Lesson Overview

LESSON 18

Compare Fractions

Lesson Objectives

Content Objectives

- Use symbols ($>$, $<$, $=$) to compare fractions with different numerators and different denominators.
- Recognize that fractions with different denominators and the same numerators represent different values.
- Use common denominators and benchmark fractions to compare fractions with different denominators.
- Recognize that to compare two fractions both must refer to the same whole.

Language Objectives

- Write fraction comparison statements using the symbols $>$, $<$, and $=$.
- Draw area models to compare two fractions.
- Orally explain how comparing both a fraction greater than $\frac{1}{2}$ and a fraction less than $\frac{1}{2}$ to $\frac{1}{2}$ can be used to determine which fraction is greater.

Prerequisite Skills

- Represent fractions with denominators 2, 3, 4, 6, or 8 using a number line or visual models.
- Identify, generate, and explain equivalent fractions.
- Express whole numbers as fractions.
- Compare fractions with the same numerators or denominators.

Standards for Mathematical Practice (SMP)

SMPs 1, 2, 3, 4, 5, and 6 are integrated in every lesson through the *Try-Discuss-Connect* routine.*

In addition, this lesson particularly emphasizes the following SMPs:

4 Model with mathematics.

5 Use appropriate tools strategically.

7 Look for and make use of structure.

*See page 363m to see how every lesson includes these SMPs.

Lesson Vocabulary

- **benchmark fraction** a common fraction that you might compare other fractions to. For example, $\frac{1}{4}$, $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{4}$ are often used as benchmark fractions.
- **common denominator** a number that is a common multiple of the denominators of two or more fractions.

Review the following key terms.

- **compare** to decide if numbers, amounts, or sizes are greater than, less than, or equal to each other.
- **denominator** the number below the line in a fraction that tells the number of equal parts in the whole.
- **fraction** a number that names equal parts of a whole. A fraction names a point on the number line.
- **greater than symbol (>)** a symbol used to compare two numbers when the first is greater than the second.
- **less than symbol (<)** a symbol used to compare two numbers when the first is less than the second.
- **numerator** the number above the line in a fraction that tells the number of equal parts that are being described.
- **unit fraction** a fraction with a numerator of 1. Other fractions are built from unit fractions.

Learning Progression

In Grade 3 students used models to compare two fractions with the same numerator or the same denominator by reasoning about their size.

In Grade 4 students extend their understanding of fractions to compare two fractions with different numerators and different denominators. Emphasis is placed on understanding that a comparison only makes sense if the two fractions have the same-sized wholes. In this lesson students use models to compare fractions by using common numerators or denominators. Students also use benchmark fractions to compare fractions. They record comparisons using the symbols $>$, $<$, and $=$.

In Grade 5 students will apply their understanding of fraction comparison when they learn to compare decimals.

Lesson Pacing Guide

Whole Class Instruction

Session	Activities	Additional
SESSION 1 Explore *45–60 min*	**Interactive Tutorial*** *(Optional)* *Prerequisite Review:* Equivalent Fractions **Comparing Fractions** • Start *5 min* • Try It *10 min* • Discuss It *10 min* • Connect It *15 min* • Close: Exit Ticket *5 min*	**Additional Practice** Lesson pages 383–384
SESSION 2 Develop *45–60 min*	**Using Common Numerators and Denominators** • Start *5 min* • Try It *10 min* • Discuss It *10 min* • Model Its *5 min* • Connect It *10 min* • Close: Exit Ticket *5 min*	**Additional Practice** Lesson pages 389–390 **Fluency** Using Common Numerators and Denominators
SESSION 3 Develop *45–60 min*	**Using a Benchmark to Compare Fractions** • Start *5 min* • Try It *10 min* • Discuss It *10 min* • Model It & Solve It *5 min* • Connect It *10 min* • Close: Exit Ticket *5 min*	**Additional Practice** Lesson pages 395–396 **Fluency** Using a Benchmark to Compare Fractions
SESSION 4 Refine *45–60 min*	**Comparing Fractions** • Start *5 min* • Example & Problems 1–3 *15 min* • Practice & Small Group Differentiation *20 min* • Close: Exit Ticket *5 min*	**Lesson Quiz** or **Digital Comprehension Check**

Lesson Materials

Lesson *(Required)* — *Per student:* 1 set of fraction tiles

Activities — *Per student:* 1 set of fraction tiles
Per pair: scissors, colored pencils, index cards
Activity Sheets: Number Lines, 1-Centimeter Grid Paper

Math Toolkit — fraction circles, fraction tiles, number lines, fraction bars, index cards, hundredths grids, tenths grids

Digital Math Tools — Fraction Models, Number Line

*We continually update the Interactive Tutorials. Check the Teacher Toolbox for the most up-to-date offerings for this lesson.

Teacher Toolbox

Small Group Differentiation

PREPARE

Ready Prerequisite Lessons

Grade 3
• Lesson 23 Find Equivalent Fractions
• Lesson 25 Use Symbols to Compare Fractions

RETEACH

Tools for Instruction

Grade 3
• Lesson 23 Find Equivalent Fractions
• Lesson 25 Use Symbols to Compare Fractions

Grade 4
• Lesson 18 Compare Fractions

REINFORCE

Math Center Activities

Grade 4
• Lesson 18 Use Fraction Vocabulary
• Lesson 18 Comparing Fractions

EXTEND

Enrichment Activity

Grade 4
• Lesson 18 Colorful Quilts

i-Ready

Independent Learning

PERSONALIZE

i-Ready Lesson*

Grade 4
• Compare Fractions

Learning Game
• *Bounce*

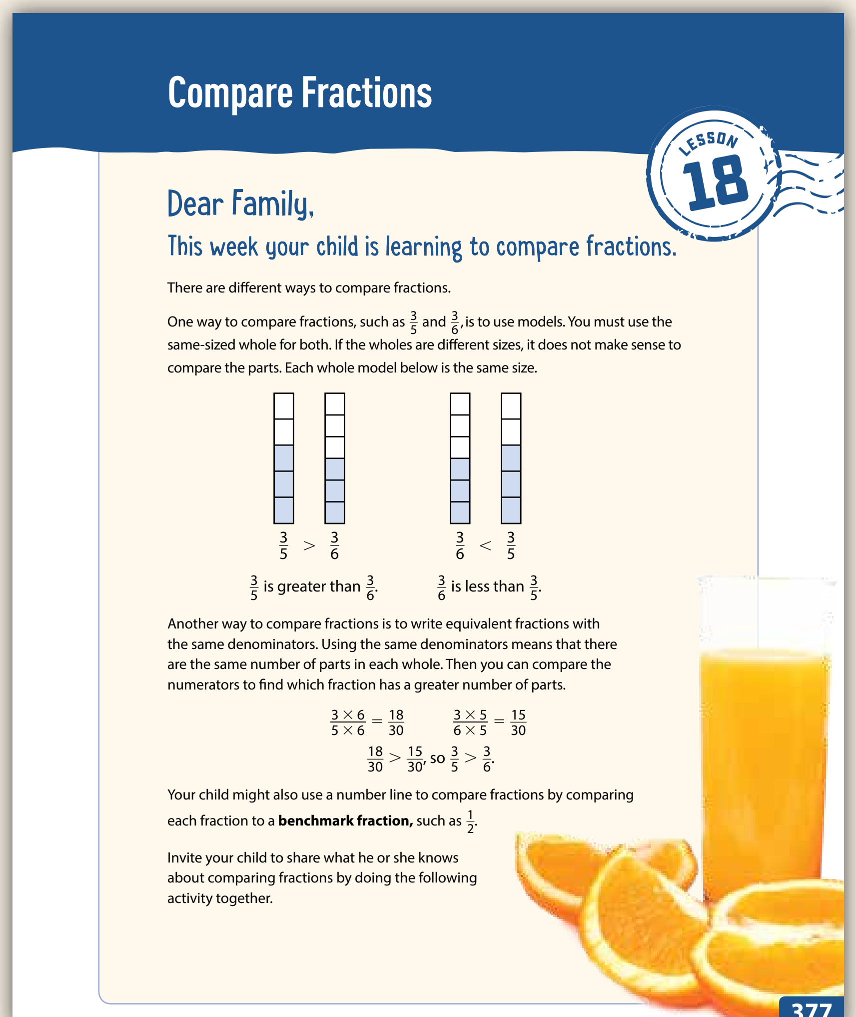
Connect to Family, Community, and Language Development

The following activities and instructional supports provide opportunities to foster school, family, and community involvement and partnerships.

Connect to Family

Use the **Family Letter**—which provides background information, math vocabulary, and an activity—to keep families apprised of what their child is learning and to encourage family involvement.

Available in Spanish
Teacher Toolbox



Compare Fractions

LESSON 18

Dear Family,

This week your child is learning to compare fractions.

There are different ways to compare fractions.

One way to compare fractions, such as $\frac{3}{5}$ and $\frac{3}{6}$, is to use models. You must use the same-sized whole for both. If the wholes are different sizes, it does not make sense to compare the parts. Each whole model below is the same size.

$\frac{3}{5} > \frac{3}{6}$ $\qquad$ $\frac{3}{6} < \frac{3}{5}$

$\frac{3}{5}$ is greater than $\frac{3}{6}$. $\qquad$ $\frac{3}{6}$ is less than $\frac{3}{5}$.

Another way to compare fractions is to write equivalent fractions with the same denominators. Using the same denominators means that there are the same number of parts in each whole. Then you can compare the numerators to find which fraction has a greater number of parts.

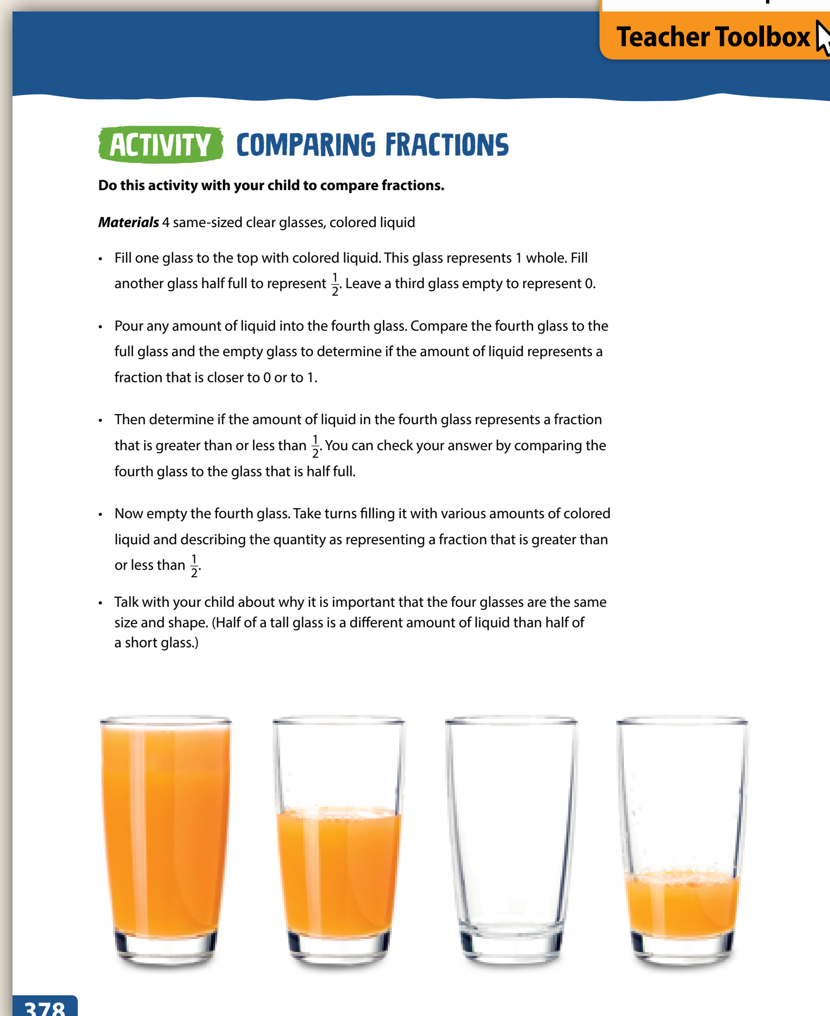
$$\frac{3 \times 6}{5 \times 6} = \frac{18}{30} \qquad \frac{3 \times 5}{6 \times 5} = \frac{15}{30}$$

$$\frac{18}{30} > \frac{15}{30}, \text{ so } \frac{3}{5} > \frac{3}{6}.$$

Your child might also use a number line to compare fractions by comparing each fraction to a **benchmark fraction,** such as $\frac{1}{2}$.

Invite your child to share what he or she knows about comparing fractions by doing the following activity together.

377



ACTIVITY COMPARING FRACTIONS

Do this activity with your child to compare fractions.

Materials 4 same-sized clear glasses, colored liquid

- Fill one glass to the top with colored liquid. This glass represents 1 whole. Fill another glass half full to represent $\frac{1}{2}$. Leave a third glass empty to represent 0.
- Pour any amount of liquid into the fourth glass. Compare the fourth glass to the full glass and the empty glass to determine if the amount of liquid represents a fraction that is closer to 0 or to 1.
- Then determine if the amount of liquid in the fourth glass represents a fraction that is greater than or less than $\frac{1}{2}$. You can check your answer by comparing the fourth glass to the glass that is half full.
- Now empty the fourth glass. Take turns filling it with various amounts of colored liquid and describing the quantity as representing a fraction that is greater than or less than $\frac{1}{2}$.
- Talk with your child about why it is important that the four glasses are the same size and shape. (Half of a tall glass is a different amount of liquid than half of a short glass.)

378

Goal

The goal of the Family Letter is to encourage students and family members to compare fractions.

- Students and family members explore how to compare fractions by using models and equivalent fractions with common denominators. Students are introduced to the term *benchmark fraction* as a strategy for comparing fractions.

Activity

Students and family members compare the amounts of liquid in 4 glasses that are the same shape and size. Three glasses are filled with varying amounts of colored liquid to represent 1, $\frac{1}{2}$, and 0. The fourth glass is filled with a random amount to compare to the amounts in the other glasses.

Math Talk at Home

Explain to students that the glasses used in the Family Letter activity need to be the same size and shape. If students do not have the materials at home to do the activity, encourage them to compare fractions in casual conversations with family members using common same-sized items they have at home.

Conversation Starters Below are additional questions students can write in their Family Letter or math journal to engage family members:

- *If you have a glass that is $\frac{1}{2}$ full, and I have a glass that is $\frac{3}{4}$ full, who has the greater amount?*
- *If I have read $\frac{1}{3}$ of a book and you have read $\frac{1}{2}$ of the same book, who has more of the book left to read?*

Connect to Community and Cultural Responsiveness

Use these activities to connect with and leverage the diverse backgrounds and experiences of all students.

Session 1 Use with *Try It*.

- Many students will be familiar with granola bars, but there may be some who cannot eat them because of food allergies. Encourage students to substitute food items they are familiar with in the problem to make it relevant to them. For example, instead of same-sized granola bars, students may visualize same-sized carrots or celery sticks. Remind students that the food items they use in the word problem must be the same size for comparing fractions.

Session 2 Use with *Apply It* problem 9.

- Ask students if they know where tomatoes or peppers originate from or if they have grown a tomato or pepper plant. Some students may respond, "the grocery store," or may have never considered where tomatoes or peppers come from. Show students pictures of tomato and pepper plants. Ask students what other fruits and vegetables they are familiar with. Possible responses include cucumbers, squash, beans, peas, okra, corn, and grapes. If there are regional fruits or vegetables students may be more familiar with, substitute these in the word problem.

Session 3 Use with *Additional Practice* problems.

- As students read the problems, encourage them to think of scenarios to make them more interesting and relevant. Model this strategy for students using problem 1. In this problem, students are asked to compare $\frac{9}{10}$ and $\frac{3}{2}$. Say: *My coach asked me to run $\frac{9}{10}$ of the track on Monday. On Tuesday, she asked me to run $\frac{3}{2}$ of the track. On which day did I run a lesser distance?* Have students share their scenarios for the problems with partners.

Session 4 Use with *Apply It* problem 6.

- Ask students if they know what a trombone is and to give examples of when they have seen one being played. Explain that a trombone is a brass instrument. To become a proficient trombonist, a person practices the trombone for many hours. In *Apply It* problem 6, the student practices his trombone $\frac{2}{6}$ of an hour. Ask students to share with the class instruments they play or would like to play. As students read the word problem, encourage them to substitute other instruments they are familiar with.

Connect to Language Development

For ELLs, use the Differentiated Instruction chart to plan and prepare for specific activities in every session.

ELL English Language Learners: Differentiated Instruction

Prepare for Session 1
Use with *Connect It*.

Levels 1–3	Levels 2–4	Levels 3–5
Speaking/Reading Read ***Connect It*** problem 2b to students. Explain that to understand the problem, students need to understand the terms in it. Write the following terms and symbols on sentence strips: *denominator*, >, <, =, and *equivalent fractions*. Display the term *denominator*. Ask students to define *denominator* to partners before sharing definitions with the group. Select one of the definitions to write on a sentence strip. Continue the process with the remaining terms. Shuffle the sentence strips. Ask students to read and match the sentence strips. Reread ***Connect It*** problem 2b. Have students restate the information in their own words.	**Writing/Reading** Choral read ***Connect It*** problem 2b. Explain that to understand the problem, students need to understand the terms in it. Have students form pairs. Ask students to write the following terms and symbols on index cards and then work with their partners to write the definitions on other cards: *compare, fractions*, >, <, =, and *equivalent fractions*. Have pairs shuffle their cards and exchange them with another group. Ask them to match the terms with the definitions. Reread ***Connect It*** problem 2b with students. Ask students to restate the information in their own words to their partners.	**Speaking/Writing** Have students read ***Connect It*** problem 2b with partners. Have partners identify the terms and symbols in the problem that they think are important for understanding the problem. Then ask students to work with their partners to explain why they think the terms and symbols they chose are important. Ask partners to write the terms, symbols, and definitions on index cards, shuffle them and then exchange them with other pairs to match. Have students reread ***Connect It*** problem 2b with partners and have them restate the information in their own words.

SESSION 1 Explore

Purpose In this session, students draw on their experience comparing fractions in order to compare two fractions with the same numerators and different denominators. They share models to explore how various solution methods are based on the number of equal parts in each whole and the sizes of the parts. They will look ahead to think about comparing fractions with different denominators by using equivalent fractions with a common denominator.

Start

Connect to Prior Knowledge

Materials For each student: 1 set of fraction tiles

Why Support students' facility with comparing fractions with the same numerators.

How Have students use fraction tiles to compare $\frac{3}{5}$ and $\frac{3}{6}$.

Compare $\frac{3}{5}$ and $\frac{3}{6}$.

Which fraction is greater?

Solution
$\frac{3}{5}$ is greater than $\frac{3}{6}$.

LESSON 18 SESSION 1 ● ○ ○ ○

Explore Comparing Fractions

Previously, you learned to compare fractions using models. Use what you know to try to solve the problem below.

Adriana and June have granola bars that are the same size. Adriana eats $\frac{2}{4}$ of her granola bar. June eats $\frac{2}{5}$ of her granola bar. Which girl eats more of her granola bar?

Learning Target

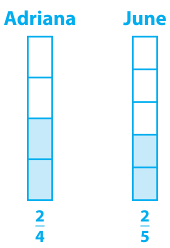
- Compare two fractions with different numerators and different denominators. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions.

SMP 1, 2, 3, 4, 5, 6, 7

TRY IT

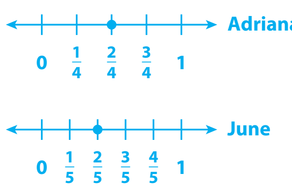
Possible student work:

Sample A



Adriana eats more of her granola bar than June does.

Sample B



$\frac{2}{4}$ is closer to 1 on the number line than $\frac{2}{5}$ is.

So, $\frac{2}{4}$ is greater than $\frac{2}{5}$.

Adriana eats more of her granola bar.

Math Toolkit

- fraction circles
- fraction tiles
- number lines
- fraction bars
- index cards
- fraction models

DISCUSS IT

Ask your partner: Do you agree with me? Why or why not?

Tell your partner: I agree with you about . . . because . . .

379

TRY IT

Make Sense of the Problem

To support students in making sense of the problem, have them show that they understand that the granola bars are the same size and that Adriana eats 2 out of 4 equal parts of her bar while June eats 2 out of 5 equal parts of her bar.

DISCUSS IT

Support Partner Discussion

To reinforce the units of fourths and fifths, encourage students to use the terms *fourths* and *fifths* as they talk together.

Look for, and prompt as necessary, for understanding of:

- both wholes as the same size
- 4 as the number of equal parts in one whole
- 5 as the number of equal parts in the other whole
- 2 as the number of parts considered in each whole

Common Misconception Look for students who think that $\frac{2}{5}$ is greater than $\frac{2}{4}$ because 5 is greater than 4. As students present solutions, have them explain how the sizes of the equal parts in the granola bars compare to each other.

Select and Sequence Student Solutions

One possible order for whole class discussion:

- fraction circles or fraction tiles modeling two fourths and two fifths
- drawings of area models showing two fourths and two fifths
- labeled number lines showing the locations of two fourths and two fifths
- writing equivalent fractions to compare two fourths and two fifths

Support Whole Class Discussion

Prompt students to note the relationship between the numbers in each model and the numbers in the problem.

Ask *How do [student name]'s and [student name]'s models show the number of equal parts in each whole? the number of parts considered?*

Listen for The models are divided into 4 and 5 equal parts or have denominators of 4 and 5. The models have 2 parts shaded, or counted, or have numerators of 2.

CONNECT IT

1 LOOK BACK

Look for understanding that Adriana eats more and that $\frac{2}{4}$ is greater than $\frac{2}{5}$ because the size of each of Adriana's two $\frac{1}{4}$-pieces is greater than the size of each of June's two $\frac{1}{5}$-pieces.

Hands-On Activity

Use fraction tiles to compare fractions with the same numerator.

If . . . *students are having difficulty comparing fractions with the same numerators,*

Then . . . *use this activity to provide a more concrete experience in reasoning about the size of the unit fractions that make up each fraction.*

Materials For each student: 1 set of fraction tiles

- Have students compare $\frac{2}{3}$ and $\frac{2}{8}$. Tell them to trace around the one-whole tile two times, so they have an outline of one whole for each fraction.
- Have students identify the unit fractions for $\frac{2}{3}\left[\frac{1}{3}\right]$ and $\frac{2}{8}\left[\frac{1}{8}\right]$ and compare the sizes of the fraction tiles for $\frac{1}{3}$ and $\frac{1}{8}$ to find which covers a larger area. $\left[\frac{1}{3}\right]$ Say: *Each third is larger than each eighth, so $\frac{2}{3}$ is greater than $\frac{2}{8}$.*
- Have students use the tiles and the one-whole outlines to build the fractions $\frac{2}{3}$ and $\frac{2}{8}$ to see that $\frac{2}{3}$ is greater than $\frac{2}{8}$.
- Repeat for additional fractions with the same numerators, such as $\frac{3}{6}$ and $\frac{3}{4}$, and $\frac{4}{5}$ and $\frac{4}{10}$.

2 LOOK AHEAD

Point out that the same symbols used to compare whole numbers are used to compare fractions. Review the meanings of the symbols $>$, $<$, and $=$. Also point out that you can compare fractions with different denominators by rewriting one or both of them to have a common denominator. Ask a volunteer to restate the definition of the term *common denominator*. Students will spend more time learning about the concept of common denominators in the Additional Practice.

Students should be able to use their work with equivalent fractions to rewrite the given fractions with a common denominator.

CONNECT IT

1 LOOK BACK

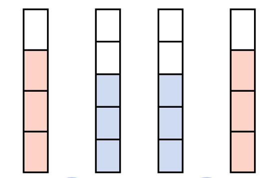
Who eats more of her granola bar, Adriana or June? Explain.

Adriana eats more. Possible explanation: Both girls eat the same number of pieces, but Adriana's two pieces are larger than June's two pieces.

2 LOOK AHEAD

Deciding who eats more of her granola bar means comparing the fractions $\frac{2}{4}$ and $\frac{2}{5}$. To compare fractions, you must use the same-sized whole.

a. Suppose you have two more granola bars that are the same size. Compare the fractions $\frac{3}{4}$ and $\frac{3}{5}$ using the area models to know who ate more. Use $>$, $<$, or $=$ to compare, just as with whole numbers.



$\frac{3}{4} > \frac{3}{5}$ $\frac{3}{5} < \frac{3}{4}$

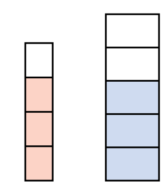
b. You can use equivalent fractions to compare fractions with different denominators. Compare $\frac{3}{4}$ and $\frac{3}{5}$. Rewrite one or both of the fractions so they have the same denominator, or a **common denominator**. Use $>$, $<$, or $=$ to compare.

$\frac{3 \times \boxed{5}}{4 \times \boxed{5}} = \frac{15}{\boxed{20}}$ $\frac{3 \times \boxed{4}}{5 \times \boxed{4}} = \frac{\boxed{12}}{20}$

$\frac{15}{\boxed{20}} > \frac{\boxed{12}}{20}$, so $\frac{3}{4} > \frac{3}{5}$.

3 REFLECT

Suppose the granola bars were different sizes. Could you still compare $\frac{3}{4}$ and $\frac{3}{5}$ in the same way? Explain.



No. Possible explanation: Because the wholes are two different sizes, comparing parts to each other does not make sense.

Close: Exit Ticket

3 REFLECT

Look for understanding that a comparison of two fractions only makes sense when the wholes are the same size.

Common Misconception **If** students do not mention that the sizes of the wholes must be the same in order to compare the fractions, **then** have them look at the different-sized wholes in the models in problem 3 and identify how the size of the wholes differ so the parts cannot be compared.

Real-World Connection

Encourage students to think about everyday places or situations in which people might need to compare fractions. Have volunteers share their ideas. Examples include following a recipe, measuring lengths of fabric, and making carpentry measurements.

SESSION 1 Additional Practice

Solutions

Support Vocabulary Development

1 Ask students to use what they know about the meaning of the term *common denominator* based on what they know about the words *common* (something two or more things share) and *denominator* (the number below the line in a fraction that tells the number of equal parts in the whole). First, have students share their definitions with partners, then ask them to share their definitions with the group. Record responses on a chart. Remind students they found a common denominator for $\frac{3}{4}$ and $\frac{3}{5}$ in *Connect It* problem 2. Ask them to tell partners what they did to find a common denominator. Record responses on the chart. If students need support verbalizing the process for finding common denominators, write the fractions $\frac{1}{3}$ and $\frac{2}{4}$ and talk through the process. As students complete the graphic organizer, encourage them to refer to the information recorded on the chart. Remind them to record information in their own words. Also, encourage students to review the *Try It* and *Connect It* information to help them show examples and non-examples of common denominators.

2 If students struggle to complete the problem, have them think through the process for finding a common denominator before solving it. Provide guiding statements to help students.

- *Think about each fraction. Circle the denominators.*
- *Think about a multiple of each denominator that can be used as a common denominator. [15]*
- *Think about the numbers you multiply 3 and 5 by to get the common denominator. [5, 3]*
- *Think about multiplying 3 by 5 in the fraction $\frac{2}{3}$. You will also multiply the numerator, 2, by 5.*
- *Think about multiplying the denominator 5 by 3 and what number you will multiply the numerator, 2, by in the fraction $\frac{2}{5}$.*
- *Think about which fraction with denominator 15 is greater and which is less. Compare the original fractions.*

Supplemental Math Vocabulary

- *fraction*
- *denominator*

Name: ______________________________

Prepare for Comparing Fractions

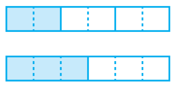
1 Think about what you know about common denominators. Fill in each box. Use words, numbers, and pictures. Show as many ideas as you can. **Possible answers:**

common denominator

In My Own Words

When two or more fractions have the same denominator, they have a common denominator.

My Illustrations



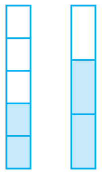
$\frac{1}{3} = \frac{2}{6}$

$\frac{1}{2} = \frac{3}{6}$

Examples

$\frac{2}{5}$, $\frac{3}{5}$, and $\frac{4}{5}$ all have a common denominator of 5.

Non-Examples



$\frac{2}{5}$ $\frac{2}{3}$

2 Compare $\frac{2}{3}$ and $\frac{2}{5}$. Rewrite the fractions so they have a common denominator.

Use >, <, or = to compare.

$\frac{2 \times \boxed{5}}{3 \times \boxed{5}} = \frac{10}{\boxed{15}}$ $\frac{2 \times \boxed{3}}{5 \times \boxed{3}} = \frac{\boxed{6}}{15}$

$\frac{10}{\boxed{15}} > \frac{\boxed{6}}{15}$, so $\frac{2}{3} > \frac{2}{5}$.

3 Assign problem 3 to provide another look at solving a problem by comparing fractions.

This problem is very similar to the problem asking which girl eats more of her granola bar. In both problems, students compare two fractions with the same numerators, but different denominators. The question asks which boy drinks more juice.

Students may want to use fraction tiles, fraction circles, or fraction bars.

Suggest that students read the problem three times, asking themselves one of the following questions each time:

- *What is this problem about?*
- *What is the question I am trying to answer?*
- *What information is important?*

Solution:

Students may draw diagrams to help compare the fractions. Since the bottles of juice are the same size, each fourth of a bottle is more than each sixth of a bottle, and so $\frac{3}{4}$ of a bottle is more than $\frac{3}{6}$ of a bottle. Donato drinks more juice.

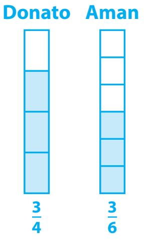
Medium

4 Have students solve the problem another way to check their answer.

3 Solve the problem. Show your work.

Donato and Aman have bottles of juice that are the same size. Donato drinks $\frac{3}{4}$ of his juice. Aman drinks $\frac{3}{6}$ of his juice. Which boy drinks more juice?

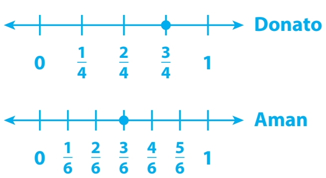
Possible student work using pictures:



Solution **Donato drinks more juice.**

4 Check your answer. Show your work.

Possible student work:



$\frac{3}{4}$ is closer to 1 on the number line than $\frac{3}{6}$ is.

So, $\frac{3}{4}$ is greater than $\frac{3}{6}$.

Donato drinks more juice.

ELL English Language Learners: Differentiated Instruction

Prepare for Session 2
Use with *Connect It*.

Levels 1–3

Speaking/Writing Use with ***Connect It*** problem 8. Ask students to brainstorm about models or strategies used to compare fractions. Encourage them to review *Try It* and *Model Its* for reminders. Make a list of responses. Point to the first model or strategy, for example, *area models*. Ask students to draw a representation for how the model or strategy is used to compare fractions. Continue the process for the remaining models or strategies. Ask: *Which model or strategy do you like best for comparing fractions?* Have students first respond verbally, then in writing: *I like to use ______ for comparing fractions.*

Levels 2–4

Speaking/Writing Use with ***Connect It*** problem 8. Ask students to brainstorm about strategies used to compare fractions. Record their responses. Have partners explain how each strategy is used to compare fractions. Listen to partner discussions. Select information to record. For example: *I heard Diana and Mila say that when you find a common denominator, you multiply both the numerator and denominator by the same number.* Ask students to identify the strategy they like best for comparing fractions and why. Provide a sentence frame: *I like to use ______ for comparing fractions because ______.*

Levels 3–5

Listening/Speaking Use with ***Connect It*** problem 8. Ask students to brainstorm about strategies used to compare fractions. Write their responses on index cards. Display a card. Have partners explain how the strategy is used to compare fractions. Shuffle the cards. Place them facedown. Call on a student to select a card. Have the rest of the students listen as the student describes how the strategy is used without naming the strategy in the description. Have students identify the strategy. Continue the process until all cards have been selected.

Purpose In this session, students solve a word problem that requires comparing two fractions with different numerators and different denominators. Students model the fractions in the problem either on paper or with manipulatives to determine the greater fraction. The purpose is to have students develop strategies to compare fractions with different numerators and different denominators.

Start

Connect to Prior Knowledge

Why Support students' facility with comparing fractions that have different numerators or different denominators.

How Determine whether fraction comparison statements are true or false.

Tell whether each comparison is *True* or *False*.

1. $\frac{2}{4} > \frac{2}{5}$
2. $\frac{3}{8} < \frac{7}{8}$
3. $\frac{2}{5} = \frac{2}{10}$

Solutions

1. True
2. True
3. False

Develop Language

Why Reinforce the meaning of the word *symbol*.

How Remind students that they have used symbols such as $+$, $-$, $\times$, and $\div$ to add, subtract, multiply, and divide. They have also used symbols such as ? and a letter to stand for an unknown number in an equation. Remind students that symbols are also used to compare numbers. Display the following symbols: $>$, $<$, $=$. Have students tell what each symbol represents.

TRY IT

Make Sense of the Problem

To support students in making sense of the problem, have them identify the fractions and the unit of measurement.

Ask *What fraction tells the weight of the grasshopper? the beetle? What unit of measurement is given for the weight of each insect?*

LESSON 18 SESSION 2

Develop Using Common Numerators and Denominators

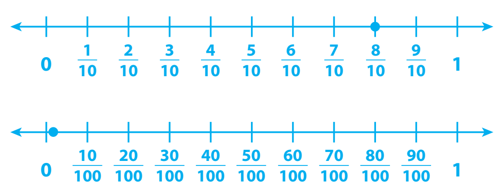
Read and try to solve the problem below.

A grasshopper weighs $\frac{2}{100}$ of an ounce. A beetle weighs $\frac{8}{10}$ of an ounce. Which weighs more?

TRY IT

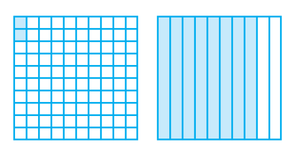
Possible student work:

Sample A



$\frac{8}{10} > \frac{2}{100}$, so the beetle weighs more.

Sample B



$\frac{2}{100}$ is much less than $\frac{8}{10}$, so the beetle weighs more than the grasshopper.

Math Toolkit
- number lines
- hundredths grids
- tenths grids
- index cards
- fraction models

DISCUSS IT

Ask your partner: How did you get started?

Tell your partner: I started by . . .

383

DISCUSS IT

Support Partner Discussion

Encourage students to use the terms *tenths* and *hundredths* as they discuss their solutions.

Support as needed with questions such as:

- *What is another way you could have solved this problem?*
- *How do you know that your answer is reasonable?*

Common Misconception Look for students who correctly describe $\frac{8}{10}$ as greater than $\frac{2}{100}$ but confuse the comparison symbols and write $<$ rather than $>$.

Select and Sequence Student Solutions

One possible order for whole class discussion:

- hundredths grids and tenths grids shaded to represent $\frac{2}{100}$ and $\frac{8}{10}$
- labeled number lines showing the locations of the two fractions
- writing equivalent fractions to compare the two fractions
- reasoning using a benchmark fraction of $\frac{1}{2}$ to compare the two fractions

Support Whole Class Discussion

Compare and connect the different representations and have students identify how they are related.

Ask *How does your model show $\frac{2}{100}$? $\frac{8}{10}$? How does your model show that the tenths and hundredths represent the same-sized whole?*

Listen for Students should recognize that accurate responses include area models divided into 100 equal parts with 2 parts shaded and into 10 equal parts with 8 parts shaded and that both models are the same size. Responses may also include number lines labeled with tenths and hundredths that show the same distance from 0 to 1.

MODEL ITS

If no student presented these models, connect them to the student models by pointing out the ways they each represent:

- the number of hundredths representing the weight of the grasshopper
- the number of tenths representing the weight of the beetle

Ask *How does each model represent hundredths? How does each model represent tenths?*

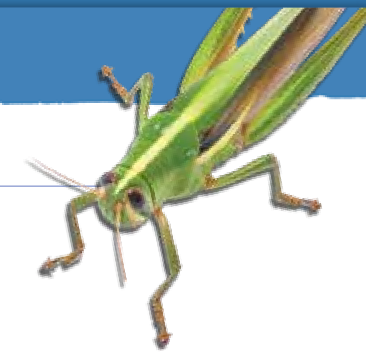
Listen for One model is divided into 100 equal parts, and the other is divided into 10 equal parts. 100 in the denominator shows the hundredths, and 10 in the denominator shows the tenths.

For the area models, prompt students to identify how the comparison of the fractions is shown.

- *Why do the models have different numbers of parts?*
- *What does the shading in each model represent?*
- *What do you observe about the size of the models?*

For a common denominator, prompt students to identify how an equivalent fraction is used.

- *What number are both 8 and 10 in $\frac{8}{10}$ multiplied by?*
- *Why is $\frac{8}{10}$ rewritten as $\frac{80}{100}$?*
- *What symbol is used to indicate equivalence?*
- *Why do you think that 100 is a better choice for the common denominator than 10?*



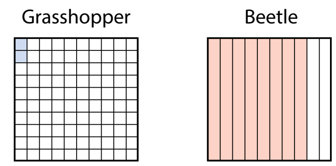
Explore different ways to understand comparing fractions.

A grasshopper weighs $\frac{2}{100}$ of an ounce. A beetle weighs $\frac{8}{10}$ of an ounce. Which weighs more?

MODEL IT

You can use models to help compare fractions.

The models show the fractions of an ounce that the grasshopper and beetle weigh.



MODEL IT

You can use a common denominator to help compare fractions.

When you compare two fractions, it helps if they have a common denominator. Fractions with the same denominator are made up of parts of the same size. The numerators tell how many of those parts each fraction has. When two fractions have the same denominator, you can compare the numerators.

Compare $\frac{2}{100}$ and $\frac{8}{10}$.

The fractions are not written with a common denominator. Find a fraction equivalent to $\frac{8}{10}$ that has a denominator of 100.

$\frac{8 \times 10}{10 \times 10} = \frac{80}{100}$

Now, compare the numerators of $\frac{2}{100}$ and $\frac{80}{100}$.

$80 > 2$

So, $\frac{80}{100} > \frac{2}{100}$ and $\frac{8}{10} > \frac{2}{100}$.

Deepen Understanding

Common Denominators

SMP 7 Use structure.

When discussing common denominators, prompt students to recognize that rewriting one of the fractions makes it easier to compare the two fractions.

Ask *Why do you multiply the denominator of $\frac{8}{10}$ by 10?*

Listen for You want to get a fraction equivalent to $\frac{8}{10}$ that has the same denominator as $\frac{2}{100}$.

Ask *Why do you also multiply the numerator of $\frac{8}{10}$ by 10?*

Listen for You need to multiply both the numerator and denominator of $\frac{8}{10}$ by the same number to find an equivalent fraction.

Generalize *How is finding common denominators useful in solving a problem about comparing fractions with different numerators and different denominators?* Have students explain their reasoning. Listen for understanding that finding an equivalent fraction with the same denominator as another fraction allows you to compare the numerators and tell which fraction is greater or less.

CONNECT IT

- Remind students that one thing that is alike about all the representations is the numbers and that one way to compare fractions is to rewrite one fraction to have the same denominator as the other fraction.
- Explain that on this page students will compare the same two fractions by rewriting one fraction to have the same numerator as the other in order to compare them.

Monitor and Confirm

1 – 5 Check for understanding that:

- the numerator 2 is multiplied by 4 to get a numerator of 8, so multiply the denominator 100 by 4 to get an equivalent fraction of $\frac{8}{400}$
- both area models are the same size
- the model divided into 400 equal parts has smaller parts than the model divided into 10 equal parts
- the model with the greater area shaded represents the greater fraction

Support Whole Class Discussion

6 Be sure students recognize that both methods—finding a common denominator or finding a common numerator—lead to the same solution: $\frac{8}{10} > \frac{2}{100}$

Ask *How does this solution compare to the solution found by using a common denominator? Why do you think this is true?*

Listen for The solutions are the same. In both cases, one fraction is rewritten as an equivalent fraction with either a common numerator or common denominator, so the relationship between the fractions $\frac{8}{10}$ and $\frac{2}{100}$ remains the same.

7 Look for the idea that the fraction with the lesser denominator has equal parts that are larger, so it is the greater fraction.

8 REFLECT

Have all students focus on the strategies used to solve this problem. If time allows, have students share their responses with a partner.

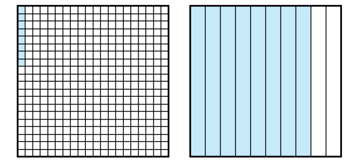
CONNECT IT

Now you will use the problem from the previous page to help you understand how to compare fractions by finding a common numerator.

1. What is an equivalent fraction for $\frac{2}{100}$ that has a numerator of 8? $\frac{8}{400}$

2. One model is divided into 400 equal parts, and the other is divided into 10 equal parts. Which model has smaller parts?

   the model divided into 400 equal parts



3. Shade 8 parts of each model.

4. Which model has a greater area shaded? the model divided into 10 equal parts

5. Which fraction is greater, $\frac{8}{400}$ or $\frac{8}{10}$? $\frac{8}{10}$

6. Which weighs more, the grasshopper or the beetle? the beetle

7. Look at the denominators of $\frac{8}{400}$ and $\frac{8}{10}$. When two fractions have the same numerator and different denominators, how do you know which fraction is greater? Explain.

   Possible explanation: The fraction with the lesser denominator has bigger parts, so it is greater.

8. **REFLECT**

   Look back at your **Try It**, strategies by classmates, **Model Its**, and the **Connect It** problems on this page. Which models or strategies do you like best for comparing fractions? Explain.

   Possible explanation: I like using the shaded models because I can see exactly how much of the model is shaded for each fraction and how big the parts are, so I can easily compare the fractions.

385

Visual Model

Use drawings of fractions with the same numerator and different denominators to compare fractions.

If . . . *students are having trouble comparing fractions with the same numerators and different denominators that are greater, such as 400,*

Then . . . *use this activity to have them compare visual models of fractions with the same numerators and different denominators that are lesser.*

Use drawings to visually model two fractions with the same numerators and different denominators in order to compare them.

- Draw two same-sized area models to represent $\frac{2}{5}$ and $\frac{2}{10}$ on the board.
- Ask: *What fractions do these area models represent?* $\left[\frac{2}{5} \text{ and } \frac{2}{10}\right]$ *Which model has a greater area shaded?* [the model with 2 parts out of 5 shaded]
- Point out that both models have the same number of parts shaded. Ask: *Why is* $\frac{2}{5} > \frac{2}{10}$? [Each fifths part is larger than each tenths part.]
- Repeat with other pairs of fractions that have the same numerators and different denominators, such as $\frac{3}{8}$ and $\frac{3}{4}$, and $\frac{5}{12}$ and $\frac{5}{10}$.

APPLY IT

For all problems, encourage students to draw some kind of model to support their thinking. Allow some leeway in precision; drawing fractional parts accurately is difficult and here precise measurements are not necessary.

9 $\frac{8}{12} < \frac{3}{4}$ or $\frac{3}{4} > \frac{8}{12}$; See possible work on the Student Worktext page. Students may also use a common denominator to rewrite $\frac{3}{4}$ as $\frac{9}{12}$ and then compare $\frac{9}{12}$ with $\frac{8}{12}$ or use a number line marked in fourths and twelfths to compare the fractions.

10 $\frac{4}{6} > \frac{2}{5}$ or $\frac{2}{5} < \frac{4}{6}$; See possible work on the Student Worktext page. Students may also use a common numerator to rewrite $\frac{2}{5}$ as $\frac{4}{10}$ and then compare $\frac{4}{10}$ with $\frac{4}{6}$.

Close: Exit Ticket

11 Morgan could have shaded 1, 2, or 3 parts of Model B; See possible explanation on the Student Worktext page.

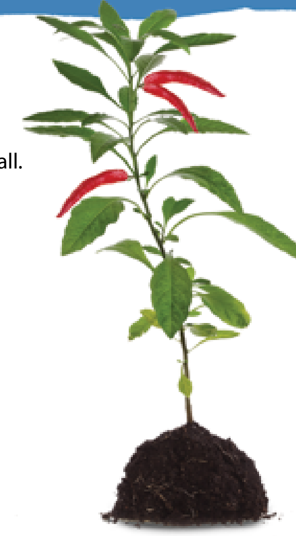
Students' solutions should indicate understanding of:

- how area models represent fractions
- how to represent equivalent fractions with area models
- using area models or equivalent fractions to compare two fractions with different denominators

Error Alert **If** students think that Morgan could have shaded 4 parts of Model B, **then** have students shade 4 parts of the tenths model in Model B and compare the shaded parts of Models A and B to see that both models have the same area shaded. Students can recognize that this indicates that the fractions have the same value and that 4 shaded parts of Model B does not represent a fraction less than the $\frac{2}{5}$ shown in Model A.

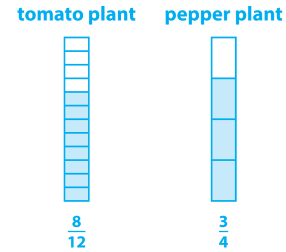
APPLY IT

Use what you just learned to solve these problems.



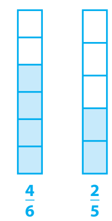
9 Mel's tomato plant is $\frac{8}{12}$ of a foot tall. Her pepper plant is $\frac{3}{4}$ of a foot tall. Compare the heights of the plants using <, >, or =. Use a model to show your comparison. Show your work.

Possible student work:



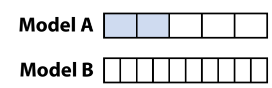
Solution $\frac{8}{12} < \frac{3}{4}$ or $\frac{3}{4} > \frac{8}{12}$

10 Compare the fractions $\frac{4}{6}$ and $\frac{2}{5}$ using <, >, or =. Use a model to show your comparison. Show your work. **Possible student work:**



Solution $\frac{4}{6} > \frac{2}{5}$ or $\frac{2}{5} < \frac{4}{6}$

11 Morgan has the two fraction models shown. Morgan shades Model B to show a fraction less than the fraction shown by Model A. How many parts of Model B could she have shaded? Explain.



Morgan could have shaded 1, 2, or 3 parts of Model B. Possible explanation: Model A represents $\frac{2}{5}$. Model B shows tenths. Since $\frac{2}{5} = \frac{4}{10}$, the number of parts she could have shaded in Model B must be less than 4.

SESSION 2 Additional Practice

Solutions

1. $\frac{3}{4} < \frac{5}{6}$; Students should shade 3 of the 4 parts of the top area model and 5 of the 6 parts of the bottom area model; See shaded area models on the student page.
Basic

2. $\frac{9}{12} < \frac{10}{12}$; Students should divide each equal part in the top area model in problem 1 into 3 equal parts and each equal part in the bottom area model into 2 equal parts for a total of 12 equal parts in both models. See divided area models in problem 1 on the student page.
Medium

3. **a.** $\frac{2 \times 4}{3 \times 4} = \frac{8}{12}$
b. $\frac{8}{12} < \frac{9}{12}$. So, $\frac{2}{3} < \frac{9}{12}$.
Medium

Name: ______________________________ LESSON 18 SESSION 2

Practice with Common Numerators and Denominators

Study the Example showing how to compare fractions by finding a common denominator. Then solve problems 1–7.

EXAMPLE

A length of ribbon is $\frac{3}{4}$ of a foot. Another length of ribbon is $\frac{5}{6}$ of a foot. Compare the lengths using a symbol.

Find a common denominator. $\frac{3 \times 3}{4 \times 3} = \frac{9}{12}$ $\frac{5 \times 2}{6 \times 2} = \frac{10}{12}$

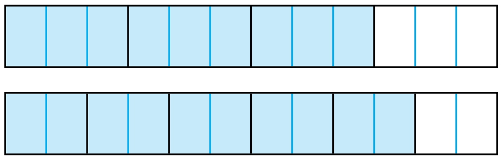
Write the equivalent fractions. $\frac{3}{4} = \frac{9}{12}$ $\frac{5}{6} = \frac{10}{12}$

Compare the numerators. $\frac{9}{12} < \frac{10}{12}$

Since $9 < 10$, that means $\frac{9}{12} < \frac{10}{12}$.

$\frac{3}{4} < \frac{5}{6}$

1. Shade the models to show $\frac{3}{4}$ and $\frac{5}{6}$. Compare the fractions. Write <, >, or =.
$\frac{3}{4}$ < $\frac{5}{6}$



2. Divide each model in problem 1 into 12 equal parts to show an equivalent fraction. Write the equivalent fractions and symbol to show the comparison.
$\frac{9}{12} < \frac{10}{12}$

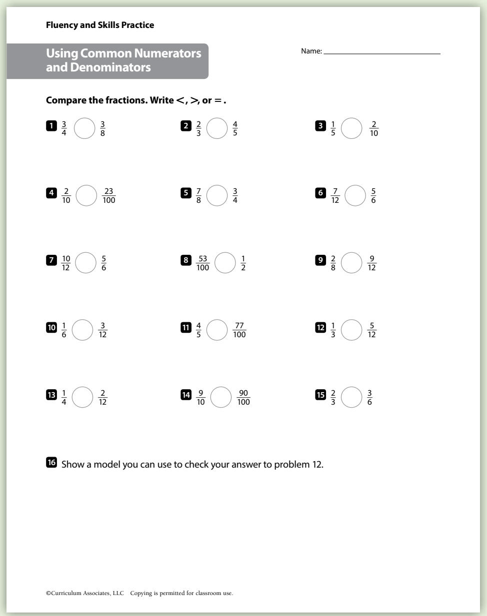
3. Compare $\frac{2}{3}$ and $\frac{9}{12}$ by finding a common denominator.
a. Write a fraction equivalent to $\frac{2}{3}$ with a denominator of 12. $\frac{2 \times 4}{3 \times 4} = \frac{8}{12}$
b. Compare the fractions. $\frac{8}{12} < \frac{9}{12}$. So, $\frac{2}{3} < \frac{9}{12}$.

387

Fluency & Skills Practice
Teacher Toolbox

Assign *Using Common Numerators and Denominators*

In this activity students practice comparing fractions. Students could compare the fractions by drawing a model or finding a common denominator. Students may encounter comparing fractions with different denominators in real-world situations, such as comparing different amounts of the same-size whole (e.g., comparing $\frac{3}{4}$ of a pie to $\frac{5}{8}$ of an equal-sized pie).


Fluency and Skills Practice

Using Common Numerators and Denominators

Name: ______________________

Compare the fractions. Write <, >, or =.

1. $\frac{3}{4}$ ◯ $\frac{3}{8}$
2. $\frac{2}{3}$ ◯ $\frac{4}{5}$
3. $\frac{1}{5}$ ◯ $\frac{2}{10}$
4. $\frac{2}{10}$ ◯ $\frac{23}{100}$
5. $\frac{7}{8}$ ◯ $\frac{3}{4}$
6. $\frac{7}{12}$ ◯ $\frac{5}{6}$
7. $\frac{10}{12}$ ◯ $\frac{5}{6}$
8. $\frac{53}{100}$ ◯ $\frac{1}{2}$
9. $\frac{2}{8}$ ◯ $\frac{9}{12}$
10. $\frac{1}{6}$ ◯ $\frac{3}{12}$
11. $\frac{4}{5}$ ◯ $\frac{77}{100}$
12. $\frac{1}{3}$ ◯ $\frac{5}{12}$
13. $\frac{1}{4}$ ◯ $\frac{2}{12}$
14. $\frac{9}{10}$ ◯ $\frac{90}{100}$
15. $\frac{2}{3}$ ◯ $\frac{3}{6}$
16. Show a model you can use to check your answer to problem 12.

©Curriculum Associates, LLC Copying is permitted for classroom use.

4 a. $\frac{1 \times 2}{5 \times 2} = \frac{2}{10}$

b. $\frac{2}{10} > \frac{2}{12}$. So, $\frac{1}{5} > \frac{2}{12}$.

Medium

5 a. $\frac{2}{5} < \frac{8}{10}$

b. $\frac{5}{12} > \frac{1}{3}$

c. $\frac{3}{5} = \frac{60}{100}$

d. $\frac{9}{100} < \frac{9}{10}$

Medium

6 **B** (**False**);
C (**True**);
E (**True**);
H (**False**);
J (**False**)

Medium

7 No; See possible explanation on the student page.

Challenge

4 Compare $\frac{1}{5}$ and $\frac{2}{12}$ by finding a common numerator.

a. Write a fraction equivalent to $\frac{1}{5}$ with a numerator of 2. $\frac{1 \times \boxed{2}}{5 \times \boxed{2}} = \frac{2}{\boxed{10}}$

b. Compare the fractions. $\frac{2}{\boxed{10}}$ (>) $\frac{2}{12}$. So, $\frac{1}{5}$ (>) $\frac{2}{12}$.

5 Compare the fractions. Use the symbols <, >, and =.

a. $\frac{2}{5}$ (<) $\frac{8}{10}$

b. $\frac{5}{12}$ (>) $\frac{1}{3}$

c. $\frac{3}{5}$ (=) $\frac{60}{100}$

d. $\frac{9}{100}$ (<) $\frac{9}{10}$

6 Tell whether each comparison is *True* or *False*.

	True	False
$\frac{2}{3} > \frac{5}{6}$	Ⓐ	Ⓑ
$\frac{4}{10} < \frac{4}{5}$	Ⓒ	Ⓓ
$\frac{70}{100} = \frac{7}{10}$	Ⓔ	Ⓕ
$\frac{1}{3} > \frac{3}{1}$	Ⓖ	Ⓗ
$\frac{3}{4} < \frac{2}{3}$	Ⓘ	Ⓙ

7 Can two fractions with the same numerator and different denominators be equal? Use words and numbers to explain.

No. Possible explanation: Fractions with the same numerator have the same number of parts, but the size of the parts is different when the denominators are different. The fractions can't be equal, because the fraction with the smaller-size parts is the smaller fraction. For example: $\frac{3}{4}$ is greater than $\frac{3}{5}$ because fourths are greater than fifths, $\frac{3}{4} > \frac{3}{5}$.

Vocabulary

common denominator a number that is a common multiple of the denominators of two or more fractions.

denominator the number below the line in a fraction that tells the number of equal parts in the whole.

numerator the number above the line in a fraction that tells the number of equal parts that are being described.

ELL English Language Learners: Differentiated Instruction

Prepare for Session 3
Use with *Apply It*.

Levels 1–3

Listening/Speaking Use with ***Apply It*** problem 7. Have students listen as you think through using a benchmark fraction to find the greater fraction. Draw a number line to use as you model the process.

- *Divide the number line into 8 equal sections.*
- *Find and label the benchmark fraction.* [$\frac{1}{2}$]
- *Find and label $\frac{4}{8}$.* [Emphasize that $\frac{4}{8}$ is located at the same position as $\frac{1}{2}$.]
- *Find and label $\frac{3}{4}$.*
- *$\frac{3}{4}$ is greater than $\frac{4}{8}$.*

Ask students to explain how to use a benchmark fraction to find the greater fraction in their own words.

Levels 2–4

Listening/Speaking Use with ***Apply It*** problem 7. Say: *Draw a number line divided into 8 equal sections*. Ask students to form pairs. Have them listen to, discuss, and respond to the following questions as they think through the process to find the greater fraction.

- *What fraction will you use as the benchmark fraction?*
- *Where will you label $\frac{4}{8}$? What connection do you see between $\frac{4}{8}$ and $\frac{1}{2}$?*
- *Where will you label $\frac{3}{4}$?*
- *Which fraction is greater? How do you know?*

Call on pairs to share their responses.

Levels 3–5

Reading Use with ***Apply It*** problem 7. Say: *Draw a number line to use as you think through the process to find the greater fraction*. Write the following questions and have students read them before solving the problem.

- *How many sections will you divide the number line into?*
- *What fraction will you use as the benchmark fraction?*
- *Where will you label $\frac{4}{8}$? What is the connection between $\frac{4}{8}$ and $\frac{1}{2}$?*
- *Where will you label $\frac{3}{4}$?*
- *Which fraction is greater? How do you know?*

Call on students to share their results.

LESSON 18

SESSION 3 Develop

Purpose In this session, students solve a problem that requires comparing two fractions with different numerators and different denominators. Students model the fractions in the problem either on paper or with manipulatives to determine the greater fraction. The purpose is to have students develop strategies for comparing fractions with different numerators and different denominators.

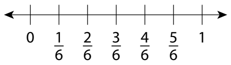
Start

Connect to Prior Knowledge

Why Review comparing fractions with the same denominators on a number line to prepare students to compare fractions with different numerators and denominators using a benchmark.

How Use a number line to compare fractions that have different numerators and the same denominator.

Tell whether each comparison is *True* or *False*.



1. $\frac{1}{6} > \frac{5}{6}$
2. $\frac{2}{6} < \frac{4}{6}$

Solutions

1. False
2. True

Develop Language

Why Practice reading fractions that have the same numerator and denominator.

How Remind students that when they see a fraction with the same numerator and denominator, the fraction names 1 whole. Write several fractions with the same numerators and denominators. Model how to read them. Tell students that the fractions name 1 whole. For example, they can read the fraction $\frac{10}{10}$ as *ten tenths*.

TRY IT

Make Sense of the Problem

To support students in making sense of the problem, have them identify the fractions and the unit of measurement.

Ask *What fraction tells the amount of time for the swimming lesson? the amount of time for homework? What unit of measurement for time is given?*

LESSON 18 SESSION 3 ● ● ● ○

Develop Using a Benchmark to Compare Fractions

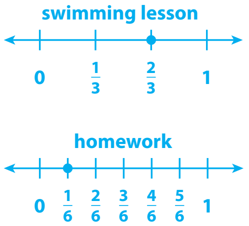
Read and try to solve the problem below.

Jasmine's swimming lesson lasts for $\frac{2}{3}$ of an hour. It takes her $\frac{1}{6}$ of an hour to do her homework. Does Jasmine spend more time on her homework or at her swimming lesson?

TRY IT

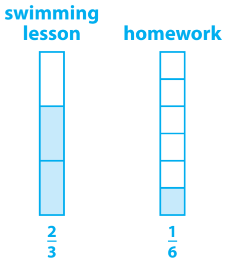
Possible student work:

Sample A



Jasmine spends more time at her swimming lesson than on her homework.

Sample B



She spends more time at her swimming lesson than she does on homework.

Math Toolkit

- fraction circles
- fraction tiles
- number lines
- fraction bars
- index cards
- fraction models

DISCUSS IT

Ask your partner: Why did you choose that strategy?

Tell your partner: I knew . . . so I . . .

389

DISCUSS IT

Support Partner Discussion

Encourage students to use the terms *thirds* and *sixths* as they discuss their solutions.

Support as needed with questions such as:

- *Have you solved a problem like this before?*
- *How is your solution method the same as or different from your partner's method?*

Common Misconception Look for students who find the correct result but reason incorrectly, thinking that $\frac{2}{3} > \frac{1}{6}$ because $2 > 1$ without taking into account the different denominators. Have students use fraction tiles to explain the comparison.

Select and Sequence Student Solutions

One possible order for whole class discussion:

- fraction circles or fraction tiles modeling two thirds and one sixth
- area models or labeled number lines representing two thirds and one sixth
- writing equivalent fractions to compare the two fractions
- reasoning using a benchmark fraction of $\frac{1}{2}$ to compare the two fractions

Support Whole Class Discussion

Compare and connect the different representations and have students identify how they are related.

Ask *Where does your model show $\frac{2}{3}$? Where does your model show $\frac{1}{6}$? How does your model show that the thirds and sixths represent the same-sized whole?*

Listen for Students should recognize that accurate responses include area models divided into 3 equal parts with 2 parts shaded and into 6 equal parts with 1 part shaded and that both models are the same size. Responses may also include labeled thirds and sixths number lines showing the same distance between 0 and 1.

MODEL IT & SOLVE IT

If no student presented these models, connect them to the student models by pointing out the ways they each represent:

- the number of thirds representing part of an hour
- the number of sixths representing part of an hour

Ask *How does each model represent the thirds? How does each model represent the sixths?*

Listen for Both number lines go from 0 to 1 and are divided into sixths, so they also show thirds.

For a number line, prompt students to identify how the comparison of the fractions is shown.

- *What does the fraction $\frac{1}{6}$ represent?*
- *How is the amount of time Jasmine swims represented?*
- *How does the number line help you compare fractions?*

For a number line with a benchmark fraction, prompt students to identify how to use a benchmark fraction to solve the problem.

- *How is this number line model the same as the other number line model? How is it different?*
- *What is the purpose of showing $\frac{1}{2}$ on the number line?*
- *How can a benchmark fraction help you compare two other fractions?*

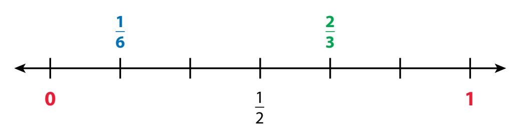
Explore different ways to understand using benchmarks to compare fractions.

Jasmine's swimming lesson lasts for $\frac{2}{3}$ of an hour. It takes her $\frac{1}{6}$ of an hour to do her homework. Does Jasmine spend more time on her homework or at her swimming lesson?

MODEL IT

You can use a number line to help you compare fractions.

The number line shows where the fractions $\frac{2}{3}$ and $\frac{1}{6}$ are compared to 0 and 1.

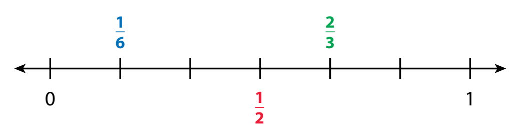


The number line shows that $\frac{1}{6}$ is closer to 0 than $\frac{2}{3}$ is.
It also shows that $\frac{2}{3}$ is closer to 1 than $\frac{1}{6}$ is.
This means that $\frac{1}{6} < \frac{2}{3}$ and $\frac{2}{3} > \frac{1}{6}$.

SOLVE IT

You can use a benchmark fraction to solve the problem.

Another way to compare fractions is by using a benchmark fraction.
Use $\frac{1}{2}$ as a benchmark to compare $\frac{1}{6}$ and $\frac{2}{3}$.



The number line shows that $\frac{1}{6}$ is less than $\frac{1}{2}$ and $\frac{2}{3}$ is greater than $\frac{1}{2}$.
So, $\frac{1}{6} < \frac{2}{3}$ and $\frac{2}{3} > \frac{1}{6}$.

Jasmine spends more time at her swimming lesson than on homework.

390

Deepen Understanding

Benchmark Fractions

SMP 4 Model with mathematics.

When discussing the model of a number line with a benchmark fraction, tell students that a *benchmark fraction* is a common fraction, such as $\frac{1}{2}$ or $\frac{3}{4}$, that you can compare other fractions to.

Ask *What does the location of $\frac{1}{2}$ on the number line compared to $\frac{1}{6}$ and to $\frac{2}{3}$ tell you about how to choose a benchmark fraction?*

Listen for When comparing fractions, it is best to use a benchmark fraction that is greater than one of the fractions and less than the other fraction.

Ask *Whole numbers can also be used as benchmarks. Would 1 be as useful as $\frac{1}{2}$ to use as a benchmark to compare the fractions $\frac{1}{6}$ and $\frac{2}{3}$? Explain.*

Listen for No, 1 would not be as useful because 1 is greater than both fractions. You would know that both fractions are less than 1, but not how the fractions compare to each other.

CONNECT IT

- Remind students that one thing that is alike about all the representations is the numbers and that one way to compare two fractions with different numerators and denominators is to use a benchmark fraction such as $\frac{1}{2}$.
- Explain that on this page students will compare two different fractions by using another benchmark instead of $\frac{1}{2}$.

Monitor and Confirm

1 – 4 Check for understanding that:

- $\frac{10}{10}$ is equal to 1
- $\frac{11}{10}$ is greater than 1 and $\frac{7}{8}$ is less than 1
- using a benchmark of 1 helps determine that $\frac{11}{10}$ is greater than $\frac{7}{8}$
- the symbol $>$ means "is greater than"

Support Whole Class Discussion

1 – 4 Tell students that these problems will prepare them to provide the explanation required in problem 5.

Be sure students understand that $\frac{1}{2}$ is not the only benchmark you can use to compare two fractions.

Ask *Why do you think that $\frac{1}{2}$ is not used as a benchmark to compare the fractions $\frac{11}{10}$ and $\frac{7}{8}$?*

Listen for The fractions $\frac{11}{10}$ and $\frac{7}{8}$ are both greater than $\frac{1}{2}$, so comparing them to $\frac{1}{2}$ does not help you tell which fraction is greater or less than the other.

Ask *Why do you think 1 is used as the benchmark to compare $\frac{11}{10}$ and $\frac{7}{8}$?*

Listen for The fraction $\frac{7}{8}$ is less than 1 and the fraction $\frac{11}{10}$ is greater than 1.

5 Look for the idea that you can use a benchmark to compare two fractions with different numerators and different denominators. This is useful especially if one fraction is greater than the benchmark and the other fraction is less than the benchmark.

6 REFLECT

Have all students focus on the strategies used to solve this problem. If time allows, have students share their responses with a partner.

CONNECT IT

Now you will solve a similar problem using 1 as a benchmark. Think about the two fractions $\frac{11}{10}$ and $\frac{7}{8}$.

1. Which fraction, $\frac{11}{10}$ or $\frac{7}{8}$, is greater than 1? $\frac{11}{10}$
2. Which fraction, $\frac{11}{10}$ or $\frac{7}{8}$, is less than 1? $\frac{7}{8}$
3. Which fraction, $\frac{11}{10}$ or $\frac{7}{8}$, is greater? Explain.
   $\frac{11}{10}$ is greater. Possible explanation: Since $\frac{11}{10}$ is greater than 1 and $\frac{7}{8}$ is less than 1, $\frac{11}{10}$ must be greater than $\frac{7}{8}$.
4. Write <, >, or = to show the comparison. $\frac{11}{10} > \frac{7}{8}$
5. Explain how you can use benchmarks to compare fractions.
   You can compare both fractions to the same number to see which fraction is greater than, less than, or equal to that benchmark. The fraction that is greater than the benchmark is greater than the fraction that is less than or equal to the benchmark.
6. **REFLECT**
   Look back at your **Try It**, strategies by classmates, and **Model It** and **Solve It**. Which models or strategies do you like best for using benchmarks to compare fractions? Explain.
   Possible explanation: I like the strategy of using a number line best because I can see that $\frac{2}{3}$ is closer to 1 than $\frac{1}{6}$ so $\frac{2}{3}$ must be greater than $\frac{1}{6}$.

Hands-On Activity

Use a number line and fraction cards to compare fractions.

If . . . *students are unsure about using a benchmark fraction to compare two fractions,*

Then . . . *use a number line and fraction cards to provide a concrete model to connect to the visual and symbolic models.*

Materials For each pair: 10 index cards labeled with the fractions $\frac{1}{3}, \frac{2}{3}, \frac{2}{4}, \frac{2}{5}, \frac{4}{6}, \frac{5}{6}, \frac{6}{8}, \frac{7}{8}, \frac{4}{10}, \frac{3}{12}$, Activity Sheet *Number Lines*

- Have students compare two fractions using the benchmark fraction, $\frac{1}{2}$.
- Have students label 0, 1, and $\frac{1}{2}$ on a number line.
- Give partners a set of fraction cards. Have one partner choose two fractions to place on the number line, one between 0 and $\frac{1}{2}$ and one between $\frac{1}{2}$ and 1. Have students explain each placement and discuss with their partners any fractions whose locations they are not sure about.
- Have the partners write a comparison statement comparing the two fractions and justify the statement in terms of $\frac{1}{2}$.
- Have partners repeat with other pairs of fractions.

APPLY IT

For all problems, encourage students to draw some kind of model to support their thinking. Allow some leeway in precision; drawing equal intervals on number lines is challenging and here exact spacing between marks on a number line is not necessary.

7 $\frac{4}{8} = \frac{1}{2}$ and $\frac{3}{4} > \frac{1}{2}$. So $\frac{3}{4} > \frac{4}{8}$. See possible work on the Student Worktext page.

8 Nathan walks a greater distance than Sarah; Students should use the benchmark number 1 to compare the fractions $\frac{10}{10}$ and $\frac{11}{12}$. See possible explanation on the Student Worktext page.

Close: Exit Ticket

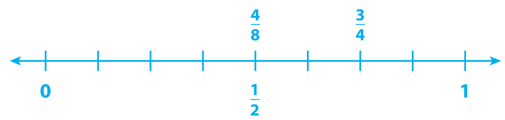
9 **B;** Students may first compare $\frac{4}{6}$ to $\frac{1}{2}$ by using a common denominator and reasoning that $\frac{1}{2} = \frac{3}{6}$ and $\frac{4}{6} > \frac{3}{6}$, so $\frac{4}{6} > \frac{1}{2}$. Students may then compare $\frac{3}{8}$ to $\frac{1}{2}$ by using a common denominator and reasoning that $\frac{1}{2} = \frac{4}{8}$ and $\frac{3}{8} < \frac{4}{8}$, so $\frac{3}{8} < \frac{1}{2}$. Because $\frac{4}{6} > \frac{1}{2}$ and $\frac{3}{8} < \frac{1}{2}$, $\frac{4}{6} > \frac{3}{8}$.

Error Alert **If** students choose A or C and incorrectly compare the fractions, **then** review how to use the benchmark fraction $\frac{1}{2}$ to compare fractions by drawing a sixths number line labeled with $\frac{1}{2}$ and an eighths number line labeled with $\frac{1}{2}$, locating $\frac{4}{6}$ and $\frac{3}{8}$ on their respective number lines and comparing the locations of both fractions to $\frac{1}{2}$.

APPLY IT

Use what you just learned to solve these problems.

7 Tell which fraction is greater, $\frac{4}{8}$ or $\frac{3}{4}$. Use the benchmark fraction $\frac{1}{2}$ to explain your answer. Show your work.



Solution $\frac{3}{4} > \frac{4}{8}$; Possible explanation: $\frac{4}{8} = \frac{1}{2}$ and $\frac{3}{4} > \frac{1}{2}$. So, $\frac{3}{4} > \frac{4}{8}$.

8 Nathan walks $\frac{10}{10}$ of a mile. Sarah walks $\frac{11}{12}$ of a mile. Who walks a greater distance? Explain. Use a benchmark number in your explanation.

Solution Nathan walks a greater distance than Sarah. Possible explanation: 10 out of 10 is equal to 1. $\frac{11}{12}$ is less than 1. So, $\frac{10}{10} > \frac{11}{12}$.

9 Use the benchmark fraction $\frac{1}{2}$ to compare the two fractions below. Which symbol correctly compares the fractions?

$\frac{4}{6} \bigcirc \frac{3}{8}$

Ⓐ <

Ⓑ >

Ⓒ =

Ⓓ +

Solutions

1 **a.** See labeled number line on the student page.

b. $\frac{3}{2}$

c. $\frac{9}{10}$

d. $\frac{9}{10} < \frac{3}{2}$; Students' explanations should include using the benchmark number 1 to compare the fractions. See possible explanation on the student page.

Medium

Name: ______________________

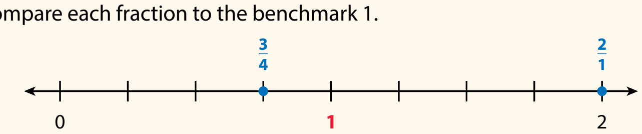
Practice Using a Benchmark to Compare Fractions

Study the Example showing how to use 1 as a benchmark to compare fractions. Then solve problems 1–4.

EXAMPLE

Carol compares $\frac{3}{4}$ and $\frac{2}{1}$. She says $\frac{3}{4} > \frac{2}{1}$ because both the numerator and the denominator in $\frac{3}{4}$ are greater than the numerator and denominator in $\frac{2}{1}$. $3 > 2$ and $4 > 1$. Is Carol correct?

Compare each fraction to the benchmark 1.

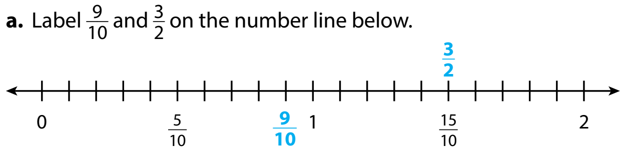


$\frac{3}{4} < 1$ and $\frac{2}{1} > 1$

$\frac{3}{4} < \frac{2}{1}$ and $\frac{2}{1} > \frac{3}{4}$. Carol is not correct.

1 Compare $\frac{9}{10}$ and $\frac{3}{2}$.

a. Label $\frac{9}{10}$ and $\frac{3}{2}$ on the number line below.



b. Which fraction is greater than 1? $\frac{3}{2}$

c. Which fraction is less than 1? $\frac{9}{10}$

d. Write $<$, $>$, or $=$ to show the comparison. Explain how you found your answer. $\frac{9}{10} < \frac{3}{2}$

Possible explanation: $\frac{9}{10}$ is less than $\frac{3}{2}$ because $\frac{9}{10}$ is less than 1 and $\frac{3}{2}$ is greater than 1.

Vocabulary

benchmark fraction a common fraction that you might compare other fractions to. For example, $\frac{1}{4}$, $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{4}$ are often used as benchmark fractions.

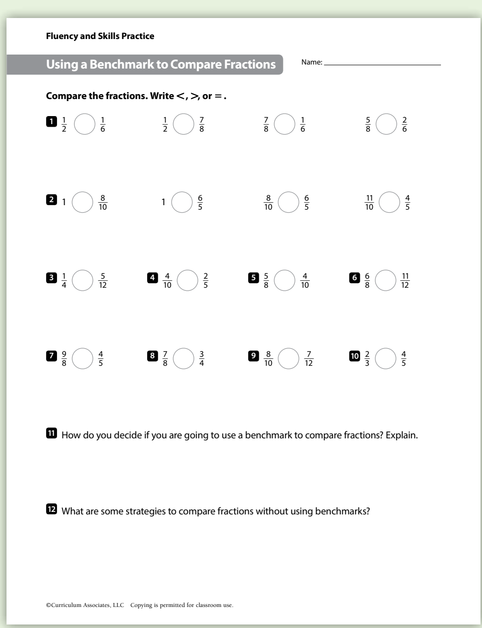
393

Fluency & Skills Practice

Teacher Toolbox

Assign Using a Benchmark to Compare Fractions

In this activity students practice comparing fractions. Students may use a benchmark, such as $\frac{1}{2}$ or 1, to compare the fractions, or they may use another strategy. It is useful for students to consider different strategies, as one strategy may be easier or more efficient to use in a real-world situation.


Fluency and Skills Practice

Using a Benchmark to Compare Fractions Name: ______________

Compare the fractions. Write $<$, $>$, or $=$.

1 $\frac{1}{2} \bigcirc \frac{1}{6}$ $\frac{1}{2} \bigcirc \frac{7}{8}$ $\frac{7}{8} \bigcirc \frac{1}{6}$ $\frac{5}{8} \bigcirc \frac{2}{6}$

2 $1 \bigcirc \frac{8}{10}$ $1 \bigcirc \frac{6}{5}$ $\frac{8}{10} \bigcirc \frac{6}{5}$ $\frac{11}{10} \bigcirc \frac{4}{5}$

3 $\frac{1}{4} \bigcirc \frac{5}{12}$ **4** $\frac{4}{10} \bigcirc \frac{2}{5}$ **5** $\frac{5}{8} \bigcirc \frac{4}{10}$ **6** $\frac{6}{8} \bigcirc \frac{11}{12}$

7 $\frac{9}{8} \bigcirc \frac{4}{5}$ **8** $\frac{7}{8} \bigcirc \frac{3}{4}$ **9** $\frac{8}{10} \bigcirc \frac{7}{12}$ **10** $\frac{2}{3} \bigcirc \frac{4}{5}$

11 How do you decide if you are going to use a benchmark to compare fractions? Explain.

12 What are some strategies to compare fractions without using benchmarks?

©Curriculum Associates, LLC Copying is permitted for classroom use.

2 **a.** See labeled number line on the student page.

b. $\frac{5}{6}$

c. $\frac{1}{3}$

d. $\frac{5}{6} > \frac{1}{3}$; Students' explanations should include using the benchmark fraction $\frac{1}{2}$ to compare the fractions. See possible explanation on the student page.

Medium

3 $\frac{7}{10} > \frac{5}{12}$; Students' explanations should include using the benchmark fraction $\frac{1}{2}$ to compare the fractions. See possible explanation on student page.

Medium

4 **True**; **1**;

True; $\frac{1}{2}$;

False; $\frac{1}{2}$;

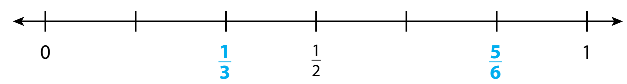
False; **1**;

False; **1**

Challenge

2 Compare $\frac{5}{6}$ and $\frac{1}{3}$ using the benchmark fraction $\frac{1}{2}$.

a. Label $\frac{5}{6}$ and $\frac{1}{3}$ on the number line below.



b. Which fraction is greater than $\frac{1}{2}$? $\frac{5}{6}$

c. Which fraction is less than $\frac{1}{2}$? $\frac{1}{3}$

d. Write <, >, or = to show the comparison. Explain how you found your answer.

$\frac{5}{6} > \frac{1}{3}$

Possible explanation: $\frac{5}{6}$ is greater than $\frac{1}{3}$ because $\frac{5}{6}$ is greater than $\frac{1}{2}$ and $\frac{1}{3}$ is less than $\frac{1}{2}$.

3 Use a benchmark fraction to compare the fractions $\frac{7}{10}$ and $\frac{5}{12}$. Explain how you found your answer.

$\frac{7}{10} > \frac{5}{12}$; Possible explanation: $\frac{1}{2}$ is equal to $\frac{5}{10}$, so $\frac{7}{10}$ is greater than $\frac{1}{2}$. $\frac{1}{2}$ is also equal to $\frac{6}{12}$, so $\frac{5}{12}$ is less than $\frac{1}{2}$. $\frac{7}{10}$ is greater than $\frac{5}{12}$.

4 Write *True* or *False* for each comparison. Then write the benchmark you could use to compare the fractions.

	True or *False*	Benchmark
$\frac{9}{8} > \frac{11}{12}$	True	1
$\frac{2}{5} < \frac{5}{6}$	True	$\frac{1}{2}$
$\frac{7}{10} < \frac{2}{4}$	False	$\frac{1}{2}$
$\frac{4}{5} > \frac{2}{2}$	False	1
$\frac{3}{2} < \frac{9}{10}$	False	1

394

ELL English Language Learners: Differentiated Instruction

Prepare for Session 4
Use with *Apply It*.

Levels 1–3

Writing Read ***Apply It*** problem 1 to students. Write *Use a Number Line, Find a Common Numerator,* and *Find a Common Denominator*. Remind students they can use these strategies to determine which fraction is greater and to find who finishes more homework. Put students into groups of 3. Ask Student A to solve the problem using *Use a Number Line,* Student B to solve it using *Find a Common Numerator,* and Student C to solve it using *Find a Common Denominator*. When students have completed the problem, ask them to compare their results. Ask: *Who finishes more problems? How do you know?* Have students write their responses using the sentence frame: ______ *finishes more problems because* ______ *is greater than* ______.

Levels 2–4

Speaking/Writing Choral read ***Apply It*** problem 1. Ask students to think of strategies they could use to solve the problem. Answers may include *use a number line, find a common numerator,* and *find a common denominator*. Divide students into three groups. Assign each group a strategy and then have them solve the problem. As they solve the problems, encourage students to discuss the process. Have groups compare their results. Ask: *Who finishes more homework problems? How do you know?* Have students write their responses using the following sentence frame: *I determined* ______ *finishes more homework problems because* ______.

Levels 3–5

Listening/Speaking Have students form pairs and read ***Apply It*** problem 1. Ask them to make a list of strategies they could use to solve the problem. Assign each pair a strategy. When they have solved the problem, ask pairs to compare their results. Ask: *How did you solve the problem using the strategy you were assigned?* Encourage students to ask questions as they listen to other students describe the process.

- *Why did you decide to use ______?*
- *What did you think of first?*
- *Did you make a prediction about which fraction was greater before you solved the problem?*

LESSON 18

SESSION 4 Refine

Purpose In this session, students solve word problems involving comparing fractions with different numerators and denominators and then discuss and confirm their answers with a partner.

Before students begin to work, use their responses to the *Check for Understanding* to determine those who will benefit from additional support.

As students complete the Example and problems 1–3, observe and monitor their reasoning to identify groupings for differentiated instruction.

Start

Check for Understanding

Materials For remediation: Activity Sheet *Number Lines*

Why Confirm understanding of comparing fractions.

How Have students compare $\frac{7}{10}$ and $\frac{4}{5}$ and use an area model or number line to explain the comparison.

Compare $\frac{7}{10}$ and $\frac{4}{5}$.
Use an area model or number line to explain the comparison.

Solution
$\frac{7}{10} < \frac{4}{5}$; Students' models should show 10 and 5 equal parts, representing $\frac{7}{10}$ as less than $\frac{4}{5}$.

LESSON 18 SESSION 4

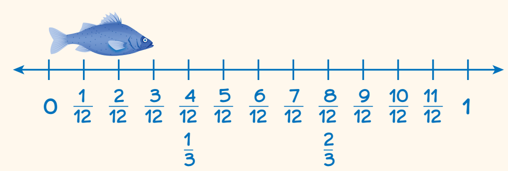
Refine Comparing Fractions

Complete the Example below. Then solve problems 1–9.

EXAMPLE

Becker catches a fish that is $\frac{3}{12}$ of a yard long. The fish has to be longer than $\frac{1}{3}$ of a yard in order to keep it. Can Becker keep the fish?

Look at how you could show your work using a number line.



Solution Since $\frac{3}{12}$ is less than $\frac{1}{3}$, Becker can't keep the fish.

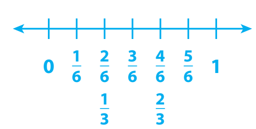
It is important that both measurements use the same unit!

PAIR/SHARE
How else could you solve this problem?

APPLY IT

1. Myron and Jane work on the same set of homework problems. Myron finishes $\frac{5}{6}$ of the problems, and Jane finishes $\frac{2}{3}$ of the problems. Who finishes more of their homework problems? Show your work.

Possible student work:



$\frac{2}{3} = \frac{4}{6}$

Since $\frac{5}{6} > \frac{4}{6}$, $\frac{5}{6} > \frac{2}{3}$.

Solution Myron finishes more homework problems.

Which strategy for comparing do you think works best with these fractions?

PAIR/SHARE
How did you and your partner decide what strategy to use to solve the problem?

395

Error Alert

If the error is . . .	Students may . . .	To support understanding . . .
$\frac{7}{10}$ is greater than $\frac{4}{5}$ because $7 > 4$	not understand that denominators must be the same in order to compare numerators.	Have students draw same-sized area models of $\frac{7}{10}$ and $\frac{4}{5}$. Point out that the size of the parts is not the same. Have students divide the model for $\frac{4}{5}$ into 10 equal parts to show an equivalent fraction with a denominator of 10. Have students write the equivalent fraction $\left[\frac{8}{10}\right]$ and then compare it to $\frac{7}{10}$.
$\frac{7}{10} = \frac{4}{5}$ because $\frac{7}{10} > \frac{1}{2}$ and $\frac{4}{5} > \frac{1}{2}$	not understand when it is appropriate to use a benchmark fraction.	Explain that when both fractions are greater than (or less than) a benchmark fraction, there is not enough information to compare. Have students label a number line with 0 and 1. Help them mark and label tenths and fifths. Have students locate $\frac{7}{10}$ and $\frac{4}{5}$ on the number line to make the comparison.

EXAMPLE

Becker cannot keep his fish because $\frac{3}{12}$ of a yard is less than $\frac{1}{3}$ of a yard; a number line showing twelfths and thirds is one way to solve the problem. Students could also solve the problem by using the common denominator 12, writing $\frac{1}{3}$ as $\frac{4}{12}$, and comparing $\frac{3}{12}$ to $\frac{4}{12}$.

Look for The twelfths and thirds must represent the same-sized whole, indicated by both measurements being described as parts of one yard.

APPLY IT

1. Myron finishes more homework problems; Students may use a number line to show equivalent thirds and sixths fractions to compare $\frac{5}{6}$ and $\frac{2}{3}$. Students could also solve the problem by multiplying both the numerator and denominator of $\frac{2}{3}$ by 2 so both fractions have a common denominator of 6.
$\frac{5}{6} > \frac{4}{6}$, so $\frac{5}{6} > \frac{2}{3}$.
DOK 2
Look for Since 6 is a multiple of 3, students may find it more efficient to use a common denominator to compare the fractions.

2. $\frac{3}{10} < \frac{7}{12}$ or $\frac{7}{12} > \frac{3}{10}$; Students should solve the problem by comparing each fraction to $\frac{1}{2}$.
$\frac{3}{10} < \frac{1}{2}$ and $\frac{7}{12} > \frac{1}{2}$.
DOK 1
Look for The fraction $\frac{3}{10}$ is less than $\frac{1}{2}$, or $\frac{5}{10}$, and the fraction $\frac{7}{12}$ is greater than $\frac{1}{2}$, or $\frac{6}{12}$.

3. **A;** Use common numerators and then look at the denominators to compare.
Explain why the other two answer choices are not correct:
C is not correct because fifths are greater than sixths.
D is not correct because $\frac{3}{6} = \frac{1}{2}$.
DOK 3

2. Compare the fractions $\frac{3}{10}$ and $\frac{7}{12}$ using the benchmark fraction $\frac{1}{2}$. Show your work.

Possible student work:

$\frac{3}{10} < \frac{1}{2}$

$\frac{7}{12} > \frac{1}{2}$

So, $\frac{3}{10} < \frac{7}{12}$.

Solution $\frac{3}{10} < \frac{7}{12}$ or $\frac{7}{12} > \frac{3}{10}$

You already know about how big $\frac{1}{2}$ is!

PAIR/SHARE
Draw a model to check your answer.

3. Janelle walks $\frac{3}{6}$ of a mile. Pedro walks $\frac{6}{10}$ of a mile. Which statement shows how to find the greater fraction?

Ⓐ $\frac{3}{6} = \frac{6}{12}$ and $\frac{6}{12} < \frac{6}{10}$

Ⓑ $\frac{3}{6} = \frac{6}{12}$ and $\frac{6}{12} > \frac{6}{10}$

Ⓒ $\frac{6}{10} = \frac{3}{5}$ and $\frac{3}{5} < \frac{3}{6}$

Ⓓ $\frac{3}{6} < \frac{1}{2}$ and $\frac{6}{10} > \frac{1}{2}$

There are several ways to compare fractions!

Tina chose Ⓑ as the correct answer. How did she get that answer?

Tina found a fraction equivalent to $\frac{3}{6}$ with a numerator of 6, but compared $\frac{6}{12}$ and $\frac{6}{10}$ incorrectly. She thought that $\frac{6}{12}$ is greater than $\frac{6}{10}$ because 12 is greater than 10.

PAIR/SHARE
How can you find the answer using a benchmark fraction?

4 **A**; Find a common denominator: 12. Multiply the numerator and denominator of $\frac{2}{3}$ by 4 and the numerator and denominator of $\frac{3}{4}$ by 3. Then compare the numerators.
DOK 1

Error Alert Students may choose B, C, or D, thinking that they can use a common numerator to compare, without recognizing that the equivalent fraction shown for $\frac{3}{4}$ is incorrect.

5 **A** (**>**);
E (**<**);
G (**>**);
K (**<**);
O (**=**)
DOK 2

6 Sam does not practice long enough; Students may compare the fractions using the benchmark fraction $\frac{1}{2}$. $\frac{5}{10} = \frac{1}{2}$ and $\frac{2}{6} < \frac{1}{2}$. So, $\frac{2}{6} < \frac{5}{10}$.
DOK 2

4 Grant uses $\frac{2}{3}$ of a cup of raisins and $\frac{3}{4}$ of a cup of almonds to make trail mix. Which statement can be used to find out if there are more raisins or almonds in the trail mix?

Ⓐ $\frac{2}{3} = \frac{8}{12}$ and $\frac{3}{4} = \frac{9}{12}$
Ⓑ $\frac{2}{3} = \frac{4}{6}$ and $\frac{3}{4} = \frac{4}{5}$
Ⓒ $\frac{2}{3} = \frac{6}{9}$ and $\frac{3}{4} = \frac{6}{12}$
Ⓓ $\frac{2}{3} = \frac{6}{9}$ and $\frac{3}{4} = \frac{6}{7}$

5 Select >, <, or = to complete a true comparison for each pair of fractions.

	>	<	=
$\frac{8}{3} \square \frac{9}{4}$	Ⓐ	Ⓑ	Ⓒ
$\frac{7}{10} \square \frac{7}{8}$	Ⓓ	Ⓔ	Ⓕ
$\frac{1}{2} \square \frac{3}{8}$	Ⓖ	Ⓗ	Ⓘ
$\frac{2}{4} \square \frac{4}{6}$	Ⓙ	Ⓚ	Ⓛ
$\frac{7}{5} \square \frac{140}{100}$	Ⓜ	Ⓝ	Ⓞ

6 Sam's music teacher tells him to practice his trombone for $\frac{5}{10}$ of an hour. Sam practices for $\frac{2}{6}$ of an hour. Does he practice long enough? Show your work.

Possible student work: $\frac{2}{6} < \frac{1}{2}$ and $\frac{5}{10} = \frac{1}{2}$, so $\frac{2}{6} < \frac{5}{10}$.

Sam **does not** practice long enough.

Differentiated Instruction

RETEACH

Hands-On Activity
Draw and cut out grid models to compare fractions.

Students struggling with comparing fractions

Will benefit from additional work with concrete representations of fractions

Materials For each pair: scissors, colored pencils, Activity Sheet *1-Centimeter Grid Paper*

- Tell each student to work with a partner to draw and cut out two 3-by-4 arrays. Explain that the arrays show the same wholes and can be used to show halves, thirds, fourths, and twelfths. Have students color part of each of their two array models and write a fraction to show each colored part. Then have students compare the fractions using >, <, or =.
- Next, have students draw and cut out two 2-by-8 arrays. Explain that they can use these two same-sized arrays to model and compare halves, fourths, and eighths fractions. Have students color part of each array, write the fraction each represents, and compare the fractions using >, <, or =.
- Ask students if they can use the 3-by-4 and 2-by-8 array models to compare eighths and twelfths. [No, because the wholes are different sizes.]

EXTEND

Challenge Activity
Compare three or more fractions.

Students who have achieved proficiency

Will benefit from deepening understanding of comparing fractions

Materials For each pair: index cards labeled with halves, thirds, fourths, fifths, sixths, eighths, tenths, and twelfths fractions

- Give each pair a set of fraction cards. Have partners set out three, four, or five fractions and order them from least to greatest.
- The strategy is to compare one fraction to another, then choose a third fraction, comparing it to each to place it correctly.

7 $\frac{5}{10} < \frac{5}{8}$; 5 of 10 equal parts is a lesser amount than 5 of 8 equal parts.

DOK 2

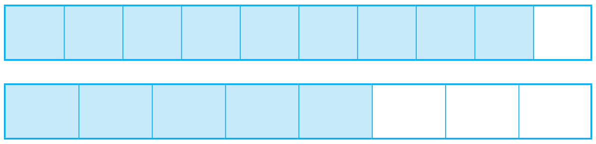
8 Rachel sells a greater fraction of her boxes of fruit; Students may draw and shade area models to compare $\frac{9}{10}$ and $\frac{5}{8}$. See possible work on the Student Worktext page.

DOK 2

7 Compare the fractions $\frac{5}{10}$ and $\frac{5}{8}$. Write the symbol >, <, or =.

$\frac{5}{10}$ < $\frac{5}{8}$

8 Rachel and Sierra have the same number of boxes of fruit to sell for a fundraiser. Each box is the same size. Rachel sells $\frac{9}{10}$ of her boxes, and Sierra sells $\frac{5}{8}$ of her boxes. Which girl sells a greater fraction of her boxes of fruit? Draw a model to show your answer. Show your work. **Possible student work:**



$\frac{9}{10}$

$\frac{5}{8}$

$\frac{9}{10}$ has a greater area shaded, so it is greater than $\frac{5}{8}$.

Rachel sells a greater fraction of her boxes of fruit.

9 MATH JOURNAL

Jeff says $\frac{3}{4}$ of a small pizza is more than $\frac{1}{3}$ of a large pizza. Alicia disagrees. Who is right? Do you have enough information to know who is right? Explain.

You don't have enough information to know who is right. Possible explanation: You need to see the size of the pizzas. If the small pizza is close in size to the large pizza, Jeff could be right. If the large pizza is a lot larger than the small pizza, Alicia could be right.

SELF CHECK Go back to the Unit 4 Opener and see what you can check off.

398

REINFORCE

Problems 4–9

Compare fractions.

All students will benefit from additional work with comparing fractions by solving problems in a variety of formats.

- Have students work on their own or with a partner to solve the problems.
- Encourage students to show their work.

PERSONALIZE

i-Ready

Provide students with opportunities to work on their personalized instruction path with *i-Ready* Online Instruction to:

- fill prerequisite gaps
- build up grade level skills

Close: Exit Ticket

9 MATH JOURNAL

Student responses should indicate understanding that the size of the wholes must be the same in order to compare two fractions and that the sizes of the pizzas differ, so there is not enough information to compare $\frac{3}{4}$ of a small pizza to $\frac{1}{3}$ of a large pizza.

Error Alert **If** students do not recognize that the size of the wholes must be the same to compare the fractions $\frac{3}{4}$ and $\frac{1}{3}$ and incorrectly think that Jeff is right, **then** have them draw a small circle divided into fourths with 3 parts shaded and a larger circle divided into thirds with 1 part shaded so they can see that the sizes of the wholes are not the same.

SELF CHECK Have students consider whether they feel they are ready to check off any new skills on the Unit 4 Opener.

LESSON 20

Add and Subtract Fractions

Lesson Objectives

Content Objectives

- Add fractions with like denominators.
- Subtract fractions with like denominators.
- Decompose fractions as a sum of fractions with the same denominators in more than one way.
- Use fraction models, number lines, and equations to represent word problems.

Language Objectives

- Draw pictures or diagrams to represent word problems involving fraction addition and subtraction.
- Use fraction vocabulary, including *numerator* and *denominator*, to explain how to add and subtract fractions with like denominators.
- Orally define and use the key mathematical terms *add, subtract, equal parts, fraction, unit fraction, numerator,* and *denominator* when reasoning and constructing arguments about fraction addition, fraction subtraction, and fraction decomposition.
- Draw models and write equations to represent ways to decompose a fraction.
- Write and solve equations to represent word problems involving fraction addition or subtraction.

Prerequisite Skills

- Understand addition as joining parts.
- Understand subtraction as separating parts.
- Know addition and subtraction basic facts.
- Understand the meaning of fractions.
- Identify numerators and denominators.
- Write whole numbers as fractions.
- Compose and decompose fractions.

Standards for Mathematical Practice (SMP)

SMPs 1, 2, 3, 4, 5, and 6 are integrated in every lesson through the *Try-Discuss-Connect* routine.*

In addition, this lesson particularly emphasizes the following SMPs:

2 Reason abstractly and quantitatively.

4 Model with mathematics.

5 Use appropriate tools strategically.

7 Look for and make use of structure.

*See page 363m to see how every lesson includes these SMPs.

Lesson Vocabulary

There is no new vocabulary. Review the following key terms.

- **denominator** the number below the line in a fraction that tells the number of equal parts in the whole.
- **fraction** a number that names equal parts of a whole. A fraction names a point on the number line.
- **numerator** the number above the line in a fraction that tells the number of equal parts that are being described.
- **unit fraction** a fraction with a numerator of 1. Other fractions are built from unit fractions.

Learning Progression

In the previous lesson students began developing an understanding of adding and subtracting fractions with like denominators. They developed an understanding of adding fractions as combining parts referring to the same whole.

This lesson extends students' understanding of fraction addition and subtraction. Here students begin to deal with addition and subtraction in the abstract. They learn to decompose fractions as a sum of fractions with the same denominators in more than one way. Students use visual models to represent word problems involving the addition and subtraction of fractions with the same whole. Students also use equations to solve word problems.

In the next lesson students will add and subtract mixed numbers with like denominators. The focus in Grade 4 is on adding and subtracting fractions with like denominators. In Grade 5, students begin to add and subtract fractions with unlike denominators.

Lesson Pacing Guide

Whole Class Instruction

Session	Instruction	Practice
SESSION 1 Explore *45–60 min*	**Interactive Tutorial*** *(Optional)* *Prerequisite Review:* Understand Adding and Subtracting Fractions **Adding and Subtracting Fractions** • Start *5 min* • Try It *10 min* • Discuss It *10 min* • Connect It *15 min* • Close: Exit Ticket *5 min*	**Additional Practice** Lesson pages 417–418
SESSION 2 Develop *45–60 min*	**Adding Fractions** • Start *5 min* • Try It & Discuss It *15 min* • Picture It & Model It *5 min* • Connect It *15 min* • Close: Exit Ticket *5 min*	**Additional Practice** Lesson pages 423–424 **Fluency** Adding Fractions
SESSION 3 Develop *45–60 min*	**Subtracting Fractions** • Start *5 min* • Try It & Discuss It *15 min* • Picture It & Model It *5 min* • Connect It *15 min* • Close: Exit Ticket *5 min*	**Additional Practice** Lesson pages 429–430 **Fluency** Subtracting Fractions
SESSION 4 Develop *45–60 min*	**Decomposing Fractions** • Start *5 min* • Try It & Discuss It *10 min* • Model Its *5 min* • Connect It *15 min* • Close: Exit Ticket *5 min*	**Additional Practice** Lesson pages 435–436 **Fluency** Decomposing Fractions
SESSION 5 Refine *45–60 min*	**Adding and Subtracting Fractions** • Start *5 min* • Example & Problems 1–3 *15 min* • Practice & Small Group Differentiation *20 min* • Close: Exit Ticket *5 min*	**Lesson Quiz** or **Digital Comprehension Check**

Teacher Toolbox

Small Group Differentiation

PREPARE

Ready Prerequisite Lessons

Grade 3
- Lesson 20 Understand What a Fraction Is
- Lesson 21 Understand Fractions on a Number Line

RETEACH

Tools for Instruction

Grade 3
- Lesson 20 Modeling Fractions
- Lesson 20 Fractions on a Number Line

Grade 4
- Lesson 20 Add and Subtract Fractions

REINFORCE

Math Center Activities

Grade 4
- Lesson 20 Make a Whole!
- Lesson 20 Different Ways to Show Sums

EXTEND

Enrichment Activity

Grade 4
- Lesson 20 Addition Grids

i-Ready

Independent Learning

PERSONALIZE

i-Ready Lesson*

Grade 4
- Add and Subtract Fractions

Learning Game
- Prerequisite: *Bounce*

Lesson Materials

Lesson *(Required)*	*Per student:* 1 set of fraction tiles or fraction circles
Activities	*Per student:* scissors, ruler, heavy paper or card stock, paper plates, markers *Per pair:* 1 set of fraction tiles or fraction circles *Activity Sheet:* Fraction Bars
Math Toolkit	counters, fraction circles, fraction tiles, fraction bars, number lines, index cards
Digital Math Tools	Fraction Models, Number Line

*We continually update the Interactive Tutorials. Check the Teacher Toolbox for the most up-to-date offerings for this lesson.

Connect to Family, Community, and Language Development

The following activities and instructional supports provide opportunities to foster school, family, and community involvement and partnerships.

Connect to Family

Use the **Family Letter**—which provides background information, math vocabulary, and an activity—to keep families apprised of what their child is learning and to encourage family involvement.

Available in Spanish
Teacher Toolbox

Add and Subtract Fractions

LESSON 20

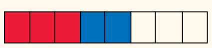
Dear Family,

This week your child is learning how to add and subtract fractions with like denominators.

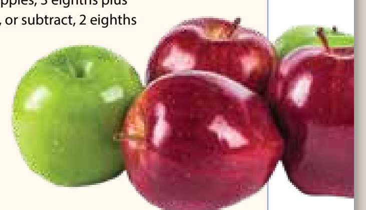
Fractions with the same number below the line have like denominators.

like denominators: $\frac{1}{4}$ and $\frac{3}{4}$ unlike denominators: $\frac{1}{2}$ and $\frac{3}{4}$

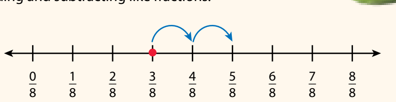
To find the sum of fractions with like denominators, understand that you are adding like units. Just as 3 apples plus 2 apples is 5 apples, 3 eighths plus 2 eighths is 5 eighths. Similarly, when you take away, or subtract, 2 eighths from 5 eighths, you have 3 eighths left.



$\frac{3}{8} + \frac{2}{8} = \frac{5}{8}$



You can also use a number line to understand adding and subtracting like fractions.



Remember that the denominator names units the same way that "apples" names units.

So, when you add two fractions with like denominators, the sum of the numerators tells how many of those units you have.

When you subtract two fractions with like denominators, the difference of the numerators tells how many of those units you have.

Invite your child to share what he or she knows about adding and subtracting fractions by doing the following activity together.

411

ACTIVITY ADDING AND SUBTRACTING FRACTIONS

Do this activity with your child to add and subtract fractions.

Materials bowl, measuring cup, ingredients shown in the recipe

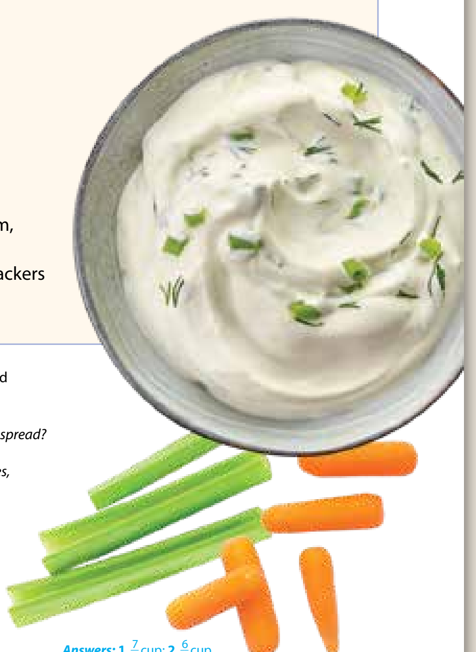
Follow the recipe below to make a creamy cracker spread or veggie dip.

Creamy Spread

Ingredients

$\frac{5}{8}$ cup cream cheese
$\frac{2}{8}$ cup sour cream
herbs
crackers or veggies

Directions
Mix the cream cheese, sour cream, and herbs together in a medium bowl. Serve immediately with crackers or sliced fresh veggies. Enjoy!



After you have made the spread, ask your child questions such as these:

1. *What fraction of a cup is the total amount of spread?*
2. *If you spread $\frac{1}{8}$ of a cup on crackers or veggies, how much spread is left?*

Make up a simple recipe using fractions for someone else in the family to make!

Answers: **1.** $\frac{7}{8}$ cup; **2.** $\frac{6}{8}$ cup

412

Goal

The goal of the Family Letter is to reinforce the concept that adding and subtracting fractions with the same denominators is like adding and subtracting parts of the same whole. The sum or difference of the numerators tells how many of those parts you have. Understanding fractions is a critical life skill necessary for hobbies and jobs such as cooking, sewing, architecture, and construction.

Activity

Students and family members work with fractional measurements of ingredients to prepare a creamy cracker spread. Look at the *Adding and Subtracting Fractions* activity and adjust it if necessary to connect with your students.

Math Talk at Home

Encourage students to discuss with their family any foods they eat at home that can be separated into equal parts, such as pizza, pie, or another favorite food their family enjoys.

Conversation Starters Below are additional conversation starters students can write in their Family Letter or math journal to engage family members:

- *Do you use a measuring cup when you cook? Can you show it to me so that I can see the markings?*
- *What favorite recipe can we make together?*
- *What do you eat that can be cut into equal parts?*

Connect to Community and Cultural Responsiveness

Use these activities to connect with and leverage the diverse backgrounds and experiences of all students.

Session 1 **Use with *Try It*.**

- You may want to ask students what type of cards or other items they collect. Substitute the collectible items students name to help them connect to the problem.

Session 2 **Use with *Try It*.**

- Ask students if they have helped build a fence or any other structure or have used any tools around their home. Share with students that before a fence or a structure is built, an architect or planner draws plans with very accurate measurements that include fractions.
- Show an image of a plan if possible. Point out that adding fractions will allow students to know how to build something so that it fits in the space available.
- Bring a measuring tape to class and point out how each inch—which is an example of a whole—can be separated into equal parts. Measure the door frame and the door of your classroom and ask students what would happen if the door was made wider than the frame.

Session 3 **Use with *Additional Practice* problem 5.**

- Many cultures record important events through weaving, tapestries, or quilting. To help build cultural connections, ask students to draw a quilt or tapestry that represents their cultural background.

Session 5 **Use anytime during the session.**

- Share with students that adding and subtracting fractions is not only used in construction or to build structures. Explain that making clothing requires careful measuring to determine how much material is needed. If time permits, have students use a tape measure to measure the length between their shoulders. Have students work in small groups to determine how much material they would need to make the shirts they are wearing.

Connect to Language Development

For ELLs, use the Differentiated Instruction chart to plan and prepare for specific activities in every session.

ELL English Language Learners: Differentiated Instruction

Prepare for Session 1
Use with *Try It*.

Levels 1–3	Levels 2–4	Levels 3–5
Listening/Speaking Read the ***Try It*** problem. As students draw representations, ask them to identify the whole by pointing to their drawings. Provide a sentence frame: *These (circles, squares) represent the* whole. Ask students to explain what *whole* means. Say: *Whole means* complete. Ask students to identify the number of cards Lynn and Paco get and the number of cards Todd gets. Help students write a fraction for the part of the pack of cards that Todd gets: $\frac{5}{12}$. Then have them identify the numerator and denominator and explain what they mean: *The 5 means* ______. *The 12 means* ______.	**Listening/Speaking** Read the ***Try It*** problem and have students form pairs. Write the following sentence frames: • *First, I identified the whole by* ______. • *Next, I found out how many total cards Lynn and Paco had by* ______. • *Then I decided the fraction for the part of the pack of cards that Todd got is* ______ *because* ______. • *My answer is reasonable because* ______. Read the sentences to the students and then have them take turns reading the sentences with their partners. Ask students to explain to their partners how they determined the fraction of the pack of cards that Todd got. Call on students to share their explanations.	**Writing/Reading** Read the ***Try It*** problem and have students form pairs. Have students solve the problem and then describe in writing the strategies they used to determine the fraction of the pack of cards that Todd got. Provide the following vocabulary for students to use in their responses: *denominator, numerator, whole, fraction*, and *reasonable*. Ask students to read what they have written to their partners.

Explore

Purpose In this session, students draw on the similarities between adding or subtracting whole numbers and adding or subtracting fractions. They share models to explore how various solution methods are based on unit fractions. They will look ahead to think about problem situations that involve subtracting from a whole.

Start

Connect to Prior Knowledge

Materials For each student: 1 set of fraction tiles or fraction circles

Why Support students' facility with composing and decomposing fractions, foreshadowing the twelfths they will work with to solve the problem.

How Have students use fraction tiles or fraction circles to show one whole built from twelfths, in any way they choose.

Use $\frac{1}{12}$s to build 1 whole. Complete the equation to show how you made 1 whole.

$\frac{\square}{12} + \frac{\square}{12} = 1$ whole

Possible Solutions

$\frac{1}{12} + \frac{11}{12}$ $\quad \frac{4}{12} + \frac{8}{12}$

$\frac{2}{12} + \frac{10}{12}$ $\quad \frac{5}{12} + \frac{7}{12}$

$\frac{3}{12} + \frac{9}{12}$ $\quad \frac{6}{12} + \frac{6}{12}$

TRY IT

Make Sense of the Problem

To support students in making sense of the problem, have them identify how many cards are in the pack they are sharing.

DISCUSS IT

Support Partner Discussion

To reinforce the units of twelfths, encourage students to use *twelfths* as they talk to each other.

Look for, and prompt as necessary, for understanding of:

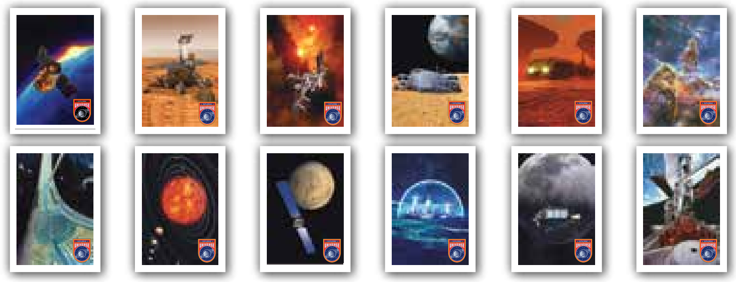
- 12 as the number of parts in the whole
- 3 and 4 as parts of the total
- a part of the total that is unknown

LESSON 20 SESSION 1 ● ○ ○ ○ ○

Explore Adding and Subtracting Fractions

Previously, you learned that adding fractions is similar to adding whole numbers. Use what you know to try to solve the problem below.

Lynn, Paco, and Todd share a pack of 12 cards. Lynn gets 4 cards, Paco gets 3 cards, and Todd gets the rest of the cards. What fraction of the pack does Todd get?



Learning Targets

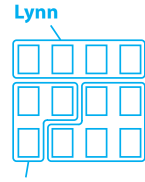
- Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model.
- Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators.

SMP 1, 2, 3, 4, 5, 6, 7

TRY IT

Possible student work:

Sample A



5 out of 12 is $\frac{5}{12}$.

Sample B

$3 + 4 = 7$. Lynn and Paco get 7 cards.

$12 - 7 = 5$. Todd gets the 5 cards left.

5 out of 12 is $\frac{5}{12}$.

Math Toolkit

- counters
- fraction circles
- fraction tiles
- fraction bars
- number lines
- fraction models

DISCUSS IT

Ask your partner: Why did you choose that strategy?

Tell your partner: At first, I thought . . .

413

Common Misconception Look for students who are not comfortable with twelfths and who try to use 3 or 4 as the denominator. As students present solutions, be sure to have them specify why they used 12 as the denominator.

Select and Sequence Student Solutions

One possible order for whole class discussion:

- physical parts showing twelfths
- drawings representing twelfths
- number lines marked in twelfths
- whole-number equations to find 5 out of 12 cards are left $\left(\frac{5}{12}\right)$

Support Whole Class Discussion

Prompt students to note the relationship between the numbers in each model and the numbers in the problem.

Ask *How do [student name]'s and [student name]'s models show the whole and the parts?*

Listen for 12 is the denominator of all the fractions. $\frac{12}{12}$ shows the whole. The parts are $\frac{4}{12}$, $\frac{3}{12}$, $\frac{7}{12}$, and $\frac{5}{12}$.

CONNECT IT

1 LOOK BACK

Look for understanding that the whole set of cards is 12 twelfths and that only some of those twelfths, $\frac{5}{12}$, are left for Todd.

Hands-On Activity
Use models to add fractions.

If . . . *students are unsure about the concept of adding fractions,*

Then . . . *use this activity to have them model similar problems.*

Materials For each student: scissors, ruler, heavy paper or card stock

- Distribute heavy paper or card stock to each student. Tell students to use scissors to cut out 12 equal-sized cards. Explain to students that the 12 cards represent one pack of cards, or one whole, and that there are 12 parts in the whole.
- Tell students to hold up 2 cards. Have students write the name of the fraction of the whole pack of cards that is represented by the 2 cards on a sheet of paper. $\left[\frac{2}{12}\right]$ Review the meaning of the fraction. [2 cards out of 12] Then repeat with 7 cards.
- Tell students to add (join) the fractions and write the sum on their sheets of paper. $\left[\frac{9}{12}\right]$ Have volunteers explain how they determined their answers.
- Repeat the activity for additional fractions, such as eighths and sixths.

2 LOOK AHEAD

Point out that sometimes students will encounter a whole that does not look like it is composed of fractional parts. Students may have to imagine the whole being cut into equal parts.

Students should be able to use fraction language to describe the whole pizza in terms of eighths, to discuss taking eighths away from the whole pizza, and to determine how many eighths are left. Students will spend more time learning about fractions in the Additional Practice.

CONNECT IT

1 LOOK BACK

Explain how you can find the fraction of the pack that Todd gets.

Possible answer: Todd gets 5 cards. There are 12 cards in the pack. The numerator tells the number of cards Todd gets, and the denominator tells the number of cards in the whole pack, so Todd gets $\frac{5}{12}$ of the pack.

2 LOOK AHEAD

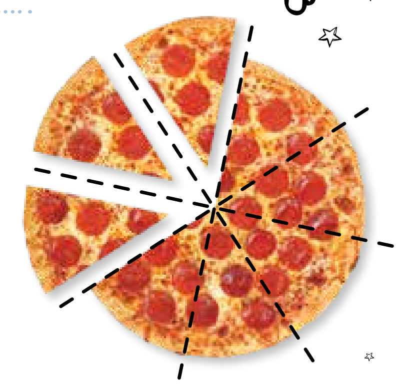
In the problem on the previous page, the whole is the pack of cards. Since there are 12 cards in the pack, each card represents $\frac{1}{12}$ of the whole. Look at the whole shown here. The whole is the pizza. It is a single object.

a. How many equal parts are shown in the pizza? **8**

b. What fraction can you use to describe each piece of pizza? $\frac{1}{8}$

c. What fraction can you use to describe the whole? $\frac{8}{8}$

d. What fraction can you use to describe the 3 pieces being taken away? $\frac{3}{8}$



3 REFLECT

How does knowing about equal parts help you add and subtract fractions?

Possible answer: Adding fractions means joining equal parts of the whole. Subtracting fractions means taking away equal parts of the whole.

Close: Exit Ticket

3 REFLECT

Look for understanding of adding or subtracting fractions as joining together or taking away parts referring to the same whole. Student responses should include references to the whole, joining equal parts of the whole to add, and taking away equal parts of the whole to subtract. Some students may use the terms *numerator* and *denominator* in their explanations.

Common Misconception **If** students do not reference the whole or are unclear in their explanations that they are adding or subtracting equal parts of the whole, **then** provide fraction tiles and have students "join" tiles to add and "take away" tiles to subtract. Discuss what students notice about the numerators and denominators.

Real-World Connection

Encourage students to think about everyday places or situations in which people might need to add or subtract fractions with like denominators. Have volunteers share their ideas. Examples include cooking, construction sites, and distances on a map.

SESSION 1 Additional Practice

Solutions

Support Vocabulary Development

1 Ask students to circle *frac* in the word *fraction*. Share with students that *frac* is a Latin root that means "break." Point out that in math, a *fraction* is a part of a whole. Ask students if they know of other words that have the root word *frac* (or *frag*). Provide examples, such as *fracture* and *fragment*.

If students struggle completing any part of the graphic organizer, have them use manipulatives, such as fraction tiles or fraction bars, to break a whole into parts.

2 Have students count the parts and ask: *How many parts does the model have? How would you describe the size of the parts?* Have students discuss the answers with partners. Then have them answer the questions in the problem. Provide a sentence frame:

The model is/is not ______ because ______.

Supplemental Math Vocabulary

- *numerator*
- *unit fraction*
- *whole*

Name: ____________________

Prepare for Adding and Subtracting Fractions

1 Think about what you know about fractions. Fill in each box. Use words, numbers, and pictures. Show as many ideas as you can. **Possible answers:**

fraction

What Is It?

A number that names the equal parts of a whole

What I Know About It

Fractions have a numerator that names the equal parts being described and a denominator that names the total number of equal parts.

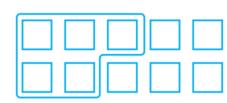
Examples

Fourths

$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$

Examples

5 tenths



Examples

$\frac{1}{2}$ $\frac{3}{8}$

$\frac{4}{12}$ $\frac{2}{3}$ $\frac{5}{4}$

2 Does the model below show eighths? Why or why not?

$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$

Possible answer: The model does not show equal parts, so it cannot be an example of eighths.

415

3 Assign problem 3 to provide another look at solving a problem with fractional parts of a set.

This problem is very similar to the problem about the set of space exploration cards. In both problems, the whole is a set, and the set is shared among three friends. The question asks what fraction of the whole is left for the third friend.

Students may want to use fraction tiles, sticky notes, or counters.

Suggest that students read the problem three times, asking themselves one of the following questions each time.

- *What is the problem about?*
- *What is the question I am trying to answer?*
- *What information is important?*

Solution:

Kara gets $\frac{4}{10}$ of the stickers. After Maria takes 2 and Jon takes 4, there are 4 stickers left. 4 out of 10 equal parts is $\frac{4}{10}$.

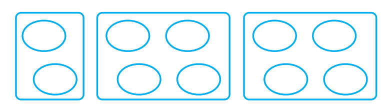
Medium

4 Have students solve the problem a different way to check their answer.

3 Solve the problem. Show your work.

Maria, Jon, and Kara share a set of 10 animal stickers. Maria gets 2 stickers, Jon gets 4 stickers, and Kara gets the rest of the stickers. What fraction of the stickers does Kara get?

Possible student work using a picture:



$\frac{2}{10}$	$\frac{4}{10}$	$\frac{4}{10}$
Maria	Jon	Kara

Solution $\frac{4}{10}$ **of the stickers**

4 Check your answer. Show your work.

Possible student work:

There are 10 stickers in all. 2 + 4 + 4 = 10, so $\frac{4}{10}$ is correct.

ELL **English Language Learners: Differentiated Instruction**

Prepare for Session 2
Use with *Connect It.*

Levels 1–3

Speaking/Writing Read ***Connect It*** problems 1–2. Ask students to draw a fence divided into 10 equal sections. Point to a section and ask: *How do I know this is $\frac{1}{10}$ of the fence?* Provide sentence frames: *1 is the numerator. 10 is the denominator. The fraction $\frac{1}{10}$ tells this is 1 out of 10 sections.* Point to and count 3 sections. Say: *Josie paints 3 sections of the fence.* Write $\frac{3}{10}$. *What does the numerator 3 in this fraction mean?* Have students answer in writing: *The numerator 3 tells me that Josie paints 3 sections of the fence.* Repeat for the numerator 4.

Levels 2–4

Speaking/Writing Read ***Connect It*** problems 1–2. Have students form pairs and explain to their partners how they know each section is $\frac{1}{10}$ of the whole. Ask: *When you see the numerator 1, what does that tell you? When you see the denominator 10, what does that tell you?* After students have discussed their thinking, have them write their responses. Write $\frac{3}{10}$ and $\frac{4}{10}$. Ask partners to discuss what is the same and different about the fractions. Then call on pairs to say how they determined the total number of sections each girl paints.

Levels 3–5

Speaking/Writing Have students form pairs and read ***Connect It*** problems 1–6 and write their responses. Ask students to discuss their thinking with partners. Extend the discussions by asking: *How would the answer be different if Josie paints 5 sections and Margo paints 1 section? What fractions would you need to solve the problem?* After students have discussed, have them work with partners to make new problems using fractions that assume Josie and Margo paint different numbers of sections of the fence.

LESSON 20

SESSION 2 Develop

Purpose In this session, students solve a problem that requires finding the sum of $\frac{3}{10}$ and $\frac{4}{10}$. Students model the fractions in the word problem either on paper or with manipulatives to represent the sum. The purpose of this problem is to have students develop strategies to add fractions.

Start

Connect to Prior Knowledge

Materials For each student: 1 set of fraction tiles or fraction circles

Why Support students' understanding of adding fractions.

How Have students use fraction tiles or fraction circles to compare two different decompositions of $\frac{3}{5}$.

Look at the two expressions.

$\frac{1}{5} + \frac{1}{5} + \frac{1}{5}$ $\qquad$ $\frac{1}{5} + \frac{2}{5}$

How are they the same?

How are they different?

Possible Solution

The expressions have the same value of $\frac{3}{5}$, but they have different addends.

Develop Language

Why Clarify the meaning of the multiple-meaning word *end*.

How Have students circle the word *end*. Explain that in this case, the *end* is the first or last part of something that is long, such as the fence in the problem or a rope. If necessary, illustrate the meaning with four students in a line. Show that there are two ends of the line with the first person being at the first end and the last person being at the last end.

TRY IT

Make Sense of the Problem

To support students in making sense of the problem, have them identify 10 as the total number of equal parts.

Ask *How many equal parts of the fence are there if Josie paints $\frac{3}{10}$ of it?*

LESSON 20 $\qquad$ SESSION 2 ● ● ○ ○ ○

Develop Adding Fractions

Read and try to solve the problem below.

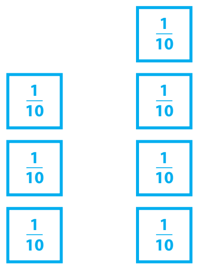
Josie and Margo are painting a fence green. Josie starts at one end and paints $\frac{3}{10}$ of the fence. Margo starts at the other end and paints $\frac{4}{10}$ of it. What fraction of the fence do they paint altogether?

TRY IT

Possible student work:

Math Toolkit
- fraction circles
- fraction tiles
- fraction bars
- number lines
- index cards
- fraction models

Sample A

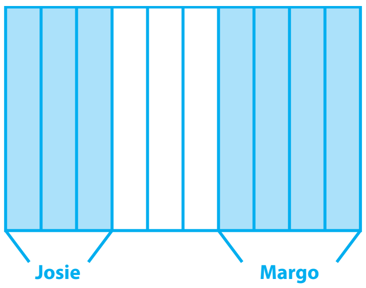


The fence is the whole. It has 10 equal parts.

Josie paints $\frac{3}{10}$. Margo paints $\frac{4}{10}$.

They paint $\frac{7}{10}$ altogether.

Sample B



Josie and Margo paint 7 out of 10 parts of the fence.

They paint $\frac{7}{10}$ altogether.

DISCUSS IT

Ask your partner: How did you get started?

Tell your partner:
A model I used was . . .
It helped me . . .

417

DISCUSS IT

Support Partner Discussion

Encourage students to use the term *tenths* as they discuss their solutions.

Support as needed with questions such as:

- *How did you get started?*
- *How would you describe your model?*

Common Misconception Look for students who write a fraction comparing the painted parts to the unpainted parts and write $\frac{7}{3}$, instead of comparing to the whole.

Select and Sequence Student Solutions

One possible order for whole class discussion:

- physical parts showing tenths
- drawings representing tenths
- whole-number solutions showing that 7 out of 10 parts are painted $\left(\frac{7}{10}\right)$
- number lines marked in tenths

Support Whole Class Discussion

Compare and connect the different representations and have students identify how they are related.

Ask *Where does your model show the total number of equal parts in the fence? the part Josie paints? the part Margo paints? the total number of tenths the two girls paint?*

Listen for Students should recognize that accurate responses include fractions with a denominator of 10 and representations that show equal parts. Responses may include 10 as the total number of equal parts, $\frac{3}{10}$ as the part Josie paints, $\frac{4}{10}$ as the part Margo paints, and $\frac{7}{10}$ as the total number of parts both girls paint.

PICTURE IT & MODEL IT

If no student presented these models, connect them to the student models by pointing out the ways they each represent:

- the whole
- the number of equal parts
- the number of parts each girl paints
- the total amount of fence painted

Ask *What number tells the number of equal parts in the whole in the picture? in the number line? Is it the same or different?*

Listen for 10 is the denominator. It tells the total number of equal parts in both the picture and the number line. Both show 10 equal parts because they represent the same whole.

For a sketch of the fence, prompt students to identify how the fence is labeled to represent the problem.

- *Is there any way that this picture is more or less helpful than the one drawn by [student name]?*
- *How is it helpful that the fence has 10 boards?*

For a number line model, prompt students to identify the greatest number on the number line as well as the number of divisions.

- *How is the number line divided?*
- *Why is it done that way?*

Explore different ways to understand adding fractions.

Josie and Margo are painting a fence green. Josie starts at one end and paints $\frac{3}{10}$ of the fence. Margo starts at the other end and paints $\frac{4}{10}$ of it. What fraction of the fence do they paint altogether?

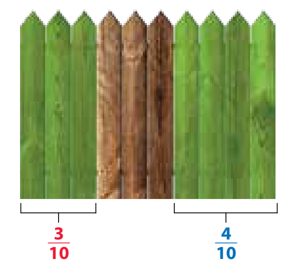
PICTURE IT

You can use a picture to help understand the problem.

Think what the fence might look like if it has 10 equal-sized parts.

Each part is $\frac{1}{10}$ of the whole.

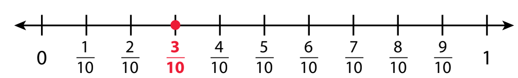
The girls paint 3 tenths and 4 tenths of the fence.



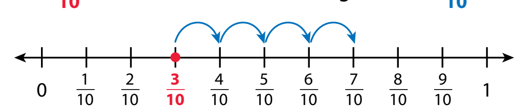
MODEL IT

You can also use a number line to help understand the problem.

The number line below is divided into tenths with a point at $\frac{3}{10}$.



Start at $\frac{3}{10}$ and count 4 tenths to the right to **add** $\frac{4}{10}$.



Deepen Understanding

Number Line Model

SMP 7 Look for structure.

When discussing the number line model, prompt students to consider how it could be used to demonstrate the commutative property.

Ask *What if you drew the starting point at $\frac{4}{10}$ instead of at $\frac{3}{10}$? Could you still model the problem?* To emphasize the point, draw a tenths number line on the board with a point at $\frac{4}{10}$.

Listen for Yes, you could count on $\frac{3}{10}$ from $\frac{4}{10}$ to find the answer.

Encourage a volunteer to come to the board and demonstrate how to find the sum.

Generalize *Do you think this is true no matter what numbers you are adding? If you were using a number line to add 3 and 4, would it be true?* Have students explain their reasoning. Listen for understanding that when adding whole numbers or fractions, the order of the addends does not matter; the sum stays the same.

CONNECT IT

Remind students that one thing that is alike about all the representations is the numbers.

Explain that on this page, students will use those numbers to write one equation that matches all the representations, including concrete, visual, and symbolic.

Monitor and Confirm

1 – 3 Check for understanding that:

- 10 is the number of equal parts
- 3 and 4 tell how many parts each girl paints
- 7 is the total number of parts painted

Support Whole Class Discussion

4 – 5 Tell students that problem 4 will prepare them to provide the explanation required in problem 6.

Be sure students understand that the problem is asking them to represent the same equation twice: once with words and once with fractions.

Ask *What part of the problem do each of the fractions in the equations show?*

Listen for $\frac{3}{10}$ is the fraction of the fence Josie paints. $\frac{4}{10}$ is the fraction Margo paints. $\frac{7}{10}$ is the fraction they paint altogether.

Ask *What is the same about the two equations?*

Listen for The numerators, 3, 4, and 7, are numbers in each equation; the denominators are words in one equation and numbers in the other.

Ask *How could you predict if the sum of $\frac{3}{10}$ and $\frac{5}{10}$ is greater than or less than the sum of $\frac{3}{10}$ and $\frac{4}{10}$ without doing the computation?*

Listen for I think it is greater because $\frac{3}{10}$ is the same in both problems, but $\frac{5}{10}$ is greater than $\frac{4}{10}$.

Explain that problem 5 is asking about adding two different fractions not shown in the fence problem.

6 Look for the idea that you add the numerators and keep the same denominator because the size of the parts does not change when you add them.

7 REFLECT Have all students focus on the strategies used to solve this problem. If time allows, have students share their responses with a partner.

CONNECT IT

Now you will use the problem from the previous page to help you understand how to add any two fractions that have the same denominator.

1 Look at **Picture It**. How do you know that each section of fence is $\frac{1}{10}$ of the whole fence?

Possible answer: The denominator tells the total number of equal-sized fence sections. There are 10 equal parts in all, so one part equals $\frac{1}{10}$.

2 What do the numerators, 3 and 4, tell you?

3 tells the number of fence sections that Josie paints. 4 tells the number of fence sections that Margo paints.

3 How many tenths of the fence do Josie and Margo paint altogether? **7**

4 Complete the equations to show what fraction of the fence Josie and Margo painted altogether.

Use words: **3 tenths** + **4 tenths** = **7** tenths

Use fractions: $\frac{3}{10}$ + $\frac{4}{10}$ = $\frac{\boxed{7}}{10}$

5 What would be the sum if the fractions were $\frac{3}{10}$ and $\frac{5}{10}$? $\frac{8}{10}$

6 Explain how you add fractions that have the same denominator.

Add the numerators of the addends to get the numerator of the sum to find how many parts you added. The denominators of the sum and the addends are the same because that tells what kind of parts you added.

7 REFLECT

Look back at your **Try It**, strategies by classmates, and **Picture It** and **Model It**. Which models or strategies do you like best for adding fractions? Explain.

Some students may respond that they like the strategy of drawing a picture because it helps them see the parts of the whole, while other students may like using a number line because it helps them see how fraction addition is like whole number addition.

419

Hands-On Activity

Connect fraction words with fraction symbols using familiar fractions, such as fourths.

If . . . *students are unsure about what the numerator and denominator name,*

Then . . . *use the activity below to connect symbolic fractions with verbal descriptions.*

Materials For each student: Activity Sheet *Fraction Bars* (3 bars for fourths, 3 bars for tenths)

- Have students shade and label 1 part of a fourths fraction bar with the words *one fourth* and the symbol $\frac{1}{4}$. Discuss the connections—for each, ask: *Where do you see the 1? Where do you see the 4?*
- Repeat with other fourths fraction bars, shading 2 parts and labeling *two fourths* and $\frac{2}{4}$, and then shading 3 parts, labeling *three fourths* and $\frac{3}{4}$. Ask: *Where is the 2? the 3? Why does 4 show up so many times?*
- Extend to tenths, using tenths fraction bars, and ask students to name, with words and numbers, $\frac{1}{10}$, $\frac{3}{10}$, and $\frac{4}{10}$.
- Prompt students to recognize that they can use drawings and words as well as numbers to keep track of the math and the meaning of fractions.

APPLY IT

For all problems, encourage students to draw some kind of model to support their thinking. Allow some leeway in precision; drawing thirds or fifths accurately is difficult and here precise measurements are not necessary.

8 $\frac{2}{3}$ of the house; Students may also show $\frac{1}{3}$ on a number line divided into thirds and count 1 tick mark to the right. They also may write the equation $\frac{1}{3} + \frac{1}{3} = \frac{2}{3}$.

9 $\frac{4}{5}$ of a meter; Students may show $\frac{1}{5}$ on a number line divided into fifths and count 3 marks to the right. They also may write the equation $\frac{1}{5} + \frac{3}{5} = \frac{4}{5}$.

Close: Exit Ticket

10 $\frac{6}{8}$ of a pound of fruit; Students may write the equation $\frac{2}{8} + \frac{4}{8} = \frac{6}{8}$. They may also show $\frac{2}{8}$ on a number line divided into eighths and count 4 marks to the right.

Students' solutions should indicate understanding of:

- adding, or joining, parts that refer to the same whole
- accurate use of visual fraction models or equations to represent the problem

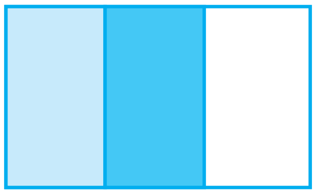
Error Alert **If** students' solutions are $\frac{6}{16}$, **then** review the structure of fractions to help them see that only numerators should be combined. Explain that denominators tell the kind of parts that are being added. Have them write *2 apples* + *4 apples* = *6 apples* on an index card and then write below it *2 eighths* + *4 eighths* = *6 eighths.*

APPLY IT

Use what you just learned to solve these problems.

8 Lita and Otis help their mom clean the house. Lita cleans $\frac{1}{3}$ of the house. Otis cleans $\frac{1}{3}$ of the house. What fraction of the house do Lita and Otis clean altogether? Show your work.

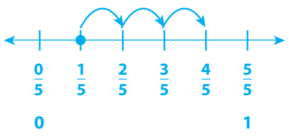
Possible student work using a model:



Solution $\frac{2}{3}$ **of the house**

9 Mark and Imani use string for a project. Mark's string is $\frac{1}{5}$ of a meter long. Imani's string is $\frac{3}{5}$ of a meter long. How long are the two strings combined? Show your work.

Possible student work using a number line:



$\frac{4}{5}$ of a meter

10 Paola makes a fruit smoothie. She uses $\frac{2}{8}$ of a pound of strawberries and $\frac{4}{8}$ of a pound of blueberries. How many pounds of fruit does she use? Show your work.

Possible student work using an equation:

$\frac{2}{8} + \frac{4}{8} = \frac{6}{8}$

Solution $\frac{6}{8}$ **of a pound of fruit**

Additional Practice

Solutions

1. Students should shade any 2 of the 12 muffin cups.
Basic

2. Students should shade any 6 of the 12 muffin cups.
Medium

3. $\frac{8}{12}$ of the tray is filled; $\frac{2}{12} + \frac{6}{12} = \frac{8}{12}$
Challenge

Name: ____________________

Practice Adding Fractions

Study the Example showing one way to add fractions. Then solve problems 1–9.

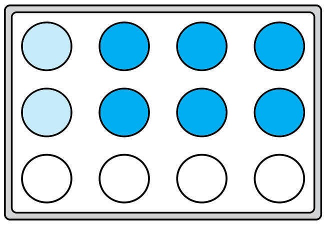
EXAMPLE

Shrina has a muffin pan that holds 12 muffins. She fills $\frac{3}{12}$ of the pan with carrot muffin batter. Then she fills $\frac{6}{12}$ with pumpkin muffin batter. What fraction of the pan does she fill?

$$\frac{3}{12} + \frac{6}{12} = \frac{9}{12}$$

So, she fills $\frac{9}{12}$ of the muffin pan.

1. Sam fills $\frac{2}{12}$ of another pan with banana muffin batter. Shade $\frac{2}{12}$ of the muffin pan diagram at the right.
Any 2 muffins may be shaded.



2. Then Sam fills $\frac{6}{12}$ with lemon muffin batter. Shade $\frac{6}{12}$ of the diagram to show this.
Any 6 muffins may be shaded.

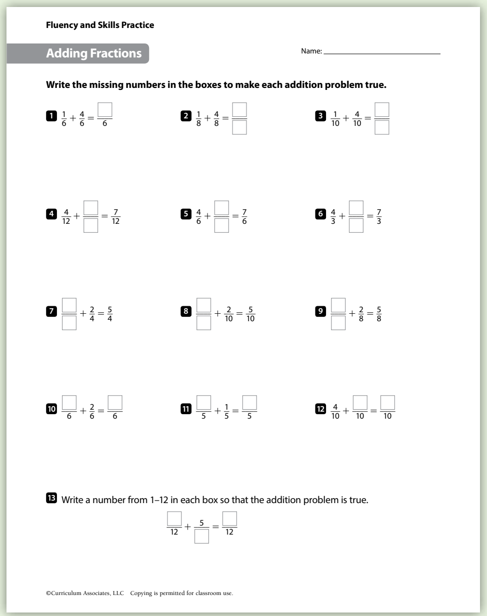
3. In problem 2, what fraction of the pan in all is filled now? $\frac{8}{12}$
Write an equation for this problem that includes your answer.
$\frac{2}{12} + \frac{6}{12} = \frac{8}{12}$

Fluency & Skills Practice

Teacher Toolbox

Assign *Adding Fractions*

In this activity students practice adding fractions with like denominators. Students may apply their understanding of adding fractions with the same denominator in real-world situations. For example, students may want to find the total distance traveled if they walked $\frac{5}{8}$ of a mile in the morning and $\frac{7}{8}$ of a mile in the afternoon.



Fluency and Skills Practice

Adding Fractions

Name: ____________________

Write the missing numbers in the boxes to make each addition problem true.

1. $\frac{1}{6} + \frac{4}{6} = \frac{\square}{6}$
2. $\frac{1}{8} + \frac{4}{8} = \frac{\square}{\square}$
3. $\frac{1}{10} + \frac{4}{10} = \frac{\square}{\square}$
4. $\frac{4}{12} + \frac{\square}{\square} = \frac{7}{12}$
5. $\frac{4}{6} + \frac{\square}{\square} = \frac{7}{6}$
6. $\frac{4}{3} + \frac{\square}{\square} = \frac{7}{3}$
7. $\frac{\square}{\square} + \frac{2}{4} = \frac{5}{4}$
8. $\frac{\square}{\square} + \frac{2}{10} = \frac{5}{10}$
9. $\frac{\square}{\square} + \frac{2}{8} = \frac{5}{8}$
10. $\frac{\square}{6} + \frac{2}{6} = \frac{\square}{6}$
11. $\frac{\square}{5} + \frac{1}{5} = \frac{\square}{5}$
12. $\frac{4}{10} + \frac{\square}{10} = \frac{\square}{10}$
13. Write a number from 1–12 in each box so that the addition problem is true.
$\frac{\square}{12} + \frac{5}{\square} = \frac{\square}{12}$

4 Tick marks should divide each whole into 8 (approximately) equal parts.
Basic

5 dot on sixth tick mark after 0 labeled $\frac{6}{8}$
Basic

6 12 arrows on the number line, six from 0 to $\frac{6}{8}$, and then six from $\frac{6}{8}$ to $\frac{12}{8}$; Some students may start at $\frac{6}{8}$ and count 6 more eighths, showing only 6 arrows from $\frac{6}{8}$ to $\frac{12}{8}$.
Medium

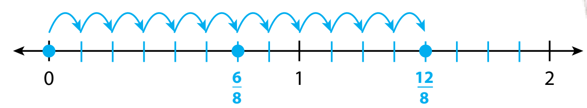
7 $\frac{12}{8}$ miles
Medium

8 $\frac{6}{8} + \frac{6}{8} = \frac{12}{8}$
Challenge

9 $\frac{10}{10}$ (or 1 whole) of the patio; Students may draw a rectangular area model divided into tenths, shading $\frac{1}{10}$ one way and $\frac{9}{10}$ another way, or they may write an equation $\frac{1}{10} + \frac{9}{10} = \frac{10}{10}$.
Challenge

Kay runs $\frac{6}{8}$ of a mile and rests. Then she runs another $\frac{6}{8}$ of a mile.

4 Divide the number line below to show eighths.



5 Label $\frac{6}{8}$ on the number line above.

6 Use arrows to show $\frac{6}{8} + \frac{6}{8}$ on the number line.

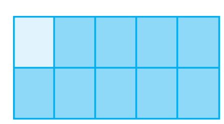
7 What is the total distance Kay runs? $\frac{12}{8}$ miles

8 Write an equation for this problem that includes your answer.

$\frac{6}{8} + \frac{6}{8} = \frac{12}{8}$

9 Jin cleans $\frac{1}{10}$ of the patio before lunch and $\frac{9}{10}$ of the patio after lunch. What fraction of the patio does Jin clean altogether? Show your work.

Possible student work:



$\frac{1}{10} + \frac{9}{10} = \frac{10}{10}$

Solution $\frac{10}{10}$ (or 1 whole) of the patio

ELL English Language Learners: Differentiated Instruction

Prepare for Session 3
Use with *Try It*.

Levels 1–3

Writing Read the ***Try It*** problem and then write *numerator* and *denominator*. Point to each word as you read it aloud. Ask students to form pairs and explain the meaning of each word to partners. Point to the fraction $\frac{5}{6}$. Ask students to identify the numerator and denominator and tell what each refers to. If students respond, "It is the top/bottom number," encourage them to provide additional information using the sentence frames: _______ *is the* _______. *It tells me* _______. Follow the same procedure for $\frac{4}{6}$. Ask students to work with their partners to write the operation needed to solve the problem and then write the equation.

Levels 2–4

Speaking/Writing Read the ***Try It*** problem with students. Divide students into groups of three to discuss what each numerator and denominator mean. Ask: *What operation will you use to solve the problem? Why?* Have students discuss their answers to the questions in their small groups. Ask them to write the procedure they followed to solve the problem. Provide sentence frames for groups to use if additional support is needed.

Levels 3–5

Writing/Speaking Have students form pairs and read the ***Try It*** problem. Ask them to discuss with their partners how they will solve it using what they know about numerators and denominators. Then have students write the steps they use to solve the problem. Ask: *How much water would Alberto have now if he put $\frac{3}{6}$ of a liter of water in the bottle from the water fountain?* After students have solved the problem, have them work with their partners to write additional scenarios to increase or decrease the amount of water in the bottle. Provide support as needed. Have pairs exchange scenarios and try to solve the problems.

LESSON 20

SESSION 3 Develop

Purpose In this session, students solve a word problem that requires finding the difference between $\frac{5}{6}$ and $\frac{4}{6}$. Students model the fractions either on paper or with manipulatives to represent the difference. The purpose is to have students develop strategies to subtract fractions.

Start

Connect to Prior Knowledge

Materials For each student: 1 set of fraction tiles or fraction circles

Why Support students' understanding of subtracting fractions.

How Have students use fraction tiles or fraction circles to find $\frac{1}{10}$ more than and $\frac{1}{10}$ less than $\frac{5}{10}$.

1. $\frac{1}{10}$ more than $\frac{5}{10}$ is ....
2. $\frac{1}{10}$ less than $\frac{5}{10}$ is ....

Solutions

1. $\frac{6}{10}$
2. $\frac{4}{10}$

Develop Language

Why Review the names of some units in the customary and the metric systems of measurement.

How Remind students that two systems of measurement are used in the United States: the customary system and the metric system. Have students try to recall some of the units for each system. Then ask them to identify the unit in the *Try It* problem and say to which system it belongs. Explain that the *liter* is a metric unit of liquid volume.

TRY IT

Make Sense of the Problem

To support students in making sense of the problem, have them identify 6 as the total number of equal parts.

Ask *How many equal parts is the liter divided into if Alberto drinks $\frac{4}{6}$ of a liter of water? How do you know?*

LESSON 20 SESSION 3 ● ● ● ○ ○

Develop Subtracting Fractions

Read and try to solve the problem below.

Alberto's water bottle has $\frac{5}{6}$ of a liter of water in it. He drinks $\frac{4}{6}$ of a liter. What fraction of a liter of water is left in the bottle?

TRY IT

Possible student work:

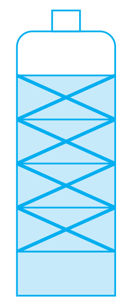
Sample A

Alberto drinks water, so you subtract to find what's left.

$\frac{5}{6} - \frac{4}{6} = \frac{1}{6}$

The water bottle has $\frac{1}{6}$ of a liter of water left.

Sample B



The water bottle has $\frac{1}{6}$ of a liter of water left.

Math Toolkit
- fraction circles
- fraction tiles
- fraction bars
- number lines
- index cards
- fraction models

DISCUSS IT

Ask your partner: Can you explain that again?

Tell your partner: I disagree with this part because . . .

423

DISCUSS IT

Support Partner Discussion

Encourage students to use the term *sixths* as they discuss their solutions.

Support as needed with questions such as:

- *Did you draw a picture or make a model to show the problem?*
- *How can you explain what the problem is asking?*

Common Misconception Students may believe that because the whole bottle is 1 liter that they must subtract either $\frac{5}{6}$ or $\frac{4}{6}$ from $\frac{6}{6}$. Have students restate the problem in their own words, using a diagram to support their explanation.

Select and Sequence Student Solutions

One possible order for whole class discussion:

- physical parts showing sixths
- drawings representing sixths
- number lines marked in sixths

Support Whole Class Discussion

Compare and connect the different representations and have students identify how they are related.

Ask *Where does your model show the total number of equal parts in the bottle? the part Alberto drinks? the part that is left?*

Listen for Students should recognize that accurate representations show 6 as the total number of equal parts; 5 out of 6 parts, or $\frac{5}{6}$, as the part of the bottle that is filled with water; 4 out of 6 parts, or $\frac{4}{6}$, as the part Alberto drinks; and 1 out of 6 parts, or $\frac{1}{6}$, as the part that is left.

PICTURE IT & MODEL IT

If no student presented these models, connect them to the student models by pointing out the ways they each represent:

- the whole
- the number of equal parts
- the number of parts Alberto drinks

Ask *What number tells the whole in the picture? in the number line? Is it the same or different?*

Listen for 6 is the denominator; it tells the total number of equal parts in both the picture and the number line. Both show 6 equal parts because they represent the same whole.

For a sketch of the water bottle, prompt students to identify how the bottle is labeled to represent the problem.

- *Is there any way that this picture is more or less helpful than the one drawn by [student name]?*
- *How is it helpful that the bottle shows 1 liter divided into sixths?*
- *Why are some parts of the bottle blue and some clear?*

For a number line model, prompt students to identify the greatest number on the number line and the number of divisions.

- *How is the number line divided?*
- *Why is the point $\frac{5}{6}$ marked?*

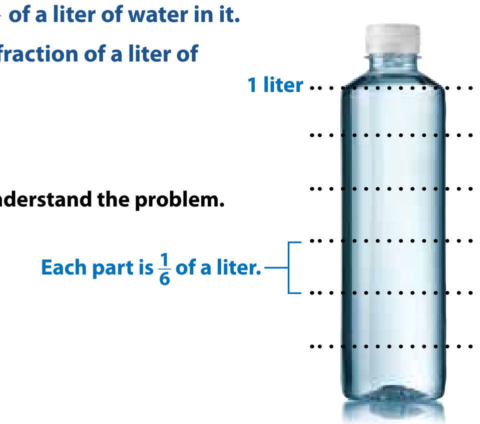
Explore different ways to understand subtracting fractions.

Alberto's water bottle has $\frac{5}{6}$ of a liter of water in it. He drinks $\frac{4}{6}$ of a liter. What fraction of a liter of water is left in the bottle?

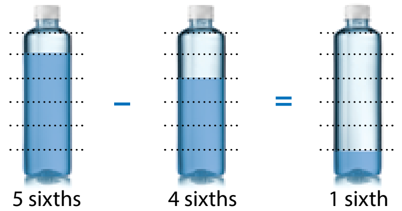
PICTURE IT

You can use a picture to help understand the problem.

The picture shows the whole liter divided into 6 equal parts.



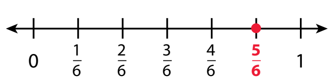
Five shaded parts show how much water is in the bottle. Alberto drinks 4 sixths of a liter, so take away 4 shaded parts. The 1 shaded part that is left shows the fraction of a liter that is left.



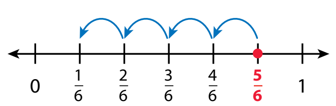
MODEL IT

You can also use a number line to help understand the problem.

The number line at the right is divided into sixths, with a point at $\frac{5}{6}$.



Start at $\frac{5}{6}$ and count back 4 sixths to **subtract** $\frac{4}{6}$.



Deepen Understanding

Connect Visual Representations to Models

SMP 4 Model with mathematics.

When discussing the number line model, prompt students to consider how it and the visual representation of the water bottle are connected.

- Draw the number line on the board. Then draw the $\frac{5}{6}$-full water bottle on its side above the number line, making sure the bottom of the bottle is aligned with 0 and each part of the bottle is aligned with a sixths tick mark.
- Have students identify that $\frac{5}{6}$ on the number line lines up with the amount of water in the bottle.
- Then cross out (or erase) 4 parts of the bottle, one part at a time, moving from right to left along the number line to show the water Alberto drinks.

Generalize To help students identify important quantities and map their relationships, ask: *What do you notice about the amount of water remaining in the water bottle?* Have students share their observations. Listen for understanding that it lines up with the $\frac{1}{6}$ mark on the number line and that both representations show that there is $\frac{1}{6}$ of a liter remaining.

CONNECT IT

Remind students that one thing that is alike about all the representations is the numbers.

Explain that on this page, students will use those numbers to write one equation that matches all the representations.

Monitor and Confirm

1 – 3 Check for understanding that:

- 6 is the number of equal parts
- 5 tells how many parts are in the bottle to start
- 4 tells how many parts Alberto drinks

Support Whole Class Discussion

4 Tell students that this problem will prepare them to provide the explanation required in problem 5.

Be sure students understand that the problem is asking them to represent the same equation twice: once with words and once with fractions.

Ask *What part of the problem do each of the fractions in the equations show?*

Listen for $\frac{5}{6}$ is the amount of water in the bottle. After Alberto drinks $\frac{4}{6}$, there is $\frac{1}{6}$ left.

Ask *What is the same about the two equations?*

Listen for The numerators, 5, 4, and 1, are numbers in each equation; the denominators are words in one equation and numbers in the other.

5 Look for the idea that you subtract the numerators and keep the same denominator because the size of the parts does not change when you subtract them.

6 REFLECT Have all students focus on the strategies used to solve this problem. If time allows, have students share their responses with a partner.

CONNECT IT

Now you will use the problem from the previous page to help you understand how to subtract any two fractions that have the same denominator.

1 In **Picture It**, why does $\frac{1}{6}$ represent 1 of the equal parts of the liter?

Possible answer: The denominator tells the number of equal parts the bottle is divided into. The water bottle is divided into 6 equal parts, so one part equals $\frac{1}{6}$.

2 What do the numerators, 5 and 4, tell you?

5 tells the number of parts of the bottle that had water to begin with.
4 tells the number of parts that Alberto drank.

3 How many sixths of a liter are left in the bottle after Alberto drinks 4 sixths? **1 sixth**

4 Complete the equations to show what fraction of a liter is left in the bottle.

Use words: **5 sixths** – **4 sixths** = **1** sixth

Use fractions: $\frac{5}{6} - \frac{4}{6} = \frac{\boxed{1}}{6}$

5 Explain how you subtract fractions with the same denominator.

Possible answer: Subtract the numerator of the amount you take away from the numerator of the starting amount to get the numerator of the answer to find how many parts you have left. The denominator of the answer is the same as the denominator of the other amounts because that tells you what kind of parts you subtracted.

6 REFLECT

Look back at your **Try It**, strategies by classmates, and **Picture It** and **Model It**. Which models or strategies do you like best for subtracting fractions? Explain.

Some students may respond that they like the strategy of using a number line because it helps them see how subtracting fractions is like subtracting whole numbers, while other students may like writing equations because it helps them solve the problem using just numbers.

425

Hands-On Activity

Use paper plates to subtract fractions.

If . . . *students are unsure about subtracting fractions,*

Then . . . *use the activity below to provide a concrete model to connect to the visual and symbolic representations.*

Materials For each student: paper plates, markers, scissors

- Distribute a paper plate, markers, and scissors to each student. Model how to divide the plate into 8 equal sections by folding the plate on top of itself three times.
- Direct students to color $\frac{5}{8}$ of the plate and then cut out that fraction of the plate. Ask students to name the fraction of the plate they have. $\boxed{\frac{5}{8}}$
- Tell students to subtract 2 eighths from the 5 eighths. Guide students to cut 2 sections from the colored portion of the plate they are holding.
- Ask students to name the fraction of the plate they are left with. $\boxed{\frac{3}{8}}$
- Write $\frac{5}{8} - \frac{2}{8} = \frac{3}{8}$ on the board.
- If time allows, repeat for other subtraction problems, such as $\frac{7}{8} - \frac{3}{8}$ and $\frac{3}{4} - \frac{1}{4}$.

APPLY IT

For all problems, encourage students to draw some kind of model to support their thinking. Allow some leeway in precision; drawing fractional parts accurately is difficult, and here precise measurements are not necessary.

7 $\frac{3}{10}$ of the lawn; $\frac{8}{10} - \frac{5}{10} = \frac{3}{10}$; Students may also show $\frac{8}{10}$ in an area model and cross out 5 tenths.

8 $\frac{1}{4}$ of the carton; $\frac{3}{4} - \frac{2}{4} = \frac{1}{4}$; Students may show $\frac{3}{4}$ on a number line divided into fourths and count back 2 fourths. They may also show $\frac{3}{4}$ in an area model and cross out 2 fourths.

Close: Exit Ticket

9 **C**; $\frac{8}{8} - \frac{4}{8} = \frac{4}{8}$

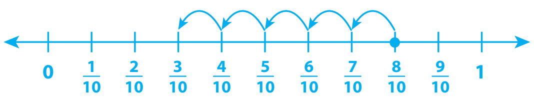
Error Alert **If** students choose A, B, or D, **then** have them identify what information is missing from the problem (the whole). Review the meaning of *denominator* (the number of equal parts in a whole) and have students explain how that relates to the missing information by writing the whole as a fraction with a denominator of 8.

APPLY IT

Use what you just learned to solve these problems.

7 Carmen has $\frac{8}{10}$ of the lawn left to mow. She mows $\frac{5}{10}$ of the lawn. Now what fraction of the lawn is left to mow? Show your work.

Possible student work using a number line:



Solution $\frac{3}{10}$ of the lawn

8 Mrs. Kirk has $\frac{3}{4}$ of a carton of eggs. She uses some for baking and has $\frac{2}{4}$ of the carton left. What fraction of the carton does she use? Show your work.

Possible student work using an equation:

$\frac{3}{4} - \frac{2}{4} = \frac{1}{4}$

Solution $\frac{1}{4}$ of the carton

9 Badru reads $\frac{4}{8}$ of a book. How much of the book does he have left to read?

Ⓐ $\frac{1}{8}$

Ⓑ $\frac{2}{8}$

Ⓒ $\frac{4}{8}$

Ⓓ $\frac{6}{8}$

Solutions

1. The number line should be divided into 10 equal sections and each tick mark labeled as tenths, as shown on the student page.
Basic

2. Arrows should start at $\frac{8}{10}$ and jump left 3 times to $\frac{5}{10}$ and then 5 times to 0, as shown on the student page.
Medium

3. $\frac{5}{10}$ of a mile
Medium

4. $\frac{8}{10} - \frac{3}{10} = \frac{5}{10}$; Students may use an addition equation, $\frac{5}{10} + \frac{3}{10} = \frac{8}{10}$.
Challenge

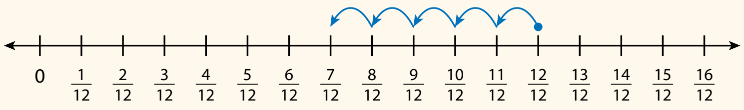
Name: ______________________ LESSON 20 SESSION 3

Practice Subtracting Fractions

Study the Example showing one way to subtract fractions. Then solve problems 1–7.

EXAMPLE

Ali buys a carton of eggs. He uses $\frac{3}{12}$ of the eggs to cook breakfast. He uses another $\frac{2}{12}$ to make a dessert for dinner. What fraction of the carton is left?



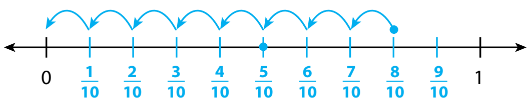
$\frac{12}{12} - \frac{3}{12} = \frac{9}{12}$

$\frac{9}{12} - \frac{2}{12} = \frac{7}{12}$

So, $\frac{7}{12}$ of the carton is left.

Keisha is at her friend's house. Her friend's house is $\frac{8}{10}$ of a mile from Keisha's home. Keisha walks $\frac{3}{10}$ of a mile toward home. Then her mother drives her the rest of the way home.

1. Divide the number line below to show tenths. Then label each tick mark.



2. Use arrows to show the problem on the number line you labeled in problem 1.

3. How far does Keisha's mother drive her? $\frac{5}{10}$ **of a mile**

4. Write an equation for this problem that includes your answer.
$\frac{5}{10} + \frac{3}{10} = \frac{8}{10}$ **or** $\frac{8}{10} - \frac{3}{10} = \frac{5}{10}$

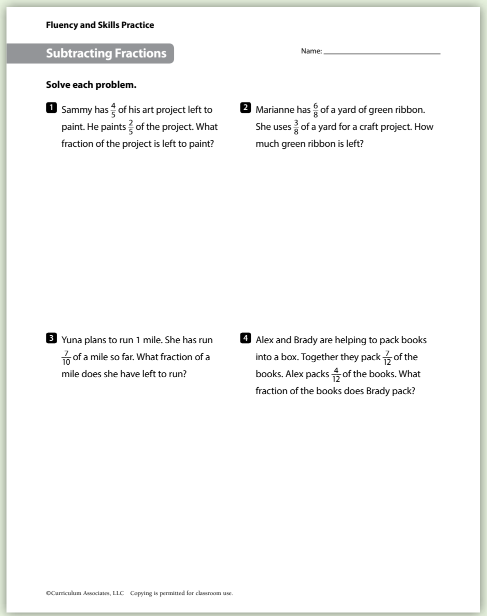
427

Fluency & Skills Practice

Teacher Toolbox

Assign *Subtracting Fractions*

In this activity students practice subtracting fractions with like denominators to solve word problems. Students may solve similar real-world problems involving the subtraction of fractions with the same denominator. Students could use visual models such as number lines or area models to solve the problems. They may also write equations.


Fluency and Skills Practice

Subtracting Fractions

Name: ______________

Solve each problem.

1. Sammy has $\frac{4}{5}$ of his art project left to paint. He paints $\frac{2}{5}$ of the project. What fraction of the project is left to paint?

2. Marianne has $\frac{6}{8}$ of a yard of green ribbon. She uses $\frac{3}{8}$ of a yard for a craft project. How much green ribbon is left?

3. Yuna plans to run 1 mile. She has run $\frac{7}{10}$ of a mile so far. What fraction of a mile does she have left to run?

4. Alex and Brady are helping to pack books into a box. Together they pack $\frac{7}{12}$ of the books. Alex packs $\frac{4}{12}$ of the books. What fraction of the books does Brady pack?

©Curriculum Associates, LLC Copying is permitted for classroom use.

5 $\frac{1}{6}$ of the quilt is white; $\frac{2}{6} + \frac{3}{6} = \frac{5}{6}$; $\frac{6}{6} - \frac{5}{6} = \frac{1}{6}$

Medium

6 $\frac{1}{8}$; Models should show $\frac{9}{8} - \frac{8}{8} = \frac{1}{8}$; See student page for a number line model. Area models should be divided into eighths and have 9 parts shaded, and 8 parts crossed out.

Basic

7 She eats $\frac{2}{6}$ of the pizza; $\frac{6}{6} - \frac{4}{6} = \frac{2}{6}$

Challenge

5 Anna makes a quilt by sewing together green, white, and yellow fabric. When she finishes, $\frac{2}{6}$ of the quilt is green, and $\frac{3}{6}$ is yellow. The rest is white. What fraction of the quilt is white? Show your work.

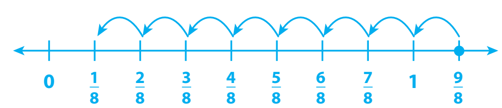
Student work may include an area model, number line, or equations, and should show that the whole quilt is $\frac{6}{6}$.

Solution **$\frac{1}{6}$ of the quilt is white.**

6 Find $\frac{9}{8} - \frac{8}{8}$.

Use a number line or an area model to show your thinking.

Possible student work:

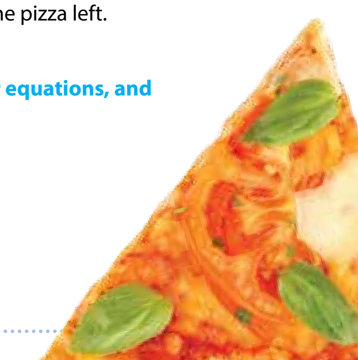


Solution **$\frac{1}{8}$**

7 Shanice has 1 whole pizza. She eats some of it and has $\frac{4}{6}$ of the pizza left. What fraction of the pizza does she eat? Show your work.

Student work may include an area model, number line, or equations, and should show that the whole pizza is $\frac{6}{6}$.

Solution **She eats $\frac{2}{6}$ of the pizza.**



ELL English Language Learners: Differentiated Instruction

Prepare for Session 4
Use with *Connect It*.

Levels 1–3

Listening/Speaking Use with ***Connect It*** problem 2. Point to the first equation for the second ***Model It***. Ask: *What do you notice about the denominators?* Provide a sentence frame for student responses: *The fractions have the same* denominator. Point to the numerators. Say: *1 + 1 + 1 + 1 + 1 = 5. What do you notice when the numerators are added?* Provide a sentence frame for student responses: *The* numerators *have a sum of 5.* Ask students to look at the remaining equations and tell what they notice about the denominators and numerators.

Levels 2–4

Speaking/Writing Use with ***Connect It*** problem 2. Ask students to look at the equations for the second ***Model It*** and then answer the following discussion questions:

- *What do you notice about the denominators?*
- *What do you notice about the numerators when you add them?*

If students respond with incomplete sentences, restate their responses in complete sentences. For example, *All the denominators are the same, 6.* Have students reread ***Connect It*** problem 2. Have them verbally respond to the question before writing their responses.

Levels 3–5

Speaking/Writing Have partners read ***Connect It*** problem 2. Provide the following discussion questions for partners to use as they review the equations for the second ***Model It***:

- *What do you observe about the equations?*
- *What do the equations have in common?*
- *How can you decompose the fraction $\frac{5}{6}$ into sums of different fractions?*

Have students reread ***Connect It*** problem 2. Encourage them to verbally respond before writing their responses. Provide the following terms for them to use in their responses: *fractions, numerator, denominator, add,* and *sum.*

LESSON 20
SESSION 4 Develop

Purpose In this session, students solve a problem that requires them to decompose a fraction into a sum of fractions in more than one way. Students model the fractions either on paper or with manipulatives to show different ways to break apart a fraction. The purpose of this problem is to have students develop strategies for decomposing fractions.

Start

Connect to Prior Knowledge

Materials For each student: 1 set of fraction tiles or fraction circles

Why Support students' facility with adding fractions.

How Have students use fraction tiles or fraction circles to find sums of two eighths fractions.

1. $\frac{1}{8} + \frac{1}{8} =$ ....
2. $\frac{1}{8} + \frac{3}{8} =$ ....
3. $\frac{1}{8} + \frac{5}{8} =$ ....

Solutions

1. $\frac{2}{8}$
2. $\frac{4}{8}$
3. $\frac{6}{8}$

Develop Language

Why Develop understanding of the term *justify*.

How Explain that to justify an answer to a math problem, students need to prove that it is correct or reasonable. Say: *When you decompose fractions, you can use a diagram to justify your answer*. Ask students to review the *Try It, Model It,* and *Connect It* activities to find examples of when they have justified an answer using a diagram or model.

TRY IT

Make Sense of the Problem

To support students in making sense of the problem, have them identify what the problem is asking them to do.

Ask *What fraction describes the amount of reading Dan has left to complete? What are the days of the week on which Dan could complete his reading? On how many of those days does he plan to complete his reading?*

LESSON 20 SESSION 4 ● ● ● ● ○

Develop Decomposing Fractions

Read and try to solve the problem below.

Dan has $\frac{5}{6}$ of his reading left to complete for the week. He plans to complete his reading on two or more days of the week from Monday to Friday. What are two different ways he could plan to complete his reading? Use a fraction to describe the part of his reading he does each day.

TRY IT

Possible student work:

Sample A:

M	T	W	Th	F
$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$
$\frac{1}{6}$	$\frac{2}{6}$	$\frac{2}{6}$	0	0

$\frac{1}{6}$ on each day Monday to Friday; $\frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{5}{6}$

$\frac{1}{6}$ on Monday, $\frac{2}{6}$ on Tuesday, and $\frac{2}{6}$ on Wednesday; $\frac{1}{6} + \frac{2}{6} + \frac{2}{6} = \frac{5}{6}$

Sample B:

$\frac{5}{6} = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6}$

$\frac{5}{6} = \frac{2}{6} + \frac{3}{6}$

Dan could do $\frac{1}{6}$ of his reading on each day Monday to Friday.

Dan could do $\frac{2}{6}$ of his reading on Monday and $\frac{3}{6}$ of his reading on Tuesday.

Math Toolkit
- counters
- fraction circles
- fraction tiles
- fraction bars
- number lines
- fraction models

DISCUSS IT

Ask your partner: Do you agree with me? Why or why not?

Tell your partner: I do not understand how . . .

429

DISCUSS IT

Support Partner Discussion

Encourage students to share what did not work for them as well as what did as they talk to each other.

Support as needed with questions such as:

- *How did you get started?*
- *How did you decide what strategy to use?*

Common Misconception Look for students who list fractions that do not have a sum of $\frac{5}{6}$. Have students use fraction tiles to show different ways to make a sum of $\frac{5}{6}$.

Select and Sequence Student Solutions

One possible order for whole class discussion:

- physical models showing two ways to decompose $\frac{5}{6}$
- drawings showing two ways to decompose $\frac{5}{6}$
- equations with two or more addends that make a sum of $\frac{5}{6}$

Support Whole Class Discussion

Compare and connect the different representations and have students identify how they are related.

Ask *Where does your model show the part of his reading that Dan does each day? Where does your model show the part of his reading that Dan completes for the week?*

Listen for Students should recognize that accurate responses include fractions with a denominator of 6 and representations with 6 equal parts. Students' responses should also include two or more addends that have a sum of $\frac{5}{6}$.

MODEL ITS

If no student presented these models, connect them to the student models by pointing out the ways they each represent:

- the whole
- the $\frac{5}{6}$ of his reading that Dan has left to complete
- different ways to break apart $\frac{5}{6}$

Ask *What does it mean to decompose a fraction? How do the models show the parts that $\frac{5}{6}$ is decomposed into?*

Listen for *Decompose* means to break a fraction into parts. The models show fractions with denominators of 6 that have a sum of $\frac{5}{6}$.

For area models, prompt students to identify how the models are labeled to represent the problem.

- *How are the two models alike? How are they different?*
- *How is $\frac{5}{6}$ shown in each model?*
- *How are the parts shown in each model?*

For equations, prompt students to recognize the strategy used to generate the list of equations.

- *What is the same about each of the equations? What is different?*
- *What do you notice about the numerators of the fractions? the denominators?*
- *What pattern do you notice in how the equations are listed?*

Explore different ways to understand decomposing fractions.

Dan has $\frac{5}{6}$ of his reading left to complete for the week. He plans to complete his reading on two or more days of the week from Monday to Friday. What are two different ways he could plan to complete his reading? Use a fraction to describe the part of his reading he does each day.

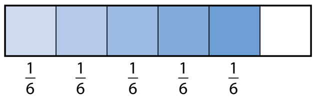
MODEL IT

You can use models to show how to decompose a fraction in different ways.

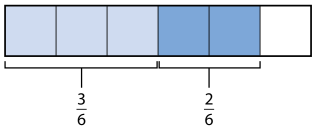
When you decompose a fraction, you break it into parts.

The models show two ways to decompose $\frac{5}{6}$.

One way:



Another way:



MODEL IT

You can also use equations to decompose a fraction in different ways.

You can list different ways to add fractions to make $\frac{5}{6}$.

$\frac{5}{6} = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6}$

$\frac{5}{6} = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{2}{6}$

$\frac{5}{6} = \frac{1}{6} + \frac{2}{6} + \frac{2}{6}$

$\frac{5}{6} = \frac{3}{6} + \frac{1}{6} + \frac{1}{6}$

$\frac{5}{6} = \frac{3}{6} + \frac{2}{6}$

$\frac{5}{6} = \frac{4}{6} + \frac{1}{6}$

Deepen Understanding

Equation Model

SMP 7 Use structure.

When discussing the second *Model It*, prompt students to consider how the first equation is the sum of unit fractions.

Ask *How is the first equation different from the other equations?*

Listen for It has the most addends, and all the addends are $\frac{1}{6}$.

Ask *How would you describe the addends in the first equation?*

Listen for All the addends are unit fractions.

Ask *If you want to add fractions to make $\frac{4}{6}$ instead of $\frac{5}{6}$, how could the first equation help you?*

Listen for $\frac{4}{6}$ could be written as a sum of 4 of these unit fractions.

Generalize *For any fraction with a numerator greater than 1, what is one way you can always decompose the fraction?* Have students explain their reasoning. Listen for understanding that a fraction with a numerator greater than 1 can always be decomposed into a sum of unit fractions with the same denominator.

CONNECT IT

- Remind students that one thing that is alike about all the representations is the numbers.
- Explain that on this page, students will use those numbers to consider general methods for decomposing a fraction into two or more parts.

Monitor and Confirm

1 – 4 Check for understanding that:

- 6 is the number of equal parts and 5 is the number of parts left
- the numerators of the addends have a sum of 5
- $\frac{4}{6}$ is the greatest amount in one day because reading is done on two or more days
- $\frac{5}{6}$ can be decomposed in different ways

Support Whole Class Discussion

5 Look for the idea that finding all the ways to make the numerator helps find all the ways to decompose a fraction.

Deepen Understanding

Decompose a Fraction

SMP 2 Reason abstractly.

To support discussion of problem 5, prompt students to consider how decomposing whole numbers can help them decompose fractions.

Ask *Look at the second* Model It. *How does the first equation show a way to make 5? How does the second equation show a way to make 5?*

Listen for In the first equation, the sum of the numerators is $1 + 1 + 1 + 1 + 1 = 5$ and, in the second, it is $1 + 1 + 1 + 2 = 5$.

Generalize *How does finding all the ways to make the numerator of a fraction help you find all the ways to decompose a fraction?* Have students explain their reasoning. Listen for understanding that when the denominators are all the same, the number of ways to make the numerator is the same as the number of ways to make the fraction.

6 REFLECT

Have all students focus on the strategies used to solve this problem. If time allows, have students share their responses with a partner.

CONNECT IT

Now you will use the problem from the previous page to help you understand how to decompose a fraction in different ways.

1. Look at the first **Model It**. How many equal parts are in each model? 6
   How many shaded parts are in each model? 5

2. Look at the equations in the second **Model It**. How can you tell if two or more fractions add to make $\frac{5}{6}$?
   Possible answer: All the fractions are sixths. If the numerators of the fractions have a sum of 5, then the fractions add to make $\frac{5}{6}$.

3. What is the greatest amount of his reading that Dan could do in one day?
   Dan could do $\frac{4}{6}$ of his reading.

4. What are two different ways that Dan could do his reading?
   Possible answer: Dan could do $\frac{1}{6}$ of his reading on each of five days. Dan could also do $\frac{3}{6}$ of his reading on one day and $\frac{2}{6}$ on another day.

5. Explain how to find all the different ways to decompose a fraction.
   Possible answer: Find all the combinations of numbers as numerators that added together equal the numerator of the fraction.

6. REFLECT
   Look back at your **Try It**, strategies by classmates, and **Model Its**. Which models or strategies do you like best for decomposing a fraction? Explain.
   Students may respond that they like drawing an area model to help them visualize ways to decompose a fraction. Other students may respond that they like making a list of equations to keep track of ways to decompose a fraction.

431

Hands-On Activity

Use fraction tiles to decompose fractions.

If . . . *students are unsure about breaking a fraction into parts,*

Then . . . *use the activity below to provide a more concrete experience.*

Materials For each pair: 1 set of fraction tiles or fraction circles

- Distribute fraction tiles or fraction circles to each pair.
- Have one student build $\frac{4}{5}$ using 4 one-fifth fraction tiles or circles. Then have the student record the relationship shown as an equation: $\frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} = \frac{4}{5}$.
- Have the partner break apart the fraction tiles in a different way and record the relationship $\left(\text{e.g., } \frac{2}{5} + \frac{2}{5} = \frac{4}{5}\right)$.
- Challenge pairs to find other ways to break apart $\frac{4}{5}$.
- Have students switch roles and repeat the activity for another fraction, such as $\frac{7}{10}$. Make sure students start by building the fraction with unit fraction tiles.

APPLY IT

For all problems, encourage students to draw some kind of model to support their thinking. Allow some leeway in the precision of students' models as drawing equal parts accurately is difficult and here precise drawings are not necessary.

7 Answers will vary. Check that the numerators of the addends have a sum of 7. Possible answer: $\frac{7}{8} = \frac{6}{8} + \frac{1}{8}$; $\frac{7}{8} = \frac{4}{8} + \frac{2}{8} + \frac{1}{8}$; $\frac{7}{8} = \frac{3}{8} + \frac{3}{8} + \frac{1}{8}$

8 **a.** $\frac{1}{4} + \frac{1}{4} + \frac{3}{4} = \frac{5}{4}$

b. $\frac{3}{4} = \frac{1}{4} + \frac{2}{4}$

c. $\frac{9}{12} = \frac{3}{12} + \frac{3}{12} + \frac{3}{12}$

Close: Exit Ticket

9 See possible diagram on the Student Worktext page. Check that students' diagrams show $\frac{1}{4} + \frac{2}{4} = \frac{3}{4}$.

Students' solutions should indicate understanding of:

- how to divide a whole into equal parts
- using a model to show the addition of fractions that refer to the same whole

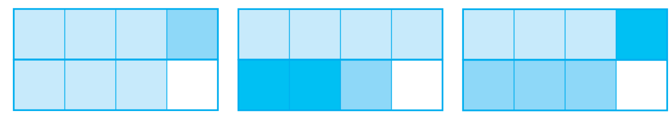
Error Alert **If** students cannot make a visual model to represent $\frac{1}{4} + \frac{2}{4} = \frac{3}{4}$, **then** have students use fraction tiles or fraction circles to model the equation and then make a sketch of the concrete model, labeling each part of the model.

APPLY IT

Use what you just learned to solve these problems.

7 Find three ways to decompose $\frac{7}{8}$ into a sum of other fractions. Draw a model for each way to show how you know the way is correct. Show your work.

Possible student work:



Solution **Possible answer:** $\frac{7}{8} = \frac{6}{8} + \frac{1}{8}$; $\frac{7}{8} = \frac{4}{8} + \frac{2}{8} + \frac{1}{8}$; $\frac{7}{8} = \frac{3}{8} + \frac{3}{8} + \frac{1}{8}$

8 Complete the equations to show a way to decompose each fraction.

a. $\frac{1}{4} + \frac{1}{4} + \frac{3}{4} = \frac{5}{4}$

b. $\frac{3}{4} = \frac{1}{4} + \frac{2}{4}$

c. $\frac{9}{12} = \frac{3}{12} + \frac{3}{12} + \frac{3}{12}$

9 Draw a diagram to justify your answer to problem 8b.

Possible student work:

$\frac{1}{4} + \frac{2}{4} = \frac{3}{4}$

432

Additional Practice

Solutions

1 **a.** $\frac{3}{5} = \frac{1}{5} + \frac{2}{5}$

b. $\frac{3}{5} = \frac{1}{5} + \frac{1}{5} + \frac{1}{5}$

Basic

2 See student page for completed model. The model should show $\frac{1}{5}$ shaded one way and $\frac{2}{5}$ shaded in another way.

Basic

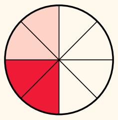
Name: ______________________

Practice Decomposing Fractions

Study the Example showing how to decompose a fraction in different ways. Then solve problems 1–5.

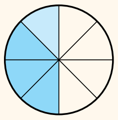
EXAMPLE

Sarah's family has $\frac{4}{8}$ of a cherry pie left over. Sarah and her sister share the leftover pie. What are two different ways that Sarah and her sister can each get some of the pie?



$\frac{2}{8} + \frac{2}{8} = \frac{4}{8}$

Sarah and her sister each get $\frac{2}{8}$ of the pie.



$\frac{1}{8} + \frac{3}{8} = \frac{4}{8}$

Sarah gets $\frac{1}{8}$ of the pie, and her sister gets $\frac{3}{8}$ of the pie.

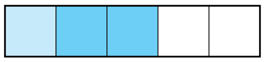
1 Complete the equations to show how to decompose $\frac{3}{5}$ in two different ways.

a. $\frac{3}{5} = \frac{1}{5} +$ $\frac{2}{5}$

b. $\frac{3}{5} = \frac{1}{5} +$ $\frac{1}{5}$ $+ \frac{1}{5}$

2 Shade the area model below to show the equation in problem 1a.

Possible shading shown.

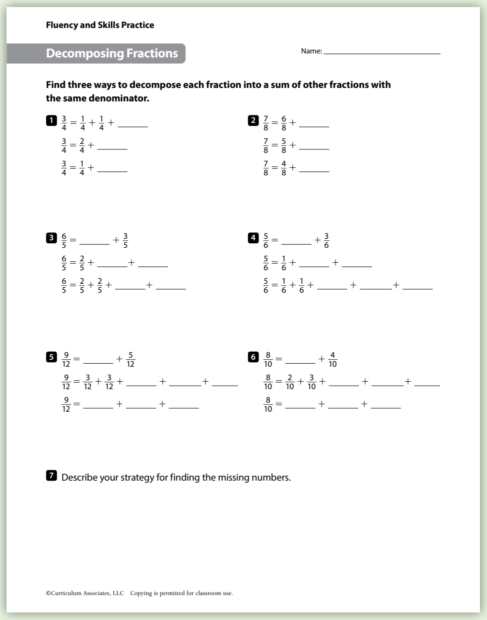


Fluency & Skills Practice

Teacher Toolbox

Assign *Decomposing Fractions*

In this activity students practice decomposing fractions into a sum of fractions with like denominators. Students may encounter real-world problems that involve decomposing a fraction into a sum of fractions. For example, if you want to find a way to put $\frac{7}{8}$ of a cup of trail mix into two snack bags, one approach is to decompose $\frac{7}{8}$ into $\frac{3}{8}$ and $\frac{4}{8}$, so you put $\frac{3}{8}$ of a cup into one bag and $\frac{4}{8}$ of a cup into the other bag.



Fluency and Skills Practice

Decomposing Fractions

Name: ______________________

Find three ways to decompose each fraction into a sum of other fractions with the same denominator.

1 $\frac{3}{4} = \frac{1}{4} + \frac{1}{4} +$ ____

$\frac{3}{4} = \frac{2}{4} +$ ____

$\frac{3}{4} = \frac{1}{4} +$ ____

2 $\frac{7}{8} = \frac{6}{8} +$ ____

$\frac{7}{8} = \frac{5}{8} +$ ____

$\frac{7}{8} = \frac{4}{8} +$ ____

3 $\frac{6}{5} =$ ____ $+ \frac{3}{5}$

$\frac{6}{5} = \frac{2}{5} +$ ____ $+$ ____

$\frac{6}{5} = \frac{2}{5} + \frac{2}{5} +$ ____ $+$ ____

4 $\frac{5}{6} =$ ____ $+ \frac{3}{6}$

$\frac{5}{6} = \frac{1}{6} +$ ____ $+$ ____

$\frac{5}{6} = \frac{1}{6} + \frac{1}{6} +$ ____ $+$ ____ $+$ ____

5 $\frac{9}{12} =$ ____ $+ \frac{5}{12}$

$\frac{9}{12} = \frac{3}{12} + \frac{3}{12} +$ ____ $+$ ____ $+$ ____

$\frac{9}{12} =$ ____ $+$ ____ $+$ ____

6 $\frac{8}{10} =$ ____ $+ \frac{4}{10}$

$\frac{8}{10} = \frac{2}{10} + \frac{3}{10} +$ ____ $+$ ____ $+$ ____

$\frac{8}{10} =$ ____ $+$ ____ $+$ ____

7 Describe your strategy for finding the missing numbers.

3 **B**; The equation is true.

C; The equation is true.

E; The equation is true.

Medium

4 Answers will vary. Possible answers: $\frac{1}{6} + \frac{1}{6} + \frac{4}{6}$, $\frac{1}{6} + \frac{2}{6} + \frac{3}{6}$, or $\frac{2}{6} + \frac{2}{6} + \frac{2}{6}$. See possible models on the student page.

Medium

5 Yes; Possible explanation: $\frac{7}{12} + \frac{1}{12} = \frac{8}{12}$ and $\frac{4}{12} + \frac{4}{12} = \frac{8}{12}$. Since both expressions have a value of $\frac{8}{12}$, $\frac{7}{12} + \frac{1}{12} = \frac{4}{12} + \frac{4}{12}$. Students may also draw models to represent each expression and show that the models represent the same part of the whole.

Challenge

3 Select all the equations that show a correct way to represent $\frac{7}{10}$.

Ⓐ $\frac{1}{10} + \frac{5}{10} = \frac{7}{10}$

Ⓑ $\frac{2}{10} + \frac{5}{10} = \frac{7}{10}$

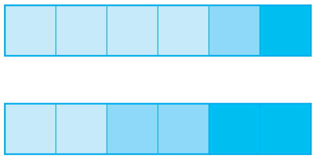
Ⓒ $\frac{1}{10} + \frac{2}{10} + \frac{4}{10} = \frac{7}{10}$

Ⓓ $\frac{1}{10} + \frac{4}{10} + \frac{3}{10} = \frac{7}{10}$

Ⓔ $\frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} = \frac{7}{10}$

4 Vijay has $\frac{6}{6}$ of a cup of raisins. He wants to put the raisins into three snack bags. What are two different ways he could put raisins into three snack bags? Use a model to show each way. Show your work.

Possible student work:



Solution **Possible answer: One way: He could put $\frac{4}{6}$ of a cup of raisins in one bag, $\frac{1}{6}$ of a cup in the second bag, and $\frac{1}{6}$ of a cup in the third bag. Another way: He could put $\frac{2}{6}$ of a cup of raisins in each snack bag: $\frac{2}{6} + \frac{2}{6} + \frac{2}{6} = \frac{6}{6}$.**

5 Is $\frac{7}{12} + \frac{1}{12}$ equivalent to $\frac{4}{12} + \frac{4}{12}$? Explain your answer.

Yes. Possible explanation: $\frac{7}{12} + \frac{1}{12} = \frac{8}{12}$ and $\frac{4}{12} + \frac{4}{12} = \frac{8}{12}$. Since both expressions have a value of $\frac{8}{12}$, $\frac{7}{12} + \frac{1}{12}$ is equivalent to $\frac{4}{12} + \frac{4}{12}$.

ELL English Language Learners: Differentiated Instruction

Prepare for Session 5
Use with *Apply It*.

Levels 1–3

Reading/Speaking Pantomime the words that name actions as you read ***Apply It*** problem 1. Write: *Add the numerators* and *Subtract the numerators*. Read the problem with students. Ask them to find the whole number and circle it. Ask: *How can you subtract $\frac{1}{3}$ from 1?* Have students discuss possible solutions with partners. Ask students to say how else they could represent 1 using fractions. Write $1 = \frac{3}{3}$. Say: *This whole number and fraction are equal. They have the same value.* Have students say: *One whole is equal to three thirds.* Ask: *How much of the smoothie does Ruth drink?* Write $\frac{1}{3}$. Have students write an equation to subtract.

Levels 2–4

Speaking/Writing Have students read ***Apply It*** problem 1. Have students retell the problem with partners. As they retell, draw a number line divided into three equal parts and label points $\frac{1}{3}$, $\frac{2}{3}$, and *1*. Ask students how else they could label the 1 on the number line and then cross out the 1 and write $\frac{3}{3}$ under it. Ask: *Why did I rewrite 1 as $\frac{3}{3}$?* Have pairs discuss why these numbers have the same value. Ask: *How can the number line be used to solve the problem? What equation can I write to represent the problem?* Ask students to work with partners to write an equation and solve the problem. Ask: *What steps did you use to solve the problem?* Write the steps with students and have them read the sentences aloud.

Levels 3–5

Reading/Writing Have students read ***Apply It*** problem 1 with their partners and discuss their ideas. Ask guiding questions as you listen to discussions: *Why do you think you need to write the whole number as a fraction? Why do the whole number 1 and the fraction $\frac{3}{3}$ have the same value? How will you write the whole number as a fraction to write an equation? Why do you use subtraction to solve the problem?* When students have solved the problem, ask them to write on strips of paper the steps they followed to solve it. Then have the partner groups shuffle the strips and exchange them with other groups. Ask students to read the strips with their partners and put them in order.

LESSON 20

SESSION 5 Refine

Purpose In this session, students solve word problems involving addition and subtraction of fractions and decomposing fractions and then discuss and confirm their answers with a partner.

Before students begin to work, use their responses to the *Check for Understanding* to determine those who will benefit from additional support.

As students complete the Example and problems 1–3, observe and monitor their reasoning to identify groupings for differentiated instruction.

Start

Check for Understanding

Why Confirm understanding of adding fractions with like denominators.

How Have students find $\frac{4}{10} + \frac{2}{10}$ using any strategy they want.

$\frac{4}{10} + \frac{2}{10} = ?$

Solution

$\frac{6}{10}$

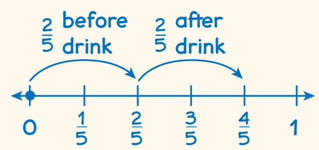
Refine Adding and Subtracting Fractions

Complete the Example below. Then solve problems 1–9.

EXAMPLE

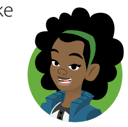
Jessica hikes $\frac{2}{5}$ of a mile on a trail before she stops to get a drink of water. After her drink, Jessica hikes another $\frac{2}{5}$ of a mile. How far does Jessica hike in all?

Look at how you could show your work using a number line.



Solution Jessica hikes $\frac{4}{5}$ of a mile.

The student used labels and "jump" arrows to show each part of the hike on a number line. It is just like adding whole numbers!



PAIR/SHARE
How else could you solve this problem?

APPLY IT

1. Ruth makes 1 fruit smoothie. She drinks $\frac{1}{3}$ of it. What fraction of the fruit smoothie is left? Show your work.

Possible student work using an equation:

$\frac{3}{3} - \frac{1}{3} = \frac{2}{3}$

Solution $\frac{2}{3}$ of the smoothie

What fraction represents the whole fruit smoothie?

PAIR/SHARE
How did you and your partner decide what fraction to start with?

Error Alert

If the error is . . .	Students may . . .	To support understanding . . .
$\frac{6}{20}$	have added both the numerators and the denominators.	Remind students that the denominator tells the kind of parts you are adding. Explain that just as 4 apples + 2 apples = 6 apples, 4 tenths + 2 tenths = 6 tenths.
$\frac{3}{10}$	have added numerators, added denominators, and then written an equivalent fraction with a denominator of 10.	Remind students that the denominator tells the kind of parts you are adding. Explain that just as 4 apples + 2 apples = 6 apples, 4 tenths + 2 tenths = 6 tenths.
$\frac{2}{10}$	have subtracted the fractions.	Remind students to read the problem carefully to be sure they are using the correct operation.
$\frac{1}{5}$	have subtracted the fractions and written an equivalent fraction.	Remind students to read the problem carefully to be sure they are using the correct operation.

EXAMPLE

Jessica hikes $\frac{4}{5}$ of a mile; The number line shown is one way to solve the problem. Students could also solve the problem by drawing a model that is divided into fifths and shading 4 parts (2 parts out of 5 and 2 parts out of 5).

Look for Add the numerators, 2 + 2.

APPLY IT

1. $\frac{2}{3}$ of the smoothie; Students could solve the problem using the equation $\frac{3}{3} - \frac{1}{3} = \frac{2}{3}$.
   DOK 1
   Look for $\frac{3}{3}$ is the fraction representing 1 whole that is hidden in the problem.

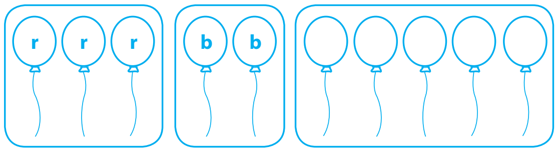
2. $\frac{5}{10}$ of the bunch; Students could solve the problem by drawing a picture of 10 balloons and labeling 3 balloons as red and 2 balloons as blue.
   DOK 2
   Look for The solution requires two steps: addition $\left(\frac{3}{10} + \frac{2}{10}\right)$ and subtraction $\left(\frac{10}{10} - \frac{5}{10}\right)$ or subtracting twice $\left(\frac{10}{10} - \frac{3}{10} - \frac{2}{10}\right)$.

3. **C**; Students could solve this problem using the equation $\frac{1}{6} + \frac{2}{6} = \frac{3}{6}$.
   Explain why the other two answer choices are not correct:
   A is not correct because you are not subtracting $\frac{1}{6}$ from $\frac{2}{6}$; this is an addition problem.
   B is not correct because $\frac{1}{3}$ is not equivalent to $\frac{3}{6}$.
   DOK 3

2. Mr. Chang has a bunch of balloons. $\frac{3}{10}$ of the bunch is red. $\frac{2}{10}$ of the bunch is blue. What fraction of the bunch is not red or blue? Show your work.

I think that there are at least two different steps to solve this problem.

Possible student work using a model:



red	blue	not red or blue
$\frac{3}{10}$	$\frac{2}{10}$	$\frac{5}{10}$

PAIR/SHARE
What other problem in this lesson is similar to this one?

Solution $\frac{5}{10}$ **of the bunch**

3. Emily eats $\frac{1}{6}$ of a bag of carrots. Nick eats $\frac{2}{6}$ of the same bag of carrots. What fraction of the bag of carrots do Emily and Nick eat altogether?

To find the fraction of the bag Emily and Nick ate together, should you add or subtract?

Ⓐ $\frac{1}{6}$

Ⓑ $\frac{1}{3}$

Ⓒ $\frac{3}{6}$

Ⓓ $\frac{3}{12}$

Rob chose Ⓓ as the correct answer. How did he get that answer?

Rob added both the numerators and the denominators.

PAIR/SHARE
Does Rob's answer make sense?

4 C; Add the number of yards Lin uses for the project and the number of yards left, $\frac{5}{8} + \frac{2}{8} = \frac{7}{8}$.
DOK 2

5 C; Find the combined amount of cake eaten, $\frac{2}{12} + \frac{3}{12} = \frac{5}{12}$. Subtract the sum from the whole, $\frac{12}{12} - \frac{5}{12} = \frac{7}{12}$.
DOK 2

Error Alert Students may not recognize this as a two-step problem and either fail to add $\frac{2}{12}$ and $\frac{3}{12}$ before subtracting, or subtract $\frac{2}{12}$ from $\frac{3}{12}$.

6 $\frac{1}{3}$ of a cup more milk than oil; Subtract the amount of oil Lee uses from the amount of milk she uses, $\frac{2}{3} - \frac{1}{3} = \frac{1}{3}$.
DOK 2

4 Lin buys some cloth. He uses $\frac{5}{8}$ of a yard for a school project. He has $\frac{2}{8}$ of a yard left. How much cloth does Lin buy?

Ⓐ $\frac{3}{8}$ of a yard

Ⓑ $\frac{7}{16}$ of a yard

Ⓒ $\frac{7}{8}$ of a yard

Ⓓ $\frac{8}{8}$ of a yard

5 Carmela cuts a cake into 12 equal-sized pieces. She eats $\frac{2}{12}$ of the cake, and her brother eats $\frac{3}{12}$ of the cake. What fraction of the cake is left?

Ⓐ $\frac{1}{12}$

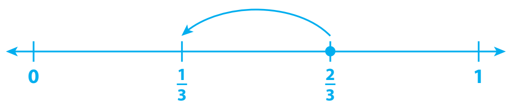
Ⓑ $\frac{5}{12}$

Ⓒ $\frac{7}{12}$

Ⓓ $\frac{12}{12}$

6 Lee makes muffins. She uses $\frac{2}{3}$ of a cup of milk and $\frac{1}{3}$ of a cup of oil. How much more milk than oil does she use? Show your work.

Possible student work:



$\frac{2}{3} - \frac{1}{3} = \frac{1}{3}$

Solution **Lee uses $\frac{1}{3}$ of a cup more milk than oil.**

Differentiated Instruction

RETEACH

Hands-On Activity
Use fraction bars to add.

Students struggling with concepts that fractions written as numbers or shown as visual models represent a part or multiple parts of a whole

Will benefit from additional work with concrete representations of fraction addition and subtraction

Materials For each student: markers, Activity Sheet *Fraction Bars* (2 bars for fourths, 2 bars for thirds, 2 bars for sixths, 2 bars for eighths)

- Distribute fourths fraction bars and markers. Tell students to color $\frac{1}{4}$ of the fraction bar. Then have them color another $\frac{1}{4}$ of the fraction bar.
- Write $\frac{1}{4} + \frac{1}{4}$ on the board. Have students use their fraction bars to show that the sum is $\frac{2}{4}$.
- Then have students color $\frac{3}{4}$ of another fourths fraction bar and cross out $\frac{2}{4}$. Write $\frac{3}{4} - \frac{2}{4}$ and have students show that the difference is $\frac{1}{4}$.
- Repeat for other fractions with denominators such as thirds, sixths, and eighths.

EXTEND

Challenge Activity
Write a problem for a given sum.

Students who achieved proficiency

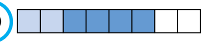
Will benefit from deepening understanding of fraction addition and subtraction

- Tell students that the sum of two fractions is $\frac{2}{5}$. However, the original fractions did not have denominators of 5.
- Challenge students to write a fraction addition problem using denominators other than 5 that has a sum of $\frac{2}{5}$.
[Possible answer: $\frac{3}{10} + \frac{1}{10}$]

7 **A**; The model shows $\frac{2}{8}$ shaded light blue for one girl's section and $\frac{4}{8}$ shaded dark blue for the other girl's section. The total shaded sections represent the total fraction of the room they paint.
D; The equation $\frac{6}{8} = \frac{3}{8} + \frac{3}{8}$ models the problem and shows that each girl could paint $\frac{3}{8}$ of the room.
E; The equation $\frac{6}{8} = \frac{5}{8} + \frac{1}{8}$ models the problem and shows that one girl could paint $\frac{5}{8}$ of the room and the other could paint $\frac{1}{8}$.
DOK 1

8 $\frac{6}{10}$ of a bucket; Possible student work using an equation: $\frac{9}{10} - \frac{3}{10} = \frac{6}{10}$
DOK 2

7 Lucy and Melody work together to paint $\frac{6}{8}$ of a room. Which models could be used to show how much of the room each girl paints?

Ⓐ 

Ⓑ 

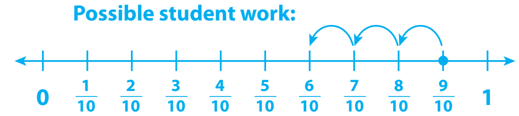
Ⓒ 

Ⓓ $\frac{6}{8} = \frac{3}{8} + \frac{3}{8}$

Ⓔ $\frac{6}{8} = \frac{5}{8} + \frac{1}{8}$

8 Cole and Max pick $\frac{9}{10}$ of a bucket of blueberries in all. Cole picks $\frac{3}{10}$ of a bucket of blueberries. What fraction of a bucket of blueberries does Max pick? Show your work.

Possible student work:



Solution $\frac{6}{10}$ **of a bucket**

9 MATH JOURNAL

Ms. Jones cuts an apple into eighths. She eats $\frac{3}{8}$ of the apple and gives the rest to her son and daughter. Describe two different ways her son and daughter can share the rest of the apple if they each have some of the apple.

Possible answer: The whole apple is $\frac{8}{8}$. $\frac{8}{8} - \frac{3}{8} = \frac{5}{8}$, so Ms. Jones's son and daughter share $\frac{5}{8}$ of the apple. They could share $\frac{5}{8}$ as $\frac{2}{8}$ for one of them and $\frac{3}{8}$ for the other, or as $\frac{1}{8}$ for one of them and $\frac{4}{8}$ for the other.

☑ SELF CHECK Go back to the Unit 4 Opener and see what you can check off.

REINFORCE

Problems 4–9

Add and subtract fractions.

All students will benefit from additional work with adding and subtracting fractions by solving problems in a variety of formats.

- Have students work on their own or with a partner to solve the problems.
- Encourage students to show their work.

PERSONALIZE

i-Ready

Provide students with opportunities to work on their personalized instruction path with *i-Ready* Online Instruction to:

- fill prerequisite gaps
- build up grade-level skills

Close: Exit Ticket

9 MATH JOURNAL

Student responses should indicate an understanding of fraction subtraction as well as an understanding that there is more than one way to decompose a fraction.

Error Alert **If** students decompose $\frac{3}{8}$ rather than $\frac{5}{8}$, **then** have students model the problem using 8 one-eighth fraction tiles or fraction circles to represent the whole apple and then model the amount Ms. Jones eats and different ways to show the amount her son and daughter can each get.

☑ SELF CHECK Have students consider whether they feel they are ready to check off any new skills on the Unit 4 Opener.

Lesson Overview

LESSON 24

Multiply Fractions by Whole Numbers

Lesson Objectives

Content Objectives

- Solve word problems that involve multiplying a fraction by a whole number.

Language Objectives

- Restate word problems involving multiplication of a whole number and a fraction.
- Draw a diagram and write an equation to represent and solve a word problem involving multiplication of a whole number and a fraction.

Prerequisite Skills

- Understand addition as joining parts.
- Understand multiplication as repeated addition.
- Recall basic multiplication facts.
- Use fraction models to add and subtract fractions with like denominators.
- Use fraction models to multiply a fraction by a whole number.
- Write equations.

Standards for Mathematical Practice (SMP)

SMPs 1, 2, 3, 4, 5, and 6 are integrated in every lesson through the *Try-Discuss-Connect* routine.*

In addition, this lesson particularly emphasizes the following SMPs:

5 Use appropriate tools strategically.

7 Look for and make use of structure.

*See page 363m to see how every lesson includes these SMPs.

Lesson Vocabulary

There is no new vocabulary. Review the following key terms.

- **denominator** the number below the line in a fraction that tells the number of equal parts in the whole.
- **fraction** a number that names equal parts of a whole. A fraction names a point on the number line.
- **multiply** to repeatedly add the same number a certain number of times. Used to find the total number of items in equal-sized groups.
- **numerator** the number above the line in a fraction that tells the number of equal parts that are being described.
- **product** the result of multiplication.

Learning Progression

In the previous lesson students drew on their understanding of whole number multiplication, multiplication as repeated addition, and visual fraction models to build conceptual understanding of multiplying a fraction by a whole number.

This lesson builds on the previous lesson as students learn to represent and solve word problems involving the multiplication of a fraction by a whole number. The focus is on understanding the process and the meaning of multiplying a fraction by a whole number.

Students will continue to build and use the skills of parsing a word problem and abstracting real-world situations into mathematical statements as they move forward in their studies of mathematics.

Lesson Pacing Guide

Whole Class Instruction

SESSION 1 **Explore** *45–60 min*	**Multiplying Fractions by Whole Numbers** • Start *5 min* • Try It *10 min* • Discuss It *10 min* • Connect It *15 min* • Close: Exit Ticket *5 min*	**Additional Practice** Lesson pages 507–508
SESSION 2 **Develop** *45–60 min*	**Multiplying Fractions by Whole Numbers** • Start *5 min* • Try It *10 min* • Discuss It *10 min* • Picture It & Model It *5 min* • Connect It *10 min* • Close: Exit Ticket *5 min*	**Additional Practice** Lesson pages 513–514 **Fluency** Multiplying Fractions by Whole Numbers
SESSION 3 **Refine** *45–60 min*	**Multiplying Fractions by Whole Numbers** • Start *5 min* • Example & Problems 1–3 *15 min* • Practice & Small Group Differentiation *20 min* • Close: Exit Ticket *5 min*	**Lesson Quiz** or **Digital Comprehension Check**

Lesson Materials

Lesson *(Required)* — *Per student:* 1 set of fraction tiles

Activities — *Per pair:* base-ten blocks (12 tens rods), 3 sets of fraction circles, 5 sticky notes, 1-month calendar

Math Toolkit — measuring spoons, fraction circles, fraction tiles, fraction bars, number lines, grid paper

Digital Math Tools — Fraction Models, Number Line

Teacher Toolbox

Small Group Differentiation

PREPARE

Ready Prerequisite Lesson

Grade 3
• Lesson 17 Solve One-Step Word Problems Using Multiplication and Division

RETEACH

Tools for Instruction

Grade 3
• Lesson 17 Multiply and Divide to Solve One-Step Word Problems

Grade 4
• Lesson 24 Multiply a Whole Number and a Fraction

REINFORCE

Math Center Activity

Grade 4
• Lesson 24 Fraction Word Problems

EXTEND

Enrichment Activity

Grade 4
• Lesson 24 Fill in the Blanks

i-Ready

Independent Learning

PERSONALIZE

i-Ready Lesson*

Grade 4
• Understand Fraction Multiplication

*We continually update the Interactive Tutorials. Check the Teacher Toolbox for the most up-to-date offerings for this lesson.

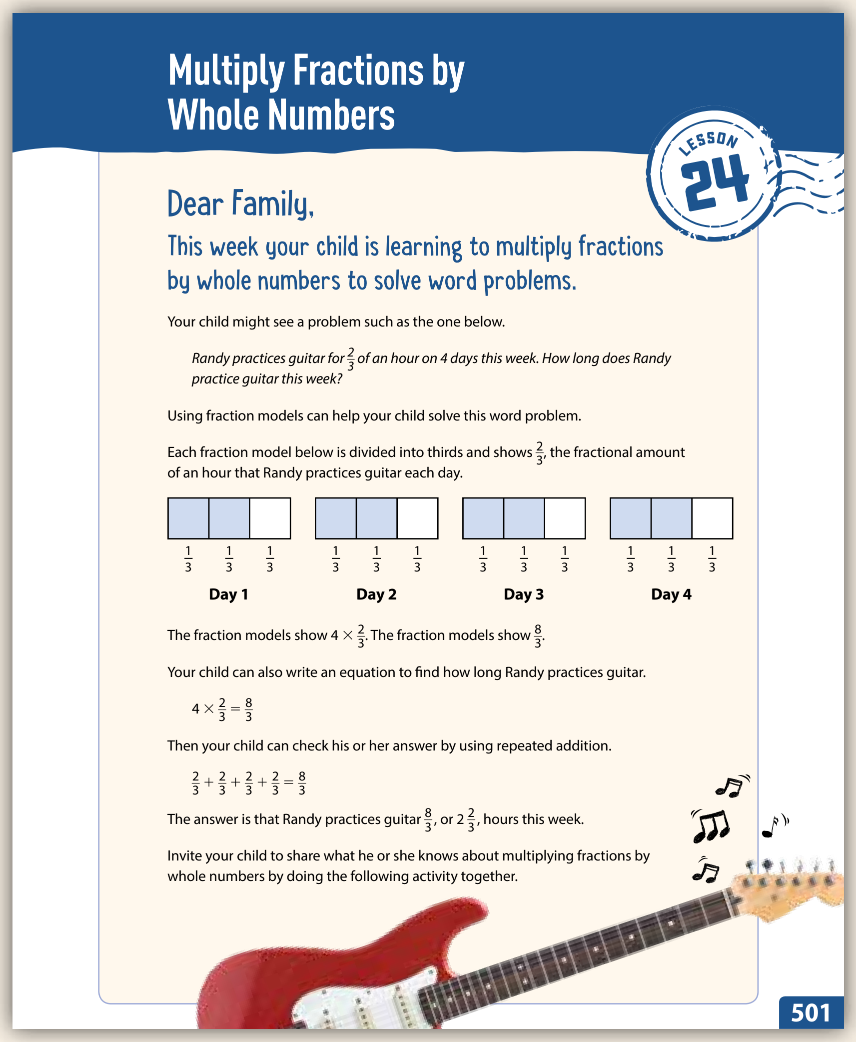
Connect to Family, Community, and Language Development

The following activities and instructional supports provide opportunities to foster school, family, and community involvement and partnerships.

Connect to Family

Use the **Family Letter**—which provides background information, math vocabulary, and an activity—to keep families apprised of what their child is learning and to encourage family involvement.

Available in Spanish
Teacher Toolbox



Multiply Fractions by Whole Numbers

LESSON 24

Dear Family,

This week your child is learning to multiply fractions by whole numbers to solve word problems.

Your child might see a problem such as the one below.

Randy practices guitar for $\frac{2}{3}$ of an hour on 4 days this week. How long does Randy practice guitar this week?

Using fraction models can help your child solve this word problem.

Each fraction model below is divided into thirds and shows $\frac{2}{3}$, the fractional amount of an hour that Randy practices guitar each day.

$\frac{1}{3}$ $\frac{1}{3}$ $\frac{1}{3}$	$\frac{1}{3}$ $\frac{1}{3}$ $\frac{1}{3}$	$\frac{1}{3}$ $\frac{1}{3}$ $\frac{1}{3}$	$\frac{1}{3}$ $\frac{1}{3}$ $\frac{1}{3}$
Day 1	**Day 2**	**Day 3**	**Day 4**

The fraction models show $4 \times \frac{2}{3}$. The fraction models show $\frac{8}{3}$.

Your child can also write an equation to find how long Randy practices guitar.

$$4 \times \frac{2}{3} = \frac{8}{3}$$

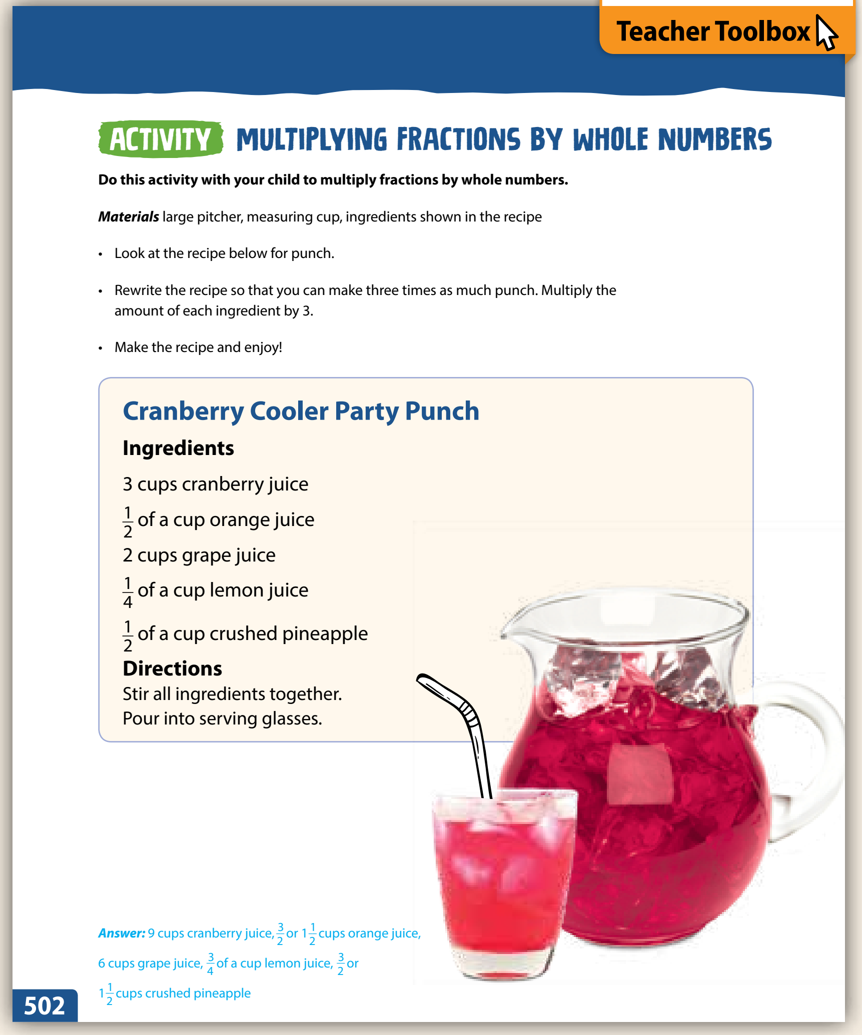
Then your child can check his or her answer by using repeated addition.

$$\frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} = \frac{8}{3}$$

The answer is that Randy practices guitar $\frac{8}{3}$, or $2\frac{2}{3}$, hours this week.

Invite your child to share what he or she knows about multiplying fractions by whole numbers by doing the following activity together.

501



ACTIVITY MULTIPLYING FRACTIONS BY WHOLE NUMBERS

Do this activity with your child to multiply fractions by whole numbers.

Materials large pitcher, measuring cup, ingredients shown in the recipe

- Look at the recipe below for punch.
- Rewrite the recipe so that you can make three times as much punch. Multiply the amount of each ingredient by 3.
- Make the recipe and enjoy!

Cranberry Cooler Party Punch

Ingredients

3 cups cranberry juice
$\frac{1}{2}$ of a cup orange juice
2 cups grape juice
$\frac{1}{4}$ of a cup lemon juice
$\frac{1}{2}$ of a cup crushed pineapple

Directions

Stir all ingredients together.
Pour into serving glasses.

Answer: 9 cups cranberry juice, $\frac{3}{2}$ or $1\frac{1}{2}$ cups orange juice, 6 cups grape juice, $\frac{3}{4}$ of a cup lemon juice, $\frac{3}{2}$ or $1\frac{1}{2}$ cups crushed pineapple

502

Goal

The goal of the Family Letter is to provide opportunities for family members to help students solve word problems that involve multiplying fractions by whole numbers. Students and family members are encouraged to use fraction models to help them solve the problems.

- When students and family members have solved the word problems, they are encouraged to check their answers by using repeated addition, a skill students have previously used to confirm answers to multiplication problems.

Activity

In the *Multiplying Fractions by Whole Numbers* activity, students and family members rewrite a recipe by multiplying the amount of each ingredient by 3. Students are encouraged to make the party punch using the revised recipe. If they do not have the ingredients available, point out that they can use other recipes they are familiar with. Suggest that students rewrite the recipes by multiplying the ingredients by the number of family members.

Math Talk at Home

Encourage students to discuss multiplying fractions by whole numbers in activities they do at home.

Conversation Starters Below are additional conversation starters students can write in their Family Letter or math journal to engage family members:

- *If we walk $\frac{3}{4}$ of a mile to and from school each day, how many miles do we walk to and from school in one week?*
- *If I drink $\frac{1}{2}$ of a cup of water every hour, how many cups of water do I drink in six hours?*

Connect to Community and Cultural Responsiveness

Use these activities to connect with and leverage the diverse backgrounds and experiences of all students.

Session 1 Use with *Try It*.

- Bring a box of crackers to the classroom. Ask students if they know how many crackers are in a serving and where they can find this information on the box. Point to the number of crackers per serving shown on the nutritional label and tell what fraction of the box the serving size is. You may ask students to brainstorm a list of food items they like to eat and note the serving size listed on the nutritional labels. Have students bring a nutritional label from home, or make copies of the nutritional labels students brainstormed. Encourage students to select a label to use for the word problem. Have them find the serving size on the label and help them determine the fraction of the box that the serving size is. Then ask students to multiply the fraction by three to find the amount in three servings.

Session 2 Use with *Additional Practice* problems 6–7.

- Ask students if they know what a flute is and if they have heard or seen a flute. Display a picture of a standard orchestral flute. Then play a recording or video of a musician playing a flute. Explain to students that flutes are popular instruments in several countries around the world. If possible, show pictures of different flutes and play flute recordings. You can share these examples:
  - In China, a traditional wooden flute is called a dizi.
  - In Japan, a shakuhachi is a flute made of bamboo. Instead of being held horizontally, the shakuhachi is held vertically.
  - In the Andes in South America, traditional flutes are also made of bamboo. There are two types of flutes found in this region called the zampoña and tara. The zampoña is made of several sizes of bamboo flutes that are attached together with string. Sound is made by blowing into the different flutes. The tara is a vertical flute that is decorated with painted traditional symbols.
  - Ask students if they play or would like to play a musical instrument. Encourage them to substitute the instruments in the word problem.

Session 3 Use with *Apply It* problem 8.

- To make the word problem relevant to students, find out what sports they are involved in or would like to play. Rewrite the word problem to reflect this interest. For example: *Brittany practices kicking soccer balls for* $\frac{2}{3}$ *of an hour on each of 3 days. For how many hours does she practice kicking soccer balls?*

Connect to Language Development

For ELLs, use the Differentiated Instruction chart to plan and prepare for specific activities in every session.

ELL English Language Learners: Differentiated Instruction

Prepare for Session 1
Use with *Connect It*.

Levels 1–3

Speaking/Reading Use with ***Connect It*** problem 3. Ask questions to help students organize their thoughts:

- *What do you like to do?*
- *How much time do you spend on (sport or activity)?*
- *How can you find out how much time you spend on (sport or activity) in three days?*

Provide sentence frames to help students record responses:

- *I like to ______.*
- *I spend ______ of an hour a day ______.*
- *I can multiply ______ by ______.*

Help students read their responses.

Levels 2–4

Listening/Writing Use with ***Connect It*** problem 3. Ask students to listen as you describe a real-world situation when you multiply a fraction by a whole number. Say: *I like to walk my dog after school. On most days, I spend* $\frac{1}{2}$ *of an hour a day walking my dog. To find out how many hours I walk my dog during the school week, I can multiply* $\frac{1}{2}$ *by 5.* Write the following sentence frames:

- *I like to ______ after school.*
- *Most days I spend ______ of an hour a day ______.*
- *To find out how many hours I ______ during the school week, I can ______.*

Encourage students to describe a real-world situation using the frames.

Levels 3–5

Reading/Writing Use with ***Connect It*** problem 3. Write the following:

- *Think of something you like to do.*
- *Identify how much time you spend doing this activity as a fraction of an hour.*
- *Figure out how you can find the total amount of time you spend on this activity in a week.*

Have students read the information and organize their thoughts. Then ask them to describe a real-world situation in which they would multiply a fraction by a whole number. Have students write their responses and read them to a partner.

LESSON 24

SESSION 1 Explore

Purpose In this session, students draw on their conceptual understanding of multiplying a fraction by a whole number. They share models to explore how various solution methods for a word problem are based on finding the total of a number of equal-sized parts in equal groups. They will look ahead to think about representing a word problem with a multiplication equation in which a fraction is multiplied by a whole number.

Start

Connect to Prior Knowledge

Materials For each student: 1 set of fraction tiles

Why Support students' understanding of multiplying a fraction by a whole number.

How Have students use fraction tiles to find $2 \times \frac{2}{10}$.

$2 \times \frac{2}{10} =$ ....

Solution
$\frac{4}{10}$

TRY IT

Make Sense of the Problem

To support students in making sense of the problem, have them show that they understand that $\frac{3}{10}$ of a box of crackers is one serving of crackers and that Bella eats 3 servings of crackers.

DISCUSS IT

Support Partner Discussion

To reinforce the units of tenths, encourage students to use *tenths* as they talk to each other.

Look for, and prompt as necessary for, understanding of:

- $\frac{3}{10}$ of a box of crackers as one serving
- 3 as the number of servings
- the fraction of the box of crackers that Ella eats as the unknown

LESSON 24 SESSION 1 ● ○ ○

Explore Multiplying Fractions by Whole Numbers

Previously, you learned about multiplying fractions by whole numbers. In this lesson, you will multiply fractions by whole numbers to solve word problems. Use what you know to try to solve the problem below.

Learning Target
- Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem.

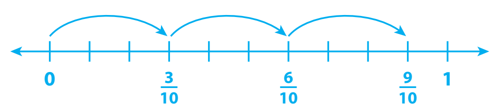
SMP 1, 2, 3, 4, 5, 6, 7

One serving of crackers is about $\frac{3}{10}$ of the whole box of crackers. Bella eats 3 servings this week. What fraction of the box of crackers does she eat?

TRY IT

Possible student work:

Sample A



$\frac{3}{10} + \frac{3}{10} = \frac{6}{10}$ $\frac{6}{10} + \frac{3}{10} = \frac{9}{10}$

Bella eats $\frac{9}{10}$ of the box of crackers.

Sample B

$\frac{3}{10} + \frac{3}{10} + \frac{3}{10} = \frac{9}{10}$

She eats $\frac{9}{10}$ of the box of crackers this week.

Math Toolkit
- fraction circles
- fraction tiles
- fraction bars
- number lines
- grid paper
- fraction models

DISCUSS IT

Ask your partner: Can you explain that again?

Tell your partner: A model I used was . . . It helped me . . .

503

Common Misconception Look for students who add 3 three times instead of adding $\frac{3}{10}$ three times. As students present solutions, have them specify the equal-sized part in each serving and the number of servings.

Select and Sequence Student Solutions

One possible order for whole class discussion:

- physical models of parts that show combining tenths
- drawings or number lines showing repeated addition of tenths
- addition equations showing repeated addition of tenths
- multiplication equations showing tenths multiplied by a whole number

Support Whole Class Discussion

Prompt students to note the relationship between the numbers in each model and the numbers in the problem.

Ask *How do [student name]'s and [student name]'s models show 1 serving of crackers? How do [student name]'s and [student name]'s models show 3 servings of crackers?*

Listen for One serving is $\frac{3}{10}$. Three servings is $\frac{3}{10} + \frac{3}{10} + \frac{3}{10}$, or 3 groups of $\frac{3}{10}$, which is $\frac{9}{10}$. $\frac{3}{10} + \frac{3}{10} + \frac{3}{10}$ is the same as $3 \times \frac{3}{10}$.

CONNECT IT

1 LOOK BACK

Look for understanding that the fraction of the box of crackers that Bella eats can be found by adding or multiplying: $\frac{3}{10} + \frac{3}{10} + \frac{3}{10}$, or $3 \times \frac{3}{10}$.

Hands-On Activity

Use tens rods to multiply a fraction by a whole number.

If . . . *students are unsure about the concept of multiplying a fraction by a whole number,*

Then . . . *use this activity to provide a more concrete experience.*

Materials For each pair: base-ten blocks (12 tens rods)

- Present this problem: One serving of pretzels is $\frac{3}{12}$ of the whole box. Eva eats 3 servings. What fraction of the box of pretzels does Eva eat?
- Distribute tens rods and have pairs display the tens rods in a row with long edges touching. Tell students they are going to model the problem using tens rods. Say: *The 12 rods represent a whole box of pretzels.* Ask: *What does 1 rod represent?* [$\frac{1}{12}$ of a box]
- Say: *One serving of pretzels is $\frac{3}{12}$ of a box.* Ask: *How can you show one serving of pretzels?* [with 3 rods]
- Have partners group the rods to show the 3 servings of pretzels that Eva eats. Ask: *What do the three groups of 3 rods represent?* [3 servings of pretzels] Have students write and solve a multiplication equation to find the fraction of a box of pretzels that Eva eats. [$3 \times \frac{3}{12} = \frac{9}{12}$]

2 LOOK AHEAD

Point out that the product of multiplying a fraction by a whole number may be a fraction less than 1, a fraction equal to 1, or a fraction greater than 1.

Students should be able to find the product of 3 and $\frac{3}{10}$ and recognize that $\frac{9}{10}$ is less than 1.

Ask *How would the answer change if Bella eats 4 servings of crackers instead of 3 servings?*

Listen for She eats $\frac{12}{10}$ of a box, or more than 1 box. She eats $1\frac{2}{10}$ boxes of crackers.

CONNECT IT

1 LOOK BACK

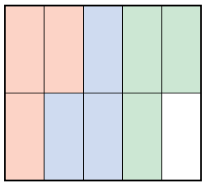
Explain how you could find the fraction of the box of crackers that Bella eats.

Possible explanation: You could multiply $\frac{3}{10}$ by 3 to get $\frac{9}{10}$.

2 LOOK AHEAD

You can multiply a fraction by a whole number to solve problems about combining equal-sized parts.

In the cracker problem, the equal-sized part is the serving size, or $\frac{3}{10}$ of the box of crackers. Bella eats 3 servings. The model at the right shows the fraction of the box of crackers that Bella eats.



$\frac{3}{10}$ of the box = 1 serving

a. You can show three times the serving size as $3 \times \frac{3}{10}$, or $\frac{3 \times 3}{10}$.

Complete the equation. $3 \times \frac{3}{10} = \frac{\boxed{9}}{\boxed{10}}$

b. When you multiply a fraction by a whole number, the answer may be a fraction less than 1 or a fraction greater than 1. You can use what you know about fractions and mixed numbers to tell between which two whole numbers the answer lies. Is the fraction of the box of crackers that Bella eats less than 1 whole or more than 1 whole? **less than 1 whole**

c. Between which two whole numbers is the fraction of the box of crackers that Bella eats? **between 0 and 1**

3 REFLECT

Describe a real situation when you might want to multiply a fraction by a whole number.

Possible answer: I practice the piano $\frac{3}{4}$ of an hour every day. I can multiply this amount of time by 7 to find how long I practice each week.

Close: Exit Ticket

3 REFLECT Look for understanding of a real-world situation in which it makes sense to multiply a fraction by a whole number to find a solution.

Common Misconception **If** students need additional support identifying a real situation that can be represented by multiplying a fraction by a whole number, **then** have them first think of a situation that involves multiplying a whole-number length by a whole number. Then have them change the whole-number length to a fractional length less than 1 and describe how the situation represents multiplying a fraction by a whole number.

Real-World Connection

Discuss with students examples of items that can be weighed or measured in fractional amounts, such as spices or fruit (fraction of a pound) or ribbon (fraction of a foot). Point out that if they had several of the items and wanted to know the total weight, or total length, then they could multiply. If possible, actually weigh or measure several such items. Then ask students to determine what 7 of a given item would weigh or measure.

Solutions

Support Vocabulary Development

1 Ask students to read the terms on the graphic organizer. Have them put checkmarks by the terms that they can define and then share their definitions with partners. If students need support providing definitions, guide them as follows:

- Ask students to draw pictures or provide examples for each term. Remind them they have used number lines and models to multiply whole numbers by whole numbers and fractions by whole numbers. Have students describe their pictures and examples.
- Write a multiplication equation that includes a fraction and a whole number. Label the fraction, multiplication symbol, whole number, and product.
- Provide vocabulary students can use in their definitions: *product, combine, denominator, numerator*, and *equal groups*.

If students need support writing definitions, provide sentence frames or starters.

2 Have students look at the model and think about an equation they can write to represent the model. Ask questions to help students complete the equation:

- *How many fraction models do you see?*
- *Is this number a whole number or a fraction?*
- *What fraction does each model represent?*
- *What is the product when you multiply the whole number and fraction?*

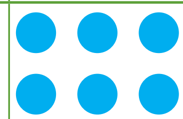
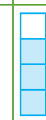
Supplemental Math Vocabulary

- *product*
- *denominator*
- *numerator*

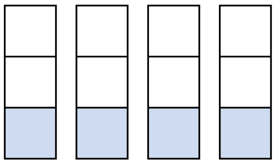
Name: ______________________

Prepare for Multiplying Fractions by Whole Numbers

1 Think about what you know about multiplying a fraction by a whole number. Fill in each box. Use words, numbers, and pictures. Show as many ideas as you can. **Possible answers:**

Word	In My Own Words	Example
multiply	**To combine equal groups to find a total**	 **$2 \times 3 = 6$**
fraction	**A number that names equal parts of a whole**	 **$\frac{3}{4}$**
whole number	**A number that has no fractional part. You use whole numbers for counting.**	 **0, 1, 2, 3, 4, . . .**

2 Complete the equation to tell how the model shows multiplying a fraction by a whole number.



4 × **$\frac{1}{3}$** = **$\frac{4}{3}$**

whole number × fraction = product

3 Assign problem 3 to provide another look at solving a problem by multiplying a fraction by a whole number.

This problem is very similar to the problem about Bella eating a fraction of a box of crackers. In both problems, students are given a word problem in which they must multiply a fraction by a whole number to solve the problem. The question asks what fraction of a box of cereal a family eats in 2 days.

Students may want to use fraction bars, use fraction tiles or circles, or draw models with pencil and paper.

Suggest that students read the problem three times, asking themselves one of the following questions each time:

- *What is this problem about?*
- *What is the question I am trying to answer?*
- *What information is important?*

Solution:

$2 \times \frac{3}{8} = \frac{6}{8}$; The family eats $\frac{6}{8}$ of the box of cereal in 2 days.

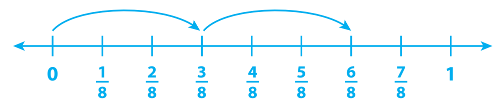
Medium

4 Have students solve the problem another way to check their answer.

3 Solve the problem. Show your work.

A family eats $\frac{3}{8}$ of a whole box of cereal each day. What fraction of the box of cereal does the family eat in 2 days?

Possible student work using a number line:



Solution **The family eats $\frac{6}{8}$ of the box of cereal.**

4 Check your answer. Show your work.

Possible student work:

$\frac{3}{8} + \frac{3}{8} = \frac{6}{8}$

506

ELL English Language Learners: Differentiated Instruction

Prepare for Session 2
Use with *Connect It*.

Levels 1–3

Reading/Speaking Use with ***Connect It*** problem 3. Have students look at the examples in ***Picture It*** and ***Model It***. Ask: *What models or strategies do you see?* [picture, fraction bars] Encourage students to point to examples on the page. Write responses and then read them aloud. Ask: *What do you need to multiply? Which model or strategy do you prefer?* Provide a sentence starter:

I like ______.

Have students work in pairs to complete the equation:

______ batches × ______ teaspoon for each batch = ______ teaspoons

Levels 2–4

Speaking/Writing Use with ***Connect It*** problem 3. Have students identify the models and strategies used for multiplying a fraction by a whole number in ***Picture It*** and ***Model It***. Have students tell the strategy or model they prefer to use to multiply a fraction by a whole number. Provide a sentence frame to record responses: *I prefer to use* ______

because ______.

Have students respond verbally and in writing. Then ask them to complete and describe the equation:

______ batches × ______ teaspoon for each batch = ______ teaspoons

Levels 3–5

Speaking/Writing Have students form pairs and read ***Connect It*** problem 3. Ask partners to brainstorm strategies they can use to multiply a fraction by a whole number and then explain to classmates. Each partner team should discuss advantages and disadvantages for their strategies and record their thoughts in a T-chart. Have students choose their favorite strategy and use it to write and solve the equation. Encourage students to write their equations with descriptions in their journal.

LESSON 24

SESSION 2 Develop

Purpose In this session, students solve a word problem that requires multiplying $\frac{2}{4}$ by 3. Students model the numbers in the problem either on paper or with manipulatives to represent the product. The purpose is to have students develop strategies for solving word problems that involve multiplication of a fraction by a whole number.

Start

Connect to Prior Knowledge

Why Support students' facility with using models to represent multiplying a fraction by a whole number.

How Have students draw a model to show $3 \times \frac{4}{5}$ and write the product.

Draw a model to show $3 \times \frac{4}{5}$. Then write the product.

Solution
Models will vary. Possible models show 3 groups of $\frac{4}{5}$ or 3 jumps of $\frac{4}{5}$ on a number line. The product is $\frac{12}{5}$.

Develop Language

Why Clarify the meaning of the word *batch*.

How Explain that in the context of the problem, *batch* means a group of cookies. Point out that each batch includes several cookies. Ask: *What do three batches refer to?* Provide a sentence starter: *Three batches refer to* _______.

TRY IT

Make Sense of the Problem

To support students in making sense of the problem, have them show that they understand that a *batch* is a quantity of items made at the same time.

Ask *What fraction of a teaspoon of vanilla is used in one batch of cookies? How many batches does James want to make?*

LESSON 24 SESSION 2

Develop Multiplying Fractions by Whole Numbers

Read and try to solve the problem below.

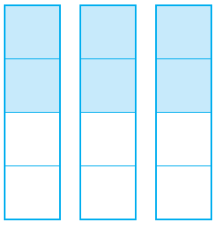
James is baking cookies. One batch of cookies uses $\frac{2}{4}$ of a teaspoon of vanilla. James wants to make 3 batches of cookies. How much vanilla does James need?

TRY IT

Math Toolkit
- fraction circles
- fraction tiles
- measuring spoons
- fraction bars
- number lines
- fraction models

Possible student work:

Sample A



$\frac{2}{4}$ $\frac{2}{4}$ $\frac{2}{4}$

$3 \times \frac{2}{4} = \frac{6}{4}$

James needs $\frac{6}{4}$ teaspoons of vanilla.

Sample B

$\frac{2}{4} + \frac{2}{4} + \frac{2}{4} = \frac{6}{4}$, or $1\frac{2}{4}$

He needs $1\frac{2}{4}$ teaspoons of vanilla.

DISCUSS IT

Ask your partner: How did you get started?

Tell your partner: I knew . . . so I . . .

507

DISCUSS IT

Support Partner Discussion

Encourage students to use the *Discuss It* questions and sentence starters on the Student Worktext page as part of their discussion.

Support as needed with questions such as:
- *Have you solved a problem like this before?*
- *What is another way you could have solved this problem?*

Common Misconception Look for students who use repeated addition and add both the numerators and denominators to get a result of $\frac{6}{12}$ rather than adding only the numerators.

Select and Sequence Student Solutions

One possible order for whole class discussion:
- physical models of parts that show combining fourths
- drawings or number lines showing repeated addition of fourths
- addition equations showing repeated addition of fourths
- multiplication equations representing the product of $\frac{2}{4}$ and 3

Support Whole Class Discussion

Compare and connect the different representations and have students identify how they are related.

Ask *How does your model show the equal-sized part? the number of equal-sized parts?*

Listen for Students should recognize that accurate representations include $\frac{2}{4}$ as the equal-sized part and 3 as the number of equal-sized parts. Representations may include shaded fraction models showing 3 groups of $\frac{2}{4}$, number lines showing 3 jumps of $\frac{2}{4}$, addition equations with 3 addends of $\frac{2}{4}$, and multiplication equations showing $\frac{2}{4}$ multiplied by 3.

PICTURE IT & MODEL IT

If no student presented these models, connect them to the student models by pointing out the ways they each represent:

- the fourths of a teaspoon used in one batch
- the number of batches
- the fourths teaspoons needed in all

Ask *How does each model represent fourths?*

Listen for The picture shows teaspoons, each labeled one fourth. The model shows fraction bars, each divided into four equal-sized parts.

For a picture of teaspoons, prompt students to identify how the picture represents the problem.

- *How is the amount of vanilla needed for one batch of cookies represented?*
- *What do the six teaspoons represent? How do you know?*

For a fraction bar model, prompt students to identify how the model represents the problem.

- *How does the first fraction bar show the amount of vanilla needed for one batch of cookies?*
- *Why are three of these same fraction bars shown?*
- *What does the shading in each fraction bar represent?*
- *What does the total of the shaded parts represent?*

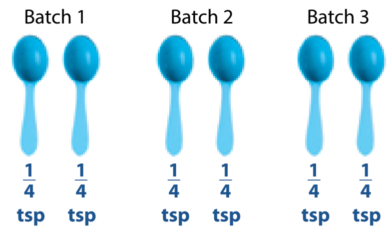
Explore different ways to understand multiplying fractions by whole numbers to solve word problems.

James is baking cookies. One batch of cookies uses $\frac{2}{4}$ of a teaspoon of vanilla. James wants to make 3 batches of cookies. How much vanilla does James need?

PICTURE IT

You can use a picture to help solve the word problem.

The picture shows six $\frac{1}{4}$ teaspoons for 3 batches.



MODEL IT

You can also use fraction bars to solve the word problem.

The fraction bar below is divided into fourths and shows $\frac{2}{4}$, the amount of vanilla in each batch.

$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$

The model below shows the amount of vanilla needed for **3 batches**.

Batch 1				Batch 2				Batch 3			
$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$

Deepen Understanding

Fraction Bar Model

SMP 7 Use structure.

Revisit the fraction bar model and prompt students to think about how a different fraction bar model could be used to represent the product.

Ask *Suppose you drew another fraction bar model to show just the shaded fourths. What would the model look like?*

Listen for The model would have 2 fourths fraction bars: one with all 4 fourths shaded and one with 2 fourths shaded.

Draw on the board the fraction bar model described.

Ask *How many fourths are shaded in each of the fraction bars in the model? What does each fraction bar represent?*

Listen for The first fraction bar has 4 fourths shaded, representing 1 whole. The second fraction bar has 2 fourths shaded, representing $\frac{2}{4}$.

Ask *What fraction and what mixed number does the model represent?*

Listen for It represents the fraction $\frac{6}{4}$ and the mixed number $1\frac{2}{4}$.

CONNECT IT

- Remind students that one thing that is alike about all the representations is the numbers.
- Explain that on this page, students will use those numbers to write a multiplication equation that matches all the representations. They will also use addition to check their answer, write the answer as a mixed number, and find the two whole numbers the answer lies between.

Monitor and Confirm

1 – 4 Check for understanding that:

- $\frac{2}{4}$ of a teaspoon of vanilla is used in each batch
- 3 is the number of batches
- the equation $3 \times \frac{2}{4} = \frac{6}{4}$ represents the problem
- multiplying $\frac{2}{4}$ by 3 is the same as adding $\frac{2}{4}$ three times

Support Whole Class Discussion

5 – 6 Tell students that problems 1 through 4 will prepare them to provide the explanation required in problem 7 and that problems 5 and 6 are about writing the amount of vanilla needed as a mixed number and determining which two whole numbers this amount of vanilla lies between.

Ask *How can you tell that the fraction $\frac{6}{4}$ can be written as a mixed number?*

Listen for The fraction is greater than 1. The numerator is greater than the denominator.

Ask *Why might it be helpful to write the answer as a mixed number instead of a fraction in this problem?*

Listen for It is easier to give the amount of vanilla as a number of whole teaspoons and fourths of a teaspoon when measuring the vanilla.

Note: Writing fractions greater than 1 as mixed numbers becomes useful when students work with decimal numbers greater than 1, in which the whole number is written to the left of the decimal point and the fractional part is written to its right.

7 Look for the idea that both models show three equal groups with equal-sized parts in each group.

8 REFLECT Have all students focus on the strategies used to solve this problem. If time allows, have students share their responses with a partner.

SESSION 2 ● ● ○

CONNECT IT

Now you will use the problem from the previous page to help you understand how to mutiply a fraction by a whole number to solve a word problem.

1. How much vanilla does James need for each batch? $\frac{2}{4}$ of a teaspoon
2. How many batches does James want to make? 3
3. Write an equation to find how many teaspoons of vanilla James needs.

   $3 \times \frac{2}{4} = \frac{6}{4}$

number of batches		teaspoon for 1 batch		teaspoons needed
3	×	$\frac{2}{4}$	=	$\frac{6}{4}$

4. Explain how you can check your answer using repeated addition.

   Possible explanation: You can add. $\frac{2}{4} + \frac{2}{4} + \frac{2}{4} = \frac{6}{4}$. The answer is the same.
5. Write the fraction that shows how many teaspoons of vanilla James needs as a mixed number. $1\frac{2}{4}$
6. Between which two whole numbers of teaspoons is the amount of vanilla James needs?

   between 1 and 2 teaspoons
7. How is the fraction bar model like the teaspoon model in showing how to multiply a fraction by a whole number?

   Possible answer: Both models show three batches, with $\frac{2}{4}$ in each batch. Both models show a total of six fourths.
8. REFLECT

   Look back at your **Try It**, strategies by classmates, and **Picture It** and **Model It**. Which models or strategies do you like best for multiplying a fraction by a whole number to solve a word problem? Explain.

   Possible answer: I like the strategy of using fraction bars because I can show a model that represents the amount needed for each batch and then add those amounts together.

509

Hands-On Activity

Use fraction circles to multiply a fraction by a whole number.

If . . . *students are unsure about finding a product that is a fraction greater than 1 and writing the product as a mixed number,*

Then . . . *use fraction circles to provide a concrete model to connect to the visual and symbolic representations.*

Materials For each pair: 3 sets of fraction circles

- Distribute the fraction circles and present this scenario: Each batch of cookies uses $\frac{3}{4}$ of a cup of sugar. How much sugar is needed for 3 batches of cookies?
- Have students use the fraction circles to represent 3 groups of $\frac{3}{4}$. Ask: *How many fourths do you have in all?* [9 fourths]
- Have students write an equation representing the situation. $\left[3 \times \frac{3}{4} = \frac{9}{4}\right]$
- Have students group the fourths to make as many whole circles as possible. Guide them to write an expression: $\frac{4}{4} + \frac{4}{4} + \frac{1}{4}$ or $1 + 1 + \frac{1}{4}$.
- Ask: *What is the total shown by your model?* $\left[2\frac{1}{4}\right]$ *What does the total represent?* [the number of cups of sugar needed to make 3 batches]

APPLY IT

For all problems, encourage students to draw some kind of model to support their thinking. Allow some leeway in precision; drawing fractional parts accurately is very difficult and here precise drawings are not necessary.

9 $\frac{24}{10}$, or $2\frac{4}{10}$, miles; See possible work on the Student Worktext page. Students may also draw a number line divided into tenths that shows 3 jumps of $\frac{8}{10}$ or write an equation showing repeated addition of $\frac{8}{10}$.

10 $\frac{10}{12}$ of a day; See possible work on the Student Worktext page. Students may also draw a visual model or a number line to show 2 groups of $\frac{5}{12}$.

Close: Exit Ticket

11 **C**; Students multiply $\frac{4}{5}$ by 9: $9 \times \frac{4}{5} = \frac{36}{5}$ or $7\frac{1}{5}$.

Students' solutions should indicate understanding of:

- using multiplication to find the total number of equal-sized parts in equal groups
- $\frac{4}{5}$ as the equal-sized part and 9 as the number of equal groups

Error Alert **If** students choose A or D, **then** provide a visual model to represent $9 \times \frac{4}{5}$ and have them count the number of fifths to find the amount of water the tank holds [36 fifths] and then write the number as a fraction and as a mixed number.

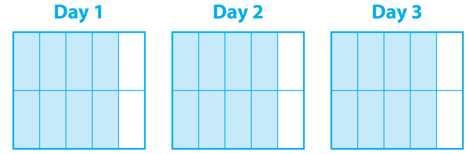
APPLY IT

Use what you just learned to solve these problems.

9 Micah jogs $\frac{8}{10}$ of a mile. Sarah jogs this same distance 3 days in a row. How far does Sarah jog altogether?

Possible student work:

Day 1 Day 2 Day 3



$\frac{8}{10}$ $\frac{8}{10}$ $\frac{8}{10}$

$3 \times \frac{8}{10} = \frac{24}{10}$

Solution $\frac{24}{10}$, or $2\frac{4}{10}$, miles

10 On Monday, Sylvia spends $\frac{5}{12}$ of a day driving to her cousin's house. On Friday, she spends the same amount of time driving home. What fraction of a day does Sylvia spend driving to her cousin's house and back?

Possible student work:

$2 \times \frac{5}{12} = \frac{10}{12}$

Solution $\frac{10}{12}$ of a day

11 Isabella fills her fish tank using a water jug. The water jug holds $\frac{4}{5}$ of a gallon of water. Isabella uses 9 full jugs to fill her fish tank. How many gallons of water does the fish tank hold?

Ⓐ $\frac{36}{45}$ gallons

Ⓑ $2\frac{3}{5}$ gallons

Ⓒ $7\frac{1}{5}$ gallons

Ⓓ $36\frac{1}{5}$ gallons

Solutions

1. $3 \times \frac{5}{6} = \frac{15}{6} = 2\frac{3}{6}$; Benson spends $\frac{15}{6}$ or $2\frac{3}{6}$ hours reading.
   Basic

2. $\frac{5}{6} + \frac{5}{6} + \frac{5}{6} = \frac{15}{6}$
   Basic

3. Katrin rides $\frac{12}{4}$, or 3, miles; Students may write the multiplication equation $4 \times \frac{3}{4} = \frac{12}{4}$.
   Medium

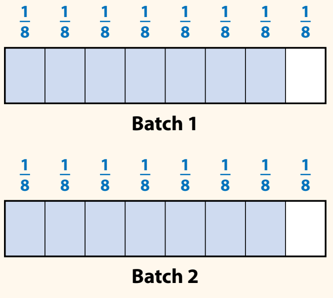
Name: ______________________________ LESSON 24 SESSION 2

Practice Multiplying Fractions by Whole Numbers

Study the Example showing how to multiply a fraction by a whole number to solve a word problem. Then solve problems 1–7.

EXAMPLE

Malik doubles a cookie recipe to make two batches of cookies. He uses $\frac{7}{8}$ of a cup of flour for each batch. How much flour does Malik use for both batches?



$2 \times \frac{7}{8} = \frac{14}{8}$

number of batches	cups per batch	cups used
2	$\frac{7}{8}$	$\frac{14}{8}$

Malik uses $\frac{14}{8}$, or $1\frac{6}{8}$, cups of flour.

1. Benson spends $\frac{5}{6}$ of an hour reading each day for 3 days. How long does Benson spend reading this week?

   $3 \times \frac{5}{6} = \boxed{\frac{15}{6}} = \boxed{2}\boxed{\frac{3}{6}}$

   Benson spends $\frac{15}{6}$ or $2\frac{3}{6}$ hours reading.

2. Show how to use repeated addition to check your answer in problem 1.

   $\frac{5}{6} + \frac{5}{6} + \frac{5}{6} = \frac{15}{6}$

3. Sabrina rides her bike $\frac{3}{4}$ of a mile. Katrin rides her bike this same distance on each of 4 days. How far does Katrin ride her bike altogether?

   $4 \times \frac{3}{4} = \frac{12}{4}$ Katrin rides $\frac{12}{4}$, or 3, miles.

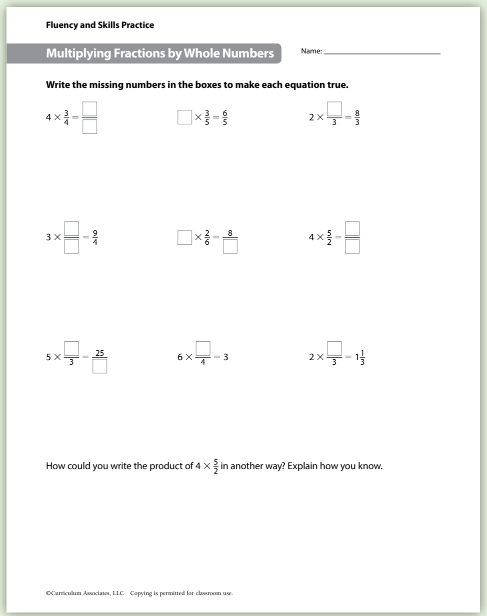
511

Fluency & Skills Practice

Teacher Toolbox

Assign *Multiplying Fractions by Whole Numbers*

In this activity students practice multiplying fractions by whole numbers. Students may apply this skill in real-world situations. For example, students may need to determine how much milk a family drinks in 3 days if they drink $\frac{5}{8}$ of a gallon each day.



Fluency and Skills Practice

Multiplying Fractions by Whole Numbers Name: ____________

Write the missing numbers in the boxes to make each equation true.

$4 \times \frac{3}{4} = \frac{\square}{\square}$ $\square \times \frac{3}{5} = \frac{6}{5}$ $2 \times \frac{\square}{3} = \frac{8}{3}$

$3 \times \frac{\square}{\square} = \frac{9}{4}$ $\square \times \frac{2}{6} = \frac{8}{\square}$ $4 \times \frac{5}{2} = \frac{\square}{\square}$

$5 \times \frac{\square}{3} = \frac{25}{\square}$ $6 \times \frac{\square}{4} = 3$ $2 \times \frac{\square}{3} = 1\frac{1}{3}$

How could you write the product of $4 \times \frac{5}{2}$ in another way? Explain how you know.

©Curriculum Associates, LLC Copying is permitted for classroom use.

4 $\frac{3}{12}$ of the day; Students may write the multiplication equation $3 \times \frac{1}{12} = \frac{3}{12}$.
Medium

5 $\frac{24}{6}$, or 4, bags; Students may write the multiplication equation $12 \times \frac{2}{6} = \frac{24}{6}$.
Medium

6 **A**; The expression represents the sum of two products: the 3 times Leslie practices the flute for $\frac{2}{6}$ of an hour and the 2 times she practices the piano for $\frac{2}{3}$ of an hour.
C; The addition expression represents the sum of the three times Leslie practices the flute for $\frac{2}{6}$ of an hour and the two times she practices the piano for $\frac{2}{3}$ of an hour.
D; The addition expression represents the sum of two products: the 3 times Leslie practices the flute for $\frac{2}{6}$ of an hour and the 2 times she practices the piano for $\frac{2}{3}$ of an hour.
Challenge

7 piano; Students may find $3 \times \frac{2}{6} = \frac{6}{6}$, or 1, hour and $2 \times \frac{2}{3} = \frac{4}{3}$, or $1\frac{1}{3}$, hours and then compare to find $1\frac{1}{3} > 1$.
Challenge

4 Jorge coaches soccer for $\frac{1}{12}$ of the day on Saturday. That day he also coaches tennis and swimming, each for the same amount of time as soccer. What fraction of the day does Jorge coach on Saturday? Show your work.

Possible student work:

$3 \times \frac{1}{12} = \frac{3}{12}$

Solution $\frac{3}{12}$ of the day

5 Greta plants flower seeds in 12 pots. She uses $\frac{2}{6}$ of a bag of flower seeds in each pot. How many bags of flower seeds does Greta use? Show your work.

Possible student work:

$12 \times \frac{2}{6} = \frac{24}{6}$

Solution $\frac{24}{6}$, or 4, bags

Leslie practices the flute for $\frac{2}{6}$ of an hour 3 times this week.
She practices piano for $\frac{2}{3}$ of an hour 2 times this week.

6 Which expressions can be used to show how much time Leslie practices both the flute and piano this week?

Ⓐ $\left(3 \times \frac{2}{6}\right) + \left(2 \times \frac{2}{3}\right)$

Ⓑ $5 \times \left(\frac{2}{6} + \frac{2}{3}\right)$

Ⓒ $\frac{2}{6} + \frac{2}{6} + \frac{2}{6} + \frac{2}{3} + \frac{2}{3}$

Ⓓ $\frac{(3 \times 2)}{6} + \frac{(2 \times 2)}{3}$

Ⓔ $\left(2 \times \frac{2}{6}\right) + \left(3 \times \frac{2}{3}\right)$

7 Which does Leslie practice for a longer amount of time, the flute or the piano? Show your work.

Flute: $3 \times \frac{2}{6} = \frac{6}{6}$, or 1, hour

Piano: $2 \times \frac{2}{3} = \frac{4}{3}$, or $1\frac{1}{3}$, hours

Solution piano

ELL English Language Learners: Differentiated Instruction

Prepare for Session 3
Use with *Apply It*.

Levels 1–3

Listening/Speaking Rewrite and then read ***Apply It*** problem 1 to students: *There are 4 tables. Each table has a bowl with $\frac{5}{8}$ of a pound of grapes. How many pounds of grapes are there altogether?* Draw simple illustrations to support student understanding. Ask: *Do you predict the answer will be greater than or less than 1 pound?* Provide the following sentence frame:

- *I predict the answer is greater than 1 pound.*

Have students share their predictions with partners. Write the equation frame:

_____ × _____ = _____.

Ask: *What whole number will you multiply by? What fraction will you write in the multiplication equation?* Have students solve the equation. Encourage them to confirm their answers with partners.

Levels 2–4

Reading/Speaking Read ***Apply It*** problem 1 with students. Have them retell the problem to partners. Ask: *Do you predict that the answer will be greater than or less than one pound? Why?* Have students share their predictions with partners. Provide the following sentence frame:

- *I predict the answer is greater than 1 pound because _____.*

Have students reread the problem and circle key information. Ask them to work with partners to solve the problem. Encourage discussion with the following question:

- *Was your prediction correct?*

Levels 3–5

Speaking/Writing Have students form pairs and read ***Apply It*** problem 1. Ask pairs to discuss the following: *Do you predict that the answer will be greater than or less than one pound? Why is your prediction reasonable?* When pairs have completed their discussions, ask them to solve the problem. Continue the discussion by asking: *Was your prediction correct? What was your thinking when you made your prediction?* Give each student an index card. Have them make a new word problem or rewrite the word problem by changing key information. Encourage students to exchange cards with other pairs and solve the new word problems.

LESSON 24

SESSION 3 Refine

Purpose In this session, students solve word problems involving multiplication of a fraction by a whole number and then discuss and confirm their answers with a partner.

Before students begin to work, use their responses to the *Check for Understanding* to determine those who will benefit from additional support.

As students complete the Example and problems 1–3, observe and monitor their reasoning to identify groupings for differentiated instruction.

Start

Check for Understanding

Materials For remediation: sticky notes, copy of Start slide

Why Confirm understanding of multiplying a fraction by a whole number.

How Have students draw a visual model and write an equation to solve a word problem that involves multiplying $\frac{2}{3}$ by 8.

Maia makes a path using 8 tiles that are each $\frac{2}{3}$ of a foot long. How long is the path?

Draw a model and write an equation to solve the problem.

Solution
The path is $5\frac{1}{3}$ feet long.
$8 \times \frac{2}{3} = \frac{16}{3}$ or $5\frac{1}{3}$

Look for models with thirds that represent a product of $\frac{16}{3}$.

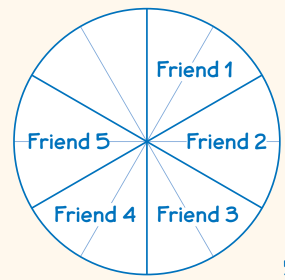
Refine Multiplying Fractions by Whole Numbers

Complete the Example below. Then solve problems 1–8.

EXAMPLE

Five friends share a pizza. Each friend eats $\frac{2}{12}$ of the pizza. How much pizza do they eat altogether?

Look at how you could show your work using a model.



$5 \times \frac{2}{12} = \frac{10}{12}$

Solution The 5 friends eat $\frac{10}{12}$ of a pizza.

The student labeled the model to show each of the 5 friends!

PAIR/SHARE
How could you write the expression in a different way?

APPLY IT

1. Each of 4 tables at a party is set with a bowl of grapes. Each bowl contains $\frac{5}{8}$ of a pound of grapes. How many pounds of grapes are there altogether? Show your work.

Possible student work:

$4 \times \frac{5}{8} = \frac{20}{8}$

Solution There are $\frac{20}{8}$, or $2\frac{4}{8}$ pounds.

Will the total weight be more or less than 1 whole pound?

PAIR/SHARE
Check your answer using repeated addition.

Error Alert

If the error is . . .	Students may . . .	To support understanding . . .
$\frac{10}{3}$	have added 8 to the numerator instead of multiplying.	Give students 8 sticky notes to represent the 8 tiles. Have students write "$\frac{2}{3}$" along the bottom of each and line up the sticky notes. Have students count by twos 8 times to determine how many thirds they have.
$\frac{8}{3}$	have counted the unshaded thirds in their model.	Have students circle the fraction in the problem. Remind students that the numerator tells how many parts you have. Ask students to model 8 groups of $\frac{2}{3}$ and count the shaded thirds to find how many thirds in all.
$\frac{2}{8}$	have drawn a fraction model showing eighths and shaded two of them.	Have students circle the fraction in the problem and draw a fraction model to represent it (a model of thirds with two of the thirds shaded). Then have students draw 8 of the same models and count the shaded parts to find the number of thirds in all.

EXAMPLE

The 5 friends eat $\frac{10}{12}$ of a pizza; The model of a circle that is divided into twelfths and the equation $5 \times \frac{2}{12} = \frac{10}{12}$ is one way to solve the problem. Students could also solve the problem by showing five jumps of $\frac{2}{12}$ on a number line divided into twelfths.

Look for Labeling a visual model to show 5 groups of $\frac{2}{12}$ helps make sense of the problem.

APPLY IT

1 There are $\frac{20}{8}$, or $2\frac{4}{8}$, pounds of grapes; Students could solve the problem using the equation $4 \times \frac{5}{8} = \frac{20}{8}$. They could also solve the problem by showing 4 shaded fraction models, each representing $\frac{5}{8}$.

DOK 2

Look for $\frac{5}{8}$ is close to $\frac{1}{2}$ and 4 halves is 2. The weight of the grapes will be more than 1 whole pound.

2 Leo paints for $\frac{8}{3}$, or $2\frac{2}{3}$, hours; Students may count 4 days and use the multiplication equation $4 \times \frac{2}{3} = \frac{8}{3}$. Students could also solve the problem by using repeated addition: $\frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} = \frac{8}{3}$. Students may write the fraction $\frac{8}{3}$ as the mixed number $2\frac{2}{3}$.

DOK 1

Look for Leo paints for $\frac{2}{3}$ of an hour on each of 4 days, so use multiplication to find the total of a number of equal-sized parts in equal groups.

3 **C**; Students could solve the problem by finding $5 \times \frac{3}{4}$ to get a result of $\frac{15}{4}$ and then writing $\frac{15}{4}$ as the mixed number $3\frac{3}{4}$ to determine that the number of miles is between 3 and 4.

Explain why the other two answer choices are not correct:

B is not correct because 2 miles is $\frac{8}{4}$ miles; the product is greater than this.

D is not correct because 4 miles is $\frac{16}{4}$ miles; the product is less than this.

DOK 3

2 Leo paints for $\frac{2}{3}$ of an hour each day on Monday, Tuesday, Thursday, and Friday. How long does Leo paint this week? Show your work.

Possible student work:

$\frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3}$ or

$4 \times \frac{2}{3} = \frac{8}{3}$

Does Leo paint for the same amount of time each day?

Solution Leo paints for $\frac{8}{3}$, or $2\frac{2}{3}$, hours.

PAIR/SHARE
Draw a model to show the problem situation.

3 Karime walks $\frac{3}{4}$ of a mile each day for 5 days. The number of miles Karime walks altogether is between which two whole numbers?

Ⓐ 0 and 1

Ⓑ 1 and 2

Ⓒ 3 and 4

Ⓓ 4 and 5

Make sure your answer is reasonable!

Lacey chose Ⓐ as the correct answer. How did she get that answer?

Possible answer: Lacey found the two whole numbers that $\frac{3}{4}$ is between instead of finding the two whole numbers that $5 \times \frac{3}{4}$ is between.

PAIR/SHARE
How did you get the answer you chose?

LESSON 24

SESSION 3 Refine

4 **C**; Multiply $\frac{5}{6}$ by 3 to get a result of $\frac{15}{6}$, or $2\frac{3}{6}$, which is between 2 and 3.
DOK 2

5 See completed table on the Student Worktext page.
DOK 1

Error Alert Students may multiply both the numerator and denominator of the fraction by the whole number and get an incorrect product of $\frac{12}{18}$, $\frac{8}{10}$, $\frac{10}{15}$, or $\frac{6}{12}$.

6 Russ buys tomatoes that weigh more; See possible work on the Student Worktext page. Students could also draw models or use number lines to show multiplication of $\frac{1}{4}$ by 6 and $\frac{1}{8}$ by 14 and then compare the products to determine which is greater.
DOK 2

SESSION 3 • • •

4 A choir concert lasts for $\frac{5}{6}$ of an hour. The choir performs 3 concerts on the weekend. Find the number of hours the choir performs on the weekend. The answer is between which two whole numbers?

Ⓐ 0 and 1

Ⓑ 1 and 2

Ⓒ 2 and 3

Ⓓ 3 and 4

5 Find the products to complete the table.

	Product
$3 \times \frac{4}{6}$	$\frac{12}{6}$ or 2
$2 \times \frac{4}{5}$	$\frac{8}{5}$ or $1\frac{3}{5}$
$5 \times \frac{2}{3}$	$\frac{10}{3}$ or $3\frac{1}{3}$
$2 \times \frac{3}{6}$	$\frac{6}{6}$ or 1

6 Morgan buys 6 tomatoes that each weigh $\frac{1}{4}$ of a pound. Russ buys 14 tomatoes that each weigh $\frac{1}{8}$ of a pound. Who buys tomatoes that weigh more? Show your work.

Possible student work:

$6 \times \frac{1}{4} = \frac{6}{4}$, and $14 \times \frac{1}{8} = \frac{14}{8}$.

I know that $\frac{6}{4} = \frac{12}{8}$ and $\frac{14}{8} > \frac{12}{8}$.

Russ buys tomatoes that weigh more.

515

Differentiated Instruction

RETEACH

Hands-On Activity

Model a fraction multiplication word problem as repeated addition.

Students struggling with word problems involving multiplying a fraction by a whole number

Will benefit from additional work with visual representations of multiplication as repeated addition

Materials For each pair: 5 sticky notes, 1-month calendar

- Present the following problem: Jalen bikes a total of $\frac{3}{5}$ of a mile to and from school, Monday through Friday. How far does Jalen bike in one week?
- Have partners circle the school days in one week, Monday through Friday. Have them label the sticky notes "$\frac{3}{5}$" and put them in the boxes on the calendar for the circled days. Ask students what these numbers represent. [the number of miles Jalen bikes each day]
- Have students used repeated addition to find how far Jalen bikes in one week. $\left[\frac{3}{5} + \frac{3}{5} + \frac{3}{5} + \frac{3}{5} + \frac{3}{5} = \frac{15}{5} \text{ miles}\right]$ Then have students write a multiplication equation for the situation. $\left[5 \times \frac{3}{5} = \frac{15}{5}\right]$

EXTEND

Challenge Activity

Write and solve a problem involving multiplication of a mixed number by a whole number.

Students who achieved proficiency

Will benefit from deepening understanding of multiplying a mixed number by a whole number

- Have students write a word problem about multiplication of a mixed number by a whole number.
- Challenge students to solve the problem. [Possible method: Write the mixed number as a fraction and then multiply.] Have students compare results.

7 **A (Yes);**
D (No);
E (Yes)
DOK 1

7 Tell whether each expression has a value of $\frac{15}{4}$.

	Yes	No
$5 \times \frac{3}{4}$	Ⓐ	Ⓑ
$1 \times \frac{5}{4}$	Ⓒ	Ⓓ
$15 \times \frac{1}{4}$	Ⓔ	Ⓕ

8 MATH JOURNAL

Use words, equations, or pictures to explain how to find the answer to the problem below.

Brittany practices hitting softballs for $\frac{2}{3}$ of an hour each day for three days. For how many hours does she practice hitting softballs?

Possible explanation: Multiply the fraction $\frac{2}{3}$ by the whole number 3 to find how many hours Brittany practices hitting softballs:

$3 \times \frac{2}{3} = \frac{6}{3}$, or 2, hours.

SELF CHECK Go back to the Unit 4 Opener and see what you can check off.

REINFORCE

Problems 4–8

Multiply fractions by whole numbers.

All students will benefit from additional work with multiplying fractions by whole numbers by solving problems in a variety of formats.

- Have students work on their own or with a partner to solve the problems.
- Encourage students to show their work.

PERSONALIZE

i-Ready

Provide students with opportunities to work on their personalized instruction path with *i-Ready* Online Instruction to:

- fill prerequisite gaps
- build up grade-level skills

Close: Exit Ticket

8 MATH JOURNAL

Student responses should indicate understanding that this situation can be represented by the multiplication expression $3 \times \frac{2}{3}$ and that the value of the expression, $\frac{6}{3}$, can be written as the whole number 2. Students should be able to interpret the result in the context of the problem and recognize that Brittany practices hitting softballs for 2 hours.

Error Alert **If** students combine the numbers in the problem to get a result of $3\frac{2}{3}$, **then** have them model the problem situation using fraction tiles to show 3 groups of $\frac{2}{3}$ and then write a multiplication equation to represent the total number of thirds.

SELF CHECK Have students consider whether they feel they are ready to check off any new skills on the Unit 4 Opener.