

Explore Points, Lines, Rays, and Angles



Learning Targets

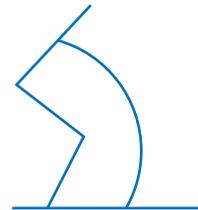
- Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.
- Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement.

SMP 1, 2, 3, 4, 5, 6

Previously, you have learned about shapes such as squares, rectangles, and triangles. Now you will learn more about what makes up these shapes. Use what you know to try to solve the problem below.

Traci tries to teach her younger sister how to draw a rectangle. Traci tells her, “Draw a shape with four straight sides.” Traci’s sister draws the shape shown.

The drawing of the shape includes 4 straight sides, but it is not a rectangle. How can Traci make her directions more clear?



TRY IT



Math Toolkit

- geoboards
- chenille stems
- rulers
- grid paper



DISCUSS IT

Ask your partner: Do you agree with me? Why or why not?

Tell your partner: I agree with you about . . . because . . .

CONNECT IT

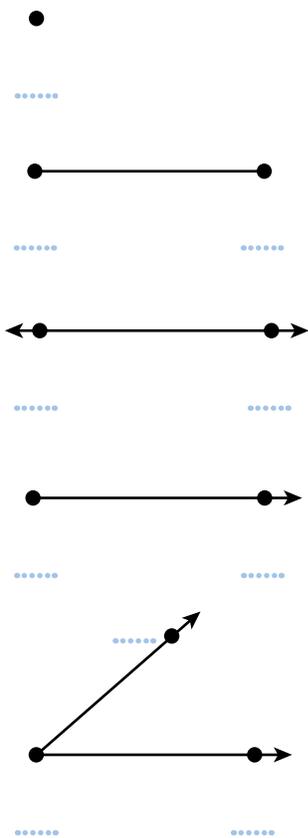
1 LOOK BACK

Explain how Traci can make her directions more clear.

2 LOOK AHEAD

Certain words in geometry are used to describe shapes in detail. Read each description and use it to label the point or points in the figure at the right.

- a. A **point** is a single location in space. A dot can show a point. You can name a point with a capital letter, such as point *A*.
- b. A **line segment** is a straight row of points that starts at one point and ends at another point. You can write "line segment *AB*" as \overline{AB} .
- c. A **line** is a straight row of points that goes on forever in both directions. You can write "line *AB*" as \overleftrightarrow{AB} .
- d. A **ray** is a straight row of points that starts at one point and goes on forever in one direction. You can write "ray *AB*" as \overrightarrow{AB} . When you name a ray, you always start with the endpoint.
- e. Rays, lines, or line segments that meet at a common point, or **vertex**, form an **angle**. You can write "angle *A*" as $\angle A$ or $\angle CAB$ or $\angle BAC$. The vertex is always the middle letter.



3 REFLECT

Does a rectangle contain lines or line segments? Explain.



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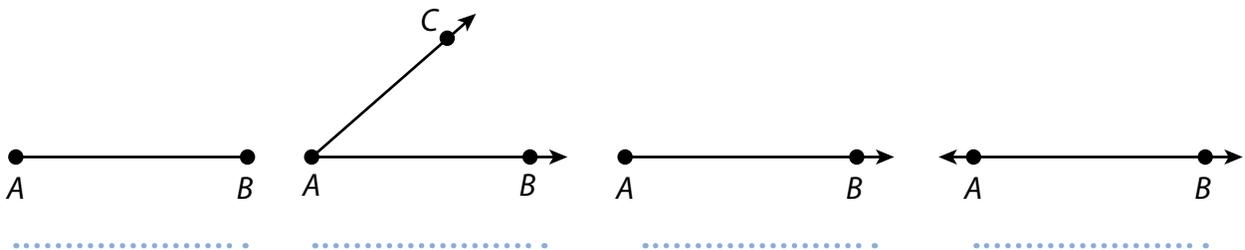
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Prepare for Points, Lines, Rays, and Angles

- 1 Think about what you know about geometric figures. Fill in each box. Use words, numbers, and pictures. Show as many ideas as you can.

Word	In My Own Words	Example
point		
line segment		
line		
ray		
angle		

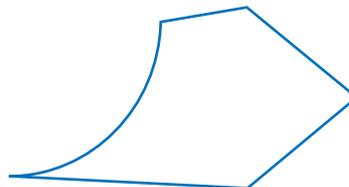
- 2 Label each figure as a *point*, *line segment*, *line*, *ray*, or *angle*.



3 Solve the problem. Show your work.

Marshall tries to teach his younger sister how to draw a square. Marshall tells her, "Draw a shape with four straight sides." Marshall's sister draws the shape shown.

The drawing of the shape includes 4 straight sides, but it is not a square. How can Marshall make his directions more clear?



Solution

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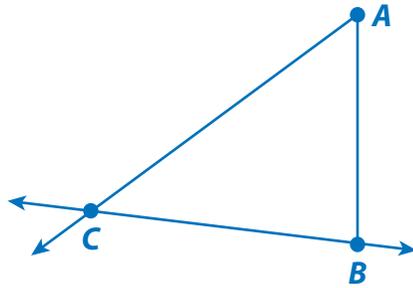
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4 Check your answer. Show your work.

Develop Points, Lines, Line Segments, and Rays

Read and try to solve the problem below.

Kent draws a shape using three different geometric figures. Describe the three geometric figures that Kent uses in his shape.



TRY IT



Math Toolkit

- chenille stems
- rulers
- tracing paper



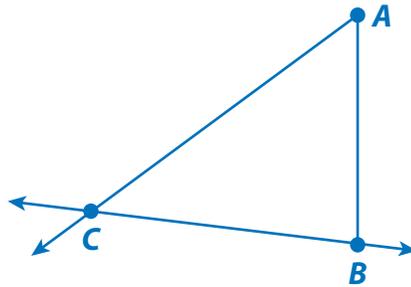
DISCUSS IT

Ask your partner: How did you get started?

Tell your partner: I started by ...

Explore different ways to understand points, lines, line segments, and rays.

Kent draws a shape using three different geometric figures. Describe the three geometric figures that Kent uses in his shape.



PICTURE IT

You can make some drawings to help describe the figures used in the shape.

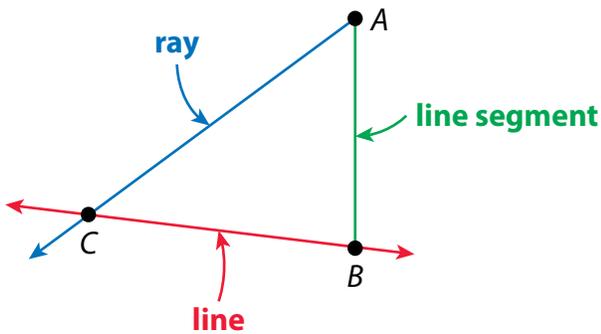
Each figure is straight. Draw the different kinds of straight rows of points that you know.



MODEL IT

You can also use words to help describe the figures used in the shape.

Label the line segment, ray, and line that are drawn as the figures in Kent's shape. Look for endpoints and arrowheads.

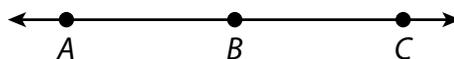


CONNECT IT

Now you will use the problem from the previous page to help you understand how to identify line segments, angles, and rays and to help you solve a similar problem.

- 1 Name a real-world example of a line segment.
- 2 When two line segments, lines, or rays meet at a point, they form an angle. Name a real-world example of an angle.
- 3 Is a beam of light from a flashlight more like a line or a ray? Explain.

- 4 The drawing below represents one line, three line segments, four rays, and one angle. Name each of these figures.



5 REFLECT

Look back at your **Try It**, strategies by classmates, and **Picture It** and **Model It**. Which models or strategies do you like best for understanding and describing points, lines, line segments, angles, and rays? Explain.

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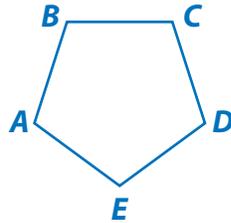
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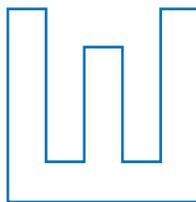
APPLY IT

Use what you just learned to solve these problems.

- 6 How many lines are in this shape? How many rays? Explain how you know.



- 7 How many line segments are in this shape? Explain how you know.



- 8 Draw and label a point, line, line segment, and ray.

Practice Points, Lines, Line Segments, and Rays

Study the Example showing a drawing with points, lines, line segments, and rays. Then solve problems 1–9.

EXAMPLE

Amy makes a drawing of a letter “A” in her math notebook. Use geometry words to describe the drawing.

There are 4 points on the drawing: point A, point B, point C, and point D.

There is a line segment from point B to point D.

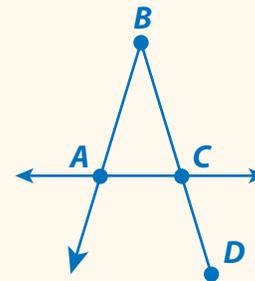
\overline{BD}

There is a line through points A and C.

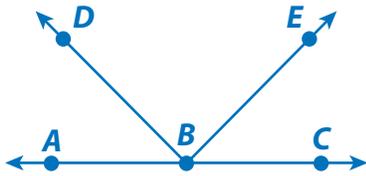
$\longleftrightarrow AC$

There is a ray from point B through point A.

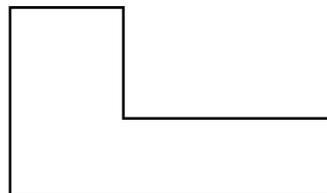
\overrightarrow{BA}



Use the drawing below to solve problems 1–4.



- 1 How many lines are in the drawing?
- 2 How many rays are in the drawing?
- 3 Write the name of the line in the drawing.
- 4 Write the names of the rays in the drawing.
- 5 Look at the shape at the right. How many line segments are in the shape?



Vocabulary

point a single location in space.



line segment a straight row of points that starts at one point and ends at another point.



line a straight row of points that goes on forever in both directions.



ray a straight row of points that starts at one point and goes on forever in one direction.



6 Label each sign below. Write *line(s)*, *line segment(s)*, or *ray(s)*.

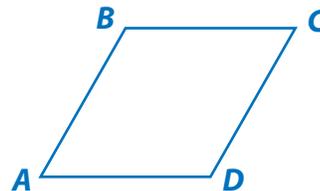


7 Look at the drawing below. Tell whether each line, line segment, ray, or angle is shown in the drawing.

	Yes	No
\overleftrightarrow{XY}	(A)	(B)
\overleftrightarrow{XZ}	(C)	(D)
\overrightarrow{WX}	(E)	(F)
\overrightarrow{YX}	(G)	(H)
\overline{ZY}	(I)	(J)
$\angle XYZ$	(K)	(L)



8 Use geometry words and symbols to describe the rhombus shown.



9 Read the description of a shape below. Then draw the shape.

- It has 3 line segments, \overline{RS} , \overline{ST} , \overline{TR} .
- Line segments \overline{RS} and \overline{TR} are the same length.
- It has 3 angles, $\angle R$, $\angle S$, and $\angle T$.

Develop Identifying Angles

Read and try to solve the problem below.

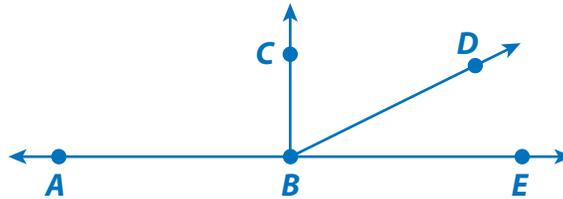
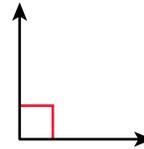
The angle shown at the right is a right angle.
A right angle is a square corner.

Look at the figure below. Name the rays that make up each of the angles listed.

1. A right angle.

2. An angle that has a smaller opening than a right angle.

3. An angle that has a wider opening than a right angle, but does not open as wide as a straight line.



TRY IT



Math Toolkit

- chenille stems
- rulers
- tracing paper



DISCUSS IT

Ask your partner: Can you explain that again?

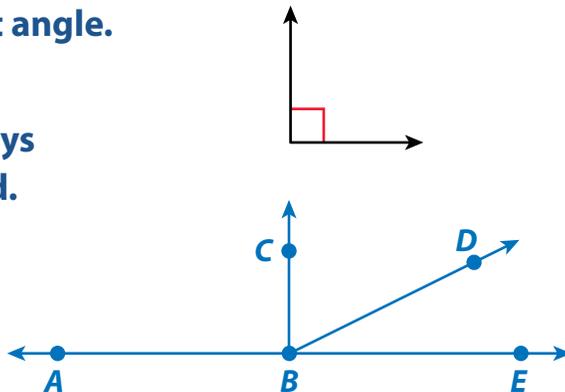
Tell your partner: I knew . . . so I . . .

Explore different ways to understand how to identify angles.

The angle shown at the right is a right angle.
A right angle is a square corner.

Look at the figure below. Name the rays that make up each of the angles listed.

1. A right angle.
2. An angle that has a smaller opening than a right angle.
3. An angle that has a wider opening than a right angle, but does not open as wide as a straight line.

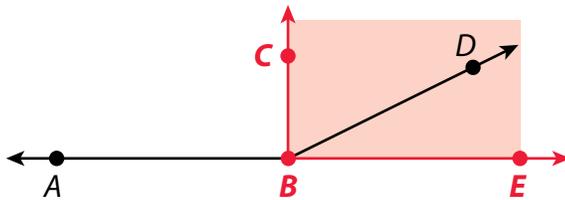


PICTURE IT

You can make a drawing to help identify different types of angles.

Use shading to find the rays that make each angle.

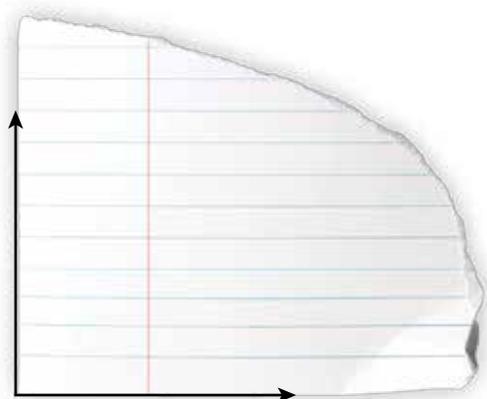
A right angle is shaded. Look at the rays along the edges of the shaded area.



MODEL IT

You can also use a model to help identify different types of angles.

Compare the opening of an angle to a right angle by holding the corner of a sheet of paper next to the angle. The angle below opens as wide as a right angle.



CONNECT IT

Now you will use the problem from the previous page to help you understand how to identify angles in figures.

- 1 **Model It** shows a right angle. Draw a right angle. Then use 3 points to name a right angle in the figure on the previous page.

- 2 An angle that has a smaller opening than a right angle is called an **acute angle**.
Name an acute angle in the figure on the previous page.
Draw an acute angle.

- 3 An angle that has a wider opening than a right angle, but does not open as wide as a straight line, is called an **obtuse angle**. Name an obtuse angle in the figure on the previous page. Draw an obtuse angle.

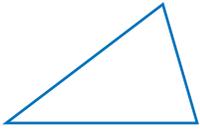
- 4 Explain how you can decide whether any angle is acute, right, or obtuse.

- 5 **REFLECT**
Look back at your **Try It**, strategies by classmates, and **Picture It** and **Model It**. Which models or strategies do you like best for identifying angles? Explain.
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APPLY IT

Use what you just learned to solve these problems.

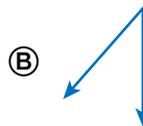
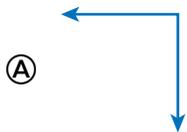
- 6 How many acute angles are in the shape below? Explain how you know.



- 7 Look at the shape below. How many obtuse angles are in the shape? Explain how you know.



- 8 Which angle is obtuse?



Practice Identifying Angles

Study the Example showing how to identify angles in a shape.
Then solve problems 1–10.

EXAMPLE

Name and describe the angles in the shape shown.

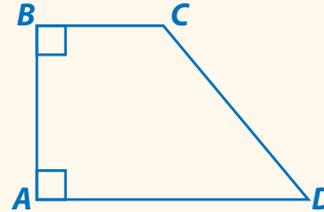
$\angle A$ is a right angle. It has a shape like a square corner.

$\angle B$ is also a right angle.

$\angle C$ is an obtuse angle. It has a wider opening than a right angle.

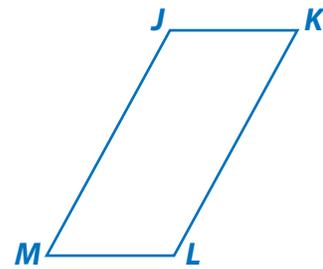
$\angle D$ is an acute angle. It has a smaller opening than a right angle.

The shape has 2 right angles, 1 acute angle, and 1 obtuse angle.

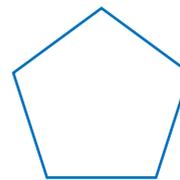


Use the shape at the right to solve problems 1–5.

- 1 How many right angles are in this shape?
- 2 How many acute angles are in this shape?
- 3 How many obtuse angles are in this shape?
- 4 Name the acute angles in the shape.
- 5 Name the obtuse angles in the shape.
- 6 Look at the shape of the sign at the right. Describe the number and kind of angles the shape has.



Jasmine draws the pentagon shown at the right. She says that all pentagons have 5 sides of equal length and 5 obtuse angles.



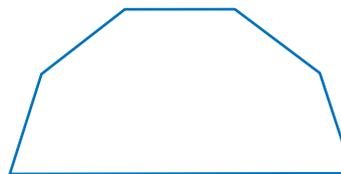
7 Draw a pentagon that is different from the one Jasmine drew. Describe the sides and angles of your pentagon.

8 In what way is Jasmine's thinking correct?

9 In what way is Jasmine's thinking incorrect?

10 Which statements correctly describe the shape below?

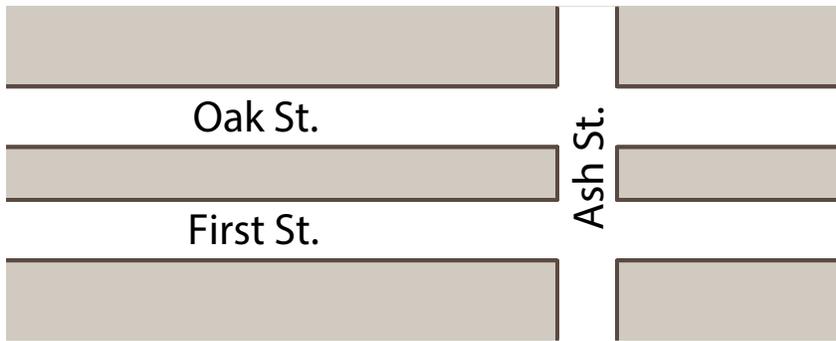
- Ⓐ The shape has acute angles.
- Ⓑ The shape has right angles.
- Ⓒ The shape has obtuse angles.
- Ⓓ The shape has 6 angles.
- Ⓔ The shape has more acute angles than obtuse angles.



Develop Parallel and Perpendicular Lines

Read and try to solve the problem below.

Jordan looks at the street map below.



Describe the relationship between Oak Street and First Street.
Then describe the relationship between Oak Street and Ash Street.

TRY IT



Math Toolkit

- geoboard
- straws
- tracing paper
- grid paper



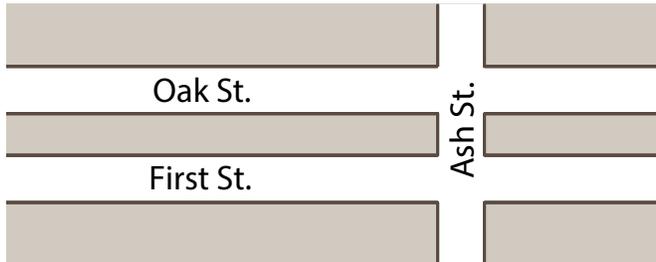
DISCUSS IT

Ask your partner: Why did you choose that strategy?

Tell your partner: At first, I thought...

Explore different ways to understand parallel and perpendicular lines and line segments.

Jordan looks at the street map below.

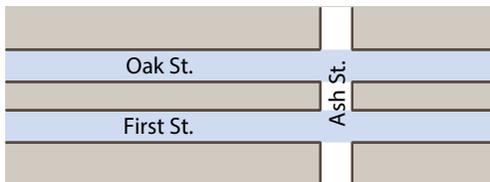


Describe the relationship between Oak Street and First Street.
Then describe the relationship between Oak Street and Ash Street.

PICTURE IT

You can use a sketch to help understand the problem.

Sketch a picture of Oak Street and First Street. Shade the streets.

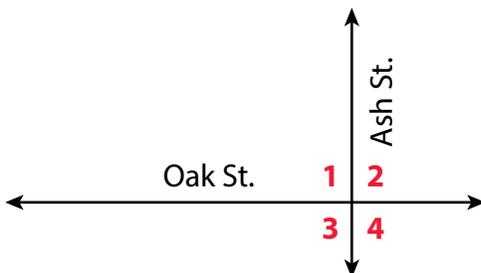


Notice that the streets do not cross.

MODEL IT

You can also use a model to help understand the problem.

Look at Oak Street and Ash Street. Think of each street as a line. When the two lines cross, they form four angles.



CONNECT IT

Now you will use the problem from the previous page to help you understand how to identify parallel and perpendicular lines.

- 1 Lines that are always the same distance apart and never cross are called **parallel lines**. Name a real-world example of parallel lines.
- 2 Suppose each street keeps going in a straight line. If Jordan travels on Oak Street and makes no turns, can he ever get to First Street? Explain.
- 3 Describe the angles that Oak Street and Ash Street make when they cross.
- 4 Lines that cross and form a right angle are called **perpendicular lines**. Name a real-world example of perpendicular lines.
- 5 Explain why 3 separate lines can all be parallel to each other, but cannot all be perpendicular to each other. Use a drawing to show your answer.

6 REFLECT

Look back at your **Try It**, strategies by classmates, and **Picture It** and **Model It**. Which models or strategies do you like best for identifying parallel and perpendicular lines? Explain.

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APPLY IT

Use what you just learned to solve these problems.

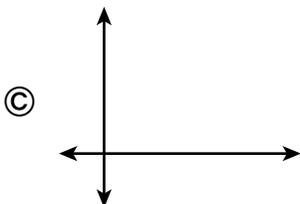
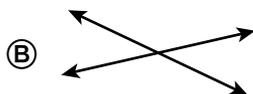
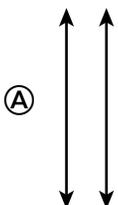
- 7 How many pairs of parallel sides does the shape below have?
Explain how you know.



- 8 How many pairs of parallel sides does the shape below have?
Explain how you know.



- 9 Which pair of lines are perpendicular?



Practice Parallel and Perpendicular Lines

Study the Example showing how to identify parallel and perpendicular lines and line segments. Then solve problems 1–6.

EXAMPLE

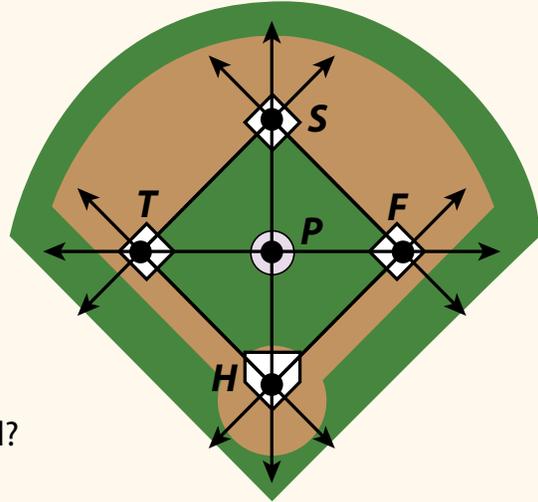
Colby draws parallel and perpendicular lines to place the bases and pitcher's mound on a drawing of a baseball field.

\overleftrightarrow{SF} and \overleftrightarrow{TH} are parallel lines.

\overleftrightarrow{ST} and \overleftrightarrow{FH} are parallel lines.

The pitcher's mound is one place where perpendicular lines cross. At what point do perpendicular lines cross at the pitcher's mound?

They cross at point P , where \overleftrightarrow{TF} crosses \overleftrightarrow{SH} .



For problems 1 and 2, use the shape at the right.

- How many pairs of parallel sides does the square have?
- Put Xs on the square where each pair of perpendicular line segments meet.
- Look at the drawing of a window at the right. Circle 3 parallel line segments in the drawing.



- 4 Look at the line segments in the letters on the tiles at the right. Fill in the table with each letter to identify parallel line segments. The first one is done for you.

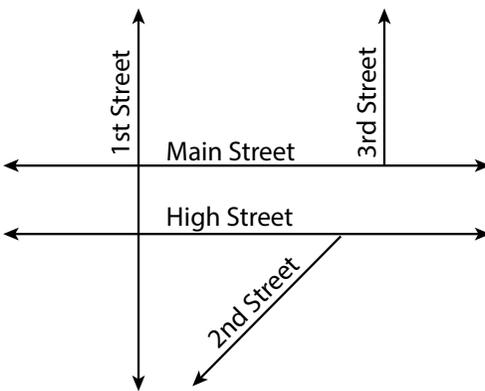


No parallel line segments	Only 1 pair of parallel line segments	More than 1 pair of parallel line segments
L		

- 5 Look at the line segments in the letters on the tiles again. Fill in the table to identify perpendicular line segments.

Only 1 pair of perpendicular line segments	Only 2 pairs of perpendicular line segments	3 pairs of perpendicular line segments

- 6 Tell whether each statement that describes the streets shown on the map below is *True* or *False*.



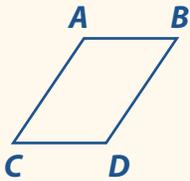
	True	False
1st and 3rd Street are perpendicular.	(A)	(B)
Main and High Street are parallel.	(C)	(D)
2nd Street is perpendicular to Main St.	(E)	(F)
1st Street is perpendicular to High St.	(G)	(H)

Refine Points, Lines, Rays, and Angles

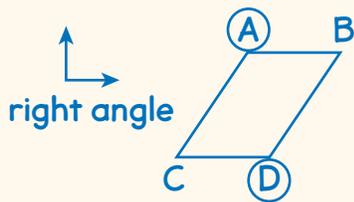
Complete the Example below. Then solve problems 1–9.

EXAMPLE

In the shape below, list each pair of parallel sides and circle the letter marking each obtuse angle.



Look at how you could show your work.



Solution

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Even if the sides of the shape went on forever, the opposite sides would never cross each other.

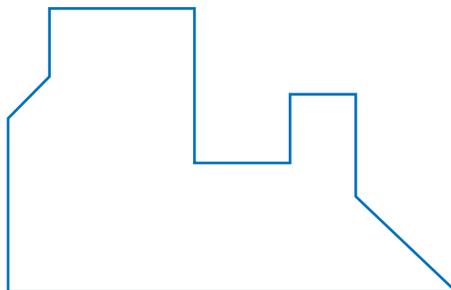


PAIR/SHARE

What kind of angles are $\angle B$ and $\angle C$? How do you know?

APPLY IT

- Put an X where each pair of perpendicular line segments meet in the shape below.

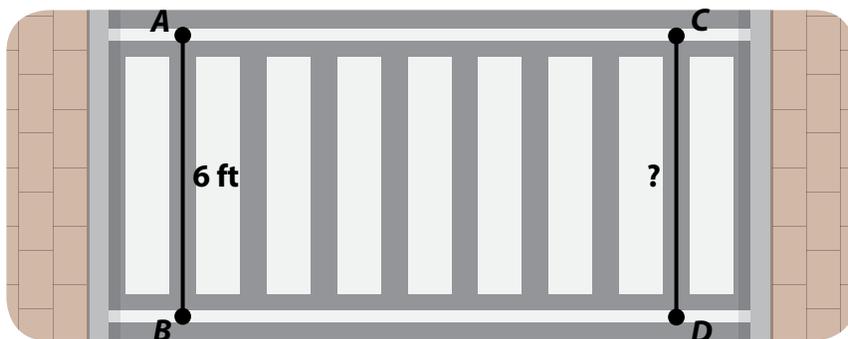


Perpendicular line segments meet to form right angles.

PAIR/SHARE

Describe the angles that are NOT marked with an X.

- 2 A crosswalk is marked with a pair of parallel line segments that extend from one side of the street to the other. The distance between the two line segments from point A to point B is 6 feet. What is the distance from point C to point D ?



Solution

- 3 Toshi cuts one fourth of a circle out of paper. How many angles does this shape have?



- (A) 0
- (B) 1
- (C) 2
- (D) 3

Esme chose (D) as the correct answer. How did she get that answer?

What facts do I know about parallel lines?



PAIR/SHARE

Can the lines still be parallel if the distance from C to D is 3 feet?

I know that it takes two rays to make an angle.

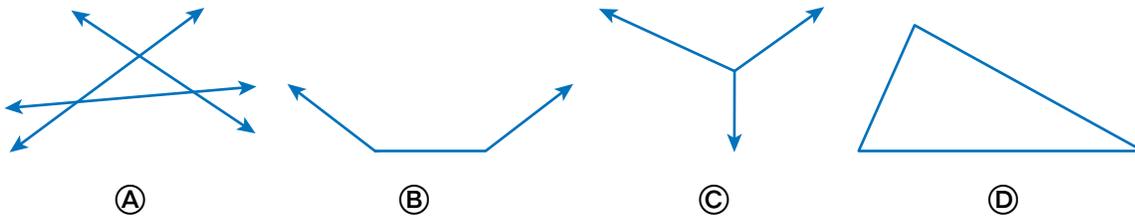
PAIR/SHARE

Does Esme's answer make sense?

4 Think about a real-world example of where a wall meets the floor and where the same wall meets the ceiling. Which term best describes what it looks like where these surfaces meet?

- (A) parallel line segments
- (B) perpendicular line segments
- (C) right angle
- (D) acute angle

5 Which drawing shows 3 lines?



6 Look at the shape below. For which terms is an example shown in the shape?

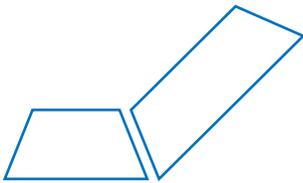


- (A) parallel line segments
- (B) perpendicular line segments
- (C) right angle
- (D) acute angle
- (E) obtuse angle

- 7 Tell whether each sentence is *True* or *False*.

	True	False
A ray goes on forever in two directions.	(A)	(B)
A line segment has exactly two endpoints.	(C)	(D)
An obtuse angle has a wider opening than a right angle.	(E)	(F)
Parallel lines meet to form an acute angle.	(G)	(H)

- 8 Liz draws the two shapes below. Use words you have learned in this lesson to describe what the shapes have in common. How are they different?



9 MATH JOURNAL

A triangle can have one pair of perpendicular sides. Can a triangle have one pair of parallel sides? Use drawings and words to explain your answer.



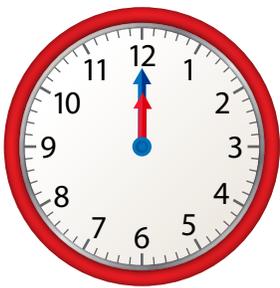
SELF CHECK Go back to the Unit 5 Opener and see what you can check off.

Explore Angles



Previously, you learned to identify angles. Now you will learn more about angles and angle measurement. Use what you know to try to solve the problem below.

Lily and Dora each turn the hour hand on a clock face. They make different angles by turning the hour hand. Who makes the greater angle? Explain how you know.



Lily's angle



Dora's angle

Learning Targets

- An angle that turns through n one-degree angles is said to have an angle measure of n degrees.
- Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.

SMP 1, 2, 3, 4, 5, 6, 7

TRY IT



Math Toolkit

- clocks
- clock face
- index cards
- sticky notes



DISCUSS IT

Ask your partner: How did you get started?

Tell your partner: I started by ...

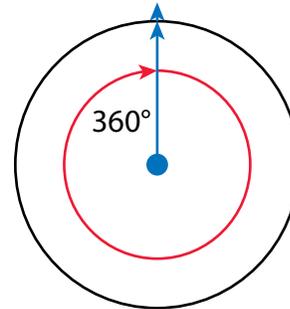
CONNECT IT

1 LOOK BACK

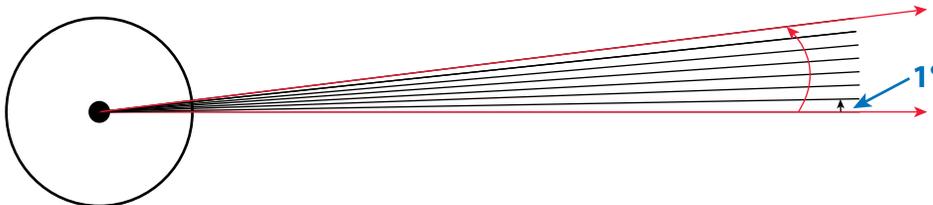
Explain how you know who makes the greater angle, Lily or Dora.

2 LOOK AHEAD

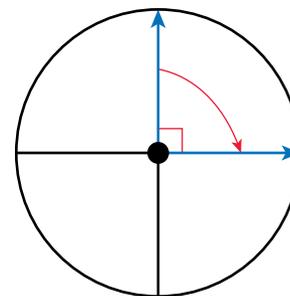
You can measure angles to compare them. A **degree** is a unit of measure for angles. Show degrees with the symbol $^{\circ}$. The angle made by a full turn of a ray in a circle measures 360 degrees, or 360° .



- a. Look at the diagram below. An angle that turns through $\frac{1}{360}$ of a circle is called a 1° angle. How many 1° angles are in a circle?



- b. The red angle in the diagram turns through part of the circle. Count to find the measure of the red angle. Write the measure of the red angle.
- c. A ray turns to form a right angle in the circle at the right. What is the measure, in degrees, of a right angle? Explain.



3 REFLECT

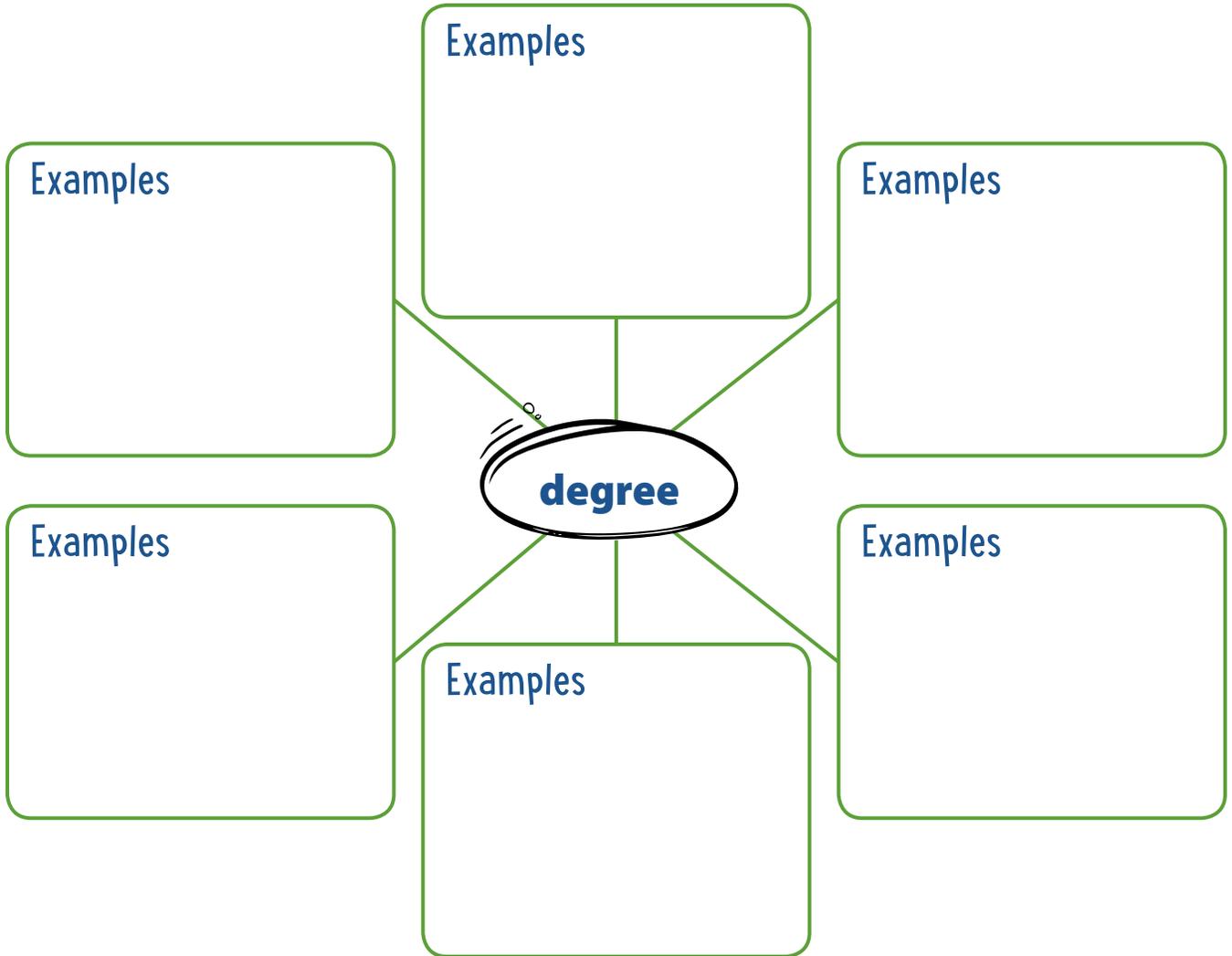
How does the way a ray turns through a circle help you think about the measure of an angle?

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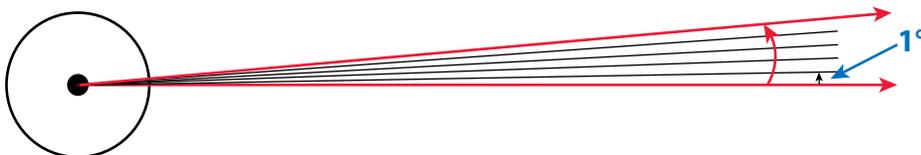
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Prepare for Angles

- 1 Think about what you know about angles. Fill in each box. Use words, numbers, and pictures. Show as many ideas as you can.



- 2 The red angle below turns through part of the circle. Count to find the measure of the red angle. Write the measure of the angle in degrees.



3 Solve the problem. Show your work.

Beau and Kong each turn the hour hand on a clock face. They make different angles by turning the hour hand. Who makes the greater angle? Explain how you know.



Beau's angle



Kong's angle

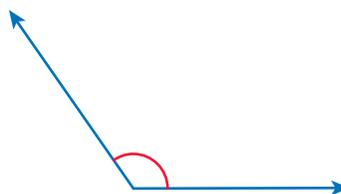
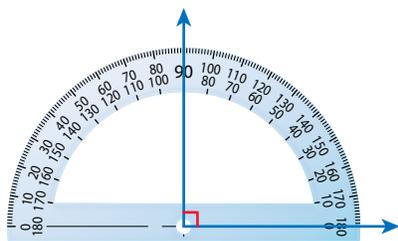
Solution

4 Check your answer. Show your work.

Develop Using a Protractor

Read and try to solve the problem below.

A protractor is a tool used to measure angles. The protractor below shows that the measure of a right angle is 90° . Kara draws the other angle below. What is the measure of Kara's angle? How can you find out?

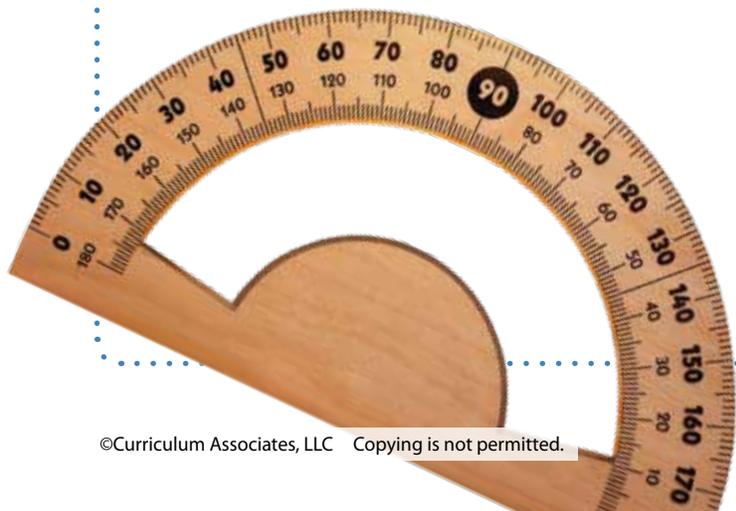


TRY IT



Math Toolkit

- protractors
- rulers
- index cards
- sticky notes



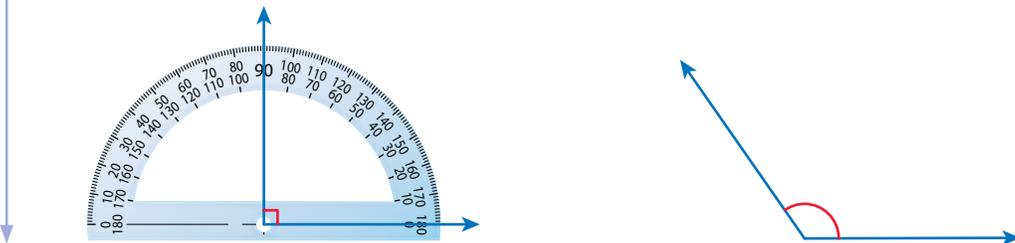
DISCUSS IT

Ask your partner: Can you explain that again?

Tell your partner: I knew . . . so I . . .

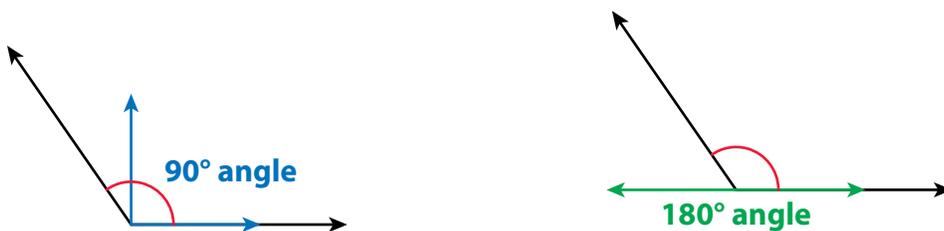
Explore different ways to understand how to use benchmarks and a protractor to measure an angle.

A protractor is a tool used to measure angles. The protractor below shows that the measure of a right angle is 90° . Kara draws the other angle below. What is the measure of Kara's angle? How can you find out?



PICTURE IT

You can use benchmarks to estimate the measure.

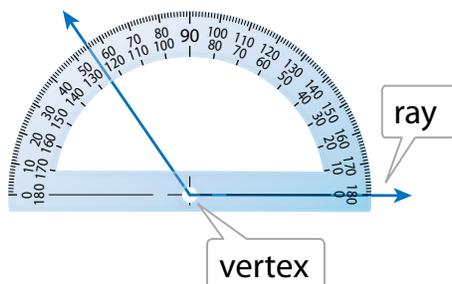


Kara's angle seems to be between 90° and 180° . It is obtuse.

MODEL IT

You can use a protractor to measure the angle.

- First, line up either mark showing 0° on the protractor with one ray of the angle.



- Next, line up the center point of the protractor with the vertex of the angle. Remember that the vertex is the point where two rays meet to form an angle.
- Then look at the other ray to read the number of degrees.

CONNECT IT

Now you will use the problem from the previous page to help you understand how to use a protractor to measure an angle.

- 1 Estimate the angle measure of Kara's angle.
- 2 Why must you line up the protractor's center point with the vertex of the angle?
- 3 Suppose you line up one ray with either mark showing 10° or 170° instead of either mark showing 0° or 180° . How would it change which mark the other ray points to?
- 4 Line up either mark showing 0° or 180° with one ray. Which mark does the other ray point to?
- 5 Which number of degrees is the measure of the angle? Explain how you know.

6 REFLECT

Look back at your **Try It**, strategies by classmates, and **Picture It** and **Model It**. Which models or strategies do you like best for measuring an angle? Explain.

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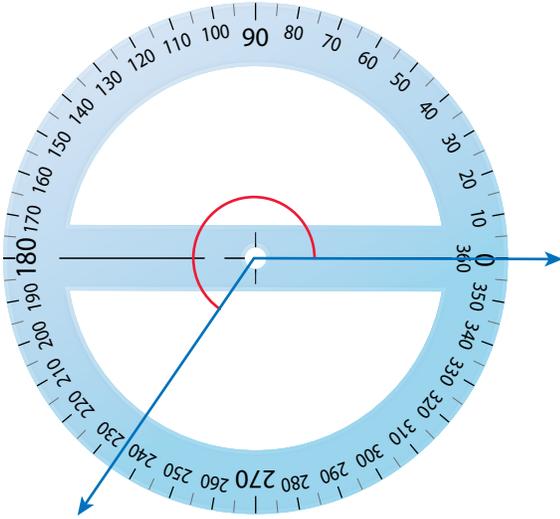
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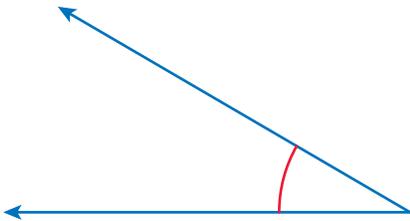
APPLY IT

Use what you just learned to solve these problems.

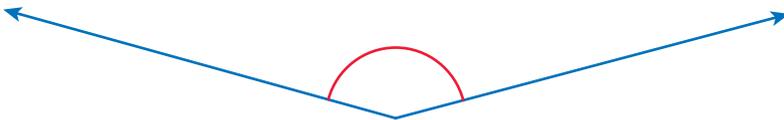
7 What is the measure, in degrees, of the angle shown?



8 What is the measure of the angle shown?



9 What is the measure of the angle shown?



Practice Using a Protractor

Study the Example showing how to use a protractor to measure an angle. Then solve problems 1–5.

EXAMPLE

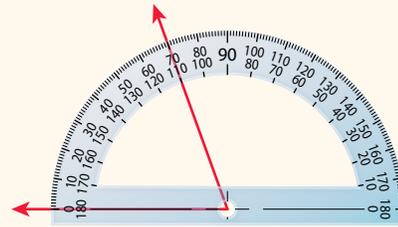
Omar draws the angle at the right. What is the measure of the angle?

Line up the 0° or the 180° mark on a protractor with one ray of the angle.

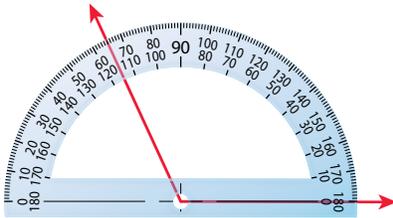
Line up the center point of the protractor with the vertex of the angle.

Look at the other ray. Read the number of degrees on the protractor. Read the number that is less than 90° .

The angle measures 70° .

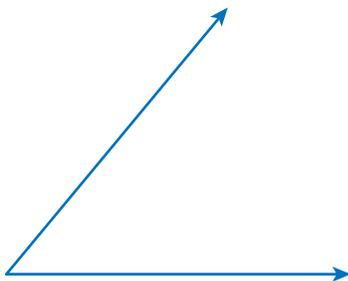


- 1 Read the number of degrees on the protractor to find the measure of the angle.



The angle measures degrees.

- 2 Use a protractor to measure the angle below.



The angle measures degrees.

Vocabulary

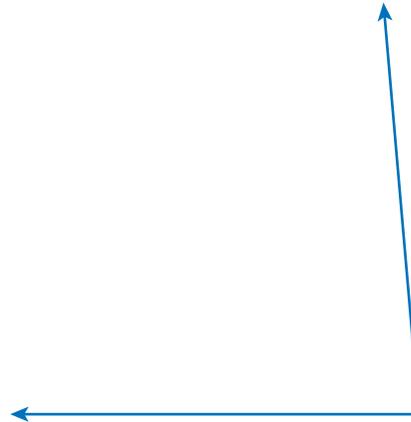
degree ($^\circ$) a unit of measure for angles.

protractor a tool used to measure angles.

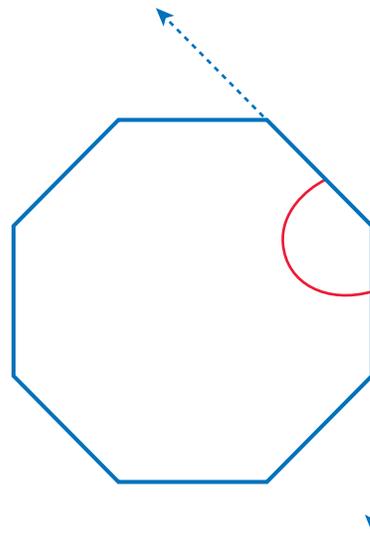
vertex the point where two rays, lines, or line segments meet to form an angle.

For problems 3–5, use a protractor to measure the angles. Write each measure.

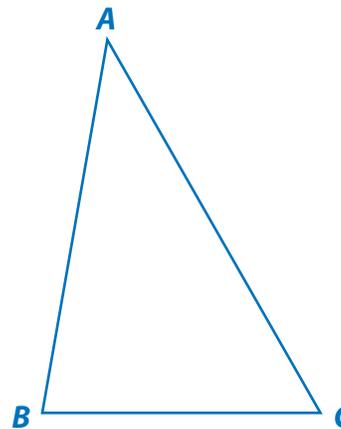
- 3 Measure the angle at the right.
The angle measures degrees.



- 4 Measure one angle of the polygon at the right.
The angle measures degrees.



- 5 Measure the angles of the triangle at the right.
Angle *A* measures degrees.
Angle *B* measures degrees.
Angle *C* measures degrees.



Develop Drawing Angles

Read and try to solve the problem below.

Draw a 30° angle. Think about using two pencils to make an angle.



TRY IT



Math Toolkit

- protractors
- rulers
- index cards
- sticky notes



DISCUSS IT

Ask your partner: Do you agree with me? Why or why not?

Tell your partner: I agree with you about ... because ...

Explore different ways to understand how to draw angles.

Draw a 30° angle. Think about using two pencils to make an angle.

PICTURE IT

You know an angle is made up of two rays with a common endpoint, called the vertex.

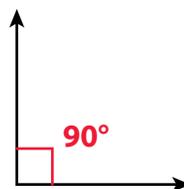
You can use two pencils to make an angle.



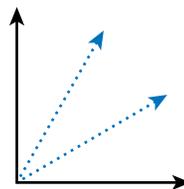
MODEL IT

You can use a benchmark angle to get an idea of what your drawing should look like.

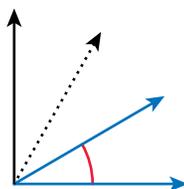
Think about a right angle. A right angle measures 90° .



You know $30 \times 3 = 90$. Imagine rays that split the 90° angle into 3 angles of equal measure.



A 30° angle opens about the same amount as the angle shown at the right.



CONNECT IT

Now you will use the problem from the previous page to help you understand how to draw angles.

1 Draw a ray on a sheet of paper. Then place the protractor's center point on the endpoint of your ray. What part of the angle is that point?

2 Keeping the protractor's center point on the endpoint of your ray, draw a point on your ray at 0° .



3 There are two marks on the protractor labeled "30." Choose the one that is 30° from your 0° mark. Draw a point at this mark.

4 Use the straight edge of the protractor to draw a ray from the vertex through the point you drew at 30° .

5 Suppose you choose the other "30" mark and draw a point at that mark. What would be the measure of your angle?

6 Think about a right angle. Compare it to the angle you drew. How wide does your angle open compared to a right angle?

7 REFLECT

Look back at your **Try It**, strategies by classmates, and **Picture It** and **Model It**. Which models or strategies do you like best for drawing angles? Explain.

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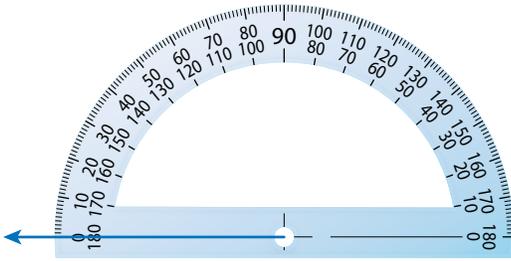
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APPLY IT

Use what you just learned to solve these problems.

- 8 Angle D measures 80° . One ray of angle D is shown. Draw another ray to make angle D .



- 9 Draw a 75° angle.

- 10 Draw a 100° angle.

Practice Drawing Angles

Study the Example showing how to draw an angle. Then solve problems 1–6.

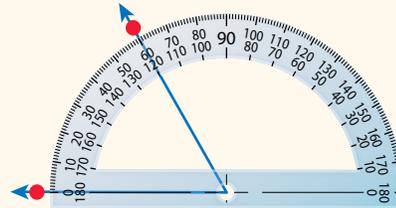
EXAMPLE

Stephanie wants to draw a 60° angle. She draws a ray and positions the endpoint of the ray on a protractor's center point. Then she lines up the protractor so the ray passes through the 0° mark on the protractor. How does she draw the other ray to form a 60° angle?

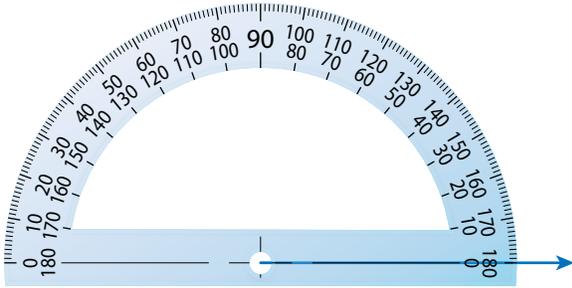
Find 60° on the protractor.

Choose the mark that is 60° from the first ray.
Draw a point at this 60° mark.

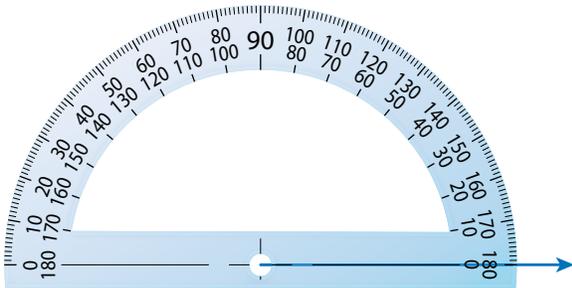
Draw a ray from the vertex through this point.



- 1 Draw a ray to show a 70° angle.



- 2 Draw a ray to show a 110° angle.



3 Draw a 160° angle.

4 Draw a 20° angle.

5 Draw a 45° angle.

6 Draw a 135° angle.

Refine Angles

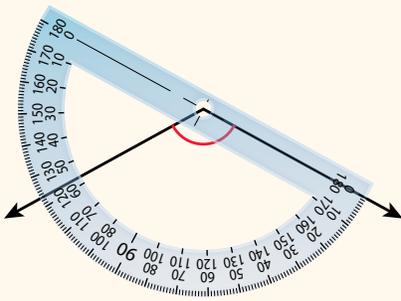
Complete the Example below. Then solve problems 1–8.

EXAMPLE

What is the measure of the angle below?



Look at how you could use a protractor to measure the angle.



Solution

The center point lines up with the vertex of the angle, and the 0° mark lines up with one ray of the angle. The other ray points to the measure of the angle.

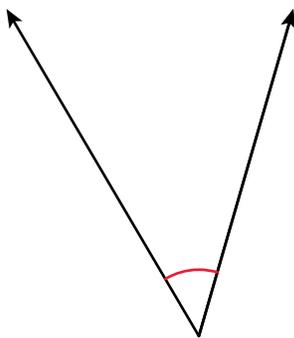


PAIR/SHARE

Does it matter which ray you choose to line up with the 0° mark?

APPLY IT

1 What is the measure of the angle below?



Solution

The angle looks like it opens less than a right angle. The measure will be less than 90° .

PAIR/SHARE

How did you and your partner decide where the vertex is?

2 Draw a 145° angle.

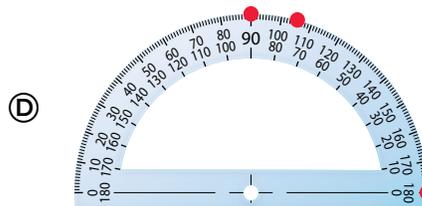
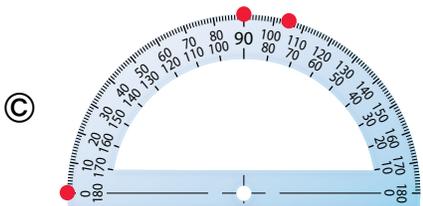
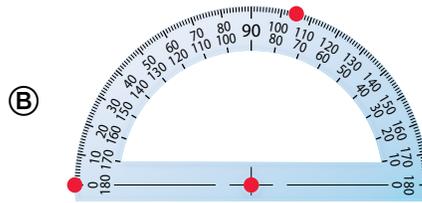
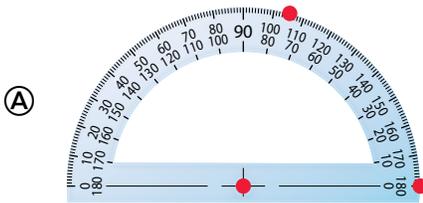
I'll need to draw two rays to make an angle.



PAIR/SHARE

If you had drawn a point at the other 0° mark, how would it change your angle?

3 Which set of points can be used to draw a 105° angle?



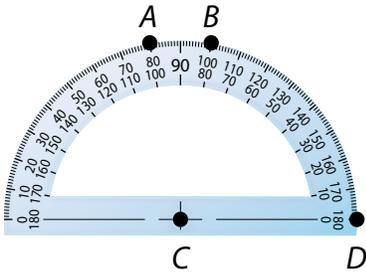
Will a 105° angle be wider or narrower than a right angle?

Mia chose © as the correct answer. How did she get that answer?

PAIR/SHARE

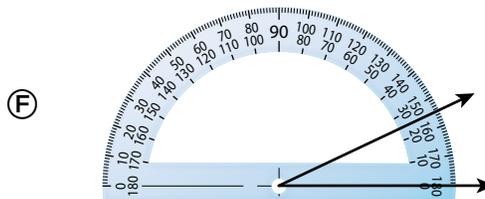
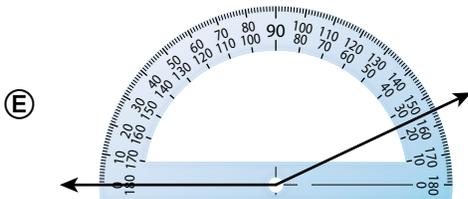
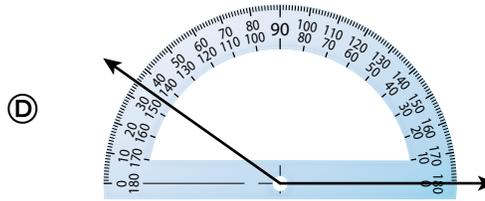
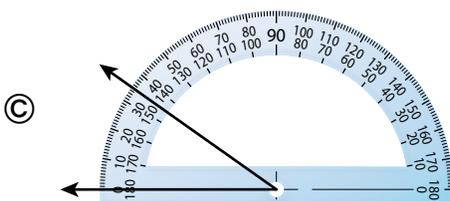
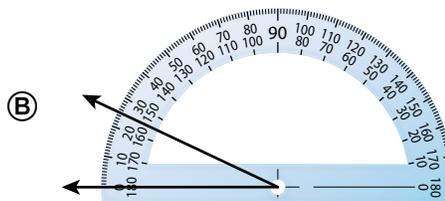
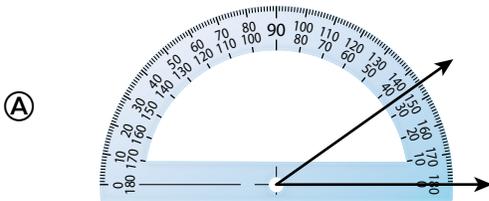
Does Mia's answer make sense?

- 4 Which point could be the vertex of an 80° angle that you could measure without moving the protractor?

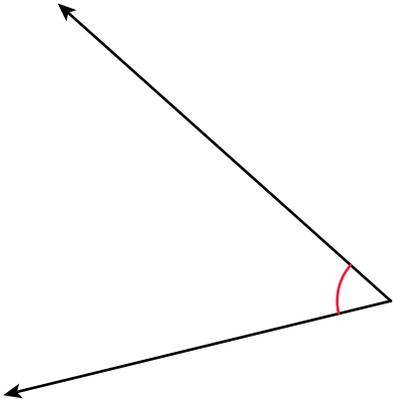


- (A) point A
- (B) point B
- (C) point C
- (D) point D

- 5 Which diagrams show a 25° angle?



- 6 What is the measure of the angle below?

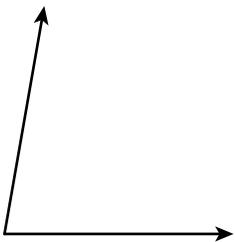


Solution

- 7 Draw a 40° angle.

8 MATH JOURNAL

Explain how you can use a protractor to measure the angle below.



SELF CHECK Go back to the Unit 5 Opener and see what you can check off.

Explore Classifying Two-Dimensional Figures



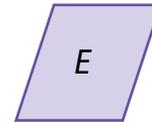
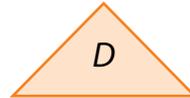
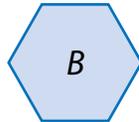
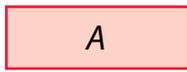
Learning Target

- Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.

SMP 1, 2, 3, 4, 5, 6, 7, 8

You have learned about parallel and perpendicular lines. Use what you know to try to solve the problem below.

Look at the shapes below. Put a check mark on all the shapes that have at least one pair of parallel sides. Put a star on all the shapes that have at least one pair of perpendicular sides. Explain how you could test your choices.



TRY IT



Math Toolkit

- pattern blocks
- rulers
- index cards
- protractors



DISCUSS IT

Ask your partner: Can you explain that again?

Tell your partner: I knew . . . so I . . .

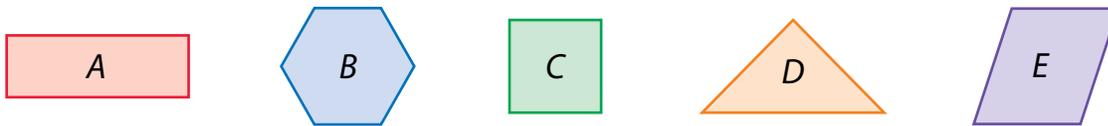
CONNECT IT

1 LOOK BACK

Which shapes have at least one pair of parallel sides and at least one pair of perpendicular sides? Explain.

2 LOOK AHEAD

Shapes with straight sides, such as triangles and quadrilaterals, are types of **polygons**. There are different ways you can sort these shapes, such as by the number of sides the shape has and by the relationships between the sides. You can also sort shapes by the kinds of angles they have.



- a. Which shapes have at least one right angle?
- b. Which shapes have at least one acute angle?
- c. Which shapes have at least one obtuse angle?

3 REFLECT

Describe the sides and angles of shape C.

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.....

.....

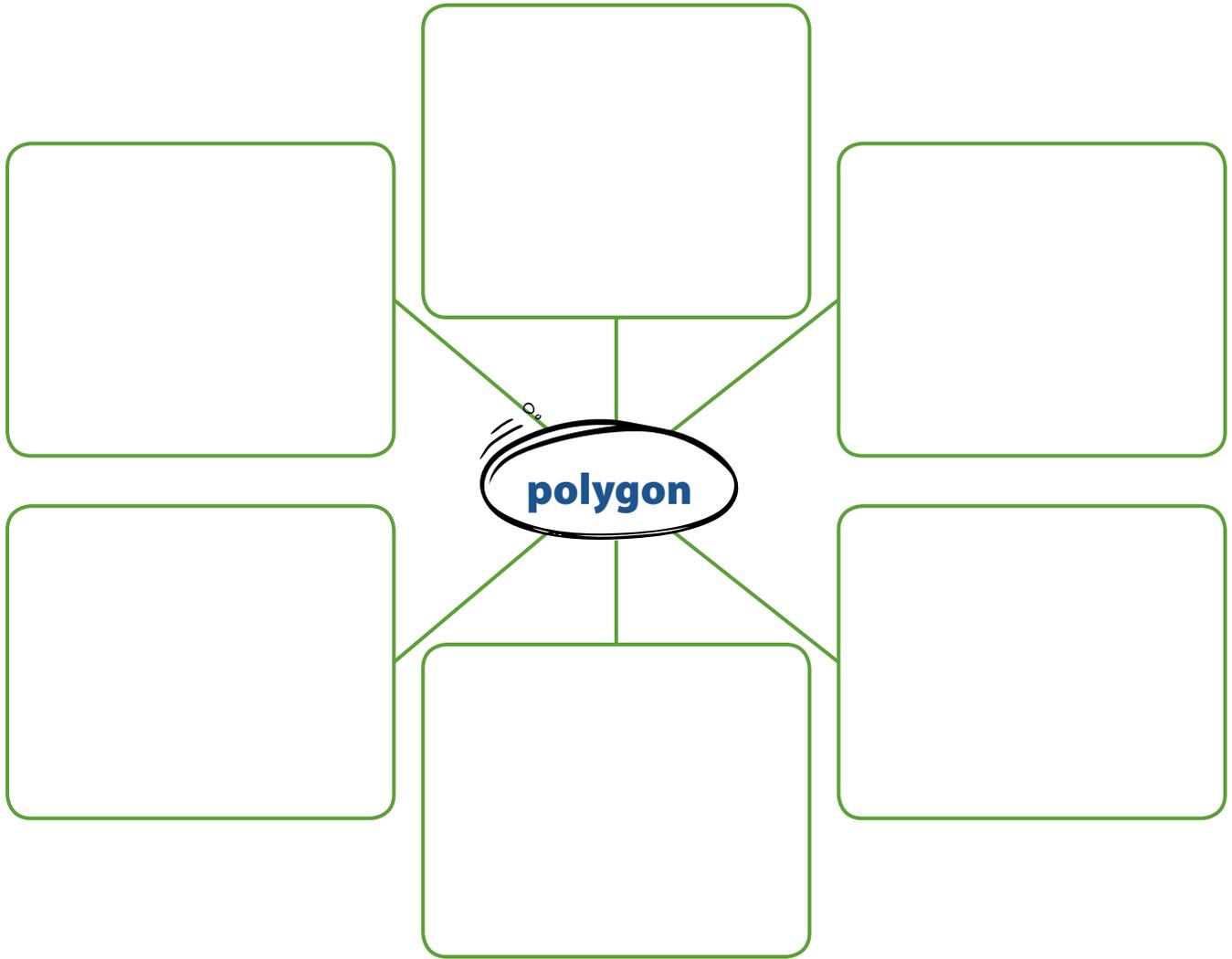
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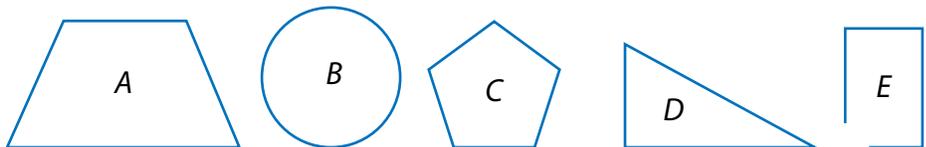


Prepare for Classifying Two-Dimensional Figures

1 Think about what you know about polygons. Fill in each box. Use words, numbers, and pictures. Show as many ideas as you can.



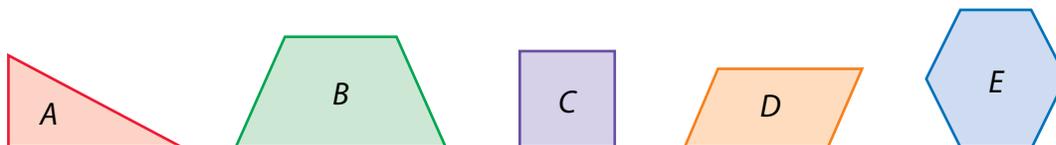
2 Which shapes are polygons?



.....

3 Solve the problem. Show your work.

Look at the shapes below. Put a check mark on all the shapes that have at least one right angle. Put a star on all the shapes that have at least one pair of parallel sides. Explain how you could test your choices.



Solution

.....

.....

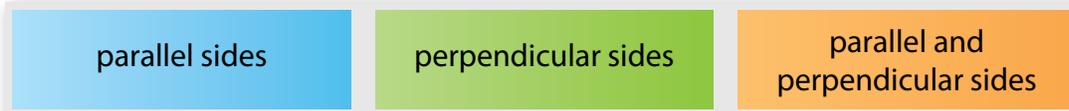
.....

4 Check your answer. Show your work.

Develop Sorting Shapes Based on Sides

Read and try to solve the problem below.

Evan plays a board game. The board is divided into three sections.



These are Evan's cards. In which sections of the board do the cards belong?



TRY IT



Math Toolkit

- pattern blocks
- rulers
- index cards
- protractors



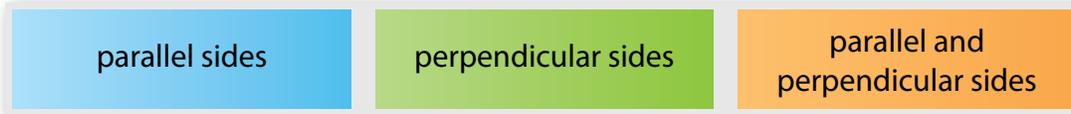
DISCUSS IT

Ask your partner: How did you get started?

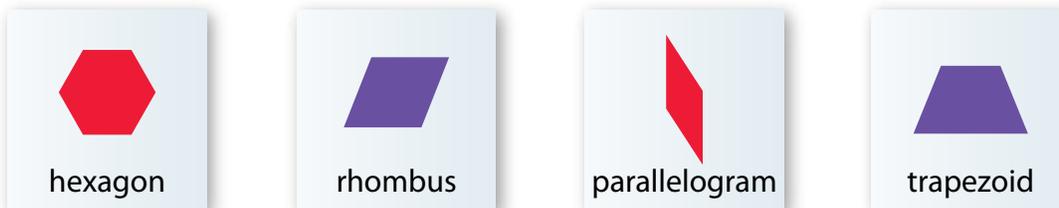
Tell your partner: I started by ...

Explore different ways to understand how to sort shapes into groups based on parallel and perpendicular sides.

Evan plays a board game. The board is divided into three sections.



These are Evan's cards. In which sections of the board do the cards belong?



PICTURE IT

You can use drawings to help sort shapes.

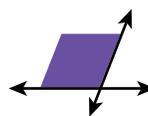
Draw a pair of parallel lines and a pair of perpendicular lines.



Draw lines along opposite sides of each shape. Compare these lines to the parallel lines you drew.



Draw lines along sides of each shape that form angles. Compare these lines to the perpendicular lines you drew.



MODEL IT

You can use a table to help sort shapes.

Make a table. Put the shape on each card in the table where the shape belongs.

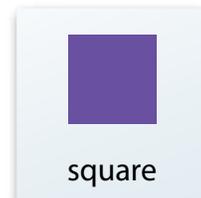
Parallel Sides	Both Parallel and Perpendicular Sides	Perpendicular Sides

Evan's cards belong in the "Parallel Sides" column of the table.

CONNECT IT

Now you will solve a problem similar to the one on the previous page to help you understand how to sort shapes into groups based on parallel and perpendicular sides. Evan gets two more cards. In which sections of the board do the cards with these shapes belong?

- 1 Evan gets a card with a square. In which section of the board does it belong?
- 2 Evan gets a card with a quadrilateral. Does the quadrilateral belong to any of the three categories on the board? If not, name a category that can be used to describe this shape.
- 3 Explain how to sort shapes based on parallel and perpendicular sides.



4 REFLECT

Look back at your **Try It**, strategies by classmates, and **Picture It** and **Model It**. Which models or strategies do you like best for sorting shapes into groups based on parallel and perpendicular sides? Explain.

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.....

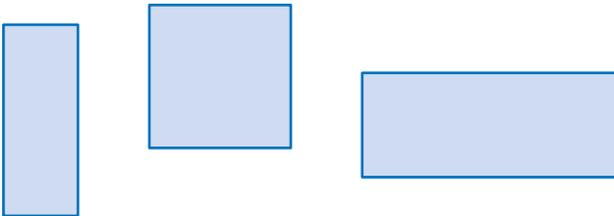
.....

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APPLY IT

Use what you just learned to solve these problems.

- 5 Describe the group that the shapes below belong in based on the kinds of sides they have.



Solution

- 6 Circle the shape below that belongs in the group: "no parallel sides."



- 7 Select all the shapes that always have pairs of perpendicular sides.

- Ⓐ hexagon
- Ⓑ parallelogram
- Ⓒ rectangle
- Ⓓ rhombus
- Ⓔ square
- Ⓕ trapezoid

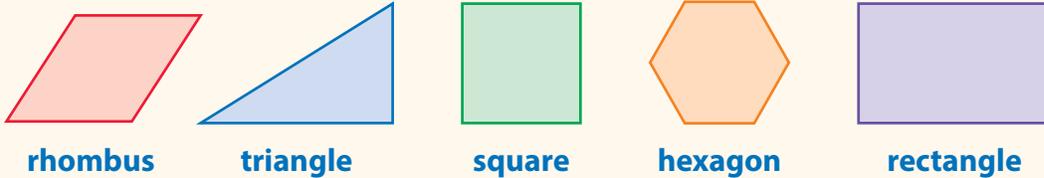


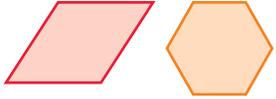
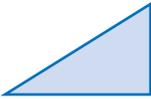
Practice Sorting Shapes Based on Sides

Study the Example showing how to sort shapes into groups based on parallel and perpendicular sides. Then solve problems 1–4.

EXAMPLE

Sort the shapes below based on parallel and perpendicular sides. Put the shapes in the table below.



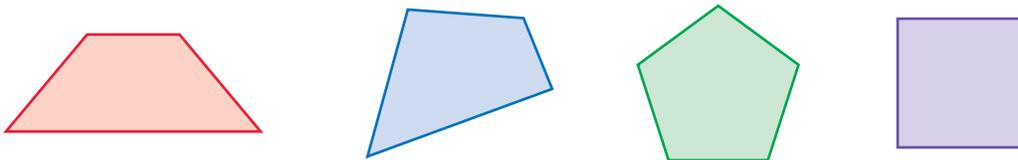
Parallel Sides	Both Parallel and Perpendicular Sides	Perpendicular Sides
		

- 1 Look at how the shapes in the Example above are sorted into groups. Then look at the shape at the right. Which group does the shape belong in?

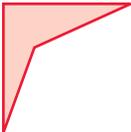


Solution

- 2 Suppose there is another group for shapes: “no parallel or perpendicular sides.” Circle the shapes below that belong in this group.



3 Select the kinds of sides each shape has.

	Parallel Sides	Perpendicular Sides
	(A)	(B)
	(C)	(D)
	(E)	(F)
	(G)	(H)

4 Select all the properties that always belong to each shape.

	Parallel Sides	Perpendicular Sides
rectangle	(A)	(B)
rhombus	(C)	(D)
square	(E)	(F)

Develop Sorting Shapes Based on Angles

Read and try to solve the problem below.

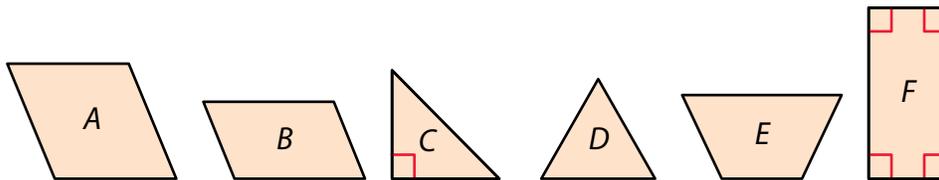
A classroom computer game shows a set of categories and a set of shapes. The player puts each shape in the correct category. Draw a line from each shape to the category it belongs in.

acute only

right only

acute and right

acute and obtuse



TRY IT



Math Toolkit

- protractors
- rulers
- index cards



DISCUSS IT

Ask your partner: Do you agree with me? Why or why not?

Tell your partner: I agree with you about . . . because . . .

Explore different ways to understand how to sort shapes into categories based on angles.

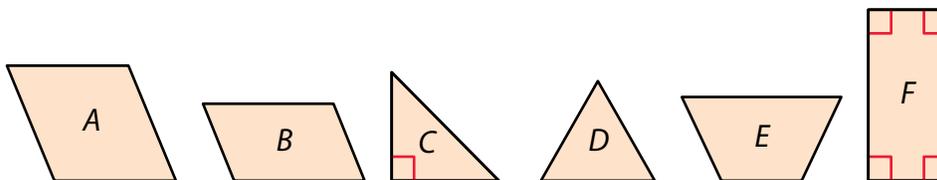
A classroom computer game shows a set of categories and a set of shapes. The player puts each shape in the correct category. Draw a line from each shape to the category it belongs in.

acute only

right only

acute and right

acute and obtuse

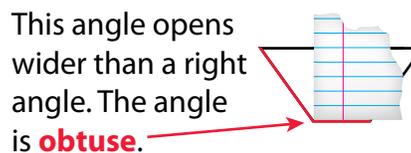


PICTURE IT

You can use a model to help sort shapes based on angles.

Use the corner of a sheet of paper as a model of a right angle. Compare each angle to the paper corner.

For example, hold up the paper corner to the trapezoid.



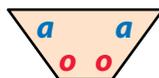
Then you can compare the paper corner to each of the other 3 angles in the trapezoid.

MODEL IT

You can label a drawing to help sort shapes based on angles.

Look at each shape. Mark each angle *a* for acute, *r* for right, or *o* for obtuse.

For example, mark the trapezoid like this:



The trapezoid has 2 acute angles and 2 obtuse angles. It belongs in the group “acute and obtuse.”

Remember to look at all of the angles in a shape before you put it in a group.

CONNECT IT

Now you will use the problem from the previous page to help you understand how to sort shapes into categories based on angles.

- 1 Look at parallelograms *A* and *B*. Check that you have drawn lines to the correct group(s). Do the two parallelograms belong to the same group? Explain.

- 2 Look at the two triangles. Check that you have drawn lines to match the triangles with their group(s). Describe the angles in each triangle.

- 3 Look at the trapezoid and rectangle. Which has right angles only?
 Look at **Picture It**. To which group does the trapezoid belong?
 Check that you have drawn lines to
 the correct group(s).

- 4 Explain how to sort shapes based on whether they have acute, right, or obtuse angles.

5 REFLECT

Look back at your **Try It**, strategies by classmates, and **Picture It** and **Model It**. Which models or strategies do you like best for sorting shapes based on angles? Explain.

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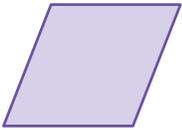
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APPLY IT

Use what you just learned to solve these problems.

- 6 Which of these groups does the rhombus below belong in: “acute angles only,” “obtuse angles only,” “right angles only,” “both acute and obtuse angles,” or “both right and obtuse angles”? Explain.



- 7 Circle the shape that has an acute angle, a right angle, and an obtuse angle.

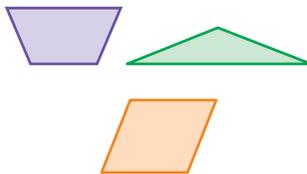


- 8 The shapes below have been sorted into two groups based on their angles. Explain how the shapes could have been sorted.

Group 1



Group 2



Practice Sorting Shapes Based on Angles

Study the Example showing how to sort shapes into groups based on angles. Then solve problems 1–5.

EXAMPLE

Label each angle in the shapes below with *a* for acute, *r* for right, and *o* for obtuse. Then draw a line from each shape to the group it belongs in.

right and acute

right and obtuse

acute and obtuse

- 1 Write the number of acute, right, and obtuse angles for each pentagon shown in the table below.

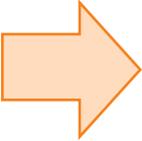
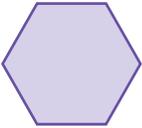
	Acute	Right	Obtuse

- 2 Explain how these pentagons are different based on their angles.

Solution

.....

3 Tell whether each shape belongs in the group described.

		Yes	No
	all right angles	(A)	(B)
	right and acute angles	(C)	(D)
	obtuse and acute angles	(E)	(F)
	right and obtuse angles only	(G)	(H)
	all obtuse angles	(I)	(J)

4 Describe a group that the two shapes at the right belong in, based on the kind of angles the shapes have.



Solution

5 Look at the shapes in problem 4. Where do they belong in the table below? Draw each shape in the column in which it belongs. Explain your answer.

Acute and Obtuse Angles	Acute and Right Angles	Obtuse and Right Angles	Acute, Right, and Obtuse Angles

Develop Sorting Triangles

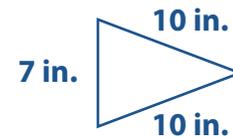
Read and try to solve the problem below.

A website sells 7 kinds of triangular flags based on sides and angles.

Flag	Equal Sides	Angles
1	3	3 acute
2	2	2 acute, 1 right
3	2	2 acute, 1 obtuse
4	2	3 acute

Flag	Equal Sides	Angles
5	0	2 acute, 1 right
6	0	2 acute, 1 obtuse
7	0	3 acute

The triangle at the right is a model for which flag number?



TRY IT



Math Toolkit

- protractors
- rulers
- index cards



DISCUSS IT

Ask your partner: Why did you choose that strategy?

Tell your partner: I do not understand how ...

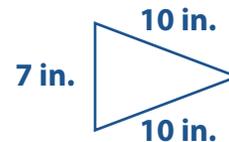
Explore different ways to understand how to sort triangles into groups based on kinds of angles and lengths of sides.

A website sells 7 kinds of triangular flags based on sides and angles.

Flag	Equal Sides	Angles
1	3	3 acute
2	2	2 acute, 1 right
3	2	2 acute, 1 obtuse
4	2	3 acute

Flag	Equal Sides	Angles
5	0	2 acute, 1 right
6	0	2 acute, 1 obtuse
7	0	3 acute

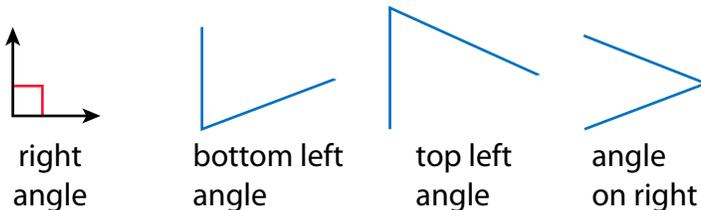
The triangle at the right is a model for which flag number?



PICTURE IT

You can use a picture to help describe the sides and angles of triangles.

Compare the angles of the triangle to a right angle. The triangle has 3 acute angles.



The triangle has 2 sides of equal length (10 in.). Flag 4 has **2 sides of equal length** and **3 acute angles**. The triangle is a model for flag 4.

The tables below show triangle names based on the number of sides of equal length and kinds of angles.

Name	Description of Sides
equilateral	3 equal sides
isosceles	2 equal sides
scalene	0 equal sides

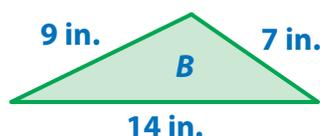
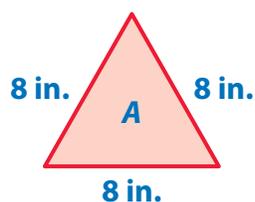
Name	Description of Angles
acute	3 acute angles
right	1 right angle
obtuse	1 obtuse angle

The triangle has 2 equal sides, so it is an **isosceles triangle**. Since it has 3 acute angles, it is an **acute triangle**.

CONNECT IT

Now you will use the problem from the previous page to help you understand how to sort triangles into groups based on kinds of angles and lengths of sides and how to name triangles.

- 1 Look back at the model for the triangular flag. Fill in the blanks to name this triangle based on its angles and sides: triangle



- 2 Look at triangle *A* above. How many sides are the same length?
 What kinds of angles does it have?
 What are two names for this triangle?
- 3 What are two names for triangle *B*?
 Can triangle *B* also be called an acute triangle? Why or why not?
- 4 Explain how to give a complete description of a triangle.

5 REFLECT

Look back at your **Try It**, strategies by classmates, and **Picture It**. Which models or strategies do you like best for sorting triangles into groups based on kinds of angles and lengths of sides and for naming triangles? Explain.

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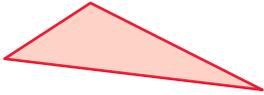
.....

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APPLY IT

Use what you just learned to solve these problems.

- 6 Give a complete description of the triangle below. Show your work.



Solution

- 7 What do the triangles below have in common? How are they different?

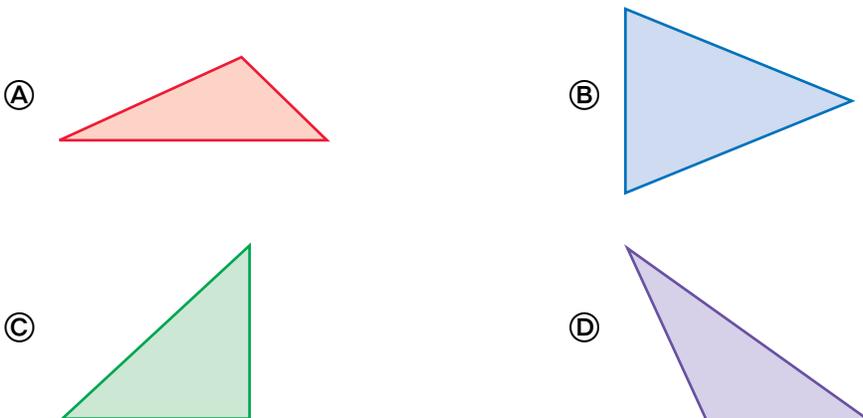


Solution

.....

.....

- 8 Which figure is an acute isosceles triangle?



Practice Sorting Triangles

Study the Example showing how to sort triangles into groups based on kinds of angles and lengths of sides. Then solve problems 1–4.

EXAMPLE

What is the same about the two triangles shown at the right? What is different?

You can sort triangles into groups based on the kinds of angles they have: acute, right, or obtuse.

You can also sort triangles based on the lengths of their sides.

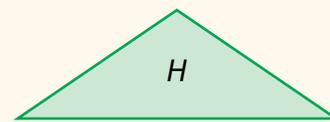
equilateral: 3 equal sides

isosceles: 2 equal sides

scalene: 0 equal sides

Triangles *B* and *H* are the same because they are both obtuse triangles. They each have 1 obtuse angle.

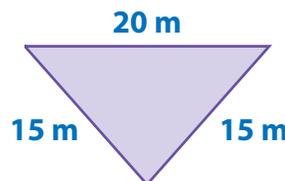
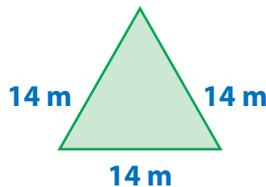
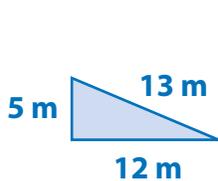
Triangles *B* and *H* are different because triangle *B* is a scalene triangle and triangle *H* is an isosceles triangle.



- 1 Look at the table. Name each triangle below based on the kinds of angles that it has and the lengths of its sides.

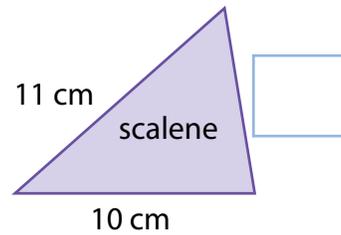
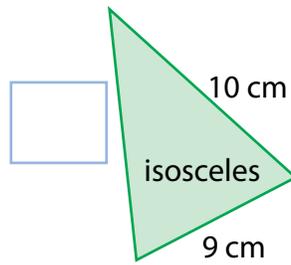
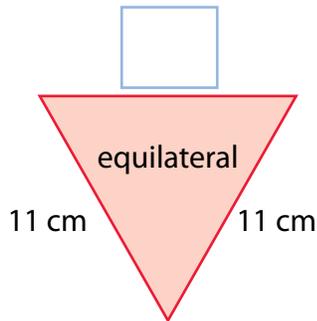
Name	Description of Angles
acute	3 acute angles
right	1 right angle
obtuse	1 obtuse angle

Name	Description of Sides
equilateral	3 equal sides
isosceles	2 equal sides
scalene	0 equal sides

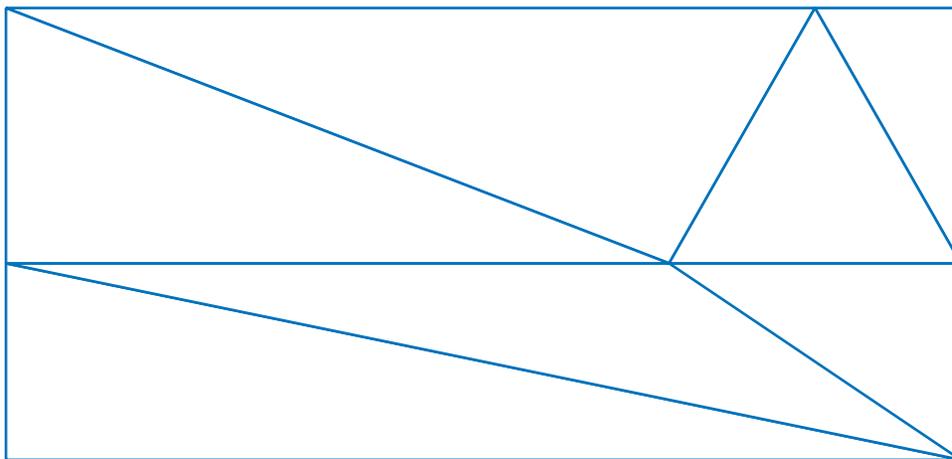


- 2 Look at the name of each triangle below. Then use the numbers in the boxes to write the missing length for one side of each triangle.

9 cm 10 cm 11 cm



- 3 Write labels inside each triangle formed by the lines in the drawing below: *a* for acute, *r* for right, *o* for obtuse, *e* for equilateral, *i* for isosceles, *s* for scalene.



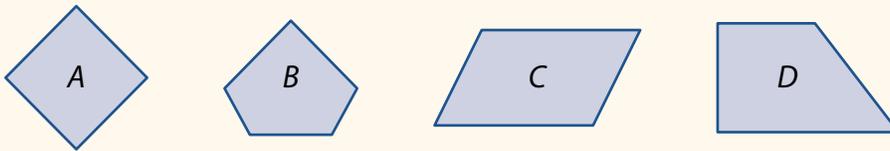
- 4 Which statements below are true?
- (A) An obtuse triangle does not have acute angles.
 - (B) A scalene triangle can be isosceles.
 - (C) Equilateral triangles are always acute.
 - (D) Isosceles triangles can be obtuse.
 - (E) Right triangles are scalene or isosceles.

Refine Classifying Two-Dimensional Figures

Complete the Example below. Then solve problems 1–7.

EXAMPLE

Do any of the shapes below have at least one pair of parallel sides and at least one right angle? If yes, list the shapes. If no, explain.



Look at how you could show your work using a table.

Shape	Parallel Sides	Right Angle
A	X	X
B		X
C	X	
D	X	X

Solution

The student listed each shape in a table and used an X to show that a shape had parallel sides or a right angle.



PAIR/SHARE

How could you test for parallel sides?

APPLY IT

- Nate and Alicia play Draw My Shape. Nate says: *My shape has 2 pairs of parallel sides, 2 acute angles, and 2 obtuse angles.* Alicia draws the rectangle below. Explain why Alicia's answer is incorrect.



Solution

You can test the angles to see if they are acute, right, or obtuse.

PAIR/SHARE

Can you have a 4-sided shape with 4 right angles and only 1 pair of parallel sides?

2 Tell how the sides and angles of the shapes below are alike and different.



square



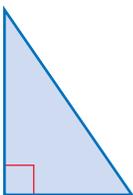
rhombus

Solution

.....

.....

3 Which is the best name for the triangle shown?



- (A) acute isosceles triangle
- (B) acute scalene triangle
- (C) right isosceles triangle
- (D) right scalene triangle

Ricky chose (B) as the correct answer. How did he get that answer?

All the square's angles look alike, but the rhombus looks like it has two different kinds of angles.



PAIR/SHARE

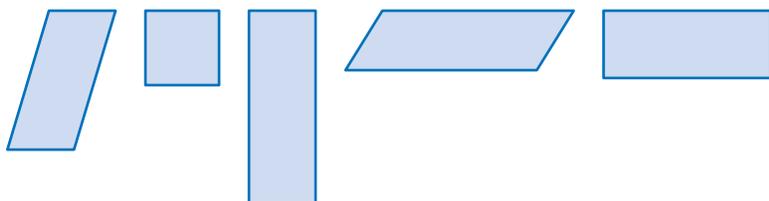
What does a rhombus have in common with a parallelogram?

How many right angles does a triangle have to have to be called a "right triangle"?

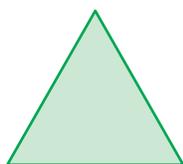
PAIR/SHARE

Could a triangle ever have 2 right angles?

- 4 Which is the best name for the group of shapes below?



- (A) shapes with acute angles
 (B) shapes with right angles
 (C) shapes with parallel sides
 (D) shapes with perpendicular sides
- 5 Sort the four shapes below. Use the characteristics shown in the table. Draw each shape in each column where it belongs. Some shapes may belong in more than one column.



equilateral
triangle



parallelogram



square



right
trapezoid

Shapes with at Least One Acute Angle	Shapes with at Least One Pair of Perpendicular Sides	Shapes with at Least One Pair of Parallel Sides

6 Tell whether each sentence is *True* or *False*.

	True	False
A right scalene triangle can have 3 different kinds of angles.	(A)	(B)
A right isosceles triangle has 2 right angles.	(C)	(D)
An equilateral triangle is also an acute triangle.	(E)	(F)
A triangle can have 2 perpendicular sides.	(G)	(H)



7 MATH JOURNAL

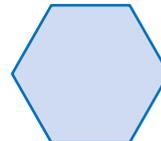
Divide the shapes below into two groups. Give each group a title that tells what all the shapes in that group have in common. Then describe another shape that belongs to each group.



quadrilateral



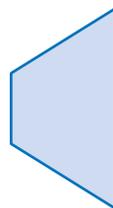
square



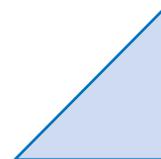
hexagon



parallelogram



trapezoid



triangle



SELF CHECK Go back to the Unit 5 Opener and see what you can check off.