Lesson Overview

LESSON 11

Multiply by One-Digit Numbers

Lesson Objectives

Content Objectives

- Multiply whole numbers of up to four digits by one-digit whole numbers.
- Use arrays, area models, and partial products to multiply.
- Use estimation to determine whether answers are reasonable.

Language Objectives

- Read aloud multiplication problems.
- Draw an array of base-ten blocks to multiply.
- Draw an area model to multiply.
- Write a solution to a multiplication problem using partial products.
- Tell how each part of an array and an area model relates to the factors, partial products, and product of a multiplication problem.

Prerequisite Skills

- Recall basic multiplication facts.
- Know properties of operations.
- Understand place value.
- Understand and use arrays and area models.

Standards for Mathematical Practice (SMP)

SMPs 1, 2, 3, 4, 5, and 6 are integrated in every lesson through the *Try-Discuss-Connect* routine.*

In addition, this lesson particularly emphasizes the following SMPs:

4 Model with mathematics.

5 Use appropriate tools strategically.

7 Look for and make use of structure.

*See page 1i to see how every lesson includes these SMPs.

Lesson Vocabulary

- **partial products** the products you get in each step of the partial-products strategy. You use place value to find partial products. For example, the partial products for 124 × 3 are 3 × 100 or 300, 3 × 20 or 60, and 3 × 4 or 12.

Review the following key terms.

- **estimate (noun)** a close guess made using mathematical thinking.
- **estimate (verb)** to give an approximate number or answer based on mathematical thinking.
- **factor** a number that is multiplied.
- **factors of a number** whole numbers that multiply together to get the given number.
- **multiple** the product of a given number and any other whole number.
- **multiplication** an operation used to find the total number of items in a given number of equal-sized groups.
- **multiply** to repeatedly add the same number a certain number of times. Used to find the total number of items in equal-sized groups.
- **product** the result of multiplication.
- **reasonable** something that makes sense when given facts are taken into account.

Learning Progression

In Grade 3 students used equations, rectangular arrays, and the properties of operations to develop an understanding of multiplication. They learned basic multiplication facts and multiplied one-digit numbers by multiples of 10.

Earlier in Grade 4 students broadened their conceptual understanding of multiplication to include the idea of multiplication as a comparison of two numbers. **In this lesson** students use arrays of base-ten blocks, area models, and partial products to multiply. They apply their understanding of place value to multiply three- and four-digit numbers by one-digit numbers.

In the next lesson students will multiply two-digit numbers by two-digit numbers.

In Grade 5 students will multiply three-digit numbers by two-digit numbers as well as multiply decimals. Students will also become familiar with using the standard algorithm for multiplication.

Lesson Pacing Guide

Whole Class Instruction

SESSION 1
Explore
45–60 min

Multiplying by One-Digit Numbers
- Start *5 min*
- Try It *10 min*
- Discuss It *10 min*
- Connect It *15 min*
- Close: Exit Ticket *5 min*

Additional Practice
Lesson pages 233–234

SESSION 2
Develop
45–60 min

Multiplying a Three-Digit Number by a One-Digit Number
- Start *5 min*
- Try It *10 min*
- Discuss It *10 min*
- Model Its *5 min*
- Connect It *10 min*
- Close: Exit Ticket *5 min*

Additional Practice
Lesson pages 239–240

Fluency
Multiplying a Three-Digit Number by a One-Digit Number

SESSION 3
Develop
45–60 min

Multiplying a Four-Digit Number by a One-Digit Number
- Start *5 min*
- Try It *10 min*
- Discuss It *10 min*
- Picture & Model It *5 min*
- Connect It *10 min*
- Close: Exit Ticket *5 min*

Additional Practice
Lesson pages 245–246

Fluency
Multiplying a Four-Digit Number by a One-Digit Number

SESSION 4
Refine
45–60 min

Multiplying by One-Digit Numbers
- Start *5 min*
- Example & Problems 1–3 *15 min*
- Practice & Small Group Differentiation *20 min*
- Close: Exit Ticket *5 min*

Lesson Quiz or **Digital Comprehension Check**

Lesson Materials

Lesson *(Required)*	none
Activities	*Per pair:* base-ten blocks (8 hundreds flats, 5 tens rods, 12 ones units), play money (25 $1 bills, 25 $10 bills, 25 $100 bills)
Math Toolkit	base-ten blocks, counters, bowls, grid paper, index cards, sticky notes
Digital Math Tools	Base-Ten Blocks, Number Line, Multiplication Models

*We continually update the Interactive Tutorials. Check the Teacher Toolbox for the most up-to-date offerings for this lesson.

Teacher Toolbox

Small Group Differentiation

PREPARE

Ready Prerequisite Lessons
Grade 3
- Lesson 8 Use Order and Grouping to Multiply
- Lesson 9 Use Place Value to Multiply
- Lesson 12 Multiplication and Division Facts

RETEACH

Tools for Instruction
Grade 3
- Lesson 8 Use Order and Grouping to Multiply
- Lesson 9 Use Place Value to Multiply
- Lesson 12 Write Multiplication and Division Facts

Grade 4
- Lesson 11 Multiply by One-Digit Numbers

REINFORCE

Math Center Activity
Grade 4
- Lesson 11 Multiplying by One-Digit Numbers

EXTEND

Enrichment Activity
Grade 4
- Lesson 11 Favorite Photos

i-Ready

Independent Learning

PERSONALIZE

i-Ready Lessons*
Grade 4
- Multiply by One-Digit Numbers, Part 1
- Multiply by One-Digit Numbers, Part 2
- Practice: Multiply by One-Digit Numbers

Learning Games
- Prerequisite: *Match*
- Prerequisite: *Cupcake*
- Prerequisite: *Pizza*

LESSON 11

Connect to Family, Community, and Language Development

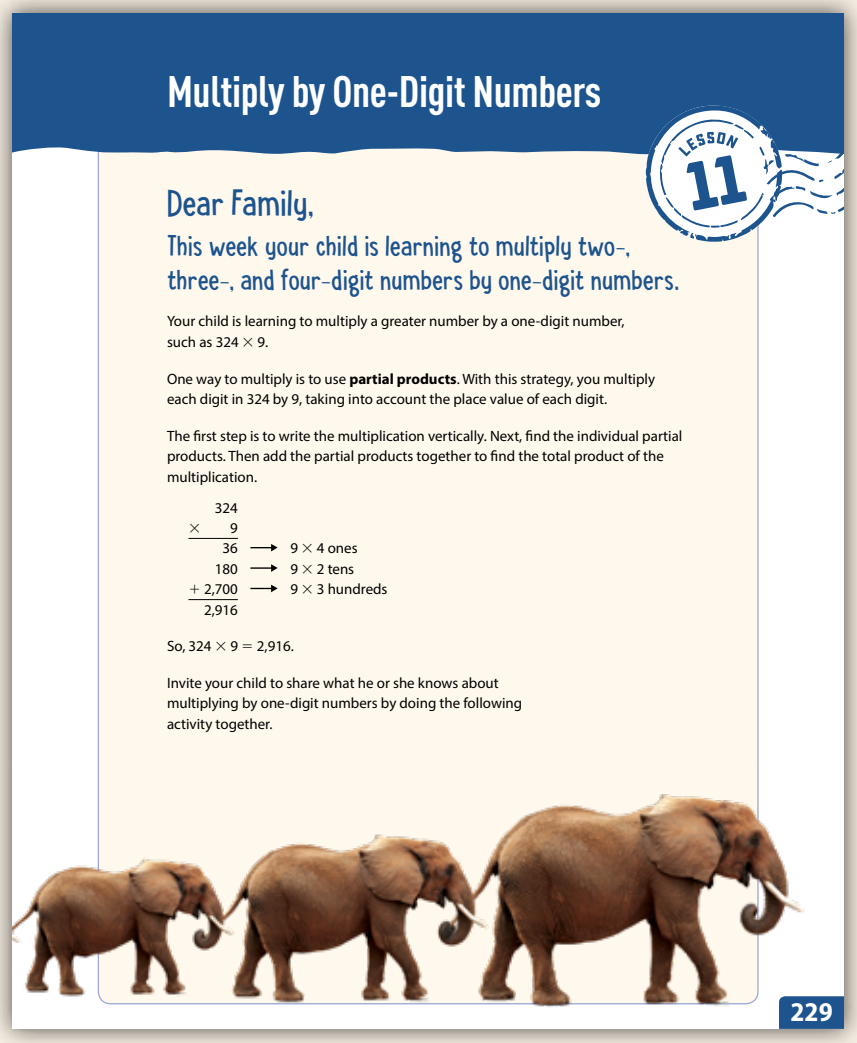
The following activities and instructional supports provide opportunities to foster school, family, and community involvement and partnerships.

Connect to Family

Use the **Family Letter**—which provides background information, math vocabulary, and an activity—to keep families apprised of what their child is learning and to encourage family involvement.

Available in Spanish

Teacher Toolbox



Multiply by One-Digit Numbers

LESSON 11

Dear Family,

This week your child is learning to multiply two-, three-, and four-digit numbers by one-digit numbers.

Your child is learning to multiply a greater number by a one-digit number, such as 324 × 9.

One way to multiply is to use **partial products**. With this strategy, you multiply each digit in 324 by 9, taking into account the place value of each digit.

The first step is to write the multiplication vertically. Next, find the individual partial products. Then add the partial products together to find the total product of the multiplication.

```
   324
 ×   9
  36   → 9 × 4 ones
 180   → 9 × 2 tens
+2,700  → 9 × 3 hundreds
 2,916
```

So, 324 × 9 = 2,916.

Invite your child to share what he or she knows about multiplying by one-digit numbers by doing the following activity together.

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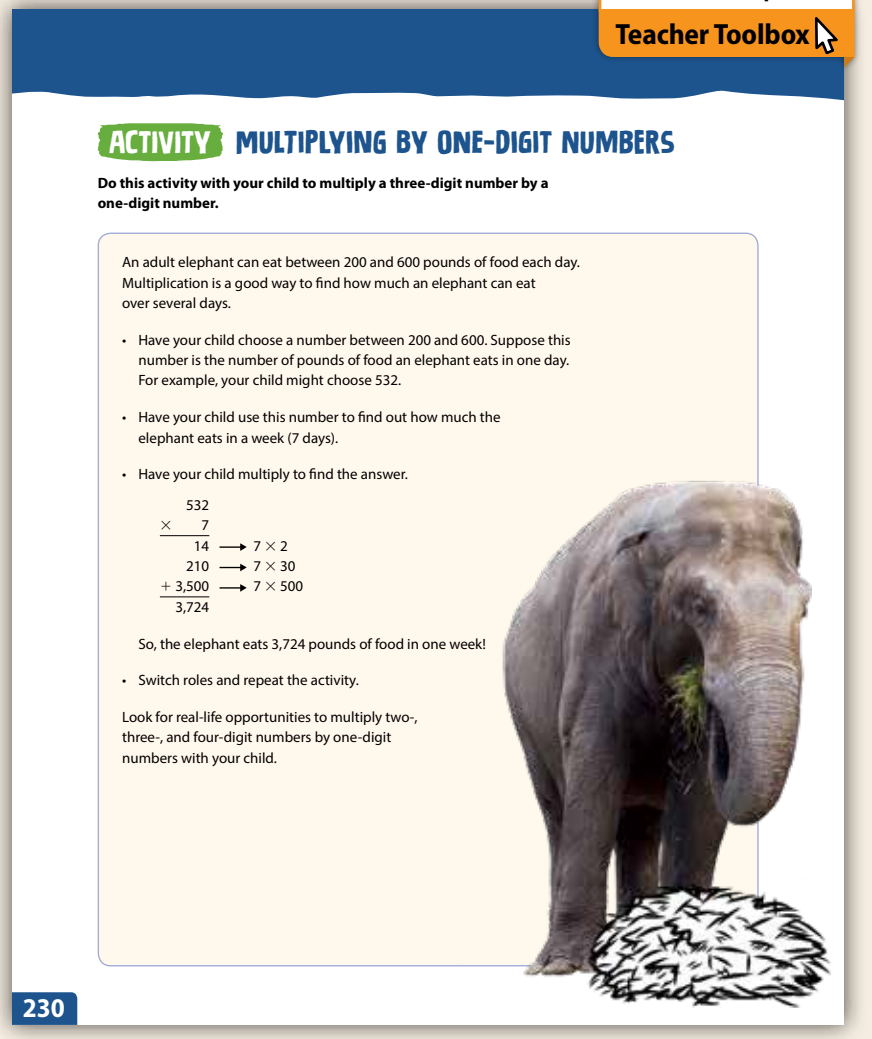

ACTIVITY MULTIPLYING BY ONE-DIGIT NUMBERS

Do this activity with your child to multiply a three-digit number by a one-digit number.

An adult elephant can eat between 200 and 600 pounds of food each day. Multiplication is a good way to find how much an elephant can eat over several days.

- Have your child choose a number between 200 and 600. Suppose this number is the number of pounds of food an elephant eats in one day. For example, your child might choose 532.
- Have your child use this number to find out how much the elephant eats in a week (7 days).
- Have your child multiply to find the answer.

```
   532
 ×   7
  14   → 7 × 2
 210   → 7 × 30
+3,500  → 7 × 500
 3,724
```

So, the elephant eats 3,724 pounds of food in one week!

- Switch roles and repeat the activity.

Look for real-life opportunities to multiply two-, three-, and four-digit numbers by one-digit numbers with your child.

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Goal

The goal of the Family Letter is to encourage students to practice multiplying two- and three-digit numbers by one-digit numbers. Students have opportunities to use their prior knowledge of multiplication using one-digit numbers.

- To carry out the multiplications, students use partial products to find the answer.

Activity

Understanding how to multiply multi-digit numbers by one-digit numbers and finding partial products prepares students for more complex multiplication problems. Look at the *Multiplying by One-Digit Numbers* activity and adjust it if necessary to connect with your students.

Math Talk at Home

Prepare students to have discussions at home with family members by providing time for them to practice the conversation starters.

Conversation Starters Below are additional conversation starters students can write in their Family Letter or math journal to engage family members:

- *If you drive 23 miles a week to and from school, how many miles do you drive in 4 weeks?*
- *If the children in our family each had to buy 225 index cards for school this year, how many index cards would we need to buy?*
- *Can you think of other multiplication problems we could solve?*

Connect to Community and Cultural Responsiveness

Use these activities to connect with and leverage the diverse backgrounds and experiences of all students.

Session 4 **Use with *Example*.**

- Ask students to share information they may know about aquariums. Some students may have aquariums at home, or there may be a school aquarium students have seen. Explain to students that there is another type of aquarium. An aquarium can be like a zoo for animals that live in the water. When people visit these aquariums, they see different kinds of fish and animals, such as turtles or seals, that live in oceans or rivers and depend on bodies of water for shelter and food. Show pictures of different fish and animals that can be seen in an aquarium. If students show an interest in a specific fish or animal found in an aquarium, adapt the word problem using this information to make the problem more relevant to student interests. For example: *An aquarium has seven hammerhead sharks. Each hammerhead shark weighs about 1,385 pounds. If each hammerhead shark weighs 1,385 pounds, how much do they weigh all together?*

Connect to Language Development

For ELLs, use the Differentiated Instruction chart to plan and prepare for specific activities in every session.

ELL English Language Learners: Differentiated Instruction

Prepare for Session 1
Use with *Connect It*.

Levels 1–3

Listening/Speaking Use with ***Connect It*** problem 2. Write $3 \times 157 =$ _____. Ask students to listen as you think through the process to solve the equation:

- *Use place value to write 157 in expanded form.* Write $100 + 50 + 7$.
- *Multiply each number by 3.* Write $(3 \times 100) + (3 \times 50) + (3 \times 7)$.
- *Add the partial products.* Write $300 + 150 + 21 = 471$.
- *$3 \times 157 = 471$* Write the product in the equation.

Have students think through the process aloud. If a student points to information, restate the response and have the student repeat it.

Levels 2–4

Reading Use with ***Connect It*** problem 2. Explain to students that they will solve the equation $3 \times 157 =$ _____ by breaking apart 157. Write the following information on sentence strips.

- *Use place value to write 157 in expanded form.*
- *Multiply each place value by the one-digit number.*
- *Add the partial products.*
- *Write the product in the equation.*

Shuffle and then display the strips. Have students read the information and put the strips in order. Encourage partners to refer to the information as they solve the equation.

Levels 3–5

Writing/Reading Use with ***Connect It*** problem 2. Have students work with partners to develop a plan of action to solve the equation $3 \times 157 =$ _____ by breaking apart 157. You may want to provide a guiding sentence, such as: *Use place value to write 157 in expanded form.* Provide each pair with sentence strips to write their plan of action. When partners have completed the task, have them exchange their sentence strips with another pair. Encourage them to read the information on the sentence strips and put them in order to solve the equation. Have them refer to the plan of action to solve the equation.

LESSON 11

SESSION 1 Explore

Purpose In this session, students draw on their ability to break apart numbers to multiply one-digit numbers by multiples of 10. They share models to explore how to multiply a one-digit number by a two-digit number. They will look ahead to think about breaking apart numbers by place value and using partial products to multiply a three-digit number by a one-digit number.

Start

Connect to Prior Knowledge

Why Support students' facility with multiplying a one-digit number by a multiple of 10 in preparation for multiplying a two-digit number by a one-digit number.

How Have students multiply one-digit numbers by multiples of 10.

Multiply.

1. $4 \times 20 =$ .....
2. $3 \times 60 =$ .....
3. $3 \times 50 =$ .....

Solutions

1. 80
2. 180
3. 150

TRY IT

Make Sense of the Problem

To support students in making sense of the problem, have them show that they understand what the problem is asking them to find.

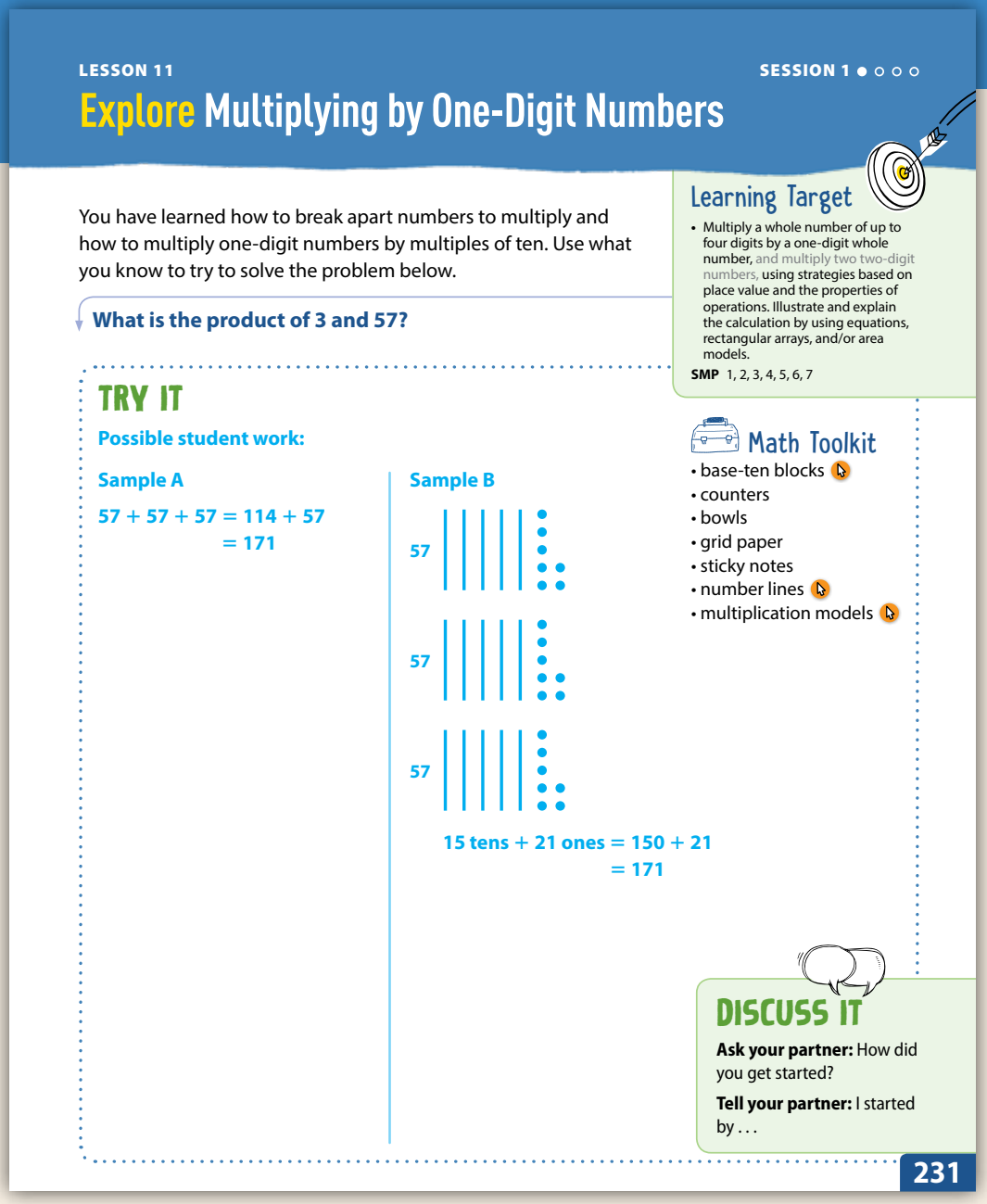
DISCUSS IT

Support Partner Discussion

To reinforce students' prior work with multiplication, encourage students to use the term *product* as appropriate as they talk to each other.

Look for, and prompt as necessary, for understanding that:

- the two-digit number has a ones and a tens place
- you can break apart a number and multiply each part and then add to get the product


LESSON 11

SESSION 1 ● ○ ○ ○

Explore Multiplying by One-Digit Numbers

You have learned how to break apart numbers to multiply and how to multiply one-digit numbers by multiples of ten. Use what you know to try to solve the problem below.

What is the product of 3 and 57?

TRY IT

Possible student work:

Sample A

57 + 57 + 57 = 114 + 57
= 171

Sample B

57

57

57

15 tens + 21 ones = 150 + 21
= 171

Learning Target

- Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

SMP 1, 2, 3, 4, 5, 6, 7

Math Toolkit

- base-ten blocks
- counters
- bowls
- grid paper
- sticky notes
- number lines
- multiplication models

DISCUSS IT

Ask your partner: How did you get started?

Tell your partner: I started by . . .

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Common Misconception Look for students who add 3 and 57 to get 60 rather than multiplying 57 by 3. As students present solutions, have them use the word *product* and explain how they used multiplication.

Select and Sequence Student Solutions

One possible order for whole class discussion:

- base-ten blocks or counters modeling 3 groups of 57
- drawings modeling 3 groups of 57
- equations showing repeated addition
- multiplication equations

Support Whole Class Discussion

Prompt students to note the relationship between the numbers in each model and the numbers in the problem.

Ask *How do [student name]'s and [student name]'s models represent 57 three times?*

Listen for There are three groups of 5 tens and three groups of 7 ones. An addition equation shows adding 57 three times. 57 is broken apart into 5 tens and 7 ones; both the 5 tens and 7 ones are multiplied by 3.

CONNECT IT

1 LOOK BACK

Look for understanding of using repeated addition or of decomposing 57 into numbers easier to multiply by 3.

Visual Model

Use drawings of base-ten blocks to multiply numbers.

If . . . *students are unsure about multiplying a two-digit number by a one-digit number,*

Then . . . *use this activity to show how the numbers can be decomposed to make them easier to work with.*

Use drawings to visually model multiplication.

- Write 2 × 86 on the board. Draw base-ten blocks to show 86: 8 tens rods and 6 ones units. Then draw a second set of blocks showing 86 below the first. Line up the tens with the tens and the ones with the ones.
- Ask: *Why are there 2 sets of blocks showing 86?* [86 is multiplied by 2.]
- Ask: *How many ones are in 86?* [6] *How many ones are there in both sets of blocks?* [12] Explain that you can multiply 6 ones by 2 to find the number of ones.
- Ask: *How many tens are in 86?* [8] *How many tens are there in both sets of blocks?* [16] Explain that you can multiply 8 tens by 2 to find the number of tens. Ask: *What number does 16 tens represent?* [160]
- Ask: *What is the sum of 12 and 160?* [172] Have students use the drawings to confirm the product.
- Repeat for other multiplication problems, such as 3 × 24 and 4 × 32.

2 LOOK AHEAD

Point out that arrays of base-ten blocks, area models, and partial products are ways to break apart numbers to multiply. Students should be able to use the visual array and the expanded form of 157 to find the product of 3 and 157.

Students should recognize that when multiplying a one-digit number by multiples of 10, 100, and 1,000, there is a relationship between the number of zeros in the product and the number of zeros in the factor.

CONNECT IT

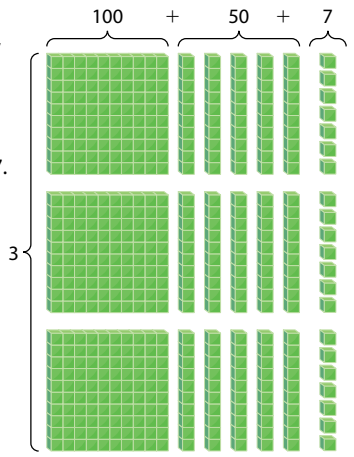
1 LOOK BACK

Explain how you found the product of 3 and 57.

I found the sum of 3 groups of 57: 57 + 57 + 57 = 171.

2 LOOK AHEAD

You can use arrays, area models, and **partial products** to break apart numbers to help you multiply. The array at the right uses base-ten blocks to show 3 × 157.



a. Write 157 in expanded form. **157 = 100 + 50 + 7**

b. Fill in the blanks below to show how to find 3 × 157.

3 × 157 = (3 × **100**) + (3 × **50**) + (3 × **7**)

= **300** + **150** + **21**

= **471**

c. What do you notice about the number of zeros in the product of 3 and 50 and in the product of 3 and 100? How many zeros would be in the product of 3 × 1,000? Explain.

Possible answer: The number of zeros in the products of 3 and 50 and 3 and 100 is the same as the number of zeros in 50 and 100. 3 × 1,000 would have three zeros in its product.

3 REFLECT

How does breaking apart the multiplication problem above by place value help you solve the problem?

Possible answer: Multiplying 3 by the value in each place makes the problem simpler. I multiply each place value and add the results.

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Close: Exit Ticket

3 REFLECT

Look for understanding that you can break apart a number into hundreds, tens, and ones, multiply each by 3, and add the results to find the product.

Common Misconception Look for students who are unclear in their explanations about breaking apart a number by place value to multiply. Have students use base-ten blocks to show 3 groups of 157, each with 1 hundred, 5 tens, and 7 ones, and count to find the number of hundreds, tens, and ones in all.

Real-World Connection

Encourage students to think about everyday situations in which they may need to multiply. Examples include calculating the number of minutes in a given number of hours and calculating the number of pennies, nickels, or dimes in a given number of dollars.

Solutions

Support Vocabulary Development

1 Ask students to brainstorm information they know about the term *product*. Remind students that when *product* is used when discussing multiplication, it refers to the result, or answer, of multiplying two numbers. Encourage students to offer definitions or equation examples of the term *product* and models or strategies they have used to find a product. Ask questions to help stimulate ideas.

- *What does the term* product *mean?*
- *How is a product found?*
- *What symbols are used in an equation with the unknown number being the product?*
- *What model could you use to find a product?*
- *How is the model used to find a product?*
- *How is finding a product like finding a sum?*

Encourage students to use all they know about products in math to complete their concept maps.

2 Have students think aloud the process for finding the product. Ask questions to help them think through the process.

- *What number could you break apart to make the equation easier to solve?*
- *How could you break apart 48 using what you know about place value?*
- *Why would you multiply both numbers you came up with by 2?*
- *Why do you add the partial products you found: 80 and 16?*

Supplemental Math Vocabulary

- *equal sign*
- *multiplication sign*
- *unknown*

Name: _____ LESSON 11 SESSION 1

Prepare for Multiplying by One-Digit Numbers

1 Think about what you know about multiplication. Fill in each box. Use words, numbers, and pictures. Show as many ideas as you can. **Possible answers:**

What Is It?

The result of multiplication

What I Know About It

A product can be found by using repeated addition or by breaking apart numbers by place value to multiply.

product

Examples

$2 \times 62 = ?$

62 62

12 tens + 4 ones = 120 + 4 = 124

Examples

$4 \times 38 = ?$

38 + 38 + 38 + 38 = 76 + 76 = 152

Examples

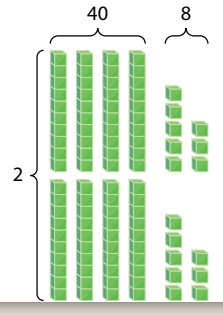
200
× 4
800

2 Fill in the blanks below to show how to find 2×48.

$2 \times 48 = (2 \times \underline{40}) + (2 \times \underline{8})$

$= \underline{80} + \underline{16}$

$= \underline{96}$



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3 Assign problem 3 to provide another look at solving a problem by multiplying a two-digit number by a one-digit number.

This problem is very similar to the problem about finding the product of 3 and 57. In both problems, students are given a multiplication problem with a one-digit factor and a two-digit factor. The question asks students to find the product of 4 and 62.

Students may want to use base-ten blocks.

Suggest that students read the problem three times, asking themselves one of the following questions each time:

- *What is this problem about?*
- *What is the question I am trying to answer?*
- *What information is important?*

Solution:
Students may use various methods to find that $4 \times 62 = 248$.
Basic

4 Have students solve the problem another way to check their answer.

3 Solve the problem. Show your work.

What is the product of 4 and 62?

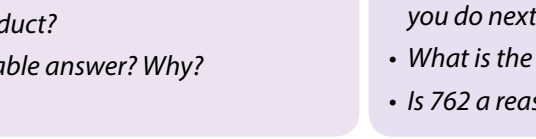
Possible student work using repeated addition:

62 + 62 + 62 + 62 = 124 + 124
= 248

Solution 4 × 62 = 248

4 Check your answer. Show your work.

Possible student work:



24 tens + 8 ones = 240 + 8
= 248

ELL English Language Learners: Differentiated Instruction

Prepare for Session 2
Use with *Connect It*.

Levels 1–3	Levels 2–4	Levels 3–5
Listening/Speaking Read ***Connect It*** problem 5 to students. Circle the terms *estimation* and *reasonable*. Write the definitions (out of order) on the board. Ask volunteers to match the terms to the definitions. Write *3 × 254 = 762*. Help partners organize their thoughts for solving the problem: • *Do you round or break apart for estimating?* • *Which number do you round?* • *Do you round the number to the nearest ten or hundred?* • *What numbers do you multiply?* • *What is the product?* • *What do you think about the answer?*	**Listening/Speaking** Read ***Connect It*** problem 5 with students. Have students circle the terms *estimation* and *reasonable* and then define the terms with partners. Instruct students to listen as you ask questions to help them organize their thoughts. • *What strategy would you use to estimate? Why?* • *Which number would you round? Why?* • *Which place value would you round the number to? Why?* • *When you round to the nearest ten, what do you do next?* • *What is the product?* • *Is 762 a reasonable answer? Why?*	**Reading/Speaking** Have students read ***Connect It*** problem 5. Distribute the questions below. Have a partner read the first question and respond. Have the next partner read the second question and respond. Ask partners to take turns until all questions have been answered. • *What strategy would you use to estimate? Why?* • *Which number would you round? Why?* • *Which place value would you round the number to? Why?* • *When you round to the nearest ten, what do you do next?* • *What is the product?* • *Is 762 a reasonable answer? Why?*

LESSON 11

SESSION 2 Develop

Purpose In this session, students solve a problem that requires them to find the product of a three-digit number and a one-digit number. Students model the multiplication either on paper or with manipulatives to find the product. The purpose of this problem is to have students develop a strategy to multiply three-digit numbers by one-digit numbers.

Start

Connect to Prior Knowledge

Why Support students' ability to multiply a two-digit number by a one-digit number by breaking apart the two-digit number by place value.

How Have students multiply a two-digit number by a one-digit number using the expanded form of the two-digit number and the distributive property of multiplication.

Multiply.

$6 \times 27 = (6 \times \_\_\_\_) + (6 \times \_\_\_\_)$

$= \_\_\_\_ + \_\_\_\_$

$= \_\_\_\_$

Solution

$6 \times 27 =$
$(6 \times 20) + (6 \times 7)$
$120 + 42$
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Develop Language

Why Clarify the meaning of the term *partial* when used to refer to *partial products*.

How Explain to students that the word *partial* means "only a part" or "incomplete." When the term *partial product* is used, it refers to a part of the product which will be put together with other parts to get the final or complete product. Refer to the *Model It* problem. Remind students that when they multiplied 4 by 3, 12 was only a part of the product—the product was incomplete. If time permits, continue the process with 3×50 and 3×200 in the equation.

TRY IT

Make Sense of the Problem

To support students in making sense of the problem, have them identify what they need to find.

Ask *What does the word* product *mean?*

Develop Multiplying a Three-Digit Number by a One-Digit Number

Read and try to solve the problem below.

What is the product of 3 and 254?

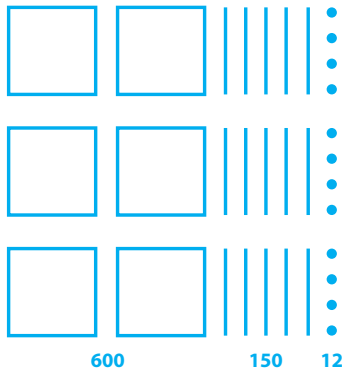
TRY IT

Possible student work:

Sample A

$3 \times 254 = (3 \times 200) + (3 \times 50) + (3 \times 4)$
$= 600 + 150 + 12$
$= 762$

Sample B



$600 + 150 + 12 = 762$
$3 \times 254 = 762$

Math Toolkit
- base-ten blocks
- grid paper
- index cards
- sticky notes
- number lines
- multiplication models

DISCUSS IT

Ask your partner: Can you explain that again?

Tell your partner: I agree with you about . . . because . . .

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DISCUSS IT

Support Partner Discussion

Encourage students to use the term *partial products* as they discuss their solutions.

Support as needed with questions such as:

- *How did you break apart the number 254 to multiply?*
- *How did you show the number of hundreds, tens, and ones in 254?*

Common Misconception Look for students who multiply each digit in 254 by 3 rather than multiplying the value of each digit by 3.

Select and Sequence Student Solutions

One possible order for whole class discussion:

- arrays of base-ten blocks modeling 3 groups of 254
- drawings modeling 3 groups of 254
- equations showing repeated addition
- multiplication equations showing breaking apart numbers by place value

Support Whole Class Discussion

Compare and connect the different representations and have students identify how they are related.

Ask *Where does your model show the number of hundreds, tens, and ones in 254? the multiplication of each place value by 3? the product?*

Listen for Students should recognize that accurate responses include representations showing 2 hundreds, 5 tens, and 4 ones in 254, each added 3 times or multiplied by 3, resulting in a product of 7 hundreds, 6 tens, and 2 ones.

MODEL ITS

If no student presented these models, connect them to the student models by pointing out the ways they each represent:

- the number of hundreds, tens, and ones in 254
- multiplying the value in each place of 254 by 3
- partial products of the expression 3×254

Ask *How do the models show breaking apart 254? using place value to multiply? partial products?*

Listen for The array shows base-ten blocks for 2 hundreds, 5 tens, and 4 ones; the multiplication shows the place values using words. The array shows 3 groups of base-ten blocks; the multiplication shows each place value multiplied by 3. The array and multiplication both show addition of the partial products 600, 150, and 12.

For an array of base-ten blocks, prompt students to identify how multiplication by place value is shown.

- *How is 254 represented?*
- *How is 3 represented?*
- *How are the numbers of hundreds, tens, and ones in the product represented?*

For multiplication using partial products, prompt students to identify how multiplication by place value is shown.

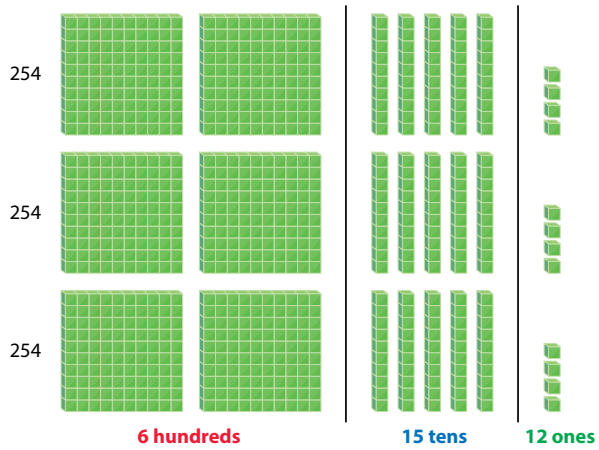
- *How is multiplication of 254 by 3 represented?*
- *How are the partial products represented?*

Explore different ways to understand multiplying a three-digit number by a one-digit number.

What is the product of 3 and 254?

MODEL IT

You can use an array of base-ten blocks to help you multiply.



$3 \times 254 = 600 + 150 + 12$

MODEL IT

You can also multiply using partial products.

$$\begin{array}{r} 254 \\ \times \quad 3 \\ \hline 12 \\ 150 \\ +\ 600 \\ \hline ? \end{array}$$

12 → 3×4 ones
150 → 3×5 tens
600 → 3×2 hundreds

The partial products are 12, 150, and 600.
The product is the sum of the partial products: $12 + 150 + 600$.

Deepen Understanding

Partial Products

SMP 7 Make use of structure.

When discussing partial products, prompt students to see that 3×254 can be written as $3 \times (200 + 50 + 4)$ using the expanded form of 254.

Ask *What idea that you learned can you use to find 3×254?*

Listen for You can multiply each addend, 200, 50, and 4, by 3.

Prompt students to perform the multiplication to find the value of the expression.

Ask *What result do you get when you multiply 200 by 3? when you multiply 50 by 3? when you multiply 4 by 3? Why do you think these results have the same value as the three partial products?*

Listen for You get 600, 150, and 12. When you write 254 in expanded form, you break apart the number by place value: 2 hundreds, 5 tens, 4 ones. When you multiply each by 3, you get the partial products.

CONNECT IT

- Remind students that one thing that is alike about all the representations is the numbers.
- Explain that on this page students will use those numbers to show how both models represent multiplying a three-digit number by a one-digit number by breaking apart numbers and using place value to multiply in order to find a product.

Monitor and Confirm

1–5 Check for understanding that:

- 600 represents 3 × 2 hundreds, 150 represents 3 × 5 tens, and 12 represents 3 × 4 ones
- add hundreds, tens, and ones in the array of base-ten blocks to find the product
- the partial products are 600, 150, and 12
- three partial products are added to find the product
- rounding 254 and multiplying by 3 gives an estimate that can be used to check for reasonableness of the answer

Support Whole Class Discussion

1–4 Tell students that these problems will prepare them to provide the explanation required in problem 6.

Be sure students understand that problem 5 is asking them to estimate to check whether the product is reasonable.

Ask *How is place value used in multiplying 254 by 3?*

Listen for 254 is broken apart by place value into 200 + 50 + 4 and then each addend is multiplied by 3.

6 Look for the idea that both models break apart the greater factor using place value and then multiply each place value by the other factor.

7 REFLECT Have all students focus on the strategies used to solve this problem. If time allows, have students share their responses with a partner.

CONNECT IT

Now you will use the problem from the previous page to help you understand how to multiply three-digit numbers by one-digit numbers.

1 In the first **Model It**, what do the numbers 600, 150, and 12 in the equation below the array represent?

Possible answer: The number 600 represents 3 × 2 hundreds, the number 150 represents 3 × 5 tens, and the number 12 represents 3 × 4 ones.

2 How can you find the product of 3 and 254 in the first **Model It**?

Possible answer: Add the hundreds, tens, and ones. 600 + 150 + 12 = 762

3 Where do you see the 6 hundreds, 15 tens, and 12 ones in the second **Model It**?

Possible answer: The 6 hundreds, 15 tens, and 12 ones are the partial products of 600, 150, and 12.

4 What is the sum of the partial products in the second **Model It**? **762**

5 How can you use estimation to check that your answer is reasonable?
Possible answer: Round 254 to 250. Multiply 250 by 3 to get 750. Since 750 is close to 762, the answer is reasonable.

6 How do both **Model Its** show breaking apart a factor to multiply?

Possible answer: Both models break apart the factor 254 by place value. The first Model It uses place-value blocks and the second Model It uses the place value of each digit in the number.

7 REFLECT

Look back at your **Try It**, strategies by classmates, and **Model Its**. Which models or strategies do you like best for multiplying a three-digit number by a one-digit number? Explain.

Possible answer: Students may respond that they like the strategy of showing an array with place-value blocks or using partial products because it helps them find the number of hundreds, tens, and ones that they can add together to find the final product.

237

Hands-On Activity

Use base-ten blocks to understand multiplying numbers.

If . . . *students are unsure about multiplying a three-digit number by a one-digit number,*

Then . . . *use the activity below to give them experience multiplying a three-digit number by a one-digit number using base-ten blocks.*

Materials For each pair: base-ten blocks (8 hundreds flats, 5 tens rods, 12 ones units)

- Give pairs the multiplication problem 4 × 213.
- The first student represents 213 with the base-ten blocks and replicates the setup to represent 4 groups of 213. The student counts the number of ones in each of the 4 groups as the partner records 4 × 3 ones = 4 × 3. The first student counts the total [12], and the partner records it.
- Partners repeat these steps with the tens rods and the hundreds flats. The resulting partial products are 12, 40, and 800.
- The first student regroups the blocks where possible to show that the product is 852. The partner confirms that 12 + 40 + 800 has a value of 852.
- Repeat the activity for 2 × 323.

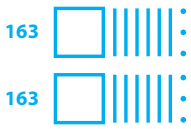
APPLY IT

Use what you just learned to solve these problems.

8 $2 \times 163 = ?$

Show your work.

Possible student work:



200 + 120 + 6 = 326

Solution **326**

9 Find the product of 5 and 738. Estimate to check that your answer is reasonable. Show your work.

Possible student work:

$$\begin{array}{r} 738 \\ \times \quad 5 \\ \hline 40 \\ 150 \\ +\ 3{,}500 \\ \hline 3{,}690 \end{array}$$

Solution **3,690; Possible estimate: 738 rounds to 700. 700 × 5 = 3,500. 3,500 is close to 3,690. My answer is reasonable.**

10 What is the product of 859 and 7? Show your work.

Possible student work:

$$\begin{array}{r} 859 \\ \times \quad 7 \\ \hline 63 \\ 350 \\ +\ 5{,}600 \\ \hline 6{,}013 \end{array}$$

Solution **6,013**

APPLY IT

For all problems, encourage students to draw some kind of model to support their thinking. Allow some leeway in precision; drawing precise representations of base-ten blocks is difficult, and precision is not required here.

8 326; See possible work on the Student Worktext page; Students may also use partial products to multiply.

9 3,690; See possible work on the Student Worktext page. Possible estimate shown on the Student Worktext page uses rounding to check for reasonableness of the product. Students may also draw an array of base-ten blocks and add hundreds, tens, and ones to find the product.
To check for reasonableness, students may also round 738 to the nearest ten and multiply 740 by 5 to get an estimated product of 3,700.

Close: Exit Ticket

10 6,013; See possible work on the Student Worktext page. Students may also use drawings of arrays of base-ten blocks and add hundreds, tens, and ones to find the product.

Students' solutions should indicate understanding of:

- breaking apart numbers by place value to multiply
- multiplying each digit in the three-digit number by the one-digit number
- adding the partial products to find the product

Error Alert **If** students' solution is the sum of $63 + 35 + 56$, **then** have them record the two factors they are multiplying next to each partial product, such as "7 × 9 ones" or "7 × 9", "7 × 5 tens" or "7 × 50," and "7 × 8 hundreds" or "7 × 800."

Solutions

1 See completed partial-product multiplication on the student page.
Basic

2 396; See possible work on the student page. Students may also use partial products to multiply and find the product.
Medium

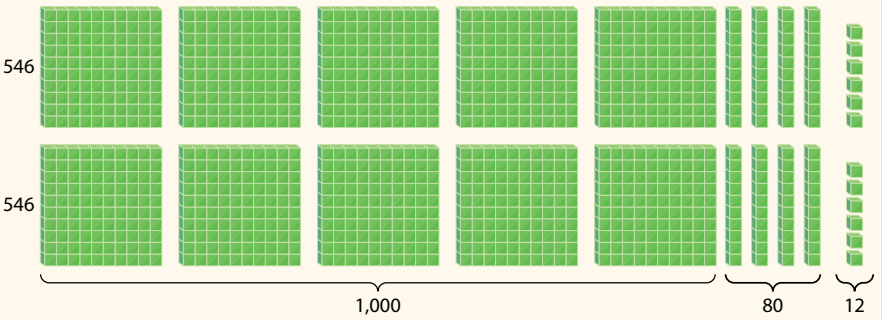
Name: _____ LESSON 11 SESSION 2

Practice Multiplying a Three-Digit Number by a One-Digit Number

Study the Example showing one way to multiply a three-digit number by a one-digit number. Then solve problems 1–5.

EXAMPLE

Find the product of 2 and 546.



$2 \times 546 = 1{,}000 + 80 + 12$
$= 1{,}092$

1 Look at the multiplication above. Use partial products to find 2×546. Fill in the boxes.

$$\begin{array}{r} 5\ 4\ 6 \\ \times\quad\ \ 2 \\ \hline 1\ 2 \\ \boxed{8}\ \boxed{0} \\ +\ 1\ ,0\ 0\ 0 \\ \hline \boxed{1}\ \boxed{0}\ \boxed{9}\ \boxed{2} \end{array}$$

→ 2 × 6 ones
→ 2 × 4 tens
→ 2 × 5 hundreds

2 $3 \times 132 = ?$

Show your work.

Possible student work:

$300 + 90 + 6 = 396$

Solution 396

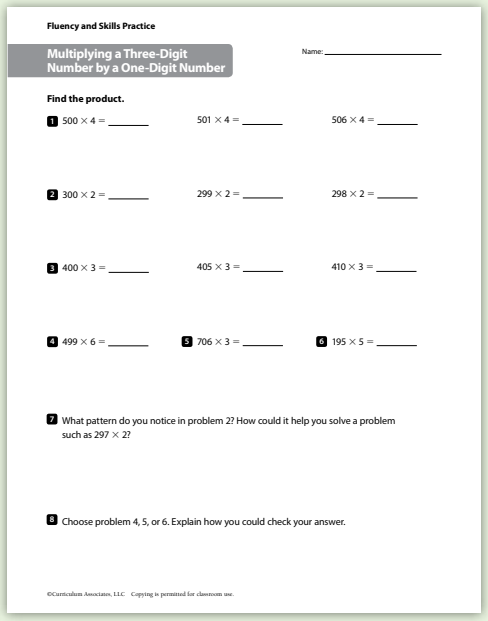
239

Fluency & Skills Practice

Teacher Toolbox

Assign *Multiplying a Three-Digit Number by a One-Digit Number*

In this activity students practice multiplying a three-digit number by a one-digit number. The first three problems have sets of related multiplication problems. Students may find that the patterns in the products of these problems are helpful in solving real-world problems that involve multiplying a multi-digit number by a one-digit number.



Fluency and Skills Practice

Multiplying a Three-Digit Number by a One-Digit Number

Name: _____

Find the product.

1 500 × 4 = _____ 501 × 4 = _____ 506 × 4 = _____

2 300 × 2 = _____ 299 × 2 = _____ 298 × 2 = _____

3 400 × 3 = _____ 405 × 3 = _____ 410 × 3 = _____

4 499 × 6 = _____ 5 706 × 3 = _____ 6 195 × 5 = _____

7 What pattern do you notice in problem 2? How could it help you solve a problem such as 297 × 2?

8 Choose problem 4, 5, or 6. Explain how you could check your answer.

3 5,490; See possible work on the Student Wortext page. Students may also draw an array of base-ten blocks and add hundreds, tens, and ones to find the product.
Medium

4 3,381; See possible work on the student page. Possible estimate shown on the student page uses rounding to check for reasonableness of the product. Students may also draw an array of base-ten blocks and add hundreds, tens, and ones to find the product. To check for reasonableness, students may also round 483 to the nearest ten and multiply 480 by 7 to get an estimated product of 3,360.
Medium

5 Possible answer: The partial product of 4 and 600 is written as 240 instead of 2,400. Find the correct product by adding the partial products 28 and 2,400 to get a product of 2,428.
Challenge

3 Find 6 × 915. Show your work.

Possible student work:

```
  915
×   6
 30
 60
+ 5,400
5,490
```

Solution 5,490

4 Find the product of 483 and 7. Estimate to check that your answer is reasonable. Show your work.

Possible student work:

```
  483
×   7
 21
560
+ 2,800
3,381
```

Solution 3,381; Possible estimate: 483 rounds to 500. 500 × 7 = 3,500. 3,500 is close to 3,381, so the answer is reasonable.

5 There is a mistake in the multiplication shown. Explain what mistake is made. Then find the correct product.

```
  607
×   4
 28
+ 240
 268
```

Solution Possible answer: The product of 4 and 600 is written as 240 instead of 2,400. The correct answer is 28 + 2,400 = 2,428.

ELL English Language Learners: Differentiated Instruction

Prepare for Session 3
Use with *Apply It*.

Levels 1–3

Reading Ask students to brainstorm the models or strategies they could use to solve the equation in ***Apply It*** problem 8. Make a list of the responses, which should include:

- *Draw a picture.*
- *Use base-ten blocks.*
- *Use partial products.*
- *Draw an area model.*

Point to and read each model/strategy with students. Divide the students into partner groups. Assign each group one model or strategy to use to solve the equation.

Levels 2–4

Listening Ask students to brainstorm models or strategies they could use to solve the equation in ***Apply It*** problem 8. Write the responses on index cards, which should include:

- *Draw a picture.*
- *Use base-ten blocks.*
- *Use partial products.*
- *Draw an area model.*

Write the models and strategies on index cards. Shuffle the cards. Have each student select a card and use the model or strategy written on it to solve the equation. When they are finished, ask them to listen to partners explain how they used the model or strategy to solve the problem.

Levels 3–5

Writing/Speaking Divide students into pairs and have them make an idea map for the models or strategies that could be used to solve the equation in ***Apply It*** problem 8. When they have completed their idea maps, have students share them with other pairs. Review the models/strategies students have written on their idea maps. Have partners give brief descriptions of how the models/strategies are used to solve the equation. Ask students to identify the model or strategy they will use to solve the problem. When students have completed the problem, have them confirm their answers with other pairs.

LESSON 11

SESSION 3 Develop

Purpose In this session, students solve a problem that requires them to multiply a four-digit number by a one-digit number. Students model the problem either on paper or with manipulatives to find the product. The purpose of this problem is to have students develop a strategy to multiply four-digit numbers by one-digit numbers.

Start

Connect to Prior Knowledge

Why Support students' ability to multiply a three-digit number by a one-digit number in preparation for multiplying a four-digit number by a one-digit number.

How Have students multiply a three-digit number by a one-digit number.

Multiply.

$4 \times 235 =$ .....

Solution
940

Develop Language

Why Clarify the meaning of the term *set*.

How Explain to students that a *set* is a group of items that belong together. Point out that different sets can have different numbers of items. Find sets of items in the classroom, such as a set of 12 cubes, 20 books, or 15 hundreds flats. Ask students if they can think of other items that come in sets. In the *Try It* problem, explain to students that there are 3 sets, or groups, of building pieces, and each set has 1,125 pieces.

TRY IT

Make Sense of the Problem

To support students in making sense of the problem, have them identify that Ezekiel has 3 building sets and that each set has 1,125 pieces.

LESSON 11 SESSION 3 ● ● ● ○

Develop Multiplying a Four-Digit Number by a One-Digit Number

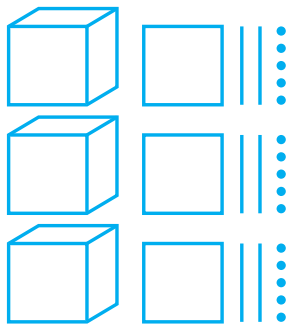
Read and try to solve the problem below.

Ezekiel has 3 building sets. Each set includes 1,125 pieces. How many pieces are in all 3 sets?

TRY IT

Possible student work:

Sample A



3,000 + 300 + 60 + 15 = 3,375
3 × 1,125 = 3,375

Sample B

```
  1,125
×     3
     15
     60
    300
+ 3,000
  3,375
```

Math Toolkit
- base-ten blocks
- grid paper
- index cards
- sticky notes
- number lines
- multiplication models

DISCUSS IT

Ask your partner: Do you agree with me? Why or why not?

Tell your partner: I disagree with this part because …

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DISCUSS IT

Support Partner Discussion

Encourage students to share what did not work for them as well as what did.

Support as needed with questions such as:

- *How did you break apart the number 1,125?*
- *How did you find the total of the thousands, hundreds, tens, and ones?*

Common Misconception Look for students who do not recognize that the digits in 1,125 in the tens, hundreds, and thousands places represent 20, 100, and 1,000, respectively. Have students write the expanded form of 1,125 to show the value of each digit.

Select and Sequence Student Solutions

One possible order for whole class discussion:

- arrays of base-ten blocks modeling 3 groups of 1,125
- drawings modeling 3 groups of 1,125
- equations showing repeated addition
- multiplication equations showing partial products

Support Whole Class Discussion

Compare and connect the different representations and have students identify how they are related.

Ask *Where does your model show the number of thousands, hundreds, tens, and ones in 1,125? the multiplication of each place value by 3? the product?*

Listen for Students should recognize that accurate responses include representations showing 1 thousand, 1 hundred, 2 tens, and 5 ones in 1,125, each added 3 times or multiplied by 3, resulting in a product of 3 thousands, 3 hundreds, 7 tens, and 5 ones.

PICTURE IT & MODEL IT

If no student presented these models, connect them to the student models by pointing out the ways they each represent:

- the number of thousands, hundreds, tens, and ones in 1,125
- multiplying the value of each place in 1,125 by 3
- partial products of 3 × 1,000, 3 × 100, 3 × 20, and 3 × 5

Ask *How are the models the same? different?*

Listen for Both show 1,125 multiplied by 3, the partial products, and multiplication by place value to calculate the partial products. The area model shows the calculations horizontally, while the partial-products model shows the calculation vertically.

Ask *Why does* Picture It *show an area model instead of an array of base-ten blocks to represent this problem?*

Listen for It takes less space to show the product of a four-digit number and a one-digit number using an area model than a model showing base-ten blocks.

For an area model, prompt students to identify how multiplication by place value is shown.

- *How is the number 1,125 shown in the area model?*
- *What does the 3 in the area model represent?*
- *What does each area represent?*
- *What does the total area represent?*

For partial products, prompt students to identify how multiplication by place value is shown.

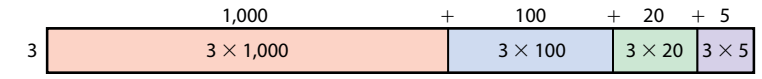
- *How is 1,125 represented?*
- *How is multiplication by 3 represented?*
- *How is the product represented?*

Explore different ways to understand multiplying a four-digit number by a one-digit number.

Ezekiel has 3 building sets. Each set includes 1,125 pieces. How many pieces are in all 3 sets?

PICTURE IT

You can use an area model to help understand the problem.



$3 \times 1{,}125 = (3 \times 1{,}000) + (3 \times 100) + (3 \times 20) + (3 \times 5)$
$= 3{,}000 + 300 + 60 + 15$

MODEL IT

You can also multiply the numbers using partial products.

$$\begin{array}{r} 1{,}125 \\ \times \quad 3 \\ \hline 15 \\ 60 \\ 300 \\ +\ 3{,}000 \\ \hline ? \end{array}$$

15 → 3 × 5 ones
60 → 3 × 2 tens
300 → 3 × 1 hundred
3,000 → 3 × 1 thousand

The partial products are 15, 60, 300, and 3,000.

The product is the sum of the partial products.

Deepen Understanding

Area Model with Partial Products

SMP 4 Model with mathematics.

When discussing the area model, prompt students to think about why this model is preferable to using an array of base-ten blocks when multiplying greater numbers.

Ask *Why do you think it is more helpful to use an area model for showing multiplication with a four-digit number than using base-ten blocks?*

Listen for With an area model, you can show each of the factors in the model along the lengths of the sides. You can show the four-digit number broken apart by place value and show each place value multiplied by 3 in the corresponding area. The area model shows the multiplication using less space than base-ten blocks.

For greater numbers, using base-ten blocks is less efficient than using an area model because it takes more space and more time to show all of the blocks that represent the product. Counting the base-ten blocks takes more time, too. It is quicker to draw an area model.

CONNECT IT

- Remind students that one thing that is alike about all the representations is the numbers.
- Explain that on this page students will use those numbers to show how both models represent multiplying a four-digit number by a one-digit number by breaking apart numbers and using place value to multiply in order to find a product.

Monitor and Confirm

1–4 Check for understanding that:

- 1,125 in expanded form is 1,000 + 100 + 20 + 5
- each place value in the expanded form of 1,125 is multiplied by the factor 3
- the sum of the partial products 3,000, 300, 60, and 15 in both models results in the product 3,375
- the place values in 1,125 can be multiplied by 3 in any order and the product remains the same

Support Whole Class Discussion

5 Tell students that this problem will prepare them to provide the explanation required in problem 6.

Be sure that students understand that problem 5 is asking them to explain how 1,125 and 3 are multiplied to find a product.

Ask *How can you use estimation to check that your answer is reasonable?*

Listen for Round 1,125 to 1,000. Multiply 1,000 by 3 to get 3,000. Since 3,000 is close to 3,375, the answer is reasonable.

Ask *How can you use addition to check your answer?*

Listen for 1,125 + 1,125 + 1,125 = 3,375.

6 Look for the idea that the digit in each place-value position of the four-digit factor is multiplied by the one-digit factor and that the sum of the partial products gives the product.

7 REFLECT Have all students focus on the strategies used to solve this problem. If time allows, have students share their responses with a partner.

CONNECT IT

Now you will use the problem from the previous page to help you understand how to multiply four-digit numbers by one-digit numbers.

1 What is the expanded form of 1,125? 1,000 + 100 + 20 + 5

2 How is the expanded form used in the equation in **Picture It**?

Each number in the expanded form is multiplied by the other factor, 3.

3 What is the sum of the numbers in the equation in **Picture It** and the sum of the partial products in **Model It**? 3,375

4 The partial products in **Model It** shows first multiplying the 3 by the value of the digit in the ones column. Would the product change if you first multiplied the 3 by the value of the digit in the thousands column? Explain.

No, the product would be the same. Possible explanation: You would add the partial products in a different order, but the sum doesn't change when you add in a different order.

5 Describe how the factor 3 is used with the factor 1,125 to find the product.

The value of the digit in each place-value position in 1,125 is multiplied by 3. Then all the partial products are added together.

6 Explain how you multiply a four-digit number by a one-digit number.

Possible explanation: Multiply the digits in each place-value position of the four-digit number by the one-digit number. Find the partial products and then add to find the final product.

7 REFLECT

Look back at your **Try It**, strategies by classmates, and **Picture It** and **Model It**. Which models or strategies do you like best for multiplying a four-digit number by a one-digit number? Explain.

Students may respond that they like the strategy of drawing an area model or using partial products because they can multiply the one-digit number by the value of each digit in the four-digit number to find either the area of each section in the area model or the four partial products. Then they add the four numbers to find the final product.

Visual Model

Use area models to multiply greater numbers.

If . . . *students are unsure about multiplying a four-digit by a one-digit number,*

Then . . . *use this activity to have them practice using area models to multiply.*

Use drawings of area models to visually model multiplication of a four-digit number by a one-digit number.

- Write 5 × 1,342 on the board. Draw an area model that shows 5 along the left side and 1,000 + 300 + 40 + 2 along the top. Divide the model into 4 areas that are roughly proportional to the amounts along the top.
- Ask: *Why is one side labeled 1,000 + 300 + 40 + 2?* [This is the expanded form of 1,342.] Explain that you use this because you want to multiply 5 by the value of each digit in 1,342. *Why is another side labeled 5?* [That is the other factor.] *What does each of the four areas of the model represent?* [Each area represents the product of 5 and one term in the expanded form of 1,342.]
- Write "5 × 1,000 =" in the first area. Ask: *What is the product of 5* and *1,000?* [5,000] Record 5,000. Repeat this step for the other three areas.
- Ask: *How do you find the product of 5 × 1,342?* [Add the areas; the sum is 6,710.]
- Repeat for other multiplication problems, such as 3 × 1,628 and 4 × 2,451.

APPLY IT

For all problems, encourage students to draw some kind of model to support their thinking. Allow some leeway in precision; drawing precise area models is difficult, and here precise drawings are not required.

8 21,368; See possible work on the Student Worktext page; Students may also use partial products to multiply.

9 27,496; See possible work on the Student Worktext page. Possible estimate shown on the Student Worktext page uses rounding as an estimation strategy to check for reasonableness of the product. Students may also use partial products to multiply. To check for reasonableness, students may also round 3,928 to the nearest hundred and multiply 3,900 by 7 to get an estimated product of 27,300.

Close: Exit Ticket

10 12,246; See possible work on the Student Worktext page; Students may also use an area model to multiply.

Students' solutions should indicate understanding of:

- breaking apart numbers by place value to multiply
- multiplying each digit in the four-digit number by the one-digit number
- adding the partial products to find the product

Error Alert **If** students are confused by the 0 in the hundreds place and multiply 6 by 2 hundred rather than by 2 thousand, **then** have them write the pairs of numbers they need to multiply to find the partial products. In this case, that would be "6 × 1 one" or "6 × 1", "6 × 4 tens" or "6 × 40", "6 × 0 hundreds" or "6 × 0", and "6 × 2 thousands" or "6 × 2,000."

APPLY IT

Use what you just learned to solve these problems.

8 5,342 × 4 = ? Show your work.

Possible student work:

	5,000	+ 300	+ 40	+ 2
4	4 × 5,000	4 × 300	4 × 40	4 × 2

5,342 × 4 = (4 × 5,000) + (4 × 300) + (4 × 40) + (4 × 2)
= 20,000 + 1,200 + 160 + 8
= 21,368

Solution 21,368

9 Find the product of 7 and 3,928. Estimate to check that your answer is reasonable. Show your work.

Possible student work:

	3,000	+ 900	+ 20	+ 8
7	7 × 3,000	7 × 900	7 × 20	7 × 8

7 × 3,928 = (7 × 3,000) + (7 × 900) + (7 × 20) + (7 × 8)
= 21,000 + 6,300 + 140 + 56
= 27,496

Solution 27,496; Possible estimate: 3,928 rounds to 4,000. 4,000 × 7 = 28,000. 28,000 is close to 27,496. My answer is reasonable.

10 2,041 × 6 = ? Show your work.

Possible student work:

2,041
× 6
6
240
+ 12,000
12,246

Solution 12,246

Solutions

1. See completed partial-product multiplication on the student page.
Basic

2. 8,215; See work on the student page.
Medium

Name: _____ LESSON 11 SESSION 3

Practice Multiplying a Four-Digit Number by a One-Digit Number

Study the Example showing one way to multiply a four-digit number by a one-digit number. Then solve problems 1–5.

EXAMPLE

Jesse's family has 4 music players. Each music player can hold 8,352 songs. What is the total number of songs all 4 music players can hold?

Use an area model.

	8,000	+ 300	+ 50	+ 2
4	4 × 8,000	4 × 300	4 × 50	4 × 2

$4 \times 8{,}352 = (4 \times 8{,}000) + (4 \times 300) + (4 \times 50) + (4 \times 2)$
$= 32{,}000 + 1{,}200 + 200 + 8$
$= 33{,}408$

All 4 music players can hold a total of 33,408 songs.

1. Complete the multiplication to use partial products to find 4 × 8,352.

```
  8,352
×     4
-------
      8   → 4 × 2 ones
    200   → 4 × 5 tens
  1,200   → 4 × 3 hundreds
+ 32,000  → 4 × 8 thousands
-------
  33,408
```

2. Show how to use partial products to find 5 × 1,643.

```
  1,643
×     5
-------
     15   → 5 × 3 ones
    200   → 5 × 4 tens
  3,000   → 5 × 6 hundreds
+ 5,000   → 5 × 1 thousand
-------
  8,215
```

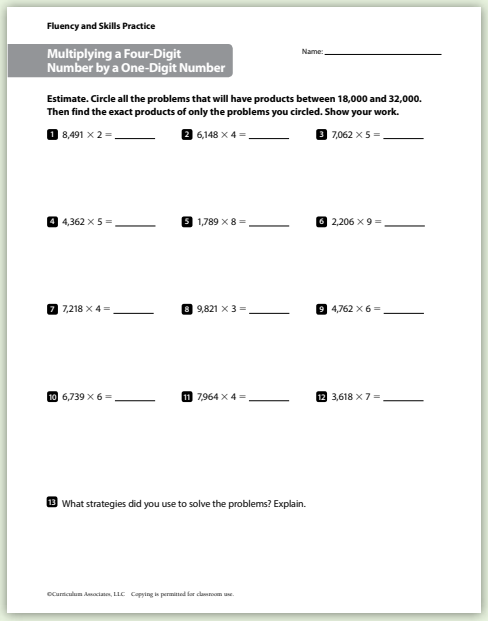
245

Fluency & Skills Practice

Teacher Toolbox

Assign *Multiplying a Four-Digit Number by a One-Digit Number*

In this activity students practice estimating and finding the product of a four-digit number and a one-digit number. It is helpful for students to practice estimating products as estimation is used in real-world situations in which an exact answer is not needed. Also, estimation may be used to check that an answer is reasonable. Students may use a variety of strategies to find the exact products, including using partial products.



Fluency and Skills Practice

Multiplying a Four-Digit Number by a One-Digit Number

Name: _____

Estimate. Circle all the problems that will have products between 18,000 and 32,000. Then find the exact products of only the problems you circled. Show your work.

1. 8,491 × 2 = _____
2. 6,148 × 4 = _____
3. 7,062 × 5 = _____
4. 4,362 × 5 = _____
5. 1,789 × 8 = _____
6. 2,206 × 9 = _____
7. 7,218 × 4 = _____
8. 9,821 × 3 = _____
9. 4,762 × 6 = _____
10. 6,739 × 6 = _____
11. 7,964 × 4 = _____
12. 3,618 × 7 = _____
13. What strategies did you use to solve the problems? Explain.

3 $(4 \times 3{,}000) + (4 \times 500) + (4 \times 60) + (4 \times 9)$; $12{,}000 + 2{,}000 + 240 + 36$; 14,276
Medium

4 $6,450; See possible work on the student page. Possible estimate shown on the student page uses rounding as an estimation strategy to check for reasonableness of the product. Students may also draw an area model to find the product. To check for reasonableness, students may also round 1,075 to the nearest hundred and multiply 1,100 by 6 to get an estimated product of 6,600.
Medium

5 **a.** Answers will vary. See possible explanation on the student page.
b. See explanation on the student page.
c. 28,269
Challenge

3 Write $4 \times 3{,}569$ in expanded form to show the place value of each digit. Then find the product.

$(4 \times 3{,}000) + (4 \times 500) + (4 \times 60) + (4 \times 9)$
$12{,}000 + 2{,}000 + 240 + 36$
14,276

4 Lee earns $1,075 each month. How much does he earn in 6 months? Estimate to check that your answer is reasonable. Show your work.

Possible student work:

$$\begin{array}{r} 1{,}075 \\ \times \quad 6 \\ \hline 30 \\ 420 \\ +\ 6{,}000 \\ \hline 6{,}450 \end{array}$$

Solution **$6,450; Possible estimate: 1,075 rounds to 1,000. 1,000 × 6 = 6,000. 6,000 is close to 6,450. My answer is reasonable.**

5 Look at Callie's work for finding $3 \times 9{,}423$.

$$\begin{array}{r} 9{,}423 \\ \times \quad 3 \\ \hline 9 \\ 60 \\ 120 \\ +\ 2{,}700 \\ \hline 2{,}889 \end{array}$$

a. Explain what Callie did wrong.

Answers will vary. Possible explanation: When multiplying 400 by 3, she wrote 120 instead of 1,200. She also wrote that 9,000 × 3 = 2,700, but it should be 27,000.

b. How can using estimation show that Callie's answer is wrong?

9,423 rounds to 9,000. 9,000 × 3 = 27,000; 27,000 is not close to 2,889. Her answer is wrong.

c. What is the correct answer?

28,269

ELL English Language Learners: Differentiated Instruction

Prepare for Session 4
Use with *Apply It*.

Levels 1–3

Writing/Reading Restate ***Apply It*** problem 8, simplifying it in the process: *Students collect 1,238 bottles each day. How many bottles do they collect in 7 days?* Have students work with partners to solve the problem. Say: *Estimate to see if your answer is reasonable.* Provide the following terms and sentence frames for students to use: *multiply*, *reasonable*, *round*, *product*, *hundred*, *8,400*.

- *I will _____ 1,238 to the nearest _____.*
- *I will _____ 1,200 by 7.*
- *The _____ is _____.*
- *My answer is _____.*

Have students read the sentences to partners.

Levels 2–4

Writing/Reading Read ***Apply It*** problem 8 with students. Have students restate the word problem in their own words and then circle information needed to solve the equation. Have students form pairs and solve the problem. Ask them to share their answers with other pairs to confirm their solutions. Provide the following terms:

- *round, nearest hundred*
- *multiply*
- *product, 8,400*
- *reasonable, because*

Have students use the terms to write four sentences explaining how they use estimation to check to see if the answers are reasonable. Ask pairs to read their explanations.

Levels 3–5

Writing/Reading Have students read ***Apply It*** problem 8 and then circle information in the word problem needed to solve it. Have them explain to partners why the information is relevant before solving the word problem. Write the following terms on index cards: *round*, *hundred*, *multiply*, *product*, *8,400*, *reasonable*. Shuffle the cards and place them facedown. Have partners take turns selecting cards. Ask them to use the terms to write sentences on sentence strips that relate to how they use estimation to check for reasonableness. When they have completed the activity, mix up the strips. Have partners read the strips and put them in sequential order.

LESSON 11

SESSION 4 Refine

Purpose In this session, students solve problems involving multiplying three- or four-digit numbers by one-digit numbers and then discuss and confirm their answers with a partner.

Before students begin to work, use their responses to the *Check for Understanding* to determine those who will benefit from additional support.

As students complete the Example and problems 1–3, observe and monitor their reasoning to identify groupings for differentiated instruction.

Start

Check for Understanding

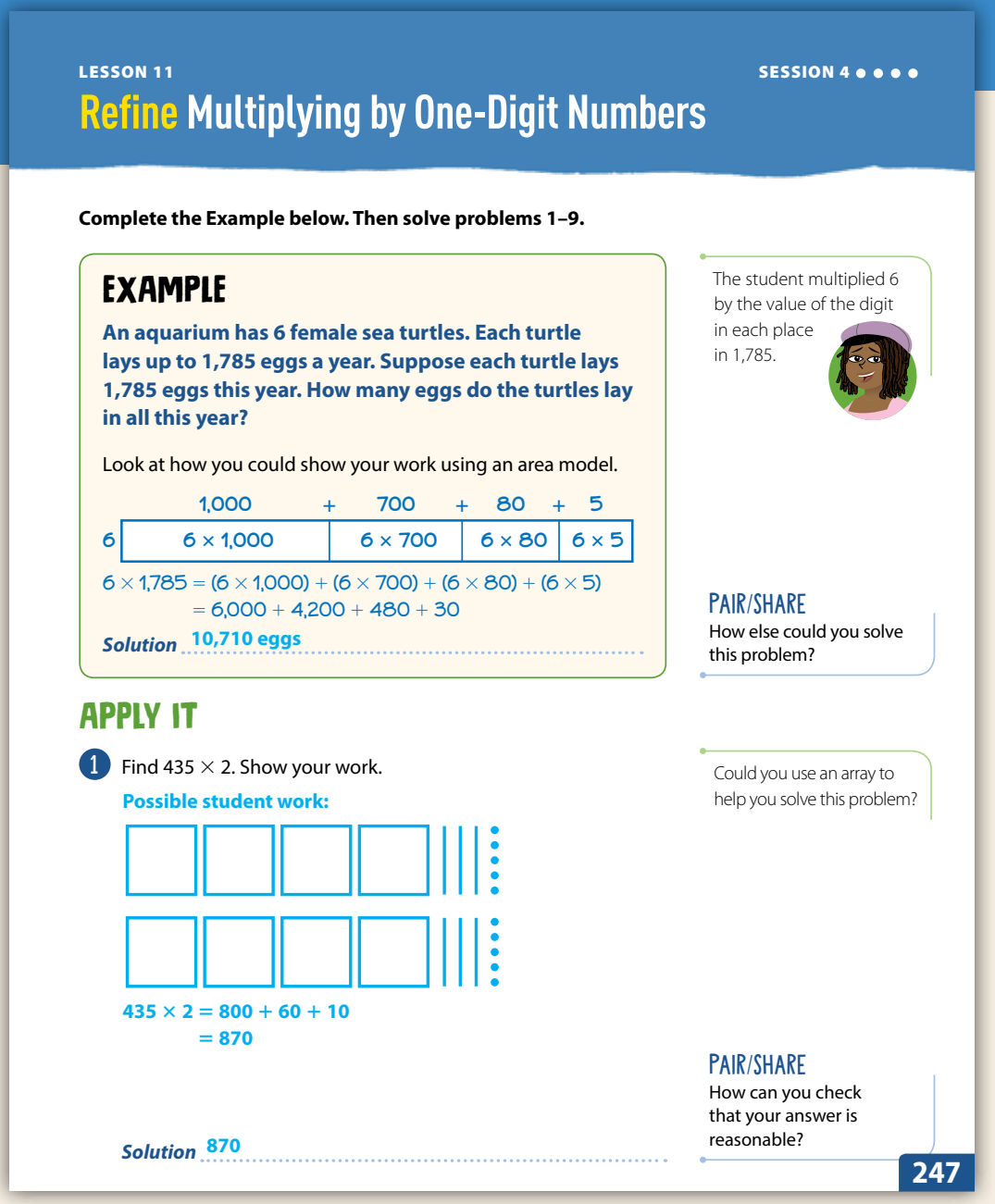
Materials For remediation: base-ten blocks (1 thousands cube, 2 hundreds flats, 4 tens rods, 8 ones units)

Why Confirm understanding of multiplying three- or four-digit numbers by one-digit numbers.

How Have students find the product of 9 and 1,248 using any strategy they want.

Find the product of 9 and 1,248.

Solution
11,232



LESSON 11 SESSION 4 ● ● ● ●

Refine Multiplying by One-Digit Numbers

Complete the Example below. Then solve problems 1–9.

EXAMPLE

An aquarium has 6 female sea turtles. Each turtle lays up to 1,785 eggs a year. Suppose each turtle lays 1,785 eggs this year. How many eggs do the turtles lay in all this year?

Look at how you could show your work using an area model.

	1,000	+ 700	+ 80	+ 5
6	6 × 1,000	6 × 700	6 × 80	6 × 5

$6 \times 1{,}785 = (6 \times 1{,}000) + (6 \times 700) + (6 \times 80) + (6 \times 5)$
$= 6{,}000 + 4{,}200 + 480 + 30$

Solution 10,710 eggs

The student multiplied 6 by the value of the digit in each place in 1,785.

PAIR/SHARE
How else could you solve this problem?

APPLY IT

1 Find 435 × 2. Show your work.

Possible student work:

$435 \times 2 = 800 + 60 + 10$
$= 870$

Solution 870

Could you use an array to help you solve this problem?

PAIR/SHARE
How can you check that your answer is reasonable?

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Error Alert

If the error is . . .	*Students may . . .*	*To support understanding . . .*
1,257	have added.	Remind student that *product* means multiplication.
135	have incorrectly found all partial products as multiplying ones by ones.	Have students use base-ten blocks to show that 1,248 is 1,000 + 200 + 40 + 8. Then have them draw an area model to show each partial product.
10,908	have incorrectly found the partial product of multiplying ones by tens as 4 × 9.	Remind students that when multiplying tens by ones, the result is 40 × 9 = 360, not 4 × 9 = 36.
10,232	have incorrectly added partial products.	Remind students that they must regroup 12 hundreds as 1 thousand and 2 hundreds when adding partial products.

EXAMPLE

10,710 eggs; An area model is shown as one way to solve the problem. Students could also solve the problem by using partial products.

Look for The value of each digit in 1,785 is multiplied by 6, and the partial products are added to find the product.

APPLY IT

1. 870; Students could solve the problem by using drawings of base-ten blocks and adding hundreds, tens, and ones to find the product.
DOK 1

   Look for An array of base-ten blocks showing 435 × 2 can be used to solve the problem.

2. 25,236; Students could solve the problem using partial products. Students could use rounding to estimate the product and check the reasonableness of the answer. 6,309 is close to 6,000; 6,000 × 4 = 24,000. 24,000 is close to 25,236, so the answer is reasonable.
DOK 2

   Look for Add the partial products to find the product and use an estimating strategy such as rounding to check for reasonableness of the answer.

3. **C**; Multiply each digit in 147 by 5 and then add the partial products.
Explain why the other two answer choices are not correct:
B is not correct because 5 × 100 = 500 and 47 × 5 is more than 5.
D is not correct because 5 should be multiplied by (100 + 40 + 7), not (100 + 80 + 1).
DOK 3

2. Find 4 × 6,309. Estimate to check that your answer is reasonable. Show your work.

How could partial products help you solve this problem?

Possible student work:

```
  6,309
×     4
     36
  1,200
+ 24,000
  25,236
```

Solution **25,236; 6,309 is close to 6,000. 6,000 × 4 = 24,000. 24,000 is close to 25,236. My answer is reasonable.**

PAIR/SHARE
Is your estimate close to your answer?

3. A hardware store has 147 containers of paint. Each container holds 5 gallons of paint. How many gallons of paint does the store have?

Multiply 5 by the value of the digit in each place in 147.

Ⓐ 235
Ⓑ 505
Ⓒ 735
Ⓓ 905

Dale chose Ⓐ as the correct answer. How did he get that answer?

Dale multiplied 5 by the tens value and 5 by the ones value. He did not multiply 5 by the hundreds value.

PAIR/SHARE
Does Dale's answer make sense?

4 **A**; $90 \times 9 = 810$
B; $405 \times 2 = 810$
E; $(5 \times 100) + (5 \times 60) + (5 \times 2) =$
$500 + 300 + 10 = 810$
DOK 1

5 **D**; Multiply each digit in 273 by 3. Find the partial products. Add to find the product.
DOK 2

Error Alert Students who choose A added 273 and 3 instead of multiplying 273 by 3.

6 5,812; Multiply each digit in 2,906 by 2. Find the partial products. Add to find the product.
DOK 2

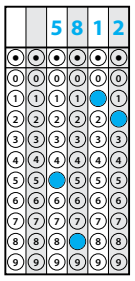
4 Select all the expressions that have a product of 810.

Ⓐ 90×9
Ⓑ 405×2
Ⓒ $(3 \times 20) + (3 \times 70)$
Ⓓ $(2 \times 400) \times (2 \times 5)$
Ⓔ $(5 \times 100) + (5 \times 60) + (5 \times 2)$

5 Mr. Larson is planning a pizza party for 273 people. He plans on 3 slices of pizza for each person. How many slices of pizza is this in all?

Ⓐ 276
Ⓑ 546
Ⓒ 619
Ⓓ 819

6 Find $2{,}906 \times 2$.



Differentiated Instruction

RETEACH

Hands-On Activity
Use play money to understand multiplying whole numbers.

Students struggling with multiplying whole numbers

Will benefit from additional work with concrete representations of place value.

Materials For each pair: play money (25 $1 bills, 25 $10 bills, and 25 $100 bills)

- Give partners an assortment of $1, $10, and $100 bills. Present multiplication problems, such as 234×3 and 123×6.
- Have students count out the greater dollar amount using the least number of bills. Then have them count out additional sets based on the other factor. Students find and record the total amount of $1s, $10s, and $100s. Connect this to finding partial products using place value.

EXTEND

Challenge Activity
Solve two-step word problems involving multiplication.

Students who have achieved proficiency

Will benefit from deepening understanding by solving two-step word problems that involve multiplication.

Have students solve the following problems.

- Brandon has 48 collectible cards. He gives 3 cards to each of his 10 friends. How many cards does Brandon have now? [18 cards]
- Mr. Rutledge takes inventory of the items on the shelves of his store. He has 9 unopened boxes of soap and 16 bars of soap on the shelf. Each unopened box of soap has 312 bars in it. How many bars of soap does Mr. Rutledge have in all? [2,824 bars]

7 Answers will vary. Possible answers for Lara's equation:

$328 \times 4 = 1{,}312$

$328 \times 5 = 1{,}640$

$328 \times 6 = 1{,}968$

$328 \times 7 = 2{,}296$

$328 \times 8 = 2{,}624$

$328 \times 9 = 2{,}952$

Possible answers for Greg's equation:

$328 \times 1 = 328$

$328 \times 2 = 656$

$328 \times 3 = 984$

DOK 2

8 8,666 water bottles; Students could solve the problem using partial products and round 1,238 to 1,200 to estimate the product and check their answer for reasonableness.

DOK 2

7 Lara says, "When you multiply a three-digit number by a one-digit number, the product is always a four-digit number." Lara writes an equation to support her statement. Greg writes an equation to show that Lara's statement is false.

Complete the equations below to show a possible equation each person could have written.

Lara's equation: $328 \times$ 4 $=$ 1,312

Greg's equation: $328 \times$ 2 $=$ 656

Answers will vary. Possible equations shown.

8 Fourth-grade students hold a recycling drive. In one week, they collect 1,238 water bottles each day. How many water bottles do the fourth graders collect that week? Estimate to check that your answer is reasonable. Show your work. [*Hint*: There are 7 days in one week.]

Possible student work shown.

```
 1,238
×    7
    56
   210
  1400
+ 7000
 8,666
```

Solution 8,666 water bottles; 1,238 rounds to 1,200. 1,200 × 7 = 8,400. 8,400 is close to 8,666, so my answer is reasonable.

9 MATH JOURNAL

Explain what strategy you would use to find 357 × 8. Then use that strategy to find the product.

Check students' methods.

Students' solutions should show that the product is 2,856.

SELF CHECK Go back to the Unit 3 Opener and see what you can check off.

REINFORCE

Problems 4–9

Multiply by one-digit numbers.

All students will benefit from additional work with multiplying by one-digit numbers by solving problems in a variety of formats.

- Have students work on their own or with a partner to solve the problems.
- Encourage students to show their work.

PERSONALIZE

i-Ready

Provide students with opportunities to work on their personalized instruction path with *i-Ready* Online Instruction to:

- fill prerequisite gaps
- build up grade-level skills

Close: Exit Ticket

9 MATH JOURNAL

Student responses should indicate understanding of breaking apart a number by place value to multiply a three-digit number by a one-digit number.

Error Alert **If** students are unable to identify a strategy to find 357 × 8, **then** have them use base-ten blocks to build an array that represents 357 × 8 and count the number of hundreds, tens, and ones to find the product.

SELF CHECK Have students consider whether they feel they are ready to check off any new skills on the Unit 3 Opener.

Lesson Overview

LESSON 12

Multiply by Two-Digit Numbers

Lesson Objectives

Content Objectives

- Multiply a two-digit number by a two-digit number.
- Use area models and partial products to multiply.
- Use estimation to determine whether an answer is reasonable.

Language Objectives

- Read aloud multiplication problems.
- Draw an area model to multiply.
- Write a solution to a multiplication problem using partial products.
- Tell how each part of an area model relates to the factors, partial products, and product of a multiplication problem.

Prerequisite Skills

- Recall basic multiplication facts.
- Know properties of operations.
- Understand place value.
- Understand and use area models.
- Multiply whole numbers of up to four digits by a one-digit whole number.

Standards for Mathematical Practice (SMP)

SMPs 1, 2, 3, 4, 5, and 6 are integrated in every lesson through the *Try-Discuss-Connect* routine.*

In addition, this lesson particularly emphasizes the following SMPs:

5 Use appropriate tools strategically.

7 Look for and make use of structure.

*See page 1i to see how every lesson includes these SMPs.

Lesson Vocabulary

There is no new vocabulary. Review the following key terms.

- **estimate (verb)** to give an approximate number or answer based on mathematical thinking.
- **factor** a number that is multiplied.
- **factors of a number** whole numbers that multiply together to get the given number.
- **multiple** the product of a given number and any other whole number.
- **multiplication** an operation used to find the total number of items in a given number of equal-sized groups.
- **multiply** to repeatedly add the same number a certain number of times. Used to find the total number of items in equal-sized groups.
- **partial products** the products you get in each step of the partial-products strategy. You use place value to find partial products. For example, the partial products for 124×3 are 3×100 or 300, 3×20 or 60, and 3×4 or 12.
- **product** the result of multiplication.
- **reasonable** something that makes sense when given facts are taken into account.

Learning Progression

In Grade 3 students developed an understanding of multiplication.

In the previous Grade 4 lesson students used arrays of base-ten blocks, area models, and partial products, as well as their understanding of place value, to multiply three- and four-digit numbers by one-digit numbers.

In this lesson students use properties of operations, area models, and partial products to multiply two-digit numbers by two-digit numbers. This helps build a foundation for Grade 5 when students will use the standard multiplication algorithm.

In Grade 5 students will multiply three-digit numbers by two-digit numbers as well as multiply decimals. Students will continue to use area models, properties of operations, and place-value understanding to multiply as they become familiar with the standard multiplication algorithm.

Lesson Pacing Guide

Whole Class Instruction

SESSION 1
Explore
45–60 min

Multiplying by Two-Digit Numbers
- Start *5 min*
- Try It *10 min*
- Discuss It *10 min*
- Connect It *15 min*
- Close & Exit Ticket *5 min*

Additional Practice
Lesson pages 255–256

SESSION 2
Develop
45–60 min

Multiplying by Two-Digit Numbers
- Start *5 min*
- Try It *10 min*
- Discuss It *10 min*
- Picture It & Model It *5 min*
- Connect It *10 min*
- Close & Exit Ticket *5 min*

Additional Practice
Lesson pages 261–262

Fluency
Multiplying by Two-Digit Numbers

SESSION 3
Refine
45–60 min

Multiplying by Two-Digit Numbers
- Start *5 min*
- Example & Problems 1–3 *15 min*
- Practice & Small Group Differentiation *20 min*
- Close: Exit Ticket *5 min*

Lesson Quiz or **Digital Comprehension Check**

Lesson Materials

Lesson *(Required)*	*Per student:* copy of Start slide (Session 2)
Activities	*Per pair:* base-ten blocks (2 hundreds flats, 14 tens rods, 55 ones units), play money (93 $1 bills, 62 $10 bills, and 6 $100 bills)
Math Toolkit	base-ten blocks, counters, cups, paper plates, grid paper
Digital Math Tool	Multiplication Models

*We continually update the Interactive Tutorials. Check the Teacher Toolbox for the most up-to-date offerings for this lesson.

Teacher Toolbox

Small Group Differentiation

PREPARE

Ready Prerequisite Lessons

Grade 3
- Lesson 8 Use Order and Grouping to Multiply
- Lesson 9 Use Place Value to Multiply
- Lesson 12 Multiplication and Division Facts

RETEACH

Tools for Instruction

Grade 3
- Lesson 8 Use Order and Grouping to Multiply
- Lesson 9 Use Place Value to Multiply
- Lesson 12 Write Multiplication and Division Facts

Grade 4
- Lesson 12 Multiply by Two-Digit Numbers

REINFORCE

Math Center Activity

Grade 4
- Lesson 12 Multiplying by Two-Digit Numbers

EXTEND

Enrichment Activity

Grade 4
- Lesson 12 Display of Cans

i-Ready

Independent Learning

PERSONALIZE

i-Ready Lessons*

Grade 4
- Multiply Two-Digit Numbers by Two-Digit Numbers
- Practice: Multiply Two-Digit Numbers

Learning Games
- Prerequisite: *Match*
- Prerequisite: *Cupcake*
- Prerequisite: *Pizza*

LESSON 12

Connect to Family, Community, and Language Development

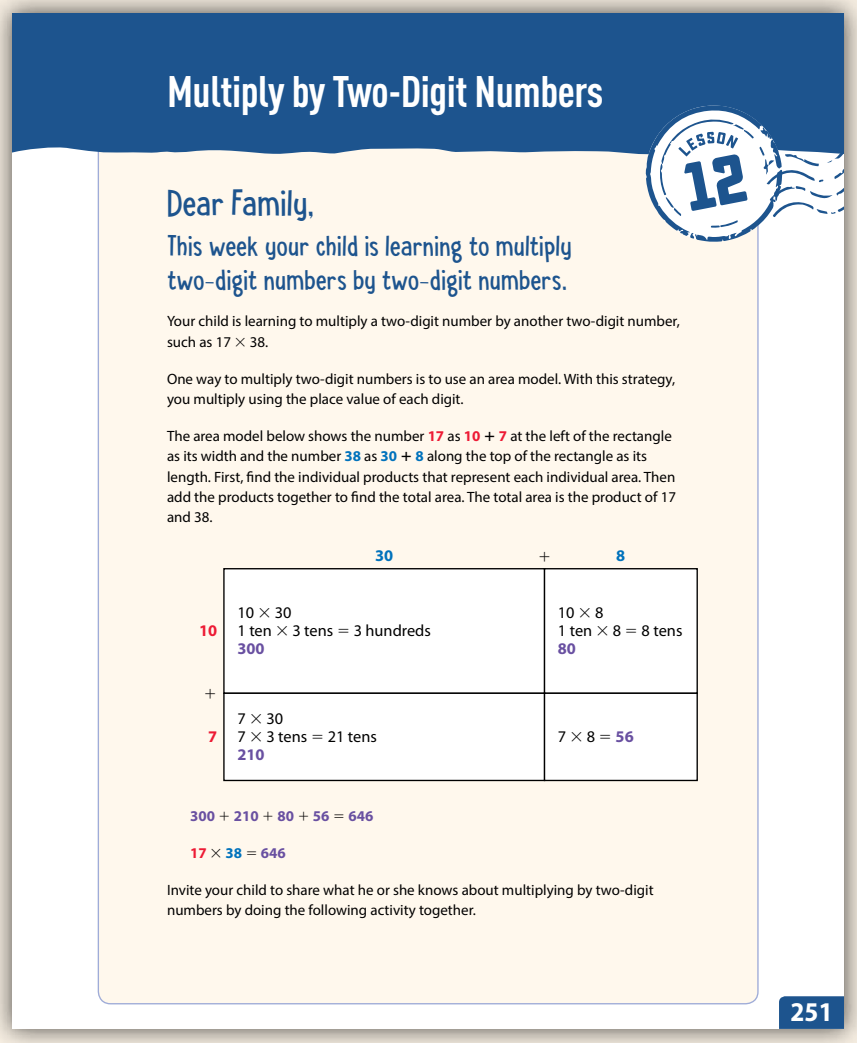
The following activities and instructional supports provide opportunities to foster school, family, and community involvement and partnerships.

Connect to Family

Use the **Family Letter**—which provides background information, math vocabulary, and an activity—to keep families apprised of what their child is learning and to encourage family involvement.

Available in Spanish

Teacher Toolbox



Multiply by Two-Digit Numbers

LESSON 12

Dear Family,

This week your child is learning to multiply two-digit numbers by two-digit numbers.

Your child is learning to multiply a two-digit number by another two-digit number, such as 17 × 38.

One way to multiply two-digit numbers is to use an area model. With this strategy, you multiply using the place value of each digit.

The area model below shows the number 17 as 10 + 7 at the left of the rectangle as its width and the number 38 as 30 + 8 along the top of the rectangle as its length. First, find the individual products that represent each individual area. Then add the products together to find the total area. The total area is the product of 17 and 38.

	30	+	8
10	10 × 30 1 ten × 3 tens = 3 hundreds 300		10 × 8 1 ten × 8 = 8 tens 80
+			
7	7 × 30 7 × 3 tens = 21 tens 210		7 × 8 = 56

300 + 210 + 80 + 56 = 646

17 × 38 = 646

Invite your child to share what he or she knows about multiplying by two-digit numbers by doing the following activity together.

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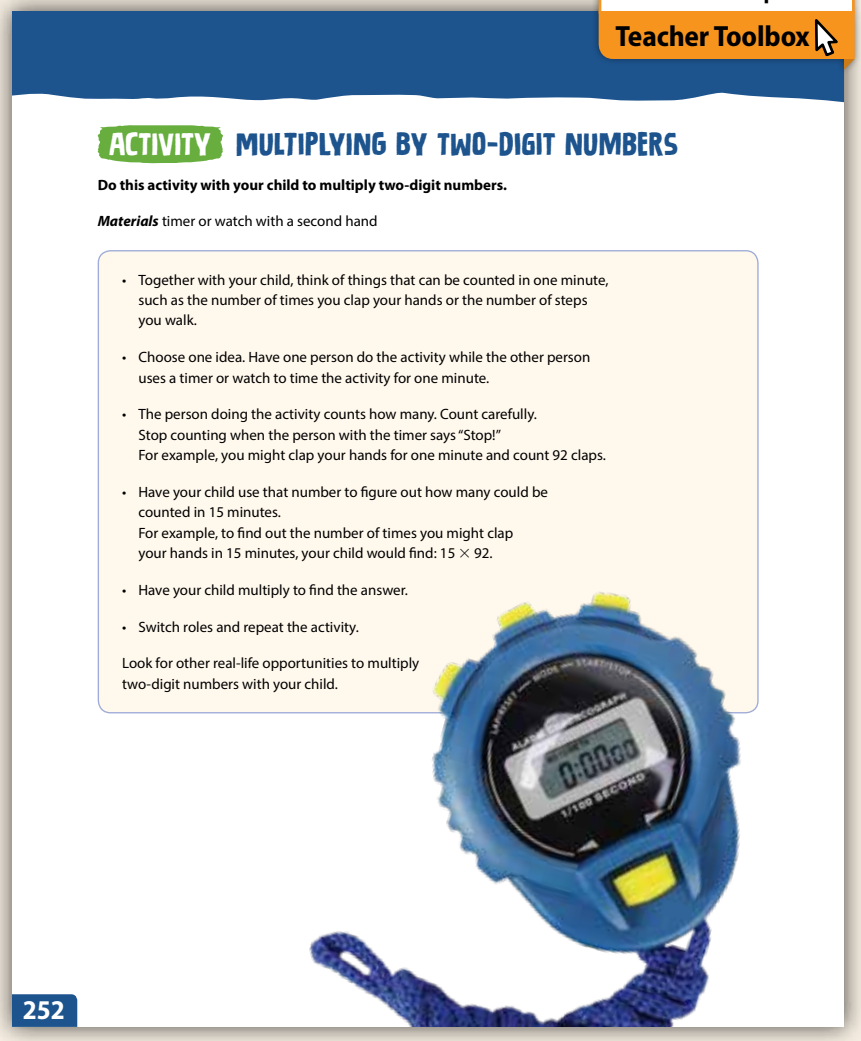

ACTIVITY MULTIPLYING BY TWO-DIGIT NUMBERS

Do this activity with your child to multiply two-digit numbers.

Materials timer or watch with a second hand

- Together with your child, think of things that can be counted in one minute, such as the number of times you clap your hands or the number of steps you walk.
- Choose one idea. Have one person do the activity while the other person uses a timer or watch to time the activity for one minute.
- The person doing the activity counts how many. Count carefully. Stop counting when the person with the timer says "Stop!" For example, you might clap your hands for one minute and count 92 claps.
- Have your child use that number to figure out how many could be counted in 15 minutes. For example, to find out the number of times you might clap your hands in 15 minutes, your child would find: 15 × 92.
- Have your child multiply to find the answer.
- Switch roles and repeat the activity.

Look for other real-life opportunities to multiply two-digit numbers with your child.

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Goal

The goal of the Family Letter is to reinforce the multiplication of two-digit numbers by two-digit numbers and to encourage students to use their background knowledge of multiplying a two-digit number by a one-digit number to find a product.

- When multiplying by two-digit numbers, an area model can be used to represent the value of each digit in the two-digit numbers to find the product.

Activity

Multiplying two-digit numbers by two-digit numbers is a skill used in everyday life. Look at the *Multiplying by Two-Digit Numbers* activity and adjust it if necessary to connect with your students.

Math Talk at Home

Encourage students to use examples of items they find at home to begin conversations about multiplying two-digit numbers by either one-digit or two-digit numbers. Encourage students to begin the conversations with the "I see" problems below, and then take turns with family members to make new multiplication problems.

Conversation Starters Below are additional conversation starters students can write in their Family Letter or math journal to engage family members:

- *I see 10 branches. I see 14 leaves on each branch. How many leaves do I see?*
- *I see 12 windows in 6 different apartments. How many windows do I see?*
- *I see 11 stacks of pennies. There are 22 pennies in each stack. How many pennies do I see?*

Connect to Community and Cultural Responsiveness

Use these activities to connect with and leverage the diverse backgrounds and experiences of all students.

Session 1 **Use with *Try It*.**

- To make the ***Try It*** problem relevant to students, encourage them to write a word problem that involves finding the product of 14 and 13. For example: *At the car dealership, there are 14 rows of new cars. In each row, there are 13 cars. How many cars are at the car dealership?* Encourage students to write their own word problems to help them build mental pictures for solving the problem. Ask them to share their word problems with partners.

Session 2 **Use with *Additional Practice Example*.**

- Explain to students that the student in the word problem takes guitar lessons to become a better guitar player. Ask students if they take lessons or practice skills to become better at doing something they enjoy. Remind students that they may have activities that they practice at home or at school. Explain to students that they can practice skills without taking private lessons. Provide examples or share a skill you have practiced.
- For example, students who like to cook may spend time with family members preparing meals to become better cooks. Students who like soccer may spend time kicking soccer balls to become better players. Students who like math may spend time solving word problems to become better mathematicians. Rewrite the word problem so that it is relevant to students by using a skill or interest they are familiar with.

Session 3 **Use with *Apply It* problem 3.**

- Display a picture of a deli and a tray of sandwiches. Ask students what they would call the deli and tray. In different parts of the United States, delis may be referred to as sandwich shops, sub shops, hoagie shops, or bodegas. Also, students may use the terms *platter*, *plate*, or *sheet* instead of *tray*. To improve student understanding of the word problem, use the terms students are familiar with, or use the familiar terms as definitions in the word problem. For example, say: *A deli, or, as we would say, a sandwich shop, is preparing trays, or platters, of sandwiches.*

Connect to Language Development

For ELLs, use the Differentiated Instruction chart to plan and prepare for specific activities in every session.

ELL English Language Learners: Differentiated Instruction

Prepare for Session 1
Use with *Try It*.

Levels 1–3

Listening/Reading Use with ***Try It***. Write the following information on sentence strips.

- *Break apart the number 13 into 10 and 3.*
- *Multiply 14 by 10.*
- *Multiply 14 by 3.*
- *Add the partial products.*

Display the sentence strips. Think aloud about the process for solving the equation $14 \times 13 = \_\_\_\_\_$. Read each strip and then complete the task. Shuffle the sentence strips. Have students read them and put them in order. Encourage students to use the sentence strips as they work through the process for solving the problem.

Levels 2–4

Listening/Reading Use with ***Try It***. Work with students to develop a process to find the product. Ask questions to help students organize their thoughts:

- *How could you break apart a number?*
- *What do you do after you break apart the number?*
- *After you multiply the numbers, what do you do?*

Write students' information on sentence strips and then shuffle them. Have students read the strips and put them in order. Encourage students to refer to the sentence strips as they solve the problem.

Levels 3–5

Reading/Writing Use with ***Try It***. Provide the following questions for partners to use as they develop a process to solve the problem:

- *How would you break apart a number? How does it help you to multiply?*
- *What do you do after you break apart the number?*
- *After you multiply the numbers, what do you do?*
- *Why do you add the partial products?*

Ask partners to write their process on sentence strips, shuffle them, and exchange them with another group. Encourage partners to read the strips, put them in order, and follow the process to solve the problem.

LESSON 12

SESSION 1 Explore

Purpose In this session, students draw on their knowledge of multiplying multi-digit numbers by one-digit numbers and of using place value and breaking apart numbers to multiply. Students share models to explore how to multiply a two-digit number by another two-digit number. They will look ahead to use basic facts and patterns of zeros in factors and products to multiply by multiples of 10 and use an area model to multiply 2 two-digit numbers.

Start

Connect to Prior Knowledge

Why Support students' facility with multiplying two-digit numbers by one-digit numbers to prepare students to multiply 2 two-digit numbers.

How Have students multiply two-digit numbers by one-digit numbers.

Multiply.

1. $23 \times 3 =$ .....
2. $47 \times 2 =$ .....
3. $61 \times 4 =$ .....

Solutions

1. 69
2. 94
3. 244

TRY IT

Make Sense of the Problem

To support students in making sense of the problem, have them show that they understand the meaning of the word *product*.

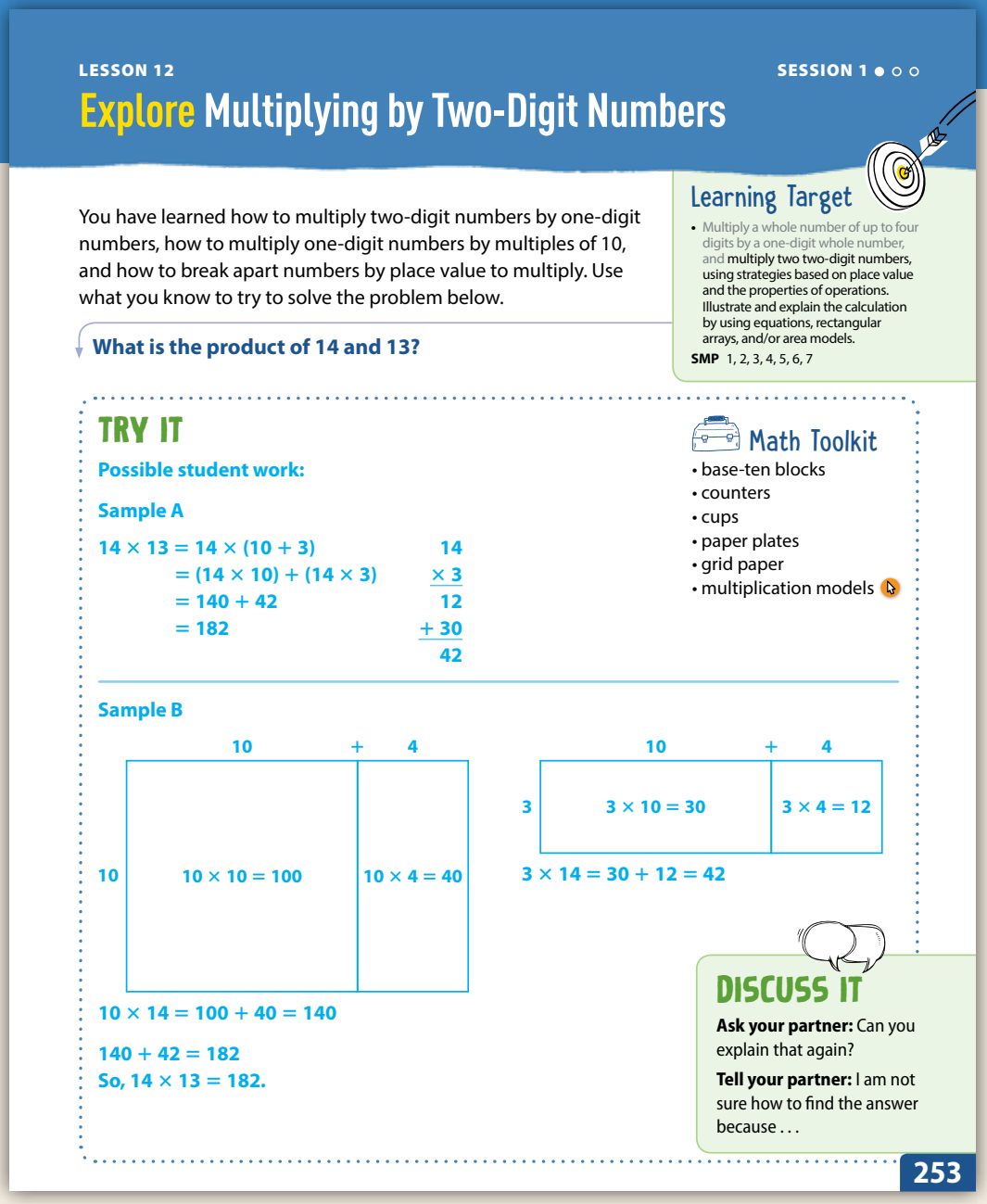
DISCUSS IT

Support Partner Discussion

To reinforce students' prior work with partial products, encourage students to use the term *partial product* as appropriate as they talk to each other.

Look for, and prompt as necessary for, understanding that:

- each two-digit number has a ones place and a tens place
- you can break apart each number and multiply to find partial products
- you can add partial products to find the product



LESSON 12 SESSION 1 ● ○ ○

Explore Multiplying by Two-Digit Numbers

You have learned how to multiply two-digit numbers by one-digit numbers, how to multiply one-digit numbers by multiples of 10, and how to break apart numbers by place value to multiply. Use what you know to try to solve the problem below.

What is the product of 14 and 13?

Learning Target

- Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

SMP 1, 2, 3, 4, 5, 6, 7

TRY IT

Possible student work:

Sample A

$14 \times 13 = 14 \times (10 + 3)$
$= (14 \times 10) + (14 \times 3)$
$= 140 + 42$
$= 182$

$$\begin{array}{r} 14 \\ \times\ 3 \\ \hline 12 \\ +\ 30 \\ \hline 42 \end{array}$$

Math Toolkit

- base-ten blocks
- counters
- cups
- paper plates
- grid paper
- multiplication models

Sample B

	10	+	4
10	10 × 10 = 100		10 × 4 = 40

$10 \times 14 = 100 + 40 = 140$

$140 + 42 = 182$

So, $14 \times 13 = 182$.

	10	+	4
3	3 × 10 = 30		3 × 4 = 12

$3 \times 14 = 30 + 12 = 42$

DISCUSS IT

Ask your partner: Can you explain that again?

Tell your partner: I am not sure how to find the answer because . . .

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Common Misconception Look for students who multiply one number by the other without taking into account the place values of the digits, such as multiplying 14 by 3 and by 1 rather than by 3 and by 10. As students present solutions, have them specify the value of each digit in the factors 14 and 13.

Select and Sequence Student Solutions

One possible order for whole class discussion:

- base-ten blocks, counters, or drawings modeling 14 groups of 13
- breaking apart one factor into tens and ones and multiplying to find partial products
- one or two area models showing partial products
- vertical multiplication of two-digit numbers showing partial products

Support Whole Class Discussion

Prompt students to note the relationship between the numbers in each model and the numbers in the problem.

Ask *How do [student name]'s and [student name]'s models represent 13 fourteen times?*

Listen for There are 14 groups of 13; 13 is added fourteen times; 13 is broken apart into 10 and 3, and each is multiplied by 14; 13 and 14 are each broken apart into tens and ones and multiplied by the tens and ones in the other factor.

CONNECT IT

1 LOOK BACK

Look for understanding of breaking apart one or both factors, multiplying to find partial products, and adding partial products to find the product.

Hands-On Activity

Use base-ten blocks to understand multiplying 2 two-digit numbers.

If . . . *students are unsure about multiplying a two-digit number by another two-digit number,*

Then . . . *use this activity to provide a concrete model of two-digit by two-digit multiplication.*

Materials For each pair: base-ten blocks (1 hundreds flat, 12 tens rods, 55 ones units)

- Present the problem 11 × 15. One partner uses base-ten blocks, and the other records the multiplication.
- Have one partner represent 15 (1 tens rod and 5 ones units) and then add more base-ten blocks to represent 15 eleven times.
- Have the partner count the number of ones units in the 11 groups as the other partner records 11 × 5 ones = 11 × 5. The first partner states the total [55 ones units = 55], and the other partner confirms it by recording the multiplication 11 × 5 = 55.
- Then have the first partner count the number of tens rods in the 11 groups as the other partner records 11 × 1 ten = 11 × 10. The first partner counts the total [11 tens rods = 110], and the other partner confirms it by recording the multiplication 11 × 10 = 110.
- Have the first partner regroup the blocks to use the fewest blocks possible. [1 hundreds flat, 6 tens rods, 5 ones units] The other partner confirms the total by recording 55 + 110 =165.
- Repeat for 11 × 13 and 12 × 14.

2 LOOK AHEAD

Point out that multiplying two-digit numbers involves breaking apart numbers by place value, using basic facts, and multiplying by multiples of 10.

Students should be able to multiply 2 two-digit numbers by completing an area model and adding partial products to find the product.

CONNECT IT

1 LOOK BACK

Explain how you found the product of 14 and 13.

Possible answer: I broke apart the 13 and multiplied 14 by 10 and 14 by 3. Then I added the products to get 182.

2 LOOK AHEAD

To multiply a two-digit number by another two-digit number, you need to understand how to multiply by multiples of 10.

a. Fill in the blanks to show how to multiply by multiples of 10.

Expression	Think of it as . . .	Think of it as . . .	Product
3 × 2	3 × 2 ones	6 ones	6
3 × 20	3 × 2 tens	6 tens	60
30 × 20	3 tens × 2 tens	6 hundreds	600
	3 × 10 × 2 × 10		
	3 × 2 × 10 × 10		
	6 × 100		

b. Complete the area model. Then add the four partial products to find 25 × 32.

	30	+	2
20	20 × 30 = 600		20 × 2 = 40
+			
5	5 × 30= 150		5 × 2 = 10

600 + 150 + 40 + 10 = 800

3 REFLECT

Suppose you want to find 30 × 30. How can you use a basic fact and breaking apart numbers to find the product of these multiples of 10?

Possible answer: Break apart 30 × 30 into 3 × 10 × 3 × 10. Change the order of factors to 3 × 3 × 10 × 10. 3 × 3 = 9; 10 × 10 = 100; 9 × 100 = 900

254

Close: Exit Ticket

3 REFLECT

Look for understanding of representing each multiple of 10 as a one-digit number multiplied by 10 and changing the order of the factors to use the basic multiplication facts 3 × 3 = 9 and 10 × 10 = 100 to find the product.

Common Misconception **If** students do not recognize that they can break apart each factor in 30 × 30 as the product of 3 × 10, **then** review how the factors 30 and 20 in the expression 30 × 20 in problem 2a are broken apart to show using a basic multiplication fact and multiplying by multiples of 10 to find the product.

Real-World Connection

Encourage students to think of everyday situations in which people may need to multiply a two-digit number by another two-digit number. Examples include calculating the number of minutes in ten or more hours and calculating the number of pennies, nickels, or dimes in ten or more dollars.

Solutions

Support Vocabulary Development

1 Ask students what *partial products* are, and when they've used them in the past to solve multiplication problems.

To guide student responses, ask questions and provide examples, as needed. Possible questions include:

- *What do you know about the words* part *and* product*?*
- Display 24 × 4. Say: *We used partial products to multiply two-digit numbers by one digit-numbers.* Have students identify the numbers and digits and guide them through the steps. Ask: *What is the first partial product? What do you do next?* Ask students to add this and other examples to the concept map.

2 Have students describe to a partner their process for completing the area model. Ask questions to guide student thinking:

- *Why are the numbers 18 and 24 broken apart by place value?*
- *When the numbers 18 and 24 are broken apart, what will you do next?*
- *When the partial products are found, what is the next step?*

Supplemental Math Vocabulary

- *place value*
- *product*
- *multiply*

Name: _____ LESSON 12 SESSION 1

Prepare for Multiplying by Two-Digit Numbers

1 Think about what you know about multiplication. Fill in each box. Use words, numbers, and pictures. Show as many ideas as you can. **Possible answers:**

What Is It?

A strategy used to multiply multi-digit numbers

What I Know About It

I can use an area model to help find partial products for a multiplication problem.

partial products

Examples

	10	+	5
10	10 × 10 = 100		10 × 5 = 50

10 × 15 = 100 + 50
= 150

Examples

12 × 15 = 12 × (10 + 5)
= (12 × 10) + (12 × 5)
= 120 + 60
= 180

Examples

37
× 5
35 5 × 7 ones
+ 150 5 × 3 tens
185

2 Complete the area model. Then add the four partial products to find 18 × 24.

	20	+	4
10	10 × 20 = **200**		10 × 4 = **40**
+			
8	8 × 20 = **160**		8 × 4 = **32**

200 + **160** + **40** + **32** = **432**

255

3 Assign problem 3 to provide another look at solving a problem by multiplying two-digit numbers.

This problem is very similar to the problem about finding the product of 14 and 13. In both problems, students are asked to multiply a two-digit number by another two-digit number. The question asks students to find the product of 16 and 12.

Suggest that students read the problem three times, asking themselves one of the following questions each time:

- *What is this problem about?*
- *What is the question I am trying to answer?*
- *What information is important?*

Solution:
Students may use any method to solve the multiplication problem. 16 × 12 = 192
Basic

4 Have students solve the problem another way to check their answer.

3 Solve the problem. Show your work.

What is the product of 16 and 12?

Possible student work:

$16 \times 12 = 16 \times (10 + 2)$
$= (16 \times 10) + (16 \times 2)$
$= 160 + 32$
$= 192$

$$\begin{array}{r} 16 \\ \times\ 2 \\ \hline 12 \\ +\ 20 \\ \hline 32 \end{array}$$

Solution $16 \times 12 = 192$

4 Check your answer. Show your work.

Possible student work:

	10	6
10	$10 \times 10 = 100$	$10 \times 6 = 60$
2	$2 \times 10 = 20$	$2 \times 6 = 12$

$100 + 60 + 20 + 12 = 100 + 80 + 12$
$= 100 + 92$
$= 192$

$16 \times 12 = 192$

ELL English Language Learners: Differentiated Instruction

Prepare for Session 2
Use with *Connect It*.

Levels 1–3	Levels 2–4	Levels 3–5
Reading/Writing Read ***Connect It*** problem 6 to students. Ask: *What strategies did you use for multiplying a two-digit number by a two-digit number?* [area model, partial products] Encourage students to confirm their responses by finding examples in ***Picture It*** and ***Model It***. Write student responses and then read them to students. Ask: *Which strategy do you like best for multiplying a two-digit number by a two-digit number?* Provide a sentence frame for students to use for their written responses: *I like using _____ to solve the multiplication equations*. Have students form pairs and practice reading their sentences to partners.	**Speaking/Writing** Read ***Connect It*** problem 6 with students. Have students brainstorm strategies used to multiply a two-digit number by a two-digit number. [area model, partial products, other strategies recommended by students] Ask: *How is an area model used to multiply a two-digit number by a two-digit number?* Have students explain their process for finding the product. Ask: *Which strategy do you like best for multiplying a two-digit number by a two-digit number? Why?* Provide a sentence frame: *I like using _____ to solve the multiplication equations because _____*. Have students use the sentence frame to provide oral responses before writing their responses.	**Speaking/Writing** Have students read ***Connect It*** problem 6 with partners. Ask partners to brainstorm strategies used to multiply a two-digit number by a two-digit number and explain their process for using each strategy. Have partners make T-charts for each strategy with the headings *Advantages* and *Disadvantages*. When partners have completed their T-charts, have them read the information to other partner groups. Encourage them to add information to their T-charts, as needed. Ask: *Which strategy do you like best for multiplying a two-digit number by a two-digit number? Why?* Call on pairs to share their T-charts with the class and to explain their thought process.

LESSON 12

SESSION 2 Develop

Purpose In this session, students solve a problem that requires finding the product of 2 two-digit numbers. Students model the multiplication either on paper or with manipulatives to find the product. The purpose of this problem is to have students develop a strategy for multiplying two-digit numbers by two-digit numbers.

Start

Connect to Prior Knowledge

Materials For each student: copy of Start slide

Why Support students' facility with breaking apart numbers by place value to multiply as well as multiplying with multiples of 10.

How Have students find the product of 2 two-digit numbers by completing expressions to break apart one factor and use partial products to find the product.

Complete to find 12 × 32.

12 × (..... + ...)

(12 × .....) + (12 × .....)

..... + ...

.....

Possible Solution

12 × (30 + 2)

(12 × 30) + (12 × 2)

360 + 24

384

Develop Language

Why Clarify the meaning of the term *row*.

How Explain to students that the word *row* can be a straight line of people or things that are next to one another. Remind students that they line up in a row, or straight line, one after another, when they go to the cafeteria or library. Ask students to give real-world examples of rows they may see at home or in school. Have students close their eyes and visualize rows of chairs in a school auditorium or cafeteria and then describe to partners what they see in their mental images.

TRY IT

Make Sense of the Problem

To support students in making sense of the problem, have them identify that there are 16 rows of chairs and that each row has 28 chairs in it.

Ask *What are you trying to find out? What do you know?*

Develop Multiplying by Two-Digit Numbers

Read and try to solve the problem below.

Folding chairs are set up in a school auditorium for a play. There are 16 rows of chairs. Each row has 28 chairs. How many folding chairs are set up for the play?

TRY IT

Math Toolkit
- base-ten blocks
- grid paper
- multiplication models

Possible student work:

Sample A

16	16	320
× 20	× 8	+ 128
120	48	448
+ 200	+ 80	
320	128	

There are 448 chairs set up for the play.

Sample B

	20	+ 8
10	10 × 20 = 200	10 × 8 = 80
+		
6	6 × 20 = 120	6 × 8 = 48

16 × 28 = 200 + 80 + 120 + 48
= 448

There are 448 folding chairs set up for the play.

DISCUSS IT

Ask your partner: Why did you choose that strategy?

Tell your partner: A model I used was . . . It helped me . . .

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DISCUSS IT

Support Partner Discussion

Encourage students to use the terms *partial product* and *hundreds, tens*, and *ones* as they discuss their solutions.

Support as needed with questions such as:

- *How did you break apart the numbers 16 and 28?*
- *How did you find the total that represents the product?*

Common Misconception Look for students who find only some, not all, of the partial products. Remind students that they must multiply the value of each digit in 16 by the value of each digit in 28.

Select and Sequence Student Solutions

One possible order for whole class discussion:

- base-ten blocks or drawings modeling 16 groups of 28
- breaking apart one factor into tens and ones and multiplying to find partial products
- area model showing partial products
- vertical multiplication of two-digit numbers showing partial products

Support Whole Class Discussion

Compare and connect the different representations and have students identify how they are related.

Ask *Where does your model show the tens and ones in 16? the tens and ones in 28? multiplication of the tens and ones in each number by the tens and ones in the other? the partial products? the product?*

Listen for Students should recognize that accurate responses include representations showing 16 and 28 broken apart into tens and ones, multiplying the value of each place in each number by the value of each place in the other to find partial products, and adding partial products to find the product.

PICTURE IT & MODEL IT

If no student presented these models, connect them to the student models by pointing out the ways they each represent:

- the number of tens and ones in 16 and 28
- multiplying the value of each place in 28 by the value of each place in 16
- partial products of 10 × 20, 10 × 8, 6 × 20, and 6 × 8

Ask *How do the models show breaking apart 16 and 28 into tens and ones? using place value to multiply? partial products?*

Listen for The area model shows 16 as 10 + 6 on the side of the model and 28 as 20 + 8 on the top; the other model shows the ones and tens in 16 and 28 next to the partial products. Both show the 2 tens and the 8 ones in 28 multiplied by the 1 ten and the 6 ones in 16 to find four partial products that are added to find the product.

For an area model, prompt students to identify how multiplication by place value is shown.

- *Where is the number of rows of chairs in the model?*
- *Why is the number of rows of chairs shown this way?*
- *Where is the number of chairs in each row?*
- *Why is the number of chairs shown this way?*

For partial products, prompt students to identify how multiplication by place value is shown.

- *How is multiplication of 28 by 16 represented?*
- *How are the partial products represented?*
- *Why do you think there are four partial products?*

Explore different ways to understand multiplying a two-digit number by a two-digit number.

Folding chairs are set up in a school auditorium for a play. There are 16 rows of chairs. Each row has 28 chairs. How many folding chairs are set up for the play?

PICTURE IT

You can use an area model to multiply two-digit numbers.

To solve this problem, multiply 28 by 16.

	20	+	8
10	10 × 20 = 200		10 × 8 = 80
+			
6	6 × 20 = 120		6 × 8 = 48

200 + 80 + 120 + 48 = ?

MODEL IT

You can also multiply two-digit numbers using partial products.

28
× 16
48 → 6 ones × 8 ones
120 → 6 ones × 2 tens
80 → 1 ten × 8 ones
+ 200 → 1 ten × 2 tens
?

Deepen Understanding

Partial Products

SMP 7 Use structure.

When discussing the partial products model, prompt students to consider the effect of changing the order of the factors. Display the vertical multiplication problem 16 × 28 with the order of the factors reversed. Work together with students to solve the problem.

Ask *Compare this multiplication with the multiplication shown in the* Model It. *What is the same about the partial products? What is different?*

Listen for The partial products are the same but in a different order.

Ask *Why does it make sense that the partial products are the same?*

Listen for You multiply the same ones and tens digits in 16 by the same ones and tens digit in 28, so the four partial products are the same.

Generalize *Do you think this is true for multiplying any two-digit number by any other two-digit number?* Have students explain their reasoning. Listen for understanding that you can multiply 2 two-digit numbers in any order and the partial products will be the same.

CONNECT IT

- Remind students that one thing that is alike about all the representations is the numbers.
- Explain that on this page, students will use those numbers to show how both models represent multiplying 2 two-digit numbers by breaking apart the numbers and using place value to multiply in order to find the partial products and the product.

Monitor and Confirm

1–3 Check for understanding that:

- 16 and 28 are broken apart by place value in both the area model and the partial products model
- each of the four sections in the area model represents one partial product
- each step in the partial products model shows the product in one section of the area model
- the sum of the four partial products, 448, is the product of 28 × 16

Support Whole Class Discussion

4–5 Be sure students understand that problem 4 is asking them about breaking apart the factor 28 in a different way and that problem 5 is asking how to check that the answer to a two-digit multiplication problem is reasonable.

Ask *Why might you be likely to break apart 28 into 10 + 10 + 8 instead of into 20 + 8? How does this affect the partial products you get and the product?*

Listen for It is easy to multiply by 10. The partial products are different, but their sum, the product, is the same. You can break apart a factor in different ways to multiply, and the product stays the same.

Ask *How is it helpful to use easier numbers to check the reasonableness of your answer to a two-digit multiplication problem?*

Listen for Multiplying with easier numbers is a quick way to check whether your answer makes sense.

5 Look for the idea that you can use nearby multiples of 10 as factors because it is easy to multiply by multiples of 10.

6 REFLECT

Have all students focus on the strategies used to solve this problem. If time allows, have students share their responses with a partner.

CONNECT IT

Now you will use the problem from the previous page to help you understand how to multiply a two-digit number by a two-digit number.

1 Why is the area model divided into four sections?

Each number in the expanded form of one factor is multiplied by each number in the expanded form of the other factor. Each section shows a product.

2 How do the four steps in the multiplication using partial products in **Model It** relate to the four sections in the area model in **Picture It**?

Each step shows the product in one section of the area model.

3 What is the sum of the partial products and also the product of 28 and 16?

448

4 Would the product change if 20 + 8 on the top of the area model were changed to 10 + 10 + 8? Explain.

No, the product would be the same. Possible explanation: Instead of a partial product of 200, you would have two partial products of 100. Instead of a partial product of 120, you would have two partial products of 60. The sum of all the partial products would still be the same.

5 How could you estimate to check the reasonableness of your answer to 28 × 16 by multiplying with easier numbers?

Possible answer: 30 × 10 = 300 and 30 × 20 = 600. My answer is between 300 and 600, so it is reasonable.

6 REFLECT

Look back at your **Try It**, strategies by classmates, and **Picture It** and **Model It**. Which models or strategies do you like best for multiplying a two-digit number by a two-digit number? Explain.

Some students may like drawing an area model because they can add the area of each section to find the product. Other students may like using partial products because they can add them to find the product.

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Hands-On Activity

Use base-ten blocks to multiply 2 two-digit numbers.

If . . . *students are unsure about multiplying two-digit numbers using partial products,*

Then . . . *use this activity to have them multiply using base-ten blocks.*

Materials For each pair: base-ten blocks (2 hundreds flats, 14 tens rods, 19 ones units)

- Distribute base-ten blocks to each pair. Have partners use the steps below to model 23 × 14 using base-ten blocks in a way similar to that of an area model.
- Model the factor 23 on a flat surface by displaying 2 tens rods and 3 ones units side by side in a single horizontal row.
- Model the factor 14 by displaying 1 tens rod and 4 ones units in a single vertical column to the left of and just beneath the horizontal row showing 23.
- Find each partial product by filling the area inside with the corresponding values: 2 hundred flats, 3 tens rods, 8 tens rods, and 12 ones units.
- The product is the value of the inside blocks. [200 + 110 + 12 = 322]
- Repeat the activity for another two-digit multiplication, such as 24 × 12.

APPLY IT

For all problems, encourage students to draw some kind of model to support their thinking. Allow some leeway in precision of student-drawn models.

7 567; See completed area model on the Student Worktext page; Students may add the partial products in any order to find the product.

8 851; See possible work on the Student Worktext page; Students may also make an area model to find the product.

Close: Exit Ticket

9 **A**; $10 \times 64 = 640$

D; $(30 + 2) \times 20 = (30 \times 20) + (2 \times 20) = 600 + 40 = 640$

E; $(40 \times 10) + (40 \times 6) = 400 + 240 = 640$

Error Alert **If** students choose B, **then** have them break apart each factor in 60×40 and rewrite the product as $(6 \times 10) \times (4 \times 10)$ and then change the order of the factors to $6 \times 4 \times 10 \times 10$, resulting in 24×100, to recognize that the product is 2,400, not 640.

APPLY IT

Use what you just learned to solve these problems.

7 Complete the area model below. Then add the partial products to find the product of 27 and 21. Show your work.

Possible student work:

	20	+ 7
20	20 × 20 = **400**	20 × 7 = **140**
+ 1	1 × 20 = **20**	1 × 7 = **7**

400 + 140 + 20 + 7 = 567

Solution **567**

8 Find 37×23. Show your work.

Possible student work:

```
  37
× 23
  21
  90
 140
+600
 851
```

Solution **851**

9 Select all the expressions that have a product of 640.

Ⓐ 10×64 Ⓑ 60×40

Ⓒ 80×80 Ⓓ $(30 + 2) \times 20$

Ⓔ $(40 \times 10) + (40 \times 6)$

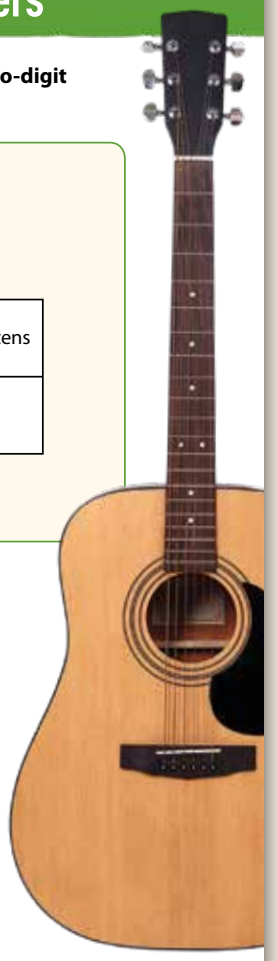
Solutions

1. See completed multiplication using partial products on the student page.
Basic

2. $71 \times 48 = 2{,}800 + 560 + 40 + 8 = 3{,}408$;
Students may add partial products in any order.
See possible student work showing an area model on the student page.
Medium

Name: _____ LESSON 12 SESSION 2

Practice Multiplying by Two-Digit Numbers



Study the Example showing how to multiply a two-digit number by a two-digit number to solve a word problem. Then solve problems 1–6.

EXAMPLE

Aaron spends 35 minutes at each guitar lesson. He has 12 guitar lessons.
How many minutes does Aaron spend at his guitar lessons?

Use an area model to multiply 35 by 12.

	30	+	5
10	10 × 30 1 ten × 3 tens = 3 hundreds **300**		10 × 5 1 ten × 5 = 5 tens **50**
+			
2	2 × 30 2 × 3 tens = 6 tens **60**		2 × 5 = **10**

300 + **50** + **60** + **10** = **420**

Aaron spends 420 minutes at his guitar lessons.

1. Look at the Example above. Use partial products to multiply 35 by 12. Fill in the blanks.

	3	5	
×	1	2	
	1	0	→ **2** ones × **5** ones
	6	**0**	→ 2 **ones** × 3 **tens**
	5	0	→ **1** ten × 5 **ones**
+	**3**	**0**	**0** → **1** ten × 3 **tens**
	4	2	0

2. Show how to use an area model to multiply 71 by 48.

	70	+	1
40	40 × 70 = 2,800		40 × 1 = 40
+			
8	8 × 70 = 560		8 × 1 = 8

Possible model shown. Students may add partial products in any order.

71 × 48 = **2,800** + **560** + **40** + **8** = **3,408**

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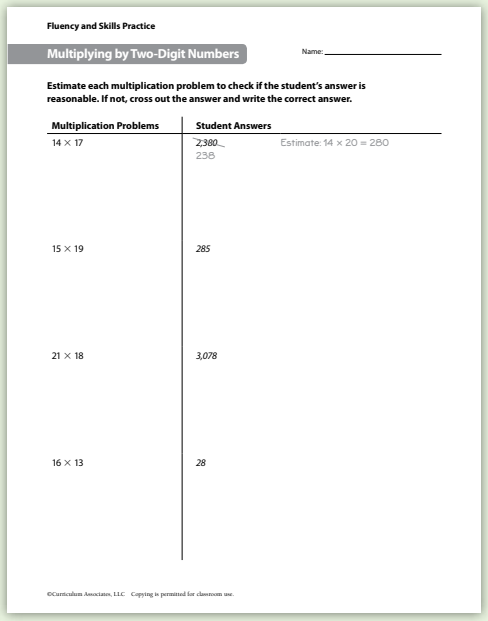
Fluency & Skills Practice

Teacher Toolbox

Assign *Multiplying by Two-Digit Numbers*

In this activity students use estimation to check that two two-digit numbers have been multiplied correctly. Students may use rounding or another estimation strategy to check the reasonableness of the given answer. If students do not think the answer is reasonable, then they multiply to find the correct product. Students may apply the same process in real-world situations. For example, suppose a bowling team buys 12 T-shirts at $14 each. The team may want to check that the total price they are charged is reasonable.



Fluency and Skills Practice

Multiplying by Two-Digit Numbers

Name: _____

Estimate each multiplication problem to check if the student's answer is reasonable. If not, cross out the answer and write the correct answer.

Multiplication Problems	Student Answers	
14 × 17	~~2,380~~ 238	Estimate: 14 × 20 = 280
15 × 19	285	
21 × 18	3,078	
16 × 13	28	

3 3,408; See multiplication with partial products on the student page.
Medium

4 **A** (**True**);
D (**False**);
F (**False**);
G (**True**)
Medium

5 1,120 glass tiles; 28 rounds to 30; 30 × 40 = 1,200. 1,200 is close to 1,120. My answer is reasonable. Students may use an area model or partial products to show that 28 × 40 = 1,120.
Medium

6 1,560 minutes; Students may show two different models such as area models and partial products or show various groupings to multiply. See possible student work on the student page.
Challenge

3 Show how to use partial products to multiply 48 by 71.

48 × 71 = 3,408

```
  48
× 71
   8
  40
 560
+ 2,800
3,408
```

4 Tell whether each equation is *True* or *False*.

	True	False
18 × 42 = (10 × 40) + (10 × 2) + (8 × 40) + (8 × 2)	Ⓐ	Ⓑ
60 × 15 = (6 × 10) + (6 × 5)	Ⓒ	Ⓓ
37 × 22 = (30 × 20) + (30 × 20) + (7 × 20) + (7 × 20)	Ⓔ	Ⓕ
99 × 11 = (1 × 9) + (1 × 90) + (10 × 9) + (10 × 90)	Ⓖ	Ⓗ

5 Mr. Greene prepares 28 bags of glass tiles for his art class. He puts 40 glass tiles in each bag. How many glass tiles does Mr. Greene use? Estimate to check that your answer is reasonable. Show your work.

Students may use an area model or partial products to show that 28 × 40 = 1,120.

Solution **1,120 glass tiles; 28 rounds to 30; 30 × 40 = 1,200. 1,200 is close to 1,120. My answer is reasonable.**

6 Stephanie has 6 classes a day at school. Each class is 52 minutes long. She goes to school 5 days a week. How much time does she spend in class each week? Show two different ways to solve this problem. Show your work.

Answers will vary. Possible answer:
Way 1: first multiply 6 × 5, then multiply 30 × 52.
Way 2: multiply 6 × 52, then 312 × 5.

Students may show 2 different models for multiplying (area model and partial products) or show various groupings to multiply.

Solution **1,560 minutes**

ELL English Language Learners: Differentiated Instruction

Prepare for Session 3
Use with *Apply It*.

Levels 1–3

Listening/Writing Use with ***Apply It*** problem 9. Display the following questions:

1. *What will the problem be about?*
2. *What 2 two-digit numbers will be used?*
3. *What will you find out?*

Read the first question. Ask students to make suggestions. If needed, show pictures, such as cookies on platters or books on shelves. Read the second question. Ask students to identify 2 two-digit numbers. Read the third question. Model using the following sentence frames to write a word problem: *I have 42 cookies on 13 plates. How many cookies do I have in all?* Ask students to make original problems using the sentence frames.

Levels 2–4

Writing/Speaking Have students work in pairs. Read ***Apply It*** problem 9 with students. Provide the following questions to guide pairs in writing their own word problem:

1. *Who is your problem about?*
2. *What is your problem about?*
3. *What 2 two-digit numbers will you multiply in the problem?*
4. *What do you want to find out?*

When partners have completed writing their word problems, have them explain to each other how they will find the answer. Suggest students use sequencing words (*first, next, then*) to help them organize their thoughts. Call on pairs to read their word problems to the class.

Levels 3–5

Writing/Reading Use with ***Apply It*** problem 9. Have students write word problems, referring to the following guiding questions as needed:

1. *Who is your problem about?*
2. *What is your problem about?*
3. *What 2 two-digit numbers will you multiply in the problem?*
4. *What do you want to find out?*

Have students exchange their word problems. Say: *Read your partner's word problem. Then explain to your partner how you will find the answer.* When students have completed the activity, have them record their word problems and explanations in their math journal.

LESSON 12

SESSION 3 Refine

Purpose In this session, students solve problems involving multiplying a two-digit number by another two-digit number, then discuss and confirm their answers with a partner.

Before students begin to work, use their responses to the *Check for Understanding* to determine those who will benefit from additional support.

As students complete the Example and problems 1–3, observe and monitor their reasoning to identify groupings for differentiated instruction.

Start

Check for Understanding

Materials For remediation: base-ten blocks (4 tens rods, 11 ones units)

Why Confirm understanding of multiplying a two-digit number by a two-digit number.

How Have students find the product of 36 and 15 using any strategy they want.

Find the product of 36 and 15.

Solution
540

LESSON 12 SESSION 3 ● ● ●

Refine Multiplying by Two-Digit Numbers

Complete the Example below. Then solve problems 1–9.

EXAMPLE

What is the product of 73 and 58?

Look at how you could show your work using partial products.

```
  73
× 58
  24  → 8 ones × 3 ones
 560  → 8 ones × 7 tens
 150  → 5 tens × 3 ones
+3,500 → 5 tens × 7 tens
```

Solution 4,234

The student added the partial products to find 73 × 58.

PAIR/SHARE
How else could you solve this problem?

APPLY IT

1 Find the product of 15 and 24. Show your work.

Possible student work:

```
  15
× 24
  20  → 4 ones × 5 ones
  40  → 4 ones × 1 ten
 100  → 2 tens × 5 ones
+200  → 2 tens × 1 ten
 360
```

Should you multiply 15 × 24 or 24 × 15?

PAIR/SHARE
How did you decide which method to use to help you solve the problem?

Solution 360

263

Error Alert

If the error is . . .	*Students may . . .*	*To support understanding . . .*
51	have added.	Remind students that *product* means multiplication.
54	have found all partial products as ones digits multiplied by ones digits.	Have students use base-ten blocks to show 36 as 30 + 6 and 15 as 10 + 5. Guide students to draw an area model to show each partial product.
270	have incorrectly found the tens by tens partial product as 3 × 10.	Remind students that when multiplying tens by tens, the result is 30 × 10 = 300, not 3 × 10 = 30.
440	have incorrectly added partial products.	Remind students that they need to regroup 14 tens as 1 hundred and 4 tens when adding partial products.

EXAMPLE

4,234; Using partial products to multiply is shown as one way to solve the problem. Students could also solve the problem by using an area model.

Look for Add the four partial products of 24, 560, 150, and 3,500 to find the product.

APPLY IT

1 360; Students could solve the problem by multiplying the value of each digit in each factor by the value of each digit in the other factor to get four partial products and then add the partial products to find the product.
DOK 1

Look for You can multiply the factors 15 and 24 in any order.

2 384; Students could solve the problem by using an area model to find partial products of 300, 60, 20, and 4 and adding the partial products to get a product of 384.
DOK 1

Look for An area model may represent the factors 12 and 32 as 10 + 2 multiplied by 30 + 2 with four sections showing partial products of 300, 60, 20, and 4.

3 **C**; Students could solve the problem by using partial products to multiply 48 by 23.

Explain why the other two answer choices are not correct:

B is not correct because the product is the sum of the four partial products, 24 + 120 + 160 + 800, not the sum of only two partial products, 24 + 800.

D is not correct because the product is the sum of 3 × (40 + 8) and 20 × (40 + 8), not the sum of 3 × (4 + 80) and 20 × (4 + 80).
DOK 3

2 What is the product of 12 and 32? Show your work.

Possible student work:

	30	+ 2
10	10 × 30 = 300	10 × 2 = 20
+ 2	2 × 30 = 60	2 × 2 = 4

300 + 60 + 20 + 4 = 384

Solution **384**

Could you use an area model to help solve the problem?

PAIR/SHARE
How could you check your answer for reasonableness?

3 A deli is preparing trays of sandwiches. There are 48 trays. Each tray has 23 sandwiches. How many sandwiches are there?

Ⓐ 240
Ⓑ 824
Ⓒ 1,104
Ⓓ 1,932

How could partial products help you solve this problem?

Nathan chose Ⓐ as the correct answer. How did he get that answer?

Nathan multiplied 2 by 40 and 2 by 8 instead of multiplying 20 by 40 and 20 by 8.

PAIR/SHARE
Does Nathan's answer make sense?

4 **D**; Find the number of minutes in 3 hours, 60 × 3 = 180. Multiply the number of blinks each minute by 180, 16 × 180 = 2,880.
DOK 2

Error Alert Students who choose A multiplied the number of blinks each minute by the number of hours, without taking into account the number of minutes in an hour.

5 4,277; Multiply the value of each digit in 47 by the value of each digit in 91 to get the four partial products of 7, 630, 40, and 3,600 and then add the partial products to get a product of 4,277.
DOK 2

6 **A**; The rectangular array represents the factor 45 as rows of 40 + 5 and the factor 15 as columns of 10 + 5. The total number of squares in the array represents the solution to 45 × 15.

D; The expression breaks apart the factors 15 and 45 by place value and represents 45 × 15 as (10 × 40) + (10 × 5) + (5 × 40) + (5 × 5).

E; The number line represents 45 × 15 as 15 jumps of 45.
DOK 2

4 A person blinks about 16 times per minute. About how many times does a person blink in 3 hours? [*Hint:* 1 hour = 60 minutes]

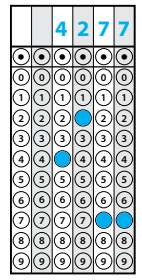
Ⓐ 48

Ⓑ 96

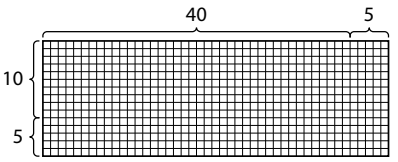
Ⓒ 960

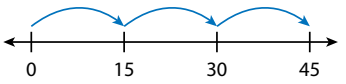
Ⓓ 2,880

5 What is the product of 47 and 91?



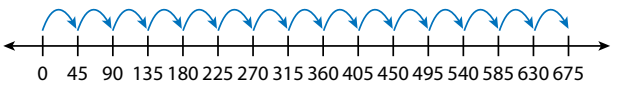
6 Which models below could represent the solution to the problem 45 × 15?

Ⓐ


Ⓑ


Ⓒ (4 × 1) + (4 × 5) + (5 × 1) + (5 × 5)

Ⓓ (10 × 40) + (10 × 5) + (5 × 40) + (5 × 5)

Ⓔ


Differentiated Instruction

RETEACH

Hands-On Activity

Use play money to understand multiplying two-digit numbers.

Students struggling with multiplying two-digit numbers

Will benefit from additional work with concrete representations of place value

Materials For each pair: play money (93 $1 bills, 62 $10 bills, and 6 $100 bills)

- Distribute $1, $10, and $100 bills to partners. Present multiplication problems in which partners multiply two-digit whole dollar amounts by two-digit numbers, such as $23 × 31 and $26 × 14.
- Have students count out the dollar amount using the least number of bills. Then have them count out the number of additional sets represented by the other factor. Students find and record the total amount of $1s and $10s. Then have students exchange ten $1s for one $10 and ten $10s for $100 as needed. Connect this to finding partial products using place value. [$23 × 31 = $713; $26 × 14 = $364]
- Have students do the same multiplication problem on paper to see the connection.

EXTEND

Challenge Activity

Solve two-step word problems involving multiplication.

Students who have achieved proficiency

Will benefit from deepening understanding by solving two-step word problems that involve multiplication

Have students solve the following problem.

- Amelia earns $12 for each hour she works as a math tutor. She works as a tutor for 16 hours one week. She also earns $25 that week for watering her neighbor's garden. How much does Amelia earn altogether that week? [$217]

7 Ian's equation: $43 \times 23 = 989$, Tia's equation: $43 \times 24 = 1{,}032$; Students may solve the problem by substituting numbers beginning with 20 in Tia's equation to identify 24 as the least number to yield a four-digit product. The number 1 less, 23, is then the greatest number to yield a three-digit product.
DOK 2

8 490 minutes; Multiply the value of each digit in 14 by the value of each digit in 35 to get the four partial products of 20, 50, 120, and 300 and then add the partial products to get a product of 490. An area model may also be used to find the partial products and product.
DOK 2

7 Complete each equation below using a factor between 20 and 30 so that:

- The missing factor in Ian's equation will give the greatest possible three-digit product.
- The missing factor in Tia's equation will give the least possible four-digit product.

Ian's equation: $43 \times$ 23 $=$ 989

Tia's equation: $43 \times$ 24 $=$ 1,032

8 Mo has 14 tutoring sessions. Each session is 35 minutes long. How many minutes does Mo spend in the 14 sessions? Show your work.

Possible student work:

$$\begin{array}{r} 14 \\ \times\ 35 \\ \hline 20 \\ 50 \\ 120 \\ +\ 300 \\ \hline 490 \end{array}$$

Solution 490 minutes

9 MATH JOURNAL

Write a word problem you can solve by multiplying 2 two-digit numbers. Solve the problem and show how to find the answer.

Possible answer: Tara buys 18 packages with 12 muffins each. How many muffins does she buy in all? Multiply 18 and 12 using partial products: $(2 \times 8) + (2 \times 10) + (10 \times 8) + (10 \times 10) = 16 + 20 + 80 + 100 = 216$. Tara buys 216 muffins.

SELF CHECK Go back to the Unit 3 Opener and see what you can check off.

REINFORCE

Problems 4–9

Multiply by two-digit numbers.

All students will benefit from additional work with multiplying by two-digit numbers by solving problems in a variety of formats.

- Have students work on their own or with a partner to solve the problems.
- Encourage students to show their work.

PERSONALIZE

i-Ready

Provide students with opportunities to work on their personalized instruction path with *i-Ready* Online Instruction to:

- fill prerequisite gaps
- build up grade level skills

Close: Exit Ticket

9 MATH JOURNAL

Student responses should indicate understanding of how to represent in words a situation involving multiplication of 2 two-digit numbers as well as how to break apart two-digit numbers by place value to multiply and how to multiply by multiples of ten.

Error Alert **If** students write an accurate word problem that involves multiplying 2 two-digit numbers but make an error in calculating the product, **then** have students draw an area model to represent the problem, break apart each factor by place value and find the four partial products and then add the partial products to find the product.

SELF CHECK Have students consider whether they feel they are ready to check off any new skills on the Unit 3 Opener.