that is alike about all the representations is they show how to replace fractions in mixed numbers that do not have same-size parts with equivalent fractions that do.
- Explain that on this page, they will explain regrouping of wholes into fractional parts in order to subtract with mixed numbers.

Monitor and Confirm

1–4 Check for understanding that:

- 12 is a common denominator of thirds and fourths
- regrouping of 1 whole of $3\frac{3}{12}$ is needed to have enough twelfths to subtract 8 twelfths
- $2\frac{15}{12}$ is equivalent to $3\frac{3}{12}$
- $3\frac{1}{4} - 1\frac{2}{3} = 1\frac{7}{12}$

Support Whole Class Discussion

2 Discuss the need for regrouping to help students to be ready to answer problem 6.

Ask *Would you need to regroup if the number being subtracted was $1\frac{1}{6}$ instead of $1\frac{2}{3}$?*

Listen for No; $1\frac{1}{6} = 1\frac{2}{12}$. $\frac{2}{12}$ is less than $\frac{3}{12}$, so there are already enough twelfths to subtract.

5 You may want to have students write a subtraction equation to represent the problem first. Then, have students use this to write the related addition equation.

Ask *What will be the two parts, or addends, of the addition equation? What will be the sum?*

Listen for The two addends will be the number subtracted and the difference, $1\frac{8}{12}$ and $1\frac{7}{12}$; together they should equal $3\frac{3}{12}$, or $3\frac{1}{4}$.

6 Look for the idea that you only need to regroup when, once the fractions have been replaced with equivalent fractions with a common denominator, the numerator of the first mixed number (the one being subtracted from) is less than the numerator of the second mixed number.

7 REFLECT Have all students focus on the strategies used to solve this problem. If time allows, have students share their preferences with a partner.

CONNECT IT

Now you will use the problem from the previous page to help you understand how to subtract mixed numbers with regrouping.

1 Look at the first set of fraction bars in **Picture It**. Why is the last bar split into 12 pieces instead of 4 pieces?

Possible answer: 12 pieces can be grouped to show either fourths or thirds.

2 Now look at the second set of fraction bars in **Picture It**. Explain why $3\frac{3}{12}$ is now shown as $2\frac{15}{12}$.

Possible answer: There were not enough twelfths to subtract $\frac{8}{12}$, so another whole was split into twelfths. $1\frac{3}{12}$ can be rewritten as $\frac{15}{12}$.

3 Look at **Model It**. How does the regrouping of $3\frac{3}{12}$ as $2\frac{15}{12}$ help you find how much longer Chloe spent at the park than riding her bike?

Possible answer: You now have enough twelfths to subtract $1\frac{8}{12}$.

4 How much longer did Chloe spend at the park than riding her bike? **$1\frac{7}{12}$** hour(s)

5 Show how you can use addition to check your answer.

Possible student work: $1\frac{7}{12} + 1\frac{8}{12} = 2\frac{15}{12} = 3\frac{3}{12}$ or $3\frac{1}{4}$

6 Do you always need to regroup when you subtract mixed numbers with unlike denominators? Explain.

Possible answer: No; First, you need to find a common denominator for the fractional parts of the mixed numbers. If the numerator of the first fractional part is greater than the numerator of the second fractional part, you do not need to regroup. If the numerator is less, then you do need to regroup.

7 REFLECT

Look back at your **Try It**, strategies by classmates, and **Picture It** and **Model It**. Which models or strategies do you like best for subtracting mixed numbers? Explain.

Possible answer: I like using fraction bars because I can use them to find a common denominator for the fractional parts of the mixed numbers and to find an equivalent mixed number with a fractional part greater than 1 if I need to regroup.

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Visual Model

Use number lines to regroup mixed numbers.

If . . . *students are unsure about regrouping mixed numbers in order to subtract,*

Then . . . *use this activity to write equivalent mixed numbers with fractions greater than 1.*

Materials For each student: Activity Sheet *Number Lines*

- Have students mark their number lines to show 0 to 5, with each whole divided into fifths. Have them label the numerals 0–5 on the number line.
- Direct them to mark and label $3\frac{1}{5}$ on the number line. Ask: *If you regroup 1 of the 3 wholes as fifths, what is the equivalent mixed number?* Prompt them to use the number line to show the answer by placing a finger at 2 on the number line (the number of wholes remaining when 1 whole is regrouped) and then counting on the number of fifths to the mark for $2\frac{1}{5}$. Have them write the equivalent mixed number found. $\left[1\frac{6}{5}\right]$
- Repeat the steps for other mixed numbers with fifths as the denominator, such as $2\frac{3}{5}$, $1\frac{4}{5}$, and $4\frac{2}{5}$.

APPLY IT

Use what you just learned to solve these problems.

8 What is $7\frac{3}{5} - \frac{9}{10}$? Show your work.

Possible student work:

$7\frac{3}{5} = 7 + \left(\frac{3 \times 2}{5 \times 2}\right) = 7\frac{6}{10} = 6\frac{16}{10}$

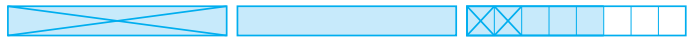
$6\frac{16}{10} - \frac{9}{10} = 6\frac{7}{10}$

Solution $6\frac{7}{10}$; **Accept any equivalent fraction.**

9 What is the difference between $2\frac{5}{8}$ and $1\frac{1}{4}$? Show your work.

Possible student work:

$1\frac{1}{4} = 1\frac{2}{8}$



Solution $1\frac{3}{8}$; **Accept any equivalent fraction.**

10 Charlie is practicing the long jump. His first jump is a distance of $16\frac{5}{6}$ feet. His second jump is a distance of $18\frac{2}{3}$ feet. How much longer is Charlie's second jump?

Ⓐ $1\frac{1}{6}$ feet

Ⓑ $1\frac{5}{6}$ feet

Ⓒ $2\frac{1}{6}$ feet

Ⓓ $2\frac{5}{6}$ feet

APPLY IT

For all problems, encourage students to draw a model to support their thinking. Allow some leeway in precision; drawing multiple fraction models all with same-size parts can be difficult.

8 $6\frac{7}{10}$ or any equivalent fraction; See possible equations used to solve the problem on the Student Worktext page. Students may also show models with the fractional parts divided into tenths. Both equation and fraction model solutions should show regrouping of $7\frac{3}{5}$. Students may use a different common denominator of 5 and 10, such as 50.

9 $1\frac{3}{8}$ or any equivalent fraction; Students may use fractions bars to show $2\frac{5}{8}$ and $1\frac{1}{4}$ as eighths. Students may also show a number line divided into eighths. They may also write the equations $1\frac{1}{4} = 1\frac{2}{8}, 2\frac{5}{8} - 1\frac{2}{8} = 1\frac{3}{8}$. Students may use a different common denominator of 8 and 4, such as 32.

Close: Exit Ticket

10 **B**; 6 is a common multiple of both denominators. Multiply both the numerator and the denominator in $\frac{2}{3}$ by 2 to write the equivalent fraction $\frac{4}{6}$. Regroup $18\frac{4}{6}$ as $17\frac{10}{6}$. Subtract: $17\frac{10}{6} - 16\frac{5}{6} = 1\frac{5}{6}$.

Error Alert **If** students choose A, C, or D, **then** review how to regroup mixed numbers so there are enough fraction parts to subtract. Have students model $2\frac{4}{6}$ using fraction bars, then divide one of the wholes into sixths. Compare $2\frac{4}{6}$ with the equivalent number shown, $1\frac{10}{6}$, emphasizing that the whole number part decreased by 1 and the fraction part increased by $\frac{6}{6}$. Have students apply this to finding the difference between $18\frac{2}{3}$ and $16\frac{5}{6}$.

Solutions

1. $2\frac{1}{8}$ or any equivalent fraction; See the completed fraction bar model on the student page.
Basic

2. $2\frac{1}{2}$; Accept any equivalent fraction. See possible student work on the student page. Students may also draw fraction models representing $6\frac{5}{6}$ showing 4 wholes and 2 sixths crossed out. They may also show a number line divided into sixths, a dot at $6\frac{5}{6}$, 4 jumps of 1 each, and 1 jump of $\frac{2}{6}$.
Medium

Name: _____ LESSON 13 SESSION 3

Practice Subtracting with Mixed Numbers

Study the Example showing how to subtract mixed numbers. Then solve problems 1–5.

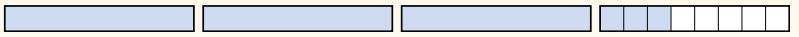
EXAMPLE

What is the difference between $3\frac{3}{8}$ and $1\frac{3}{4}$?

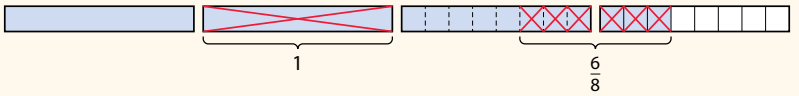
You can show $3\frac{3}{8} - 1\frac{3}{4}$ using fraction bars.

Rewrite the mixed numbers using common denominators. $3\frac{3}{8} - 1\frac{6}{8}$

Model $3\frac{3}{8}$. Divide the last fraction bar into eighths.



Divide one more fraction bars into eighths so there are enough eighths to subtract.



Find the difference: $2\frac{11}{8} - 1\frac{6}{8} = 1\frac{5}{8}$.

1. Now use the fraction bars to find $3\frac{3}{8} - 1\frac{1}{4}$. Show your work.

$3\frac{3}{8} - 1\frac{1}{4} =$ $2\frac{1}{8}$; Accept any equivalent fraction.

2. What is $6\frac{5}{6} - 4\frac{1}{3}$? Show your work.

Possible answer:

$\frac{1}{3} = \frac{2}{6}$

$6 - 4 = 2; \frac{5}{6} - \frac{2}{6} = \frac{3}{6}$

Solution $2\frac{1}{2}$; Accept any equivalent fraction.

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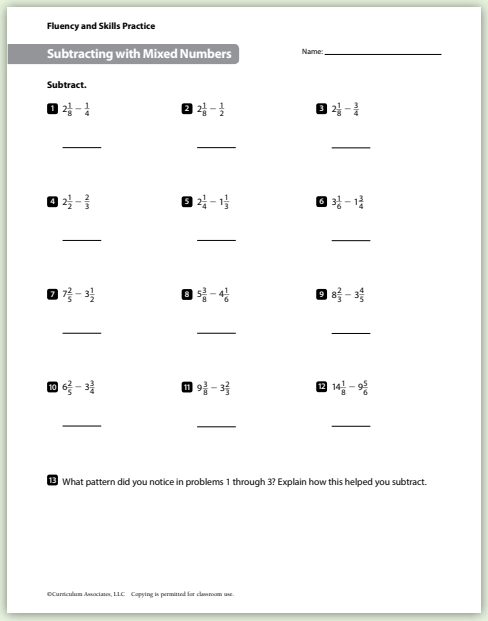
Fluency & Skills Practice

Teacher Toolbox

Assign *Subtracting with Mixed Numbers*

In this activity students subtract either fractions or mixed numbers from mixed numbers. Students may apply their understanding of subtraction with mixed numbers in real-world contexts. For example, students may need to find the difference of the lengths of two plumbing pipes or the difference between the masses of two animals. Identifying patterns in subtraction of mixed numbers helps students improve their mental math skills.



Fluency and Skills Practice

Subtracting with Mixed Numbers

Name: _____

Subtract.

1. $2\frac{1}{8} - \frac{1}{4}$ _____
2. $2\frac{1}{8} - \frac{1}{2}$ _____
3. $2\frac{1}{8} - \frac{3}{4}$ _____
4. $2\frac{1}{2} - \frac{2}{3}$ _____
5. $2\frac{1}{4} - 1\frac{1}{3}$ _____
6. $3\frac{1}{6} - 1\frac{3}{4}$ _____
7. $7\frac{2}{5} - 3\frac{1}{2}$ _____
8. $5\frac{3}{8} - 4\frac{1}{6}$ _____
9. $8\frac{2}{3} - 3\frac{4}{5}$ _____
10. $6\frac{2}{5} - 3\frac{3}{4}$ _____
11. $9\frac{3}{8} - 3\frac{2}{3}$ _____
12. $14\frac{1}{8} - 9\frac{5}{6}$ _____

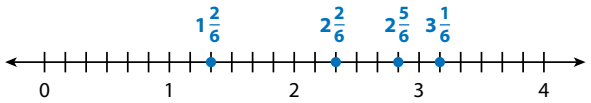
13. What pattern did you notice in problems 1 through 3? Explain how this helped you subtract.

3 **a.** $1\frac{2}{6} = \frac{8}{6}$
b. $2\frac{5}{6} = 1\frac{11}{6}$
c. $2\frac{2}{6} = 1\frac{8}{6}$
d. $3\frac{1}{6} = 2\frac{7}{6}$
Medium

4 $1\frac{5}{6}$; Accept any equivalent fraction. Students may rewrite the expression showing like denominators: $3\frac{2}{6} - 1\frac{3}{6}$, then replace $3\frac{2}{6}$ with a form that includes a fraction greater than 1 so there are enough sixths: $2\frac{8}{6} - 1\frac{3}{6}$. They may also use number lines or fraction models to find the difference.
Medium

5 $5\frac{1}{8}$ pounds; Accept any equivalent fraction. Students may rewrite all the numbers using eighths as the common denominator for the fractions, add the weights of the two books: $\frac{6}{8} + \frac{4}{8} = \frac{10}{8}$, or $1\frac{2}{8}$, then subtract that from the backpack weight: $6\frac{3}{8} - 1\frac{2}{8}$. They may use a different common denominator of 2, 4, and 8. They may also use fraction models or number lines to represent the problem.
Challenge

3 Sometimes it is helpful to rewrite mixed numbers in a form that includes a fraction greater than 1. Use the number line to write the missing numbers.



a. $1\frac{2}{6} = \frac{8}{6}$

b. $2\frac{5}{6} = 1\frac{11}{6}$

c. $2\frac{2}{6} = 1\frac{8}{6}$

d. $3\frac{1}{6} = 2\frac{7}{6}$

4 What is $3\frac{1}{3} - 1\frac{1}{2}$? Show your work.

Students might use models, number lines, equations, or some other method to find $3\frac{1}{3} - 1\frac{1}{2}$.

Solution **$1\frac{5}{6}$; Accept any equivalent fraction.**

5 Emil's backpack weighs $6\frac{3}{8}$ pounds. He removes a book that weighs $\frac{3}{4}$ pound. Then he removes a book that weighs $\frac{1}{2}$ pound. How much does Emil's backpack weigh now? Show your work.

Students might use models, equations, or some other method to find the answer. They may first add the weights of the two books $\left(\frac{3}{4} + \frac{1}{2}\right)$ and subtract the combined weight from $6\frac{3}{8}$ pounds, or they may subtract the weight of each book individually from the weight of the backpack.

Solution **$5\frac{1}{8}$ pounds; Accept any equivalent fraction.**

ELL English Language Learners: Differentiated Instruction

Prepare for Session 4
Use with *Apply It*.

Levels 1–3

Speaking/Listening Read ***Apply It*** problem 3 to students. Ask them to point to the total area of Cara's bathroom floor. Have students complete:

- *The total floor area is* ___$3\frac{2}{3}$ square yards___.

Ask students to point to the size of the rug. Say that the rug covers *a part* of the floor. Have students complete:

- *Cara's rug covers an area of* ___$1\frac{1}{4}$ square yards___.

Ask students to point to the question and pay attention to the word *not*. Explain that they need to determine what part is *not* covered. Ask students to work in pairs to draw a model and solve the problem.

Levels 2–4

Reading/Writing Read ***Apply It*** problem 3 with students. Provide the sentence frames below:

- *The total area of the bathroom floor is* ___$3\frac{2}{3}$ square yards___.
- *The rug covers an area of* ___$1\frac{1}{4}$ square yards___.
- *I can calculate the part of the floor that is not covered by subtracting* ___$1\frac{1}{4}$___ *from* ___$3\frac{2}{3}$___.

Have students work in pairs to solve the problem and explain their thinking:

- *My answer is reasonable because* _____.

Levels 3–5

Reading/Writing Have students read ***Apply It*** problem 3 with a partner. Have each student write the strategy he or she will use to determine what part of the bathroom floor is not covered.

Ask students to read what they have written to their partners and discuss their strategies. Then have them work together to solve the problem.

LESSON 13

SESSION 4 Refine

Purpose In this session students solve word problems involving subtracting fractions and mixed numbers with unlike denominators and then discuss and confirm their answers with a partner.

Before students begin to work, use their responses to the *Check for Understanding* to determine those who will benefit from additional support.

As students complete the Example and problems 1–3, observe and monitor their reasoning to identify groupings for differentiated instruction.

Start

Check for Understanding

Why Confirm understanding of subtracting fractions.

How: Have students find $2\frac{7}{10} - \frac{2}{3}$ using any strategy they want.

Use any strategy to subtract.

$2\frac{7}{10} - \frac{2}{3} = ?$

Solution

$2\frac{1}{30}$ or any equivalent fraction

Refine Subtracting Fractions

Study the Example below. Then solve problems 1–8.

EXAMPLE

The first chapter of Henry's book is $5\frac{2}{3}$ pages long. The second chapter is $8\frac{2}{5}$ pages long. How much longer is the second chapter than the first chapter?

Look at how you could show your work using equations.

$$5\frac{2}{3} = 5 + \left(\frac{2\times5}{3\times5}\right) = 5\frac{10}{15}$$

$$8\frac{2}{5} = 8 + \left(\frac{2\times3}{5\times3}\right) = 8\frac{6}{15} = \left(7 + \frac{15}{15}\right) + \frac{6}{15} = 7\frac{21}{15}$$

$$\begin{array}{r} 7\frac{21}{15} \\ -\,5\frac{10}{15} \\ \hline 2\frac{11}{15} \end{array}$$

Solution $2\frac{11}{15}$ **pages longer**

After finding a common denominator, the student used a vertical format to subtract.

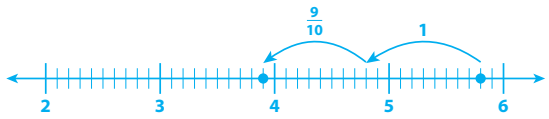
PAIR/SHARE

Explain why $8\frac{6}{15}$ is regrouped as $7\frac{21}{15}$.

APPLY IT

1 What is $5\frac{4}{5} - 1\frac{9}{10}$? Show your work.

Possible student work:



Solution $3\frac{9}{10}$; **Accept any equivalent fraction.**

Should you regroup $5\frac{4}{5}$ before you subtract?

PAIR/SHARE

Explain how your model shows regrouping.

Error Alert

If the error is . . .	*Students may . . .*	*To support understanding . . .*
$2\frac{5}{30}$	not have multiplied each numerator by the factor used to produce the common denominator.	Have students draw models to show $\frac{7}{10}$ and $\frac{2}{3}$, then divide each model into thirtieths. Use the models to illustrate the fractions in the problem and find the difference.
$2\frac{5}{7}$	have subtracted the numerators and subtracted the denominators because they may not understand what a denominator means.	Remind students that only same-size parts can be subtracted. Together write $\frac{7}{10}$ and $\frac{2}{3}$ as equivalent fractions with 30 as the common denominator. $\left[\frac{21}{30}, \frac{20}{30}\right]$ Write the expression *21 thirtieths − 20 thirtieths* on the board and below it write the expression $\frac{21}{30} - \frac{20}{30}$. Guide students in writing the value of the top expression and using it to find the value of the bottom expression. Emphasize all the parts are the same size (as shown by the denominator) and only the numerators are subtracted.

EXAMPLE

$2\frac{11}{15}$ pages longer; The equations shown are one way to solve the problem. Students could also solve the problem by drawing models or number lines divided into a common denominator of 3 and 5.

Look for Writing the problem in a vertical format is similar to lining up decimals by place value.

APPLY IT

1. $3\frac{9}{10}$ or any equivalent fraction; Students could solve the problem using a number line divided into tenths, marking $5\frac{4}{5}$ with a dot, and showing jumps to the left of 1 whole and 9 tenths.
   DOK 1

   Look for When the fractional part of the number being subtracted is greater than the fractional part of the total, you need to regroup 1 whole before subtracting.

2. $\frac{1}{12}$ or any equivalent fraction; Students could solve the problem by representing it with the missing addend equation $\frac{3}{4} + ? = \frac{5}{6}$, then writing a related subtraction expression with a common denominator: $\frac{10}{12} - \frac{9}{12}$.
   DOK 2

   Look for 12 and 24 are two possible common denominators for fourths and sixths.

3. **C**; Students could solve the problem using the equations $3\frac{2}{3} = 3\frac{8}{12}$, $1\frac{1}{4} = 1\frac{3}{12}$, and $3\frac{8}{12} - 1\frac{3}{12} = 2\frac{5}{12}$.
   Explain why the other two answer choices are not correct:

   A is not correct because only the whole numbers were subtracted.

   B is not correct because the numerators were not multiplied by the appropriate numbers before subtracting.
   DOK 3

2. What number do you add to $\frac{3}{4}$ to get $\frac{5}{6}$? Show your work.

   Possible student work:

   $\frac{3}{4} + ? = \frac{5}{6}$

   $\frac{5}{6} - \frac{3}{4} = \frac{10}{12} - \frac{9}{12} = \frac{1}{12}$

   What common multiple of 6 and 4 can you use as a common denominator?

   Solution **$\frac{1}{12}$; Accept any equivalent fraction.**

   PAIR/SHARE
   Explain why you chose the model you did to solve this problem.

3. Cara's bathroom floor has an area of $3\frac{2}{3}$ square yards. She lays down a rug that has an area of $1\frac{1}{4}$ square yards. What area of her floor is NOT covered by the rug?

   What equivalent mixed number could you subtract?

   Ⓐ 2 square yard

   Ⓑ $2\frac{1}{12}$ square yards

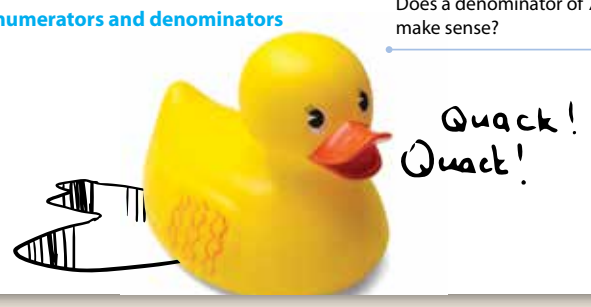
   Ⓒ $2\frac{5}{12}$ square yards

   Ⓓ $2\frac{3}{7}$ square yards

   Jordan chose Ⓓ as the correct answer. How did he get that answer?

   Possible answer: First he subtracted the whole numbers. Then he added both the numerators and denominators of the fractional parts.

   PAIR/SHARE
   Does a denominator of 7 make sense?



4 **B;** $\frac{3}{5} - \frac{1}{3} = \frac{9}{15} - \frac{5}{15} = \frac{4}{15}$.
DOK 1

5 **A** (Yes);
D (No);
F (No);
G (Yes)
DOK 2

6 $8\frac{3}{14}$; Write equivalent mixed numbers with a common denominator, $12\frac{5}{7} = 12\frac{10}{14}$, $4\frac{1}{2} = 4\frac{7}{14}$. Subtract the known addend from the sum to find the missing addend, $12\frac{10}{14} - 4\frac{7}{14}$.
DOK 2

Error Alert Students may think because *sum* is used in the problem, they should add the two mixed numbers. They get an incorrect answer of $16\frac{17}{14}$, or $17\frac{3}{14}$.

4 What is the value of the expression $\frac{3}{5} - \frac{1}{3}$?

Ⓐ $\frac{2}{2}$
Ⓑ $\frac{4}{15}$
Ⓒ $\frac{2}{5}$
Ⓓ $\frac{6}{15}$

5 Mackenzie's footprint is $\frac{7}{12}$ foot long. Her dad's footprint is $1\frac{1}{6}$ feet long. Can the equation be used to find how much longer Mackenzie's dad's footprint is than Mackenzie's?

	Yes	No
$1\frac{2}{12} - \frac{7}{12} = ?$	Ⓐ	Ⓑ
$\frac{7}{12} + 1\frac{1}{6} = ?$	Ⓒ	Ⓓ
$1\frac{1}{6} - \frac{2}{3} = ?$	Ⓔ	Ⓕ
$\frac{14}{12} - \frac{7}{12} = ?$	Ⓖ	Ⓗ

6 The sum of $4\frac{1}{2}$ and what number is $12\frac{5}{7}$? Show your work.

Possible student work:

$12\frac{5}{7} = 12\frac{10}{14}$
$4\frac{1}{2} = 4\frac{7}{14}$
$12\frac{10}{14} - 4\frac{7}{14} = 8\frac{3}{14}$

Solution $8\frac{3}{14}$

Differentiated Instruction

RETEACH

Hands-On Activity
Use fraction bars to rewrite and subtract fractions.

Students struggling with concepts that fractions can be replaced with equivalent fractions

Will benefit from additional work with representations of equivalent fractions

Materials For each student: Activity Sheet *Fraction Bars* (thirds, fourths, tenths)

- Have students write $\frac{1}{3} - \frac{1}{6}$ and shade a thirds fraction bar to model $\frac{1}{3}$. Have them identify 6 as a common multiple for the denominators 3 and 6, and fold the bar to show sixths.
- Have students replace $\frac{1}{3}$ in the expression with the equivalent fraction now shown by the shaded part of the bar. $\left[\frac{2}{6}\right]$ Instruct students to subtract $\frac{1}{6}$ by crossing out one shaded section of the bar, and record the difference. $\left[\frac{1}{6}\right]$
- Use fourths and tenths bars to repeat the activity for two more subtraction expressions in which one denominator is a multiple of the other, such as $\frac{3}{4} - \frac{3}{8}$ and $\frac{4}{5} - \frac{6}{10}$.

EXTEND

Challenge Activity
Write subtraction problems with unlike denominators.

Students who have achieved proficiency

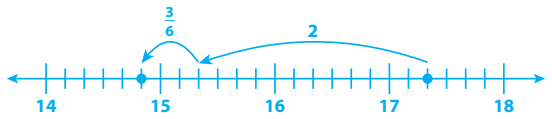
Will benefit from deepening understanding of subtracting with mixed numbers

- Have students write subtraction problems with unlike fractions and mixed numbers. Each student writes a problem and its solution. Partners exchange problems and solve.
- Partners discuss the solutions and come to an agreement on what the correct solutions are.

7 Carter has an older sister and a younger brother.

Part A Carter's sister is $17\frac{1}{3}$ years old. Carter is $2\frac{1}{2}$ years younger than his sister. What is Carter's age in years? Show your work.

Possible student work:



Solution $14\frac{5}{6}$ **years; Accept any equivalent fraction.**

Part B Carter's brother is $5\frac{3}{4}$ years younger than their sister. How much older is Carter than his brother? Show your work.

Possible student work:

Carter's brother: $17\frac{1}{3} - 5\frac{3}{4} = 17\frac{4}{12} - 5\frac{9}{12} = 16\frac{16}{12} - 5\frac{9}{12} = 11\frac{7}{12}$

Difference in age: $14\frac{5}{6} - 11\frac{7}{12} = 14\frac{10}{12} - 11\frac{7}{12} = 3\frac{3}{12}$ **or** $3\frac{1}{4}$

Solution $3\frac{1}{4}$ **years older; Accept any equivalent fraction.**

8 MATH JOURNAL

Find the difference between $1\frac{2}{5}$ and $\frac{9}{10}$. Explain how you solved the problem.

$\frac{5}{10}$ **or** $\frac{1}{2}$**; Possible explanation: The common denominator is 10. When I changed** $1\frac{2}{5}$ **to** $1\frac{4}{10}$**, I saw that there were not enough tenths to subtract** $\frac{9}{10}$**. I regrouped the whole as** $\frac{10}{10}$ **and added it to** $\frac{4}{10}$**. So,** $1\frac{4}{10} = \frac{14}{10}$ **and** $\frac{14}{10} - \frac{9}{10} = \frac{5}{10}$**.**

SELF CHECK Go back to the Unit 2 Opener and see what you can check off.

7 **Part A**

$14\frac{5}{6}$ years; Accept any equivalent fraction; See possible student work on the Student Worktext page. Subtract $2\frac{1}{2}$ years from the sister's age of $17\frac{1}{3}$ years.

Part B

$3\frac{1}{4}$ years older; Accept any equivalent fraction; See possible student work on the Student Worktext page. First, find the brother's age by subtracting $5\frac{3}{4}$ years from the sister's age, then subtract the brother's age from Carter's age to find how much older Carter is.

DOK 2

REINFORCE

Problems 4–8

Subtract fractions.

All students will benefit from additional work with subtracting fractions by solving problems in a variety of formats.

- Have students work on their own or with a partner to solve the problems.
- Encourage students to show their work.

PERSONALIZE

i-Ready

Provide students with opportunities to work on their personalized instruction path with *i-Ready* Online Instruction to:

- fill prerequisite gaps
- build up grade-level skills

Close: Exit Ticket

8 MATH JOURNAL

Student responses should indicate understanding that subtracting a fraction from a mixed number requires using a common denominator, and that when there are not enough fractional parts to subtract, you regroup 1 whole into fractional parts, writing an equivalent mixed number with a fraction greater than 1 before subtracting.

Error Alert **If** students, after writing mixed numbers with a common denominator, subtract from whichever numerator is greater, **then** have students circle the mixed number on the left (the one to be subtracted from) and identify whether or not there are enough fractional parts to subtract the fractions.

SELF CHECK Have students consider whether they feel they are ready to check off any new skills on the Unit 2 Opener.

Lesson Overview

LESSON 14

Add and Subtract in Word Problems

Lesson Objectives

Content Objectives

- Add and subtract fractions and mixed numbers with unlike denominators to solve word problems.
- Add and subtract decimals to hundredths to solve word problems.
- Use benchmark fractions to estimate fraction sums and differences.
- Use rounded decimals to estimate decimal sums and differences.
- Use estimation to check whether a solution is reasonable.

Language Objectives

- Draw diagrams or write equations to represent word problems involving fraction or decimal addition or subtraction.
- Estimate sums and differences of fractions or decimals and justify by comparing the numbers used to estimate to benchmark fractions or rounded decimals.
- Tell whether a solution is reasonable by comparing the result to an estimate.

Prerequisite Skills

- Compare a fraction to a benchmark fraction.
- Add and subtract fractions and mixed numbers with unlike denominators.
- Round decimals.
- Compare decimals.
- Add and subtract decimals to hundredths.
- Write equations to represent and solve word problems.

Standards for Mathematical Practice (SMP)

SMPs 1, 2, 3, 4, 5, and 6 are integrated in every lesson through the *Try-Discuss-Connect* routine.*

In addition, this lesson particularly emphasizes the following SMPs:

4 Model with mathematics.

5 Use appropriate tools strategically.

6 Attend to precision.

*See page 1i to see how every lesson includes these SMPs.

Lesson Vocabulary

There is no new vocabulary. Review the following key terms.

- **benchmark fraction** a common fraction that you might compare other fractions to. For example, $\frac{1}{4}$, $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{4}$ are often used as benchmark fractions.
- **common denominator** a number that is a common multiple of the denominators of two or more fractions.
- **equivalent fractions** two or more different fractions that name the same part of a whole or the same point on a number line.

Learning Progression

In previous lessons students added and subtracted fractions and mixed numbers with unlike denominators. They found a common denominator and wrote equivalent fractions in order to add and subtract. Students also rounded decimals to given place values, and added and subtracted decimals to hundredths.

In this lesson students apply their understanding of adding and subtracting fractions with unlike denominators and decimals to solving word problems. They estimate sums and differences of fractions and mixed numbers, add and subtract fractions and mixed numbers with unlike denominators, and check whether the results are reasonable in the context of a problem. They use estimation strategies based on benchmark fractions, which they learned about in Grade 4. They also use estimation strategies with rounded decimals to check the reasonableness of decimal sums and differences.

In later grades students will solve word problems involving adding and subtracting rational numbers.

Lesson Pacing Guide

Whole Class Instruction

SESSION 1
Explore
45–60 min

Interactive Tutorial* *(Optional)*
Prerequisite Review:
Add and Subtract Fractions

Adding and Subtracting in Word Problems
- Start *5 min*
- Try It *10 min*
- Discuss It *10 min*
- Connect It *15 min*
- Close: Exit Ticket *5 min*

Additional Practice
Lesson pages 273–274

SESSION 2
Develop
45–60 min

Estimating in Word Problems with Fractions
- Start *5 min*
- Try It *10 min*
- Discuss It *10 min*
- Model Its *5 min*
- Connect It *10 min*
- Close: Exit Ticket *5 min*

Additional Practice
Lesson pages 279–280

Fluency
Estimating in Word Problems with Fractions

SESSION 3
Develop
45–60 min

Using Estimation with Decimals
- Start *5 min*
- Try It *10 min*
- Discuss It *10 min*
- Picture It & Model It *5 min*
- Connect It *10 min*
- Close: Exit Ticket *5 min*

Additional Practice
Lesson pages 285–286

Fluency
Using Estimation with Decimals

SESSION 4
Refine
45–60 min

Adding and Subtracting in Word Problems
- Start *5 min*
- Example & Problems 1–3 *15 min*
- Practice & Small Group Differentiation *20 min*
- Close: Exit Ticket *5 min*

Lesson Quiz or **Digital Comprehension Check**

Lesson Materials

Lesson *(Required)*	*Per student:* 1 set of fraction tiles or circles, base-ten blocks (5 hundreds flats, 15 tens rods) *Per pair:* 1 set of fraction tiles
Activities	*Per pair:* 1 set of fraction tiles, play money (6 dollar bills, 15 dimes, 5 quarters) *Per group:* 1 set of fraction circles *Activity Sheet:* Number Lines
Math Toolkit	fraction tiles, fraction circles, fraction bars, grid paper, number lines, base-ten blocks, base-ten grid paper, decimal grids, thousandths decimal place-value charts
Digital Math Tools	Fraction Models, Number Line

Teacher Toolbox

Small Group Differentiation

PREPARE

Ready Prerequisite Lessons

Grade 4
- Lesson 20 Add and Subtract Fractions
- Lesson 21 Add and Subtract Mixed Numbers

RETEACH

Tools for Instruction

Grade 4
- Lesson 20 Add and Subtract Fractions
- Lesson 21 Add and Subtract Mixed Numbers

Grade 5
- Lesson 14 Solve Word Problems with Fractions and Decimals

REINFORCE

Math Center Activities

Grade 5
- Lesson 14 Estimate Fraction Sums and Differences
- Lesson 14 Use Fraction Vocabulary

EXTEND

Enrichment Activity

Grade 5
- Lesson 14 Baking Cookies

i-Ready

Independent Learning

PERSONALIZE

i-Ready Lessons*

Grade 5
- Add and Subtract Decimals
- Practice: Subtract Decimals
- Add and Subtract Fractions in Word Problems

Learning Game
- Prerequisite: *Bounce*

*We continually update the Interactive Tutorials. Check the Teacher Toolbox for the most up-to-date offerings for this lesson.

Connect to Family, Community, and Language Development

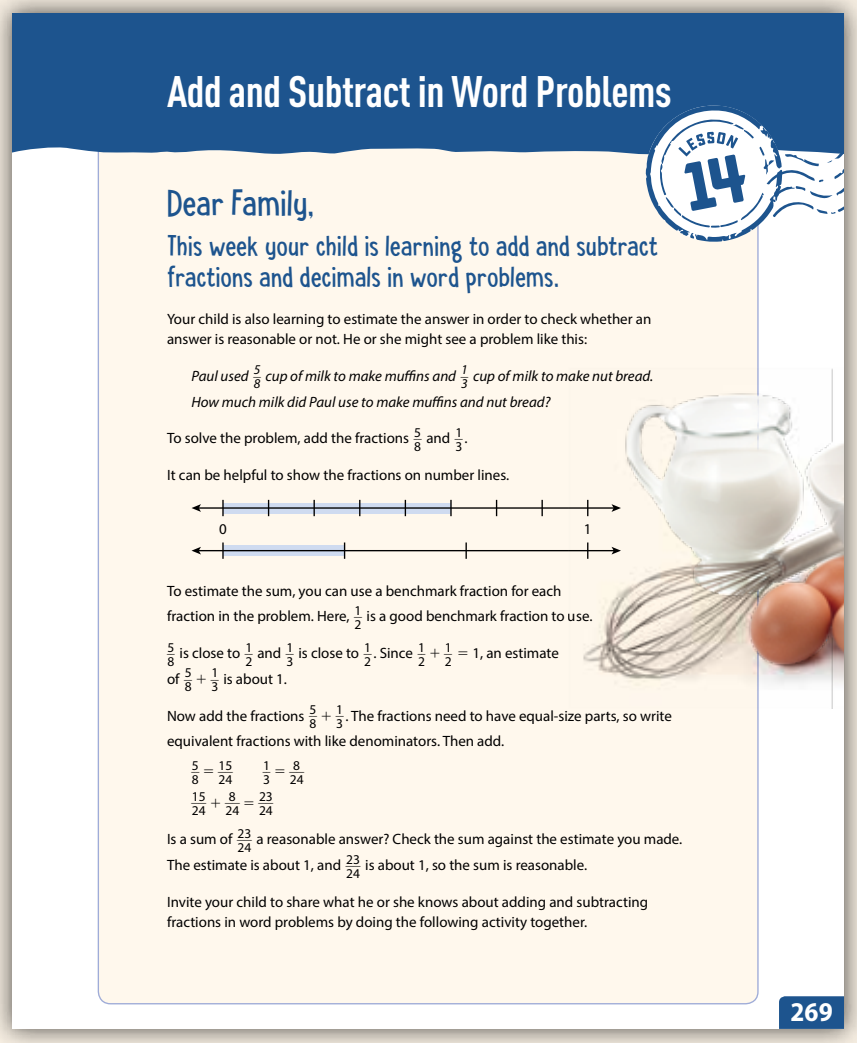
The following activities and instructional supports provide opportunities to foster school, family, and community involvement and partnerships.

Connect to Family

Use the **Family Letter**—which provides background information, math vocabulary, and an activity—to keep families apprised of what their child is learning and to encourage family involvement.

Available in Spanish

Teacher Toolbox



Add and Subtract in Word Problems

LESSON 14

Dear Family,

This week your child is learning to add and subtract fractions and decimals in word problems.

Your child is also learning to estimate the answer in order to check whether an answer is reasonable or not. He or she might see a problem like this:

Paul used $\frac{5}{8}$ cup of milk to make muffins and $\frac{1}{3}$ cup of milk to make nut bread.
How much milk did Paul use to make muffins and nut bread?

To solve the problem, add the fractions $\frac{5}{8}$ and $\frac{1}{3}$.

It can be helpful to show the fractions on number lines.

To estimate the sum, you can use a benchmark fraction for each fraction in the problem. Here, $\frac{1}{2}$ is a good benchmark fraction to use.

$\frac{5}{8}$ is close to $\frac{1}{2}$ and $\frac{1}{3}$ is close to $\frac{1}{2}$. Since $\frac{1}{2} + \frac{1}{2} = 1$, an estimate of $\frac{5}{8} + \frac{1}{3}$ is about 1.

Now add the fractions $\frac{5}{8} + \frac{1}{3}$. The fractions need to have equal-size parts, so write equivalent fractions with like denominators. Then add.

$\frac{5}{8} = \frac{15}{24}$ $\frac{1}{3} = \frac{8}{24}$
$\frac{15}{24} + \frac{8}{24} = \frac{23}{24}$

Is a sum of $\frac{23}{24}$ a reasonable answer? Check the sum against the estimate you made. The estimate is about 1, and $\frac{23}{24}$ is about 1, so the sum is reasonable.

Invite your child to share what he or she knows about adding and subtracting fractions in word problems by doing the following activity together.

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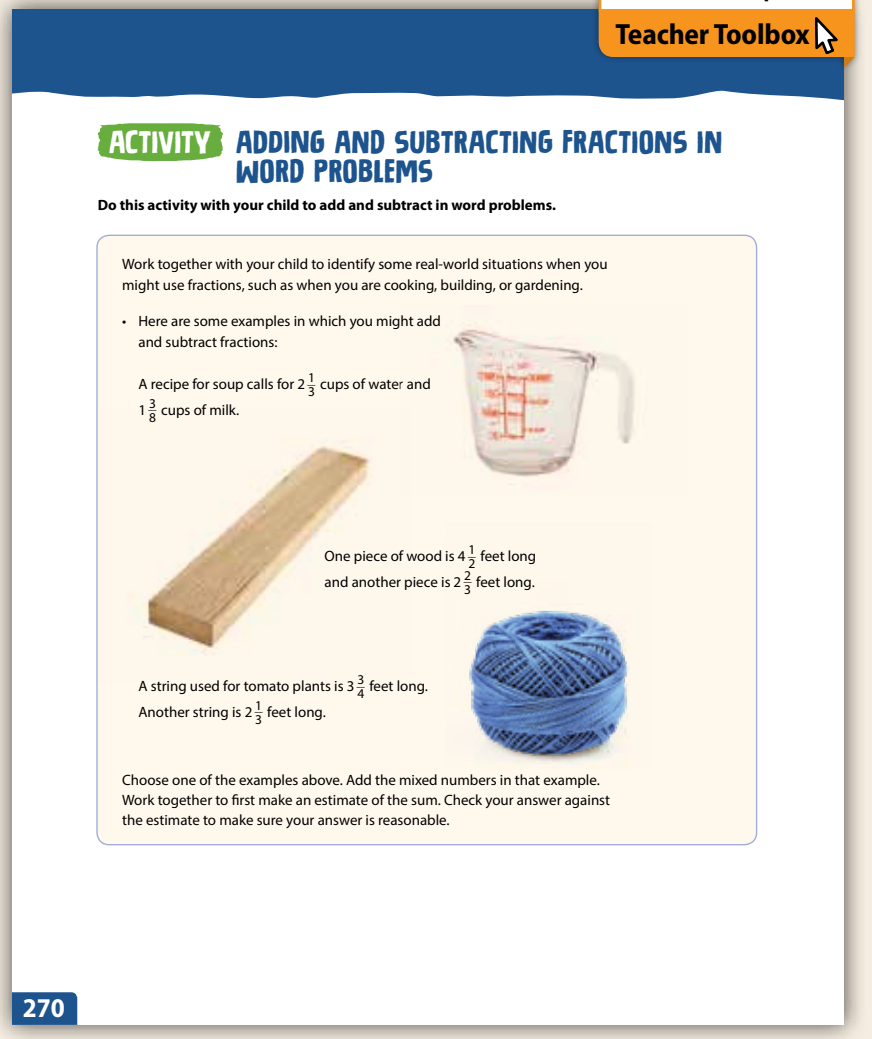

ACTIVITY ADDING AND SUBTRACTING FRACTIONS IN WORD PROBLEMS

Do this activity with your child to add and subtract in word problems.

Work together with your child to identify some real-world situations when you might use fractions, such as when you are cooking, building, or gardening.

- Here are some examples in which you might add and subtract fractions:

A recipe for soup calls for $2\frac{1}{3}$ cups of water and $1\frac{3}{8}$ cups of milk.

One piece of wood is $4\frac{1}{2}$ feet long and another piece is $2\frac{2}{3}$ feet long.

A string used for tomato plants is $3\frac{3}{4}$ feet long. Another string is $2\frac{1}{3}$ feet long.

Choose one of the examples above. Add the mixed numbers in that example. Work together to first make an estimate of the sum. Check your answer against the estimate to make sure your answer is reasonable.

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Goal

The goal of the Family Letter is to provide opportunities to use estimation in word problem situations that involve adding or subtracting fractions. The skills discussed are also applied to decimals in the lesson.

Activity

Understanding how to add and subtract fractions and how to use estimation with these operations will help students apply these skills in real-life situations. Use the *Adding and Subtracting Fractions in Word Problems* activity and adjust if needed to connect with your students. Students can include decimals in the activity as well.

Math Talk at Home

Encourage students to look for objects at home that have fractions or decimals printed on them and make up stories about the objects that involve addition and subtraction.

Conversation Starters Below are additional conversation starters students can write in their Family Letter or math journal to engage family members:

- *Do you ever add or subtract fractions or decimals at work or at home?*
- *How do you estimate when adding or subtracting fractions or decimals?*
- *Let's make up a story together that has to do with adding or subtracting fractions or decimals.*

Connect to Community and Cultural Responsiveness

Use these activities to connect with and leverage the diverse backgrounds and experiences of all students.

Sessions 3 and 4 **Use anytime during these sessions.**

- Encourage students to think of real-life situations where they might need to add fractions or decimals. Point out that an example is measuring the water consumption of a household. Water usage is measured by meters that are placed at homes and businesses. These meters measure monthly water usage to the nearest hundredth of a unit. The previous month's reading (the number displayed by the meter at the end of the month) is subtracted from the latest month's reading, and the difference is used to calculate monthly water usage.
- Encourage students to search for sample water bills online and have them generate word problems about water usage using the information they have gathered.

Session 4 **Use with *Apply It* problem 2.**

- Some students may be more familiar with soccer than football. Show pictures of both sports and other sports students may like. Invite students to share what they know about the equipment for each sport. Students can research weights of different items, for example: balls, helmets, shoes, and shin pads. Encourage students to use some of their findings to write a new word problem.

Connect to Language Development

For ELLs, use the Differentiated Instruction chart to plan and prepare for specific activities in every session.

ELL English Language Learners: Differentiated Instruction

Prepare for Session 1
Use with *Connect It*.

Levels 1–3	Levels 2–4	Levels 3–5
Reading/Speaking Read the ***Try It*** problem aloud. Clarify any confusing information. Ask students to circle the fractions in the problem. Have them form pairs and use gestures, words and pictures to discuss the problem. Allow time for students to engage in productive struggle to solve the problem, then model a solution using fraction bars. Think aloud as you model the problem, and pause to elicit responses from students. For example, you might say: *The fractions have unlike denominators. What do I need to do?* [find a common denominator] $\frac{3}{8}$ *and* $\frac{9}{24}$ *are called* _equivalent fractions_.	**Reading/Speaking** Read the ***Try It*** problem with students. Have students form pairs to make sense of the problem. Provide sentence frames to support students as the think aloud: • *I need to find how much* _water Aleena used to water the roses and geraniums_. • *To* _add_ *fractions with* _unlike_ *denominators, I need to find a* _common_ *denominator.* Allow time for students to work together to solve the problem. Group them with another pair of students to compare solution strategies.	**Reading/Speaking** Have students read the ***Try It*** problem and form pairs to discuss possible solutions strategies. Provide a bank of terms to support discussion, such as: *common denominator, equivalent fractions, multiple, numerator, addition*. After students solve the problem independently, have them explain their solution in writing using at least three terms from the word bank. The have students meet with their pairs to compare answers and written explanations of how they solved the problem.

LESSON 14

SESSION 1 Explore

Purpose In this session students draw on prior knowledge to solve a word problem involving adding fractions with unlike denominators. They identify which operation to use and share models to explore how various solution methods are based on finding equivalent fractions. They will look ahead to think about using benchmark fractions to estimate sums to assess the reasonableness of their answers.

Start

Connect to Prior Knowledge

Materials For each student: 1 set of fraction tiles or circles

Why Support students' facility with identifying common denominators to prepare for solving word problems involving fractions.

How Have students use fraction tiles or circles to identify a common denominator for each pair of fractions.

Identify a common denominator for each pair of fractions.

$\frac{2}{3}$ and $\frac{1}{4}$ $\frac{5}{6}$ and $\frac{1}{8}$ $\frac{1}{3}$ and $\frac{1}{8}$

Possible Solutions

$\frac{2}{3}$ and $\frac{1}{4}$: twelfths or twenty-fourths; $\frac{5}{6}$ and $\frac{1}{8}$: twenty-fourths or forty-eighths; $\frac{1}{3}$ and $\frac{3}{8}$: twenty-fourths or forty-eighths

TRY IT

Make Sense of the Problem

To support students in making sense of the problem, have them identify the amount of water Aleena uses for her roses and the amount of water Aleena uses for her geraniums.

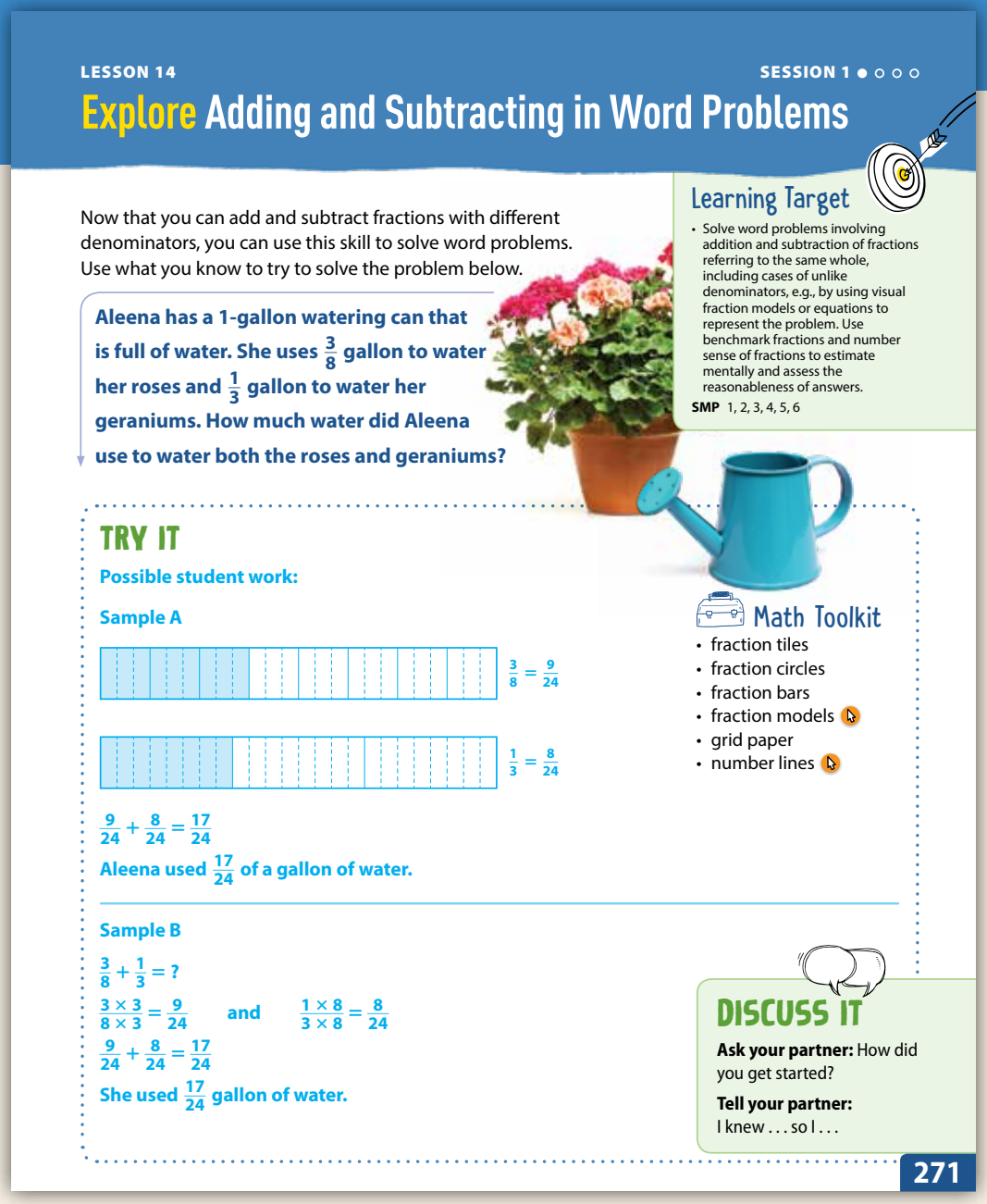
DISCUSS IT

Support Partner Discussion

To reinforce that the problem involves fractions with unlike denominators, encourage students to discuss the size of the parts in each fraction.

Look for, and prompt as necessary for, understanding of:

- $\frac{3}{8}$ and $\frac{1}{3}$ as parts to add to find the total
- $\frac{3}{8}$ and $\frac{1}{3}$ as fractions with unlike denominators



Common Misconception Look for students who find how much water is left in the watering can by subtracting the amounts used from 1. As students present solutions, have them specify why they chose the operation they used.

Select and Sequence Student Solutions

One possible order for whole class discussion:

- concrete models to show the fractions
- drawings to show equivalent fractions and the sum
- number lines marked in twenty-fourths
- equations to show equivalent fractions and the sum

Support Whole Class Discussion

Prompt students to note the relationship between the numbers in each model and the numbers in the problem.

Ask *How do [student name]'s and [student name]'s models show each addend and how to find equivalent fractions with like denominators?*

Listen for A whole can be divided into 24 equal parts to show both eighths and thirds. 24 is a common denominator for $\frac{3}{8}$ and $\frac{1}{3}$. $\frac{3}{8} = \frac{9}{24}$ and $\frac{1}{3} = \frac{8}{24}$.

CONNECT IT

1 LOOK BACK

Look for understanding that you need to find equivalent fractions to add and that the amount of water used was $\frac{17}{24}$ of a gallon.

Visual Model

Use a number line to find a sum.

If . . . *students are unsure about the concept of adding fractions with unlike denominators,*

Then . . . *use this activity to model a sum.*

Materials For each student: Activity Sheet *Numbers Lines*

- Have students use two number lines, one divided into eighths and the other into thirds, to model the amounts of water in the *Try It* problem. Have them start at 0 and shade segments to show $\frac{3}{8}$ gallon and $\frac{1}{3}$ gallon.
- Ask students to think about putting the two segments together on the eighths number line to show the sum $\frac{3}{8} + \frac{1}{3}$. Ask: *Are the number lines divided into same-size parts?* [No] *How could you divide the eighths number line into same-size parts to show adding thirds?* [Divide each eighth into thirds.]
- Have students add tick marks to their eighths number line to divide each eighth into thirds. Ask: *What size are the parts of the whole now?* [twenty-fourths] *How many of these parts are equivalent to* $\frac{1}{3}$*?* [8]
- Have students shade a segment of length $\frac{1}{3}$ on the eighths number line to show the sum $\frac{3}{8} + \frac{1}{3}$. Discuss how the number line shows $\frac{3}{8} + \frac{1}{3} = \frac{9}{24} + \frac{8}{24} = \frac{17}{24}$.

2 LOOK AHEAD

Point out that you can estimate sums and differences of fractions using benchmark fractions that are easier to work with. Students should be able to locate $\frac{1}{3}$ between $\frac{1}{4}$ and $\frac{1}{2}$, as all are unit fractions. Have them discuss how they located $\frac{3}{8}$ between benchmark fractions. Then have several students share their answers for part c.

CONNECT IT

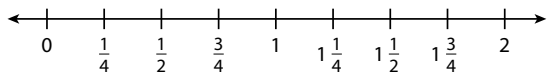
1 LOOK BACK

Explain how you found how much water Aleena used to water both the roses and the geraniums.

Possible answer: I found equivalent fractions and added; $\frac{9}{24} + \frac{8}{24} = \frac{17}{24}$. Aleena used $\frac{17}{24}$ gallon of water.

2 LOOK AHEAD

A benchmark fraction is a common fraction that you can easily compare to other fractions. The number line below shows the location of some benchmark fractions between 0 and 2. You can use these fractions to estimate sums and differences.



a. Between which two benchmark fractions is $\frac{3}{8}$? How do you know?

Between $\frac{1}{4}$ and $\frac{1}{2}$; Possible answer: $\frac{3}{8}$ is between $\frac{2}{8}$ and $\frac{4}{8}$, and the benchmarks $\frac{1}{4}$ and $\frac{1}{2}$ are equivalent to $\frac{2}{8}$ and $\frac{4}{8}$.

b. Between which two benchmark fractions is $\frac{1}{3}$? How do you know?

Between $\frac{1}{4}$ and $\frac{1}{2}$; Possible answer: $\frac{1}{4}$, $\frac{1}{3}$, and $\frac{1}{2}$ are all unit fractions, so you can order the fractions from least to greatest in reverse order of their denominators.

c. Use your answers from parts a and b above to find a low estimate for the sum $\frac{3}{8} + \frac{1}{3}$ and a high estimate for the sum $\frac{3}{8} + \frac{1}{3}$. Explain your reasoning.

Possible answer: Low: $\frac{1}{4} + \frac{1}{4} = \frac{1}{2}$; high: $\frac{1}{2} + \frac{1}{2} = 1$; The sum should be between the sum of the lesser benchmarks and the sum of the greater benchmarks.

3 REFLECT

The actual sum of any two fractions will be somewhere between a low estimate and a high estimate for the sum. How does the actual sum you found in problem 1 compare to your low estimate and high estimate for the sum?

Possible answer: The estimated sum is between $\frac{1}{2}$ and 1. The actual sum of $\frac{17}{24}$ is between the estimates because $\frac{17}{24}$ is greater than $\frac{1}{2}$ and less than 1.

Close: Exit Ticket

3 REFLECT

Look for understanding of how the actual sum, $\frac{17}{24}$, lies between the low estimate of $\frac{1}{2}$ and the high estimate of 1 because $\frac{1}{2} = \frac{12}{24}$ and $1 = \frac{24}{24}$.

Common Misconception **If** students do not recognize that the actual sum lies between the low and high estimates of the sum, **then** have them write the equivalent fractions in twenty-fourths for each number from 0 to 1 on the number line. Then have students plot the estimates and the actual sum on the number line.

Real-World Connection

Encourage students to think about everyday places or situations where people might need to add or subtract fractions with unlike denominators. Have volunteers share their ideas. Examples: following a recipe, measuring to cut lengths of wood or fabric for a project, measuring distances.

Solutions

Support Vocabulary Development

1 Have students say *benchmark fraction*. Assess prior knowledge by asking students to discuss what they know about the word *benchmark*. Listen to assess if perceived knowledge is correct. Explain that a *benchmark* is something that is used for comparing and measuring things, such as a common fraction that can be used to compare other fractions.

Ask students to work in small groups to complete the graphic organizer. Call on volunteers to share what they wrote under *In My Own Words*. Correct any misconceptions and ask students to revise their graphic organizers if necessary.

2 Read the problem. Have students circle on the number line the two benchmark fractions closest to $\frac{5}{8}$. Have them tell a partner how they located $\frac{5}{8}$ on the number line before writing their answer to the question.

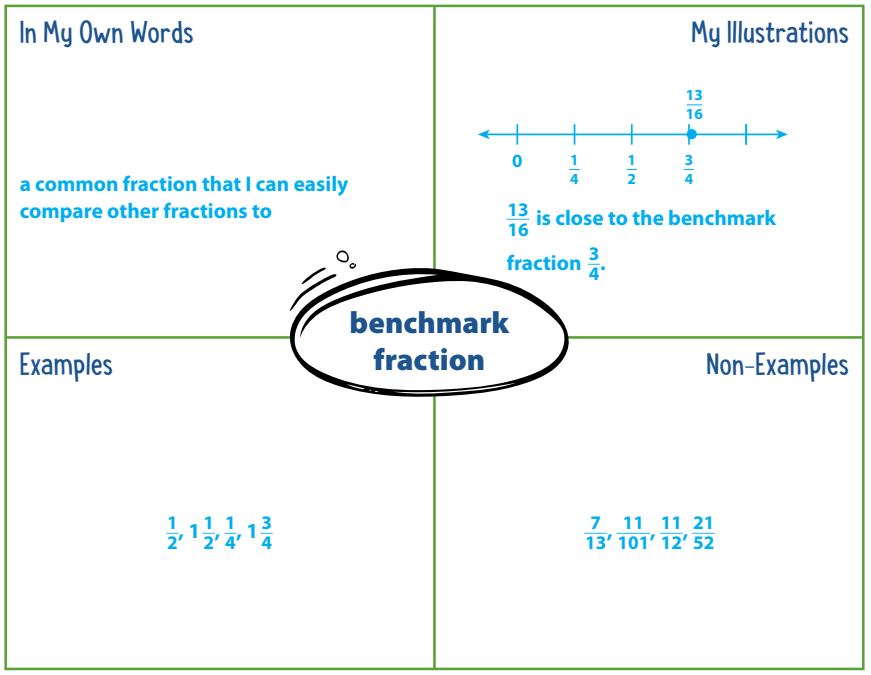
Supplemental Math Vocabulary

- *estimating*
- *rounding*

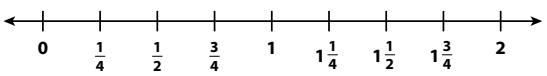
Name: _____ LESSON 14 SESSION 1

Prepare for Adding and Subtracting in Word Problems

1 Think about what you know about benchmark fractions. Fill in each box. Use words, numbers, and pictures. Show as many ideas as you can. **Possible answers:**



2 Between which two benchmark fractions is $\frac{5}{8}$? How do you know?



Between $\frac{1}{2}$ and $\frac{3}{4}$; Possible explanation: $\frac{5}{8}$ is between $\frac{4}{8}$ and $\frac{6}{8}$, and the benchmarks $\frac{1}{2}$ and $\frac{3}{4}$ are equivalent to $\frac{4}{8}$ and $\frac{6}{8}$.

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3 Solve the problem. Show your work.

Hai has a 1-gallon jug of water. He drinks $\frac{1}{8}$ gallon of water before lunch and $\frac{2}{3}$ gallon of water after lunch. How much water did Hai drink all day?

Possible student work using pictures:

$\frac{1}{8} = \frac{3}{24}$

$\frac{2}{3} = \frac{16}{24}$

$\frac{3}{24} + \frac{16}{24} = \frac{19}{24}$

Solution **Hai drank $\frac{19}{24}$ gallon of water.**

4 Check your answer. Show your work.

Possible student work:

$\frac{1}{8} + \frac{2}{3} = ?$

$\frac{1 \times 3}{8 \times 3} = \frac{3}{24}$ and $\frac{2 \times 8}{3 \times 8} = \frac{16}{24}$

$\frac{3}{24} + \frac{16}{24} = \frac{19}{24}$

He drank $\frac{19}{24}$ gallon of water.

3 Assign problem 3 to provide another look at solving a problem with fractions that have different denominators.

This problem is very similar to the problem about finding how much water Aleena uses to water both the roses and geraniums. In both problems, students need to add fractions with unlike denominators to solve the problem. The question asks for the total amount of water Hai drank during the day.

Students may want to use fraction bars, fraction circles, or fraction tiles.

Suggest that students read the problem three times, asking themselves one of the following questions each time:

- *What is this problem about?*
- *What is the question I am trying to answer?*
- *What information is important?*

Solution: $\frac{1}{8} = \frac{3}{24}$ and $\frac{2}{3} = \frac{16}{24}$; $\frac{3}{24} + \frac{16}{24} = \frac{19}{24}$; Hai drank $\frac{19}{24}$ of a gallon of water.

Medium

4 Have students solve the problem another way to check their answer.

ELL English Language Learners: Differentiated Instruction

Prepare for Session 2
Use with *Connect It*.

Levels 1–3

Listening/Writing Use with ***Connect It*** problem 5. Write the following sentence frames and provide vocabulary for students to complete the sentences with a partner. Vocabulary: *estimate*, *benchmark fractions*, *close*, *reasonable*

- *A benchmark helps me* ___estimate___.
- *I can check my answer by comparing it to the sum or difference of the* ___benchmark fractions___.
- *I need to see if my answer is* ___close___ *to the estimate.*
- *If my answer is close to the estimate, then the answer is* ___reasonable___.

Levels 2–4

Speaking/Writing Use with ***Connect It*** problem 5. Remind students that something is *reasonable* when it makes sense. Provide the following sentence frames to partners:

- *I can check my answer by* _____.
- *If my answer is close to the estimate, then the answer is* _____.

After partners complete their sentences, ask them to explain to another pair of students why their answer is reasonable.

Levels 3–5

Reading/Speaking Use with ***Connect It*** problem 5. Ask students to read the problem and work with a partner to explain how they can tell whether an answer is reasonable and why. Provide the following vocabulary for students to use in their conversations: *estimate*, *benchmark fractions*, *close*, *reasonable*.

LESSON 14

SESSION 2 Develop

Purpose In this session students solve a problem that requires them to estimate and find the difference between $3\frac{1}{2}$ and $1\frac{1}{3}$. Students model the mixed numbers on paper or with manipulatives to find an estimated difference and the actual difference. The purpose of this problem is to have students develop strategies for using benchmark fractions to estimate solutions for word problems involving fraction addition or subtraction.

Start

Connect to Prior Knowledge

Materials For each pair: 1 set of fraction tiles

Why Support students' facility with subtracting fractions with unlike denominators.

How Have students use fraction tiles to show each difference. Discuss how they found equivalent fractions with like denominators.

Model each difference.

$\frac{7}{8} - \frac{1}{4} =$ .....

$\frac{3}{4} - \frac{1}{2} =$ .....

$\frac{1}{2} - \frac{1}{3} =$ .....

Solutions

$\frac{5}{8}; \frac{1}{4}; \frac{1}{6}$

Develop Language

Why Reinforce understanding of the term *reasonable*.

How Explain that something is *reasonable* if it "makes sense." Tell students that when they check their answers to make sure they are reasonable, they compare the answer against an estimate and against the context of the word problem. If the answer is close to the estimate and makes sense in context, then the answer is reasonable.

TRY IT

Make Sense of the Problem

To support students in making sense of the problem, have them identify that they are asked to both estimate and calculate the amount of chicken left, as well as tell if their answer is reasonable.

Ask *Will Frankie have more than $3\frac{1}{2}$ pounds or less than $3\frac{1}{2}$ pounds after he makes fajitas?*

LESSON 14 SESSION 2 ● ● ○ ○

Develop Estimating in Word Problems with Fractions

Read and try to solve the problem below.

Frankie purchases a $3\frac{1}{2}$-pound bag of chicken. He uses $1\frac{1}{3}$ pounds of chicken for fajitas. How many pounds of chicken does Frankie have left? Estimate and solve. Tell if your answer is reasonable.

TRY IT

Possible student work:

Sample A

$1\frac{1}{3}$ is about $1\frac{1}{4}$.

Math Toolkit
- fraction tiles
- fraction circles
- fraction bars
- fraction models
- grid paper
- number lines

Frankie has about $2\frac{1}{4}$ pounds of chicken left.

$3\frac{1}{2} = 3\frac{3}{6}; 1\frac{1}{3} = 1\frac{2}{6}$

$3\frac{3}{6} - 1\frac{2}{6} = 2\frac{1}{6}$. $2\frac{1}{6}$ is close to $2\frac{1}{4}$, so my answer is reasonable.

Sample B

Solve: $3\frac{1}{2} = 3\frac{3}{6}; 1\frac{1}{3} = 1\frac{2}{6}$

$3\frac{3}{6} - 1\frac{2}{6} = 2\frac{1}{6}$

Frankie has $2\frac{1}{6}$ pounds of chicken left.

Estimate: $1\frac{1}{3}$ is about $1\frac{1}{2}$.

$3\frac{1}{2} - 1\frac{1}{2} = 2$

Frankie has about 2 pounds of chicken left.

$2\frac{1}{6}$ pounds is close to 2 pounds, so my answer is reasonable.

DISCUSS IT

Ask your partner: Do you agree with me? Why or why not?

Tell your partner: I disagree with this part because . . .

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DISCUSS IT

Support Partner Discussion

Encourage students to use the term *benchmark fraction* as they discuss their estimates with each other.

Support as needed with questions such as:

- *How did you get started?*
- *Why did you choose the strategy you used?*

Common Misconception Look for students who add the mixed numbers instead of subtracting them. Ask them to draw a visual model of the $3\frac{1}{2}$ pounds that Frankie buys and then think about the operation involved in "using" $1\frac{1}{3}$ of those pounds.

Select and Sequence Student Solutions

One possible order for whole class discussion:

- concrete models to show estimates and/or the difference
- drawings to show estimates and/or the difference
- number lines used to find estimates and/or the difference
- equations used to find estimates and/or the difference

Support Whole Class Discussion

Compare and connect the different models and have students describe how they are related.

Ask *How did you estimate the difference?*

Listen for Students should recognize that accurate responses include a description of how benchmark fractions or "friendlier" numbers were used.

Ask *How does your model show the difference?*

Listen for Students should recognize that accurate representations include showing how to use equivalent fractions to subtract fractions with like denominators. Representations may include fraction tiles, drawings, or equations to show how to subtract equivalent fractions: $3\frac{3}{6} - 1\frac{2}{6} = 2\frac{1}{6}$.

MODEL ITS

If no student presented these models, connect them to the student models by pointing out the ways they represent:

- benchmark fractions for estimating
- a method for finding a common denominator
- a way to subtract mixed numbers

Ask *What benchmark fractions and numbers are shown on the number lines?*

Listen for The number line uses 0, $\frac{1}{2}$, 1, $1\frac{1}{2}$, 2, $2\frac{1}{2}$, 3, $3\frac{1}{2}$, 4 as benchmarks.

For the number line used for estimating, prompt students to identify how the mixed numbers in the problem relate to the number line.

- *Is the mixed number for pounds of chicken Frankie purchased on or between tick marks on the number line?*
- *Is the mixed number for pounds of chicken Frankie used for fajitas on or between tick marks on the number line?*

For the number line used for solving, prompt students to discuss how the solution is modeled.

- *Why does this* Model It *mention that* $2 \times 3 = 6$*? How is this used on the number line?*
- *Where do you see each number of pounds of chicken in the model?*

Explore different ways to understand estimating with fractions.

Frankie purchases a $3\frac{1}{2}$-pound bag of chicken. He uses $1\frac{1}{3}$ pounds of chicken for fajitas. How many pounds of chicken does Frankie have left? Estimate and solve. Tell if your answer is reasonable.

MODEL IT

You can use a number line to find benchmark fractions to estimate.

$3\frac{1}{2}$ is already a benchmark fraction you can use to estimate.

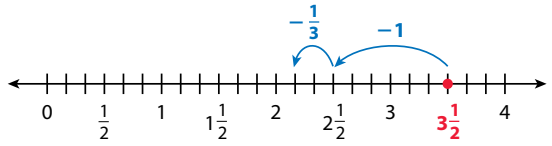
You can find a benchmark fraction close to $1\frac{1}{3}$ to help you estimate the difference.

MODEL IT

You can model the problem with a number line.

Since $2 \times 3 = 6$, the fractions in the problem, $3\frac{1}{2}$ and $1\frac{1}{3}$, can be rewritten using a common denominator of 6. $3\frac{1}{2} = 3\frac{3}{6}$, and $1\frac{1}{3} = 1\frac{2}{6}$.

The number line below is divided into sixths. It shows starting with a total of $3\frac{1}{2}$ pounds and then making two jumps to the left for the $1\frac{1}{3}$ pounds of chicken used.



You can rewrite $3\frac{1}{2} - 1\frac{1}{3}$ as $3\frac{3}{6} - 1\frac{2}{6}$.

Deepen Understanding

Number Line Models

SMP 5 Use tools.

When discussing the number line models, prompt students to consider how the scale chosen for each number line makes it a useful tool for a specific purpose.

Ask *What are the scales used for the number lines in the two* Model Its*? What is each number line best-suited for?*

Listen for The first number line is divided into halves. It is good for estimating—you can plot $3\frac{1}{2}$ easily and see that when you jump back $1\frac{1}{3}$, you land between tick marks for 2 and $2\frac{1}{2}$. The second number line is divided into sixths. Because 6 is a common denominator for the fractions in the problem, it is easy to show the exact difference on this number line.

Generalize *What should you think about when choosing a scale for a number line?* Listen for understanding that when estimating with fractions, you may want to divide a number line with tick marks at benchmark fractions. To find an exact answer, dividing into parts using a common denominator is useful.

CONNECT IT

- Remind students that the number line models can be used to both estimate and solve the problem.
- Explain that on this page, they will use the number line models to compare the actual answer to the estimate to see if their answer is reasonable.

Monitor and Confirm

1 – 3 Check for understanding that:

- $1\frac{1}{3}$ is between the benchmark numbers 1 and $1\frac{1}{2}$
- $1\frac{1}{2}$ (or 1) is a good benchmark choice for $1\frac{1}{3}$
- 2 $\left(\text{or } 2\frac{1}{2}\right)$ is a good estimate of the difference
- $2\frac{1}{6}$ pounds is the actual difference

Support Whole Class Discussion

4 Have students share their answers to problem 4 to recognize that estimates may be greater than or less than the actual difference.

Ask *Could you get a different answer depending on what benchmark you used for $1\frac{1}{3}$?*

Listen for The actual difference could be greater than or less than the estimate depending on what number you choose as a benchmark for $1\frac{1}{3}$. Choose $1\frac{1}{2}$ and the actual difference $2\frac{1}{6}$ will be greater than the estimate, 2. Choose 1 and the actual difference will be less than the estimate, $2\frac{1}{2}$.

Ask *Suppose that after Frankie makes fajitas, he wants to know if he has enough chicken left to make a recipe that calls for 2 pounds of chicken. Would he need to find the exact difference? Could he use an estimate to decide?*

Listen for Both the low estimate and the high estimate show that he will have at least 2 pounds left. He would not need to find the exact difference.

5 Look for the idea that fractions can be replaced by benchmark fractions to estimate a sum or difference, and that if the estimate is close to the actual sum or difference, it can show the answer is reasonable.

6 REFLECT Have all students focus on the strategies used to solve this problem. If time allows, have students share their preferences with a partner.

CONNECT IT

Now you will use the problem from the previous page to help you understand how to use estimation with adding and subtracting fractions.

1 Look at the first **Model It**. Identify the closest half on each side of $1\frac{1}{3}$.

$1\frac{1}{3}$ is greater than 1 and less than $1\frac{1}{2}$.

Why are halves a good choice for benchmark fractions for $1\frac{1}{3}$?

Possible answer: because you are subtracting from $3\frac{1}{2}$

2 Use one of the benchmark fractions for $1\frac{1}{3}$ that you found in problem 1 to estimate how many pounds of chicken Frankie has left. Write a subtraction equation to show your estimated difference.

For benchmark of 1: $3\frac{1}{2} - 1 = 2\frac{1}{2}$. For benchmark of $1\frac{1}{2}$: $3\frac{1}{2} - 1\frac{1}{2} = 2$.

3 Use the common denominator found in the second **Model It** to find the actual difference. How many pounds of chicken does Frankie have left?

$3\frac{1}{2} - 1\frac{1}{3} = 3\frac{3}{6} - 1\frac{2}{6} = 2\frac{1}{6}$. Frankie has $2\frac{1}{6}$ pounds of chicken left.

4 Is the actual difference greater than or less than your estimate in problem 2? Why?

Possible answer: Greater than; I subtracted a benchmark fraction of $1\frac{1}{2}$, which is greater than $1\frac{1}{3}$, so the actual difference is greater than my estimate.

5 Explain how you can check if a fraction sum or difference is reasonable.

Possible answer: Estimate the sum or difference using benchmark fractions. Compare the actual answer to the estimate. If the answer is close to the estimate, then it is reasonable.

6 REFLECT

Look back at your **Try It**, strategies by classmates, and **Model Its**. Which models or strategies do you like best for estimating with fractions? Explain.

Possible answer: I like using a number line because I can see all the benchmark fractions. Then I can pick the benchmark fractions that are closest to the fractions I am using to make my estimate.

Hands-On Activity

Use fraction circles to estimate with fractions.

If . . . *students have difficulty with using benchmark fractions to estimate,*

Then . . . *use this activity to model "easy" sums and differences.*

Materials For each pair: 1 set of fraction tiles

- Have students model the sum $\frac{1}{3} + \frac{3}{4}$ with fraction tiles. [1 one-third and 3 one-fourths] Have students replace the $\frac{1}{3}$ tile with a $\frac{1}{4}$ tile. Ask: *Is the $\frac{1}{4}$ tile close in size to the $\frac{1}{3}$ tile?* [yes]
- Discuss estimating the sum of $\frac{1}{4} + \frac{3}{4}$. Ask: *Could you find the sum of $\frac{1}{4}$ and $\frac{3}{4}$ more easily than the sum of $\frac{1}{3}$ and $\frac{3}{4}$? Why?*
- Have students explain how to find the sum $\frac{1}{4} + \frac{3}{4}$ either mentally or with fraction circles. $\left[\frac{4}{4}, \text{ or } 1\right]$ Then have them use a common denominator to find the actual sum of $\frac{1}{3}$ and $\frac{3}{4}$ using $\frac{4}{12}$ and $\frac{9}{12}$. Discuss how the actual sum $\frac{13}{12}$, or $1\frac{1}{12}$, is close to the estimated sum, 1, so the actual sum is reasonable.
- Repeat by comparing the estimated and actual difference for $\frac{3}{4} - \frac{1}{3}$.

APPLY IT

Use what you just learned to solve these problems.

7 Tim's bean sprout grew $3\frac{3}{8}$ inches. Teegan's bean sprout grew $2\frac{3}{4}$ inches. How many more inches did Tim's bean sprout grow than Teegan's? Estimate to tell if your solution is reasonable. Show your work.

Possible student work:

Estimate: Round each amount to the closest $\frac{1}{2}$ inch: $3\frac{1}{2} - 3 = \frac{1}{2}$.

Actual difference:

$2\frac{3}{4} = 2\frac{6}{8}$

$3\frac{3}{8} - 2\frac{6}{8} = 2\frac{11}{8} - 2\frac{6}{8} = \frac{5}{8}$

$\frac{5}{8}$ inch is close to $\frac{1}{2}$ inch, so my answer is reasonable.

Tim's bean sprout grew **$\frac{5}{8}$ inch; Accept any equivalent fraction.** more than Teegan's bean sprout.

8 Samantha likes to run at least 5 miles each day. She plans a new course: from home to the park is $1\frac{1}{3}$ miles, from the park to the library is $2\frac{2}{5}$ miles, and from the park to home is $\frac{2}{3}$ mile. Will Samantha run at least 5 miles on this new course?

Use only estimation to decide. Then explain if you are confident in your estimate or if you need to find an actual sum. Show your work.

Students' reasons for estimating or finding actual sums may vary. Check that estimates are accurate. Possible student work:

$1\frac{1}{3}$ is about $1\frac{1}{2}$, $2\frac{2}{5}$ is about $2\frac{1}{2}$, and $\frac{2}{3}$ is about 1. $1\frac{1}{2} + 2\frac{1}{2} + 1 = 5$. Since all of my benchmark fractions are greater than the actual numbers, my estimate of 5 is high. Samantha will not run at least 5 miles. I do not need to add the actual numbers.

Samantha **will not** run at least 5 miles.

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APPLY IT

For all problems, encourage students to draw some kind of model to support their thinking. Allow some leeway in precision; drawing precise representations of fractions with area models or number lines can be difficult.

7 Estimate: $\frac{1}{2}$-inch or other reasonable amount; Actual: $\frac{5}{8}$ inch or an equivalent fraction; See possible equations on the Student Worktext page. Students may also use fraction tiles or fraction bars to find differences.

Close: Exit Ticket

8 Samantha will not run at least 5 miles; See possible estimates and explanation on the Student Worktext page. Explanations should include reasoning that supports whether the actual sum, $1\frac{5}{15} + 2\frac{6}{15} + \frac{10}{15} = 4\frac{6}{15}$, needs to be found.

Students' solutions should indicate understanding of:

- using benchmark numbers to estimate
- adding fractions
- assessing reasonableness of actual results with estimates

Error Alert **If** students say Samantha will run at least 5 miles, **then** they may be estimating by rounding each number up to the next whole number ($2 + 3 + 1 = 6$) rather than using benchmark fractions to estimate.

Solutions

1. $1\frac{5}{8}$: Shade one whole bar. Divide the other bar into 8 equal parts and shade 5 parts.
$1\frac{1}{2}$: Shade one whole bar. Divide the other bar into 2 equal parts and shade 1 part.
Students should see that the shaded part for $1\frac{5}{8}$ is longer than the shaded part for $1\frac{1}{2}$.
Basic

2. $3\frac{3}{8}$ inches or any equivalent fraction; The answer is reasonable because $3\frac{3}{8}$ is close to $3\frac{1}{4}$, as estimated in the Example. See possible equations for finding the sum on the student page.
Medium

Name: _____ LESSON 14 SESSION 2

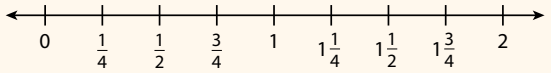
Practice Estimating in Word Problems with Fractions

Study the Example showing how to estimate a sum using benchmark fractions. Then solve problems 1–5.

EXAMPLE

David grew $1\frac{3}{4}$ inches last year and $1\frac{5}{8}$ inches this year. Estimate how much he grew in the two years.

You can estimate $1\frac{3}{4} + 1\frac{5}{8}$ using benchmark fractions. The number line below shows common fractions used as benchmark fractions to estimate sums and differences.

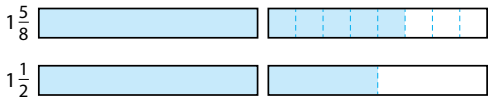


$1\frac{3}{4}$ is one of the benchmark fractions. $1\frac{5}{8}$ is a little greater than $1\frac{1}{2}$. Estimate using $1\frac{1}{2}$.

$1\frac{3}{4} + 1\frac{1}{2} = 1\frac{3}{4} + 1\frac{2}{4} = 2\frac{5}{4}$, or $3\frac{1}{4}$.

The sum is a little greater than $3\frac{1}{4}$, so David grew a little more than $3\frac{1}{4}$ inches.

1. Look at the Example. Shade the fraction bars below to show that $1\frac{5}{8}$ is a little greater than $1\frac{1}{2}$. **Possible student work shown.**



2. Find the actual sum $1\frac{3}{4} + 1\frac{5}{8}$ to determine how much David grew in two years. Use the estimate to explain how you know your answer is reasonable. Show your work.

Possible student work:

$1\frac{3}{4} = 1\frac{6}{8}$; $1\frac{6}{8} + 1\frac{5}{8} = 2\frac{11}{8}$; $2\frac{11}{8} = 3\frac{3}{8}$

Solution **$3\frac{3}{8}$ inches or any equivalent fraction. The answer is a little greater than $3\frac{1}{4}$, as estimated.**

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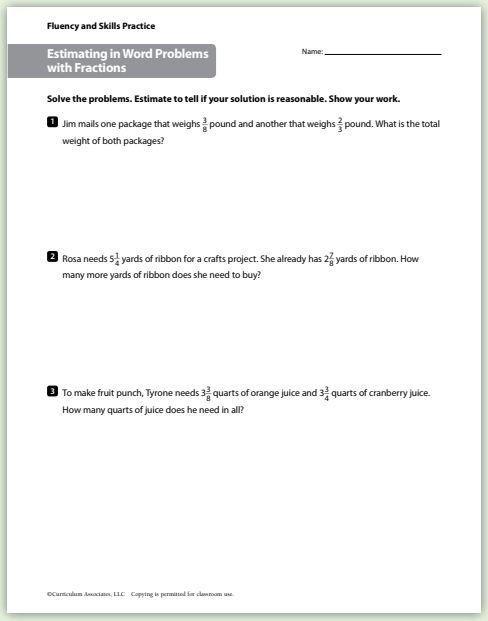
Fluency & Skills Practice

Teacher Toolbox

Assign *Estimating in Word Problems with Fractions*

In this activity students solve real-world problems involving addition and subtraction of fractions and mixed numbers with unlike denominators. They then estimate to evaluate whether their solutions are reasonable. Using estimation to judge the reasonableness of solutions is a valuable habit of mind to develop when dealing with mathematical situations in daily life.



Fluency and Skills Practice

Estimating in Word Problems with Fractions

Name: _____

Solve the problems. Estimate to tell if your solution is reasonable. Show your work.

1. Jim mails one package that weighs $\frac{3}{8}$ pound and another that weighs $\frac{2}{3}$ pound. What is the total weight of both packages?

2. Rosa needs $5\frac{1}{4}$ yards of ribbon for a crafts project. She already has $2\frac{7}{8}$ yards of ribbon. How many more yards of ribbon does she need to buy?

3. To make fruit punch, Tyrone needs $3\frac{3}{8}$ quarts of orange juice and $3\frac{3}{4}$ quarts of cranberry juice. How many quarts of juice does he need in all?

3 Accept estimates between $2\frac{1}{4}$ and 3; See the student page for one example of using benchmark fractions to estimate. Students may also estimate with $4\frac{2}{4} - 2\frac{1}{4}$ or $5 - 2$.
Challenge

4 $2\frac{5}{12}$ cups; Students may explain that $2\frac{5}{12}$ is close to their estimate of $2\frac{1}{2}$ (or other reasonable estimate) when explaining reasonableness.
Medium

5 See possible explanation on the student page. Students may also use benchmark fractions in a subtraction equation, $4\frac{1}{2} - 3\frac{1}{4} = 1\frac{1}{4}$, to explain why $2\frac{1}{12}$ is not a reasonable difference.
Medium

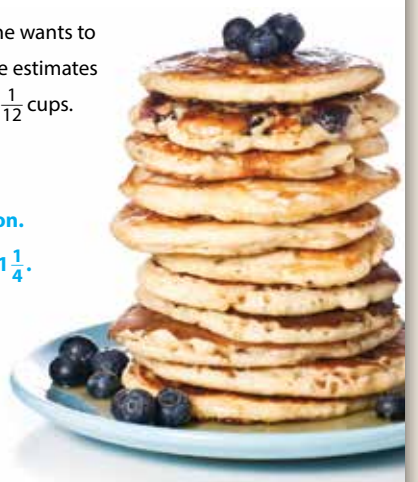
Irene makes $4\frac{2}{3}$ cups of pancake batter. She splits the batter into 2 bowls. She mixes blueberries into $2\frac{1}{4}$ cups of batter and walnuts into the rest of the batter.

3 Estimate how much of the batter has walnuts in it. Explain your estimate.
Possible answer: $2\frac{1}{2}$ cups; $4\frac{2}{3}$ is close to $4\frac{3}{4}$, and $2\frac{1}{4}$ is a benchmark fraction. $4\frac{3}{4} - 2\frac{1}{4} = 2\frac{2}{4}$, or $2\frac{1}{2}$.

4 Find the actual amount of batter that has walnuts in it.
Explain how you know your answer is reasonable. Show your work.
Students might use fraction strips, equations, or some other method to subtract $2\frac{1}{4}$ from $4\frac{2}{3}$.

Solution **$2\frac{5}{12}$ cups. Explanations will vary but should indicate that the answer $2\frac{5}{12}$ is close to the estimate from problem 3.**

5 Irene makes a second batch of $3\frac{1}{4}$ cups of pancake batter. She wants to know how much more batter she made in the first batch. She estimates that the difference between the sizes of the two batches is $2\frac{1}{12}$ cups. Explain why this estimate is *not* reasonable.
Answers will vary. Possible explanation: I can use the benchmark fraction $4\frac{1}{2}$ for $4\frac{2}{3}$. $3\frac{1}{4}$ is a benchmark fraction. $4\frac{1}{2} - 3\frac{1}{4} = 1\frac{1}{4}$. An estimate of $2\frac{1}{12}$ is much greater than $1\frac{1}{4}$. So, it is not a reasonable estimate.



ELL English Language Learners: Differentiated Instruction

Prepare for Session 3
Use with *Apply It*.

Levels 1–3

Speaking/Writing Use with ***Apply It*** problem 9. Read the problem with students and have them circle the total amount of money that Vinh has. Provide the following sentence frames and have students choral read them after you. Then ask students to complete the sentences with a partner and use the information to work together to solve the problem.

- *Vinh has a total of* __$15__ .
- *Vinh already spent* __$8.57__ *and* __$2.34__ .
- *Vinh wants to buy a salad for* __$5.25__ .

Levels 2–4

Reading/Writing Use with ***Apply It*** problem 9. Have students read the problem with a partner. Ask them to write a list of what Vinh has ordered and to write simple sentences to describe their process and the result. Provide the sentence frames below as one approach to the problem:

- *First, I will* __round__ *the price of each item.*
- *Then, I will* __add the rounded numbers__ .
- *I* __compare__ *the total to $15.*
- *Vinh* __does not have__ *enough money.*

When complete, encourage students to read their sentences to another set of partners.

Levels 3–5

Reading/Writing Use with ***Apply It*** problem 9. Have students read the problem independently. Encourage them to write the steps they will follow to solve the problem.

Provide the following vocabulary words for students to use in their writing: *estimate*, *add*, *rounding*, *justify*.

LESSON 14

SESSION 3 Develop

Purpose In this session students solve a problem that requires them to estimate and find the sum of 3.74 and 3.65. Students model the decimal addition on paper or with manipulatives to find an estimated sum and the actual sum. The purpose of this problem is to have students develop strategies for estimating solutions for word problems involving decimal addition or subtraction.

Start

Connect to Prior Knowledge

Materials For each student: base-ten blocks (5 hundreds flats and 15 tens rods)

Why Support student facility with adding with decimals to tenths to prepare for estimation with decimals to tenths.

How Have students use base-ten blocks to show each sum. Discuss any regrouping needed.

Use base-ten blocks to show each sum.

3.5 + 2.4 = ...

1.7 + 2.7 = ...

2.3 + 0.8 = ...

Solutions

5.9; 4.4; 3.1

Develop Language

Why Provide practice using the terms *estimate* and *estimation*.

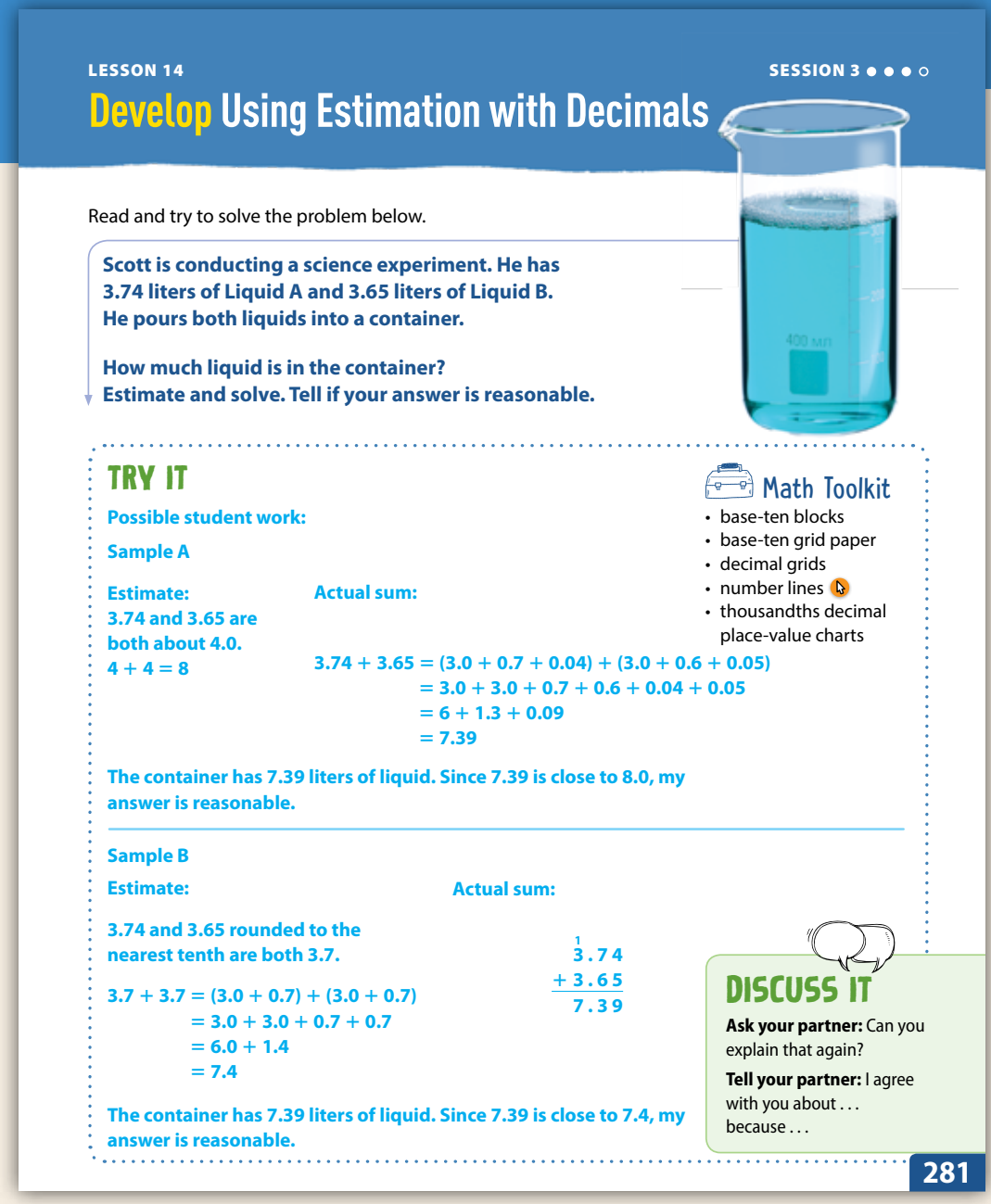
How Tell students that an *estimate* is an approximation. Say: *When you use estimation, you get an approximate answer.* Have students find the word *estimation* in *Apply It* problem 8. Ask: *What does the problem ask you to do first? What type of answer will you get? What will you do to check your estimate?*

TRY IT

Make Sense of the Problem

To support students in making sense of the problem, have them identify that they are asked to both estimate and calculate the amount of liquid in the container.

Ask *How can you use the estimate to see if your solution is reasonable?*



LESSON 14 SESSION 3 ● ● ● ○

Develop Using Estimation with Decimals

Read and try to solve the problem below.

Scott is conducting a science experiment. He has 3.74 liters of Liquid A and 3.65 liters of Liquid B. He pours both liquids into a container.

How much liquid is in the container?
Estimate and solve. Tell if your answer is reasonable.

TRY IT

Possible student work:

Sample A

Estimate:
3.74 and 3.65 are both about 4.0.
4 + 4 = 8

Actual sum:

$3.74 + 3.65 = (3.0 + 0.7 + 0.04) + (3.0 + 0.6 + 0.05)$
$= 3.0 + 3.0 + 0.7 + 0.6 + 0.04 + 0.05$
$= 6 + 1.3 + 0.09$
$= 7.39$

The container has 7.39 liters of liquid. Since 7.39 is close to 8.0, my answer is reasonable.

Sample B

Estimate:

3.74 and 3.65 rounded to the nearest tenth are both 3.7.

$3.7 + 3.7 = (3.0 + 0.7) + (3.0 + 0.7)$
$= 3.0 + 3.0 + 0.7 + 0.7$
$= 6.0 + 1.4$
$= 7.4$

The container has 7.39 liters of liquid. Since 7.39 is close to 7.4, my answer is reasonable.

Actual sum:

$$\begin{array}{r} {\scriptstyle 1}\\ 3.74\\ +\ 3.65\\ \hline 7.39 \end{array}$$

Math Toolkit

- base-ten blocks
- base-ten grid paper
- decimal grids
- number lines
- thousandths decimal place-value charts

DISCUSS IT

Ask your partner: Can you explain that again?

Tell your partner: I agree with you about . . . because . . .

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DISCUSS IT

Support Partner Discussion

Encourage students to use the term *about* when discussing estimated solutions.

Support as needed with questions such as:

- *Why did you choose the estimation strategy you used?*
- *What operation did you use? Why?*

Common Misconception Look for students who choose decimals that are still difficult to compute with when estimating. Ask: *What decimals are equivalent to benchmark fractions? What decimals might be easier to add or subtract with? Why?* Encourage students to choose numbers they can add mentally.

Select and Sequence Student Solutions

One possible order for whole class discussion:

- concrete models to show estimates and/or the sum
- drawings to show estimates and/or the sum
- number lines used to find estimates and/or the sum
- equations used to find estimates and/or the sum

Support Whole Class Discussion

Compare and connect the different models and have students describe how they are related.

Ask *How did you choose numbers for your estimate?*

Listen for Students should recognize that good choices for estimates use numbers that are easy to add and close to the actual addends. Responses may include rounding to the nearest whole number, to the nearest tenth, or to the nearest 0.25.

Ask *What strategy did you use to find the actual sum?*

Listen for Student may describe the range of models and strategies based on place value and properties of operations that they explored in previous lessons on adding and subtracting decimals.

PICTURE IT & MODEL IT

If no student presented these models, connect them to the student models by pointing out the ways they each represent:

- the addends in the problem
- a way to estimate the sum

Ask *How do the decimal grids and number line model show the addends in the problem?*

Listen for The decimal grids use shading to show the whole number and hundredths in each number. The number line shows each addend as a point.

For the decimals grids, prompt students to think about how the picture supports estimating.

- *How do decimal grids show whole numbers?*
- *Between which two whole numbers is each addend?*
- *How do the grids show which whole number is closer to the addend?*

For the number line model, prompt students to consider how the labels support estimating.

- *What does each tick mark on the number line represent? Which tick marks are labeled with numbers?*
- *Between which two labeled tick marks does the point for each addend lie?*
- *How can you tell if each addend is closer to 3.5 or to 4.0?*

Explore different ways to understand estimating with decimals.

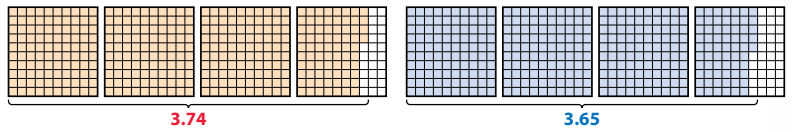
Scott is conducting a science experiment. He has 3.74 liters of Liquid A and 3.65 liters of Liquid B. He pours both liquids into a container.

How much liquid is in the container? Estimate and solve. Tell if your answer is reasonable.

PICTURE IT

You can picture an estimate of the problem using decimal grids.

Both 3.74 liters and 3.65 liters are about 4 liters.



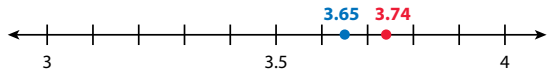
An estimate of the sum is 4 + 4.

The actual sum can be found by finding 3.74 + 3.65.

MODEL IT

You can use a number line to help estimate.

3.74 and 3.65 are both between 3.5 and 4.



An estimate of the sum is 3.5 + 3.5.

The actual sum can be found by finding 3.74 + 3.65.

Deepen Understanding

Number Line Model

SMP 6 Attend to precision.

Prompt students to consider how to use the number line model to look at the precision of the estimates in *Picture It* and *Model It*.

Ask *Which sum, 4.0 + 4.0 or 3.5 + 3.5, produces an estimate closer to the actual sum 3.74 + 3.65? How could the number line help you decide?*

Listen for You can estimate the sum using either 4 + 4 or 3.5 + 3.5. The number line shows that both 3.65 and 3.74 are closer to 3.5 than to 4.0. Because each addend is closer to 3.5, the sum 3.5 + 3.5, or 7.0, is a more precise estimate than the sum 4 + 4, or 8.

Generalize *Is it always better to choose numbers closer to the actual addends when estimating? Think about the actual addends 3.74 and 3.65 again. What would be the pros and cons of using 3.7 + 3.7 for estimating instead of 3.5 + 3.5?* Have students explain their reasoning. Listen for understanding that 3.7 + 3.7 would produce a closer estimate than 3.5 + 3.5, but that the sum 3.7 + 3.7 is not as easy to find with mental math.

CONNECT IT

- Remind students that one thing that is alike about all the representations is they show ways to replace the original addends with numbers that are easier to compute with.
- Explain that on this page, they will use those numbers to explain how to check the reasonableness of a decimal sum.

Monitor and Confirm

2 and 4 Check for understanding that:

- the actual sum is 7.39 L
- if both addends in the estimated sum are greater than the actual addends, the estimated sum would also be greater than the actual sum
- if both addends in the estimated sum are less than the actual addends, the estimated sum would also be less than the actual sum

Support Whole Class Discussion

1 and 3 Tell students that these problems will prepare them to provide the explanation required in problem 5.

Be sure students understand the problems by asking them to think about what numbers to use when estimating.

Ask *Both problems ask why a number is good to use as an addend for an estimate of the sum. How can different numbers, 4 and 3.5, both be good numbers to use for an estimated sum for this problem?*

Listen for Good numbers are ones that are close to the actual addends but easy to compute with. The numbers 4 and 3.5 are both close to 3.74 and 3.65 and are easy to add. Different numbers can be used for estimating.

5 Look for the idea that you should estimate with numbers close to the actual numbers in the problem to find an estimated sum or difference. Students should explain that if the actual sum or difference is close to their estimate, then the answer is reasonable.

6 REFLECT Have all students focus on the strategies used to solve this problem. If time allows, have students share their preferences with a partner.

CONNECT IT

Now you will use the problem from the previous page to help you understand how to estimate with decimals.

1 Look at **Picture It**. Why is 4 a good number to use for each addend in an estimated sum?

Possible answer: Each addend is close to 4, and 4 is an easy number to use for adding.

2 Is the actual sum of the measures of Liquids A and B in **Picture It** less than or greater than the estimated sum? Why?

Less than; Possible answer: The actual sum is 7.39 liters. The estimated sum is 8 liters. Each addend in the estimated sum is greater than the corresponding addend in the actual sum, so this will be a high estimate of the actual sum.

3 Look at **Model It**. Why is 3.5 a good number to use for each addend in an estimated sum?

Possible answer: Each addend is close to 3.5, and 3.5 is an easy number to use for adding.

4 Is the actual sum of the measures of Liquids A and B in **Model It** less than or greater than the estimated sum? Why?

Greater than; Possible answer: The actual sum is 7.39 liters. The estimated sum is 7 liters. Each addend in the estimated sum is less than the corresponding addend in the actual sum, so this will be a low estimate of the actual sum.

5 Explain how you can check if a decimal sum or difference is reasonable.

Possible answer: Estimate the sum or difference using numbers near the actual decimals that are easier to compute with. Compare the actual answer to the estimate. If the answer is close to the estimate, it is reasonable.

6 REFLECT

Look back at your **Try It**, strategies by classmates, and **Picture It** and **Model It**. Which models or strategies do you like best for estimating with decimals? Explain.

Possible answer: I like using a number line because I can find numbers that are close to the numbers that I am adding.

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Hands-On Activity

Use money to model estimating with decimals.

If . . . *students have difficulty estimating decimal sums and differences,*

Then . . . *use this activity to relate decimal amounts to money.*

Materials For each pair: play money (6 dollar bills, 15 dimes, 5 quarters)

- Display the problem 2.13 + 3.84. Tell students that they can estimate the sum by thinking about rounding amounts of money to find \$2.13 + \$3.84.
- Ask: *What place value are you rounding to when you round to the nearest dollar? To the nearest dime?* [ones; tenths] *When you round to the nearest quarter, are you rounding to a place value?* [No; you are rounding to the nearest $\frac{1}{4}$ of a dollar.]
- Have students model an estimate for 2.13 + 3.84 using dollars and dimes. [5 dollars and 9 dimes, or 5.90] If needed, ask: *Is \$2.13 closer to \$2.10 or \$2.20? Is \$3.84 closer to \$3.80 or \$3.90?*
- Have students model an estimate for 2.13 + 3.84 using dollars and quarters. [5 dollars and 4 quarters, or 6 dollars; 6] If needed, ask: *Is \$2.13 closer to \$2.00 or \$2.25? Is \$3.84 closer to \$3.75 or to \$4.00?*
- Repeat for other decimal sums, such as 1.42 + 2.79 and 4.31 + 0.96.

APPLY IT

For all problems, encourage students to think about whether a whole number or decimal would be better to use for estimating the sum or difference.

7 Estimate: 3 meters or other reasonable amount; Actual: 3.25 meters; See possible work on the Student Worktext page. Students may also use $16 - 12.5 = 3.5$ as an estimate. When finding the actual difference, students may also count on from 12.6 to 15.85 to find how many more meters of netting Sean needs.

8 Erika does not need to pick more apples; See possible explanation on the Student Worktext page. Explanations should include reasoning that supports whether an actual sum, $2.56 + 1.18 + 2.79 = 6.53$, needs to be found.

Close: Exit Ticket

9 Vinh does not have enough money; See possible work on the Student Worktext page. Students may also choose to find the actual sum, $\$8.57 + \$2.34 + \$5.25 = \$16.16$, to determine whether Vinh has enough money.

Students' solutions should indicate understanding of:

- computing estimated or actual decimal sums
- choosing appropriate addends to use for estimating
- comparing numbers, including decimals

Error Alert **If** students say Vinh has enough money based on their estimate of \$15, **then** have them compare each addend in their estimated sum with the actual addends. Ask them to explain if their estimate is a low estimate or a high estimate. With a low estimate, discuss how the actual cost of Vinh's food is higher than \$15.

APPLY IT

Use what you just learned to solve these problems.

7 Sean has 12.6 meters of garden netting. He needs 15.85 meters to enclose his garden. How many more meters of netting does Sean need to buy? Estimate to tell if your solution is reasonable. Show your work. **Possible student work:**

Estimate: 16 − 13 = 3
3.25 is close to 3. My answer is reasonable.

Solution:

$$\begin{array}{r} 15.85 \\ -\ 12.60 \\ \hline 3.25 \end{array}$$

Sean needs **3.25** meters more of garden netting.

8 Erika wants at least 6 kilograms of apples for a recipe. She picks a 2.56-kilogram bag of red apples, a 1.18-kilogram bag of green apples, and a 2.79-kilogram bag of yellow apples. Does Erika need to pick more apples?

Use estimation only to decide. Then explain if you are confident in your estimate or if you need to find an actual sum. Show your work.

Students' reasons for estimating or finding actual sums may vary. Check that estimates are accurate. Possible student work:
A low estimate for the sum is 2.5 + 1 + 2.5 = 6.0. Since the actual mass of each bag of apples is more than its estimate, Erika will have more than 6 kilograms. I do not need to add the actual numbers.

Erika **does not need** to pick more apples.

9 Vinh is having lunch at a café. He has \$15 in his pocket and has already ordered a sandwich for \$8.57 and a drink for \$2.34. Vinh also wants to buy a side salad for \$5.25. Does he have enough money? Estimate or find an exact answer to solve. Show your work.

Possible student work:
I can use an estimate to figure out if Vinh has enough money. Even if I use a low estimate for each item, it comes out to \$8 + \$2 + \$5 = \$15. Since the actual price of each item is more than the estimate, I know that Vinh does not have enough money.

Vinh **does not have** enough money.

Solutions

1. An estimate is enough; See a possible explanation on the student page.
Basic

2. 2.77 liters; Subtract the amount Kamala has, 2.73 liters, from the amount she wants to have, 5.5 liters.
Medium

Name: _____ LESSON 14 SESSION 3

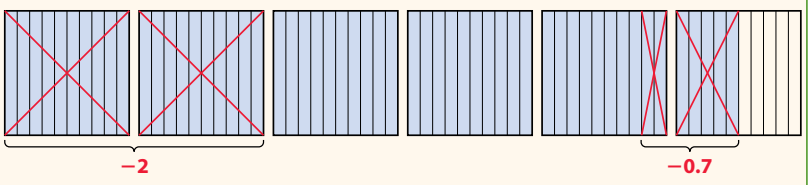
Practice Using Estimation with Decimals

Study the Example showing how to estimate a difference using decimal grids. Then solve problems 1–4.

EXAMPLE

Kamala has 2.73 liters of lemonade. She wants to have about 5.5 liters for her party. About how much more lemonade does Kamala need?

One way to estimate is to round to the nearest tenth.
5.5 is given to the nearest tenth. **2.73** is about **2.7**.



Kamala needs about 2.8 liters more of lemonade.

1. Look at the Example. Does this situation require an exact answer, or is the estimate enough? Explain.

An estimate is enough; Possible answer: The problem says Kamala wants about, not exactly, 5.5 L of lemonade.

2. Suppose Kamala wants to have exactly 5.5 liters of lemonade for her party. How much more lemonade does she need? Show your work.

Possible student work:
5.5 − 2.0 = 3.5
3.5 − 0.7 = 2.8
2.80 − 0.03 = 2.77

Kamala needs **2.77** liters more of lemonade.

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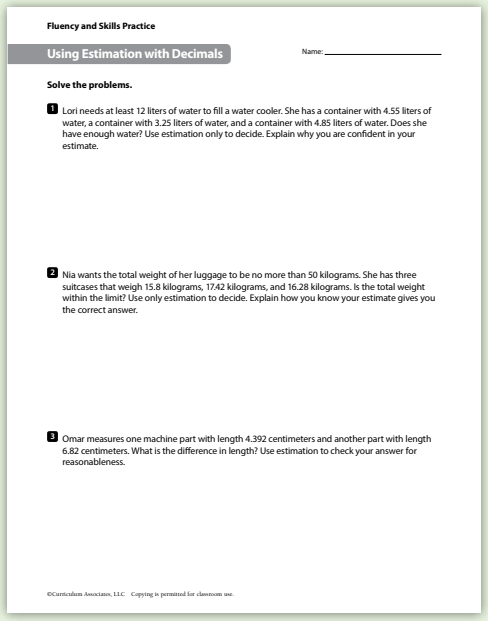
Fluency & Skills Practice

Teacher Toolbox

Assign *Using Estimation with Decimals*

In this activity students use estimation in the context of real-world problems involving the addition or subtraction of decimals. Estimation is an efficient method of solving problems in real-world situations when an exact answer is not required, as when determining the rough sum or difference of costs between differently priced goods.



Fluency and Skills Practice

Using Estimation with Decimals

Name: _____

Solve the problems.

1. Lori needs at least 12 liters of water to fill a water cooler. She has a container with 4.55 liters of water, a container with 3.25 liters of water, and a container with 4.85 liters of water. Does she have enough water? Use estimation only to decide. Explain why you are confident in your estimate.

2. Nia wants the total weight of her luggage to be no more than 50 kilograms. She has three suitcases that weigh 15.8 kilograms, 17.42 kilograms, and 16.28 kilograms. Is the total weight within the limit? Use only estimation to decide. Explain how you know your estimate gives you the correct answer.

3. Omar measures one machine part with length 4.392 centimeters and another part with length 6.82 centimeters. What is the difference in length? Use estimation to check your answer for reasonableness.

3 Ryan and Sarah are looking at cell phone plans. They could share a group plan that costs $119.95 per month, or they could each pay for an individual plan that costs $62.77 per month.

a. Estimate which choice would cost less for Ryan and Sarah. Explain why.

Group plan; Possible answer: The individual plans cost more than $60 each, so two individual plans would be more than $120, which is greater than the cost of the group plan.

b. How much money could they save per month by paying for the choice that costs less instead of the choice that costs more? Show your work.

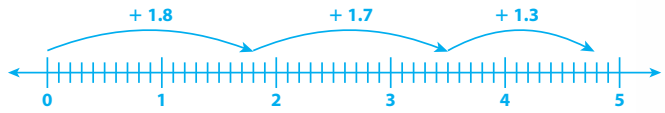
Possible student work:

$$\begin{array}{r} {}^{1}62.{}^{1}77 \\ +\,62.77 \\ \hline 125.54 \end{array} \qquad \begin{array}{r} {}^{1}{}^{14}{}^{1414}\cancel{125}.\cancel{54} \\ -\,119.95 \\ \hline 5.59 \end{array}$$

Ryan and Sarah can save **$5.59** by choosing a(n) **group** plan.

4 Chris wants to make at least 4.5 pounds, but no more than 5 pounds, of berry salad. He finds a carton of raspberries that weighs 1.83 pounds, a carton of blueberries that weighs 1.5 pounds, a carton of blackberries that weighs 1.72 pounds, and a carton of strawberries that weighs 1.29 pounds. If Chris wants to use three different types of berries, what is one combination of cartons he could buy? Explain. Show your work.

Possible student work:



Solution **Accept all combinations except raspberries, blackberries, and blueberries (5.05 pounds). Possible answer: Chris could buy raspberries, blackberries, and strawberries. By estimating each weight to the nearest tenth of a pound, I found a total weight of 1.8 + 1.7 + 1.3 = 4.8. So, the estimated weight of these three cartons is 4.8 pounds, which is between 4.5 pounds and 5 pounds.**

3 **a.** Group plan; Students may estimate the cost of each individual plan as $60 and add $60 + $60 for an estimate of $120.

b. $5.59; group; When finding the cost of two individual plans, students may also break apart decimals and combine whole numbers and decimals separately before combining results to find the sum. They may count on from the cost of the group plan to the cost of two individual plans to find the difference.

Medium

4 Any combinations of berries except raspberries, blackberries, and blueberries (1.83 + 1.72 + 1.5); See the student page for possible student work. Look for explanations that support the given combination.

Challenge

ELL English Language Learners: Differentiated Instruction

Prepare for Session 4
Use with *Apply It*.

Levels 1–3

Speaking/Listening Use with ***Apply It*** problem 1. Read the problem aloud to students and ask them to circle the amount of blue paint and the amount of yellow paint. Provide the following sentence frames and ask students to read after you:

- *Parker has* $3\frac{1}{2}$ *ounces of blue paint.*
- *Parker has* $1\frac{2}{5}$ *ounces of yellow paint.*

Ask students to circle the term *compute*. Remind students that *compute* means to calculate an answer. Have students work with a partner to solve the problem.

Levels 2–4

Reading/Writing Use with ***Apply It*** problem 1. Have students read the problem with a partner.

Ask students to make a two-column, three-row chart. Label the columns *Low Benchmark* and *High Benchmark*. Label the rows *Blue, Yellow,* and *Green*.

Have students work with a partner to complete the chart. Then provide the following sentence frames for them to write a solution to the problem:

- *The number of ounces of green paint will be between _____ and _____.*
- *My answer is reasonable because _____.*

Levels 3–5

Reading/Writing Use with ***Apply It*** problem 1. Have students read the problem independently. Then ask them to work with a partner to write out a strategy they will use to first estimate and then calculate how much green paint Parker makes. Have them solve the problem.

Ask them to read what they have written to another set of partners and discuss their strategies.

LESSON 14

SESSION 4 Refine

Purpose In this session students solve word problems involving finding both estimated and actual sums and differences and then discuss and confirm their answers with a partner.

Before students begin to work, use their responses to the *Check for Understanding* to determine those who will benefit from additional support.

As students complete the Example and problems 1–3, observe and monitor their reasoning to identify groupings for differentiated instruction.

Start

Check for Understanding

Materials For remediation: 1 set of fraction tiles

Why Confirm understanding of adding and subtracting with fractions in word problems.

How Give students the word problem involving adding $1\frac{3}{4}$ and $1\frac{1}{3}$. Have them estimate and solve using any strategy they want.

Arnetta reads her book for $1\frac{3}{4}$ hours. Then she goes to a soccer practice for $1\frac{1}{3}$ hours. How much time does she spend on these two activities? Estimate and solve. Tell if your answer is reasonable.

Possible Solution

$1\frac{3}{4} + 1\frac{1}{4} = 3$; Time spent: $3\frac{1}{12}$ hours; An estimate of 3 is close to the sum I found, $3\frac{1}{12}$, so my answer is reasonable.

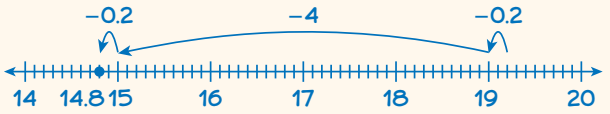
Refine Adding and Subtracting in Word Problems

Complete the Example below. Then solve problems 1–7.

EXAMPLE

Steven buys a movie ticket for $14.75 and then buys some snacks for the movie. He spends a total of $19.23. About how much did Steven spend on snacks?

Look at how you could show your work using a number line.



Solution about $4.40

The student rounded to the nearest tenth to estimate the answer.

PAIR/SHARE
How do you know if the amount spent on snacks will be more or less than $4?

APPLY IT

1 Parker mixes $3\frac{1}{2}$ ounces of blue paint with $1\frac{2}{5}$ ounces of yellow paint to make green paint for the leaves of a tree. How many ounces of green paint did Parker make?

Estimate and then compute. Explain how you know your result is reasonable. Show your work.

Student work for estimating will vary.

Actual sum: $3\frac{1}{2} + 1\frac{2}{5} = 3\frac{5}{10} + 1\frac{4}{10} = 4\frac{9}{10}$

Will there be a little more than $4\frac{1}{2}$ ounces or a little less than $4\frac{1}{2}$ ounces of green paint?

Solution **Estimates and explanations will vary, but the estimate should be about 5. The actual sum is $4\frac{9}{10}$ ounces.**

PAIR/SHARE
Was your estimate more than or less than the actual answer? By how much?

Error Alert

If the error is . . .	*Students may . . .*	*To support understanding . . .*
$2\frac{4}{7}$	have added numerators and added denominators.	Use fraction tiles to model $1\frac{3}{4}$ and $1\frac{1}{3}$. Point out that thirds and fourths do not combine to make sevenths; rather, it is necessary to find a smaller part that both thirds and fourths can be split into. Work with students to identify a common denominator. [12] Use fraction tiles to determine that $\frac{3}{12}$ make $\frac{1}{4}$ and $\frac{4}{12}$ make $\frac{1}{3}$. Model each fraction using twelfths and then add.
$\frac{5}{12}$	have subtracted instead of added.	Discuss how there are two activities done one after the other with no overlap in time spent, so you need to add, not subtract, to find the total time spent on the two activities. If needed, provide examples of similar whole number addition situations for comparison.
missing estimates in student work	not know how to use benchmark fractions to estimate.	Discuss how you can use a benchmark fraction such as $1\frac{1}{4}$ for $1\frac{1}{3}$ to add $1\frac{3}{4} + 1\frac{1}{4}$ for an estimated sum of 3. Discuss using the estimate to check for reasonableness.

EXAMPLE

About $4.40; rounding to the nearest tenth and using a number line as shown is one way to solve the problem. Students could also solve the problem by rounding to the nearest $0.25, and finding the difference between $14.75 and $19.25 by adding on amounts of $0.25, $4, and $0.25 for an estimate of $4.50.

Look for Your estimate will be different depending on whether you round to the nearest dime ($0.10), the nearest quarter ($0.25), or nearest dollar ($1.00).

APPLY IT

1 $4\frac{9}{10}$ ounces; Students may use 10 as the common denominator and add. Check that student estimates are about 5.

DOK 2

Look for There will be a little more than $4\frac{1}{2}$ ounces of green paint because $3\frac{1}{2} + 1$ is $4\frac{1}{2}$. $\frac{2}{5}$ more ounces of yellow paint make the amount a little more than $4\frac{1}{2}$ ounces.

2 About 2.0 kilograms; Students could round the decimals to the nearest 0.5. Students may also solve the problem by using $3 - 1$ to estimate a difference of 2 kilograms. Accept reasonable estimates.

DOK 2

Look for The exact difference may be more or less than the estimated difference depending on whether a higher or lower estimate was used for the mass of the helmet and the mass of the item compared to the helmet, the football.

3 **C**; Students could solve the problem by explaining that $3\frac{5}{9}$ is a little more than $3\frac{1}{2}$, so the difference of $5\frac{1}{2} - 3\frac{5}{9}$ is a little less than 2. Explain why the other two answer choices are not correct:

A is not correct because the answer is close to 2.

B is not correct because this answer, like **A**, is too low an estimate.

DOK 3

2 Jaime's football has a mass of 0.435 kilograms. His football helmet has a mass of 2.57 kilograms. Estimate how much more the mass of the helmet is than the mass of the football. Explain your estimate. Show your work.

Accept reasonable estimates. Possible student work:
Estimate each mass to the nearest 0.5 kilogram.
0.435 is about 0.5.
2.57 is about 2.5.
2.5 − 0.5 = 2.0

I could think about rounding to the nearest half or whole kilogram, instead of to the nearest tenth or hundredth.

PAIR/SHARE
How does the exact difference compare to your estimate?

Solution **Possible answer: about 2.0 kilograms**

3 Which is a reasonable estimate for the difference $5\frac{1}{2} - 3\frac{5}{9}$?

Ⓐ between $\frac{1}{2}$ and 1

Ⓑ between 1 and $1\frac{1}{2}$

Ⓒ between $1\frac{1}{2}$ and 2

Ⓓ between 2 and $2\frac{1}{2}$

How can you use benchmark fractions to estimate the difference?

Elise chose Ⓓ as the correct answer. How did she get that answer?

Possible answer: She estimated that $3\frac{5}{9}$ is close to $3\frac{1}{2}$. Then she subtracted $3\frac{1}{2}$ from $5\frac{1}{2}$ to get 2. Since $3\frac{5}{9}$ is a little more than $3\frac{1}{2}$, she estimated the difference as a little more than 2, but she should have estimated it as a little less than 2.

PAIR/SHARE
Does Elise's answer make sense?

4 **B**; $3\frac{3}{4} - 3\frac{3}{16} = 3\frac{12}{16} - 3\frac{3}{16}$, so the actual difference is $\frac{9}{16}$ inch. $\frac{9}{16}$ is $\frac{1}{16}$ from $\frac{1}{2}$; the other estimates are all farther from $\frac{9}{16}$ than $\frac{1}{2}$.

DOK 2

Error Alert Students who choose A may have chosen the least estimate instead of comparing each estimate to the actual difference. Have students show all four estimates and the actual difference on a number line.

5 About $52; Students may round each amount up to the next dollar and explain that doing so produces an estimate greater than the actual cost. Students may also estimate using $4.50 + $33.50 + $12.50 for an estimated sum of $50.50. They may explain that the actual cost will be more because they used low estimates for each price. Accept reasonable estimates and explanations.

DOK 2

4 William compares monthly rainfall amounts for the summer months using the table below.

Month	Monthly Rainfall
June	$3\frac{3}{16}$ inches
July	$3\frac{3}{4}$ inches
August	$3\frac{1}{2}$ inches

Which estimate is closest to the actual difference between the rainfall amounts for June and July?

Ⓐ $\frac{1}{4}$ inch

Ⓑ $\frac{1}{2}$ inch

Ⓒ 1 inch

Ⓓ $1\frac{1}{2}$ inches

5 Carter is at the school store. He wants to buy a pack of notebooks that costs $4.79, a calculator that costs $33.54, and a tablet case that costs $12.67. About how much money does Carter plan to spend at the school store? Will the actual cost be more or less than your estimate? Explain. Show your work.

Possible student work:
$4.79 is about $5; $33.54 is about $34; and $12.67 is about $13.
5 + 34 + 13 = 52

Solution **Possible answer: Carter plans to spend about $52. The actual cost will be less than my estimate because I used high estimates for each price.**

Differentiated Instruction

RETEACH

Hands-On Activity
Use fraction tiles to add mixed numbers.

Students struggling with solving word problems with mixed numbers,

Will benefit from additional work with concrete representations

Materials For each group: fraction circles (3 wholes, 4 one-fourths, 16 one-eighths)

- Organize students into groups and distribute fraction circles. Give students a problem: *After the 5th-grade pizza party, there are* $1\frac{3}{4}$ *cheese pizzas and* $2\frac{5}{8}$ *vegetable pizzas left over. How many pizza boxes does the clean-up committee need to store the leftover pizza?*
- Have students use fraction circles to model the leftover amounts of pizza. Discuss using a common denominator, eighths, to combine the groups. After regrouping, students should have 4 wholes and 3 eighths. Students should be able to state that the clean-up committee needs 5 boxes, one for each whole pizza and one for the $\frac{3}{8}$ of a pizza that remain.
- Repeat with a subtraction situation: *The clean-up committee eats the equivalent of* $1\frac{1}{2}$ *pizzas from the boxes of leftover pizza. Now how much pizza is left?*

EXTEND

Challenge Activity
Estimate with word problems.

Students who have achieved proficiency

Will benefit from deepening understanding of estimation

- In groups of 3, students write word problems involving addition or subtraction of mixed numbers with unlike denominators, or decimals.
- Using one problem, each student estimates the solution and draws a model to illustrate. Students work together to solve the problem and compare the answer to the estimates.
- Repeat with the other problems.

6 A certain liquid boils at 175.62°F. The liquid is currently at 68.8°F. Jimmy says that the temperature needs to rise by about 125°F to boil.

Part A Without finding the actual difference, explain why Jimmy's estimate is or is not reasonable.

Possible explanation: Jimmy's answer is not reasonable because 175.62 is close to 175 and 68.8 is close to 70. 175 − 70 = 105. Jimmy's answer should be close to 105°F.

Part B Find the actual amount the temperature must rise for the liquid to boil. Show your work.

Possible student work:

$$\begin{array}{r} 1\,\overset{6}{\cancel{7}}\,\overset{14}{\cancel{5}}.\overset{16}{\cancel{6}}2 \\ -\ \ 68.80 \\ \hline 106.82 \end{array}$$

Solution **The temperature needs to rise 106.82°F.**

7 MATH JOURNAL

Ramona has $4\frac{1}{5}$ gallons of red paint. She knows she needs at least $12\frac{3}{4}$ gallons to paint her whole house. Use benchmark fractions to explain about how much more paint Ramona needs.

Possible explanation: $\frac{1}{5}$ is close to the benchmark fraction $\frac{1}{4}$, so use $4\frac{1}{4}$ for $4\frac{1}{5}$. $12\frac{3}{4} - 4\frac{1}{4} = 8\frac{2}{4}$. She needs about $8\frac{2}{4}$, or $8\frac{1}{2}$, gallons of paint.

SELF CHECK Go back to the Unit 2 Opener and see what you can check off.

6 **Part A**

Jimmy's answer is not reasonable; See possible explanation on the Student Worktext page.

Part B

106.82°F; Subtract the current temperature from the boiling point temperature. See possible work on the Student Worktext page. Students may also count on to find the number of degrees the temperature must rise: 68.8 + 100 = 168.8; 168.8 + 1.2 = 170; 170 + 5.62 = 175.62. 100 + 1.2 + 5.62 = 106.82.

DOK 2

REINFORCE

Problems 4–7

Add and subtract with fractions and decimals in word problems.

All students will benefit from additional work with adding and subtracting with fractions and decimals in word problems by solving problems in a variety of formats.

- Have students work on their own or with a partner to solve the problems.
- Encourage students to show their work.

PERSONALIZE

i-Ready

Provide students with opportunities to work on their personalized instruction path with *i-Ready* Online Instruction to:

- fill prerequisite gaps
- build up grade-level skills

Close: Exit Ticket

7 MATH JOURNAL Student responses should indicate understanding of how to use benchmark fractions to estimate a difference involving subtracting mixed numbers.

Error Alert **If** students find a common denominator and subtract to find the actual amount of paint needed, **then** review that benchmark fractions are common fractions, such as halves or fourths, that can be used for estimating. Have students explain why $4\frac{1}{4}$ would be a good benchmark number to use for $4\frac{1}{5}$ in this problem, which calls for finding the difference between $12\frac{3}{4}$ gallons and $4\frac{1}{5}$ gallons.

SELF CHECK Have students consider whether they feel they are ready to check off any new skills on the Unit 2 Opener.