Lesson Overview

LESSON 31

Classify Quadrilaterals

Lesson Objectives

Content Objectives

- Identify quadrilaterals and their attributes.
- Draw quadrilaterals, given attributes.
- Compare and contrast attributes of quadrilaterals.
- Identify shared attributes of different quadrilaterals.
- Categorize quadrilaterals according to attributes.
- Identify and draw quadrilaterals that do not belong to a given category.

Language Objectives

- Define the key vocabulary terms *attribute*, *parallel*, *parallelogram*, *quadrilateral*, *rectangle*, and *rhombus* to discuss reasoning.
- Draw a quadrilateral with given attributes.

Prerequisite Skills

- Identify sides and angles of quadrilaterals, including right angles.
- Understand that all quadrilaterals do not look the same.
- Know the attributes of quadrilaterals.

Standards for Mathematical Practice (SMP)

SMPs 1, 2, 3, 4, 5, and 6 are integrated in every lesson through the *Try-Discuss-Connect* routine.*

In addition, this lesson particularly emphasizes the following SMPs:

1 Make sense of problems and persevere in solving them.

5 Use appropriate tools strategically.

7 Look for and make use of structure.

*See page 455i to see how every lesson includes these SMPs.

Lesson Vocabulary

- **attribute** any characteristic of an object or shape, such as number of sides or angles, lengths of sides, or angle measures.
- **parallel** always the same distance apart.
- **parallelogram** a quadrilateral with opposite sides parallel and equal in length.

Review the following key terms.

- **quadrilateral** a two-dimensional closed shape with exactly 4 sides and 4 angles.
- **rectangle** a quadrilateral with 4 right angles. Opposite sides of a rectangle are the same length.
- **rhombus** a quadrilateral with all sides the same length.
- **right angle** an angle that looks like the corner of a square.

Learning Progression

In the previous lesson students identified and compared attributes or characteristics of two-dimensional shapes and grouped shapes into categories based on their attributes.

In this lesson students consider how categories of shapes are related as they classify quadrilaterals. They identify quadrilaterals as four-sided shapes and recognize that other attributes of quadrilaterals distinguish one shape from another. Students identify parallelograms, rectangles, and rhombuses based on attributes, such as the number of right angles, presence of parallel sides, and sides and pairs of sides that are the same length. Students compare attributes of squares and rectangles and come to understand that although all squares are rectangles, not all rectangles are squares. Students also name and draw quadrilaterals based on given attributes.

In Grade 4 students extend classifying geometric figures to include classifying hexagons, trapezoids, and triangles. Students will classify and name triangles based on the lengths of their sides as well as by the kinds of angles they have.

Lesson Pacing Guide

Whole Class Instruction

SESSION 1
Explore
45–60 min

Interactive Tutorial* *(Optional)*
Prerequisite Review:
Understand Categories of Shapes

Classifying Quadrilaterals
- Start *5 min*
- Try It *10 min*
- Discuss It *10 min*
- Connect It *15 min*
- Close: Exit Ticket *5 min*

Additional Practice
Lesson pages 691–692

SESSION 2
Develop
45–60 min

Comparing Quadrilaterals
- Start *5 min*
- Try It *10 min*
- Discuss It *10 min*
- Picture It & Model It *5 min*
- Connect It *10 min*
- Close: Exit Ticket *5 min*

Additional Practice
Lesson pages 697–698

Fluency
Comparing Quadrilaterals

SESSION 3
Develop
45–60 min

Naming and Drawing Quadrilaterals
- Start *5 min*
- Try It *10 min*
- Discuss It *10 min*
- Model It & Solve It *5 min*
- Connect It *10 min*
- Close: Exit Ticket *5 min*

Additional Practice
Lesson pages 703–704

Fluency
Naming and Drawing Quadrilaterals

SESSION 4
Refine
45–60 min

Classifying Quadrilaterals
- Start *5 min*
- Example & Problems 1–3 *15 min*
- Practice & Small Group Differentiation *20 min*
- Close: Exit Ticket *5 min*

Lesson Quiz
or **Digital Comprehension Check**

Lesson Materials

Lesson *(Required)*	none
Activities	*Per student:* ruler, 4 straws, 1 piece of string or yarn about a yard long, scissors, crayons or markers *Per pair:* 5 sheets of blank unlined paper *Per group:* 4 index cards *Activity Sheets:* 1-Centimeter Grid Paper**, Dot Paper
Math Toolkit	geoboards, rubber bands, rulers, 1-centimeter grid paper, 1-inch grid paper, dot paper, index cards, sticky notes, colored pencils, toothpicks

**Used for more than one activity.

*We continually update the Interactive Tutorials. Check the Teacher Toolbox for the most up-to-date offerings for this lesson.

Teacher Toolbox

Small Group Differentiation

PREPARE

Ready Prerequisite Lesson
Grade 2
- Lesson 28 Recognize and Draw Shapes

RETEACH

Tools for Instruction
Grade 2
- Lesson 28 Draw and Describe Shapes

Grade 3
- Lesson 31 Categories of Plane Figures

REINFORCE

Math Center Activity
Grade 3
- Lesson 31 Quadrilaterals

EXTEND

Enrichment Activity
Grade 3
- Lesson 31 Shape Search

i-Ready

Independent Learning

PERSONALIZE

i-Ready Lesson*
Grade 3
- Classify and Compare Quadrilaterals

Connect to Family, Community, and Language Development

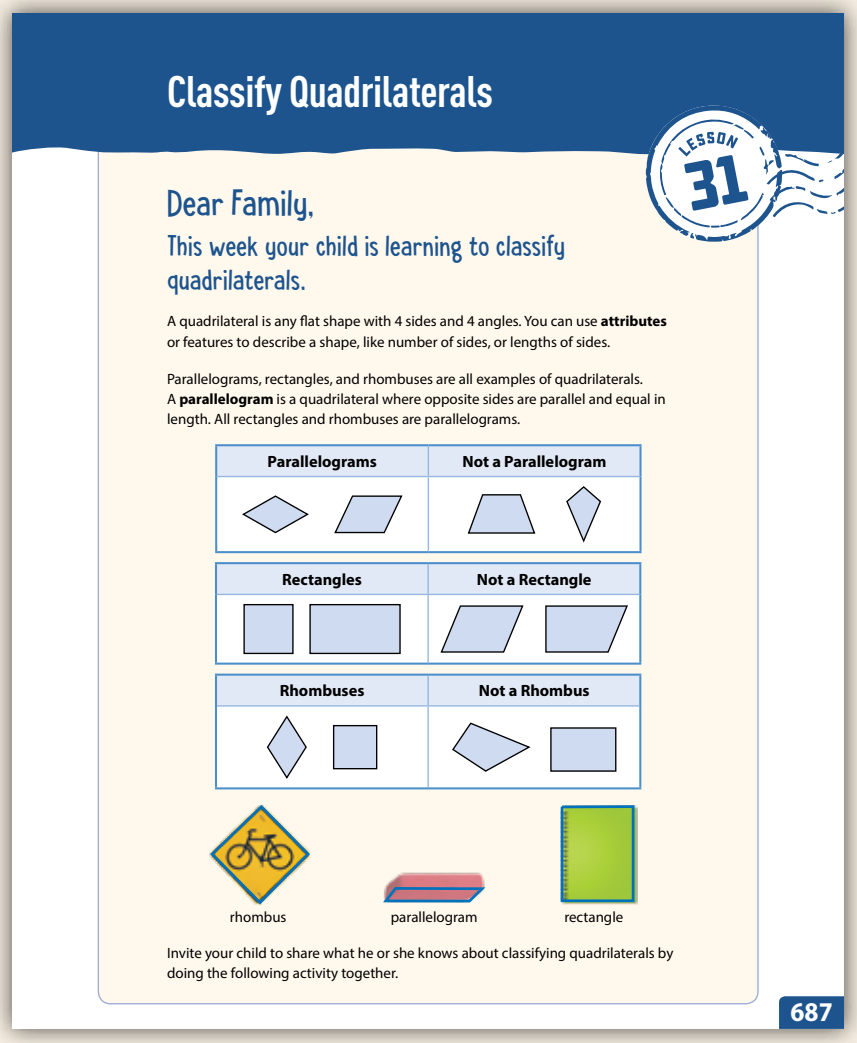
The following activities and instructional supports provide opportunities to foster school, family, and community involvement and partnerships.

Connect to Family

Use the **Family Letter**—which provides background information, math vocabulary, and an activity—to keep families apprised of what their child is learning and to encourage family involvement.

Available in Spanish

Teacher Toolbox



Classify Quadrilaterals

LESSON 31

Dear Family,

This week your child is learning to classify quadrilaterals.

A quadrilateral is any flat shape with 4 sides and 4 angles. You can use **attributes** or features to describe a shape, like number of sides, or lengths of sides.

Parallelograms, rectangles, and rhombuses are all examples of quadrilaterals. A **parallelogram** is a quadrilateral where opposite sides are parallel and equal in length. All rectangles and rhombuses are parallelograms.

Parallelograms	Not a Parallelogram

Rectangles	Not a Rectangle

Rhombuses	Not a Rhombus

rhombus parallelogram rectangle

Invite your child to share what he or she knows about classifying quadrilaterals by doing the following activity together.

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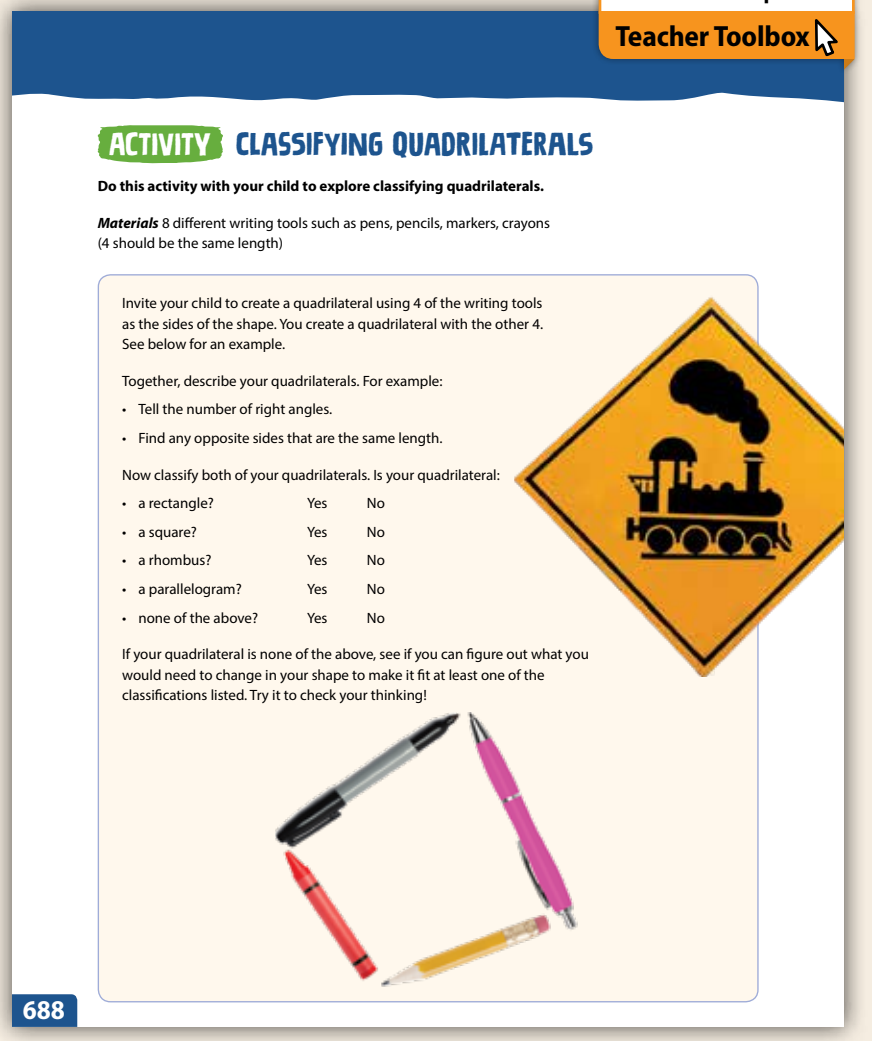

ACTIVITY CLASSIFYING QUADRILATERALS

Do this activity with your child to explore classifying quadrilaterals.

Materials 8 different writing tools such as pens, pencils, markers, crayons (4 should be the same length)

Invite your child to create a quadrilateral using 4 of the writing tools as the sides of the shape. You create a quadrilateral with the other 4. See below for an example.

Together, describe your quadrilaterals. For example:

- Tell the number of right angles.
- Find any opposite sides that are the same length.

Now classify both of your quadrilaterals. Is your quadrilateral:

- a rectangle? Yes No
- a square? Yes No
- a rhombus? Yes No
- a parallelogram? Yes No
- none of the above? Yes No

If your quadrilateral is none of the above, see if you can figure out what you would need to change in your shape to make it fit at least one of the classifications listed. Try it to check your thinking!

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Goal

The goal of the Family Letter is to introduce the formal geometric term *quadrilateral* to classify flat (two-dimensional) four-sided shapes.

- Quadrilaterals are classified by identifying attributes such as the length of sides and number of right angles. Examples and non-examples of parallelograms, rectangles, and rhombuses are provided. Shapes are compared to find relationships between them.

Activity

Look at the *Classifying Quadrilaterals* activity and adjust it if necessary to connect with your students.

Math Talk at Home

Encourage students to talk with their family members about quadrilaterals and their attributes, or features. Ask students to note which quadrilaterals they are able to locate in their homes. Have students also note which quadrilaterals are found most often.

Conversation Starters Below are additional conversation starters students can write in their Family Letter or math journal to engage family members:

- *Name all of the quadrilaterals you know.*
- *What are some common attributes of quadrilaterals?*

Connect to Community and Cultural Responsiveness

Use these activities to connect with and leverage the diverse backgrounds and experiences of all students.

Session 1 **Use with *Connect It*.**

- Display the word *parallel*. Identify the three ℓs in the word. Say: *The ℓs in the word* parallel *are all parallel to one another*. Extend the top and bottom of each ℓ to show parallel lines.

  Say: *Lines and sides of shapes are parallel when they are always the same distance apart.*

  Point out that parallel lines will never meet or cross, even if they continue on and on. Have students locate shapes in the classroom that have opposite parallel sides. Align a pointer or straightedge along each side to extend the parallel lines to show that they do not and will not meet.

Sessions 3 and 4 **Use with either session.**

- Distribute 10 toothpicks to pairs of students. Display different quadrilateral shapes and a three-column table labeled: *Equal Sides*, *Unequal Sides*, and *Parallel Sides*. Have students try to use four toothpicks to make each shape. Determine if the shape can be made with four sides of equal length. If the shape cannot be constructed with four complete toothpicks, have students use additional toothpicks to complete the shape. (Do not allow students to break the toothpicks.) Have pairs discuss which attributes each shape possesses. Write the names of the shapes in the table according to the attributes it possesses.

Connect to Language Development

For ELLs, use the Differentiated Instruction chart to plan and prepare for specific activities in every session.

English Language Learners: Differentiated Instruction

Prepare for Session 1
Use with *Connect It*.

Levels 1–3	Levels 2–4	Levels 3–5
Listening/Speaking Read ***Connect It*** problem 2a. Have students circle the word *parallel*. Gesture with your arms to model horizontal and vertical parallel lines. Have students replicate your gesture. Say: *Parallel sides stay an equal distance apart. Parallel sides do not intersect, or touch.* Display and overlay lines on a square and a triangle to illustrate *parallel sides* and *intersect*. Display, read, and have students discuss and complete the sentence frame: *Parallel* *sides do not touch or* *intersect* .	**Listening/Speaking** Read ***Connect It*** problem 2a. Have students circle the word *parallel*. Gesture with your arms to model horizontal and vertical parallel lines. Have students replicate the gesture. Say: *Parallel sides stay an equal distance apart. They do not intersect, or touch.* Display and overlay lines on shapes to illustrate *intersect*. Point to and identify opposite sides of a square. Ask: *Do the lines intersect? Will the lines ever intersect? Explain.* Display and have students discuss and complete the sentence frame: *Parallel* *sides do not touch or* *intersect* .	**Listening/Speaking** Have students read ***Connect It*** problem 2a and circle the word *parallel*. Gesture with your arms to model horizontal and vertical parallel lines. Have students replicate the gesture. Say: *Parallel sides stay an equal distance apart. They do not intersect, or touch.* Display and overlay lines on shapes to illustrate *intersect*. Ask: *Do the opposite sides of a square intersect? Will the lines ever intersect? Explain.* Display and have students discuss and complete the sentence frame: *Parallel* *sides do not touch or* *intersect* .

LESSON 31

SESSION 1 Explore

Purpose In this session students draw on what they know about the characteristics of different quadrilaterals. They compare a rhombus with a rectangle to explore how quadrilaterals can share some characteristics but not others. They will look ahead to define *attribute* and think about classifying quadrilaterals by attributes.

Start

Connect to Prior Knowledge

Why Prepare students to compare quadrilaterals using their sides and angles.

How Have students describe the sides and angles of a square.

Describe the sides and angles of a square.

Possible Solution

A square has 4 sides of the same length and 4 right angles.

TRY IT

Make Sense of the Problem

To support students in making sense of the problem, have them show that they understand that they are to compare the characteristics of two different types of quadrilaterals: a rhombus and a rectangle.

DISCUSS IT

Support Partner Discussion

To reinforce the concept of comparison, encourage students to use the terms *same* and *different* as they talk to each other.

Look for, and prompt as necessary for, understanding that:

- they should compare characteristics of sides and angles
- the shapes share some characteristics
- some characteristics are unique to the rhombus
- some characteristics are unique to the rectangle

LESSON 31 SESSION 1 ● ○ ○ ○

Explore Classifying Quadrilaterals

Previously you compared shapes and put them into groups. In this lesson you will learn how to group quadrilaterals. Use what you know to try to solve the problem below.

A rhombus is one kind of quadrilateral. A rectangle is another kind of quadrilateral. How are a rhombus and a rectangle the same? How are they different?

rhombus rectangle

Learning Target

- Understand that shapes in different categories may share attributes, and that the shared attributes can define a larger category. Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.

SMP 1, 2, 3, 4, 5, 6, 7

TRY IT

Possible student work:

Sample A

same:
- four sides
- four angles
- opposite sides are the same length

different:
- only rhombus has four sides all the same length
- only rectangle has four right angles

Math Toolkit

- geoboards
- rubber bands
- grid paper
- index cards
- sticky notes

Sample B

	same	different
sides	• four sides • opposite sides are the same length	• only rhombus has four sides the same length
angles	• four angles	• only rectangle has four right angles

DISCUSS IT

Ask your partner: Can you explain that again?

Tell your partner: I knew . . . so I . . .

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Common Misconception Look for students who are unsure of how to describe the quadrilaterals. As students present solutions, have them specify how they knew the characteristics of each shape.

Select and Sequence Student Solutions

One possible order for whole class discussion:

- all characteristics for each quadrilateral listed first, then comparisons made
- similarities and differences of sides separated from similarities and differences of angles
- all similarities grouped together and all differences grouped together

Support Whole Class Discussion

Prompt students to note the characteristics of each shape being compared.

Ask *How do [student name]'s and [student name]'s comparisons each show the similarities between the shapes? How do they show the differences?*

Listen for Students who listed all characteristics of each quadrilateral separately may have used marks such as checks or circles to distinguish similarities from differences. Other students may have grouped the characteristics under headings such as *same* and *different*.

CONNECT IT

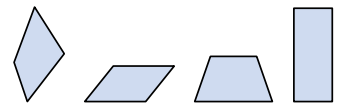
1 LOOK BACK

How are a rhombus and a rectangle alike? How are they different?

Both have 4 sides and 2 pairs of opposite sides that are the same length. Only the rhombus has 4 sides that are all the same length and only the rectangle has 4 right angles.

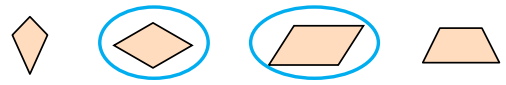
2 LOOK AHEAD

A quadrilateral is a shape with 4 sides and 4 angles. The shapes to the right are quadrilaterals. You can name a quadrilateral based on its attributes. An **attribute** is a way to describe a shape.



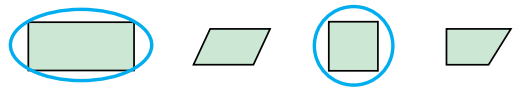
a. A quadrilateral is a **parallelogram** if it has the attributes *both pairs of opposite sides are the same length* and *opposites sides are parallel*. Sides are **parallel** if they are always the same distance apart.

Circle the parallelograms:



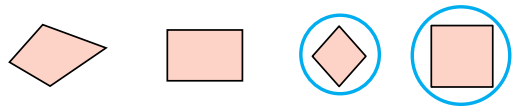
b. A quadrilateral is a rectangle if it has 4 right angles. A rectangle also has 2 pairs of opposite sides that are parallel and the same length.

Circle the rectangles:



c. A quadrilateral is a rhombus if it has 4 sides that are all the same length. A rhombus also has 2 pairs of parallel sides.

Circle the rhombuses:



3 REFLECT

List 3 attributes a quadrilateral could have.

Possible answer: 4 right angles, 2 pairs of parallel sides, opposite sides the same length

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CONNECT IT

1 LOOK BACK

Look for understanding that both the rectangle and rhombus have 4 sides and 4 angles but the lengths of the sides and presence of right angles differ.

Hands-On Activity

Draw a rectangle and a rhombus to compare.

If . . . *students are unsure whether their comparisons apply to other rectangles and rhombuses,*

Then . . . *use this activity to have them draw their own quadrilaterals to compare.*

Materials For each student: ruler, Activity Sheet *1-Centimeter Grid Paper*

- Have students draw a rectangle (that is not a square) by tracing along gridlines.
- To draw a rhombus, tell students to choose a point to be the center of their rhombus, where two grid lines cross. Have them make two marks along one of those intersecting lines, an equal distance above and below the center, and two more marks along the other line, a different equal distance (to avoid drawing a square) to the left and right of the center. Using the ruler, they can draw the rhombus by connecting the four marks.
- Have students compare the sides and angles of the quadrilaterals they drew. Ask: *What do the shapes have in common? In what ways are they different? Do your observations agree with those you made on the previous page?*
- Repeat the activity with rectangles and rhombuses of different sizes.

2 LOOK AHEAD

If you did not use the term *attribute* when teaching, you can now connect *attribute* to *characteristic* or whatever word you did use. Students will become more familiar with the term *attribute* in the Additional Practice. Also point out that two quadrilaterals from the same category can have different shapes.

Students should notice that they circled a square as a rectangle in problem 2b and also as a rhombus in problem 2c. Remind them that some shapes can be described by more than one name or fall into more than one category.

Close: Exit Ticket

3 REFLECT

Look for understanding of the meaning of the term *attribute* and a sample of the types of attributes discussed on this page. Student responses should include descriptions of sides and/or angles, including references to sides that are parallel or the same length and right angles.

Common Misconception **If** students are confused by the phrase "both pairs of opposite sides are the same length," **then** explain that each pair can be a different length, such as the 2 pairs of opposite sides in a rectangle, or the two pairs can be the same length, such as the 2 pairs of opposite sides in the rhombus. Elicit that when the 1 pair of opposite sides is the same length as the other pair of opposite sides, the shape can also be described as having "4 sides the same length."

Real-World Connection

Ask students to give examples of quadrilaterals they see in the classroom, at home, or outside (e.g., windows, doors, desktops, books, computer screens, road signs, remote controls, tabletops, and paper). Point out that many of these objects are three-dimensional and that you are focusing on the two-dimensional surfaces.

Solutions

Support Vocabulary Development

1 Ask students to chorally read the titles in the top row of the table as you point to them. Read each of the terms in the first column and have students chorally repeat. Ask students to explain the meaning of the word *attribute*, and then to describe the attributes of each of the shapes in the first column. If students need more support, have them review the information in *Connect It* problem 2a–c before discussing this table.

2 Pair students. Read the problem aloud. Have students circle *parallelograms* and refer to the table to support their understanding of the term. Have students compare the shapes in problem 2 with the examples they drew in the table. Ask: *Which shape is not included in the table?* [the triangle] *How is that shape different from the shapes in the table?* [A triangle has three sides and the all the shapes in the table have four sides.]

Have students work with a partner to label each of the shapes, referring to the table and the *Connect It* for support. Remind students that a parallelogram is any shape with four sides that has two pairs of parallel sides. Support students in understanding that more than one label may apply to each shape.

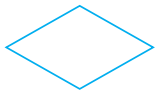
Supplemental Math Vocabulary

- *attribute*
- *parallel*
- *parallelogram*
- *quadrilateral*
- *rectangle*
- *rhombus*
- *right angle*

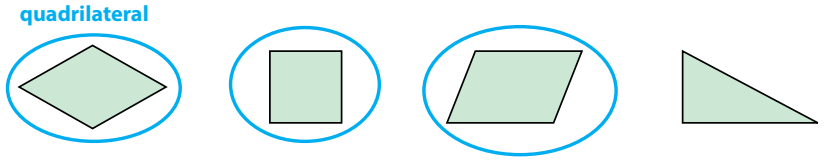
Name: _____ LESSON 31 SESSION 1

Prepare for Classifying Quadrilaterals

1 Think about what you know about quadrilaterals. Fill in each box. Use words, numbers, and pictures. Show as many ideas as you can. **Possible answers:**

Word	In My Own Words	Example
quadrilateral	**a flat shape with 4 sides and 4 angles**	
attribute	**a way to describe a shape**	**number of sides, length of sides, number of angles, number of right angles**
parallelogram	**a quadrilateral with opposite sides parallel and the same length**	
rectangle	**a quadrilateral with 4 right angles; also a parallelogram**	
rhombus	**a quadrilateral with all 4 sides the same length; also a parallelogram**	

2 Circle the parallelograms. What other word above describes your circled shapes?
quadrilateral



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3 Assign problem 3 to provide another look at comparing and contrasting the attributes of two different quadrilaterals.

This problem is very similar to the problem about comparing and contrasting the attributes of a rhombus and a rectangle. In both problems, students are asked how two different quadrilaterals are the same and how they are different. The question asks how a parallelogram and a square are the same and how are they different.

Students may want to use pattern blocks or household objects that are shaped like different quadrilaterals.

Suggest that students read the problem three times, asking themselves one of the following questions each time:

- *What is this problem about?*
- *What is the question I am trying to answer?*
- *What information is important?*

Solution: Both shapes have four sides and four angles. In both shapes, opposite sides are the same length and parallel. Only the square has all four sides the same length. Only the square has four right angles.
Medium

4 Have students solve the problem a different way to check their answer.

3 Solve the problem. Show your work.

A parallelogram is one kind of quadrilateral. A square is another kind of quadrilateral. How are a parallelogram and a square the same? How are they different?


parallelogram


square

Possible student work using a table:

	same	different
sides	• 4 sides • opposite sides are the same length • opposite sides are parallel	• only square has 4 sides the same length
angles	• 4 angles	• only square has 4 right angles

Solution Both shapes have 4 sides and 4 angles with opposite sides the same length and parallel. Only the square has 4 sides the same length and 4 right angles.

4 Check your answer. Show your work.

Possible student work:

same:
- 4 sides
- 4 angles
- opposite sides are the same length
- opposite sides are parallel

different:
- only square has 4 sides all the same length
- only square has 4 right angles

ELL English Language Learners: Differentiated Instruction

Prepare for Session 2
Use with *Apply It*.

Levels 1–3

Listening/Speaking Pair students. Read ***Apply It*** problem 8. Say: *Circle the words and phrases* trapezoid, quadrilateral, one pair of parallel sides, *and* two right angles. Ask: *How do you know a trapezoid has four sides?* [It is a quadrilateral.] *Use your arms and show parallel sides*. Display the following four shapes: a square, a parallelogram, a trapezoid with no right angles, and a trapezoid with two right angles.

Inventory each shape for the required attributes named in the problem. Have students draw and label the shape after the correct trapezoid is identified.

Levels 2–4

Listening/Speaking Pair students. Read ***Apply It*** problem 8. Say: *Circle the words and phrases* trapezoid, quadrilateral, one pair of parallel sides, *and* two right angles. Ask: *How do you know a trapezoid has four sides?* [It is a quadrilateral.] *Use your arms and show parallel sides.* Display the following four shapes: a square, a parallelogram, a trapezoid with no right angles, and a trapezoid with two right angles.

Say: *Analyze with your partner which shape has the required attributes named in the problem*. Have students draw and label the shape after the correct trapezoid is identified and its attributes are analyzed.

Levels 3–5

Listening/Speaking Have pairs read ***Apply It*** problem 8. Say: *Circle the words and phrases* trapezoid, quadrilateral, one pair of parallel sides, *and* two right angles. Ask: *How do you know a trapezoid has four sides?* [It is a quadrilateral.] Display the following four shapes: a square, a parallelogram, a trapezoid with no right angles, and a trapezoid with two right angles.

Say: *Analyze with your partner which shape has the required attributes named in the problem*. Have students draw, label, and describe the correct trapezoid after it is identified and its attributes are analyzed.

LESSON 31

SESSION 2 Develop

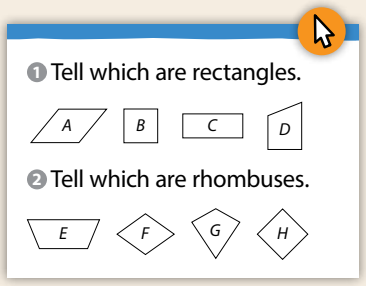
Purpose In this session students compare the attributes of rectangles and squares to discover the relationship between the two. The purpose of this problem is to have students explore the concept that a quadrilateral can be classified in more than one way and that some classifications are included in others.

Start

Connect to Prior Knowledge

Why Support students' progress in learning to classify quadrilaterals.

How Have students identify rectangles and rhombuses by inspection.



Solutions

1. *B, C*
2. *F, H*

Develop Language

Why Clarify the meaning of the phrase *2 pairs.*

How Explain that a pair is a set of two things and so 2 pairs are two sets of two things. Say: *A pair of shoes has two shoes. Two pairs of shoes have four shoes.* Have students suggest other things that come in pairs. For each, ask: *How many _____ are in a pair of _____?* [2] *What is the total number of _____ in two pairs?* [4] Have students complete the following sentence frame:

A parallelogram has __2 pairs__ of parallel opposite sides. It has a total of __4__ sides.

TRY IT

Make Sense of the Problem

To support students in making sense of the problem, have them identify that they are comparing the attributes of a square and a rectangle.

Develop Comparing Quadrilaterals

Read and try to solve the problem below.

Is a square a rectangle?

Is a rectangle a square?

TRY IT

Possible student work:

Sample A

Is a square a rectangle?
- 4 right angles? ✓
- opposite sides the same length? ✓

A square is a rectangle.

Is a rectangle a square?
- 4 right angles? ✓
- 4 sides the same length? ✗

A rectangle is not always a square.

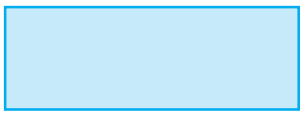
Math Toolkit
- geoboards
- rubber bands
- grid paper
- dot paper
- colored pencils

Sample B

square

always has:
- 4 right angles
- 4 sides the same length

rectangle



always has:
- 4 right angles
- opposite sides the same length

A square is a rectangle.
A rectangle is not always a square.

DISCUSS IT

Ask your partner: Do you agree with me? Why or why not?

Tell your partner: I agree with you about . . . because . . .

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DISCUSS IT

Encourage students to use the term *attributes* as they discuss their solutions.

Support as needed with questions such as:

- *What attributes did you consider when comparing the square with the rectangle?*
- *How did you use the attributes of each quadrilateral to answer the questions?*

Common Misconception Look for students who think a quadrilateral can have only one name rather than one type of quadrilateral being a special case of another type of quadrilateral. Have students explain in their own words why a square is also a rectangle, but not all rectangles are squares.

Select and Sequence Student Solutions

One possible order for whole class discussion:

- attributes mentioned limited to all right angles and all sides the same length
- thorough lists of attributes for both shapes
- tables listing and comparing several attributes

Support Whole Class Discussion

Compare and connect the different explanations and have students identify what they have in common.

Ask *How did you look for similarities and differences between the two shapes? How did your comparison help you answer the questions?*

Listen for Students may have made a list or table of the attributes of squares and rectangles in order to find what they do and do not have in common. If one shape has all the attributes of the other shape, then it belongs to both groups.

PICTURE IT & MODEL IT

If no student presented these models, connect them to the student models by pointing out the ways they each show:

- the attributes that both shapes have in common
- the attributes that belong to only one shape
- whether a shape always or only sometimes has an attribute

Ask *How are the attributes listed? What do the shapes have in common? Are there any attributes that either shape does not have or does not always have?*

Listen for Both shapes have 4 sides and 4 angles, 4 right angles, 2 pairs of parallel sides, and 2 pairs of sides that are the same length. Squares have all 4 sides the same length, but rectangles do not always have this attribute.

For the pictures with lists of attributes, prompt students to explain how the lengths of the sides of the shapes compare.

- *If a shape has 4 sides the same length, does it also have 2 pairs of sides the same length?*
- *If a shape has 2 pairs of sides the same length, does it also have 4 sides the same length?*

For the table of attributes, prompt students to tell whether each attribute always applies to each shape.

- *Do* all *squares or only* some *squares have each attribute?*
- *Do* all *rectangles or only* some *rectangles have each attribute?*

Explore different ways to understand comparing quadrilaterals.

Is a square a rectangle?

Is a rectangle a square?

PICTURE IT

You can use a drawing to compare quadrilaterals.

All quadrilaterals have 4 sides and 4 angles.

4 right angles
2 pairs of parallel sides
4 sides the same length

4 right angles
2 pairs of parallel sides
2 pairs of opposite sides the same length

MODEL IT

You can use a table to compare quadrilaterals.

Shape	4 sides 4 angles	4 right angles	2 pairs of parallel sides	2 pairs of opposite sides that are the same length	4 sides that are the same length
Square	✓	✓	✓	✓	✓
Rectangle	✓	✓	✓	✓	sometimes

Deepen Understanding

Classify Quadrilaterals

SMP 1 Make sense of problems.

When discussing the table, prompt students to think of real-world examples of one group being a part of another group.

Ask *Are all students in this class students at this school? Are all students at this school members of this class? Which group do more people belong to? Explain.*

Listen for All students in the class are students at the school, but not all students in the school are in the class. More people belong to *students at this school* because it contains those in *members of this class* plus others.

Ask *How are the groups* people who live in Florida *and* people who live in the U.S. *related? Is one group part of another? Are the groups the same?*

Listen for *People who live in Florida* is part of *people who live in the U.S.* because everyone in the first group is also in the second group. The groups are not the same because not everyone who lives in the U.S. lives in Florida.

Generalize *How is this like classifying quadrilaterals?* Some quadrilateral groups are parts of other groups. For example, squares are also rectangles.

CONNECT IT

- Remind students that one thing that is alike about all of the solutions is that they all use attributes to compare the rectangle and square.
- Explain that on this page, students will use the attributes to determine whether all squares are rectangles and whether all rectangles are squares.

Monitor and Confirm

1 Check for understanding that:

- all squares have 4 sides the same length
- not all rectangles have 4 sides the same length, but some rectangles do

Support Whole Class Discussion

2 – 3 Tell students that these problems will prepare them to provide the explanations in problems 4 and 5.

Be sure students understand that the problems are asking them to identify whether one shape has all the attributes of the other shape.

Ask *Which shape has all the attributes of the other shape? How do you know?*

Listen for The square has all the attributes of the rectangle, because *4 sides the same length* is a special case of *2 pairs of sides the same length*.

4 – 5 Look for the understanding that every square is a rectangle because it has all the attributes of a rectangle and that not every rectangle is a square because not all rectangles have 4 sides the same length.

6 REFLECT Have all students focus on the strategies used to solve this problem. If time allows, have students share their preferences with a partner.

CONNECT IT

Now you will use the problem from the previous page to help you understand how to compare quadrilaterals.

1 What is an attribute of a square that is not an attribute of every rectangle?

4 sides that are the same length

2 Does every rectangle have all the attributes of a square? **no**

3 Does every square have all the attributes of a rectangle? **yes**

4 Is every square a rectangle? Explain why or why not.

Yes; Possible explanation: A square has all the attributes of a rectangle, so it is a rectangle.

5 Is every rectangle a square? Explain why or why not.

No; Possible explanation: Sometimes a rectangle can also be a square, but only if all 4 sides are the same length.

6 REFLECT

Look back at your **Try It**, strategies by classmates, and **Picture It** and **Model It**. Which models or strategies do you like best for comparing quadrilaterals? Explain.

Some students may prefer to draw the shapes, so that they can visually check which attributes are the same and which are different. Other students may choose to organize and compare attributes in a table.

Hands-On Activity

Show all squares are rectangles but not all rectangles are squares.

If . . . *students struggle with the idea that all squares are rectangles but not all rectangles are squares,*

Then . . . *have them construct several rectangles and squares and check which ones fit into both categories.*

Materials For each pair: 5 sheets of blank unlined paper, Activity Sheet *1-Centimeter Grid Paper*

- Have students cover each edge of the grid paper with a sheet of blank paper to form a quadrilateral with 4 right angles on the grid paper. They can align an edge of each blank paper with the gridlines to ensure right angles.
- On another sheet of paper, they should make a table with four columns: *Length*, *Width*, *Is it a rectangle?* and *Is it a square?* Instruct students to move the blank papers to form at least 3 rectangles and 3 squares, completing a row of their table for each shape.
- Discuss the results as a class. Students should start to see that every square is also a rectangle, but not every rectangle is a square.

APPLY IT

For all problems, encourage students to make a list of attributes to support their thinking.

7 The first and last figures should be circled. The second and third figures do not have 4 sides of the same length, and the third figure also does not have 4 right angles.

8 See possible figure on Student Worktext page. Look for a quadrilateral with two adjacent right angles. The other two angles should not be right angles.

Close: Exit Ticket

9 The first, third, and fourth figures should be circled. The second figure does not have 4 right angles or 2 pairs of opposite sides the same length.

Students' solutions should indicate understanding that:

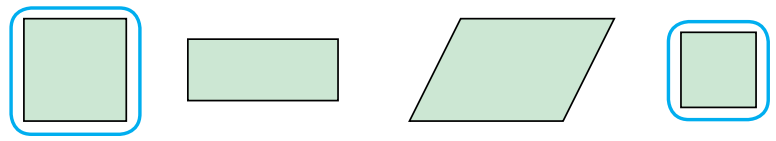
- a quadrilateral with four right angles is a rectangle
- a square is also a rectangle

Error Alert **If** students do not choose the square, **then** have them list the attributes of a rectangle. Prompt students to recognize that 4 sides the same length can be thought of as 2 pairs of sides the same length, where one pair is the same length as the other pair.

APPLY IT

Use what you just learned to solve these problems.

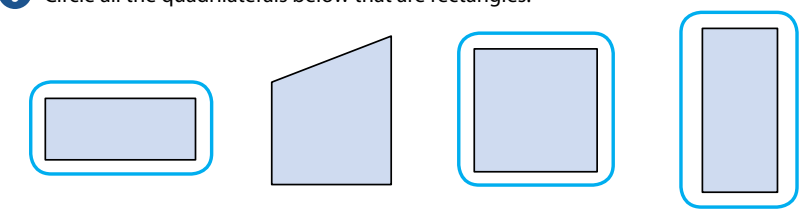
7 Circle all the quadrilaterals below that are squares.



8 One way to define a trapezoid is a quadrilateral with exactly one pair of parallel sides. Draw a trapezoid with two right angles.

Possible student work:

9 Circle all the quadrilaterals below that are rectangles.



Solutions

1. No; Possible explanation: Shape *A* does not have any parallel sides, so it cannot be a parallelogram.
Medium

2. square; Possible explanation: A square is both a rhombus and a rectangle, and rhombuses and rectangles are both parallelograms. Therefore, a square is a parallelogram.
Medium

3. rhombus; rectangle
Basic

Name: _____ LESSON 31 SESSION 2

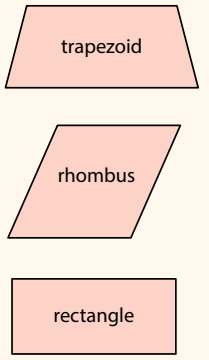
Practice Comparing Quadrilaterals

Study the Example showing how to compare quadrilaterals.
Then solve problems 1–7.

EXAMPLE

Which of these shapes are parallelograms?

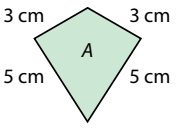
You can list the attributes of a parallelogram in a table.
Check if each shape always has these attributes.



Attribute	Trapezoid	Rhombus	Rectangle
4 sides	yes	yes	yes
4 angles	yes	yes	yes
2 pairs of parallel sides	no	yes	yes
2 pairs of sides that are the same length	no	yes	yes

A rhombus and a rectangle have all the attributes of a parallelogram.

1. Is shape *A* a parallelogram? Explain.



Possible answer: Shape *A* is not a parallelogram. It has many of the attributes of a parallelogram, but not 2 pairs of parallel sides.

2. What is another kind of quadrilateral that is also a parallelogram? Explain.

Possible answer: A square is also a parallelogram. It has all the attributes of a parallelogram.

3. Fill in the blanks. Use information from the table above.

Every **rhombus** is a parallelogram.

Every **rectangle** is a parallelogram.

Vocabulary

attribute a way to describe a shape, like number of sides or length of sides.

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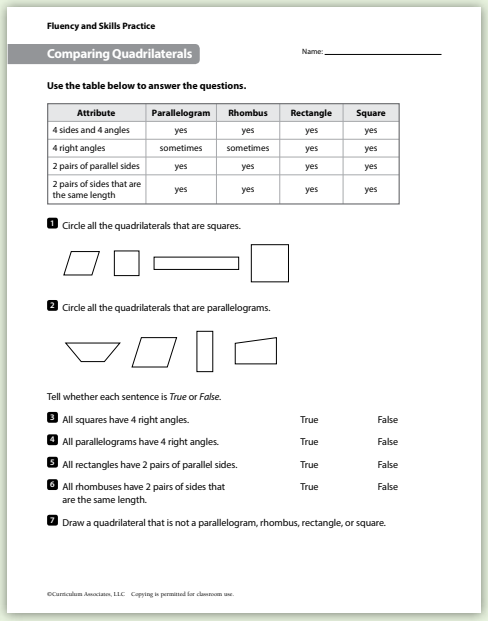
Fluency & Skills Practice Teacher Toolbox

Assign *Comparing Quadrilaterals*

In this activity students practice comparing quadrilaterals using their attributes. Students should be able to recognize real-world examples of each quadrilateral. For example, students may want to determine which quadrilateral is most commonly found in their home or classroom. Or, they may wish to identify the quadrilaterals used to create buildings, bridges, and other structures.



Fluency and Skills Practice

Comparing Quadrilaterals

Name: _____

Use the table below to answer the questions.

Attribute	Parallelogram	Rhombus	Rectangle	Square
4 sides and 4 angles	yes	yes	yes	yes
4 right angles	sometimes	sometimes	yes	yes
2 pairs of parallel sides	yes	yes	yes	yes
2 pairs of sides that are the same length	yes	yes	yes	yes

1. Circle all the quadrilaterals that are squares.

2. Circle all the quadrilaterals that are parallelograms.

Tell whether each sentence is *True* or *False*.

3. All squares have 4 right angles. True False
4. All parallelograms have 4 right angles. True False
5. All rectangles have 2 pairs of parallel sides. True False
6. All rhombuses have 2 pairs of sides that are the same length. True False
7. Draw a quadrilateral that is not a parallelogram, rhombus, rectangle, or square.

4 The second figure (square) and third figure (rhombus) should be circled.
Medium

5 The first figure (rectangle) and third figure (square) should be circled.
Basic

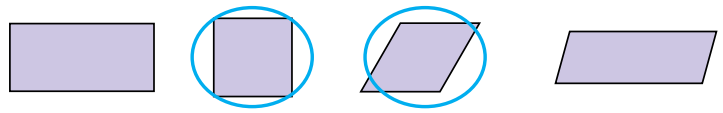
6 **A** (**True**), Squares have 4 right angles, so they are rectangles;
C (**True**), Rectangles have 2 pairs of opposite sides that are parallel and the same length, so they are parallelograms;
F (**False**), Not all parallelograms have 4 right angles;
H (**False**), Not all quadrilaterals have parallel sides;
I (**True**), All parallelograms have 4 sides and 4 angles.
Medium

7 Yes; Possible explanation: Only some rectangles have 4 sides the same length, but all squares do.
Challenge

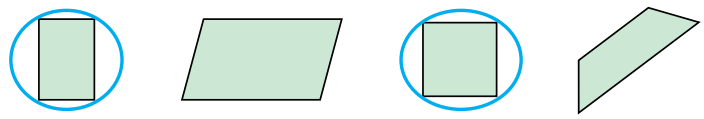
Use the table to solve problems 4–7.

Attribute	Parallelogram	Rhombus	Rectangle	Square
4 sides and 4 angles	yes	yes	yes	yes
4 right angles	sometimes	sometimes	yes	yes
2 pairs of parallel sides	yes	yes	yes	yes
2 pairs of sides that are the same length	yes	yes	yes	yes

4 Circle all the quadrilaterals that are rhombuses.



5 Circle all the quadrilaterals that are rectangles.



6 Tell whether each sentence is *True* or *False*.

	True	False
All squares are rectangles.	Ⓐ	Ⓑ
All rectangles are parallelograms.	Ⓒ	Ⓓ
All parallelograms are rectangles.	Ⓔ	Ⓕ
All quadrilaterals are parallelograms.	Ⓖ	Ⓗ
All parallelograms are quadrilaterals.	Ⓘ	Ⓙ

7 Jaime says that some rectangles are not squares. Do you agree? Explain.

Yes; Possible explanation: A rectangle is a square only if it has 4 sides of the same length. Rectangles only sometimes have 4 sides of the same length.

ELL English Language Learners: Differentiated Instruction

Prepare for Session 3
Use with *Apply It*.

Levels 1–3

Listening/Speaking Pair students. Read ***Apply It*** problem 7. Say: *Circle the words and phrases* quadrilateral, at least one right angle, *and* not a rectangle. Ask the following questions, pausing after each to give students a chance to discuss the question with a partner: *How many sides must the shape have?* [4] *How do you know?* [It is a quadrilateral.] *How many right angles must the shape have?* [1] *Can it have more?* [yes] *Can the shape have four right angles? Explain.* [No, a shape with four right angles is a rectangle.] (Model) Say: *Draw a capital L on your page. This is a right angle. How many more lines do you need to draw?* [2] *Draw slanted lines to be sure you do not draw right angles.* Select students to share their drawings.

Levels 2–4

Listening/Speaking Pair students. Read ***Apply It*** problem 7. Say: *Circle the words and phrases* quadrilateral, at least one right angle, *and* not a rectangle. Ask the following questions, pausing after each to give students a chance to discuss the question with a partner: *How many sides must the shape have?* [4] *How do you know?* [It is a quadrilateral.] *How many right angles must the shape have?* [1] *Can it have more?* [yes] *Can the shape have four right angles? Explain.* [No, a shape with four right angles is a rectangle.] *Draw a right angle on your page. How can you be sure not to draw a rectangle?* [Draw slanted lines.] Select students to share their drawings.

Levels 3–5

Listening/Speaking Have pairs read ***Apply It*** problem 7. Say: *Circle the words and phrases* quadrilateral, at least one right angle, *and* not a rectangle. Ask the following questions, pausing after each to give students a chance to discuss the question with a partner: *How many right angles must the shape have?* [1] *Can it have more?* [yes] *Can the shape have four right angles? Explain.* [No, a shape with four right angles is a rectangle.] *Draw a right angle on your page. How can you be sure not to draw a rectangle?* [Draw slanted lines.] Select students to share their drawings.

LESSON 31

SESSION 3 Develop

Purpose In this session students solve a problem that requires using given attributes to classify a quadrilateral. Students review the attributes to identify a more specific name for the quadrilateral. The purpose of this problem is to have students develop a strategy for classifying quadrilaterals.

Start

Connect to Prior Knowledge

Why Reinforce the concept that different quadrilaterals can share some attributes and not others.

How Have students identify the attributes that belong to squares but not to all parallelograms.

Name two attributes of a square that are NOT attributes of all parallelograms.

Solution

4 sides the same length and 4 right angles

Develop Language

Why Clarify the meaning of the term *arrange*.

How Display the word *arrange*. Say: Arrange *means to place items in a certain position or order.* Select four items. Have volunteers arrange the items from tallest (or longest) to shortest. Repeat with other items and ask students to arrange them in various ways, such as in a circle, in a line, and from largest to smallest.

TRY IT

Make Sense of the Problem

To support students in making sense of the problem, have them identify the given attributes that will help them classify the quadrilateral.

Ask *What do you know about the sides of the shape? What do you know about the angles of the shape?*

Develop Naming and Drawing Quadrilaterals

Read and try to solve the problem below.

I have a quadrilateral. It has 4 sides that are all the same length. It does not have any right angles. What is the name of my shape?

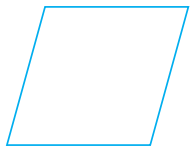
TRY IT

Possible student work:

Sample A

square: 4 sides the same length ✓, 4 right angles ✗
rectangle: 4 right angles ✗
rhombus: 4 sides the same length ✓, does not need to have right angles ✓
It's a rhombus.

Sample B



4 sides the same length → could be rhombus or square
no right angles → cannot be square
The shape is a rhombus.

Math Toolkit
- geoboards
- rubber bands
- rulers
- grid paper
- dot paper
- toothpicks

DISCUSS IT

Ask your partner: Do you agree with me? Why or why not?

Tell your partner: I agree with you about . . . because . . .

DISCUSS IT

Support Partner Discussion

Encourage students to use the term *attributes* as they discuss their solutions.

Support as needed with questions such as:

- *Did you find more than one type of quadrilateral with the given attributes?*
- *Were you able to rule out any possibilities? How?*

Common Misconception Look for students who classify the quadrilateral as a parallelogram without realizing that they can be more specific. Have students make a Venn diagram to classify quadrilaterals. Help them use the diagram to understand that rectangles, squares, and rhombuses are specific types of parallelograms.

Select and Sequence Student Solutions

One possible order for whole class discussion:

- concrete models of the shape
- drawings of the shape
- lists of attributes with no model

Support Whole Class Discussion

Compare and connect the different solution methods and have students identify the common steps.

Ask *How does each solution show the attributes of the quadrilateral?*

Listen for A drawing or model should show a quadrilateral with 4 sides the same length and no right angles. A written list or table should include both of these attributes.

MODEL IT & SOLVE IT

If no student presented these models, connect them to the student models by pointing out the ways they each represent:

- the given attributes
- the types of quadrilaterals considered

Ask *How does each model reflect the information given in the problem? Where does it show that the attributes were used to consider specific types of quadrilaterals?*

Listen for The model shows the attributes, while the solution lists them. Both *Model It* and *Solve It* use the absence of right angles to eliminate the square as a possible classification for the shape.

For the model, prompt students to describe how the toothpicks reflect the attributes given in the problem.

- *How does the model show that the sides are all the same length?*
- *How does the model show that there are no right angles?*

For the list of attributes, prompt students to consider other quadrilaterals.

- *Which quadrilaterals have 4 sides the same length? Are these quadrilaterals possible solutions, or can you rule them out?*
- *Which quadrilaterals have right angles? Are these quadrilaterals possible solutions, or can you rule them out?*

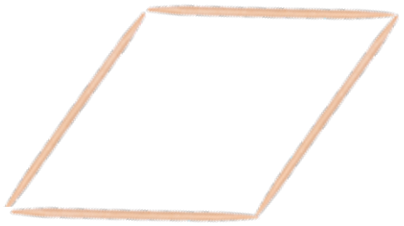
Explore different ways to understand naming and drawing quadrilaterals.

I have a quadrilateral. It has 4 sides that are all the same length. It does not have any right angles. What is the name of my shape?

MODEL IT

You can make a model to help name a quadrilateral.

Choose 4 toothpicks all the same length. Arrange them to look like a quadrilateral. Make sure there are no right angles.



It does not have any right angles, so it is not a square.

SOLVE IT

You can make a list of the attributes to help you name a quadrilateral.

Look at the model above. Think about everything you know about this shape.

- It is a quadrilateral, so it has 4 sides and 4 angles.
- It has 4 sides that are all the same length.
- It does not have any right angles, so it is not a square.

Using the list of attributes, you can name the shape.

Deepen Understanding

Quadrilaterals and Their Attributes

SMP 5 Use tools.

When discussing the toothpick model of the quadrilateral, ask students to describe ways they might create models with different attributes.

Ask *How would you model a quadrilateral with 2 pairs of sides the same length?*

Listen for Students may describe a model like the one shown here but use 2 pairs of toothpicks, each pair a different length.

Ask *How would you model a quadrilateral with right angles?*

Listen for You can use the corner of a piece of paper to make sure toothpicks meet at right angles or lay the toothpicks along crossing lines on grid paper. You could also draw right on the grid paper.

Ask *How would you model a quadrilateral with parallel sides?*

Listen for Parallel sides can be drawn on grid paper, dot paper, or lined paper, or by tracing both edges of a ruler. You can also lay toothpicks along parallel lines on grid paper.

CONNECT IT

- Remind students that one thing that is alike about all the solutions is the attributes considered.
- Explain that on this page, students will use the attributes of a quadrilateral to classify it.

Monitor and Confirm

1 Check for understanding that:

- the shape on the previous page is a rhombus because all 4 sides are the same length
- the shape is not a square because it does not have right angles

Support Whole Class Discussion

2–3 Be sure students understand that these problems are asking them to classify the quadrilateral given in problem 2.

Ask *What attributes can you check for that are related to the sides of the shape? What attributes can you check for that are related to the angles?*

Listen for Students can check the number of sides, whether any sides are parallel, and which sides (if any) are the same length. They can check the number of angles and whether the shape has any right angles.

4 Look for a quadrilateral that does not have all 4 sides the same length, does not have 4 right angles, and has at most 1 pair of parallel sides.

5 REFLECT Have all students focus on the strategies used to solve this problem. If time allows, have students share their preferences with a partner.

CONNECT IT

Now you will use the problem from the previous page to help you understand how to name and draw quadrilaterals by looking at their attributes.

1 What is the name of the shape described on the previous page? How do you know?

rhombus; Possible answer: The only shapes with four sides that are all the same length are the rhombus and the square. The shape is not a square because it has no right angles, so it must be a rhombus.

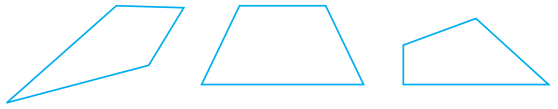
2 Look at the shape to the right. Is it a quadrilateral? Explain why or why not.

Yes; Possible explanation: It has 4 sides and 4 angles.

3 Is the shape a parallelogram? Is it a rectangle? Is it a rhombus? Explain.

No, no, no; Possible explanation: The shape does not have parallel sides, so it is not a parallelogram. It does not have 4 right angles, so it is not a rectangle. Its sides are not all the same length, so it is not a rhombus.

4 Draw a different quadrilateral that is NOT a parallelogram, a rectangle, or a rhombus. **Possible answers:**



5 REFLECT

Look back at your **Try It**, strategies by classmates, and **Model It** and **Solve It**. Which models or strategies do you like best for naming and drawing quadrilaterals? Explain.

Some students may prefer making a model of a shape based on its given attributes to help them visually recognize the type of quadrilateral described. Others might prefer organizing the attributes in a list or table and checking them off to identify the quadrilateral.

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Hands-On Activity

Build a quadrilateral with 4 sides the same length.

If . . . *students have trouble visualizing the different shapes a quadrilateral with 4 sides the same length can make,*

Then . . . *use this activity to have them construct a tractable model.*

Materials For each student: 4 straws, 1 piece of string or yarn about a yard long, scissors

- Ask students to cut their 4 straws to be the same length. Then have them thread the string through all 4 straws and tie the ends of the string together so that the straws all meet end to end.
- Instruct students to lay their construction flat on their desks so that the straws form a quadrilateral. Invite them to stretch the quadrilateral in different directions to change the angle measures, taking note of how the shape changes.
- Have students change the shape to make one angle a right angle. Ask: *When you make one right angle, what happens to the other angles?* [They become right angles.] *How would you classify this quadrilateral now?* [It is a square.]
- Repeat the activity with straws cut to a different same-length.

APPLY IT

For all problems, encourage students to list any attributes that could help them answer the questions.

6 The second and third figures should be circled because neither one has 4 right angles.

7 See Student Worktext page for possible shapes. Look for a quadrilateral with either 1 or 2 right angles.

Close: Exit Ticket

8 parallelogram; See Student Worktext page for possible shape. Look for a parallelogram that is not a rectangle or a rhombus.

Students' solutions should indicate understanding that:

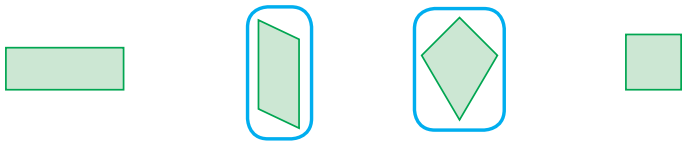
- a parallelogram has 2 pairs of opposite sides that are the same length
- a quadrilateral does not have to have any right angles or all sides the same length to be a parallelogram

Error Alert **If** students draw and name a rectangle, rhombus, or square, **then** have them list the attributes of the quadrilateral they drew and elicit from them that not all of those attributes were specified in the problem.

APPLY IT

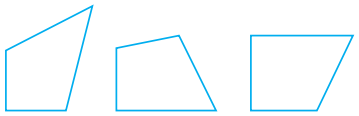
Use what you just learned to solve these problems.

6 Circle all the quadrilaterals below that have 2 pairs of sides the same length, but are not rectangles.



7 Draw a quadrilateral that has at least 1 right angle, but is not a rectangle.

Possible answers:



8 Draw a quadrilateral in which all sides are not the same length, opposite sides are the same length, and there are no right angles. Then name the quadrilateral.

Possible answer:

Solution parallelogram

Solutions

1. 4 sides and 4 angles
   Basic

2. 1 pair of parallel sides
   Basic

3. 2 right angles
   Basic

4. No, the shape has no sides the same length.
   Basic

5. quadrilateral
   Medium

Name: _____ LESSON 31 SESSION 3

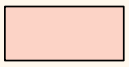
Practice Naming and Drawing Quadrilaterals

Study the Example showing how to name a quadrilateral.
Then solve problems 1–9.

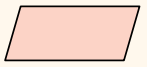
EXAMPLE

Justin is drawing a quadrilateral with opposite sides that are the same length. All 4 sides are not the same length. What quadrilaterals can Justin draw?

Make a drawing to see what the quadrilaterals might look like.



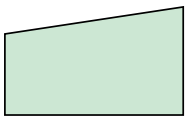
Opposite sides are the same length.
The shape has 4 right angles.



Opposite sides are the same length.
The shape has no right angles.

Justin can draw a rectangle or a parallelogram.

Use the shape on the right to answer problems 1–5.



1. One wall of a shed looks like the shape on the right. How many sides and angles does the shape have?
   The shape has 4 sides and 4 angles.

2. How many parallel sides does the shape have?
   The shape has 1 pair of parallel sides.

3. How many right angles does the shape have?
   The shape has 2 right angles.

4. Does the shape have 2 pairs of sides the same length?
   No, the shape has no sides the same length.

5. Circle all the words you can use to name this shape.
   quadrilateral parallelogram rectangle

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Fluency & Skills Practice

Teacher Toolbox

Assign *Naming and Drawing Quadrilaterals*

In this activity students practice naming and drawing types of quadrilaterals from a description of their attributes. Students may need to use a similar strategy in real-world situations such as describing a table top with no right angles or a picture with four equal sides to someone who doesn't know the names of specific quadrilaterals.

Fluency and Skills Practice

Naming and Drawing Quadrilaterals

Name: _____

Read and solve the problems. Show your work.

1. Draw a quadrilateral with at least 2 right angles that is not a rectangle.
2. Draw a quadrilateral in which all sides are the same length and there are no right angles. Then name the quadrilateral.
3. Draw a quadrilateral in which all sides are not the same length, opposite sides are the same length, and all angles are right angles. Then name the quadrilateral.
4. Draw a quadrilateral in which all sides are the same length and all angles are right angles. Then name the quadrilateral.
5. Draw a quadrilateral with 2 sides that are the same length, opposite sides are not the same length, and no right angles.
6. Draw a quadrilateral with all sides that are not the same length and no right angles.
7. Explain how you named the shape in problem 4.

6 *D*; rectangle
Medium

7 *A*; rhombus
Medium

8 *B*; hexagon
Medium

9 See student page for possible shape and explanation. Look for a square; The shape is a quadrilateral, a parallelogram, a rectangle, a rhombus, and a square.
Challenge

Use the clues and shapes *A*–*E* to solve problems 6–8.

6 I have 4 sides. I am a parallelogram.
I have all right angles.
I am not a square.

I am shape D.

I am a rectangle.

7 I am a quadrilateral.
I do not have any right angles.
My sides are all the same length.

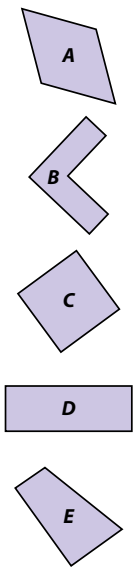
I am shape A.

I am a rhombus.

8 I have more than 1 right angle.
Some of my sides are the same length.
I am not a quadrilateral.

I am shape B.

I am a hexagon.



9 Draw a quadrilateral that has at least 3 right angles, 2 pairs of parallel sides, and all sides the same length. Write all of the possible names for your shape. Tell why the names fit.

Possible drawing:

Students should realize that if this quadrilateral has at least 3 right angles, it must have 4 right angles. Since all sides are the same length, they should draw a square.

Possible explanation: The shape has 4 sides of the same length, opposite sides that are parallel, and 4 right angles. The shape is a square, so it is also a rhombus (all squares are rhombuses), a rectangle (all squares are rectangles), a parallelogram (all rectangles are parallelograms), and a quadrilateral (all parallelograms are quadrilaterals).

ELL English Language Learners: Differentiated Instruction

Prepare for Session 4
Use with *Apply It*.

Levels 1–3

Speaking/Writing Choral read ***Apply It*** problem 2. Give pairs a rectangular piece of paper and a pair of scissors. Model how to fold the paper from top to bottom on a slant. Have students replicate and cut along the fold line. Say: *Tell how the shapes are alike.* [four sides, 2 right angles, one pair of parallel sides] *Is either shape a parallelogram?* If students are not sure, display a parallelogram. Ask: *What feature must a parallelogram have?* [two pairs of parallel sides] Have students complete this sentence starter in writing.

The quadrilaterals are not parallelograms because _____.

Levels 2–4

Speaking/Writing Choral read ***Apply It*** problem 2. Give pairs a rectangular piece of paper and a pair of scissors. Model how to fold the paper from top to bottom on a slant. Have students replicate and cut along the fold line. Say: *Tell how the shapes are alike. Is either shape a parallelogram? Explain.* If students are unsure, display a parallelogram. Ask: *What feature must a parallelogram have?* [two pairs of parallel sides] Say: *With your partner, write a sentence that explains why the shapes are not parallelograms.*

Levels 3–5

Speaking/Writing Have pairs take turns reading ***Apply It*** problem 2 aloud. Give pairs a rectangular piece of paper and a pair of scissors. Model how to fold the paper from top to bottom on a slant. Have students replicate and cut along the fold line. Ask pairs to discuss. Say: *Tell how the shapes are alike. Is either shape a parallelogram? Explain.* Have students draw a parallelogram. Elicit that a parallelogram has two pairs of parallel sides. Say: *With your partner, write a sentence that explains why the shapes are not parallelograms.*

LESSON 31

SESSION 4 Refine

Purpose In this session students solve word problems involving classifying quadrilaterals and then discuss and confirm their answers with a partner.

Before students begin work, use their responses to the *Check for Understanding* to determine those who will benefit from additional support.

As students complete the Example and problems 1–3, observe and monitor their reasoning to identify groupings for differentiated instruction.

Start

Check for Understanding

Why Confirm understanding of classifying quadrilaterals.

How Have students classify a quadrilateral given a list of attributes.

Identify the quadrilateral with these attributes:

- 4 right angles
- 2 pairs of parallel sides
- 2 pairs of sides the same length
- all sides not the same length

Solution
rectangle

LESSON 31 SESSION 4 ● ● ● ●

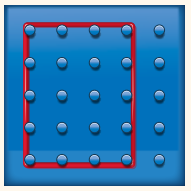
Refine Classifying Quadrilaterals

Complete the Example below. Then solve problems 1–9.

EXAMPLE

A patio has 2 pairs of sides that are the same length. All sides are not the same length, but it does have 4 right angles. What shape is the patio?

Look at how you could show your work using a model.



Solution The patio is a rectangle.

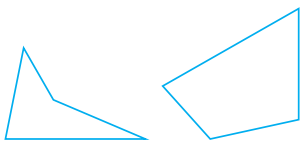
The student used a geoboard to model the shape. Now you can see what the shape looks like.

PAIR/SHARE
How else could you model the shape?

APPLY IT

1 Draw a quadrilateral that has no sides the same length and no right angles. Show your work.

Possible student answers:



The shape you draw will not be a rectangle or a square. It will not be a parallelogram or a rhombus.

PAIR/SHARE
What is a different shape you can draw to solve the problem?

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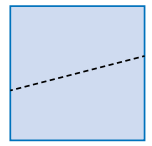
Error Alert

If the error is . . .	*Students may . . .*	*To support understanding . . .*
parallelogram	not have been as specific as possible.	Have students list the attributes in the description of the shape that identify it as a parallelogram. Have them use the remaining attributes to refine their answer.
rhombus	have confused attributes for a rhombus with attributes for a rectangle.	Instruct students to draw a parallelogram, a rectangle, a rhombus, and a square. Have them read the description again and check each shape for each attribute.
square	have confused the fact that all squares are rectangles to mean that all rectangles are squares.	Remind students that a square is a rectangle, but a rectangle is not always a square. The shape does not have all sides the same length, so it is not a square.

2 Friona cut along the dashed line shown on the shape below. She knows that she made two quadrilaterals.



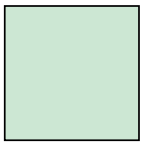
Is either of Friona's quadrilaterals a parallelogram? Explain why or why not.

Solution **Neither of Friona's quadrilaterals is a parallelogram; Possible explanation: Neither shape has 2 pairs of parallel sides.**

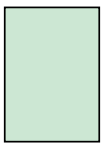
It may help to list the attributes of a parallelogram.

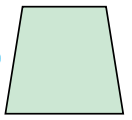
PAIR/SHARE
List the attributes of each of Friona's quadrilaterals.

3 Which shape is NOT a rectangle?

Ⓐ 

Ⓑ 

Ⓒ 

Ⓓ 

What are the attributes of each shape?

Ari chose Ⓐ as the correct answer. How did he get that answer?

Possible answer: Ari forgot that a square has all the attributes of a rectangle, so a square is always also a rectangle.

PAIR/SHARE
What are four ways to name the shape Ari chose?

EXAMPLE

The patio is a rectangle; the rectangle shown is one possible shape for the patio. Students could also draw a rectangle with different dimensions, but not a square.

Look for A model or drawing illustrates the attributes.

APPLY IT

1 See Student Worktext page for possible shapes. Students should draw shapes that have 4 sides, 4 angles, no sides the same length, and no right angles.
DOK 2

Look for List the quadrilaterals the shape cannot be.

2 No; Possible explanation: Friona's quadrilaterals are not parallelograms because they do not have 2 pairs of parallel sides.
DOK 2

Look for A parallelogram has 4 sides and 4 angles, 2 pairs of parallel sides, and 2 pairs of sides the same length.

3 **D**; Students could solve the problem by listing the attributes of a rectangle and checking each shape for those attributes.

Explain why the other two answer choices are not correct:

B and **C** are not correct because they each show shapes with 4 sides, 4 right angles, 2 pairs of parallel sides, and 2 pairs of sides the same length. These shapes are rectangles.
DOK 2

4 **C**; A rhombus does not have to have 4 right angles.
DOK 1

5 parallelogram; The figures are quadrilaterals (they have 4 sides and 4 angles) and parallelograms (they have 2 pairs of parallel sides). Parallelogram is more specific.
DOK 2

Error Alert Students may classify the shapes in problem 5 as quadrilaterals if they are not specific enough.

6 Check students' shapes and reasoning. The quadrilateral must be either a rectangle or a square. All squares are rectangles and all rectangles are parallelograms, so a square would belong to all three groups and a rectangle would belong to two of the groups. Explanations should list the attributes of the shape chosen.
DOK 3

4 A rhombus must have all of these attributes except which one?

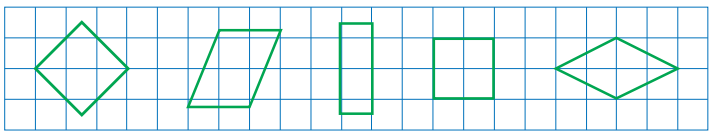
Ⓐ 4 sides that are the same length

Ⓑ 2 pairs of parallel sides

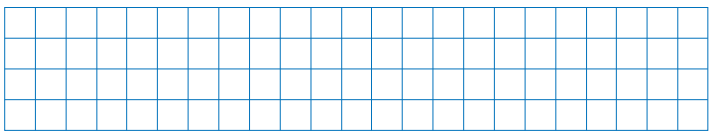
Ⓒ 4 right angles

Ⓓ 4 sides and 4 angles

5 What is the best name that describes all the shapes below? **parallelogram**



6 Use the grid below. Draw a quadrilateral that belongs to at least two of these groups: *parallelogram*, *rectangle*, or *square*. Explain why your shape belongs to these groups. Show your work.



Quadrilaterals will vary, but must be either a rectangle or a square. Explanations will vary. Look for understanding that a square is also a rectangle and also a parallelogram, and a rectangle is also a parallelogram.

Differentiated Instruction

RETEACH

Hands-On Activity
Play a game guessing quadrilaterals from their attributes.

Students struggling with classifying quadrilaterals

Will benefit from additional work with attributes.

Materials For each group of 4: 4 index cards

- Have groups label each card with one shape (*parallelogram*, *rectangle*, *rhombus*, *square*) and place them facedown in a pile. One member draws a card and looks at it, making sure none of the others see it.
- Another player asks whether the quadrilateral has a particular attribute, such as *4 right angles*. After the student answers "yes" or "no", the player who asked the question can try to name the quadrilateral. Players take turns asking about attributes until someone guesses the quadrilateral.
- Repeat the game until all the cards have been used and all the players have had a chance to choose a card.

EXTEND

Challenge Activity
Draw a quilt pattern.

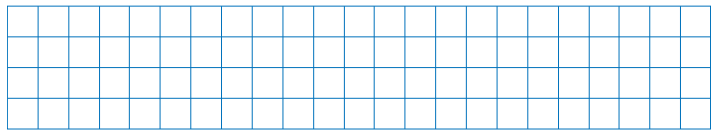
Students who have achieved proficiency

Will benefit from deepening understanding of quadrilaterals and their attributes.

Materials For each student: crayons or markers, Activity Sheet *1-Centimeter Grid Paper* or Activity Sheet *Dot Paper*

- Show students how to draw a quilt pattern by drawing squares and rectangles that cover a section of their paper with no gaps. Have them create their own repeating patterns.
- Challenge them to also use parallelograms that do not have right angles. They can draw a parallelogram by drawing a pair of parallel sides the same length.

7 Use the grid below. Draw a quadrilateral that does NOT belong to any of these groups: *parallelogram*, *rectangle*, or *square*. Explain why your shape does not belong to any of these groups. Show your work.



Quadrilaterals will vary, but cannot be a parallelogram, a rectangle, a rhombus, or a square. Explanations will vary. Look for understanding of what makes a quadrilateral a parallelogram, a rectangle, or a square, and understanding of how to describe and draw a quadrilateral that is none of these.

8 Tell whether each sentence is *True* or *False*.

	True	False
All rhombuses are quadrilaterals.	Ⓐ	Ⓑ
All rectangles are squares.	Ⓒ	Ⓓ
All parallelograms are rectangles.	Ⓔ	Ⓕ
All quadrilaterals are parallelograms.	Ⓖ	Ⓗ
All squares are rhombuses.	Ⓘ	Ⓙ

9 MATH JOURNAL

Jess says that a square cannot be a rectangle because a rectangle has 2 long sides and 2 short sides. Is he correct? Explain.

No. Possible explanation: One pair of opposite sides of a rectangle does not have to be longer than the other pair. When all 4 sides are the same length, a rectangle is also a square.

SELF CHECK Go back to the Unit 6 Opener and see what you can check off.

7 Check students' shapes and reasoning. The quadrilateral cannot be any kind of parallelogram. Rectangles and squares are also parallelograms, so, if the quadrilateral is not a parallelogram, then it does not belong to any of the three groups listed. Explanations should point out that the quadrilateral does not have the attributes of any of the shapes listed.
DOK 3

8 **A** (**True**), All rhombuses have 4 sides and 4 angles, so they are quadrilaterals;
D (**False**), Not all rectangles have 4 sides the same length;
F (**False**), Not all parallelograms have 4 right angles;
H (**False**), Not all quadrilaterals have 2 pairs of parallel sides;
I (**True**), All squares have 4 sides the same length, so they are rhombuses.
DOK 2

REINFORCE

Problems 4–9
Classify quadrilaterals.

All students will benefit from additional work with classifying quadrilaterals by solving problems in a variety of formats.

- Have students work on their own or with a partner to solve the problems.
- Encourage students to show their work.

PERSONALIZE

i-Ready

Provide students with opportunities to work on their personalized instruction path with *i-Ready* Online Instruction to:

- fill prerequisite gaps
- build up grade-level skills

Close: Exit Ticket

9 MATH JOURNAL

Student responses should indicate understanding that having 4 sides the same length is a special case of having 2 pairs of sides that are the same length, and therefore all squares are rectangles.

Error Alert **If** students agree with Jess, **then** have them draw a square and mark the 2 pairs of opposite sides. Elicit from them that the sides in each pair are the same length, and therefore the square has 2 pairs of sides the same length. Discuss the concept that a square is a special type of rectangle.

SELF CHECK Have students consider whether they feel they are ready to check off any new skills on the Unit 6 Opener.

Lesson Overview

LESSON 33

Partition Shapes into Parts with Equal Areas

Lesson Objectives

Content Objectives

- Partition a shape into equal areas.
- Express the area of each equal part as a unit fraction of the whole shape.
- Partition the same shape in different ways.

Language Objectives

- Draw lines to separate a rectangle into same-sized smaller rectangles.
- Draw lines to separate a shape into same-sized smaller parts.
- Shade a given fraction of a shape that has been divided into equal parts.

Prerequisite Skills

- Recognize and identify two-dimensional shapes.
- Partition a shape into two, three, or four equal parts.
- Understand the meaning of fractions.
- Express equal parts of shapes as halves, thirds, and fourths.

Standards for Mathematical Practice (SMP)

SMPs 1, 2, 3, 4, 5, and 6 are integrated in every lesson through the *Try-Discuss-Connect* routine.*

In addition, this lesson particularly emphasizes the following SMPs:

4 Model with mathematics.

5 Use appropriate tools strategically.

7 Look for and make use of structure.

*See page 455i to see how every lesson includes these SMPs.

Lesson Vocabulary

There is no new vocabulary. Review the following key terms.

- **area** the amount of space inside a closed two-dimensional figure. Area is measured in square units such as square centimeters.
- **equivalent fractions** two or more different fractions that name the same part of a whole or the same point on a number line.
- **fraction** a number that names equal parts of a whole. A fraction names a point on the number line.
- **unit fraction** a fraction with a numerator of 1. Other fractions are built from unit fractions.

Learning Progression

In Grade 2 students divided shapes into equal parts, used fraction language such as *halves*, *thirds*, and *fourths* to describe the equal parts, and recognized that the combined equal parts make up the whole. **In previous Grade 3 lessons** students developed an understanding of fractions and learned to name fractions as the number of equal parts in the whole.

In this lesson students first divide rectangles into equal parts. They recognize that equal parts have equal areas by combining their understanding of fractions as equal parts of a whole with their understanding of area of rectangles. For example, students recognize that each equal part of a rectangle divided into six equal parts has an area that is $\frac{1}{6}$ of the whole rectangle. Students use models such as folded sheets of rectangular paper and rectangles divided into rows of same-sized squares to develop an understanding of equal fractional parts having equal areas. Students then extend this knowledge to other shapes such as circles and hexagons.

In Grade 4 students will reason about symmetry in a figure by folding and cutting along a line to determine whether the parts are mirror images. Students will also identify and draw lines of symmetry in a figure.

Lesson Pacing Guide

Whole Class Instruction

SESSION 1 **Explore** *45–60 min*	**Partitioning Shapes into Parts with Equal Areas** • Start *5 min* • Try It *10 min* • Discuss It *10 min* • Connect It *15 min* • Close: Exit Ticket *5 min*	**Additional Practice** Lesson pages 741–742
SESSION 2 **Develop** *45–60 min*	**Partitioning Shapes into Equal Parts** • Start *5 min* • Try It *5 min* • Discuss It *10 min* • Model It & Solve It *5 min* • Connect It & Apply It *15 min* • Close: Exit Ticket *5 min*	**Additional Practice** Lesson pages 747–748 **Fluency** Partitioning Shapes into Equal Parts
SESSION 3 **Refine** *45–60 min*	**Partitioning Shapes into Parts with Equal Areas** • Start *5 min* • Example & Problems 1–3 *15 min* • Practice & Small Group Differentiation *20 min* • Close: Exit Ticket *5 min*	**Lesson Quiz** or **Digital Comprehension Check**

Lesson Materials

Lesson *(Required)*	*Activity Sheet:* 1-Inch Grid Paper
Activities	*Per student:* 2 sheets of blank paper, crayons *Per group:* 24 unit tiles, 5 cards labeled with the unit fractions $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{6}$, and $\frac{1}{8}$ *Activity Sheets:* 1-Inch Grid Paper, 1-Centimeter Grid Paper
Math Toolkit	unit tiles, fraction tiles, grid paper, dot paper, sticky notes, colored pencils
Digital Math Tool	Fraction Models

Teacher Toolbox

Small Group Differentiation

PREPARE

Ready Prerequisite Lesson

Grade 2
- Lesson 30 Partition Rectangles

RETEACH

Tools for Instruction

Grade 2
- Lesson 30 Fill a Rectangle with Squares

Grade 3
- Lesson 33 Partitioning Shapes

REINFORCE

Math Center Activities

Grade 3
- Lesson 33 Equal Areas
- Lesson 33 Divide Shapes

EXTEND

Enrichment Activity

Grade 3
- Lesson 33 Designing a New Home

i-Ready

Independent Learning

PERSONALIZE

i-Ready Lesson*

Grade 3
- Divide Shapes into Parts with Equal Areas

*We continually update the Interactive Tutorials. Check the Teacher Toolbox for the most up-to-date offerings for this lesson.

Connect to Family, Community, and Language Development

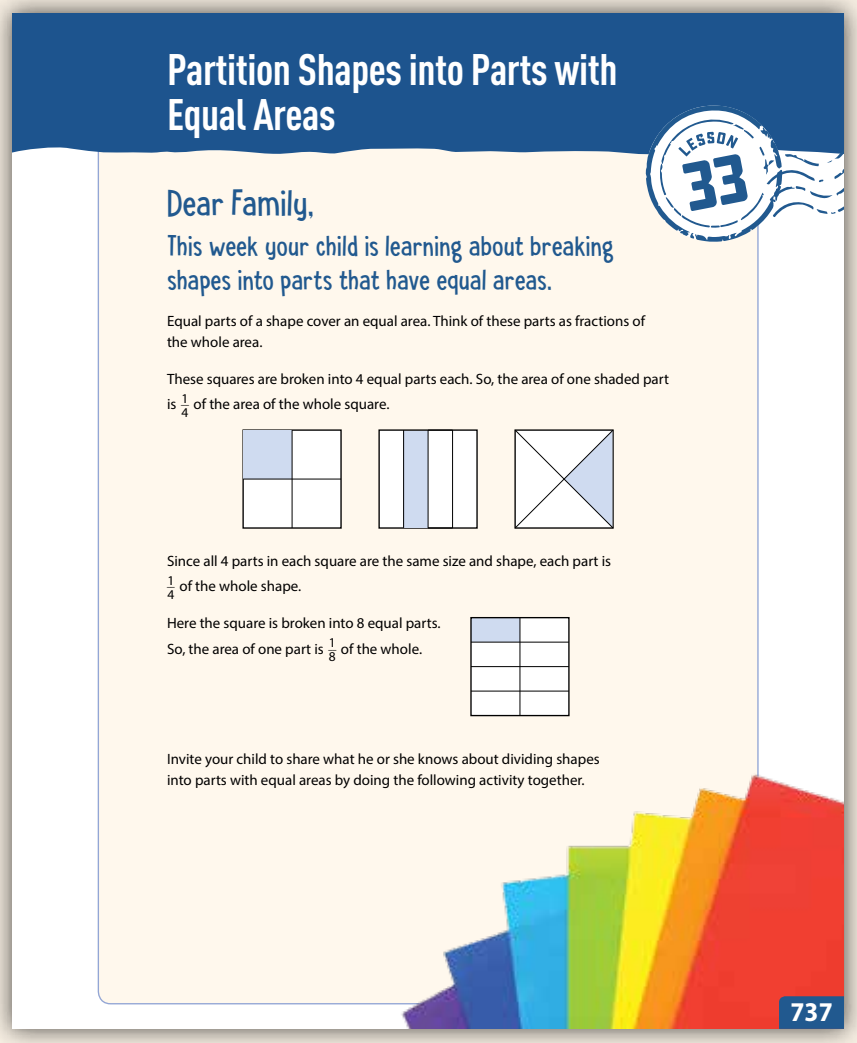
The following activities and instructional supports provide opportunities to foster school, family, and community involvement and partnerships.

Connect to Family

Use the **Family Letter**—which provides background information, math vocabulary, and an activity—to keep families apprised of what their child is learning and to encourage family involvement.

Available in Spanish

Teacher Toolbox



Partition Shapes into Parts with Equal Areas

LESSON 33

Dear Family,

This week your child is learning about breaking shapes into parts that have equal areas.

Equal parts of a shape cover an equal area. Think of these parts as fractions of the whole area.

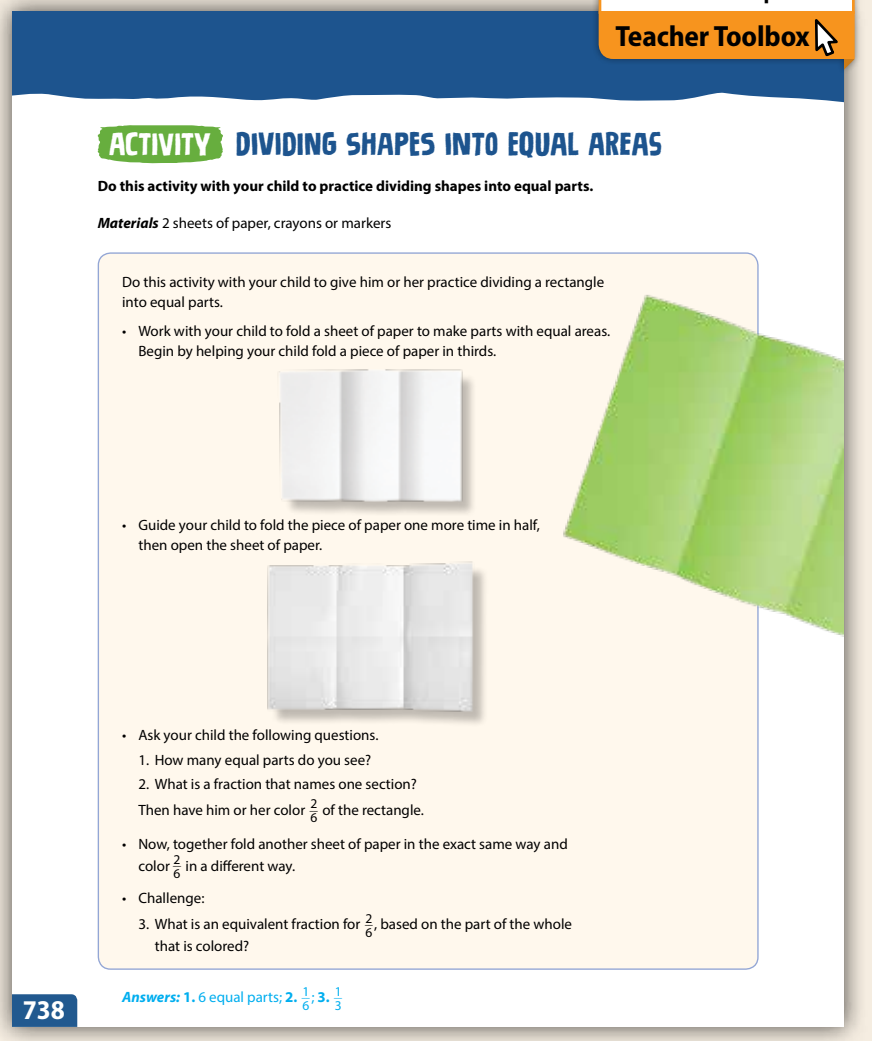
These squares are broken into 4 equal parts each. So, the area of one shaded part is $\frac{1}{4}$ of the area of the whole square.

Since all 4 parts in each square are the same size and shape, each part is $\frac{1}{4}$ of the whole shape.

Here the square is broken into 8 equal parts. So, the area of one part is $\frac{1}{8}$ of the whole.

Invite your child to share what he or she knows about dividing shapes into parts with equal areas by doing the following activity together.

737



ACTIVITY DIVIDING SHAPES INTO EQUAL AREAS

Do this activity with your child to practice dividing shapes into equal parts.

Materials 2 sheets of paper, crayons or markers

Do this activity with your child to give him or her practice dividing a rectangle into equal parts.

- Work with your child to fold a sheet of paper to make parts with equal areas. Begin by helping your child fold a piece of paper in thirds.
- Guide your child to fold the piece of paper one more time in half, then open the sheet of paper.
- Ask your child the following questions.
  1. How many equal parts do you see?
  2. What is a fraction that names one section?

  Then have him or her color $\frac{2}{6}$ of the rectangle.
- Now, together fold another sheet of paper in the exact same way and color $\frac{2}{6}$ in a different way.
- Challenge:
  3. What is an equivalent fraction for $\frac{2}{6}$, based on the part of the whole that is colored?

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Answers: **1.** 6 equal parts; **2.** $\frac{1}{6}$; **3.** $\frac{1}{3}$

Goal

The goal of the Family Letter is to show that equal parts of a geometric shape have equal areas. The concepts of area, fractions as parts of a whole, unit fractions, and equivalent fractions are reviewed.

Activity

Students and family members will divide a rectangle into equal parts.

Look at the *Dividing Shapes into Equal Areas* activity and adjust it if necessary to connect with your students.

Math Talk at Home

Encourage students to talk with their family members about area and fractions. Facilitate a brainstorming session with students to generate a list of household items that can be used to discuss dividing shapes into equal areas. Some examples include cheese, paper towels, napkins, slices of bread, tinfoil, and wax paper.

Conversation Starters Below are additional conversation starters students can write in their Family Letter or math journal to engage family members.

- *How many different ways can a napkin/paper towel be folded into halves? What are the different ways?*
- *How many different ways can a slice of bread be cut into fourths? What are the different ways?*

Connect to Community and Cultural Responsiveness

Use these activities to connect with and leverage the diverse backgrounds and experiences of all students.

Session 1 **Use throughout the session.**

- Display pictures of different types of ceiling and floor tiling. Relate same-sized square tiles to the unit squares used to measure area in previous lessons. Ask students to identify the pictures with same-sized square tiles. Explain that same-sized tiles represent equal parts of the whole ceiling or floor. Point out that tiles of different sizes do not have equal areas. Invite students to describe the tiling at school and tell if the tiles represent equal parts of the whole.
- Give each student a rectangular piece of paper and have them replicate your steps. Display one piece and fold it in half vertically. Shade one half of the paper. Fold the paper in half again horizontally. Unfold and verify the shaded half is $\frac{2}{4}$. Display: $\frac{1}{2} = \frac{2}{4}$.
- Refold the paper and fold in half a third time vertically. Unfold and verify the shaded half is $\frac{4}{8}$. Display: $\frac{1}{2} = \frac{2}{4} = \frac{4}{8}$. Ask: *What pattern do you see in these three fractions? What fraction will you make that is equal to* $\frac{1}{2}$ *if you fold the piece of paper a fourth time?* Display students' suggestions. Have students fold the paper a fourth time. Affirm that the fraction $\frac{8}{16}$ is equal to $\frac{1}{2}$.

Sessions 1–3 **Use throughout the lesson.**

- Create a display on a poster showing different ways to fold paper into parts with equal areas. Throughout the lesson, invite students to fold square or rectangular paper into different numbers of parts with equal areas. Encourage students to discuss how the papers are folded and what fractions of the area are shaded. Have students choose two same-sized papers with shaded areas and compare these fractions using $=$, $<$, and $>$. Select students to add their papers to the display. Refer to the display throughout the lesson to support understanding.

Session 3 **Use throughout the session.**

- Repeat the paper folding task explained in Session 1. This time use a circular piece of paper. Show students a picture of a pizza. Fold the circle in half 3 times to make 8 equal parts. Facilitate a discussion about what fraction will be eaten if someone eats 1 slice of the circle/pizza. Repeat the paper folding task with a hexagonal piece of paper. Target a different fraction to explore, such as $\frac{1}{4}$.

Connect to Language Development

For ELLs, use the Differentiated Instruction chart to plan and prepare for specific activities in every session.

ELL English Language Learners: Differentiated Instruction

Prepare for Session 1
Use with *Try It*.

Levels 1–3	Levels 2–4	Levels 3–5
Listening/Speaking Pair students. Read ***Try It***. Give three square sticky notes to each student. Say: *The notes are squares like on your page.* Model how to fold a square vertically and have students replicate. Unfold and trace the crease from the fold. Ask: *What unit fraction describes each part of the square?* $\left[\frac{1}{2}\right]$ Have the students label each part and stick the note to the page in their books. Say: *You folded the paper in half vertically. With your partner, find another way to fold the paper into two equal parts.* Select students to show how they folded the note. Continue until horizontal and diagonal folds are suggested. Have students label each fractional part $\frac{1}{2}$ and place the notes in their books.	**Listening/Speaking** Pair students. Read ***Try It***. Give three square sticky notes to each student. Say: *What shape are these papers? How are they like the shapes in your book?* Fold a note vertically, unfold and trace the crease from the fold. Have students replicate. Ask: *What unit fraction describes each part of the square?* $\left[\frac{1}{2}\right]$ *Label each part and stick the note to the page in your book.* Have students discuss with partners other ways they can divide a square into two equal parts. Invite students to show and describe how they folded the other two sticky notes. Provide words, such as *diagonally* and *horizontally,* if needed. Have students label each fractional part $\frac{1}{2}$ and place the notes in their books.	**Listening/Speaking** Pair students and have them read ***Try It***. Give three square sticky notes to each student. Ask: *What shape are these papers? How are they like the shapes in your book? What unit fraction describes each part of this shape after it has been divided into two equal parts?* Pause for students to answer. Ask: *How do you know?* Have students take turns telling partners the different ways they can fold the squares into two equal parts. Have students label each fractional part and place the notes on the page in their books. Select pairs to share and discuss how they folded the sticky notes and how they know what fraction each part represents.

LESSON 33

SESSION 1 Explore

Purpose In this session students draw on what they know about fractions and area. They compare models to explore the different ways that one half of a square can be shaded. They will look ahead to think about different ways to divide a shape into equal parts.

Start

Connect to Prior Knowledge

Why Prepare students to divide squares into equal parts and name one part with a fraction.

How Have students identify the fractions represented by area models.

What fraction is shown by each area model?

1. 
2. 

Solutions

1. $\frac{1}{3}$
2. $\frac{3}{4}$

TRY IT

Make Sense of the Problem

To support students in making sense of the problem, have them show that they understand that each square should be divided into two equal-sized parts in a different way.

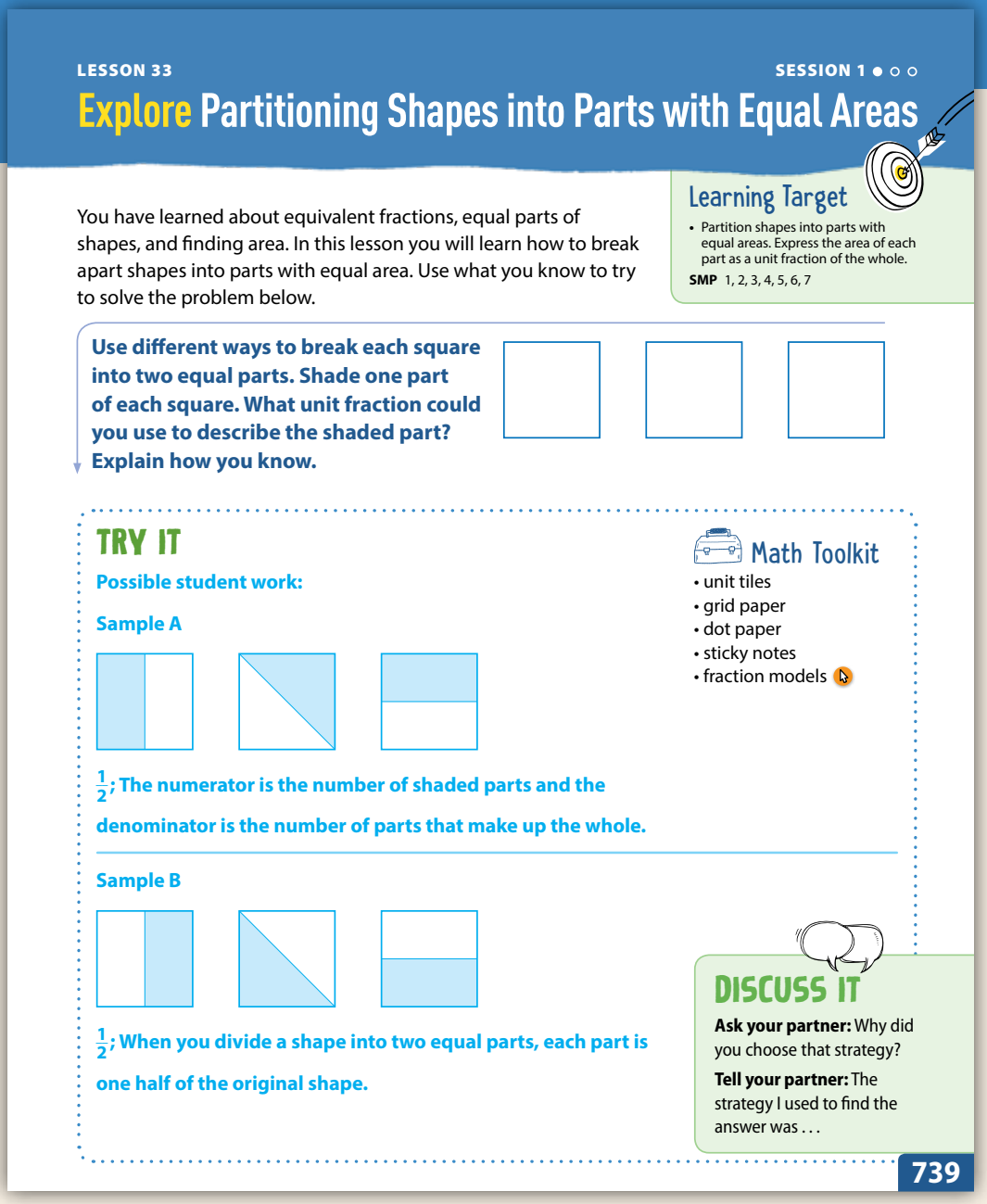
DISCUSS IT

Support Partner Discussion

To reinforce the concept that the parts should be equal in size, encourage students to use the term *equal* as they talk to each other.

Look for, and prompt as necessary for, understanding that:

- each square should be divided in a different way
- each square should be divided into 2 equal parts with 1 part shaded
- they should use a unit fraction to describe the shaded part



LESSON 33 SESSION 1 ● ○ ○

Explore Partitioning Shapes into Parts with Equal Areas

Learning Target

- Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole.

SMP 1, 2, 3, 4, 5, 6, 7

You have learned about equivalent fractions, equal parts of shapes, and finding area. In this lesson you will learn how to break apart shapes into parts with equal area. Use what you know to try to solve the problem below.

Use different ways to break each square into two equal parts. Shade one part of each square. What unit fraction could you use to describe the shaded part? Explain how you know.

TRY IT

Possible student work:

Sample A

Math Toolkit

- unit tiles
- grid paper
- dot paper
- sticky notes
- fraction models

$\frac{1}{2}$; **The numerator is the number of shaded parts and the denominator is the number of parts that make up the whole.**

Sample B

$\frac{1}{2}$; **When you divide a shape into two equal parts, each part is one half of the original shape.**

DISCUSS IT

Ask your partner: Why did you choose that strategy?

Tell your partner: The strategy I used to find the answer was . . .

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Common Misconception Look for students who are not sure how to divide the square so that the parts are the same size. As students present solutions, have them specify how they know that the two parts of each square are equal.

Select and Sequence Student Solutions

One possible order for whole class discussion:

- explanations that refer to the two parts having the same shape
- explanations that use the terms *numerator* and *denominator*
- explanations that refer to the two parts having the same area

Support Whole Class Discussion

Prompt students to compare the two parts of each model.

Ask *How are the two parts of [student name]'s model the same? How are they different? How can you tell they are equal?*

Listen for The shape and size of each half are the same. They are in different positions within the model and, as in the case of two triangles, may be rotated or flipped. Two figures that are the same size and shape have equal areas because they cover the same amount of space.

CONNECT IT

1 LOOK BACK

Look for understanding that the numerator represents the number of parts shaded and the denominator represents the number of parts that make a whole.

Visual Model

Use grid paper to check that two halves of a square have the same area.

If . . . *students are unsure about the concept of dividing a square into equal parts,*

Then . . . *use this activity to have them check that the areas of the parts are equal.*

Materials For each student: Activity Sheet *1-Inch Grid Paper*

- Have students draw a 4-by-4 square on the grid paper and divide the square into two equal parts.
- Ask students to find the area of each part by counting the grid squares. [8 square units] If students drew a diagonal, help them to pair the partial grid squares in order to count whole units.
- Repeat the activity with a same-sized square but dividing it in a different way. Prompt students to recognize that each half is always 8 square units, regardless of the shapes of the halves.

2 LOOK AHEAD

Point out that rectangles *A* and *B* show the same fraction because the wholes are the same size and there are the same number of equal parts in each whole and the same number of parts are shaded. Prompt students to recognize that rectangles *C* and *D* have a similar relationship for the same reason.

CONNECT IT

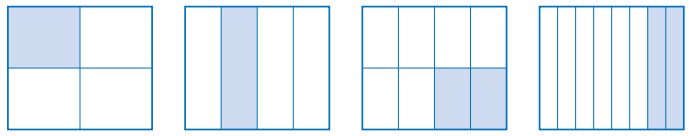
1 LOOK BACK

Explain how you know the unit fraction that names the shaded part of each square.

Possible answer: For each square, 1 part is shaded and 2 parts make a whole. So, $\frac{1}{2}$ of each square is shaded.

2 LOOK AHEAD

You can break apart the same shape into equal parts in a lot of ways. You can use fractions to describe the area that each part covers. Look at the rectangles below. The shaded areas of all four rectangles are both alike and different.



a. What fraction of the area of rectangle *A* is shaded? $\frac{1}{4}$

What fraction of the area of rectangle *B* is shaded? $\frac{1}{4}$

What fraction of the area of rectangle *C* is shaded? $\frac{2}{8}$

What fraction of the area of rectangle *D* is shaded? $\frac{2}{8}$

b. For rectangles *C* and *D*, what unit fraction is equivalent to the fraction shown by the shaded parts? $\frac{1}{4}$

3 REFLECT

What is the same about the areas shown by the shading in the four rectangles above? What is different? **Possible answer: All rectangles have $\frac{1}{4}$ of their areas shaded. Rectangles *A* and *B* are broken into 4 parts in different ways, each with 1 part shaded. Rectangles *C* and *D* are broken into 8 parts in different ways, each with 2 parts shaded.**

Close: Exit Ticket

3 REFLECT

Look for understanding that the same amount, $\frac{1}{4}$, of each rectangle is shaded, though the number and shape of the parts vary from rectangle to rectangle.

Common Misconception **If** students do not see that $\frac{1}{4} = \frac{2}{8}$, **then** draw rectangles *C* and *D* on a sheet of paper. Use a black marker to trace over the outlines and the lines that divide the rectangles into fourths so that the rectangles appear to be identical to rectangles *A* and *B* with additional lines drawn in to show eighths.

Real-World Connection

Ask students to think of everyday situations that require dividing rectangles or other shapes into equal parts. Examples may include cutting a sheet cake or a pan of brownies into equal-sized servings, cutting a pizza or a pie into equal-sized slices, dividing a board or a sheet of plywood into equal parts for construction or crafting, and making Bingo or Tic-Tac-Toe game boards. Make sure students understand the relationship between two-dimensional shapes and the three-dimensional objects they might identify.

Solutions

Support Vocabulary Development

1 Ask student to point to and read *unit fraction* chorally. Remind students that unit fractions were explored, drawn, and labeled in previous lessons. Explain that the word *unit* comes from the Latin root *ūnus* that means *one*. Invite students to suggest ways to remember the meaning of unit.

Ask students to share examples of unit fractions. Display the unit fractions that are shared. Ask: *What is the same in all unit fractions?* [The numerator is 1.] *All of the denominators are different. What does the denominator represent?* [It represents the total number of parts in the whole.]

Have students work with partners and discuss the examples they plan to include in the boxes. Remind students to use words, numbers, and pictures to complete the examples.

2 Have students number the parts of the rectangle beginning with the shaded part. Display and have students replicate on the student page:

$$\frac{\text{numerator}}{\text{denominator}} = \frac{\text{number of shaded parts}}{\text{total number of parts}} = \frac{?}{?}$$

Say: *Complete this equation with the unit fraction that describes the picture.*

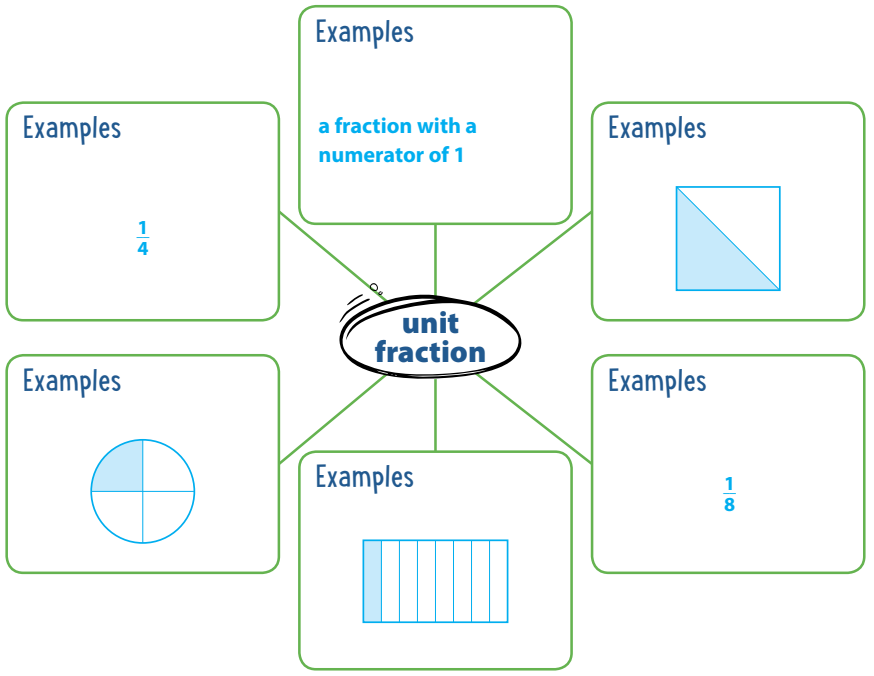
Supplemental Math Vocabulary

- *area*
- *equivalent fraction*
- *fraction*

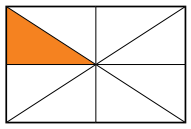
Name: _____ LESSON 33 SESSION 1

Prepare for Partitioning Shapes into Parts with Equal Areas

1 Think about what you know about fractions and shapes. Fill in each box.
Use words, numbers, and pictures. Show as many ideas as you can. **Possible answers:**



2 Look at the rectangle below.



What unit fraction names the shaded part of the rectangle? $\frac{1}{8}$

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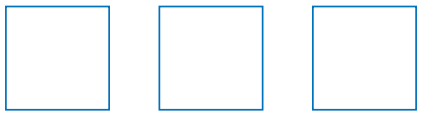
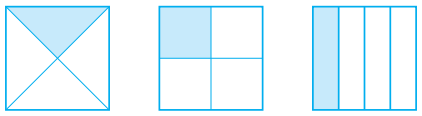
3 Solve the problem. Show your work.

Use different ways to break each square below into four equal parts. Shade one part of each square. What unit fraction could you use to describe the shaded part? Explain how you know.



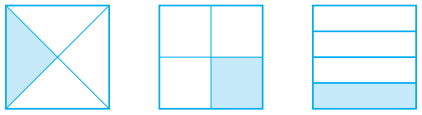
Possible student work using pictures:



Solution **$\frac{1}{4}$; The numerator is the number of shaded parts and the denominator is the number of parts that make up the whole.**

4 Check your answer. Show your work.

Possible student work using pictures:



$\frac{1}{4}$; When you divide a shape into 4 equal parts, each part is one fourth of the whole shape.

3 Assign problem 3 to provide another look at dividing shapes into parts with equal areas.

This problem is very similar to the problem about breaking squares into two equal parts. In both problems, students are asked to break squares into a number of equal parts, shade one part, and say what unit fraction could be used to describe the shaded part. The question asks students to break squares into four equal parts.

Students may want to use geoboards and rubber bands or paper and scissors.

Suggest that students read the problem three times, asking themselves one of the following questions each time:

- *What is this problem about?*
- *What is the question I am trying to answer?*
- *What information is important?*

Solution: Students can divide the square into four equal parts with parts shaped like triangles, squares, or rectangles.

$\frac{1}{4}$; the numerator is the number of shaded parts and the denominator is the number of parts that make up the whole.

Medium

4 Have students solve the problem a different way to check their answer.

ELL English Language Learners: Differentiated Instruction

Prepare for Session 2
Use with *Apply It*.

Levels 1–3

Listening/Speaking Read ***Apply It*** problem 9. Display shape *A*. Model and explain how to label the shape with a fraction so that the numerator is the number of shaded parts and the denominator is the total number of parts in the shape. Ask: *What fraction of the shape is shaded? Is the fraction equivalent to $\frac{1}{2}$? How do you know?* Display the equation $\frac{1}{2} = \frac{2}{4}$. Explain that $\frac{1}{2}$ and $\frac{2}{4}$ are equivalent fractions. Ask students to tell other fractions that are equivalent to $\frac{1}{2}$. Say: *In all of these fractions, the numerator is one half of the denominator.*

Have students work with partners to label the other shapes with fractions.

Levels 2–4

Listening/Speaking Read ***Apply It*** problem 9. Display shape *A*. Model and explain how to label the shape with a fraction so that the numerator is the number of shaded parts and the denominator is the total number of parts in the shape. Display the equivalent fractions: $\frac{2}{4}, \frac{3}{6}, \frac{4}{8}$. Ask: *What do you notice about the numerator and denominator of each fraction?* Encourage students to express that the numerator is $\frac{1}{2}$ of the denominator. Say: *These fractions are equivalent to $\frac{1}{2}$.*

Have students work with partners to label the other shapes with fractions. Ask partners to discuss how they know which shapes show $\frac{1}{2}$ of the area shaded.

Levels 3–5

Listening/Speaking Pair students and have them read ***Apply It*** problem 9. Display shape *A*. Ask: *What fraction of the shape is shaded?* Display the fractions: $\frac{2}{4}, \frac{3}{6}, \frac{4}{8}$. Ask: *What do you notice about the numerator and denominator of each fraction?* [The numerator is $\frac{1}{2}$ of the denominator.] Say: *These fractions are equivalent to $\frac{1}{2}$. What do the denominators of these fractions have in common?* Have pairs discuss why there is no fraction equivalent to $\frac{1}{2}$ that has a denominator of 3.

Have pairs label the other shapes with fractions and discuss how they know which shapes show $\frac{1}{2}$ of the area shaded.

LESSON 33

SESSION 2 Develop

Purpose In this session students solve a problem that requires coloring $\frac{1}{4}$ of a paper that is divided into eight equal parts. Students model the situation either on paper or with manipulatives to show how eighths can be used to make $\frac{1}{4}$. The purpose of this problem is to show students that there are several different ways to show a given fraction.

Start

Connect to Prior Knowledge

Materials For each student: Activity Sheet *1-Inch Grid Paper*

Why Support students' facility with dividing a shape into equal parts.

How Have students divide a square into four equal parts and use a fraction to describe one of the parts.

On grid paper, draw a square with side lengths of 4 units.

Divide your square into 4 equal parts and shade 1 part.

What fraction describes the shaded part?

Solution

$\frac{1}{4}$; Check students' drawings.

Develop Language

Why Clarify the meaning of *total area*.

How Remind students that *total* means "complete or whole". Have students find the phrase *total area* in *Apply It* problems 7 and 8. Have students use the following sentence frame to tell what it means to find the total area in each problem: *I need to find the fraction of the _____ that each part represents.* Have students point to the total or complete area.

TRY IT

Make Sense of the Problem

To support students in making sense of the problem, have them identify how many times the paper is folded.

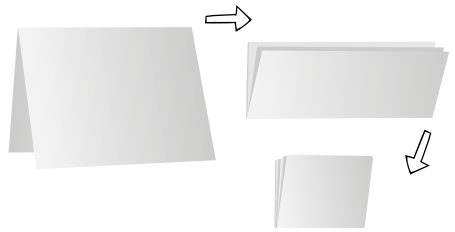
Ask *How many times does Brett fold the piece of paper? How much of the paper does he color red?*

Develop Partitioning Shapes into Equal Parts

Read and try to solve the problem below.

Brett folded a piece of paper three times as shown. He then colored $\frac{1}{4}$ of the total area of the paper red. How could he have colored his paper?

Explain how you know your way is right.



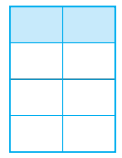
TRY IT

Possible student work:

Sample A

There are 4 rows on the paper.

Brett could have colored 1 of the 4 rows to show $\frac{1}{4}$.

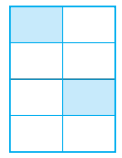


Sample B

There are 8 equal parts when you unfold the paper.

Brett could have colored 2 of the 8 parts to show $\frac{1}{4}$.

$\frac{2}{8}$ is the same as $\frac{1}{4}$.



Math Toolkit

- fraction tiles
- grid paper
- colored pencils
- fraction models

DISCUSS IT

Ask your partner: Do you agree with me? Why or why not?

Tell your partner: I agree with you about . . . because . . .

DISCUSS IT

Support Partner Discussion

Encourage students to use the term *fourths* as they discuss their solutions.

Support as needed with questions such as:

- *Did you draw a picture to help you answer the question?*
- *How did you decide how many parts Brett colored red?*

Common Misconception Look for students who think the paper must be divided into exactly 4 parts to show $\frac{1}{4}$ rather than the 8 parts made by the folds shown in the problem. Have students follow the given directions to fold a sheet of paper and verify that 2 of 8 equal parts covers $\frac{1}{4}$ of the sheet.

Select and Sequence Student Solutions

One possible order for whole class discussion:

- the 2 parts colored are adjacent sections in the corner of the paper
- the 2 parts colored are adjacent sections in the middle of the paper
- the 2 parts colored are nonadjacent sections

Support Whole Class Discussion

Compare and connect the different solutions and have students identify how they are related.

Ask *How many parts is the paper divided into? How many parts should be colored? Which parts are colored?*

Listen for The paper is divided into 8 parts, so 2 parts should be colored to show $\frac{1}{4}$. It does not matter which 2 parts are colored.

MODEL IT & SOLVE IT

If no student presented these models, connect them to the student models by pointing out the ways they each represent:

- the number of parts the paper is divided into
- the amount of paper to be colored

Ask *How many parts is the paper divided into? What fraction of the paper is each part?*

Listen for The paper is divided into 8 parts, so each part is $\frac{1}{8}$ of the paper.

For a paper model, prompt students to think about how the number of folds determines the number of parts the paper is divided into.

- *How many parts does the paper have after folding it the first time? The second time? The third time?*
- *How is the number of folds related to the number of parts the paper is divided into?*

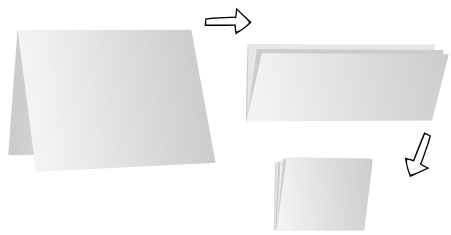
For the solution, prompt students to plan how they will color some of the 8 parts to show $\frac{1}{4}$.

- *What fraction of the rectangle is each part?*
- *How many eighths are equal to $\frac{1}{4}$?*
- *How can you compare the fractions using the = symbol?*

Explore different ways to understand dividing shapes into equal parts.

Brett folded a piece of paper three times as shown. He then colored $\frac{1}{4}$ of the total area of the paper red. How could he have colored his paper?

Explain how you know your way is right.



MODEL IT

You can act out the problem and make a model.

Fold a piece of paper in half three times as Brett did.

Unfold the paper.

This is what the paper looks like, divided into equal parts.

SOLVE IT

You can use equivalent fractions to solve the problem.

The paper has 8 equal parts.

You need to color a number of parts red so that $\frac{1}{4}$ of the area of the paper is colored.

Think of a fraction equivalent to $\frac{1}{4}$ to help solve the problem. You can compare numbers using $<$, $>$, or $=$. Since your fraction will be equivalent to $\frac{1}{4}$, you can compare the fractions using $=$.

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Deepen Understanding

Divide Shapes into Equal Parts

SMP 4 Model with mathematics.

As you discuss the paper model, prompt students to think about how other fractions can be modeled by folding paper.

Ask *Suppose Brett folded the paper 4 times instead of 3 times. How many parts would the paper be divided into? How do you know?*

Listen for The paper would have 16 parts. Each of the 8 parts would be divided into 2 parts, and $8 \times 2 = 16$.

Ask *Suppose you fold a sheet of paper into thirds two different times. How many parts will the paper be divided into? How do you know?*

Listen for The paper will have 9 parts. Each of the 3 parts from the first folding will be divided into 3 parts after the second folding because $3 \times 3 = 9$.

Ask *How could you fold a paper to divide it into six equal parts?*

Listen for You could fold the paper into thirds and then in half (or in half and then into thirds).

CONNECT IT

- Remind students that the one thing that is alike about all the representations is the numbers.
- Explain that on this page, students will use those numbers to determine how $\frac{1}{4}$ of the paper can be colored and why there are many different solutions to the problem.

Monitor and Confirm

1 – 3 Check for understanding that:

- the paper is divided into 4 rows of 2
- 1 row out of 4 rows is $\frac{1}{4}$ of the paper
- a row of 2 squares out of 8 squares is $\frac{2}{8}$ of the paper

Support Whole Class Discussion

4 Tell students that this problem will help them provide the explanation required in problem 5.

Be sure that students understand that the problem is asking them for an alternate way to color $\frac{1}{4}$ of the paper and that there are several possible answers.

Ask *Which parts could Brett have colored? How do you know that these parts have the same total area as one row?*

Listen for Any 2 parts can be colored. All 8 parts of the paper are the same size and shape, so they have the same area. So, any 2 parts will have the same area as the 2 parts in one row.

5 Look for the idea that as long as the total area of the colored parts is $\frac{1}{4}$ the total area of the paper, it does not matter which parts are colored or whether they are next to each other.

6 REFLECT Have all students focus on the strategies used to solve this problem. If time allows, have students share their preferences with a partner.

CONNECT IT

Now you will use the problem from the previous page to help you understand how to divide shapes into equal parts.

1 How many equal parts are on the paper? 8 How many in 1 row? 2

Suppose Brett colors 1 row. What fraction of the paper does he color? $\frac{2}{8}$

What fraction of the paper is NOT colored? $\frac{6}{8}$

Use <, >, or = to compare the fraction of the paper that is colored and the fraction that is not colored. $\frac{2}{8} < \frac{6}{8}$

2 What fraction of the paper is 1 row? Explain.

$\frac{1}{4}$; Possible explanation: There are 4 equal rows in the whole, so 1 row is $\frac{1}{4}$ of the whole.

3 Does Brett color $\frac{1}{4}$ of the area of the paper? Use your answers above to explain.

Yes; Possible explanation: 1 row is $\frac{2}{8}$ of the paper in problem 1 and $\frac{1}{4}$ of the paper in problem 2. $\frac{2}{8}$ and $\frac{1}{4}$ are equivalent fractions, so $\frac{2}{8} = \frac{1}{4}$.

4 How else could Brett have colored $\frac{1}{4}$ of the paper?

Possible answer: He could have colored the top 2 parts in the first column of the paper.

5 To color $\frac{1}{4}$ of the paper, must Brett color parts that are next to each other? Explain.

No; Possible explanation: All 8 parts of the paper are equal. As long as he colors any 2 parts, $\frac{1}{4}$ of the whole paper will be covered.

6 REFLECT

Look back at your **Try It**, strategies by classmates, and **Model It** and **Solve It**. Which models or strategies do you like best for dividing shapes into equal parts? Explain.

Some students may prefer making concrete models by folding paper. Others might be comfortable enough with symmetry to know intuitively where to draw lines that divide shapes equally.

745

Hands-On Activity

Divide a paper rectangle into parts by folding.

If . . . *students are unsure how 2 or more parts can represent a unit fraction,*

Then . . . *use this activity to give them a concrete model to manipulate.*

Materials For each student: 2 sheets of blank paper, crayons

- Instruct students to fold one sheet of paper in half two times and shade the part of the paper that is showing. Then have them unfold the paper and use a fraction to describe how much of the paper is shaded. $\left[\frac{1}{4}\right]$
- Tell students to refold the paper along the previous folds and then fold it in half one more time. Then have them unfold the paper and use a different fraction to describe how much of the paper is shaded. $\left[\frac{2}{8}\right]$
- Repeat the activity with the second sheet of paper, but have students shade the paper after just one fold. Repeat the second step twice to find three different fractions that describe the shaded part. $\left[\frac{1}{2}, \frac{2}{4}, \frac{4}{8}\right]$

APPLY IT

For all problems, encourage students to be prepared to explain how they got their answers.

7 $\frac{1}{8}$; See Student Worktext page for possible student work. Look for the rectangle to be divided into 8 equal parts.

8 $\frac{1}{8}$; See Student Worktext page for possible student work. Look for the rectangle to be divided into 8 equal parts in a different way than done in problem 7.

Close: Exit Ticket

9 **B**, 1 out of 2 parts shaded is $\frac{1}{2}$;

D, $\frac{2}{4} = \frac{1}{2}$;

F, $\frac{4}{8} = \frac{1}{2}$

Error Alert **If** students choose answer choices A or E, **then** redraw each shape, shading 2 adjacent parts instead of the ones shown. Trace around each shaded section and around the other 2-part sections that have the same shape to show that 1 out of 4 sections is shaded in choice A and that 1 out of 3 sections is shaded in choice E.

APPLY IT

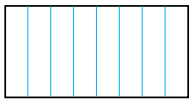
Use what you just learned to solve these problems.

7 Divide this rectangle into 8 equal parts. What fraction of the total area of the rectangle is each part?

Possible student work.

Solution $\frac{1}{8}$

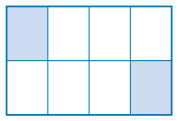
8 Show a different way to divide the rectangle from problem 7 into 8 equal parts. What fraction of the total area of the rectangle is each part?

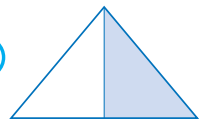


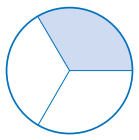
Possible student work.

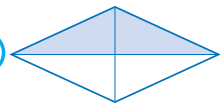
Solution $\frac{1}{8}$

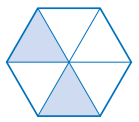
9 Each shape is divided into equal parts. Which shapes show $\frac{1}{2}$ of the area shaded?

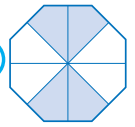
Ⓐ 

Ⓑ 

Ⓒ 

Ⓓ 

Ⓔ 

Ⓕ 

Solutions

1. 6
Basic

2. 2
Basic

3. $\frac{3}{6}$, or $\frac{1}{2}$
Basic

4. $\frac{1}{6}$; See student page for possible division.
Medium

5. $\frac{2}{6}$, or $\frac{1}{3}$; Possible explanation: There are 3 rows and 1 row is shaded, so $\frac{1}{3}$ is shaded.
Medium

Name: _____ LESSON 33 SESSION 2

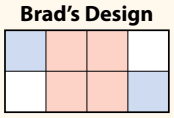
Practice Partitioning Shapes into Equal Parts

Study the Example showing how to divide rectangles into equal parts. Then solve problems 1–10.

EXAMPLE

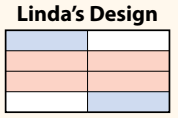
Brad and Linda each cover a same-sized board with mosaic tiles. Here are the designs they made. What part of Brad's design is red tiles? What part of Linda's design is red tiles?

Brad's Design



2 rows of 4 tiles = 8 tiles
$\frac{4}{8}$, or $\frac{1}{2}$, of the tiles are red.

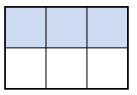
Linda's Design



4 rows of 2 tiles = 8 tiles
$\frac{4}{8}$, or $\frac{1}{2}$, of the tiles are red.

1. How many equal parts are in rectangle *A*? 6

A

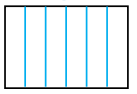


2. How many rows are in rectangle *A*? 2

3. What fraction of the total area of rectangle *A* is shaded? $\frac{3}{6}$, or $\frac{1}{2}$

4. Use rectangle *B* to show another way to divide a rectangle into 6 equal parts. What unit fraction is each part? $\frac{1}{6}$

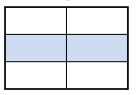
B



Possible answer:

5. What fraction of the total area of rectangle *C* is shaded? Tell how you know.

C



There are 6 equal parts and 2 are shaded, so $\frac{2}{6}$, or $\frac{1}{3}$, of the area is shaded.

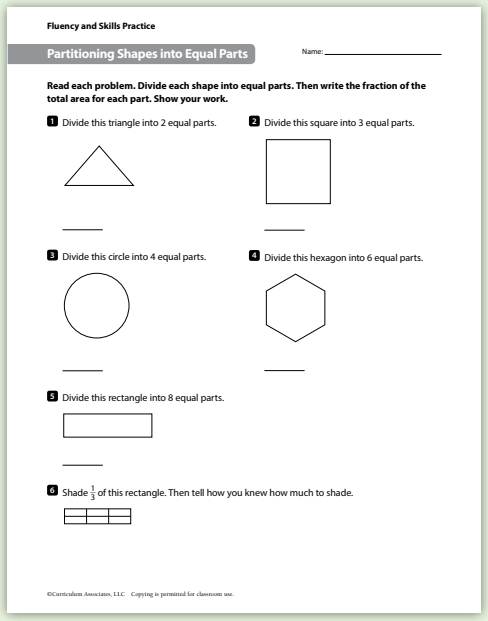
747

Fluency & Skills Practice

Teacher Toolbox

Assign *Partitioning Shapes into Equal Parts*

In this activity students practice dividing shapes into equal parts. Then they identify the fraction of the total area that each equal part represents. Students may apply the same concept in real-world situations. For example, a group of students may want to divide the area of a mural into equal parts so that each student paints the same amount and then say what fraction of the mural each student painted.



Fluency and Skills Practice

Partitioning Shapes into Equal Parts

Name: _____

Read each problem. Divide each shape into equal parts. Then write the fraction of the total area for each part. Show your work.

1. Divide this triangle into 2 equal parts.

_____

2. Divide this square into 3 equal parts.

_____

3. Divide this circle into 4 equal parts.

_____

4. Divide this hexagon into 6 equal parts.

_____

5. Divide this rectangle into 8 equal parts.

_____

6. Shade $\frac{1}{3}$ of this rectangle. Then tell how you knew how much to shade.

6 $\frac{1}{8}$
Medium

7 **A** (**True**);
D (**False**), $\frac{1}{4}$ is shaded;
E (**True**);
H (**False**), Each row is $\frac{1}{2}$.
Medium

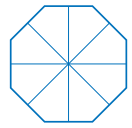
8 See student page for possible divisions.
Medium

9 See student page for possible shading.
Medium

10 $\frac{1}{4} = \frac{2}{8}$
Medium

11 See student page for possible division and shading; Possible explanation: 2 of the 6 parts are shaded, so $\frac{2}{6}$ of the hexagon is shaded. $\frac{2}{6} = \frac{1}{3}$
Challenge

6 The octagon is divided into equal parts. What fraction of the total area of the octagon is each part? $\frac{1}{8}$



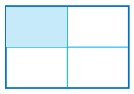
7 Compare squares *X* and *Y*. Tell whether each statement is *True* or *False*.

	True	False
$\frac{1}{2}$ of shape *X* is shaded.	Ⓐ	Ⓑ
$\frac{1}{2}$ of shape *Y* is shaded.	Ⓒ	Ⓓ
Each row of shape *X* is $\frac{1}{4}$ of the whole square.	Ⓔ	Ⓕ
Each row of shape *Y* is $\frac{1}{4}$ the whole square.	Ⓖ	Ⓗ

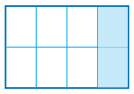
X

Y

8 Divide rectangle *S* into 4 equal parts and divide rectangle *T* into 8 equal parts. **Possible answer:**



S

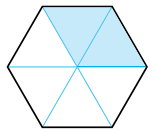


T

9 Shade $\frac{1}{4}$ of the area of each rectangle in problem 8. **See possible answer above.**

10 Use <, >, or = to compare the shaded parts of the rectangles in problem 8.
$\frac{1}{4}$ (=) $\frac{2}{8}$

11 Divide the hexagon into 6 equal triangles. Then shade $\frac{1}{2}$ or $\frac{1}{3}$ of the area of the hexagon. Tell how you know $\frac{1}{2}$ or $\frac{1}{3}$ of the area is shaded.



Possible answer: I shaded 2 of the 6 same-sized triangles.
So, $\frac{2}{6}$, or $\frac{1}{3}$, of the hexagon is shaded.

ELL English Language Learners: Differentiated Instruction

Prepare for Session 3
Use with *Apply It*.

Levels 1–3

Listening/Speaking Read ***Apply It*** problem 4. Ask: *What shape is this problem about?* Read the problem again. Say: *Circle what is true about the rectangle.* Display the given information from the problem:

1) *It is divided into same-sized squares* □.
2) *Four squares are shaded:* ■.
3) *The shaded squares are $\frac{1}{2}$ the whole area.*

Guide students to draw the shape that is described. Display the four shaded squares of the rectangle. Say: *Four squares are $\frac{1}{2}$ the area of the rectangle. How many squares make up the other half?* Complete the rectangle to display the other half of it. Ask: *How many squares make up the whole rectangle?*

Levels 2–4

Listening/Speaking Read ***Apply It*** problem 4. Ask: *What shape is this problem about?* Have students read the problem. Say: *Circle what is true about the rectangle.* Display:

1) *It is divided into same-sized squares.*
2) *Four squares are shaded.*
3) *The shaded squares are $\frac{1}{2}$ the whole area.*

Guide students to draw the shape that is described. Display the four shaded squares of the rectangle. Say: *Four squares are $\frac{1}{2}$ the area of the rectangle. How many squares make up the other half? How do you know?* [The two halves have equal area.] Ask: *How many squares make up the whole rectangle?* Have students describe the shape they drew to partners and compare.

Levels 3–5

Listening/Speaking Have students read ***Apply It*** problem 4. Ask: *What shape is this problem about?* Have students work with partners. Ask partners to identify the given information in the problem and draw what they know about the rectangle.

Provide the following sentence frame:

- *Four squares are _____ the area of the rectangle.*

Ask: *How many squares make up the whole rectangle?* Have students draw the rectangle that is described in the problem. Have partners compare their rectangles.

LESSON 33

SESSION 3 Refine

Purpose In this session students solve word problems involving dividing shapes into equal parts and then discuss and confirm their answers with a partner.

Before students begin work, use their responses to the *Check for Understanding* to determine those who will benefit from additional support.

As students complete the Example and problems 1–3, observe and monitor their reasoning to identify groupings for differentiated instruction.

Start

Check for Understanding

Why Confirm understanding of partitioning shapes into parts with equal areas.

How Have students divide a rectangle into 6 equal parts and shade $\frac{1}{2}$ of the rectangle.

Draw a rectangle and divide it into 6 equal parts.

Shade $\frac{1}{2}$ of the rectangle.

Possible Solution

Check students' work. 3 of the 6 parts should be shaded.

Refine Partitioning Shapes into Parts with Equal Areas

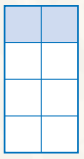
Complete the Example below. Then solve problems 1–8.

EXAMPLE

A rectangular game board is divided into same-sized squares. There are 4 rows. Each row has 2 squares. What fraction of the total area of the game board does each row cover?

Look at how you could show your work using a model.

1 row out of 4 rows is shaded.



Solution Each row covers $\frac{1}{4}$ of the area of the game board.

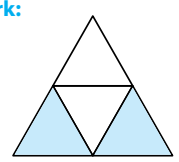
The student used a model to make a model of the game board.

PAIR/SHARE
How could you solve the problem without using a model?

APPLY IT

1 The triangle is divided into equal parts. How does the area of one part compare to the area of the whole triangle? Shade $\frac{1}{2}$ of the triangle.

Possible student work:



Solution The area of one part is $\frac{1}{4}$ the area of the whole triangle.

How many smaller triangles are there?

PAIR/SHARE
What is a different way to shade $\frac{1}{2}$ of the triangle?

Error Alert

If the error is . . .	*Students may . . .*	*To support understanding . . .*
1 part is shaded	have shaded only 1 part because $\frac{1}{2}$ is a unit fraction.	Review that $\frac{1}{2}$ means "1 out of 2 parts." Elicit that the denominator of a fraction tells how many parts are in the whole. Have students identify 2 equal parts of the rectangle and identify the number of smaller parts in it.
2 parts are shaded	have used the denominator of $\frac{1}{2}$ to decide how many parts to shade.	Elicit or review the meaning of *numerator* and *denominator*. Have students identify 2 equal parts of the rectangle and identify the number of smaller parts in it.

EXAMPLE

$\frac{1}{4}$; the model shown is one way to solve the problem. Students could also solve the problem by shading a different row.

Look for Draw a model with 4 rows of 2, with 1 row shaded.

APPLY IT

1. The area of one part is $\frac{1}{4}$ the area of the whole triangle; See Student Worktext page to check that 2 parts are shaded.
   DOK 1

   Look for Any 2 parts can be shaded.

2. 2; See Student Worktext page for possible shading.
   DOK 1

   Look for It takes 2 parts to cover $\frac{1}{3}$ of the circle.

3. **D**; Students could solve the problem by identifying the fraction with a numerator that represents 1 square and a denominator that represents the number of equal squares the rectangle is divided into.

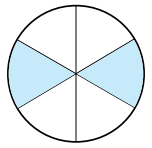
   Explain why the other two answer choices are not correct:

   B is not correct because $\frac{1}{3}$ is 2 out of 6 parts.

   C is not correct because the rectangle is divided into 6 parts, not 4 parts.

   DOK 3

2. Shade $\frac{1}{3}$ of the circle below. How many same-sized parts cover $\frac{1}{3}$ of the circle? Show your work.



Answers will vary.
Possible shading shown.

Remember that $\frac{1}{3}$ means 1 out of 3 equal parts.

Solution **2 parts**

PAIR/SHARE
What fraction of the whole circle is each part?

3. A rectangle is equally divided into 2 rows. Each row is divided into 3 same-sized squares. What fraction of the total area of the rectangle is each square?

   Ⓐ $\frac{1}{2}$

   Ⓑ $\frac{1}{3}$

   Ⓒ $\frac{1}{4}$

   Ⓓ $\frac{1}{6}$

How many squares are in the whole rectangle?

Ben chose Ⓐ as the correct answer. How did he get that answer?

Possible answer: Ben found the fraction of the total area of the rectangle that 1 row covers.

PAIR/SHARE
What do you think Ben was thinking when he chose his answer?

4 **C**; Half the rectangle is made up of 4 squares, so the whole rectangle is made up of 4×2 squares, or 8 squares.
DOK 2

Error Alert Students may choose A if they misread the problem and think that the 4 squares make up the whole rectangle instead of only half.

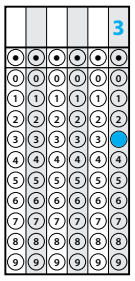
5 The whole rectangle is made up of 6 squares, so, to shade $\frac{1}{2}$ the area of the rectangle, you would shade 3 squares; $6 \div 2 = 3$.
DOK 2

6 2; See Student Worktext page for possible shadings; $\frac{1}{3}$ of a rectangle is 2 squares, so students can shade any 2 of the 6 squares in each rectangle.
DOK 1

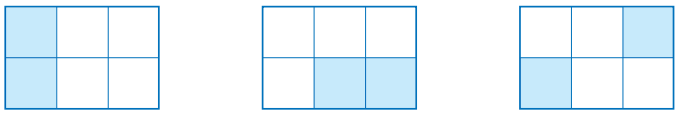
4 A rectangle is divided into same-sized squares. Four of the squares are shaded. The area of the shaded parts is $\frac{1}{2}$ the area of the whole rectangle. How many squares make up the whole rectangle?

Ⓐ 2 squares
Ⓑ 4 squares
Ⓒ 8 squares
Ⓓ 16 squares

5 A rectangle is divided into 6 same-sized squares. How many squares cover $\frac{1}{2}$ of the area of the rectangle?



6 The rectangles below are all the same size. Dani wants to shade $\frac{1}{3}$ of the area of each rectangle. Use the rectangles below to show three different ways to shade $\frac{1}{3}$. **Answers will vary. Possible student work:**



How many squares do you need to shade to cover $\frac{1}{3}$ of the area of one of the rectangles?

2 squares

Differentiated Instruction

RETEACH

Hands-On Activity
Use unit tiles to form rectangles and identify fractional areas.

Students struggling with area models that show fractions

Will benefit from additional work with rectangles divided into equal parts.

Materials For each group: 24 unit tiles, 5 cards labeled with the unit fractions $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{6}$, and $\frac{1}{8}$

- Students in each group take turns selecting a fraction card. They use all the tiles to build a rectangle and then break apart a section that represents the unit fraction on the card.
- Students in the group discuss the work, decide if it is correct, and work together to correct any inaccurate representations.
- Repeat the activity with a different number of tiles.

EXTEND

Challenge Activity
Find the area of equal-sized pieces of pizza.

Students who have achieved proficiency

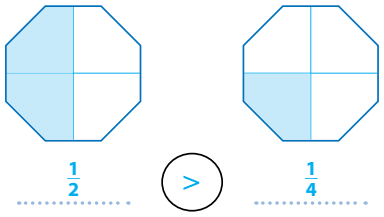
Will benefit from deepening understanding of dividing shapes into equal parts.

Materials For each student: Activity Sheet
1-Centimeter Grid Paper

- Present the following situation: There is a rectangular pizza cut into 8 equal-sized pieces. The pizza is 12 inches wide and 16 inches long.
- Have students draw a picture of the pizza on the grid paper. (Let each grid square represent 1 square inch.) Challenge them to divide it into 8 equal parts and find the area of each part. [24 square inches]

7 $\frac{1}{2}; \frac{1}{4}; \frac{1}{2} > \frac{1}{4}$; See Student Worktext page for possible divisions and shadings. Each must be divided into 4 equal parts and one must have 1 part shaded $\left(\frac{1}{4}\right)$ and the other must have 2 parts shaded $\left(\frac{1}{2}\right)$. The shaded parts of the shapes should be correctly compared using $<$ or $>$.
DOK 2

7 Divide each octagon into 4 equal parts. Then shade one or more parts of each to show two different unit fractions. Write the fraction under each octagon. Then compare the fractions using $<$, $>$, or $=$. **Possible student work.**



8 MATH JOURNAL

Suppose you divide a hexagon into 6 equal parts. Explain how you could shade the parts to show three different unit fractions.

Possible answer: I could shade one part to show $\frac{1}{6}$. I could shade 2 parts to show $\frac{2}{6}$ or $\frac{1}{3}$. I could shade 3 parts to show $\frac{3}{6}$ or $\frac{1}{2}$.

SELF CHECK Go back to the Unit 6 Opener and see what you can check off.

REINFORCE

Problems 4–8

Partition shapes into parts with equal areas.

All students will benefit from additional work with partitioning shapes into parts with equal areas by solving problems in a variety of formats.

- Have students work on their own or with a partner to solve the problems.
- Encourage students to show their work.

PERSONALIZE

i-Ready

Provide students with opportunities to work on their personalized instruction path with *i-Ready* Online Instruction to:

- fill prerequisite gaps
- build up grade-level skills

Close: Exit Ticket

8 MATH JOURNAL

Student responses should indicate understanding that each part represents $\frac{1}{6}$ of the hexagon. They can shade 1 part to show $\frac{1}{6}$, 2 parts to show $\frac{1}{3}$, and 3 parts to show $\frac{1}{2}$.

Error Alert **If** students only identify the fraction $\frac{1}{6}$, then review the other unit fractions they have worked with $\left(\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \text{and } \frac{1}{8}\right)$ and check to see if any of them can be shown by shading parts of the model.

SELF CHECK Have students consider whether they feel they are ready to check off any new skills on the Unit 6 Opener.