Lesson Overview

LESSON 2

Find Volume Using Unit Cubes

Lesson Objectives

Content Objectives

- Find the volume of a rectangular prism in various cubic units by filling it with unit cubes and counting them or by counting the number of unit cubes in one layer and multiplying by the number of layers.
- Find volume by counting improvised units.
- Recognize that the volume of a unit cube depends on the measurement unit used for its dimensions.
- Determine the third dimension of a rectangular prism given its volume and two dimensions.

Language Objectives

- Describe orally or in writing the connection between volume, layers, and unit cubes filling a rectangular prism.
- Summarize similarities and differences between volume, area, and length, and between the units used to measure them.

Prerequisite Skills

- Understand that volume is measured using unit cubes and that a unit cube has a volume of 1 cubic unit.
- Be familiar with customary and metric units of measurement.
- Recall addition and multiplication facts.

Standards for Mathematical Practice (SMP)

SMPs 1, 2, 3, 4, 5, and 6 are integrated in every lesson through the *Try-Discuss-Connect* routine.*

In addition, this lesson particularly emphasizes the following SMPs:

5 Use appropriate tools strategically.

6 Attend to precision.

7 Look for and make use of structure.

*See page 1i to see how every lesson includes these SMPs.

Lesson Vocabulary

There is no new vocabulary. Review the following key terms.

- **cubic unit** the volume of a unit cube.
- **face** a flat surface of a solid shape.
- **rectangular prism** a solid figure with six rectangular faces.
- **unit cube** a cube with side lengths of 1 unit. A unit cube is said to have one cubic unit of volume, and can be used to measure the volume of a solid figure.
- **volume** the amount of space inside a solid figure. Volume is measured in cubic units such as cubic inches.

Learning Progression

In the previous lesson students learned about volume as an attribute of three-dimensional figures. They understood volume as the amount of space inside a solid figure. Students used unit cubes to find the volume of rectangular prisms, using both an addition strategy and a multiplication strategy based on the number of unit cubes that fill one layer and the number of layers.

In this lesson students extend their understanding of volume in solid figures to include volumes expressed in cubic inches, cubic centimeters, and cubic feet as well as improvised units. Students use visual models of rectangular prisms, filled or partially filled with unit cubes, to determine volume. Students recognize that the product of the length and width of the prism represents the number of cubes in one layer and that the height of the prism represents the number of layers. Students also find the unknown measure of one dimension (length, width, or height) when given a rectangular prism's volume and the measures of its other two dimensions.

In the next lesson students will use volume formulas to find the volume of rectangular prisms and other three-dimensional figures composed of two or more rectangular prisms.

In Grade 6 students will find volumes of solid figures with fractional side lengths.

Lesson Pacing Guide

Whole Class Instruction

SESSION 1
Explore
45–60 min

Finding Volume Using Unit Cubes
- Start *5 min*
- Try It *10 min*
- Discuss It *10 min*
- Connect It *15 min*
- Close: Exit Ticket *5 min*

Additional Practice
Lesson pages 19–20

SESSION 2
Develop
45–60 min

Finding Volume Using Unit Cubes
- Start *5 min*
- Try It *10 min*
- Discuss It *10 min*
- Picture It & Model It *5 min*
- Connect It *10 min*
- Close: Exit Ticket *5 min*

Additional Practice
Lesson pages 25–26

Fluency
Finding Volume
Using Unit Cubes

SESSION 3
Refine
45–60 min

Finding Volume Using Unit Cubes
- Start *5 min*
- Example & Problem 1–3 *15 min*
- Practice & Small Group Differentiation *20 min*
- Close: Exit Ticket *5 min*

Lesson Quiz
or **Digital Comprehension Check**

Lesson Materials

Lesson *(Required)*	*Per student:* 12 centimeter cubes
Activities	*Per student:* 20 inch cubes, scissors, tape *Per group:* 60 unit cubes *Activity Sheets:* Box Template, 1-Centimeter Grid Paper
Math Toolkit	unit cubes, 1-centimeter grid paper, isometric dot paper, square sticky notes

*We continually update the Interactive Tutorials. Check the Teacher Toolbox for the most up-to-date offerings for this lesson.

Teacher Toolbox

Small Group Differentiation

PREPARE

Ready Prerequisite Lesson

Grade 4
- Lesson 16 Find Perimeter and Area

RETEACH

Tools for Instruction

Grade 4
- Lesson 16 Perimeter and Area

Grade 5
- Lesson 2 Measure Volume

REINFORCE

Math Center Activities

Grade 5
- Lesson 2 Same Volume, Different Shape
- Lesson 2 Find the Prism

EXTEND

Enrichment Activity

Grade 5
- Lesson 2 Packing Boxes

i-Ready

Independent Learning

PERSONALIZE

i-Ready Lessons*

Grade 5
- Understand and Measure Volume
- Practice: Measure Volume

Connect to Family, Community, and Language Development

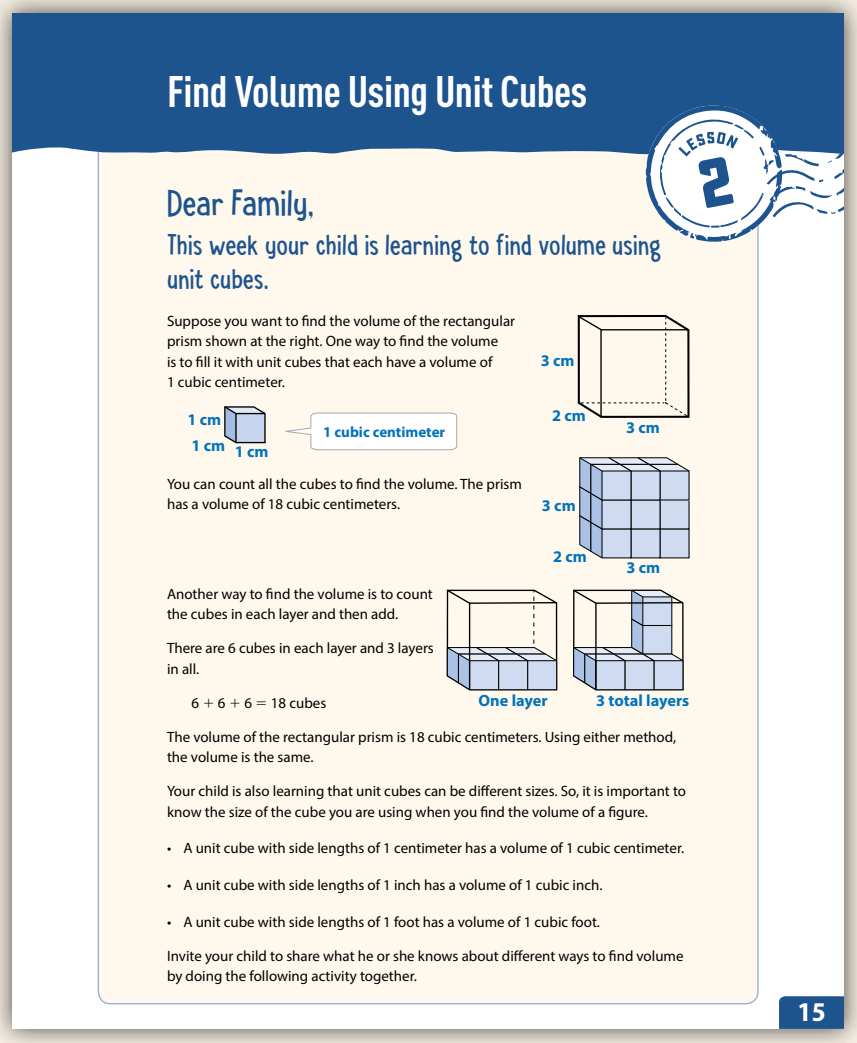
The following activities and instructional supports provide opportunities to foster school, family, and community involvement and partnerships.

Connect to Family

Use the **Family Letter**—which provides background information, math vocabulary, and an activity—to keep families apprised of what their child is learning and to encourage family involvement.

Available in Spanish

Teacher Toolbox



Find Volume Using Unit Cubes

LESSON 2

Dear Family,

This week your child is learning to find volume using unit cubes.

Suppose you want to find the volume of the rectangular prism shown at the right. One way to find the volume is to fill it with unit cubes that each have a volume of 1 cubic centimeter.

1 cm, 1 cm, 1 cm — 1 cubic centimeter

3 cm, 2 cm, 3 cm

You can count all the cubes to find the volume. The prism has a volume of 18 cubic centimeters.

3 cm, 2 cm, 3 cm

Another way to find the volume is to count the cubes in each layer and then add.

There are 6 cubes in each layer and 3 layers in all.

$6 + 6 + 6 = 18$ cubes

One layer — 3 total layers

The volume of the rectangular prism is 18 cubic centimeters. Using either method, the volume is the same.

Your child is also learning that unit cubes can be different sizes. So, it is important to know the size of the cube you are using when you find the volume of a figure.

- A unit cube with side lengths of 1 centimeter has a volume of 1 cubic centimeter.
- A unit cube with side lengths of 1 inch has a volume of 1 cubic inch.
- A unit cube with side lengths of 1 foot has a volume of 1 cubic foot.

Invite your child to share what he or she knows about different ways to find volume by doing the following activity together.

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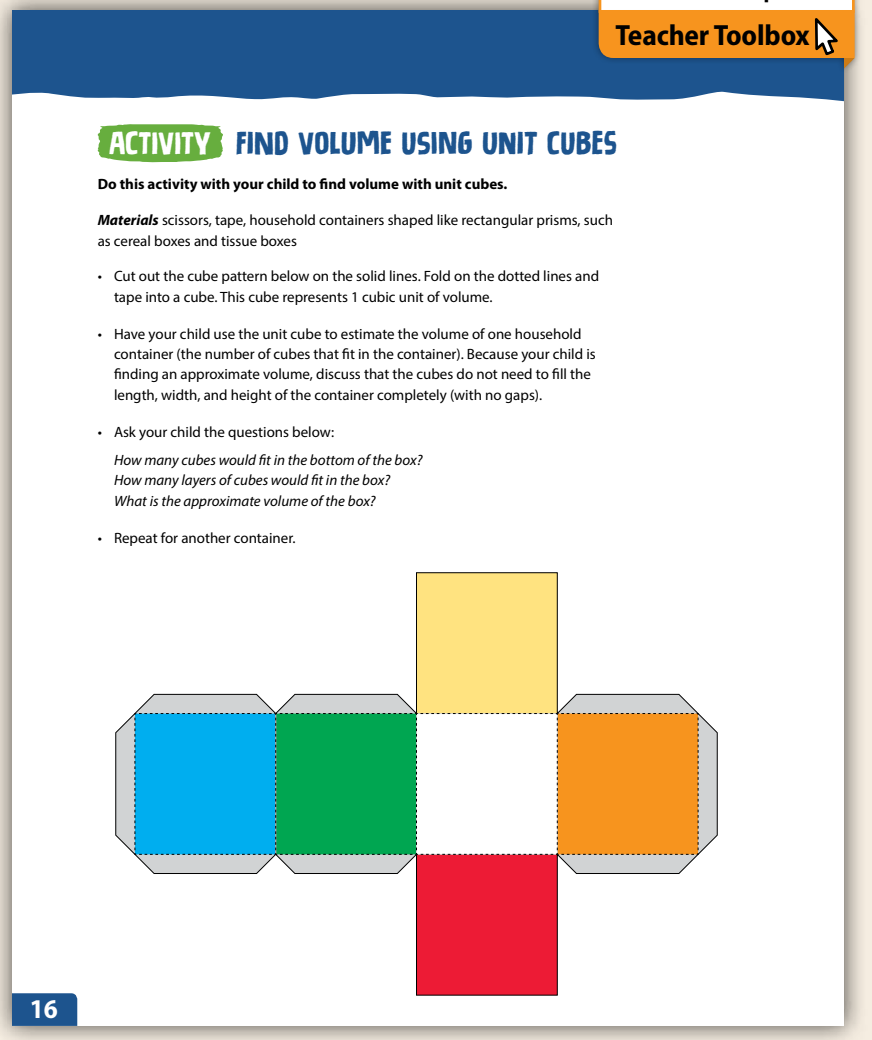

ACTIVITY FIND VOLUME USING UNIT CUBES

Do this activity with your child to find volume with unit cubes.

Materials scissors, tape, household containers shaped like rectangular prisms, such as cereal boxes and tissue boxes

- Cut out the cube pattern below on the solid lines. Fold on the dotted lines and tape into a cube. This cube represents 1 cubic unit of volume.
- Have your child use the unit cube to estimate the volume of one household container (the number of cubes that fit in the container). Because your child is finding an approximate volume, discuss that the cubes do not need to fill the length, width, and height of the container completely (with no gaps).
- Ask your child the questions below:

  How many cubes would fit in the bottom of the box?
  How many layers of cubes would fit in the box?
  What is the approximate volume of the box?

- Repeat for another container.

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Goal

The goal of the Family Letter is to further the exploration of the measurement of volume using unit cubes.

Activity

Calculating how much space is available to fill is a common task in daily life. The activity provides opportunities for students to explore the volume of household items that are shaped like rectangular prisms. Look at the *Find Volume Using Unit Cubes* activity and adjust if needed to connect with your students.

Math Talk at Home

Encourage students to talk with their families about items in their household that are shaped like rectangular prisms.

Conversation Starters Below are conversation starters students can write in their Family Letter or math journal to engage family members:

- *What role does volume have in how food is packaged and shipped in boxes? For example, how does volume affect how many cereal boxes can be put inside a larger box for shipping?*
- *What unit of measure do you believe may be used to measure the volume of the box needed to ship these items?*

Connect to Community and Cultural Responsiveness

Use these activities to connect with and leverage the diverse backgrounds and experiences of all students.

Session 1 **Use with *Try It*.**

- Tell students that many of the products that are sold in the United States are made in other countries. Point out that items are transported across the world in containers or very large boxes. If time permits, show images of cargo containers on boats. Before reading the ***Try It,*** have students connect finding volume to determining how many smaller boxes would fit in a container.

Session 2 **Use with *Model It*.**

- Students might be familiar with the use of storage units, perhaps through ones their families use, their friends' families use, or that they have seen on TV or in their country of origin. Point out that to make the most out of the space a storage unit holds, it's important to stack items in it efficiently, such as placing boxes on top of each other in layers. Ask students to describe the best way to store boxes in a storage unit. Encourage them to say how the volume of a storage unit and how many boxes can be stored in it are related.

Connect to Language Development

For ELLs, use the Differentiated Instruction chart to plan and prepare for specific activities in every session.

ELL English Language Learners: Differentiated Instruction

Prepare for Session 1
Use with *Connect It*.

Levels 1–3	Levels 2–4	Levels 3–5
Listening/Speaking Work with students to solve ***Connect It*** problem 2. Use a ruler or measuring tape to demonstrate the size of an inch, centimeter and foot. Have students use gestures to show the approximate volume of a unit cube for each unit of measurement. Point to the illustrations in the table as you present each unit of volume. Encourage students to notice the abbreviations for each unit. Say the plural forms of each unit of measurement as students repeat: *cubic inches, cubic centimeters, cubic feet*. Explain that the plural form is used when the number before the unit is greater than 1.	**Listening/Speaking** Have students work with a partner to read ***Connect It*** problem 2. Have them say aloud the name of each unit and, with fingers or hands, show the approximate size of each of the three cubic units. Display the singular forms of each unit (inch, centimeter, foot). Ask partners to name the plural forms of each unit of measurement. Call on volunteers to share. [inches, centimeters, feet] Ask: *Which units will you use to express the volume of Carl's box?* [cubic inches]	**Listening/Speaking** Have students read ***Connect It*** problem 2. Partner students to retell the information presented in the table using their own words. Remind students to use numbers, words and units to express volume with precision as they respond to the problem. Have students solve the problem individually, then compare answers.

LESSON 2

SESSION 1 Explore

Purpose In this session students draw on what they know about using unit cubes to measure volume. They share models to explore using unit cubes with a specific edge length to find volume. They will look ahead to think about expressing volume in a variety of cubic units.

Start

Connect to Prior Knowledge

Why Activate students' knowledge of different units used to measure area, in preparation for measuring volume in various specific units.

How Have students find and compare the areas of two rectangles with dimensions that involve the same numbers but different units.

Find each rectangle's area.

What is the same and what is different about the areas?

Solution
6 square inches,
6 square centimeters;
Listen for Same: both areas are 6 square units; Different: units are squares inches and square centimeters.

TRY IT

Make Sense of the Problem

To support students in making sense of the problem, have them identify that the length of each edge of the unit cube is 1 foot.

DISCUSS IT

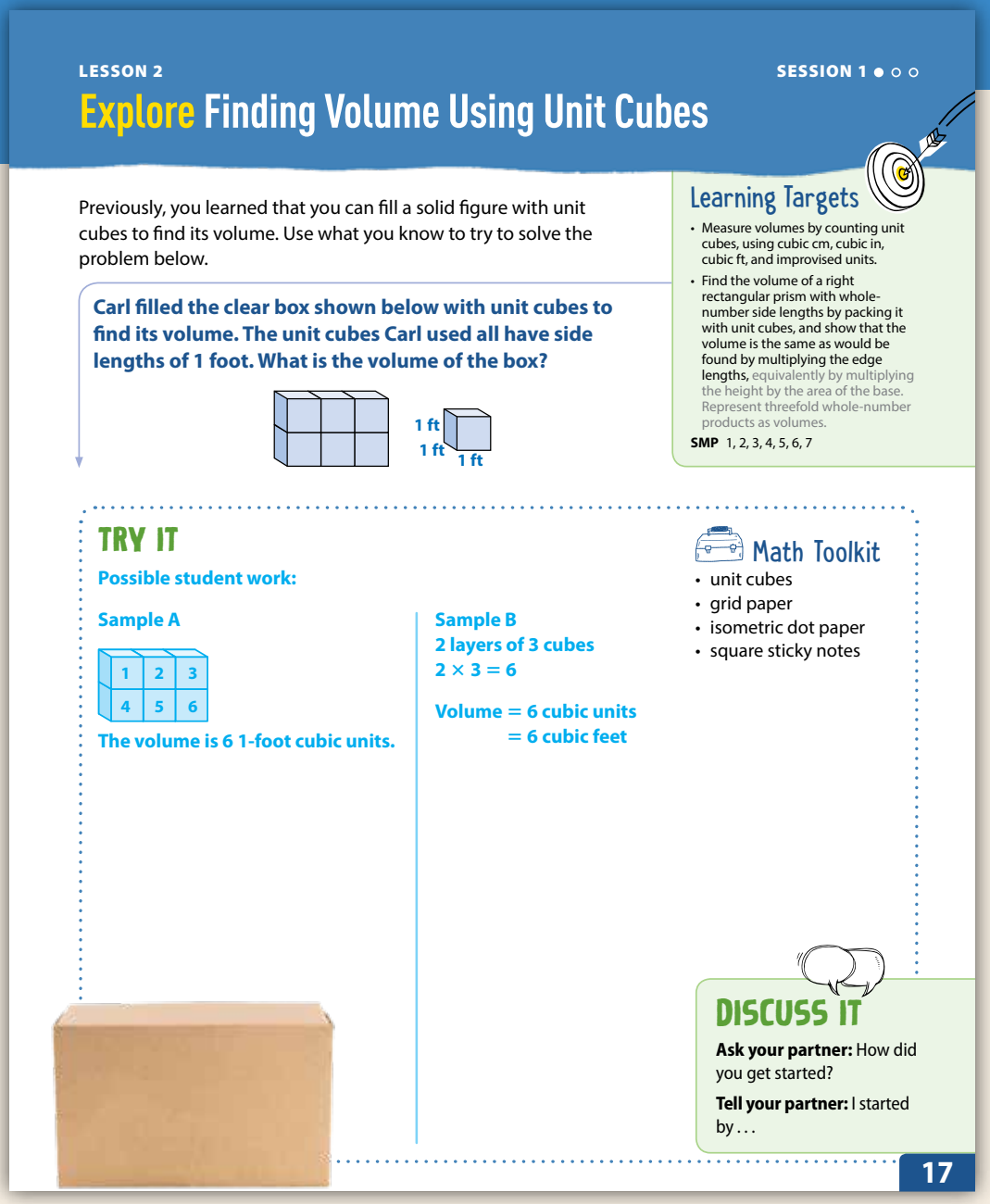
Support Partner Discussion

Encourage students to use the term *1 cubic foot* as they talk to each other.

Look for, and prompt as necessary for, understanding of:

- feet as the unit for the edge length of the unit cube
- *1 cubic foot* as the volume of the unit cube
- finding volume by counting each unit cube as 1 cubic foot

Common Misconception Look for students who are not comfortable distinguishing among units of length, area, and volume and who may use *feet* or *square feet* to describe the volume. As students present solutions, have them explain how they decided what to call the unit of volume.



LESSON 2 SESSION 1 ● ○ ○

Explore Finding Volume Using Unit Cubes

Previously, you learned that you can fill a solid figure with unit cubes to find its volume. Use what you know to try to solve the problem below.

Carl filled the clear box shown below with unit cubes to find its volume. The unit cubes Carl used all have side lengths of 1 foot. What is the volume of the box?

1 ft
1 ft
1 ft

Learning Targets

- Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.
- Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes.

SMP 1, 2, 3, 4, 5, 6, 7

TRY IT

Possible student work:

Sample A

1	2	3
4	5	6

The volume is 6 1-foot cubic units.

Sample B
2 layers of 3 cubes
2 × 3 = 6

Volume = 6 cubic units
= 6 cubic feet

Math Toolkit

- unit cubes
- grid paper
- isometric dot paper
- square sticky notes

DISCUSS IT

Ask your partner: How did you get started?

Tell your partner: I started by . . .

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Select and Sequence Student Solutions

One possible order for whole class discussion:

- evidence of counting, such as writing the numbers 1–6 on the diagram or building the figure with unit cubes
- counting cubes in one layer and using repeated addition to find volume
- counting cubes in one layer and then multiplying by 2 to find volume
- use of units of volume, such as *cubic units, 1-foot cubes*, or *cubic feet*

Support Whole Class Discussion

Prompt students to note the relationship between the numbers in each model and the unit cubes in the diagram in the problem.

Ask *How do [student name]'s and [student name]'s models show finding the total number of 1-foot cubes in the figure?*

Listen for Count each cube to find a volume of 6 1-foot cubes. Count the cubes in one layer (3) and use addition to find a volume of 6 1-foot cubes ($3 + 3 = 6$). Count the cubes in one layer (3), count the number of layers (2), and multiply to find a volume of 6 1-foot cubes ($2 \times 3 = 6$).

CONNECT IT

1 LOOK BACK

Look for understanding of cubic feet as the unit of volume and that the volume is the total number of 1-foot unit cubes that fill the box.

Hands-On Activity

Use unit cubes to find volume.

For all . . . *students to make sense of the concept of using cubic units to measure volume,*

Use . . . *this activity to have them find the volume of a box by packing it with unit cubes.*

Materials For each student: 20 inch cubes, scissors, tape, Activity Sheet *Box Template*

- Have students cut out and assemble the open box.
- Show students a single inch cube. Ask: *What does it mean that this is an inch cube?* [The length of each edge is 1 in.] Have students use their inch cubes to find the volume of the box. [18 cubic inches]
- Ask students to explain why the volume is 18 cubic inches. Listen for understanding that the space inside the box is filled without gaps or overlaps by 18 inch cubes and that the unit of measure is called *cubic inches*.

2 LOOK AHEAD

Point out that the unit of volume is based on the unit used for the length of each edge of the unit cube.

Ask *The unit cubes in the table are not shown to scale. If you had a 1-inch unit cube and a 1-foot unit cube on your desk, how would they compare?*

Listen for There are 12 inches in 1 foot, so a 1-foot unit cube is bigger than a 1-inch unit cube. The 1-foot unit cube would be 12 times taller, 12 times longer, and 12 times wider than the 1-inch unit cube.

Ask *Suppose you did not have any standard-size unit cubes available. What might you use as an improvised unit for measuring volume?*

Listen for You could use other objects all of the same size, such as rectangular erasers or food packages. If you used nonrectangular shapes that would leave gaps, such as marbles, you would get an approximate volume.

CONNECT IT

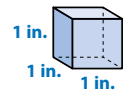
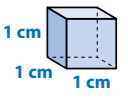
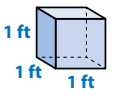
1 LOOK BACK

Describe the measurement unit Carl should use and explain how he can find the volume of the box.

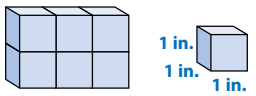
Cubes that are 1 foot long on each side; Possible explanation: Carl could count all the 1-foot cubes and record the total he counted. He could also multiply the number of 1-foot cubes in each layer by the number of layers in the box: 2 × 3 = 6, so the volume is 6 cubic feet.

2 LOOK AHEAD

Volume is measured in cubic units. Here are some cubic units you might use to measure volume.

Unit of Volume	cubic inch	cubic centimeter	cubic foot
Unit Cube			

Carl has another box as shown below. What is its volume? **6 cubic inches**



3 REFLECT

What is the same and what is different about the volume of Carl's two boxes?

The volume of each box is 6 cubic units, but one is measured in cubic feet and the other is measured in cubic inches.

Close: Exit Ticket

3 REFLECT

Look for understanding of different-sized units of volume. Student responses should include noting that although the same *number* (6) represents the volume of the two boxes, the boxes do not have the same *volume* because the units are different. Some students may note that the box of volume 6 cubic inches is much smaller than the box of 6 cubic feet.

Common Misconception **If** students give an answer such as 11 cubic inches or 22 cubic inches, indicating that they counted the number of visible faces (either in the picture or in total), rather than counting the number of cubes, **then** provide unit cubes and have students build a model of Carl's box to count the number of cubes. Discuss that each unit cube is one unit of volume.

Real-World Connection

Encourage students to think about everyday places or situations in which people might use volume. Have volunteers share their ideas.
Examples: determining how many bags of soil you need for a garden box, determining how many packages will fit in a delivery truck.

Solutions

Support Vocabulary Development

1 Ask students to look at the term *rectangular prism*.

As preparation for filling out the concept web, have students go back through the first two lessons and identify rectangular prisms in the diagrams and illustrations. Have volunteers direct the class to specific illustrations in the book and have the class agree or disagree that it is a rectangular prism. You can also do this with solid or flat geometrical figures displayed around the classroom.

2 Have students look at the picture in problem 2 and discuss with a partner why they can use a made-up unit like a box of pencils to measure volume. Provide the following sentence frame to guide students:

You can use boxes of pencils to measure volume because _____.

Supplemental Math Vocabulary

- *cube*
- *unit of volume*
- *cubic inch*
- *cubic centimeter*
- *cubic foot*

Name: _____ LESSON 2 SESSION 1

Prepare for Finding Volume Using Unit Cubes

1 Think about what you know about solid figures. Fill in each box.
Use words, numbers, and pictures. Show as many ideas as you can. **Possible answers:**

What Is It?

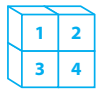
a solid figure with 6 rectangular sides

What I Know About It

I can fill a rectangular prism with unit cubes to find the volume.

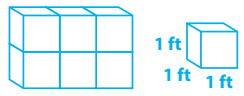
rectangular prism

Examples



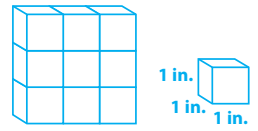
The volume is 4 cubic units.

Examples



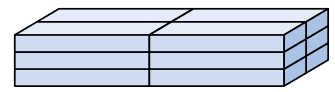
The volume is 6 cubic feet.

Examples



The volume is 9 cubic inches.

2 Instead of using cubic units to measure volume, Paulina wants to measure volume using boxes of pencils. What is the volume of this rectangular prism using boxes of pencils as the unit of measure?



12 boxes of pencils

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3 Assign problem 3 to provide another look at solving a problem by finding the volume of a rectangular prism filled with unit cubes.

This problem is very similar to the problem about Carl filling a box with unit cubes to find its volume. In both problems, students are shown a picture of a rectangular prism where one of the dimensions is 1 unit. The question asks for the volume of the box Jan filled with unit cubes.

Students may want to use unit cubes or grid paper.

Suggest that students read the problem three times, asking themselves one of the following questions each time:

- *What is this problem about?*
- *What is the question I am trying to answer?*
- *What information is important?*

Solution:

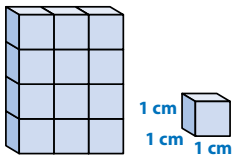
The box has 4 layers of 3 cubes. $4 \times 3 = 12$, so the volume of the box is 12 cubic centimeters.

Medium

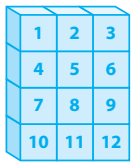
4 Have students solve the problem another way to check their answer.

3 Solve the problem. Show your work.

Jan filled the box shown below with unit cubes to find its volume. The unit cubes Jan used all have side lengths of 1 centimeter. What is the volume of the box?



Possible student work using a picture:



Solution **The volume of the box is 12 cubic centimeters.**

4 Check your answer. Show your work.

Possible student work:
4 layers of 3 cubes
$4 \times 3 = 12$

Volume = 12 cubic units
= 12 cubic centimeters

The volume of the box is 12 cubic centimeters.

ELL English Language Learners: Differentiated Instruction

Prepare for Session 2
Use with *Connect It*.

Levels 1–3

Listening/Speaking To work through ***Connect It*** problems 1–6 with students, use unit cubes or similar manipulatives to review the meaning of *layer*. Have students model the ***Model It*** illustrations with the manipulatives as they respond to the following questions: *How many cubes are in one layer? How many layers are there? How many cubes in all?*

For problem 6, encourage students to use gestures, numbers, symbols, pictures and words to explain their thinking. Model fluent speaking by restating students answers in complete sentences.

Levels 2–4

Speaking/Writing Ask students to form pairs and read ***Connect It*** problems 1–6. Make sure students realize that these problems refer to the rectangular prism on the previous page. Provide unit cubes or similar manipulatives so that partners can model the problem and discuss their answers before they write them down.

Provide sentence frames to help students organize their thoughts in writing for problem 6:

- *Find the number of unit cubes in one layer by multiplying _____ times _____.*
- *Then multiply by _____ to find the total volume.*

Levels 3–5

Reading/Speaking Ask students to form pairs and read through ***Connect It*** problems 1–6. Make sure they understand that these problems refer to the rectangular prism on the previous page. Write the terms *layer*, *volume*, *multiply*, and *cubic centimeter*, and ask students to use them as they answer the problems. After they finish, ask a student from each pair to share their answer to problem 6.

SESSION 2 Develop

Purpose In this session students solve a problem that requires finding the volume of a rectangular prism with a length of 4 centimeters, a width of 2 centimeters, and a height of 3 centimeters. Students model the prism either on paper or with manipulatives to develop a strategy for using the dimensions of a rectangular prism to find its volume.

Start

Connect to Prior Knowledge

Materials For each student: 12 centimeter cubes

Why Support students in connecting the dimensions of a rectangular prism to the number of unit cubes that fill the prism without gaps.

How Have students build prisms with a volume of 12 cubic centimeters and record the dimensions of their prisms.

Build a prism with a volume of 12 cubic centimeters. Record the prism's dimensions.

length = ...

width = ...

height = ...

Possible Solutions

12 cm, 1 cm, 1 cm
6 cm, 2 cm, 1 cm
4 cm, 3 cm, 1 cm
3 cm, 2 cm, 2 cm

Develop Language

Why Deepen understanding of the term *strategy*.

How Have students circle the term *strategy* in the *Discuss It*. Asks students to share where they have used this term before outside of math class. Explain to students that a strategy is a plan for taking action or to achieve a goal. Point out that in math a strategy is a plan for finding a solution to a problem.

TRY IT

Make Sense of the Problem

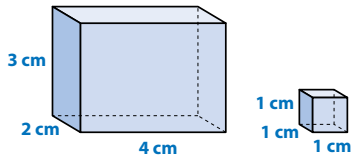
To support students in making sense of the problem, have them identify that the rectangular prism has a length of 4 centimeters, a width of 2 centimeters, and a height of 3 centimeters.

Ask *What does each label on the prism mean?*

Develop Finding Volume Using Unit Cubes

Read and try to solve the problem below.

Abigail uses cardboard to build a rectangular prism like the one shown below. What is the volume of the prism?

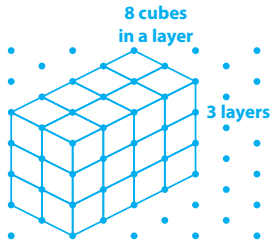


TRY IT

Possible student work:

Sample A
Fill the bottom of the box with 2 rows of 4 cubes.
That's 8 cubes.
There are 2 more layers of 8 cubes.
8 + 8 + 8 = 24
Volume = 24 cubic centimeters

Sample B



8 × 3 = 24
The volume is 24 cubic centimeters.

Math Toolkit
- unit cubes
- 1-cm grid paper
- isometric dot paper

DISCUSS IT

Ask your partner: Why did you choose that strategy?

Tell your partner: I knew . . . so I . . .

21

DISCUSS IT

Support Partner Discussion

Encourage students to use the term *cubic centimeters* as they discuss their solutions.

Support as needed with questions such as:

- *How did you begin to think about this problem?*
- *Where in the problem did you find the information you need?*

Common Misconception Look for students who use the length and width to find the number of cubes that would fill the bottom of the prism (one layer) but do not multiply by the number of layers.

Select and Sequence Student Solutions

One possible order for whole class discussion:

- models of the full prism built with unit cubes
- models or drawings showing one horizontal layer and one column of 3 cubes
- models or drawings that use vertical "layers"
- equations that show repeated addition
- equations that show multiplication

Support Whole Class Discussion

Compare and connect methods that use horizontal layers to methods that use vertical layers.

Ask *Suppose you stand the box up so that it is 4 centimeters tall. Does the volume of the box remain the same? Explain your reasoning.*

Listen for The volume remains the same because the number of unit cubes that fit in the box does not change.

PICTURE IT & MODEL IT

If no student presented these models, connect them to the student models by pointing out the ways they each show:

- the number of cubes in a layer
- the number of layers
- how the number of cubes in a layer and the number of layers are related to the dimensions shown on the prism

Ask *How are 1-centimeter cubes used to show volume in the picture of the prism completely filled with cubes? in the model of the prism partially filled with cubes?*

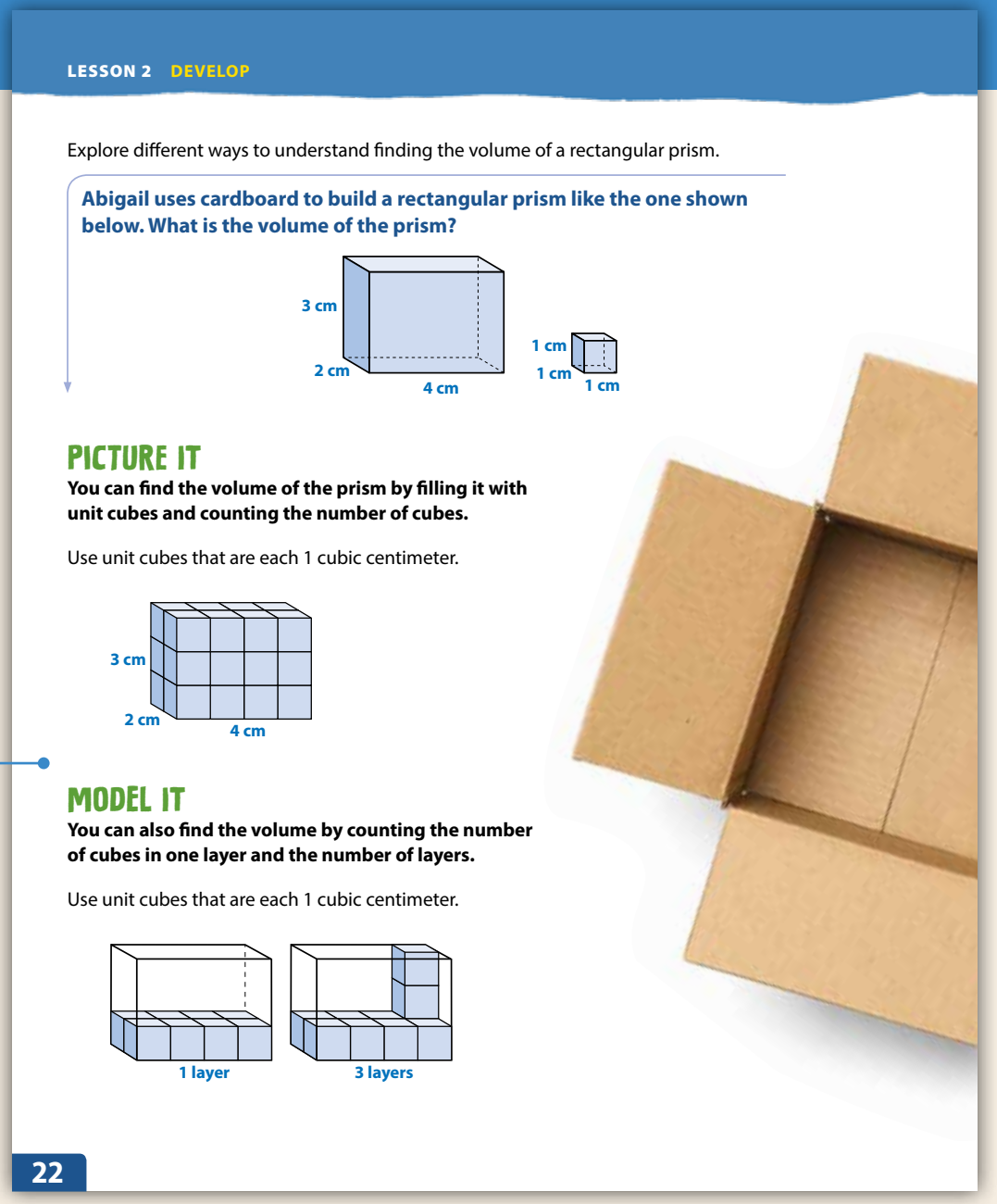
Listen for The number of cubes that completely fill the prism shows its volume. The number of cubes in one layer and the stacked cubes showing 3 layers can be used to find volume.

For the completely filled prism, prompt students to identify how the 1-centimeter cube is related to the dimensions labeled on the prism.

- *Why are 4 cubes shown as filling the length of the prism? 2 cubes as filling its width? 3 cubes as filling its height?*

For the partially filled prism, prompt students to identify how the 1-centimeter cube is related to one layer of cubes and to the number of layers.

- *Why is only one layer of cubes shown in the left prism? Would another layer show a different number of cubes?*
- *How does the right prism show 3 layers?*



Deepen Understanding

Units of Volume

SMP 6 Attend to precision.

When discussing the unit cube model of the prism, introduce students to exponent notation, such as cm^3 or $in.^3$, used for cubic units.

Ask *The volume of each unit cube in the model is 1 cubic centimeter. You may see this volume written as* 1 cm^3. *What do you think the small, raised 3 means?*

Listen for The unit of volume has three dimensions, each of which is measured in centimeters.

Tell students that the small, raised 3 is called an *exponent*. They will learn more about exponents in a later lesson. Ask: *How do you think you would use an exponent to write a volume of 35 cubic centimeters? to write a volume of 20 cubic feet?* Invite volunteers to write the abbreviations on the board. [35 cm^3; 20 ft^3]

Ask *What advantages do you see to exponent notation?*

Listen for It gives you a compact way to write volume measurements, using the same abbreviations you use for length measurements.

CONNECT IT

- Remind students that one thing that is alike about all the representations is the numbers.
- Explain that on this page they will use those numbers to write multiplication expressions that help them find volume.

Monitor and Confirm

1 – 5 Check for understanding that:

- there are 8 unit cubes in one layer
- there are 3 layers
- the volume is 24 cubic centimeters

Support Whole Class Discussion

2 – 4 Tell students that these problems will prepare them to provide the explanation required in problem 6.

Be sure students understand that these problems are asking them to think about how to use multiplication with the dimensions of a rectangular prism to find the volume of the prism.

Ask *Which dimensions of the prism help you find the number of cubes in one layer? Explain.*

Listen for The length tells you the number of cubes that fit in one row of the bottom layer, and the width tells you the number of rows. You multiply length by width to find the number of cubes in one layer.

Ask *How is the number of layers related to one of the dimensions of the prism?*

Listen for The number of layers is the same as the height of the prism, in centimeters.

6 Look for the idea that to find the volume of a rectangular prism, you multiply the length and width to find the number of cubes in one layer and then multiply by the number of layers, or the height, to find the total number of cubes that fill the prism.

7 REFLECT Have all students focus on the strategies used to solve this problem. If time allows, have students share their preferences with a partner.

CONNECT IT

Now you will use the problem from the previous page to help you understand how to find the volume of a rectangular prism.

1 Look at the model in **Picture It** on the previous page. Count the number of cubes in one layer. There are 8 cubes in one layer.

2 How could you find the number of cubes in one layer without counting the cubes?
Possible answer: You can multiply the length by the width. 4 × 2 = 8

3 Once you know how many cubes are in one layer, what else do you need to know to find the volume? **how many layers there are**

4 There are 8 cubes in each layer and there are 3 layers. What multiplication expression can you write to find the volume of the prism? **8 × 3**

5 What is the volume of Abigail's rectangular prism? **24 cubic centimeters**

6 Explain how you can use multiplication to find the volume of a rectangular prism.
Possible answer: First, you can multiply the length by the width to find how many unit cubes are in 1 layer. Then you can multiply by the number of layers to find the total volume.

7 REFLECT

Look back at your **Try It**, strategies by classmates, and **Picture It** and **Model It**. Which models or strategies do you like best for finding the volume of a rectangular prism? Explain.

Some students may prefer building a model and counting cubes because they like to work with a three-dimensional figure. Other students may prefer to draw a diagram to recognize the number of cubes per layer and the number of layers, so they can use repeated addition or multiplication to calculate the volume.

Visual Model

Connect the area of a rectangle to the volume of a prism.

If . . . *students are unsure how the length and width of the prism relate to the rows of cubes that fill one layer,*

Then . . . *use this activity to connect the area of the bottom of the prism to the number of cubes that cover the bottom.*

Materials For each student: Activity Sheet *1-Centimeter Grid Paper*

- Have students draw and label a 4 cm-by-2 cm rectangle.
- Ask students how to use the length and width to find the area of the rectangle. [length × width = 4 × 2 = 8; area = 8 square centimeters]
- Tell students to think of the rectangle as the bottom of a prism with a length of 4 cm and a width of 2 cm. Ask students how the number of cubes that fill one layer of the prism relates to the number squares inside the rectangle. [They are the same; the number of cubes and the number of squares are both 8.]
- Ask students how to use the number of cubes that fit in one layer to find the volume of a prism with a length of 4 cm and a width of 2 cm. [Multiply 8 by the number of layers; the volume will be in cubic centimeters.]

APPLY IT

For all problems, encourage students to write equations that support their thinking.

8 27 cubic inches; Students may count the cubes in one layer, count the number of layers, and then use repeated addition or multiplication to find the volume. Students use the dimensions of the unit cube shown to identify the unit of volume as cubic inches. Some students may write the solution as 27 $in.^3$.

9 24 cubic feet; Students may count the cubes in one layer, count the number of layers, and then use repeated addition or multiplication to find the volume. Students use the size of the unit cube given in the problem to identify the unit of volume as cubic feet. Some students may write the solution as 24 ft^3.

Close: Exit Ticket

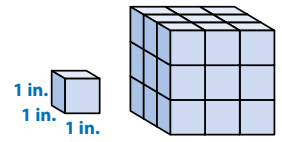
10 **C**; (number of cubes in one layer) × (number of layers) = (2 × 3) × 6 = 6 × 6 = 36, so the volume is 36 cubic centimeters.

Error Alert **If** students choose A, B, or D, **then** have them build a model of the box using unit cubes and review that you multiply the length and width to find the number of cubes in one layer and then multiply by the number of layers, or the height, to find the volume. Also review that the unit for volume is based on the unit for the edge length of the unit cube.

APPLY IT

Use what you just learned to solve these problems.

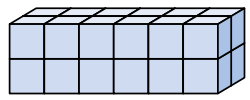
8 What is the volume of the rectangular prism at the right? Show your work.



Possible student work:
3 × 3 = 9
9 cubes in 1 layer
9 + 9 + 9 = 27

Solution **27 cubic inches**

9 Mr. Wong finds the volume of a box by filling it with 1-foot unit cubes, as shown below. What is the volume of Mr. Wong's box? Show your work.

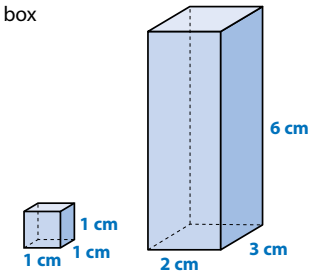


Possible student work:
6 × 2 = 12
12 cubes in 1 layer
12 × 2 = 24

Solution **24 cubic feet**

10 Jamila wants to find the volume of the rectangular box at the right. What is the volume of the box?

Ⓐ 18 cubic units
Ⓑ 30 cubic centimeters
Ⓒ 36 cubic centimeters
Ⓓ 36 cubic inches



Solutions

1. 2 layers with 6 cubes in each layer;
6 cubes + 6 cubes = 12 cubes;
12 cubic centimeters
Basic

2. 2 layers with 10 cubes in each layer;
2 layers × 10 cubes = 20 cubes;
20 cubic feet
Basic

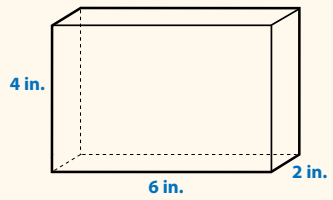
Name: _____ LESSON 2 SESSION 2

Practice Finding Volume Using Unit Cubes

Study the Example showing how to use layers to find the volume of a rectangular prism. Then solve problems 1–7.

EXAMPLE

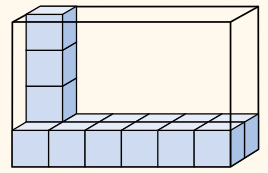
Keith uses this box to store his colored markers.
What is the volume of the box?



Think about filling the box with 1-inch cubes.
One layer has 2 rows of 6 cubes, or 12 cubes.
There are 4 layers of cubes.

$12 + 12 + 12 + 12 = 48$, or $12 \times 4 = 48$

The volume of the box is 48 cubic inches.



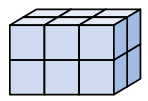
1. Prism *G* is filled with unit cubes that have side length 1 centimeter.

There are 2 layers with 6 cubes in each layer.

6 cubes + 6 cubes = 12 cubes

The volume is 12 cubic centimeters.

G



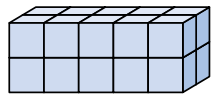
2. Prism *H* is filled with unit cubes that have side length 1 foot.

There are 2 layers with 10 cubes in each layer.

2 × 10 cubes = 20 cubes

The volume is 20 cubic feet.

H



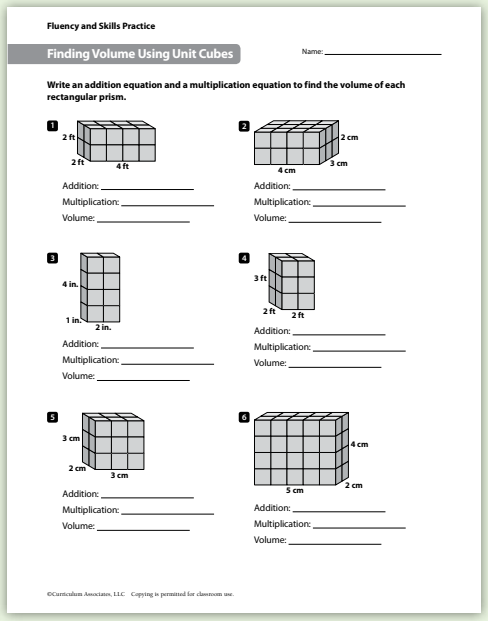
25

Fluency & Skills Practice

Teacher Toolbox

Assign *Finding Volume Using Unit Cubes*

In this activity students write addition and multiplication equations to find the volume of rectangular prisms using the dimensions of length, width, and height. Students may encounter real-world situations that involve developing equations to find the volume of rectangular prisms, such as determining the volume of garden soil needed to fill a planter or the number of boxes that will fit in a display case.



Fluency and Skills Practice

Finding Volume Using Unit Cubes

Name: _____

Write an addition equation and a multiplication equation to find the volume of each rectangular prism.

1. 2 ft, 2 ft, 4 ft
Addition: _____
Multiplication: _____
Volume: _____

2. 4 cm, 3 cm, 2 cm
Addition: _____
Multiplication: _____
Volume: _____

3. 4 in., 1 in., 2 in.
Addition: _____
Multiplication: _____
Volume: _____

4. 3 ft, 2 ft, 2 ft
Addition: _____
Multiplication: _____
Volume: _____

5. 3 cm, 2 cm, 3 cm
Addition: _____
Multiplication: _____
Volume: _____

6. 4 cm, 5 cm, 2 cm
Addition: _____
Multiplication: _____
Volume: _____

3 16 cubic feet; Students may use the given dimensions to visualize filling the prism with 2 layers of 8 cubes. Some students may write the solution as 16 ft^3.
Medium

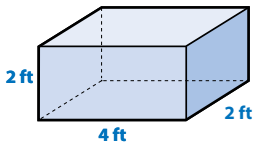
4 6 juice boxes; Students may multiply the number of juice boxes in one layer, 2, by the number of layers of juice boxes that would fit in the box, 3.
Medium

5 Answers will vary. See possible answer on student page.
Medium

6 Box *E*; Possible explanation: volume of Box *D* = 15 cubic units, volume of Box *E* = 16 cubic units
Medium

7 Answers will vary. See possible answer on student page.
Challenge

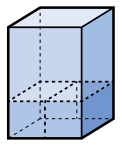
3 What is the volume of the rectangular prism at the right? Show your work.



Possible student work:
Fill the prism with 1-foot unit cubes.
2 layers × 8 cubes = 16 cubes

Solution **16 cubic feet**

4 Jenn noticed that she can fit two juice boxes side by side on the bottom of this box. She can make two more layers like the one shown to fill the box. Using a juice box as a measure of volume, what is the volume of the larger box?



Possible student work:
Fill the prism with same-sized juice boxes.
3 layers × 2 juice boxes = 6 juice boxes

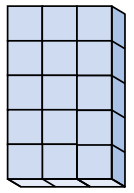
Solution **6 juice boxes**

5 A box is 2 inches long, 1 inch wide, and 6 inches tall. What is the relationship between the volume of this box and the one in problem 4? Tell how you know.

Possible answer: This box has the same length and width as the box in problem 4, but the height is doubled. This box is made of 2 of the boxes in problem 4, so its volume is twice the volume of that box.

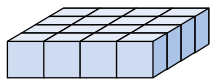
6 Box *D* and Box *E* are made from unit cubes of the same size. Which has a greater volume, Box *D* or Box *E*? Explain.

Box *E* has a greater volume. Possible explanation: Box *D* has 5 layers of 3 cubes each for a volume of 15 cubic units. Box *E* has 1 layer of 16 cubes for a volume of 16 cubic units.



7 Add a layer to Box *D* and compare the volume of the new Box *D* to the volume of Box *E*.

Possible answer: With 1 more layer, Box *D* would have a volume of 18 cubic units, which is 2 cubic units greater than the volume of Box *E*.



ELL English Language Learners: Differentiated Instruction

Prepare for Session 3
Use with *Apply It*.

Levels 1–3

Speaking/Writing Read ***Apply It*** problem 1 to students as you point to each dimension in the diagram. Use gestures to review the meanings of long, wide, and high. Have students label the box with the terms to support understanding of the problem. Encourage students to share any multiplication equations they used to solve the problem. Prompt students with frames such as:

- $2 \times 6 =$ _____
- _____ $\times 4 =$ _____

If students forget to write the units as part of the final answer, then add the units yourself: *cubic centimeters*.

Levels 2–4

Speaking/Writing Have students read ***Apply It*** problem 1 with a partner. Ask them to underline important information and label the illustration with the appropriate dimensions. If needed, have students use sentence frames from the previous session to remember how to use multiplication to solve the problem.

- *Find the number of unit cubes in one layer by multiplying _____ times _____.*
- *Then multiply by _____ to find the total volume.*

Levels 3–5

Speaking/Writing Have students form pairs and discuss the process they will follow to solve ***Apply It*** problem 1. Provide the following terms: *layer*, *length*, *height*, *width*, *multiply*, and *solution*.

Ask students to write down the steps they used to find the volume of the box using complete sentences. Have pairs compare their responses with another group and discuss how their responses are similar or different. Monitor their discussions and ensure students identify the correct steps.

LESSON 2

SESSION 3 Refine

Purpose In this session students solve word problems involving the use of unit cubes to find volume and then discuss and confirm their answers with a partner.

Before students begin to work, use their responses to the *Check for Understanding* to determine those who will benefit from additional support.

As students complete the Example and problems 1–3, observe and monitor their reasoning to identify groupings for differentiated instruction.

Start

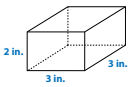
Check for Understanding

Materials For remediation: 20 unit cubes

Why Confirm understanding of finding volume.

How Have students find the volume of the prism using any strategy they want.

Find the prism's volume.



Describe layers of unit cubes that would fill the prism.

Solution

18 cubic inches;

Possible descriptions:

2 layers of 1-inch cubes, with each layer having 3 rows of 3 cubes; 3 layers of 6 1-inch cubes

LESSON 2 SESSION 3 ● ● ●

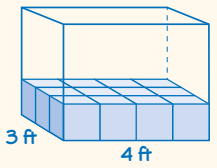
Refine Finding Volume Using Unit Cubes

Complete the Example below. Then solve problems 1–9.

EXAMPLE

Pedro has a storage box with a volume of 36 cubic feet. He knows that the box is 4 feet long and 3 feet wide. How high is the box?

Look at how you could show your work using a drawing and multiplication facts.



$4 \times 3 = 12$

$12 \times \square = 36$

$12 \times 3 = 36$

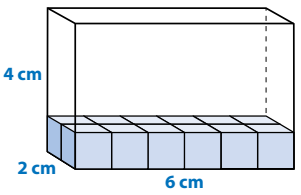
Solution 3 feet

The student started by finding the number of cubes in the bottom layer.

PAIR/SHARE
Could you solve this problem another way?

APPLY IT

1 A box measures 6 centimeters long, 2 centimeters wide, and 4 centimeters high. What is the volume of the box? Show your work.



Possible student work:
$6 \times 2 = 12$
$12 \times 4 = 48$

Solution 48 cubic centimeters

How many layers of cubes will there be in the box?

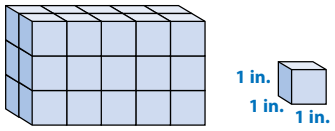
PAIR/SHARE
Can you use multiplication to solve this problem?

27

Error Alert

If the error is . . .	*Students may . . .*	*To support understanding . . .*
9 cubic inches	have found the volume of cubes that would fill one layer.	Have students use cubes to build a model of the prism. Have them count the number of cubes aloud as they build the prism. Then have them compare their answer to the number of cubes counted. Ask students to identify the error.
8 cubic inches	have added the dimensions.	Have students use cubes to build a model of the prism. Have them count the number of cubes aloud as they build the prism. Then have them compare their answer to the number of cubes counted. Ask students to identify the error.
21 cubic inches	have drawn in lines to show unit cubes and counted the number of visible faces of cubes.	Have students use cubes to build a model of the prism. Have them count the number of cubes aloud as they build the prism. Then have them compare their answer to the number of cubes counted. Ask students to identify their error.

2 Kamala made the figure below using cubes. What is the volume of Kamala's figure? Show your work.



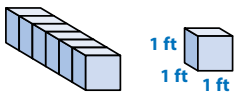
How many cubes are there in each layer?

Possible student work:
Each layer has 2 cubes × 5 = 10 cubes.
10 cubes × 3 = 30 cubes

Solution **30 cubic inches**

PAIR/SHARE
How did you decide which method to use to solve the problem?

3 What is the volume of the rectangular prism below?



There is more than one way to find the volume of a rectangular prism.

Ⓐ 6 square feet
Ⓑ 6 cubic feet
Ⓒ 8 square feet
Ⓓ 8 cubic feet

Nam chose Ⓓ as the correct answer. How did he get that answer?

Possible answer: Nam added the dimensions (6 + 1 + 1 = 8) together instead of multiplying.

PAIR/SHARE
Does Nam's answer make sense?

EXAMPLE

3 feet; A drawing and using multiplication facts is shown as one way to solve the problem. Students could also solve the problem by finding how many layers of 12 they need to get a sum or product of 36.

Look for You multiply length by width to find the number of cubes in one layer.

APPLY IT

1 48 cubic centimeters; Students could solve the problem by multiplying the length and width to find the number of cubes in one layer and then multiplying by the number of layers (the height). Some students may write the solution as 48 cm^3.
DOK 2

Look for There are 4 layers. Each cube has an edge length of 1 centimeter, so 4 cubes will fill the height of the prism.

2 30 cubic inches; Students could solve the problem by counting the cubes to find the dimensions of the prism, multiplying the length by the width to find the number of cubes in one layer, and then multiplying the product by the number of layers (the height). Some students may write the solution as 30 $in.^3$.
DOK 1

Look for You can count cubes shown in the figure to find the length, width, and height of the prism.

3 **B**; Students could solve the problem by counting 6 cubes. One layer of 6 1-foot cubes has a volume of 6 cubic feet.

Explain why the other two answer choices are not correct:

A is not correct because volume is measured in cubic units, not square units.

C is not correct because you need to multiply, not add, the dimensions, and volume is measured in cubic units, not square units.
DOK 3

4 **B**; Count the 3 cubes at the front and the 2 cubes along the side and then multiply to find the number of cubes in the bottom layer. Or count the number of cubes in the top layer.
DOK 2

5 **A**; Count the cubes that fill the length and width of one layer and then multiply 5 by 6 to find 30 cubes in one layer. Then multiply by 4, the number of layers.

C; Each layer is 30 cubic feet. Add the cubic feet of the four layers.
DOK 2

Error Alert Students who choose B, D, or E may have added some or all of the dimensions instead of multiplying.

6 The product of the length and width must be equal to 24 ÷ 2, or 12.

B (No);

C (Yes);

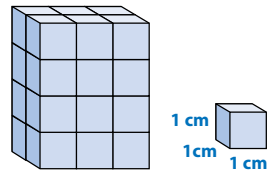
F (No);

G (Yes);

I (Yes)
DOK 2

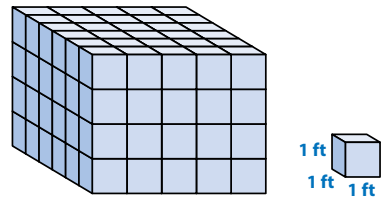
4 How many 1-centimeter unit cubes are in the bottom layer of the rectangular prism at the right?

Ⓐ 3
Ⓑ 6
Ⓒ 12
Ⓓ 24



5 Which expressions can be used to find the volume, in cubic feet, of the rectangular prism at the right?

Ⓐ 30×4
Ⓑ $(5 + 6) \times 4$
Ⓒ $30 + 30 + 30 + 30$
Ⓓ $5 + 6 + 4$
Ⓔ $30 + 4$



6 Flora has a rectangular gift box that has a volume of 24 cubic inches. The box is 2 inches tall. Determine if the gift box could have the given length and width.

	Yes	No
Length: 11 inches; Width: 11 inches	Ⓐ	Ⓑ
Length: 4 inches; Width: 3 inches	Ⓒ	Ⓓ
Length: 10 inches; Width: 2 inches	Ⓔ	Ⓕ
Length: 6 inches; Width: 2 inches	Ⓖ	Ⓗ
Length: 12 inches; Width: 1 inch	Ⓘ	Ⓙ

Differentiated Instruction

RETEACH

Hands-On Activity

Use unit cubes to relate the dimensions of a rectangular prism and its volume.

Students struggling with relating the dimensions of a prism to the number of cubes in one layer and the number of layers

Will benefit from additional work with connecting length, width, and height to counting unit cubes

Materials For each group: 60 unit cubes

- Have one student build one layer of a rectangular prism. Have the group use the layer to find the length and width of the prism.
- Have the builder add layers to build the prism as high as possible. Have the group decide what the height of the prism is. Then have the students find the volume of the prism.
- Repeat the activity with a different student as builder.

EXTEND

Challenge Activity

Find the volume of a box given dimensions in different units of measure.

Students who have achieved proficiency

Will benefit from deepening understanding of different-sized units of volume

- Challenge students to find the volume, in cubic feet, of a box that has a length of 6 feet, a width of 3 feet, and a height of 24 inches. Remind students to include the unit of measure in their answer. [36 cubic feet]
- Challenge students to find the volume, in cubic inches, of a box that has a length of 2 feet, a width of 1 foot, and a height of 8 inches. Remind students to include the unit of measure in their answer. [2,304 cubic inches]

7 Figure *B*; Students may count cubes to find the dimensions of each prism and then multiply the prism's length, width, and height to find its volume; see possible student work on the Student Worktext page.

DOK 2

8 **Part A**

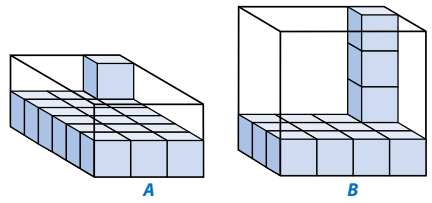
Drawings will vary; dimensions, in centimeters, should be any three numbers whose product is 40; possible dimensions: 20 cm × 2 cm × 1 cm.

Part B

See possible explanation on the Student Worktext page. Students may also explain that if you get the same product when you multiply the length, width, and height of each prism, then the prisms have the same volume.

DOK 3

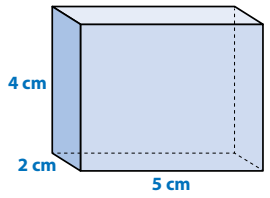
7 Both figures are filled with unit cubes of the same size. Which rectangular prism has the greater volume, Figure *A* or Figure *B*? Show your work.



Possible student work:
Volume of *A* = (3 × 6) × 2 = 18 × 2 = 36
Volume of *B* = (4 × 3) × 4 = 12 × 4 = 48

Figure *B* has the greater volume.

8 **Part A** Mato drew the rectangular prism shown below. Draw and label a different rectangular prism with the same volume as Mato's prism.



Drawings will vary, but the dimensions, in centimeters, should be any set of 3 numbers whose product is 40.

Part B Explain how you know that the volume of your prism is the same as the volume of Mato's prism.

Possible answer: The volume of Mato's prism is 40 cubic centimeters. He used 10 1-centimeter cubes for each layer and there are 4 layers. The volume of my prism is also 40 cubic centimeters.

9 MATH JOURNAL

Jorge uses 1-centimeter cubes to make a rectangular prism. Each layer of his prism is 5 cubes long and 2 cubes wide. His prism has 6 layers. Explain two ways to find the volume of Jorge's prism.

Possible answer: You can find the volume by finding the number of centimeter cubes in 1 layer (2 × 5 = 10), and then adding that number six times, once for each layer: 10 + 10 + 10 + 10 + 10 + 10 = 60. You can also multiply the number of cubes in one layer by the number of layers: 6 × 10 = 60; the volume is 60 cubic centimeters.

SELF CHECK Go back to the Unit 1 Opener and see what you can check off.

REINFORCE

Problems 4–9

Find volume using unit cubes.

All students will benefit from additional work with finding volume using unit cubes by solving problems in a variety of formats.

- Have students work on their own or with a partner to solve the problems.
- Encourage students to show their work.

PERSONALIZE

i-Ready

Provide students with opportunities to work on their personalized instruction path with *i-Ready* Online Instruction to:

- fill prerequisite gaps
- build up grade-level skills

Close: Exit Ticket

9 MATH JOURNAL

60 cubic centimeters; Student explanations should clearly describe methods, such as using repeated addition and multiplying the number of cubes per layer by the number of layers. Students should recognize that the unit of volume is based on the unit of length given for the side of the unit cube.

Error Alert **If** students do not correctly name the unit of volume as cubic centimeters, **then** show students a centimeter cube and ask them what attribute of the cube is described by *1 centimeter* [length of each edge], by *1 square centimeter* [area of each face], and by *1 cubic centimeter* [volume of the cube].

SELF CHECK Have students consider whether they feel they are ready to check off any new skills on the Unit 1 Opener.

Lesson Overview

LESSON 3

Find Volume Using Formulas

Lesson Objectives

Content Objectives

- Find the volume of a rectangular prism by multiplying its height by the area of its base.
- Solve real-world and mathematical problems involving volumes of rectangular prisms by applying the formulas $V = \ell \times w \times h$ and $V = b \times h$.
- Use addition to find volumes of solid figures composed of two non-overlapping rectangular prisms.

Language Objectives

- Explain the relationships between the formulas $V = \ell \times w \times h$ and $V = b \times h$ and describe how to use the formulas.
- List information related to volume given in diagrams of rectangular prisms.
- Write appropriate formulas to solve a word problem about volume of rectangular prisms.
- Draw lines on diagrams to divide solid figures into two non-overlapping rectangular prisms.
- Listen to the arguments of others about volume and ask questions to clarify or build on their ideas.

Prerequisite Skills

- Recall basic multiplication facts.
- Apply the associative property of multiplication.
- Know the formula to find the area of a rectangle: $A = \ell \times w$.
- Determine units used when measuring volume.
- Decompose shapes.

Standards for Mathematical Practice (SMP)

SMPs 1, 2, 3, 4, 5, and 6 are integrated in every lesson through the *Try-Discuss-Connect* routine.*

In addition, this lesson particularly emphasizes the following SMPs:

2 Reason abstractly and quantitatively.

5 Use appropriate tools strategically.

7 Look for and make use of structure.

8 Look for and express regularity in repeated reasoning.

*See page 1i to see how every lesson includes these SMPs.

Lesson Vocabulary

- **base (of a prism)** one side of a prism, usually considered to be the side shown as the bottom of the prism. In the volume formula $V = b \times h$, b represents the area of the base of the prism.

Review the following key terms.

- **area** the amount of space inside a closed two-dimensional figure. Area is measured in square units such as square centimeters.
- **cubic unit** the volume of a unit cube.
- **formula** a mathematical relationship that is expressed in the form of an equation. For example, $A = \ell \times w$.
- **volume** the amount of space inside a solid figure. Volume is measured in cubic units such as cubic inches.

Learning Progression

In the previous lesson students extended their understanding of the volume of a solid figure to include volumes expressed in cubic inches, cubic centimeters, and cubic feet. Students used visual models of rectangular prisms, filled or partially filled with unit cubes, to determine volume. Students recognized that the product of the length and width of the prism represents the number of cubes in one layer and that the height of the prism represents the number of layers.

In this lesson students explain how volume formulas are related to finding volume as (number of cubes per layer) × (number of layers). They use volume formulas to find the volume of rectangular prisms and solid figures composed of non-overlapping rectangular prisms. Students recognize volume as additive, understanding that the volume of a solid figure is the combined volume of the rectangular prisms that compose the solid figure, with no gaps or overlaps.

In Grade 6 students will continue using these techniques to find volumes of solid figures with fractional side lengths.

Lesson Pacing Guide

Whole Class Instruction

SESSION 1 Explore *45–60 min*	**Finding Volume Using Formulas** • Start *5 min* • Try It *10 min* • Discuss It *10 min* • Connect It *15 min* • Close: Exit Ticket *5 min*	**Additional Practice** Lesson pages 35–36
SESSION 2 Develop *45–60 min*	**Finding Volume Using Formulas** • Start *5 min* • Try It *10 min* • Discuss It *10 min* • Picture It & Model It *5 min* • Connect It *10 min* • Close: Exit Ticket *5 min*	**Additional Practice** Lesson pages 41–42 **Fluency** Finding Volume Using Formulas
SESSION 3 Develop *45–60 min*	**Breaking Apart Figures to Find Volume** • Start *5 min* • Try It *10 min* • Discuss It *10 min* • Model Its *5 min* • Connect It *10 min* • Close: Exit Ticket *5 min*	**Additional Practice** Lesson pages 47–48 **Fluency** Breaking Apart Figures to Find Volume
SESSION 4 Refine *45–60 min*	**Finding Volume Using Formulas** • Start *5 min* • Example & Problem 1–3 *15 min* • Practice & Small Group Differentiation *20 min* • Close: Exit Ticket *5 min*	**Lesson Quiz** or **Digital Comprehension Check**

Lesson Materials

Lesson *(Required)*	*Per student:* 24 unit cubes *Per pair:* 60 unit cubes
Activities	*Per student:* base-ten blocks (5 hundreds flats) *Per pair:* 30 unit cubes, 25 inch cubes, strip of paper, 2 rectangular boxes of the same height, tape, sticky notes *Activity Sheet:* L-Shaped Figures
Math Toolkit	unit cubes, grid paper, isometric dot paper

Teacher Toolbox

Small Group Differentiation

PREPARE

Ready Prerequisite Lesson

Grade 4
- Lesson 16 Find Perimeter and Area

RETEACH

Tools for Instruction

Grade 4
- Lesson 16 Perimeter and Area

Grade 5
- Lesson 3 Calculate Volume

REINFORCE

Math Center Activities

Grade 5
- Lesson 3 Use Volume Vocabulary
- Lesson 3 Volume of Composite Figures

EXTEND

Enrichment Activity

Grade 5
- Lesson 3 Swimming Pools

i-Ready

Independent Learning

PERSONALIZE

i-Ready Lessons*

Grade 5
- Measure Volume Using Formulas
- Practice: Volume of Rectangular Prisms
- Practice: Volume of Composite Figures

*We continually update the Interactive Tutorials. Check the Teacher Toolbox for the most up-to-date offerings for this lesson.

Connect to Family, Community, and Language Development

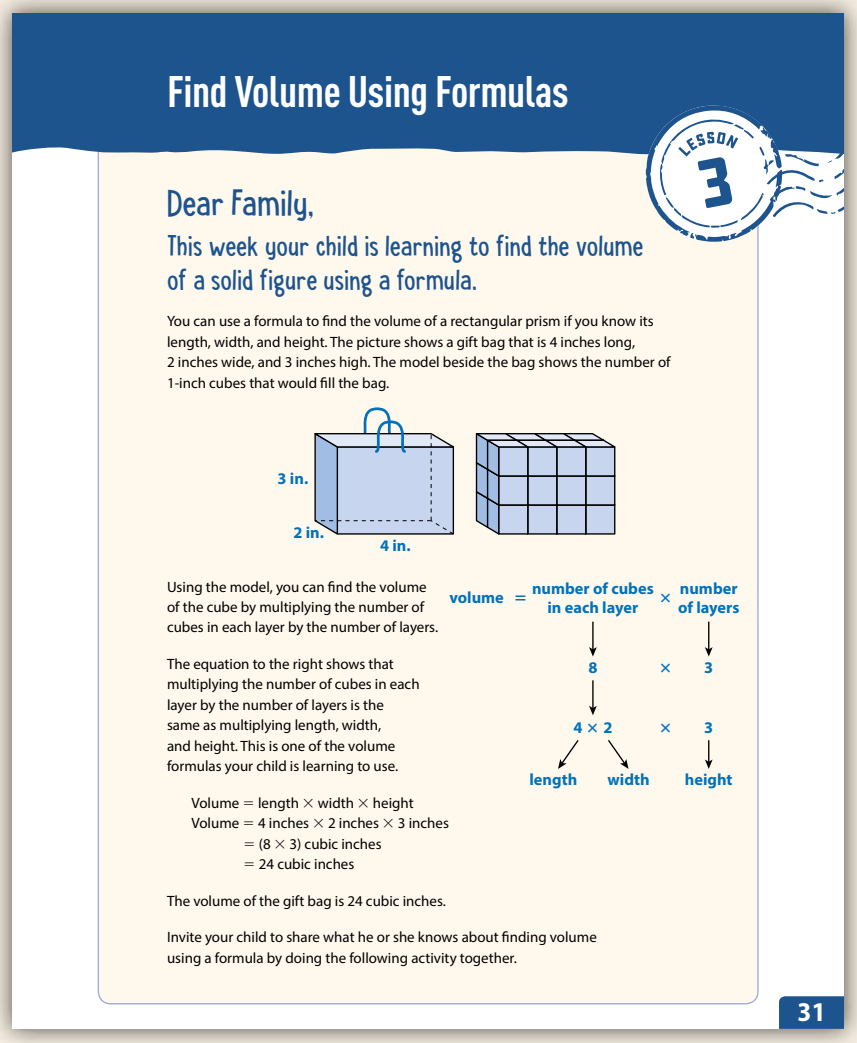
The following activities and instructional supports provide opportunities to foster school, family, and community involvement and partnerships.

Connect to Family

Use the **Family Letter**—which provides background information, math vocabulary, and an activity—to keep families apprised of what their child is learning and to encourage family involvement.

Available in Spanish

Teacher Toolbox



Find Volume Using Formulas

LESSON 3

Dear Family,

This week your child is learning to find the volume of a solid figure using a formula.

You can use a formula to find the volume of a rectangular prism if you know its length, width, and height. The picture shows a gift bag that is 4 inches long, 2 inches wide, and 3 inches high. The model beside the bag shows the number of 1-inch cubes that would fill the bag.

3 in.
2 in.
4 in.

Using the model, you can find the volume of the cube by multiplying the number of cubes in each layer by the number of layers.

volume = number of cubes in each layer × number of layers

8 × 3

4 × 2 × 3

length × width × height

The equation to the right shows that multiplying the number of cubes in each layer by the number of layers is the same as multiplying length, width, and height. This is one of the volume formulas your child is learning to use.

Volume = length × width × height
Volume = 4 inches × 2 inches × 3 inches
= (8 × 3) cubic inches
= 24 cubic inches

The volume of the gift bag is 24 cubic inches.

Invite your child to share what he or she knows about finding volume using a formula by doing the following activity together.

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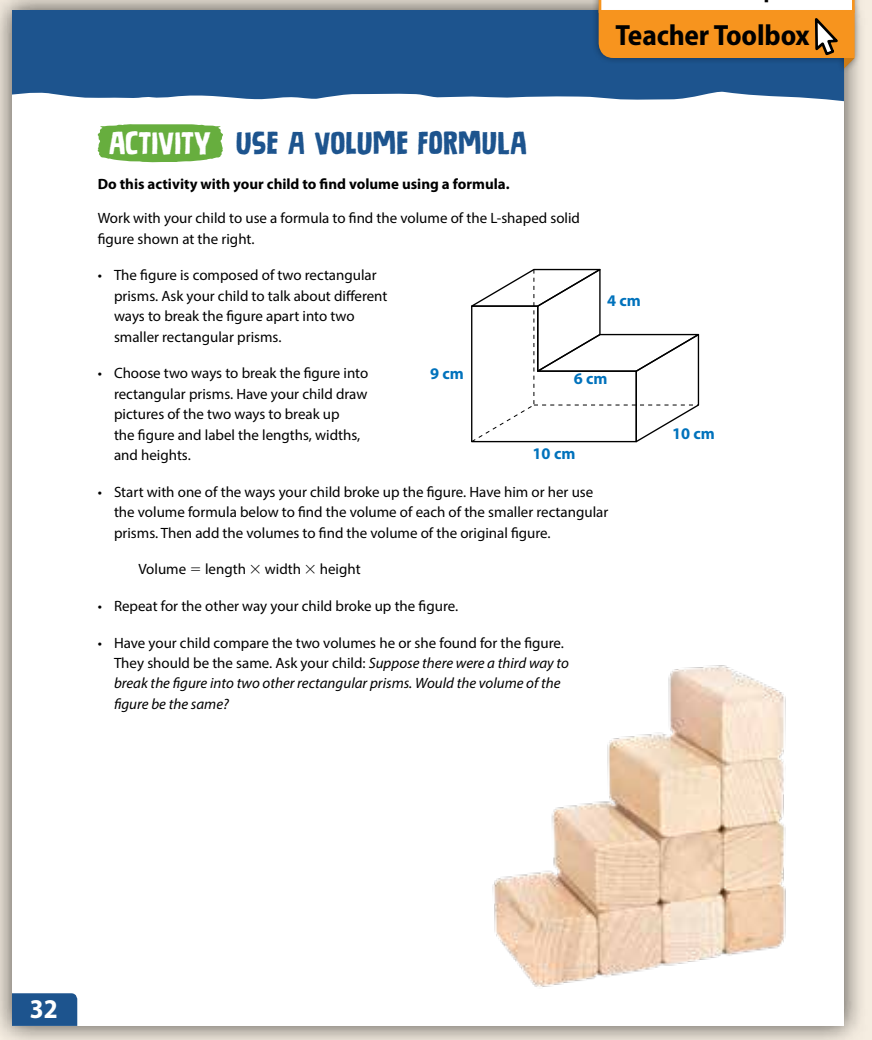

ACTIVITY USE A VOLUME FORMULA

Do this activity with your child to find volume using a formula.

Work with your child to use a formula to find the volume of the L-shaped solid figure shown at the right.

- The figure is composed of two rectangular prisms. Ask your child to talk about different ways to break the figure apart into two smaller rectangular prisms.
- Choose two ways to break the figure into rectangular prisms. Have your child draw pictures of the two ways to break up the figure and label the lengths, widths, and heights.

4 cm
9 cm
6 cm
10 cm
10 cm

- Start with one of the ways your child broke up the figure. Have him or her use the volume formula below to find the volume of each of the smaller rectangular prisms. Then add the volumes to find the volume of the original figure.

  Volume = length × width × height

- Repeat for the other way your child broke up the figure.
- Have your child compare the two volumes he or she found for the figure. They should be the same. Ask your child: *Suppose there were a third way to break the figure into two other rectangular prisms. Would the volume of the figure be the same?*

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Goal

The goal of this letter is to explain the connection between the use of unit cubes to find volume and the use of a formula to find volume.

Activity

Students have learned to use a formula to find the area of a two-dimensional plane figure. That skill will be extended to three-dimensional solid figures by adding a third measurement. The area formula will be extended from $\ell \times w$ to the volume formula of $\ell \times w \times h$. Look at the activity *Use a Volume Formula* and adjust as needed to connect with students.

Math Talk at Home

Encourage students to have a conversation with family members about objects in the house that are rectangular prisms or composed of rectangular prisms.

Conversation Starters Below are conversation starters students can write in their Family Letter or math journals to engage family members:

- *What is an object in our home that is a rectangular prism?*
- *What unit or units of measurement do you know?*
- *What units can we use to measure the volume of this object?*

Connect to Community and Cultural Responsiveness

Use these activities to connect with and leverage the diverse backgrounds and experiences of all students.

Session 1 **Use with *Try It*.**

- Point out that many people shop online. Ask students if their family members order items online. Encourage students to say what kinds of items their family members purchase online. Examples might include household products, food, electronics, and gifts. Ask students to discuss how an online store might determine what size box to use when shipping items ordered. Display different size boxes and place items inside them that match their size. Guide students to observe that the size of an item determines what size box it is placed inside. If time permits, visit an online store that calculates how much space is left in your box as you order and add items to it.

Session 2 **Use with *Model It*.**

- Point out that, like a pencil holder, many containers are rectangular prisms. Encourage students to think about containers they use at school or at home to store items. Help students think of examples and then explain how you would calculate the volume and what unit of measure would be most appropriate to use.

Session 3 **Use with *Model It*.**

- Students might be familiar with gardens, either at home or in their neighborhood. Encourage students to think about what gardens are used for and to name vegetables that are commonly grown in gardens, such as tomatoes, potatoes, and green beans. Point out that gardens often have to use space efficiently in order to ensure vegetables have enough room to grow. Call on volunteers to list adjectives that describe gardens they have at home, have seen in their neighborhood, or have seen on TV. Possible adjectives include *rectangular, square, green, brown,* and *colorful*.

Connect to Language Development

For ELLs, use the Differentiated Instruction chart to plan and prepare for specific activities in every session.

ELL English Language Learners: Differentiated Instruction

Prepare for Session 1
Use with *Try It*.

Levels 1–3

Reading/Speaking Read the ***Try It*** problem to students. Direct students' attention to the illustration. Ask: *What information do you need to know to find the volume of Becky's model?* [the dimensions; the length, width, and height] Give students time to find and label the length, width, and height of the prism. Then, have them share their answers using sentence frames:

- *The length of the prism is _____.*
- *The width of the prism is _____.*
- *The height of the prism is _____.*

Organize students into small groups to find the volume. Encourage them to use the strategies they know to solve the problem.

Levels 2–4

Reading/Speaking Ask a volunteer to read the ***Try It*** problem aloud. As the other students listen, have them underline important information in the problem such as *1-inch cubes, rectangular prism,* and *volume*. Direct students' attention to the illustration. Ask them to think about how the underlined words relate to the illustration. Call on volunteers to share ideas. Organize students into pairs to discuss solution strategies and solve the problem. Encourage partners to name the dimensions of the prism using the sentence frame:

- *The _____ of the prism is _____.*

Levels 3–5

Reading/Speaking Have students read the ***Try It*** problem and underline important information they need to solve the problem and plan a solution strategy. Before solving, organize students into partners to share their methods for finding volume.

Encourage students to critique each other's reasoning and ask questions to clarify misunderstandings. If students need support when asking questions, offer sentence frames such as:

- *Why did you _____?*
- *Which operation did you use _____?*
- *How did you know _____?*
- *How did you find _____?*

LESSON 3

SESSION 1 Explore

Purpose In this session students draw on what they know about finding the volume of a rectangular prism represented by a unit cube model. They will share strategies, helping them recognize the repeated reasoning that underlies volume calculations. They will look ahead to think about finding volume by using a formula.

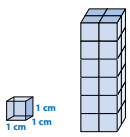
Start

Connect to Prior Knowledge

Why Support students' facility in identifying the dimensions of a rectangular prism from a diagram, preparing them to recognize the structure of volume formulas.

How Have students write an equation for the volume of a rectangular prism. Students use information in the diagram to determine the dimensions of the prism and the unit of volume.

Write an equation to find the volume of the prism.



Possible Solution
$2 \times 2 \times 6 = 24$;
volume = 24 cubic centimeters

TRY IT

Make Sense of the Problem

To support students in making sense of the problem, clarify that Becky can use her model to plan the size of the gift bag she is making. Have students identify that Becky's model is made from 1-inch unit cubes.

DISCUSS IT

Support Partner Discussion

To reinforce the meaning of each dimension, encourage students to use *length*, *width*, and *height* as they discuss the problem.

Look for, and prompt as necessary for, understanding of:

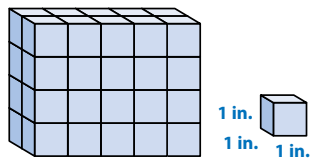
- 5 inches as the length of the prism
- 2 inches as the width of the prism
- 4 inches as the height of the prism

LESSON 3 SESSION 1 ● ○ ○ ○

Explore Finding Volume Using Formulas

Previously, you learned how to find volume by counting unit cubes and by using addition and multiplication strategies. Now you will learn how to find volume using a formula. Use what you know to try to solve the problem below.

Becky uses 1-inch cubes to create a model for a small paper gift bag she is making. Her model is a rectangular prism. What is the volume of Becky's model?



Learning Targets

- Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems.
- Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.

SMP 1, 2, 3, 4, 5, 6, 7, 8

TRY IT

Possible student work:

Sample A

$5 \times 2 = 10$, so 1 layer is 10 cubes.

$10 \times 4 = 40$, so 4 layers is 40 cubes.

Volume = 40 cubic inches

Sample B

Front side: 20 1-inch cubes
Back side: 20 1-inch cubes

$20 + 20 = 40$

The volume is 40 cubic inches.

Math Toolkit

- unit cubes
- grid paper
- isometric dot paper

DISCUSS IT

Ask your partner: Do you agree with me? Why or why not?

Tell your partner: I agree with you about . . . because . . .

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Common Misconception Look for students who are not comfortable with relating volume to the operation of multiplication and might try to add the dimensions. As students present solutions, have them justify how they found the volume.

Select and Sequence Student Solutions

One possible order for whole class discussion:

- building a model and counting the cubes to find the volume
- solutions that identify the number of cubes per layer and the number of layers
- solutions that specify the length, width, and height of the prism
- equations that use repeated addition
- equations that use multiplication

Support Whole Class Discussion

Prompt students to note the relationship between the numbers in each model and the unit cubes in the diagram in the problem.

Ask *How do [student name]'s and [student name]'s models show the dimensions of the prism?*

Listen for Each layer is 5 cubes long and 2 cubes wide and the prism is 4 cubes high; the length is 5 inches, the width is 2 inches, and the height is 4 inches.

CONNECT IT

1 LOOK BACK

Look for understanding that Becky's model has a volume of 40 cubic inches and for students to describe a method of finding the volume that does not involve counting each cube in her model.

Hands-On Activity

Use hundreds flats to model prisms.

If . . . *students are unsure about the concept of multiplying to find volume,*

Then . . . *use concrete models to relate volume to repeated addition of layers and to multiplication.*

Materials For each student: 5 hundreds flats

- Give each student from 2 to 5 hundreds flats.
- Ask students to find the area of the top surface of the flat. [100 square units] Have students share their strategies, which might include counting by 10s, or multiplying 10 by 10.
- Tell students to think of the flat as a three-dimensional prism. Ask students how the area of the two-dimensional surface of the flat relates to the number of unit cubes that make up the three-dimensional flat. [The area in square units is the same as the number of unit cubes (100).]
- Have each student stack their own hundreds flats to make a multi-layer rectangular prism. Ask students to find the volume of their prism. Have volunteers share their strategies, which may include repeated addition or multiplying the area of the surface by the height.
- Repeat activity with other numbers of flats.

2 LOOK AHEAD

Write the formulas $V = b \times h$ and $V = \ell \times w \times h$ on the board. Ask a volunteer to explain what a *formula* is and to give another example of a formula. Students may mention area and perimeter formulas.

Ask another volunteer to explain the new term *base* and the meaning of the letter *b* in the volume formula $V = b \times h$.

Students should recognize how the volume formulas capture the pattern of the volume calculations they have been doing during the previous two lessons as they worked with unit cubes models. They will explore these connections further in the next session.

CONNECT IT

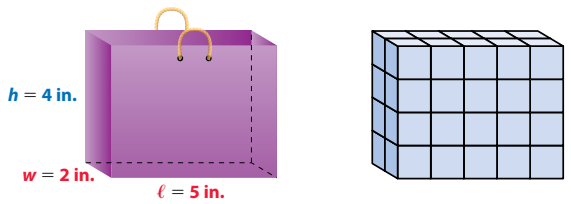
1 LOOK BACK

Explain how you can find the volume of Becky's model without counting each cube.

Possible answer: There are 10 cubes in every layer and there are 4 layers, so there are 40 cubes in Becky's model.

2 LOOK AHEAD

Becky's finished gift bag is shown below. You can find the volume with a formula. Use the letters ℓ, ***w***, and ***h*** to represent the **length**, **width**, and **height**.



a. The bottom of a rectangular prism can be called its **base**. The letter ***b*** is used to represent the **area of the base**.

Write an equation using the area of the base to find the bag's volume ***V***. Then complete the formula with a letter.

40 = 10 × 4

V = ***b*** × ***h***

b. Use the letters shown in the formula below to help you complete another equation for the volume of the bag. Then complete the formula.

40 = 5 × 2 × 4

V = ℓ × ***w*** × ***h***

3 REFLECT

How are the two volume formulas you wrote above alike? How are they different?

Possible answer: Both formulas use multiplication to find the total number of cubic units. In the formula $V = b \times h$ you find the area of the base first, using the same length and width that are in the formula $V = \ell \times w \times h$.

Close: Exit Ticket

3 REFLECT

Look for understanding of each letter in the formulas. Student responses should mention the operation of multiplication and that in both formulas all three dimensions of a prism are multiplied. Students might compare the equations they wrote for Becky's gift bag using the formulas and note that the 10 in the first equation comes from the product 5×2, which also appears in the second equation.

Common Misconception **If** students think the formula $V = b \times h$ only involves two dimensions of a prism, **then** give students two hundreds flats stacked as a prism. Ask them where they see the product 10×10 represented in the prism as well as where that product would appear in the formula $V = b \times h$.

Real-World Connection

Encourage students to think about everyday places or situations in which people might need to find the volume of a solid figure that can be modeled by a rectangular prism or a combination of two rectangular prisms. Have volunteers share their ideas. Examples: volume of a rectangular baking pan, total capacity of a refrigerator and freezer, volume of an L-shaped swimming pool.

Solutions

Support Vocabulary Development

1 Ask students to look at the word *formula*. Have students review their notes and discuss with a partner what they think the term means. Ask them to say if they have used the term in other subject areas or at home. Encourage students to share examples of using the word *formula*.

Have students share their definition of *formula* before creating the illustrations and examples. Clarify any misconceptions and state that a *formula* is an equation that shows how quantities are related and uses letters to represent the quantities.

2 Have students look at the figure in problem 2 and discuss how many layers and cubes it has. Provide sentence frames to guide students:
The rectangular prism has _____ layers.
Each layer has _____ cubes.
There are ____ cubes total.
The volume is ____ cubic units.

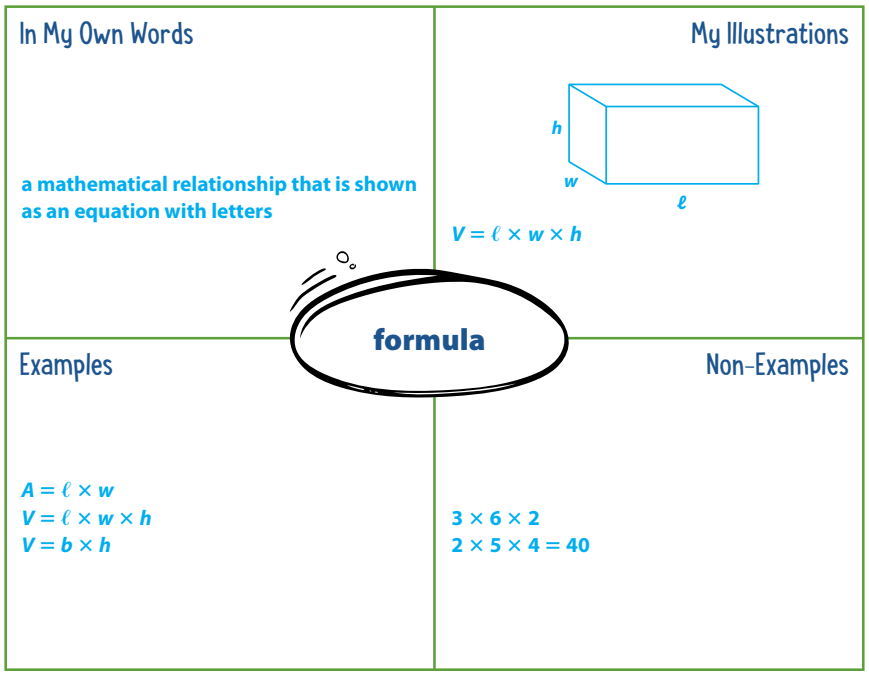
Supplemental Math Vocabulary

- *algorithm*
- *equation*
- *complex figure*
- *base*

Name: _____ LESSON 3 SESSION 1

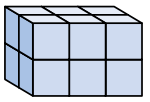
Prepare for Finding Volume Using Formulas

1 Think about what you know about formulas. Fill in each box. Use words, numbers, and pictures. Show as many ideas as you can. **Possible answers:**



2 Find the volume of the rectangular prism. Explain.

Possible answer: There are 6 unit cubes in each layer. 6 + 6 = 12. So, the volume of the prism is 12 cubic units.



1 unit cube

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3 Assign problem 3 to provide another look at solving a problem by finding the volume of a rectangular prism.

This problem is very similar to the problem about Becky's model for a small gift bag. In both problems, students are given a picture of a rectangular prism made up of unit cubes and are asked to find its volume. The question asks for the volume of Adrien's model.

Students may want to use unit cubes or grid paper.

Suggest that students read the problem three times, asking themselves one of the following questions each time:

- *What is this problem about?*
- *What is the question I am trying to answer?*
- *What information is important?*

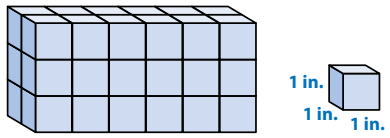
Solution:

6 × 2 = 12, so the bottom layer has 12 cubes.
3 × 12 = 36, so 3 layers is 36 cubes. The volume of Adrien's model is 36 cubic inches.
Medium

4 Have students solve the problem another way to check their answer.

3 Solve the problem. Show your work.

Adrien uses 1-inch cubes to create a model for a small box he is making. His model is a rectangular prism. What is the volume of Adrien's model?



Possible student work:
6 × 2 = 12, so 1 layer is 12 cubes.
12 × 3 = 36, so 3 layers is 36 cubes.

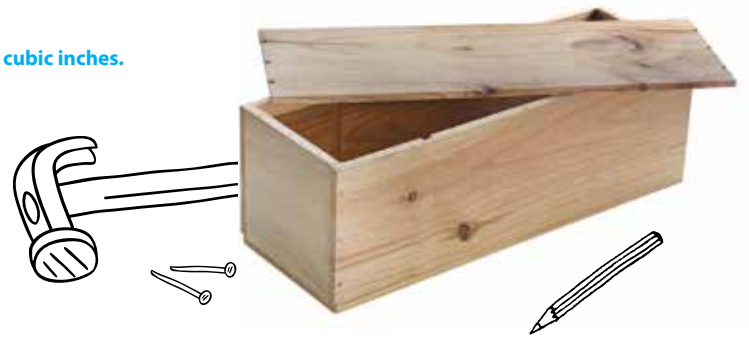
Solution **The volume of the model is 36 cubic inches.**

4 Check your answer. Show your work.

Possible student work:
Front side: 18 1-inch cubes
Back side: 18 1-inch cubes

18 + 18 = 36

The volume is 36 cubic inches.



ELL English Language Learners: Differentiated Instruction

Prepare for Session 2
Use with *Try It*.

Levels 1–3

Listening/Speaking Read the ***Try It*** problem. Have students point to the first sentence and choral read it after you. Ask: *What shape is Gareth's cup?* [rectangular] Next, have them look at the second sentence and choral read it after you. Ask: *What does the sentence tell you about the cup?* [how long, wide and high; length, width, and height; the dimensions] Organize students into small groups to share ideas and solve the problem. Then call on students to explain their ideas with gestures, numbers, pictures, or words.

Levels 2–4

Speaking/Writing Organize students into pairs to read, discuss, and solve the ***Try It*** problem. Have one partner read the problem aloud and the other partner restate the problem in his or her own words. Allow time for students to talk about solution strategies and create representations such as pictures or equations. Provide sentence frames for students to explain their thinking:

- *There is a _____ that is _____ inches long, _____ inches wide, _____ inches high.*
- *I can _____ length, width, and height to find the _____.*

After students complete their sentences, call on volunteers to read their sentences aloud.

Levels 3–5

Reading/Writing Have students form pairs and read the ***Try It*** problem. Ask pairs to work together to write the strategy they would use to solve the problem. Provide the sequence words *first, next,* and *then* for guidance. Ask students to write their strategy in complete sentences. Have students share their strategies with another pair. Ask pairs to compare strategies. If both pairs solve the problem in the same way, challenge them to think of a new way to solve the problem.

LESSON 3

SESSION 2 Develop

Purpose In this session students solve a problem that requires finding the volume of a real-world object given its length, width, and height. Students model the volume of the pencil cup in the word problem either on paper or with manipulatives. The purpose of this problem is to transition students from concrete and visual strategies for finding volume to using abstract volume formulas.

Start

Connect to Prior Knowledge

Materials For each student: 24 unit cubes

Why Reinforce the meaning of the term *base of a rectangular prism* and prepare students to use volume formulas.

How Have students build rectangular prisms with a volume of 24 cubic units and identify the area of the base and the height.

Model a rectangular prism with a volume of 24 cubic units.

What is the area of the base of your prism? $b =$ .....

What is the height of your prism? $h =$ .....

Possible Solutions

$b = 24$ square units, $h = 1$ unit;
$b = 12$ square units, $h = 2$ units;
$b = 8$ square units, $h = 3$ units;
$b = 6$ square units, $h = 4$ units

Develop Language

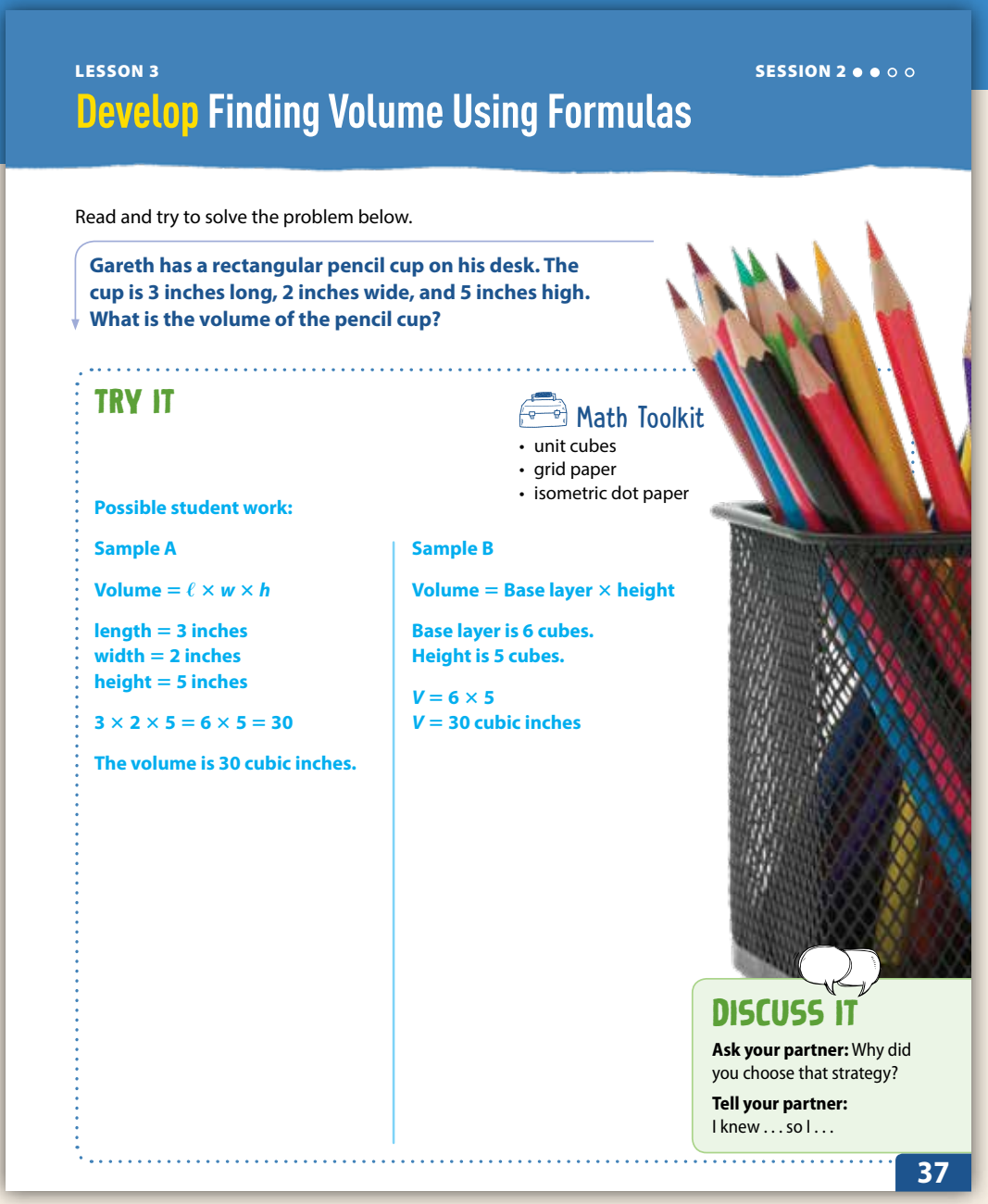
Why Develop understanding of the word *dimension*.

How Have students circle the word *dimension* when they work with *Model It*. Tell students that the dimensions of an object are its measurements, such as its length, height, and width. Point out that an object's dimensions tell you about its size and shape. Ask students to say what measurements provide information about the size of a piece of paper. Point out that these measurements are the paper's dimensions.

TRY IT

Make Sense of the Problem

To support students in making sense of the problem, have them identify that the pencil cup is a rectangular prism with a length of 3 inches, a width of 2 inches, and a height of 5 inches.


LESSON 3 SESSION 2 ● ● ○ ○

Develop Finding Volume Using Formulas

Read and try to solve the problem below.

Gareth has a rectangular pencil cup on his desk. The cup is 3 inches long, 2 inches wide, and 5 inches high. What is the volume of the pencil cup?

TRY IT

Math Toolkit
- unit cubes
- grid paper
- isometric dot paper

Possible student work:

Sample A

Volume $= \ell \times w \times h$

length = 3 inches
width = 2 inches
height = 5 inches

$3 \times 2 \times 5 = 6 \times 5 = 30$

The volume is 30 cubic inches.

Sample B

Volume = Base layer × height

Base layer is 6 cubes.
Height is 5 cubes.

$V = 6 \times 5$
$V = 30$ cubic inches

DISCUSS IT

Ask your partner: Why did you choose that strategy?

Tell your partner: I knew . . . so I . . .

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DISCUSS IT

Support Partner Discussion

Encourage students to use the term *base* when talking about the bottom of the pencil cup.

Support as needed with questions such as:

- *How did your model or drawing of the pencil cup help you try to solve the problem?*
- *What strategy or model did your partner use to find the volume of the pencil cup?*
- *What worked for you, and what did not work for you?*

Common Misconception Look for students who confuse finding the area of the base of the pencil cup with finding the pencil cup's volume. Ask students to build the layer of 1-inch unit cubes that fills the base of the pencil cup and compare the number of cubes to the area of the base and the volume of the pencil cup.

Select and Sequence Student Solutions

One possible order for whole class discussion:

- a physical model, or drawing, of the pencil cup made from unit cubes
- a drawing of the pencil cup with dimensions labeled
- numbers written in a formula as length times width times height
- numbers written in a formula as area of the base times height

Support Whole Class Discussion

Compare and connect different representations and have students identify how they are related.

Ask *How does your model show the length, the width, and the height of the pencil cup?*

Listen for Students should recognize that accurate representations show 3 as the length, 2 as the width, and 5 as the height; for example, a 3-by-2-by-5 unit cube model, a drawing with the dimensions written on the edges, or a formula with the letters ℓ, w, and h replaced by 3, 2, and 5.

PICTURE IT & MODEL IT

If no student presented these models, connect them to the student models by pointing out the ways they each represent:

- the length, the width, and the height
- the area of the base and the height

Ask *How are all the models alike?*

Listen for All the models show that the pencil cup is a rectangular prism with dimensions of 3 inches, 2 inches, and 5 inches.

For *Picture It,* prompt students to tell how the picture represents the pencil cup.

- *What dimensions do the unit cubes stand for?*
- *How are the base layer and the top layer alike?*

For *Model It,* prompt students to think about how the drawing and the formulas are related.

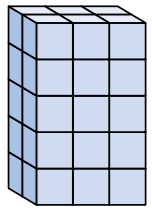
- *How does this model show the dimensions of the pencil cup?*
- *What numbers would you use to replace the ℓ, the w, and the h in the first formula?*
- *What numbers would you use to replace the b and the h in the second formula? Why?*

Explore different ways to understand using the dimensions of a rectangular prism to find its volume.

Gareth has a rectangular pencil cup on his desk. The cup is 3 inches long, 2 inches wide, and 5 inches high. What is the volume of the pencil cup?

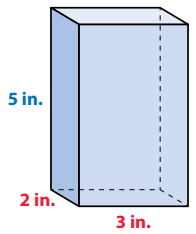
PICTURE IT

You can picture the pencil cup as a rectangular prism made up of 1-inch cubes.



MODEL IT

You can draw a model of the pencil cup and label its dimensions. Then use a volume formula.



$V = \ell \times w \times h$ or $V = b \times h$

Deepen Understanding

Associative Property

SMP 2 Reason abstractly and quantitatively.

When discussing the volume formula $V = \ell \times w \times h$, prompt students to consider using volume to represent the associative property of multiplication.

Ask a student to write the product that represents the volume of the pencil cup on the board. [$3 \times 2 \times 5$] Then ask: *Does the order in which you multiply the factors of this product affect the result?* To clarify, insert parentheses around 3×2 and write the product a second time, inserting parentheses around 2×5.

Have two volunteers come to the board and complete the products, multiplying the factors inside the parentheses first and then multiplying by the third factor.

$(3 \times 2) \times 5 = 6 \times 5$ $\quad$ $3 \times (2 \times 5) = 3 \times 10$

$= 30$ $\quad$ $= 30$

Ask *How does each order for multiplying show a different way of thinking of the unit cube model as equal layers of cubes?*

Listen for $(3 \times 2) \times 5 = 6 \times 5$ shows thinking of the model as 5 layers of 6 cubes, and $3 \times (2 \times 5) = 3 \times 10$ shows thinking of it as 3 layers of 10 cubes.

CONNECT IT

- Remind students that one thing that is alike about all the representations is the dimensions.
- Explain that on this page, students will use those dimensions to explain how the unit cube model leads to the formulas that can be used to find the volume of any rectangular prism.

Monitor and Confirm

1 – 3 Check for understanding that:

- 6 cubes are in every layer, including the base layer, which can be found by multiplying the length times the width
- 5 tells how many layers, or the height
- 30 is the volume, which can be found by multiplying the area of the base times the height

Support Whole Class Discussion

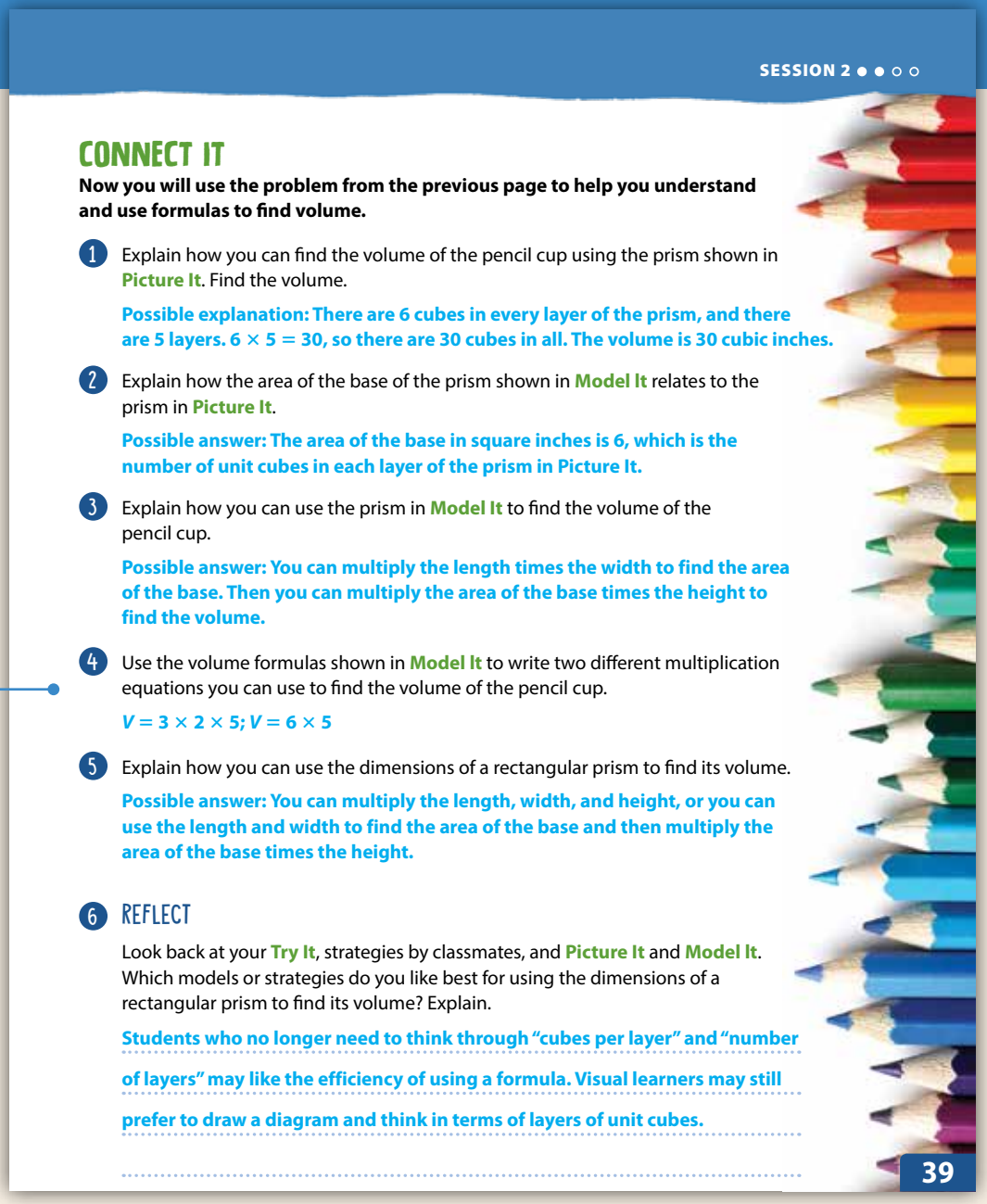
4 Tell students that this problem will prepare them to provide the explanation required in problem 5.

Be sure students understand that the problem is asking them to replace the letters in the formulas with the numbers in the picture and model.

> **Ask** *What do the letters in the formulas mean?*
>
> ***Listen for*** In the first formula, the ℓ stands for the length, the w stands for the width, and the h stands for the height. In the second formula, the b refers to the area of the base and the h refers to the height.

5 Look for the idea that you multiply the three dimensions of a rectangular prism to find its volume. You can also think of this as multiplying the area of the base times the height.

6 REFLECT Have all students focus on the strategies used to solve this problem. If time allows, have students share their preferences with a partner.



CONNECT IT

Now you will use the problem from the previous page to help you understand and use formulas to find volume.

1 Explain how you can find the volume of the pencil cup using the prism shown in **Picture It**. Find the volume.

Possible explanation: There are 6 cubes in every layer of the prism, and there are 5 layers. 6 × 5 = 30, so there are 30 cubes in all. The volume is 30 cubic inches.

2 Explain how the area of the base of the prism shown in **Model It** relates to the prism in **Picture It**.

Possible answer: The area of the base in square inches is 6, which is the number of unit cubes in each layer of the prism in Picture It.

3 Explain how you can use the prism in **Model It** to find the volume of the pencil cup.

Possible answer: You can multiply the length times the width to find the area of the base. Then you can multiply the area of the base times the height to find the volume.

4 Use the volume formulas shown in **Model It** to write two different multiplication equations you can use to find the volume of the pencil cup.

$V = 3 \times 2 \times 5$; $V = 6 \times 5$

5 Explain how you can use the dimensions of a rectangular prism to find its volume.

Possible answer: You can multiply the length, width, and height, or you can use the length and width to find the area of the base and then multiply the area of the base times the height.

6 REFLECT

Look back at your **Try It**, strategies by classmates, and **Picture It** and **Model It**. Which models or strategies do you like best for using the dimensions of a rectangular prism to find its volume? Explain.

Students who no longer need to think through "cubes per layer" and "number of layers" may like the efficiency of using a formula. Visual learners may still prefer to draw a diagram and think in terms of layers of unit cubes.

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Hands-On Activity

Connect dimension words with the symbols in the formulas.

If . . . *students are unsure about what the symbolic letters in the formula represent,*

Then . . . *use this activity to connect the letters in the formula with the actual dimensions in a model.*

Materials For each pair: 30 unit cubes, strip of paper

- Ask students to build a layer that is 3-by-2 unit cubes on top of a sheet of paper. Have them label a side with 3 unit cubes by writing *length* on the paper next to that side. Have them circle the letter *l*. Then ask them to label a side with 2 unit cubes by writing *width* on the paper and circling the *w*.
- Have students write *base* across the top of the paper. Discuss how the 3-by-2 layer they made can be the base layer for a prism.
- Ask students to make four more 3-by-2 layers and stack them on the original base layer on the paper. Then have them write *height* on a strip of paper and circle the letter *h* before resting the label against the stacked unit cubes.
- Draw attention to the first formula. Have students relate the letters in their models with the letters in the first formula. Repeat for the second formula.

APPLY IT

For all problems, encourage students to draw a model to support their thinking. Allow some leeway in precision; some right angles might not be drawn with 90° corners; some dimensions might not be to scale.

7 420 cubic centimeters; Students may draw the rectangular jewelry box and label its dimensions: length is 12 cm, width is 7 cm, and height is 5 cm. They may also use the formula $V = \ell \times w \times h$. Some students may write the solution as 420 cm^3.

8 120 cubic inches; Students may also draw the book and label its dimensions: length is 12 in., width is 10 in., and height is 1 in. They may also use the formula $V = b \times h$. Some students may write the solution as 120 $in.^3$.

Close: Exit Ticket

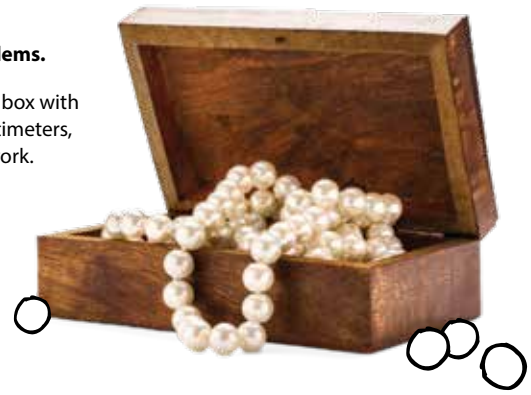
9 **A**; Divide the volume by the given dimension to find the product of the other two dimensions;
$100 \div 5 = 20$ and $1 \times 20 = 20$.

D; Check that the product of the three dimensions is the given volume;
$4 \text{ m} \times 5 \text{ m} \times 5 \text{ m} = 100 \text{ m}^3$.

Error Alert **If** students choose B, C, and/or E, **then** review the formulas $V = \ell \times w \times h$ and $V = b \times h$. Suggest that students consider the given dimension, 5 meters, as the height, h. Discuss how to use each formula to determine whether a given set of dimensions work for a prism with a volume of 100 cubic meters and one dimension of 5 meters.

APPLY IT

Use what you just learned to solve these problems.



7 What is the volume of a rectangular jewelry box with a length of 12 centimeters, a width of 7 centimeters, and a height of 5 centimeters? Show your work.

Possible student work:
area of base = 12 × 7 = 84
height = 5

V* = *b* × *h
= 84 × 5 = 420

Solution **420 cubic centimeters**

8 How much space is taken up by a book that is 12 inches long, 10 inches wide, and 1 inch tall? Show your work.

Possible student work:
V* = ℓ × *w* × *h
= 12 × 10 × 1 = 120

Solution **120 cubic inches**

9 A rectangular prism has a volume of 100 cubic meters. One of the dimensions is 5 meters. Which pairs of measurements could be the other two dimensions of the prism?

Ⓐ 1 meter, 20 meters
Ⓑ 5 meters, 10 meters
Ⓒ 10 meters, 10 meters
Ⓓ 4 meters, 5 meters
Ⓔ 20 meters, 20 meters

Solutions

Some students may choose to write volumes using exponent notation for the units. For example, they may write 12 in.^{3} instead of 12 cubic inches.

1 **a.** See completed diagram on student page.

b. 12 cubic inches; students may write the equation $4 \times 3 \times 1 = 12$.
Basic

2 20 cubic feet; Students may find the area of base as $b = 2$ feet $\times$ 2 feet $= 4$ square feet; $V = b \times h = 4 \times 5 = 20$; the volume is 20 cubic feet.
Medium

Name: _____ LESSON 3 SESSION 2

Practice Finding Volume Using Formulas

Study the Example showing how to use formulas to find the volume of a rectangular prism. Then solve problems 1–7.

EXAMPLE

Gwen puts her leftover food in a rectangular container. The container is 6 inches long, 5 inches wide, and 2 inches tall. What is the volume of the container?

Use the formula *volume* = *length* × *width* × *height*.

$V = \ell \times w \times h = 6 \times 5 \times 2$, or 60 cubic inches

Or use the formula *volume* = *area of the base* × *height*.

The *area of the base* is the same as the *length* × *width*.

$b = 6 \times 5$, or 30

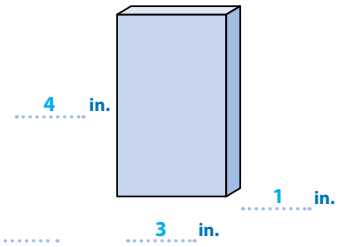
$V = b \times h = 30 \times 2$, or 60 cubic inches

1 Ted's box is 4 inches tall, 3 inches long, and 1 inch wide.

a. Label the picture of the box with its dimensions.

b. What is the volume of the box? Show your work.

Possible student work:
$4 \times 3 \times 1 = 12$



Solution 12 cubic inches

2 A rectangular prism has a square base with sides that are 2 feet long. The height of the prism is 5 feet. What is the volume of the prism? Show your work.

Possible student work:
area of base = 4 square feet
$4 \times 5 = 20$

Solution 20 cubic feet

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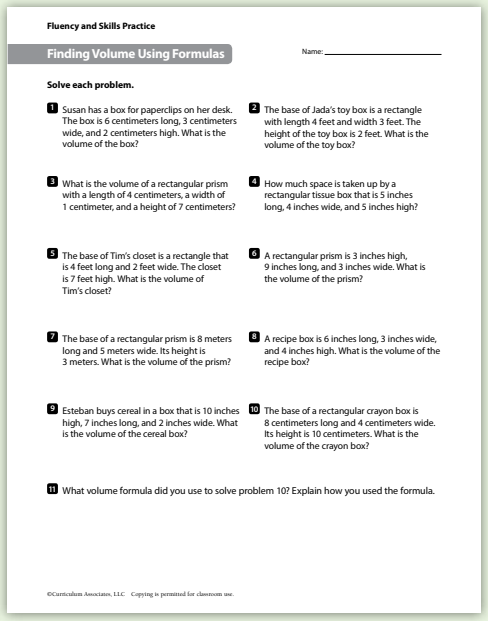
Fluency & Skills Practice Teacher Toolbox

Assign *Finding Volume Using Formulas*

In this activity students solve word problems that involve finding the volume of rectangular prisms for which all three dimensions are known. Students may encounter real-world situations that involve finding the volume of rectangular prisms of known dimensions. For example, a student might need to determine how much sand is needed to fill a sandbox or how much concrete is needed to construct a driveway.



Fluency and Skills Practice

Finding Volume Using Formulas

Name: _____

Solve each problem.

1. Susan has a box for paperclips on her desk. The box is 6 centimeters long, 3 centimeters wide, and 2 centimeters high. What is the volume of the box?
2. The base of Jada's toy box is a rectangle with length 4 feet and width 3 feet. The height of the toy box is 2 feet. What is the volume of the toy box?
3. What is the volume of a rectangular prism with a length of 4 centimeters, a width of 1 centimeter, and a height of 7 centimeters?
4. How much space is taken up by a rectangular tissue box that is 5 inches long, 4 inches wide, and 5 inches high?
5. The base of Tim's closet is a rectangle that is 4 feet long and 2 feet wide. The closet is 7 feet high. What is the volume of Tim's closet?
6. A rectangular prism is 3 inches high, 9 inches long, and 3 inches wide. What is the volume of the prism?
7. The base of a rectangular prism is 8 meters long and 5 meters wide. Its height is 3 meters. What is the volume of the prism?
8. A recipe box is 6 inches long, 3 inches wide, and 4 inches high. What is the volume of the recipe box?
9. Esteban buys cereal in a box that is 10 inches high, 7 inches long, and 2 inches wide. What is the volume of the cereal box?
10. The base of a rectangular crayon box is 8 centimeters long and 4 centimeters wide. Its height is 10 centimeters. What is the volume of the crayon box?
11. What volume formula did you use to solve problem 10? Explain how you used the formula.

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3 Elon's shed is 10 feet long, 6 feet wide, and 8 feet tall. What is the volume of the shed? Show your work.

Possible student work:
10 × 6 × 8 = 480

Solution **480 cubic feet**

4 The base of a rectangular prism has a length of 4 centimeters and has a width of 2 centimeters. The height of the prism is 3 centimeters. What is the volume of the prism? Show your work.

Possible student work:
8 × 3 = 24

Solution **24 cubic centimeters**

5 What is the volume of a box that is 8 inches long, 2 inches wide, and 6 inches tall? Show your work.

Possible student work:
8 × 2 × 6 = 96

Solution **96 cubic inches**

6 The base of a rectangular prism is a rectangle that is 7 inches long and 5 inches wide. Its height is 10 inches. Write two different equations that you can use to find the volume.

Possible equations:
7 × 5 × 10 = 350 and 35 × 10 = 350

7 Jin has two boxes. Box *A* has dimensions of 6 centimeters, 5 centimeters, and 9 centimeters. Box *B* has dimensions of 4 centimeters, 10 centimeters, and 7 centimeters. Which box holds more? Explain.

Box *B* holds more; Possible explanation: The volume of Box *A* is 6 × 5 × 9, or 270 cubic centimeters. The volume of Box *B* is 4 × 10 × 7, or 280 cubic centimeters. Box *B* has a greater volume, so it holds more.

3 480 cubic feet;
Possible equation: $10 \times 6 \times 8 = 480$
Medium

4 24 cubic centimeters;
Possible equations: area of base $= 4 \times 2 = 8$, volume $= 8 \times 3 = 24$
Medium

5 96 cubic inches;
Possible equation: $8 \times 2 \times 6 = 96$
Medium

6 $7 \times 5 \times 10 = 350$
$35 \times 10 = 350$
Medium

7 Box *B* holds more; Student responses should include finding the volume of each box and comparing the two volumes to determine that Box *B* holds more. Box *A* volume: $6 \times 5 \times 9$, or 270 cubic centimeters. Box *B* volume: $4 \times 10 \times 7$, or 280 cubic centimeters. Box *B* has a greater volume, so it holds more.
Challenge

ELL English Language Learners: Differentiated Instruction

Prepare for Session 3
Use with *Try It*.

Levels 1–3

Listening/Speaking Read the ***Try It*** problem aloud. Have students read the first sentence aloud. Help them understand "raised garden bed" by providing photos or sharing images online. Say: *A raised garden is a garden that is built off the ground rather than directly on it*. Have students circle the word *diagram* and say it aloud. Say: *A diagram is a drawing of an object and its measurements*. Have students point to the last sentence and circle the word *fill*. Say: *When you fill something, you need to know its volume.*

Provide a sentence frame to help students think about the problem.

- *In this problem, I need to find out how many _____ of soil Bethany needs.*

Levels 2–4

Listening/Speaking Read the ***Try It*** problem aloud. Have students form pairs and restate the problem in their own words with their partners. Guide students with questions and provide sentence frames as needed.

- *What type of shape is the raised garden bed?*
- *How do you know?*
- *What does Bethany want to do?*
- *What do you need to calculate?*
- *The raised garden bed is _____.*
- *I know this because _____.*
- *Bethany wants to _____.*
- *I need to calculate _____.*

Call on students to read the completed sentences.

Levels 3–5

Speaking/Writing Read the ***Try It*** problem aloud and have students form pairs. Ask: *What does the diagram show? What unit is used for the measurements?* Have students respond in complete sentences. Ask students to work with their partners to write the problem in their own words. After students have completed their writing, ask them questions to check their comprehension. For example:

- *What do you need to find?*
- *Can the problem be solved with the information you wrote?*

LESSON 3

SESSION 3 Develop

Purpose In this session students solve a problem that requires finding the volume of a solid figure composed of two rectangular prisms. Students model ways to break apart the solid figure to deepen their understanding of volume as additive and to develop a strategy for finding the volume of a solid figure composed of non-overlapping rectangular prisms.

Start

Connect to Prior Knowledge

Materials For each pair: 60 unit cubes

Why Support students' understanding of volume as additive and prepare them for breaking apart solid figures into rectangular prisms.

How Have students build a solid figure from two non-overlapping prisms and find the combined volume of the prisms.

Make two rectangular prisms so the base of one prism is longer and wider than the base of the other. Place the smaller prism on top of the larger prism. What is the combined volume of the prisms?

Possible Solution

a 3 × 2 × 4 prism on top of a 5 × 3 × 2 prism; 54 cubic units

Develop Language

Why Emphasize the difference between *drawing* and *diagram*.

How Have students point to the word *diagram*. Say: *A diagram is a drawing that explains something. What does the diagram explain about the garden bed?* Have students brainstorm other things they could explain with a diagram.

TRY IT

Make Sense of the Problem

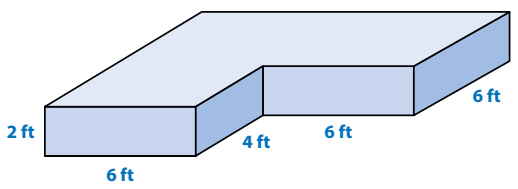
To support students in making sense of the problem, clarify what a *raised* garden bed is.

Ask *What is the height of Bethany's raised garden bed?*

Develop Breaking Apart Figures to Find Volume

Read and try to solve the problem below.

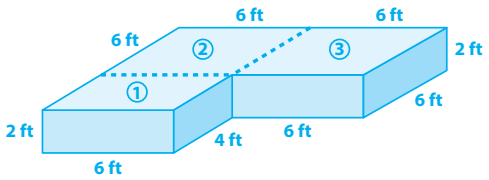
Bethany has a raised garden bed. The diagram shows its measurements. All the corners are right angles. If she fills the bed to the top with soil, how many cubic feet of soil will Bethany need?



TRY IT

Possible student work:

Sample A



$V = \ell \times w \times h$

① $V = 6 \times 4 \times 2 = 48$ — 48 cubic feet
② $V = 6 \times 6 \times 2 = 72$ — 72 cubic feet
③ $V = 6 \times 6 \times 2 = 72$ — + 72 cubic feet
192 cubic feet

Sample B

Divide figure into left and right parts.

Left side
$\ell = 6, w = 10, h = 2$
$V = b \times h = 60 \times 2 = 120$

Right side
$\ell = 6, w = 6, h = 2$
$V = b \times h = 36 \times 2 = 72$

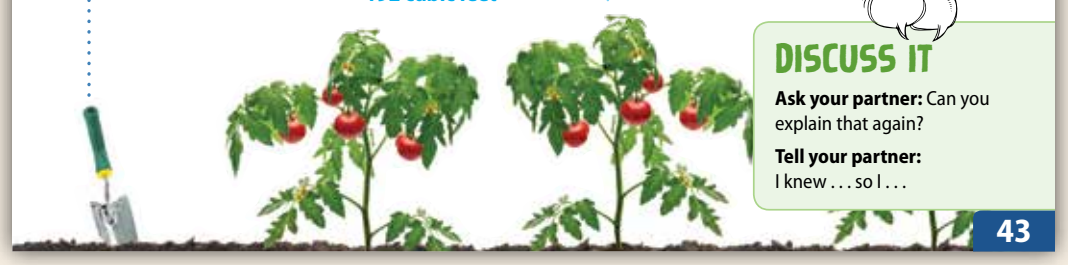
Total volume = 120 + 72 = 192
She needs 192 cubic feet of soil.

Math Toolkit
- unit cubes
- grid paper
- isometric dot paper

DISCUSS IT

Ask your partner: Can you explain that again?

Tell your partner: I knew . . . so I . . .



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DISCUSS IT

Support Partner Discussion

Encourage students to discuss the diagram given in the problem.

Support as needed with questions such as:

- *How did you use the measurements on the diagram?*
- *Did you and your partner use the same approach to the problem?*

Common Misconception Look for students who have trouble finding a missing dimension of the garden bed. If students use 6 feet as the length of back edge, ask them to specify whether the back edge is longer or shorter than the front edge.

Select and Sequence Student Solutions

One possible order for whole class discussion:

- physical models or drawings showing unit cubes
- drawings with lines drawn to show decomposing the figure into three parts or into two parts
- equations that use a volume formula
- representations that use subtraction

Support Whole Class Discussion

Compare and connect the different representations and have students identify how they are related.

Ask *How does your model represent the garden bed as a combination of rectangular prisms?*

Listen for Students should recognize that accurate representations break the garden bed into non-overlapping prisms with a height of 2 feet. Responses may include seeing the figure as composed of two or three prisms that are "added" or as one 12 ft × 10 ft × 2 ft prism with a 6 ft × 4 ft × 2 ft prism "subtracted."

MODEL ITS

If no student presented these models, connect them to the student models by pointing out the ways they each represent:

- decomposing Bethany's garden bed into parts that are non-overlapping rectangular prisms
- the dimensions of each part

Ask *How are the two ways of breaking apart the garden bed similar? How are they different?*

Listen for Both models break the garden bed into two rectangular prisms. One way breaks the bed into a front part and a back part, and the other way breaks the bed into a left side and a right side. The prisms in the two decompositions have different dimensions.

For the first model, prompt students to understand the equation that labels the back edge of the garden bed.

- *Three line segments in the original figure are labeled 6 ft. Which other line segments in the first model have a length of 6 ft?*
- *Where does each 6 in the equation 6 ft + 6 ft = 12 ft come from?*

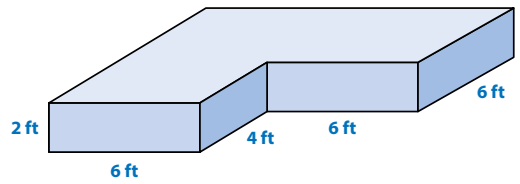
For the second model, prompt students to explain the equation that labels the left edge of the garden bed and examine the expression for the total volume.

- *How do you know the left edge of the garden bed has a length of 10 ft?*
- *How does the expression represent the total volume of the garden bed?*

Explain that when evaluating an expression, the operations in parentheses are performed first.

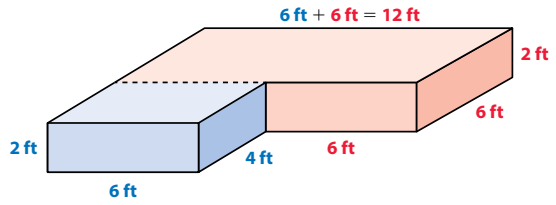
Explore different ways to understand finding the volume of a solid figure by breaking it apart into two rectangular prisms.

Bethany has a raised garden bed. The diagram shows its measurements. All the corners are right angles. If she fills the bed to the top with soil, how many cubic feet of soil will Bethany need?



MODEL IT

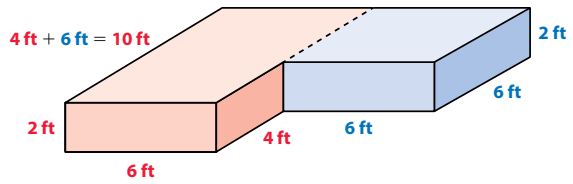
You can break the garden bed into two rectangular prisms this way.



One rectangular prism measures **6 feet** × **4 feet** × **2 feet**.
The other rectangular prism measures **12 feet** × **6 feet** × **2 feet**.

MODEL IT

You can also break the garden bed into two rectangular prisms in a different way.



One rectangular prism measures **10 feet** × **6 feet** × **2 feet**.
The other rectangular prism measures **6 feet** × **6 feet** × **2 feet**.

Total volume: (**10 feet** × **6 feet** × **2 feet**) + (**6 feet** × **6 feet** × **2 feet**)

Deepen Understanding

Decompose Solid Figures

SMP 7 Look for structure.

When discussing the decompositions shown in the models, prompt students to consider how to use a subtraction model to find the volume of a solid figure.

Have students look at the original figure of the garden bed in the problem.

Ask *Instead of thinking of the L-shaped garden bed as two rectangular prisms joined together, how can you think of it as a single rectangular prism with a "missing piece?"*

Listen for You can picture filling in the right front corner. If you extend the front and right sides until they meet, you form one rectangular prism.

Draw the base of the garden bed on the board. Invite a volunteer to draw in the "missing piece" of the base and label its length and width. [6 ft, 4 ft]

Now ask: *How could you use subtraction to find the volume of the garden bed?* [Subtract the volume of the 6 ft × 4 ft × 2 ft prism that would fill in the right front corner from the volume of the larger 12 ft × 10 ft × 2 ft prism.]

CONNECT IT

- Remind students that one thing that is alike about all the models is that they use decompositions of the garden bed into non-overlapping prisms.
- Explain that on this page students will use the models to write equations to find the volume of the garden bed.

Monitor and Confirm

1–3 Check for understanding that:

- volume of a rectangular prism $= \ell \times w \times h$
- adding the volumes of the non-overlapping rectangular prisms gives the volume of the garden bed
- the volume of garden bed is 192 cubic feet

Support Whole Class Discussion

3–4 Tell students that these two problems prepare them to provide the explanation required in problem 5.

Be sure students understand that problem 3 refers to the decomposition shown in the first *Model It*, while problem 4 refers to the decomposition shown in the second *Model It*. Remind students that when evaluating the expression in problem 4, perform the operations in parentheses first. Encourage students to write a similar expression for the first *Model It*.

Ask *How is the equation you wrote in problem 4 similar to the equations you wrote in problem 3? How is it different?*

Listen for In each problem, you use the volume formula $V = \ell \times w \times h$ to find the volumes of the rectangular prisms, and you add the volumes to find the volume of the garden bed. In problem 3, the two areas are found separately and then added together. In problem 4 the total area is found with a single expression. The numbers you use in the formula are different for each decomposition of the garden bed, but the total volume is the same.

5 Look for the idea that how you break a solid figure into non-overlapping rectangular prisms does not affect the volume of the whole.

6 REFLECT Have all students focus on the strategies used to solve this problem. If time allows, have students share their preferences with a partner.

CONNECT IT

Now you will use the problem from the previous page to help you understand ways to break apart a solid figure to find its volume.

1 Look at the first **Model It**. How can you find the volume of each rectangular prism?

Possible answer: Multiply length by width by height.

2 How can you find the volume of the entire garden bed?

Possible answer: Add the volumes of the prisms that make up the solid figure.

3 What is the volume of the entire garden bed? Show your work. **192 cubic feet**

One part: $6 \times 4 \times 2 = 48$
Other part: $12 \times 6 \times 2 = 144$ **Total: $48 + 144 = 192$**

4 Now look at the second **Model It**. Show how to use the expression to find the volume of the garden bed if you break it apart this way. When you evaluate an expression, do the operations in the parentheses first.

$(10 \times 6 \times 2) + (6 \times 6 \times 2) = 120 + 72 = 192$
Total: 192 cubic feet

5 Do you need to break apart a solid figure in a certain way to find its volume? Use the problem from the previous page to explain your reasoning.

No; Possible explanation: As long as you can break the figure into non-overlapping parts and find the volume of each part, you can add the volumes to obtain the total volume.

6 REFLECT

Look back at your **Try It**, strategies by classmates, and **Model Its**. Which models or strategies do you like best for breaking apart solid figures to find volume? Explain.

Students who have trouble finding the dimensions of the parts when dividing a figure into two prisms may prefer dividing an L-shaped figure into three non-overlapping prisms, as in Try It Sample A, because they find it easier to recognize the dimensions of the parts. Other students may prefer breaking the figure into two parts so they only calculate two volumes before adding.

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Hands-On Activity

Decompose solid figures composed of rectangular prisms.

If . . . *students are unsure about breaking apart a solid figure in different ways,*

Then . . . *use this activity to provide a concrete experience.*

Materials For each pair: 25 inch cubes, Activity Sheet *L-Shaped Figures*

- Organize students into pairs. Explain that the inch grid paper shows two copies of the base of an L-shaped solid figure with a height of 1 inch.
- Have one student build the L-shaped figure by filling one copy of the base with inch cubes.
- Have the other student separate the L-shaped figure that the first student built into two rectangular prisms. Have the student show the decomposition on the grid paper by drawing a line inside the outline of the base.
- Have the first student write the volume of each of the two prisms inside the corresponding part of the base and find the volume of the L-shaped figure.
- Have the partners trade roles and repeat the activity using the other copy of the base on the grid paper, finding a different way to break apart the figure.
- If time allows, let partners draw their own bases on grid paper and repeat.

APPLY IT

For all problems, encourage students to use a model to support their thinking. Allow some leeway in precision; some right angles might not be drawn with 90° corners; some dimensions might not be to scale.

7 480 cubic meters; See possible work on Student Worktext page. Students may also draw a model of the pool divided into two rectangular prisms with the dimensions labeled. They may also use the formula $V = \ell \times w \times h$.

8 21,000 cubic centimeters; See possible work on Student Worktext page. Students may also divide the prism into an upper part (30 cm × 10 cm × 10 cm) and a lower part (30 cm × 30 cm × 20 cm).

Close: Exit Ticket

9 72 cubic feet; See possible work on Student Worktext page. Students may also divide the prism into a left part (6 ft × 3 ft × 2 ft) and a right part (3 ft × 2 ft × 6 ft).

Students' solutions should indicate understanding of:

- dividing the solid figure into non-overlapping rectangular prisms
- determining the dimensions of the prisms
- using the dimensions in a volume formula to find the volume of each prism
- adding the resulting volumes to find the solution

Error Alert **If** students struggle with determining the dimensions of two prisms that make up the figure, **then** have them build the figure using cubes and then identify the dimensions of the separate rectangular prisms.

APPLY IT

Use what you just learned to solve these problems.

7 The Recreation Center has an L-shaped pool. One part of the pool is 8 meters by 6 meters. The other part is 12 meters by 6 meters. The whole pool is 4 meters deep. What is the volume of the entire pool? Show your work.

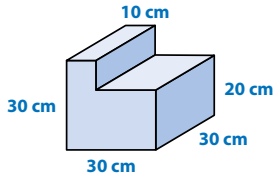
Possible student work:
Volume = $b \times h$

Volume of one part = $48 \times 4 = 192$
Volume of other part = $72 \times 4 = 288$

$288 + 192 = 480$

Solution **480 cubic meters**

8 What is the volume of the solid figure below? Show your work.



Possible student work:

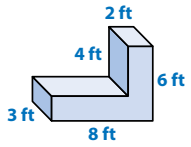
$V = \ell \times w \times h$

Left part: $V = 30 \times 10 \times 30 = 9{,}000$
Right part: $V = 30 \times 20 \times 20 = 12{,}000$

$9{,}000 + 12{,}000 = 21{,}000$

Solution **21,000 cubic centimeters**

9 What is the volume of the solid figure below? Show your work.



Possible student work:
Upper part: $V = 3 \times 2 \times 4 = 24$
Lower part: $V = 8 \times 3 \times 2 = 48$

$24 + 48 = 72$

Solution **72 cubic feet**

Solutions

1. 8 ft × 5 ft × 3 ft, or 120 cubic feet
Basic

2. 20 cubic feet; Find the width of Prism *C* (4 ft) by subtracting the two 2-foot lengths from the length of Prism *D* (8 ft). Then use the volume formula.
Medium

3. 140 cubic feet; Add the volumes of Prisms *D* and *C*.
Basic

Name: _____ LESSON 3 SESSION 3

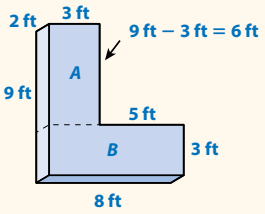
Practice Breaking Apart Figures to Find Volume

Study the Example showing how to break apart a solid figure into rectangular prisms and find its volume. Then solve problems 1–8.

EXAMPLE

Molly wants to know how much soil she needs to fill her two-tiered planter, shown below. What is the volume of the planter?

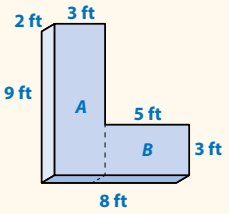
You can break the figure into two rectangular prisms in different ways.



Prism *A* measures 6 ft × 3 ft × 2 ft.
Volume of Prism *A* = 36 cubic feet

Prism *B* measures 8 ft × 3 ft × 2 ft.
Volume of Prism *B* = 48 cubic feet

Volume of planter = 36 + 48, or 84
The volume is 84 cubic feet.



Prism *A* measures 9 ft × 3 ft × 2 ft.
Volume of Prism *A* = 54 cubic feet

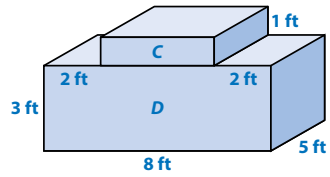
Prism *B* measures 5 ft × 3 ft × 2 ft.
Volume of Prism *B* = 30 cubic feet

Volume of planter = 54 + 30, or 84
The volume is 84 cubic feet.

1. Show how to find the volume of Prism *D*.
8 ft × 5 ft × 3 ft, or 120 cubic feet

2. Find the volume of Prism *C*.
volume = 5 ft × 4 ft × 1 ft, or 20 cubic feet

3. What is the volume of the whole figure?
120 + 20 = 140; volume = 140 cubic feet



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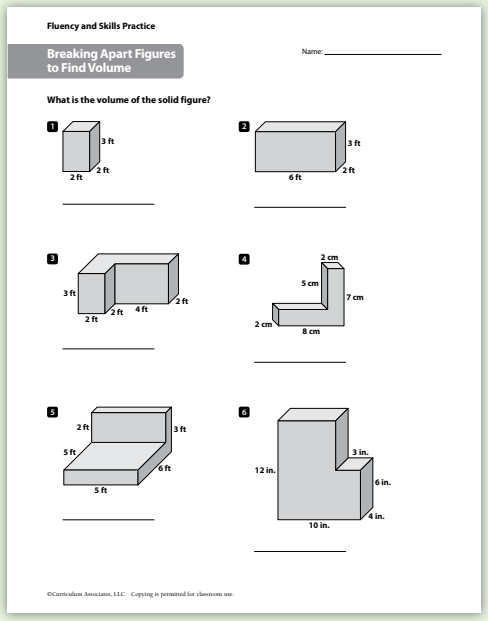
Fluency & Skills Practice

Teacher Toolbox

Assign *Breaking Apart Figures to Find Volume*

In this activity students find the volume of composite solids by breaking them apart into rectangular prisms, calculating the volume of these prisms, and then finding the sum of these volumes. Students may encounter real-world situations that involve finding the volumes of composite solids, such as finding the total volume of a pool with a shallow end and a deep end.


Fluency and Skills Practice

Breaking Apart Figures to Find Volume

Name: _____

What is the volume of the solid figure?

1. 3 ft, 2 ft, 2 ft
2. 3 ft, 6 ft, 2 ft
3. 3 ft, 2 ft, 2 ft, 4 ft, 2 ft
4. 2 cm, 5 cm, 7 cm, 2 cm, 8 cm
5. 2 ft, 3 ft, 5 ft, 6 ft, 5 ft
6. 3 in., 12 in., 6 in., 4 in., 10 in.

4 See Figures *A* and *B* on student page for possible answers. For either Figure *A* or Figure *B*, students should draw a vertical line between the rectangular prism on the left and the rectangular prism on the right. For the remaining figure, students should draw a horizontal line between the top and bottom rectangular prisms.
Basic

5 120 cubic inches; Answers will vary. See student page for a solution for dividing Figure *A* into a left part and a right part with a vertical line.
Medium

6 120 cubic inches; Answers will vary. See student page for a solution for dividing Figure *B* into a top part and a bottom part with a horizontal line.
Medium

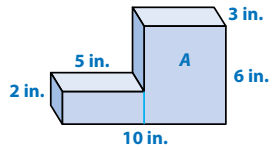
7 60 cubic feet; See student page for work that divides the figure into upper and lower prisms.
Medium

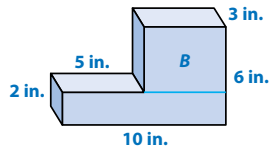
8 90 cubic meters; See student page for work that divides the figure into left, middle, and right prisms.
Challenge

4 Draw lines in Figures *A* and *B* to separate them into two rectangular prisms. Do each in a different way.

5 Show how to find the volume of Figure *A*.

Possible student work:
10 in. − 5 in. = 5 in.
5 in. × 3 in. × 6 in. = 90 cubic inches
5 in. × 3 in. × 2 in. = 30 cubic inches
90 + 30 = 120, so volume = 120 cubic inches





6 Show how to find the volume of Figure *B*.

Possible student work:
6 in. − 2 in. = 4 in. and 10 in. − 5 in. = 5 in.
5 in. × 3 in. × 4 in. = 60 cubic inches
10 in. × 3 in. × 2 in. = 60 cubic inches

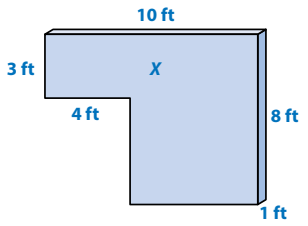
60 + 60 = 120, so volume = 120 cubic inches

7 What is the volume of Figure *X*? Show your work.

Possible student work:
10 ft × 3 ft × 1 ft = 30 cubic feet
6 ft × 5 ft × 1 ft = 30 cubic feet

30 + 30 = 60

Solution **60 cubic feet**

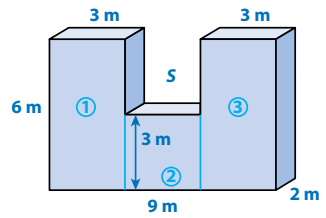


8 Show how to break Figure *S* into three rectangular prisms. Then find the volume of Figure *S*. Show your work.

Possible student work:
Prism 1: 3 m × 2 m × 6 m = 36 cubic meters
Prism 2: 3 m × 2 m × 3 m = 18 cubic meters
Prism 3: same as Prism 1

36 + 18 + 36 = 90

Solution **90 cubic meters**



ELL English Language Learners: Differentiated Instruction

Prepare for Session 4
Use with *Apply It*.

Levels 1–3	Levels 2–4	Levels 3–5
Speaking/Listening Read ***Apply It*** problem 1 and have the students choral read. Remind students to break down problems one sentence at a time. Explain that cement is a liquid material that turns hard over time. Say: *People fill things with cement to build sidewalks or buildings. When you fill something, you need to know the volume.* Ask questions and provide sentence frames to guide students through the process: • *What do you need to find? I need to find how much _____ I need to fill the sidewalk.* • *What do you need to calculate? I need to calculate _____.* Call on students to read the completed sentences.	**Reading/Speaking** Have students form pairs and read ***Apply It*** problem 1. Write the following questions on the board and have students take turns answering them with their partners: • *What type of angles does the diagram have?* • *Why do I need to know the type of angles the diagram has?* • *What do they use to make the sidewalk?* • *What do you need to calculate?* Have students restate the problem in their own words. Provide guidance as needed for students to use complete sentences.	**Reading/Writing** Have students form pairs and read ***Apply It*** problem 1. Have partners work together to describe the problem in their own words. Ask them to create a two-column chart with the following headers: *What we know* and *What we need to find out*. Have students complete the chart using complete sentences. Then call on pairs to read their charts to the class.

LESSON 3

SESSION 4 Refine

Purpose In this session students solve word problems involving finding volume using formulas and then discuss and confirm their answers with a partner.

Before students begin to work, use their responses to the *Check for Understanding* to determine those who will benefit from additional support.

As students complete the Example and problems 1–3, observe and monitor their reasoning to identify groupings for differentiated instruction.

Start

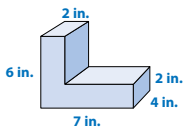
Check for Understanding

Materials For remediation: copy of Start slide, 2 different colored pencils, 88 unit cubes

Why Confirm understanding of finding volume using formulas.

How Have students find the volume of the L-shaped figure using any strategy they want.

Find the volume of this L-shaped solid figure.



Solution
88 cubic inches

Refine Finding Volume Using Formulas

Complete the Example below. Then solve problems 1–9.

EXAMPLE

Coen is making a clay vase. He wants the interior of the vase to be a rectangular prism with base 3 inches long and 3 inches wide. He wants the vase to hold 45 cubic inches of water. How tall should Coen make the vase?

Look at how you could use a formula to solve the problem.

$$\text{Volume} = \ell \times w \times h$$
$$45 = 3 \times 3 \times h$$
$$45 = 9 \times h$$
$$45 \div 9 = h$$
$$5 = h$$

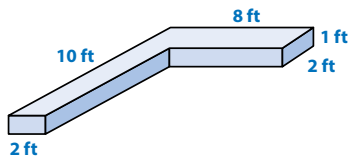
Solution 5 inches

The student wrote an equation using the formula for volume. The height is the unknown.

PAIR/SHARE
Did you and your partner solve the problem the same way?

APPLY IT

1 The diagram shows the dimensions of a cement walkway, where all of the sides meet at right angles. What is the total volume of cement needed to make the walkway? Show your work.



After you break apart the figure, how will you find the missing measurements?

Possible student work:

$10 \times 2 \times 1 = 20$
$6 \times 2 \times 1 = 12$
$20 + 12 = 32$

or

$8 \times 2 \times 1 = 16$
$8 \times 2 \times 1 = 16$
$16 + 16 = 32$

Solution 32 cubic feet

PAIR/SHARE
How did you and your partner decide how to break apart the solid figure?

Error Alert

If the error is . . .	*Students may . . .*	*To support understanding . . .*
56 cubic inches	have only found the volume of the bottom prism.	Have students use colored pencils to shade the two rectangular prisms they broke apart the figure into. Explain that the volume of the entire figure is the sum of the volumes of the two colored prisms. Prompt students to recognize and correct their error.
672 cubic inches	have multiplied the dimensions labeled in the diagram.	Use unit cubes to build an L-shaped model with the given dimensions. Have students count the total number of cubes in the figure. Emphasize that this is the volume of the figure. Then have students break the model into two rectangular prisms. Direct students to count the number of cubes in each prism. Point out that the sum of the volumes of the two prisms is equal to the volume they found by counting the total number of cubes in the figure. Prompt students to recognize and correct their error.

EXAMPLE

5 inches; Using the formula $V = \ell \times w \times h$ and a division equation is shown as one way to solve the problem. Students could also use the formula $V = b \times h$.

Look for You know two dimensions and the volume. The height is the unknown value. Divide the volume by the product of the length and width.

APPLY IT

1. 32 cubic feet; Students could solve the problem by dividing the figure of the walkway into left and right prisms or into front and back prisms. See possible equations for each method on the Student Worktext page.
   DOK 2

   Look for For a division into left and right prisms, the 8-foot-long back edge of the walkway is separated into parts of lengths 2 ft and 6 ft. For a division into front and back prisms, the 10-foot-long left edge of the walkway is separated into parts of lengths 8 ft and 2 ft.

2. 7 meters; Students could solve the problem by using the formula $V = \ell \times w \times h$.
   DOK 2

   Look for The width is 2 meters and the height is 3 meters; length is the unknown dimension.

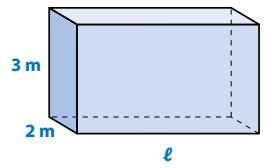
3. **C**; Students could solve the problem by using the formula $V = \ell \times w \times h$. Each dimension is 2 feet, so the volume is 2 feet $\times$ 2 feet $\times$ 2 feet.

   Explain why the other two answer choices are not correct:

   A is not correct because only two dimensions were multiplied, 2×2, instead of three dimensions, $2 \times 2 \times 2$.

   D is not correct because the three dimensions were added and then multiplied by 2, instead of the three dimensions being multiplied, $2 \times 2 \times 2$.
   DOK 3

2. The rectangular prism shown below has a volume of 42 cubic meters. What is the length of the prism? Show your work.



What dimensions are labeled on the prism?

Possible student work:

Volume $= \ell \times w \times h$
$42 = \ell \times 2 \times 3$
$42 = \ell \times 6$
$42 \div 6 = \ell$
$7 = \ell$

Solution **7 meters**

PAIR/SHARE
How could you check your answer?

3. A cube is a rectangular prism whose side lengths are all the same. What is the volume of a cube with a side length of 2 feet?

   Ⓐ 4 cubic feet

   Ⓑ 6 cubic feet

   Ⓒ 8 cubic feet

   Ⓓ 12 cubic feet

   Danny chose Ⓑ as the correct answer. How did he get that answer?

   Possible answer: Danny added the length, width, and height (2 + 2 + 2 = 6) instead of multiplying.

Each side of a cube is a square.

PAIR/SHARE
Does Danny's answer make sense?

4 **C**; The length and width are both 5 cm;
5 cm × 5 cm × 7 cm = 175 cubic centimeters.
DOK 2

5 **B**; Break apart the mold horizontally into a 5 in. by 2 in. by 3 in. prism and a 7 in. by 5 in. by 3 in. prism.

C; Break apart the mold vertically into a 5 in. by 2 in. by 6 in. prism and a 5 in. by 5 in. by 3 in. prism.
DOK 2

Error Alert Students who choose A have not accounted for the 5 in. by 2 in. by 3 in. part on the lower left side, while students who choose D have counted that part twice.

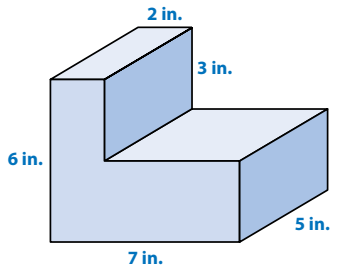
6 1,100 cubic meters; The prisms are identical, so the length of the left prism is 10 m; find the volume of one rectangular prism and then multiply the result by 2.
DOK 2

4 A rectangular prism has a square base with side lengths of 5 centimeters and a height of 7 centimeters. What is the volume of the prism?

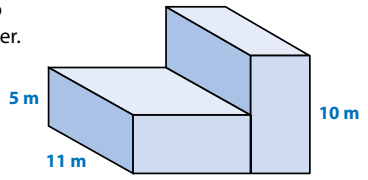
Ⓐ 35 cubic centimeters
Ⓑ 140 cubic centimeters
Ⓒ 175 cubic centimeters
Ⓓ 245 cubic centimeters

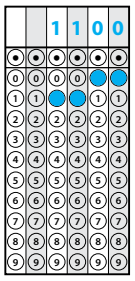
5 The diagram below shows the measurements of a mold used to make sandcastles. Which expressions can be used to find the volume of the mold, in cubic inches?

Ⓐ (5 × 2 × 3) + (5 × 5 × 3)
Ⓑ (5 × 2 × 3) + (7 × 5 × 3)
Ⓒ (5 × 2 × 6) + (5 × 5 × 3)
Ⓓ (5 × 2 × 6) + (7 × 5 × 3)
Ⓔ (5 × 2 × 3) + (7 × 5 × 6)



6 The diagram shows the dimensions of two identical rectangular prisms joined together. What is the combined volume, in cubic meters, of the two prisms?





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Differentiated Instruction

RETEACH

Hands-On Activity

Use a model to find the volume of an L-shaped figure.

Students struggling with breaking apart complex figures to find volume

Will benefit from additional work with a concrete model of a solid figure composed of two rectangular prisms

Materials For each pair: 2 rectangular boxes of the same height, tape, sticky notes

- Draw a model of an L-shaped aquarium and label the dimensions in feet.
- Distribute two boxes, tape, and sticky notes to each pair of students. Have students use the boxes to build a model of the aquarium. Tell students to use sticky notes to label the edges of their model with their dimensions.
- Have students use their model to describe how they could break apart the aquarium into two rectangular prisms. Then have them find the volume of the aquarium.
- Repeat with other dimensions.

EXTEND

Challenge Activity

Reason about a solid figure composed of two cubes.

Students who have achieved proficiency

Will benefit from deepening understanding of volume comparisons

- Tell students there is a solid figure composed of a cube with edge lengths of 2 inches stacked on top of a larger cube. The edge lengths of the larger cube are twice as long as the edge lengths of the smaller cube.
- Ask students: *How many times as great is the volume of the solid figure than the volume of the 2-inch cube?* [9 times as great]

7 Answers will vary, but the product of the three dimensions must equal 60. For example:

6 feet × 1 feet × 10 feet

5 feet × 2 feet × 6 feet

4 feet × 3 feet × 5 feet

DOK 2

8 360 cubic feet; Students may divide the prism into a left part (12 ft × 6 ft × 4 ft) and a right part (6 ft × 6 ft × 2 ft) or into a top part (18 ft × 6 ft × 2 ft) and a bottom part (12 ft × 6 ft × 2 ft). See possible equations for each method on the Student Worktext page.

DOK 2

7 A cardboard box has a volume of 60 cubic feet. Give three different sets of measurements that could be the dimensions of the box.

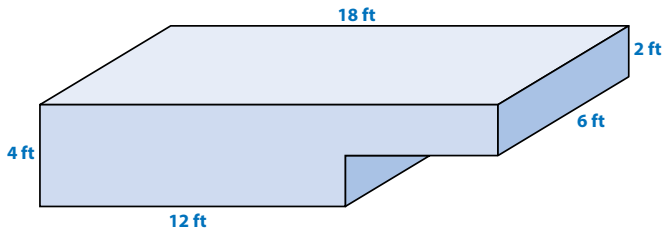
6 feet × 1 feet × 10 feet

5 feet × 2 feet × 6 feet

4 feet × 3 feet × 5 feet

Sample answers shown. Accept any answer where the product of the three numbers is 60.

8 Rami designed a small pond for a restaurant. The diagram below shows the measurements of the pond. How many cubic feet of water are needed to fill the pond? Show your work.



Possible student work:

$12 \times 6 \times 4 = 288$
$6 \times 6 \times 2 = 72$
$288 + 72 = 360$

or

$18 \times 6 \times 2 = 216$
$12 \times 6 \times 2 = 144$
$216 + 144 = 360$

360 cubic feet

9 MATH JOURNAL

Describe a real-world object that can be modeled by a rectangular prism and give its dimensions. Use a formula to find the volume of the object.

Possible answer: A shoe box has a length of 15 inches, a width of 7 inches, and a height of 4 inches. $V = \ell \times w \times h = 15 \times 7 \times 4 = 420$. The volume of the shoe box is 420 cubic inches.

SELF CHECK Go back to the Unit 1 Opener and see what you can check off.

REINFORCE

Problems 4–9

Find volume using formulas.

All students will benefit from additional work finding volume by solving problems in a variety of formats.

- Have students work on their own or with a partner to solve the problems.
- Encourage students to show their work.

PERSONALIZE

i-Ready

Provide students with opportunities to work on their personalized instruction path with *i-Ready* Online Instruction to:

- fill prerequisite gaps
- build up grade-level skills

Close: Exit Ticket

9 MATH JOURNAL

Student responses should include a description of an appropriate rectangular object with length, width, and height and also should show correct use of a volume formula. The volume of the object should be stated with the correct cubic unit.

Error Alert **If** students multiply only two of the dimensions when using the formula $V = b \times h$, **then** review the meaning of each letter, stating that b stands for the *area of the base*. Then ask students to work with a rectangular prism found in the classroom. Have them use the length and width of the prism to find the area of the base and then use the formula $V = b \times h$ to find the volume.

SELF CHECK Have students consider whether they feel they are ready to check off any new skills on the Unit 1 Opener.