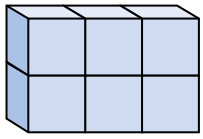
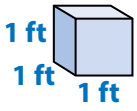
Explore Finding Volume Using Unit Cubes

Previously, you learned that you can fill a solid figure with unit cubes to find its volume. Use what you know to try to solve the problem below.

Carl filled the clear box shown below with unit cubes to find its volume. The unit cubes Carl used all have side lengths of 1 foot. What is the volume of the box?





Learning Targets

- Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.
- Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes.

SMP 1, 2, 3, 4, 5, 6, 7

TRY IT

Math Toolkit

- unit cubes
- grid paper
- isometric dot paper
- square sticky notes

DISCUSS IT

Ask your partner: How did you get started?

Tell your partner: I started by . . .

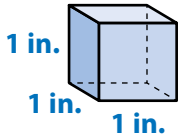
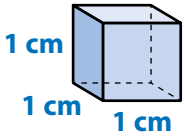
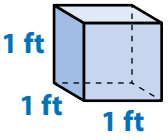
CONNECT IT

1 LOOK BACK

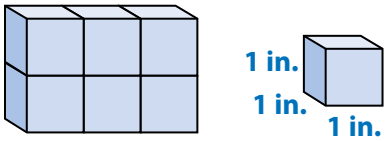
Describe the measurement unit Carl should use and explain how he can find the volume of the box.

2 LOOK AHEAD

Volume is measured in cubic units. Here are some cubic units you might use to measure volume.

Unit of Volume	cubic inch	cubic centimeter	cubic foot
Unit Cube			

Carl has another box as shown below. What is its volume?



3 REFLECT

What is the same and what is different about the volume of Carl's two boxes?

Prepare for Finding Volume Using Unit Cubes

1 Think about what you know about solid figures. Fill in each box.
Use words, numbers, and pictures. Show as many ideas as you can.

What Is It?

What I Know About It

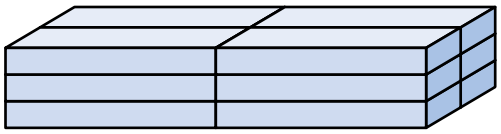
rectangular prism

Examples

Examples

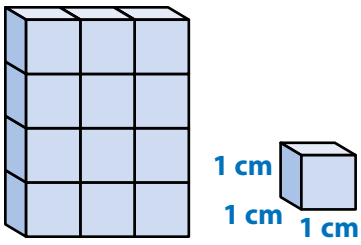
Examples

2 Instead of using cubic units to measure volume, Paulina wants to measure volume using boxes of pencils. What is the volume of this rectangular prism using boxes of pencils as the unit of measure?



3 Solve the problem. Show your work.

Jan filled the box shown below with unit cubes to find its volume. The unit cubes Jan used all have side lengths of 1 centimeter. What is the volume of the box?



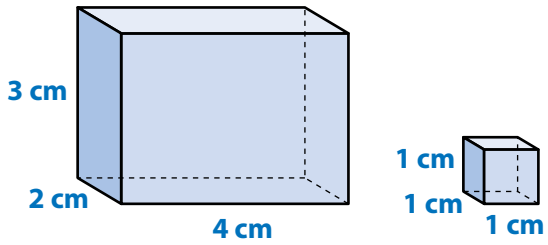
Solution

4 Check your answer. Show your work.

Develop Finding Volume Using Unit Cubes

Read and try to solve the problem below.

Abigail uses cardboard to build a rectangular prism like the one shown below. What is the volume of the prism?



TRY IT

Math Toolkit

- unit cubes
- 1-cm grid paper
- isometric dot paper

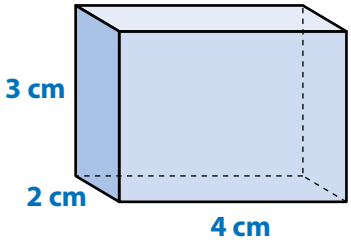
DISCUSS IT

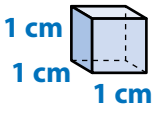
Ask your partner: Why did you choose that strategy?

Tell your partner: I knew . . . so I . . .

Explore different ways to understand finding the volume of a rectangular prism.

Abigail uses cardboard to build a rectangular prism like the one shown below. What is the volume of the prism?

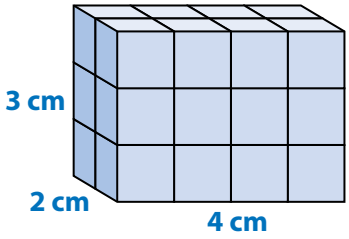




PICTURE IT

You can find the volume of the prism by filling it with unit cubes and counting the number of cubes.

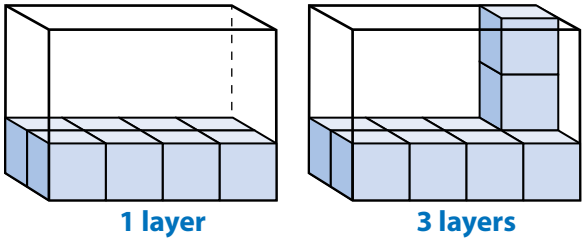
Use unit cubes that are each 1 cubic centimeter.



MODEL IT

You can also find the volume by counting the number of cubes in one layer and the number of layers.

Use unit cubes that are each 1 cubic centimeter.



CONNECT IT

Now you will use the problem from the previous page to help you understand how to find the volume of a rectangular prism.

1. Look at the model in **Picture It** on the previous page. Count the number of cubes in one layer. There are _____ cubes in one layer.

2. How could you find the number of cubes in one layer without counting the cubes?

3. Once you know how many cubes are in one layer, what else do you need to know to find the volume?

4. There are 8 cubes in each layer and there are 3 layers. What multiplication expression can you write to find the volume of the prism? _____

5. What is the volume of Abigail's rectangular prism? _____

6. Explain how you can use multiplication to find the volume of a rectangular prism.

7 REFLECT

Look back at your **Try It**, strategies by classmates, and **Picture It** and **Model It**. Which models or strategies do you like best for finding the volume of a rectangular prism? Explain.

_____

_____

_____

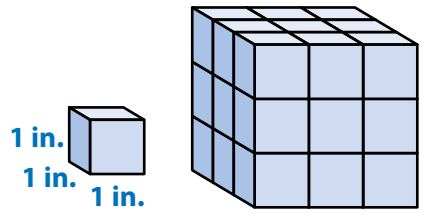
_____

_____

APPLY IT

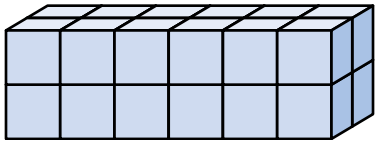
Use what you just learned to solve these problems.

8 What is the volume of the rectangular prism at the right? Show your work.



Solution

9 Mr. Wong finds the volume of a box by filling it with 1-foot unit cubes, as shown below. What is the volume of Mr. Wong's box? Show your work.



Solution

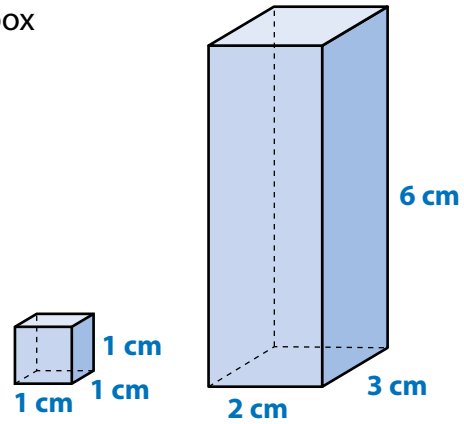
10 Jamila wants to find the volume of the rectangular box at the right. What is the volume of the box?

Ⓐ 18 cubic units

Ⓑ 30 cubic centimeters

Ⓒ 36 cubic centimeters

Ⓓ 36 cubic inches

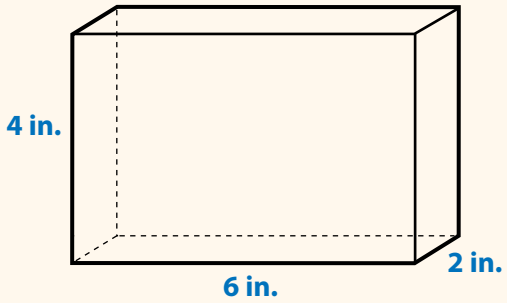


Practice Finding Volume Using Unit Cubes

Study the Example showing how to use layers to find the volume of a rectangular prism. Then solve problems 1–7.

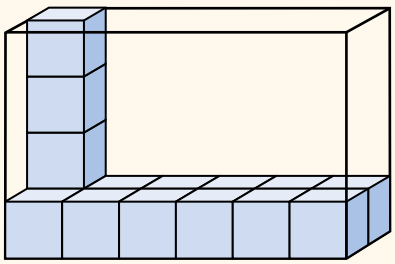
EXAMPLE

Keith uses this box to store his colored markers. What is the volume of the box?



Think about filling the box with 1-inch cubes. One layer has 2 rows of 6 cubes, or 12 cubes. There are 4 layers of cubes.

$12 + 12 + 12 + 12 = 48$, or $12 \times 4 = 48$



The volume of the box is 48 cubic inches.

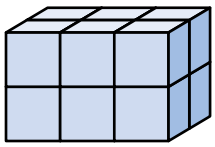
1 Prism *G* is filled with unit cubes that have side length 1 centimeter.

There are _____ layers with _____ cubes in each layer.

_____ cubes + _____ cubes = _____ cubes

The volume is _____.

G



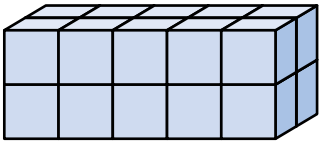
2 Prism *H* is filled with unit cubes that have side length 1 foot.

There are _____ layers with _____ cubes in each layer.

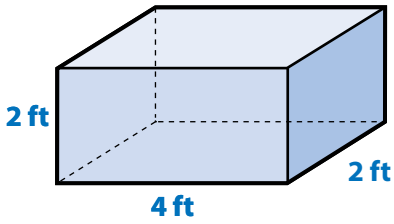
_____ × _____ cubes = _____ cubes

The volume is _____.

H

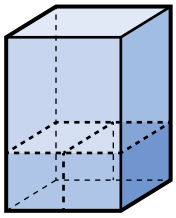


3 What is the volume of the rectangular prism at the right? Show your work.



Solution

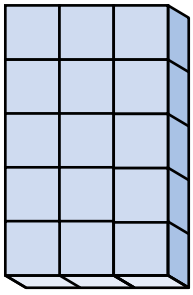
4 Jenn noticed that she can fit two juice boxes side by side on the bottom of this box. She can make two more layers like the one shown to fill the box. Using a juice box as a measure of volume, what is the volume of the larger box?



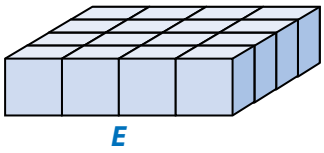
Solution

5 A box is 2 inches long, 1 inch wide, and 6 inches tall. What is the relationship between the volume of this box and the one in problem 4? Tell how you know.

6 Box *D* and Box *E* are made from unit cubes of the same size. Which has a greater volume, Box *D* or Box *E*? Explain.



7 Add a layer to Box *D* and compare the volume of the new Box *D* to the volume of Box *E*.



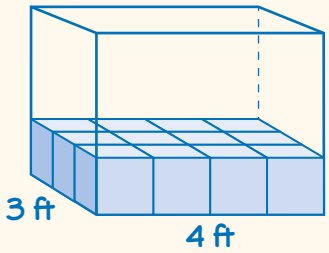
Refine Finding Volume Using Unit Cubes

Complete the Example below. Then solve problems 1–9.

EXAMPLE

Pedro has a storage box with a volume of 36 cubic feet. He knows that the box is 4 feet long and 3 feet wide. How high is the box?

Look at how you could show your work using a drawing and multiplication facts.



$4 \times 3 = 12$

$12 \times \square = 36$

$12 \times 3 = 36$

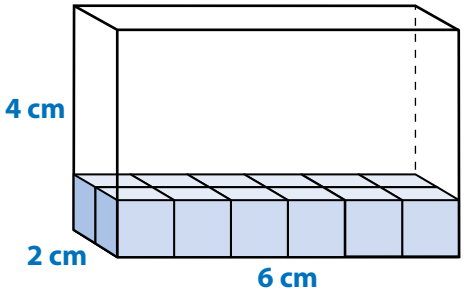
Solution

The student started by finding the number of cubes in the bottom layer.

PAIR/SHARE
Could you solve this problem another way?

APPLY IT

1 A box measures 6 centimeters long, 2 centimeters wide, and 4 centimeters high. What is the volume of the box? Show your work.

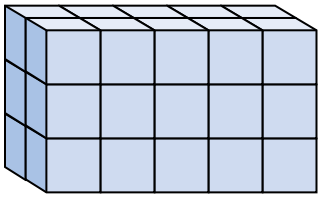


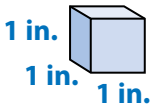
Solution

How many layers of cubes will there be in the box?

PAIR/SHARE
Can you use multiplication to solve this problem?

2 Kamala made the figure below using cubes. What is the volume of Kamala's figure? Show your work.



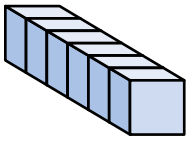


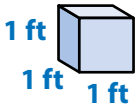
How many cubes are there in each layer?

PAIR/SHARE
How did you decide which method to use to solve the problem?

Solution

3 What is the volume of the rectangular prism below?





There is more than one way to find the volume of a rectangular prism.

Ⓐ 6 square feet

Ⓑ 6 cubic feet

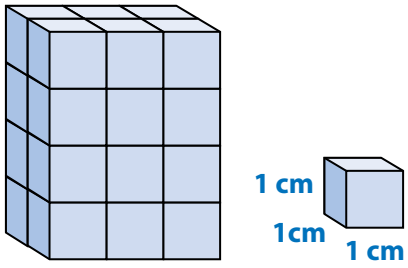
Ⓒ 8 square feet

Ⓓ 8 cubic feet

Nam chose Ⓓ as the correct answer. How did he get that answer?

PAIR/SHARE
Does Nam's answer make sense?

4 How many 1-centimeter unit cubes are in the bottom layer of the rectangular prism at the right?



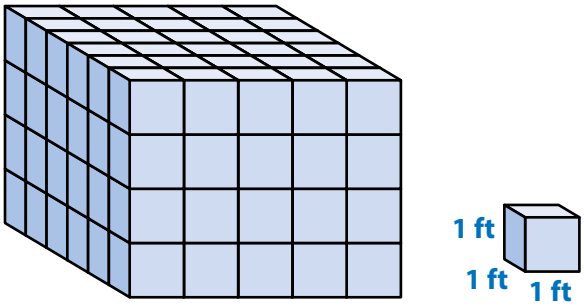
Ⓐ 3

Ⓑ 6

Ⓒ 12

Ⓓ 24

5 Which expressions can be used to find the volume, in cubic feet, of the rectangular prism at the right?



Ⓐ 30×4

Ⓑ $(5 + 6) \times 4$

Ⓒ $30 + 30 + 30 + 30$

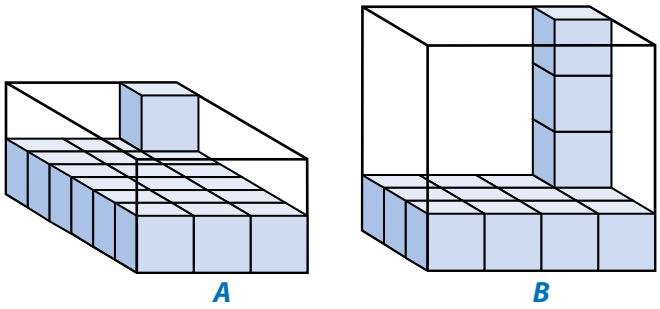
Ⓓ $5 + 6 + 4$

Ⓔ $30 + 4$

6 Flora has a rectangular gift box that has a volume of 24 cubic inches. The box is 2 inches tall. Determine if the gift box could have the given length and width.

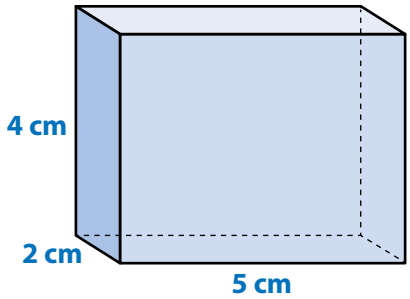
	Yes	No
Length: 11 inches; Width: 11 inches	Ⓐ	Ⓑ
Length: 4 inches; Width: 3 inches	Ⓒ	Ⓓ
Length: 10 inches; Width: 2 inches	Ⓔ	Ⓕ
Length: 6 inches; Width: 2 inches	Ⓖ	Ⓗ
Length: 12 inches; Width: 1 inch	Ⓘ	Ⓙ

7 Both figures are filled with unit cubes of the same size. Which rectangular prism has the greater volume, Figure *A* or Figure *B*? Show your work.



..... has the greater volume.

8 **Part A** Mato drew the rectangular prism shown below. Draw and label a different rectangular prism with the same volume as Mato's prism.



Part B Explain how you know that the volume of your prism is the same as the volume of Mato's prism.

9 MATH JOURNAL

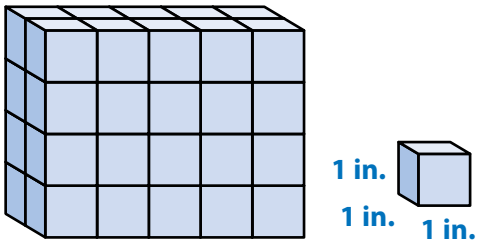
Jorge uses 1-centimeter cubes to make a rectangular prism. Each layer of his prism is 5 cubes long and 2 cubes wide. His prism has 6 layers. Explain two ways to find the volume of Jorge's prism.

SELF CHECK Go back to the Unit 1 Opener and see what you can check off.

Explore Finding Volume Using Formulas

Previously, you learned how to find volume by counting unit cubes and by using addition and multiplication strategies. Now you will learn how to find volume using a formula. Use what you know to try to solve the problem below.

Becky uses 1-inch cubes to create a model for a small paper gift bag she is making. Her model is a rectangular prism. What is the volume of Becky's model?



Learning Targets

- Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems.
- Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.

SMP 1, 2, 3, 4, 5, 6, 7, 8

TRY IT

Math Toolkit

- unit cubes
- grid paper
- isometric dot paper

DISCUSS IT

Ask your partner: Do you agree with me? Why or why not?

Tell your partner: I agree with you about . . . because . . .

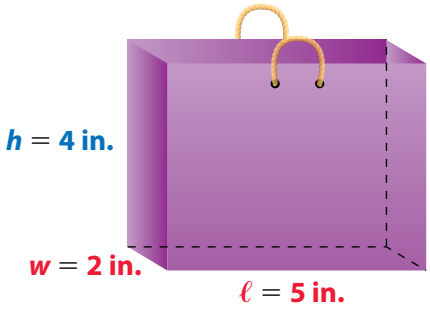
CONNECT IT

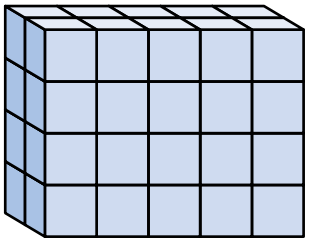
1 LOOK BACK

Explain how you can find the volume of Becky's model without counting each cube.

2 LOOK AHEAD

Becky's finished gift bag is shown below. You can find the volume with a formula. Use the letters ℓ, ***w***, and ***h*** to represent the **length**, **width**, and **height**.





a. The bottom of a rectangular prism can be called its **base**. The letter ***b*** is used to represent the **area of the base**.

Write an equation using the area of the base to find the bag's volume ***V***. Then complete the formula with a letter.

..... = ... × .....

V = ***b*** × .....

b. Use the letters shown in the formula below to help you complete another equation for the volume of the bag. Then complete the formula.

..... = ... × ..... × .....

V = ℓ × ***w*** × .....

3 REFLECT

How are the two volume formulas you wrote above alike? How are they different?

.....

.....

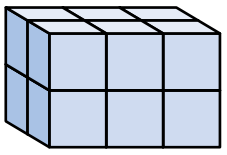
.....

Prepare for Finding Volume Using Formulas

1 Think about what you know about formulas. Fill in each box. Use words, numbers, and pictures. Show as many ideas as you can.

In My Own Words	My Illustrations
formula	
Examples	Non-Examples

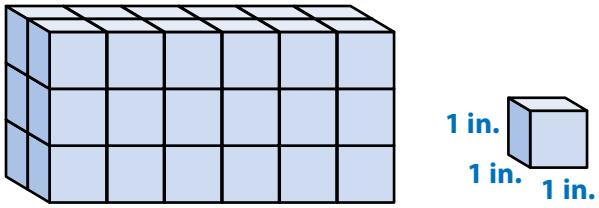
2 Find the volume of the rectangular prism. Explain.



1 unit cube

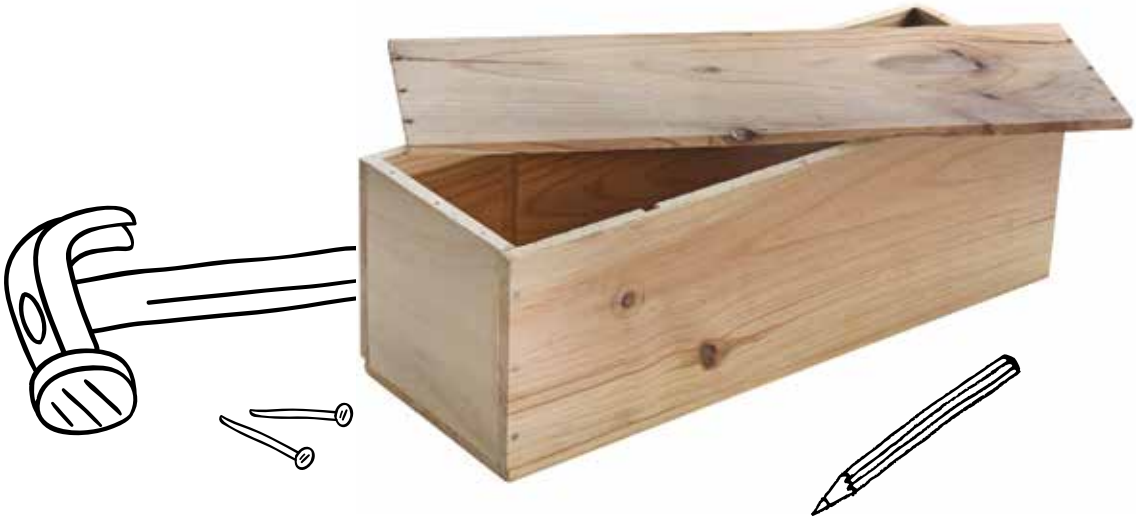
3 Solve the problem. Show your work.

Adrien uses 1-inch cubes to create a model for a small box he is making. His model is a rectangular prism. What is the volume of Adrien's model?



Solution

4 Check your answer. Show your work.



Develop Finding Volume Using Formulas

Read and try to solve the problem below.

Gareth has a rectangular pencil cup on his desk. The cup is 3 inches long, 2 inches wide, and 5 inches high. What is the volume of the pencil cup?

TRY IT

Math Toolkit

- unit cubes
- grid paper
- isometric dot paper

DISCUSS IT

Ask your partner: Why did you choose that strategy?

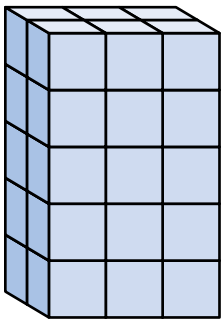
Tell your partner: I knew . . . so I . . .

Explore different ways to understand using the dimensions of a rectangular prism to find its volume.

Gareth has a rectangular pencil cup on his desk. The cup is 3 inches long, 2 inches wide, and 5 inches high. What is the volume of the pencil cup?

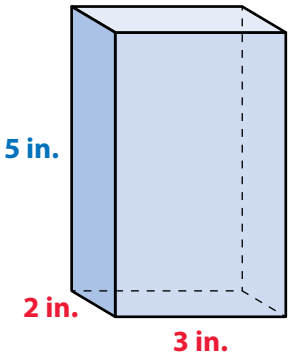
PICTURE IT

You can picture the pencil cup as a rectangular prism made up of 1-inch cubes.



MODEL IT

You can draw a model of the pencil cup and label its dimensions. Then use a volume formula.



$V = \ell \times w \times h$ or $V = b \times h$

CONNECT IT

Now you will use the problem from the previous page to help you understand and use formulas to find volume.

1. Explain how you can find the volume of the pencil cup using the prism shown in **Picture It**. Find the volume.

2. Explain how the area of the base of the prism shown in **Model It** relates to the prism in **Picture It**.

3. Explain how you can use the prism in **Model It** to find the volume of the pencil cup.

4. Use the volume formulas shown in **Model It** to write two different multiplication equations you can use to find the volume of the pencil cup.

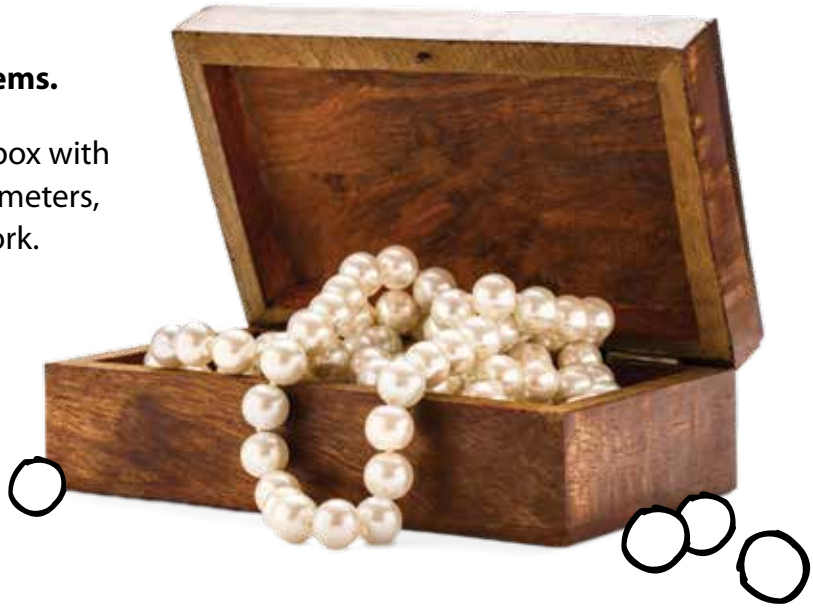
5. Explain how you can use the dimensions of a rectangular prism to find its volume.

6 REFLECT

Look back at your **Try It**, strategies by classmates, and **Picture It** and **Model It**. Which models or strategies do you like best for using the dimensions of a rectangular prism to find its volume? Explain.

APPLY IT

Use what you just learned to solve these problems.



7 What is the volume of a rectangular jewelry box with a length of 12 centimeters, a width of 7 centimeters, and a height of 5 centimeters? Show your work.

Solution

8 How much space is taken up by a book that is 12 inches long, 10 inches wide, and 1 inch tall? Show your work.

Solution

9 A rectangular prism has a volume of 100 cubic meters. One of the dimensions is 5 meters. Which pairs of measurements could be the other two dimensions of the prism?

Ⓐ 1 meter, 20 meters

Ⓑ 5 meters, 10 meters

Ⓒ 10 meters, 10 meters

Ⓓ 4 meters, 5 meters

Ⓔ 20 meters, 20 meters

Practice Finding Volume Using Formulas

Study the Example showing how to use formulas to find the volume of a rectangular prism. Then solve problems 1–7.

EXAMPLE

Gwen puts her leftover food in a rectangular container. The container is 6 inches long, 5 inches wide, and 2 inches tall. What is the volume of the container?

Use the formula *volume* = *length* × *width* × *height*.

$V = \ell \times w \times h = 6 \times 5 \times 2$, or 60 cubic inches

Or use the formula *volume* = *area of the base* × *height*.

The *area of the base* is the same as the *length* × *width*.

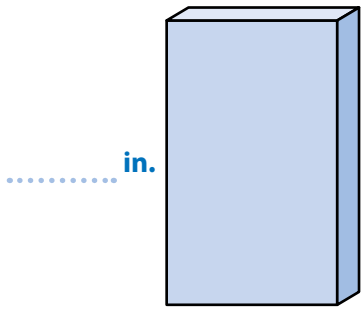
$b = 6 \times 5$, or 30

$V = b \times h = 30 \times 2$, or 60 cubic inches

1 Ted's box is 4 inches tall, 3 inches long, and 1 inch wide.

a. Label the picture of the box with its dimensions.

b. What is the volume of the box? Show your work.



Solution .....

2 A rectangular prism has a square base with sides that are 2 feet long. The height of the prism is 5 feet. What is the volume of the prism? Show your work.

Solution .....

3 Elon's shed is 10 feet long, 6 feet wide, and 8 feet tall. What is the volume of the shed? Show your work.

Solution

4 The base of a rectangular prism has a length of 4 centimeters and has a width of 2 centimeters. The height of the prism is 3 centimeters. What is the volume of the prism? Show your work.

Solution

5 What is the volume of a box that is 8 inches long, 2 inches wide, and 6 inches tall? Show your work.

Solution

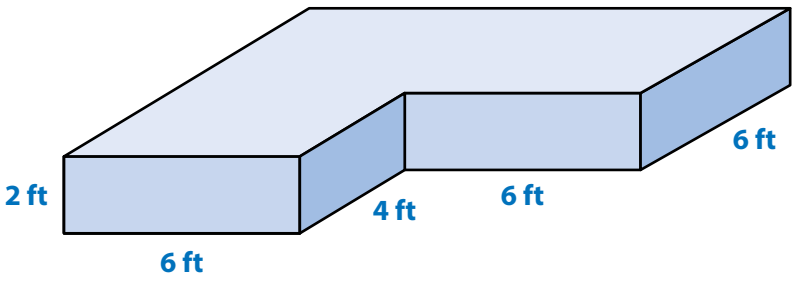
6 The base of a rectangular prism is a rectangle that is 7 inches long and 5 inches wide. Its height is 10 inches. Write two different equations that you can use to find the volume.

7 Jin has two boxes. Box *A* has dimensions of 6 centimeters, 5 centimeters, and 9 centimeters. Box *B* has dimensions of 4 centimeters, 10 centimeters, and 7 centimeters. Which box holds more? Explain.

Develop Breaking Apart Figures to Find Volume

Read and try to solve the problem below.

Bethany has a raised garden bed. The diagram shows its measurements. All the corners are right angles. If she fills the bed to the top with soil, how many cubic feet of soil will Bethany need?



TRY IT

Math Toolkit

- unit cubes
- grid paper
- isometric dot paper

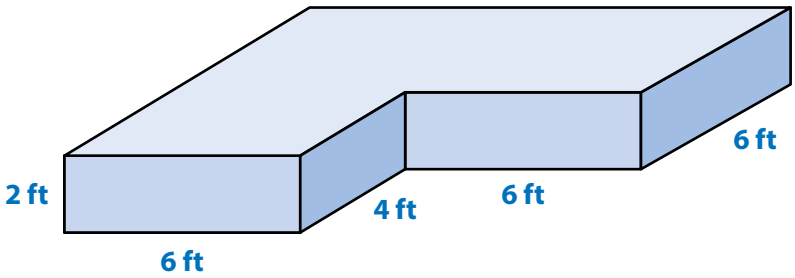
DISCUSS IT

Ask your partner: Can you explain that again?

Tell your partner: I knew . . . so I . . .

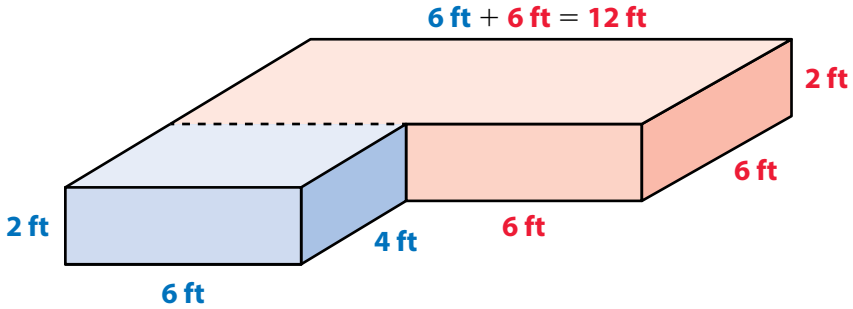
Explore different ways to understand finding the volume of a solid figure by breaking it apart into two rectangular prisms.

Bethany has a raised garden bed. The diagram shows its measurements. All the corners are right angles. If she fills the bed to the top with soil, how many cubic feet of soil will Bethany need?



MODEL IT

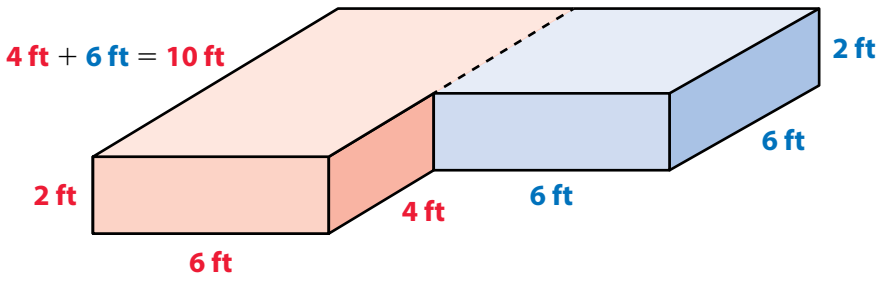
You can break the garden bed into two rectangular prisms this way.



One rectangular prism measures **6 feet** × **4 feet** × **2 feet**.
The other rectangular prism measures **12 feet** × **6 feet** × **2 feet**.

MODEL IT

You can also break the garden bed into two rectangular prisms in a different way.



One rectangular prism measures **10 feet** × **6 feet** × **2 feet**.
The other rectangular prism measures **6 feet** × **6 feet** × **2 feet**.

Total volume: (**10 feet** × **6 feet** × **2 feet**) + (**6 feet** × **6 feet** × **2 feet**)

CONNECT IT

Now you will use the problem from the previous page to help you understand ways to break apart a solid figure to find its volume.

1. Look at the first **Model It**. How can you find the volume of each rectangular prism?

2. How can you find the volume of the entire garden bed?

3. What is the volume of the entire garden bed? Show your work.

4. Now look at the second **Model It**. Show how to use the expression to find the volume of the garden bed if you break it apart this way. When you evaluate an expression, do the operations in the parentheses first.

5. Do you need to break apart a solid figure in a certain way to find its volume? Use the problem from the previous page to explain your reasoning.

6 REFLECT

Look back at your **Try It**, strategies by classmates, and **Model Its**. Which models or strategies do you like best for breaking apart solid figures to find volume? Explain.

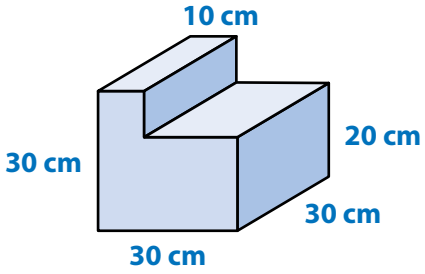
APPLY IT

Use what you just learned to solve these problems.

7 The Recreation Center has an L-shaped pool. One part of the pool is 8 meters by 6 meters. The other part is 12 meters by 6 meters. The whole pool is 4 meters deep. What is the volume of the entire pool? Show your work.

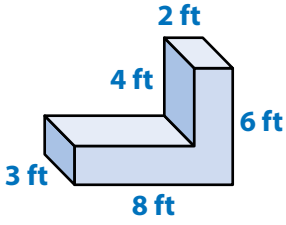
Solution

8 What is the volume of the solid figure below? Show your work.



Solution

9 What is the volume of the solid figure below? Show your work.



Solution

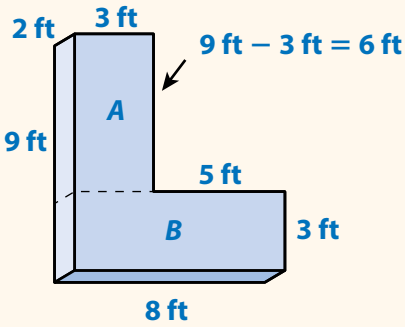
Practice Breaking Apart Figures to Find Volume

Study the Example showing how to break apart a solid figure into rectangular prisms and find its volume. Then solve problems 1–8.

EXAMPLE

Molly wants to know how much soil she needs to fill her two-tiered planter, shown below. What is the volume of the planter?

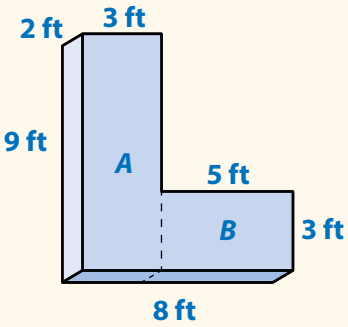
You can break the figure into two rectangular prisms in different ways.



Prism *A* measures 6 ft × 3 ft × 2 ft.
Volume of Prism *A* = 36 cubic feet

Prism *B* measures 8 ft × 3 ft × 2 ft.
Volume of Prism *B* = 48 cubic feet

Volume of planter = 36 + 48, or 84
The volume is 84 cubic feet.

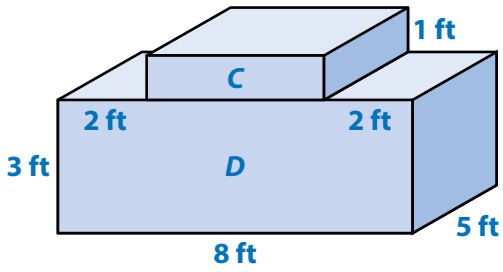


Prism *A* measures 9 ft × 3 ft × 2 ft.
Volume of Prism *A* = 54 cubic feet

Prism *B* measures 5 ft × 3 ft × 2 ft.
Volume of Prism *B* = 30 cubic feet

Volume of planter = 54 + 30, or 84
The volume is 84 cubic feet.

1. Show how to find the volume of Prism *D*.

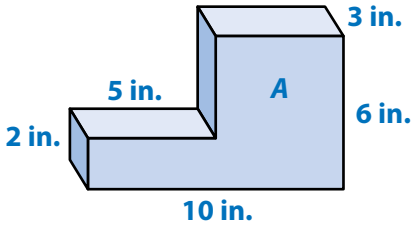


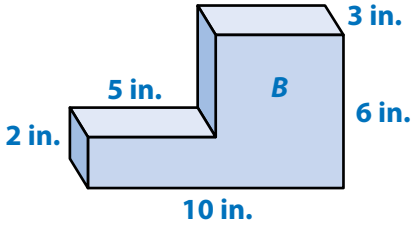
2. Find the volume of Prism *C*.

3. What is the volume of the whole figure?

4 Draw lines in Figures *A* and *B* to separate them into two rectangular prisms. Do each in a different way.

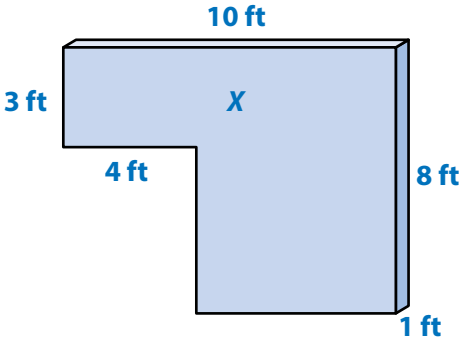
5 Show how to find the volume of Figure *A*.





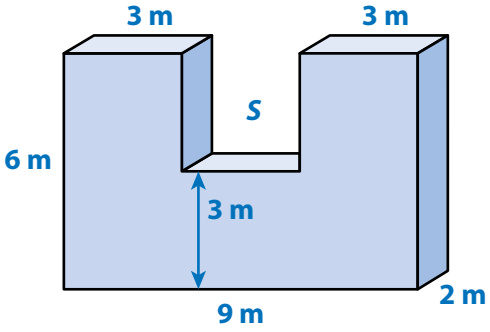
6 Show how to find the volume of Figure *B*.

7 What is the volume of Figure *X*? Show your work.



Solution

8 Show how to break Figure *S* into three rectangular prisms. Then find the volume of Figure *S*. Show your work.



Solution

Refine Finding Volume Using Formulas

Complete the Example below. Then solve problems 1–9.

EXAMPLE

Coen is making a clay vase. He wants the interior of the vase to be a rectangular prism with base 3 inches long and 3 inches wide. He wants the vase to hold 45 cubic inches of water. How tall should Coen make the vase?

Look at how you could use a formula to solve the problem.

$$\text{Volume} = \ell \times w \times h$$
$$45 = 3 \times 3 \times h$$
$$45 = 9 \times h$$
$$45 \div 9 = h$$
$$5 = h$$

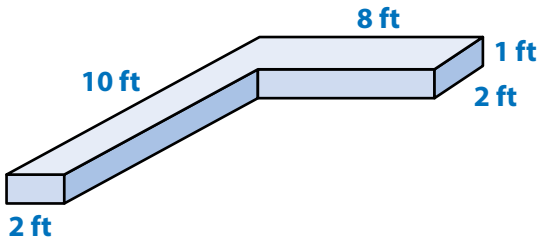
Solution .....

The student wrote an equation using the formula for volume. The height is the unknown.

PAIR/SHARE
Did you and your partner solve the problem the same way?

APPLY IT

1 The diagram shows the dimensions of a cement walkway, where all of the sides meet at right angles. What is the total volume of cement needed to make the walkway? Show your work.

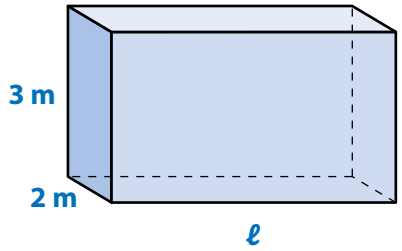


After you break apart the figure, how will you find the missing measurements?

PAIR/SHARE
How did you and your partner decide how to break apart the solid figure?

Solution .....

2 The rectangular prism shown below has a volume of 42 cubic meters. What is the length of the prism? Show your work.



What dimensions are labeled on the prism?

PAIR/SHARE
How could you check your answer?

Solution

3 A cube is a rectangular prism whose side lengths are all the same. What is the volume of a cube with a side length of 2 feet?

Ⓐ 4 cubic feet

Ⓑ 6 cubic feet

Ⓒ 8 cubic feet

Ⓓ 12 cubic feet

Danny chose Ⓑ as the correct answer. How did he get that answer?

Each side of a cube is a square.

PAIR/SHARE
Does Danny's answer make sense?

4 A rectangular prism has a square base with side lengths of 5 centimeters and a height of 7 centimeters. What is the volume of the prism?

Ⓐ 35 cubic centimeters

Ⓑ 140 cubic centimeters

Ⓒ 175 cubic centimeters

Ⓓ 245 cubic centimeters

5 The diagram below shows the measurements of a mold used to make sandcastles. Which expressions can be used to find the volume of the mold, in cubic inches?

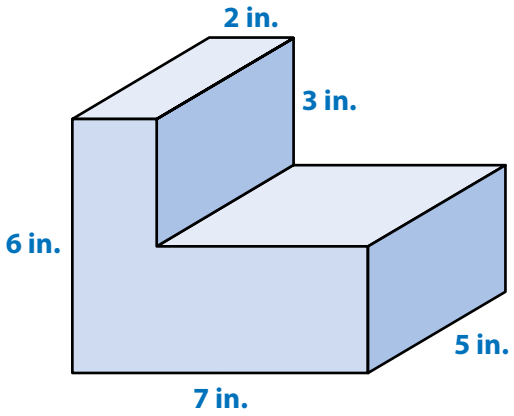
Ⓐ $(5 \times 2 \times 3) + (5 \times 5 \times 3)$

Ⓑ $(5 \times 2 \times 3) + (7 \times 5 \times 3)$

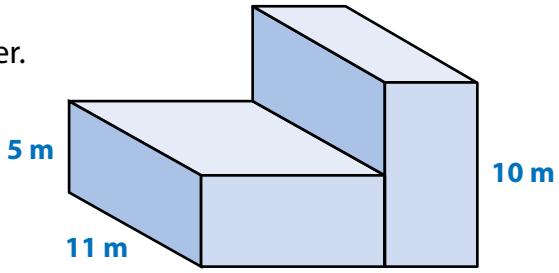
Ⓒ $(5 \times 2 \times 6) + (5 \times 5 \times 3)$

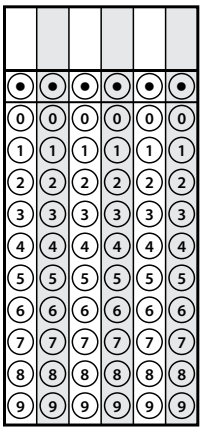
Ⓓ $(5 \times 2 \times 6) + (7 \times 5 \times 3)$

Ⓔ $(5 \times 2 \times 3) + (7 \times 5 \times 6)$



6 The diagram shows the dimensions of two identical rectangular prisms joined together. What is the combined volume, in cubic meters, of the two prisms?





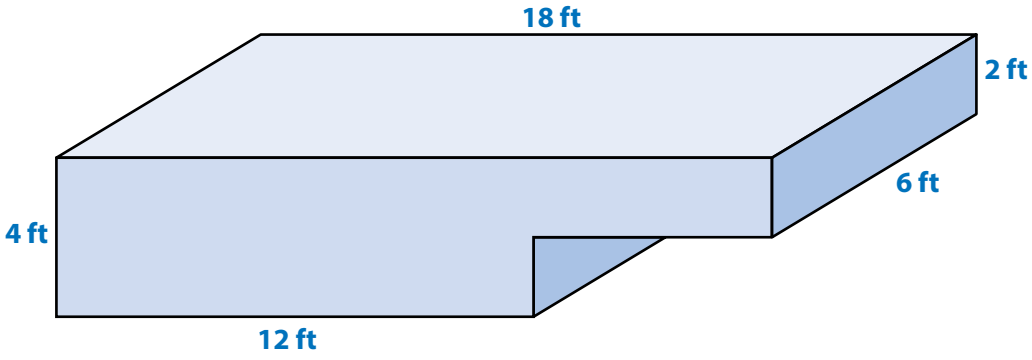
7 A cardboard box has a volume of 60 cubic feet. Give three different sets of measurements that could be the dimensions of the box.

..... feet × ..... feet × ..... feet

..... feet × ..... feet × ..... feet

..... feet × ..... feet × ..... feet

8 Rami designed a small pond for a restaurant. The diagram below shows the measurements of the pond. How many cubic feet of water are needed to fill the pond? Show your work.



..... cubic feet

9 MATH JOURNAL

Describe a real-world object that can be modeled by a rectangular prism and give its dimensions. Use a formula to find the volume of the object.

SELF CHECK Go back to the Unit 1 Opener and see what you can check off.