Lesson Overview

LESSON 20

Understand What a Fraction Is

Lesson Objectives

Content Objectives

- Understand that a fraction is a whole divided into some number of equal parts.
- Understand and recognize the parts of a fraction.
- Understand that unit fractions are the building blocks of fractions in the same way that 1 is the building block of whole numbers.

Language Objectives

- Write the fraction shown by an area model.
- Shade an area model to represent a given unit fraction.
- Shade area models to represent a variety of fractions.
- Orally define and use the key mathematical terms *denominator*, *fraction*, *numerator*, and *unit fraction* when describing reasoning to a partner.

Prerequisite Skills

- Partition circles and rectangles into two, three, or four equal shares.
- Describe parts of circles and rectangles using the words *halves*, *thirds*, *fourths*, *half of*, *a third of*, etc., including describing the wholes as two halves, etc.

Standards for Mathematical Practice (SMP)

SMPs 1, 2, 3, 4, 5, and 6 are integrated in every lesson through the *Try-Discuss-Connect* routine.*

In addition, this lesson particularly emphasizes the following SMPs:

2 Reason abstractly and quantitatively.

6 Attend to precision.

*See page 455i to see how every lesson includes these SMPs.

Lesson Vocabulary

- **denominator** the number below the line in a fraction that tells the number of equal parts in the whole.
- **fraction** a number that names equal parts of a whole. A fraction names a point on the number line.
- **numerator** the number above the line in a fraction that tells the number of equal parts that are being described.
- **unit fraction** a fraction with a numerator of 1. Other fractions are built from unit fractions.

Learning Progression

In Grade 2 students used fraction language to describe dividing shapes into equal parts. They divided squares, circles, and rectangles into equal parts and named the parts as halves, thirds, and fourths. Through their work with models, students began to understand the concept of dividing a whole into equal parts.

In Grade 3 students develop a more formal understanding of fractions.

In this lesson students focus on the meaning of fractions and name fractions by the number of equal parts in the whole, such as *sixths* or *eighths*. Students learn about the structure of fractions, identifying the denominator as the number of equal parts in the whole and the numerator as the number of parts being considered. Students identify unit fractions, such as $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{6}$, and $\frac{1}{8}$, by using models with one part shaded out of a number of equal parts. Students apply their understanding of unit fractions to understand greater fractions that are built from unit fractions, such as $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{6}$, and $\frac{5}{8}$.

This lesson builds a foundation **for subsequent Grade 3 lessons** that develop an understanding of fractions as numbers on a number line and introduce the concepts of equivalent fractions and comparing fractions by reasoning about their size.

In Grade 4 students will use their understanding of fractions and fraction equivalency to add and subtract fractions.

Lesson Pacing Guide

Whole Class Instruction

SESSION 1
Explore
45–60 min

What a Fraction Is
- Start *5 min*
- Model It *10 min*
- Discuss It *5 min*
- Model It *10 min*
- Discuss It *10 min*
- Close: Exit Ticket *5 min*

Additional Practice
Lesson pages 461–462

SESSION 2
Develop
45–60 min

Describing Parts of a Whole with Fractions
- Start *5 min*
- Model It: Write Fractions from Models *5 min*
- Discuss It *5 min*
- Model It: Draw Models of Fractions *5 min*
- Discuss It *5 min*
- Connect It *15 min*
- Close: Exit Ticket *5 min*

Additional Practice
Lesson pages 465–466

Fluency
Describing Parts of a Whole with Fractions

SESSION 3
Refine
45–60 min

Ideas About What a Fraction Is
- Start *5 min*
- Apply It *35 min*
- Close: Exit Ticket *5 min*

Lesson Quiz or **Digital Comprehension Check**

Lesson Materials

Lesson *(Required)*	*Per student:* 4 unit tiles, 10 color tiles (5 yellow, 5 red) *Activity Sheet:* Circle and Square
Activities	*Per student:* 3 inch tiles, 1 paper square, 2 crayons *Activity Sheet:* 1-Inch Grid Paper
Digital Math Tool	Fraction Models

*We continually update the Interactive Tutorials. Check the Teacher Toolbox for the most up-to-date offerings for this lesson.

Teacher Toolbox

Small Group Differentiation

PREPARE

Ready Prerequisite Lesson

Grade 2
- Lesson 29 Understand Partitioning Shapes into Halves, Thirds, and Fourths

RETEACH

Tools for Instruction

Grade 2
- Lesson 30 Make Equal Shares

Grade 3
- Lesson 20 Modeling Fractions

REINFORCE

Math Center Activities

Grade 3
- Lesson 20 Write the Fraction
- Lesson 20 Show Fractions

EXTEND

Enrichment Activity

Grade 3
- Lesson 20 Colorful Quilts

i-Ready

Independent Learning

PERSONALIZE

i-Ready Lesson*

Grade 3
- Understand What a Fraction Is

Learning Game
- *Bounce*

LESSON 20

Connect to Family, Community, and Language Development

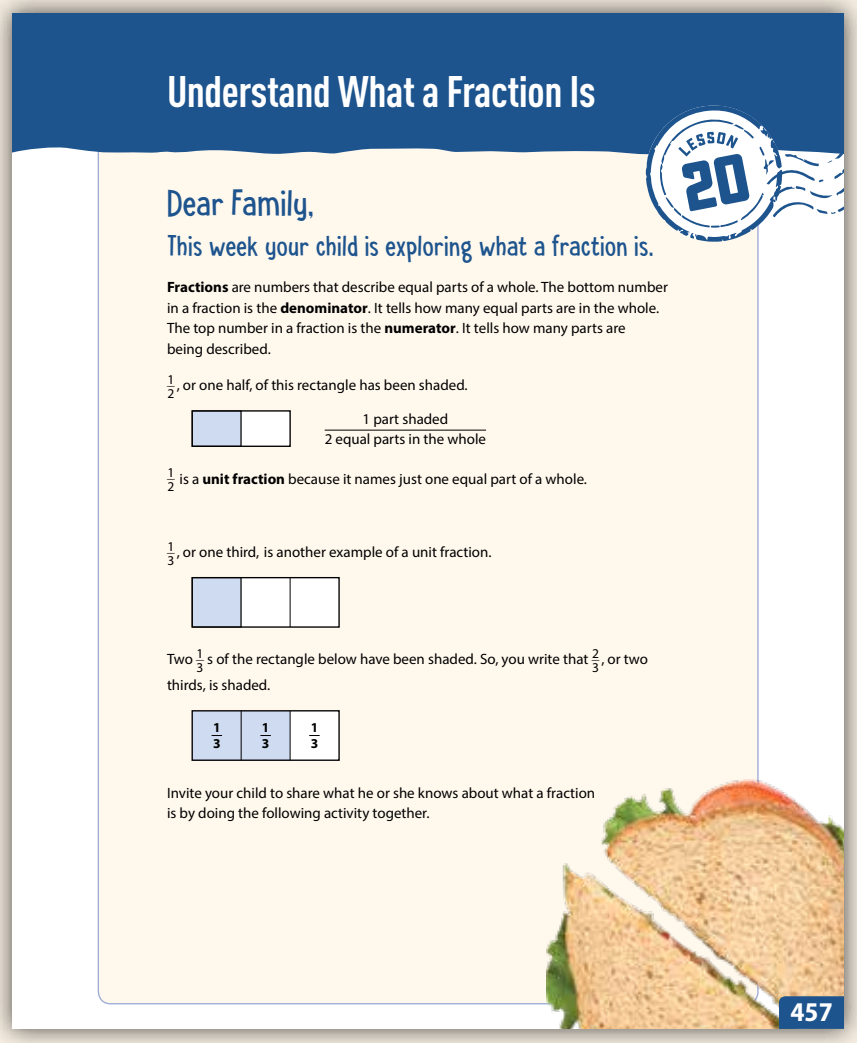
The following activities and instructional supports provide opportunities to foster school, family, and community involvement and partnerships.

Connect to Family

Use the **Family Letter**—which provides background information, math vocabulary, and an activity—to keep families apprised of what their child is learning and to encourage family involvement.

Available in Spanish

Teacher Toolbox



Understand What a Fraction Is

LESSON 20

Dear Family,

This week your child is exploring what a fraction is.

Fractions are numbers that describe equal parts of a whole. The bottom number in a fraction is the **denominator**. It tells how many equal parts are in the whole. The top number in a fraction is the **numerator**. It tells how many parts are being described.

$\frac{1}{2}$, or one half, of this rectangle has been shaded.

$\frac{1 \text{ part shaded}}{2 \text{ equal parts in the whole}}$

$\frac{1}{2}$ is a **unit fraction** because it names just one equal part of a whole.

$\frac{1}{3}$, or one third, is another example of a unit fraction.

Two $\frac{1}{3}$s of the rectangle below have been shaded. So, you write that $\frac{2}{3}$, or two thirds, is shaded.

Invite your child to share what he or she knows about what a fraction is by doing the following activity together.

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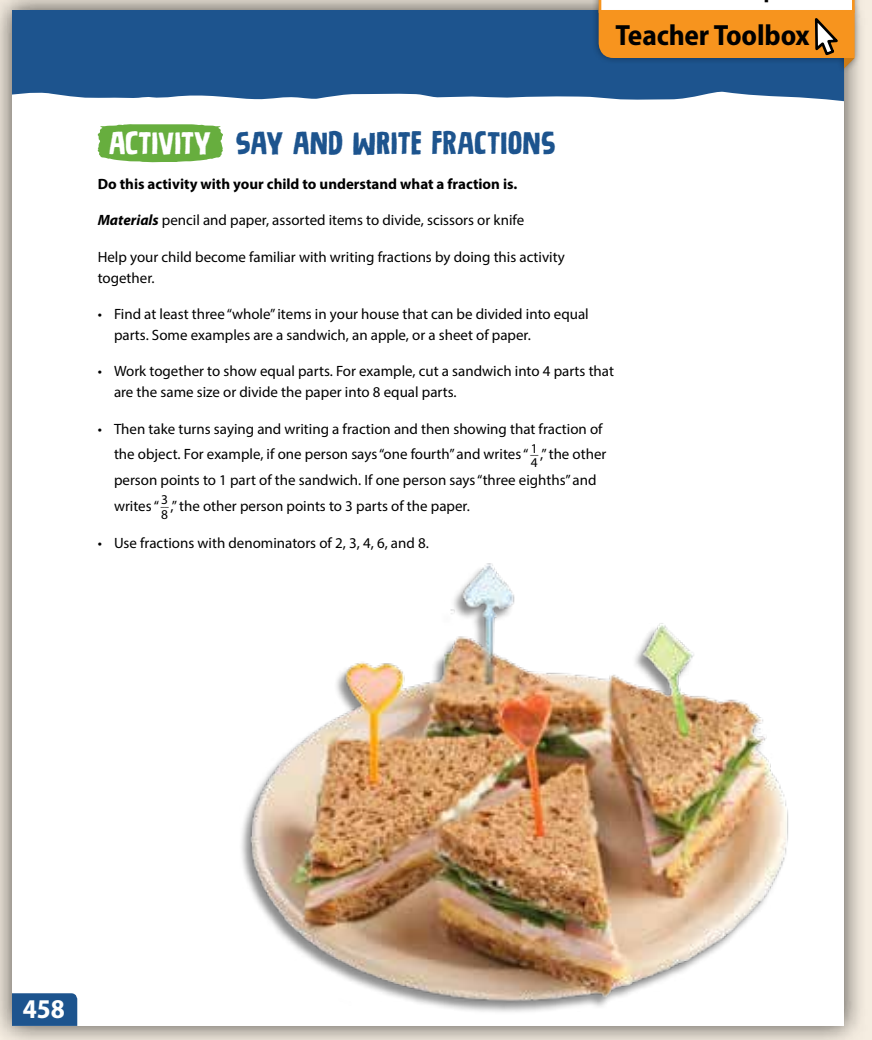

ACTIVITY SAY AND WRITE FRACTIONS

Do this activity with your child to understand what a fraction is.

Materials pencil and paper, assorted items to divide, scissors or knife

Help your child become familiar with writing fractions by doing this activity together.

- Find at least three "whole" items in your house that can be divided into equal parts. Some examples are a sandwich, an apple, or a sheet of paper.
- Work together to show equal parts. For example, cut a sandwich into 4 parts that are the same size or divide the paper into 8 equal parts.
- Then take turns saying and writing a fraction and then showing that fraction of the object. For example, if one person says "one fourth" and writes "$\frac{1}{4}$," the other person points to 1 part of the sandwich. If one person says "three eighths" and writes "$\frac{3}{8}$," the other person points to 3 parts of the paper.
- Use fractions with denominators of 2, 3, 4, 6, and 8.

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Goal

The goal of the Family Letter is to explain the parts of a fraction. Unit fractions are fractions that represent one equal part of a whole. Parts of a whole can be represented by combining unit fractions.

Activity

Look at the *Say and Write Fractions* activity and adjust it if necessary to connect with your students.

Math Talk at Home

Encourage students to talk with their family members about fractions. Have them brainstorm a list of "whole" items commonly found at home that can be used to explore fractions. Examples include a bar of soap, a paper plate, and a sandwich.

Conversation Starters Below are additional conversation starters students can write in their Family Letter or math journal to engage family members:

- *What were you doing the last time you used fractions?*
- *What kinds of foods make you think of fractions? Why?*
- *How do you remember the difference between the numerator and the denominator of a fraction?*

Connect to Community and Cultural Responsiveness

Use these activities to connect with and leverage the diverse backgrounds and experiences of all students.

Sessions 1–3 **Use throughout the lesson.**

- Prior to Session 1, *Explore What a Fraction Is*, have students scan the lesson for items that are divided into equal parts. Display the items in a list on chart paper. Ask: *How are these items the same? How are they different?* Generalize that food items are the most common examples for displaying fractions. Ask students to look for real-world situations in which fractions are used or can be used. Ask students to bring in a sample of an item that can represent a fraction. Display a half-cup measuring cup and a stick-of-butter wrapper with the fractions visible. Place in a class *Fractions in Action* collection. Explain why each of the items is an example of *Fractions in Action*.
- As students add to the collection, facilitate brief "one-on-one interviews" with students for the class to observe. After two or three interviews, encourage students to act as the facilitators and ask the questions you have modeled. For example:
  - *What fractions are part of your example?*
  - *Why is this an example of* Fractions in Action*?*
  - *How are fractions being used?*
  - *Why are the fractions important?*

  Spread out the interviews and explanations of students' items over the course of Sessions 1–3.

Connect to Language Development

For ELLs, use the Differentiated Instruction chart to plan and prepare for specific activities in every session.

ELL English Language Learners: Differentiated Instruction

Prepare for Session 1
Use with *Model It*.

Levels 1–3	Levels 2–4	Levels 3–5
Listening/Speaking Read ***Model It*** problem 1a aloud. Have students circle *one third*. Say *one third* and have students repeat. Point to the first figure and say: *Count and number the equal parts of the shape. Write the numbers under the figure. There are three equal parts in the whole.* Display 3 as the denominator of a fraction. Say: *Count and number the shaded equal parts. Write the number on top of the figure. There is one equal part shaded.* Write 1 as the numerator of the fraction. Read the fraction *one third* again. Have students write the fraction and decide whether to circle the shape. Repeat the process with the remaining shapes.	**Listening/Speaking** Read ***Model It*** problem 1a aloud. Pair students. Have students circle *one third*. Say *one third* and have students repeat. Point to the first figure and say: *Count and number the equal parts of the shape. Write the numbers under the figure. How many equal parts are shaded? Write the number on top of the figure.* Display and read $\frac{1}{3}$. Have students write and read the fraction and decide whether or not to circle the shape. Say: *With your partner, decide the fraction for each shape and whether or not to circle it.* Select pairs to share their fractions and decisions for the remaining shapes.	**Listening/Speaking** Have pairs read ***Model It*** problem 1a aloud. Point to the first figure and say: *Count and number the equal parts of the shape. Write the numbers under the figure. How many equal parts are shaded? Write the number on top of the figure.* Display and read $\frac{1}{3}$. Have students write and read the fraction and decide whether or not to circle the shape. Say: *With your partner, decide the fraction for each shape and whether or not to circle it.* For shapes that are not circled, have students explain why.

LESSON 20

SESSION 1 Explore

Purpose In this session students explore what a fraction is. They learn how to name a fraction. Then students explore how unit fractions can help you to understand fractions with numerators greater than 1.

Start

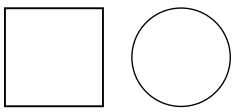
Connect to Prior Knowledge

Materials For each student: Activity Sheet *Circle and Square*

Why Support students' facility with dividing circles and rectangles into 2, 3, or 4 equal named parts.

How Have students divide a circle and a square into fourths.

Divide the circle and the square on your paper into fourths.



Solutions

Look for the circle and the square to be divided into 4 equal parts.

MODEL IT

Read the question at the top of the Student Worktext page. Remind students that they have seen shapes divided into 2, 3, or 4 equal parts. Elicit what each of these parts is called. [half, third, fourth]

1 – 3 Tell students that they are going to use what they know about equal parts to write and name fractions. Then clarify the task and have students complete the problems.

Common Misconception **If** students write $\frac{3}{3}$ for the fraction in problem 2, **then** have students join 1 red and 2 blue connecting cubes to make one piece. Ask: *How many parts in all? How many red parts?*

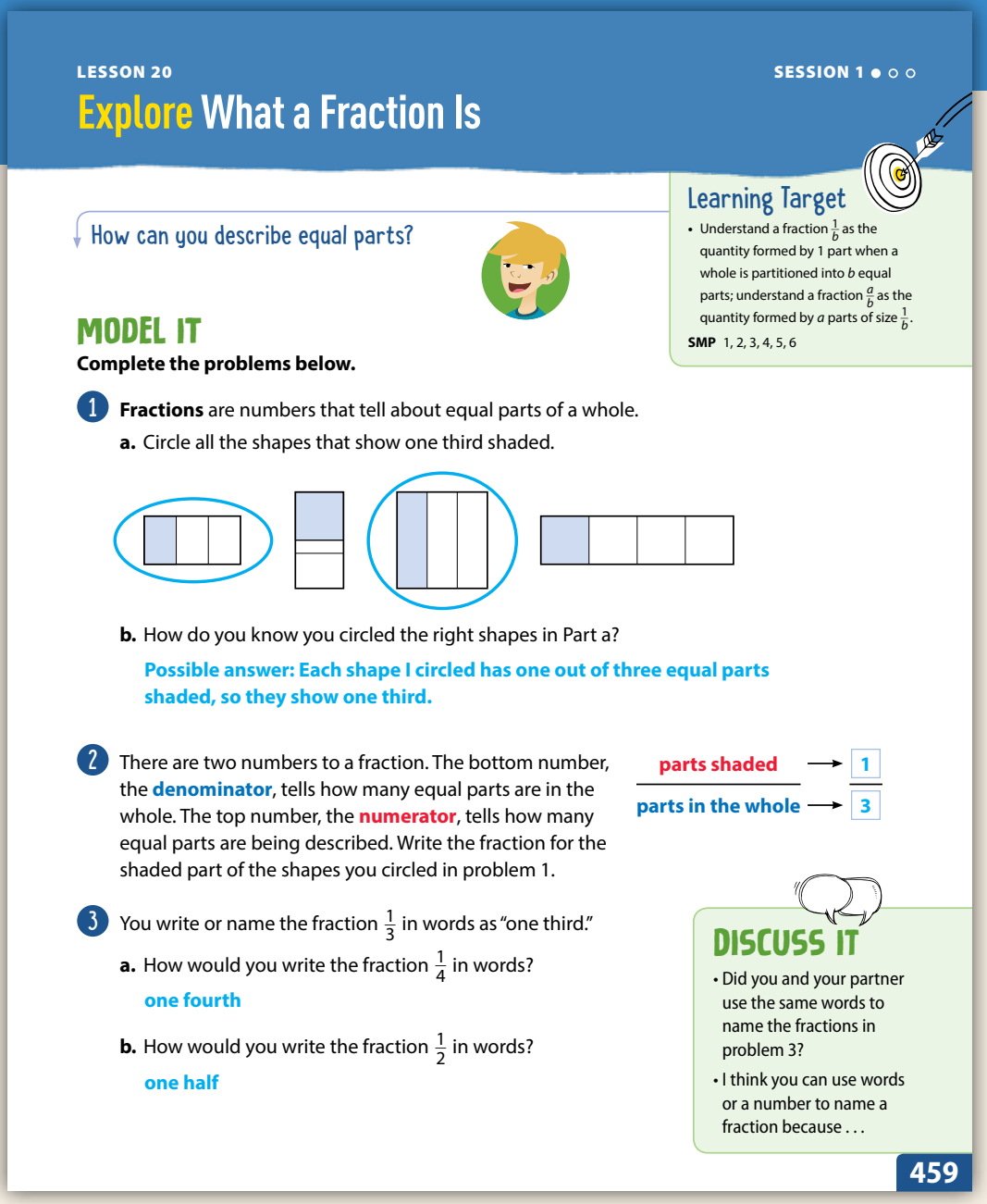
DISCUSS IT

Support Partner Discussion

Encourage students to discuss what each number in a fraction represents.

Look for understanding of:

- the denominator, or bottom number, is the total number of parts
- the numerator, or top number, is the number of parts being described



LESSON 20 SESSION 1 ● ○ ○

Explore What a Fraction Is

How can you describe equal parts?

Learning Target

- Understand a fraction $\frac{1}{b}$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction $\frac{a}{b}$ as the quantity formed by a parts of size $\frac{1}{b}$.

SMP 1, 2, 3, 4, 5, 6

MODEL IT

Complete the problems below.

1 **Fractions** are numbers that tell about equal parts of a whole.

a. Circle all the shapes that show one third shaded.

b. How do you know you circled the right shapes in Part a?

Possible answer: Each shape I circled has one out of three equal parts shaded, so they show one third.

2 There are two numbers to a fraction. The bottom number, the **denominator**, tells how many equal parts are in the whole. The top number, the **numerator**, tells how many equal parts are being described. Write the fraction for the shaded part of the shapes you circled in problem 1.

parts shaded → 1

parts in the whole → 3

3 You write or name the fraction $\frac{1}{3}$ in words as "one third."

a. How would you write the fraction $\frac{1}{4}$ in words?

one fourth

b. How would you write the fraction $\frac{1}{2}$ in words?

one half

DISCUSS IT

- Did you and your partner use the same words to name the fractions in problem 3?
- I think you can use words or a number to name a fraction because . . .

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Support Whole Class Discussion

Prompt students to practice using fraction terms, such as *equal parts*, *numerator*, and *denominator* in their discussions.

Ask *Why do you need two numbers (numerator and denominator) to show a fraction?*

Listen for The two numbers give different information. The denominator tells how many equal parts are in the whole. The numerator tells how many of those parts you are talking about.

Ask *How did you know how to write $\frac{1}{4}$ in words? Is there another word you could have used?*

Listen for There is a 1 on the top, so I know to write the word *one* first. The equal parts word for 3 is *third,* which was used for $\frac{1}{3}$, so I used the equal parts word for 4, *fourth*. I could have also used the word *quarter* because 4 quarters equal 1 whole. So, the fraction $\frac{1}{4}$ can be written as *one fourth* or *one quarter*.

Ask *How did you know how to write $\frac{1}{2}$ in words?*

Listen for The equal parts word for 2 is *half*, so the fraction $\frac{1}{2}$ can be written as *one half*.

MODEL IT

4 – 5 Tell students that they will now think about the special name for a fraction with a numerator of 1 and then write and name a fraction with a numerator greater than 1. Clarify the task and have students complete the problems.

Common Misconception **If** students write $\frac{3}{1}$ for problem 5c, **then** have them draw a rectangle and divide it into 4 equal parts. Ask students to name the number of equal parts in the whole before asking them to shade 3 parts. Have students use the words *numerator* and *denominator* as they describe the whole and shaded parts.

DISCUSS IT

Support Partner Discussion

Again, encourage students to discuss what each number in a fraction represents as they refer to their answers to problem 5.

Look for understanding of:

- the denominator shows the number of parts in the whole, 4
- the numerator shows the number of shaded parts, 3
- each part represents one fourth, so three parts represent three fourths

Hands-On Activity

Focus on equal parts.

If . . . *students are struggling to connect the word and number forms of a fraction,*

Then . . . *use this activity to have them practice making and naming fractional parts.*

Materials For each student: 1 paper square

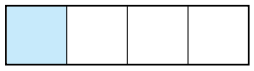
- Ask students to fold their paper into 2 equal parts. Have them unfold and point to one part. Ask: *What is the fraction for this part in words?* [one half] *As a number?* $\left[\frac{1}{2}\right]$
- Have students fold their paper into 4 equal parts. Have them unfold and point to one of the four parts. Ask: *What is the fraction for this part in words?* [one fourth] *As a number?* $\left[\frac{1}{4}\right]$
- You may wish to challenge students by having them fold the paper into 8 equal parts.

MODEL IT

Complete the problems below.

4 A **unit fraction** has a 1 in the numerator. It names 1 part of a whole.

Shade $\frac{1}{4}$ of the model below. **Possible student work:**



5 Look at the same model again.

a. Shade three fourths of the model. **Possible student work:**

b. How could you count each fourth you shaded to also name the fraction? Fill in the missing fourths.

1 fourth, 2 fourths, 3 fourths

c. Write the fraction for the parts you shaded in Part a.

$$\frac{\text{parts shaded} \rightarrow 3}{\text{parts in the whole} \rightarrow 4}$$

d. How would you name the fraction from Part c in words?

three fourths

DISCUSS IT

- Count by $\frac{1}{4}$s up to one whole. How do you know when to stop?
- I think counting by $\frac{1}{4}$s is like counting whole numbers because . . .
- I think counting by $\frac{1}{4}$s is different from counting whole numbers because . . .

6 REFLECT

Explain why the denominator does not change when you are counting by the unit fraction $\frac{1}{4}$ to reach $\frac{3}{4}$.

The parts are all fourths, so the denominator doesn't change.

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Support Whole Class Discussion

Prompt students to think about the connection between fractions with numerators greater than 1 and their related unit fraction.

Ask *How is counting by $\frac{1}{4}$s the same as counting by ones? How is it different?*

Listen for It is the same because you are counting by ones. It is different because when you count by unit fractions, you count by one part, not one whole.

Ask *What is the same about all unit fractions? What is different?*

Listen for Numerators are always 1; denominators are any number (other than 0).

Close: Exit Ticket

6 REFLECT

Look for understanding that the number of equal parts in the whole does not change, so the denominator does not change—it remains "fourths."

Common Misconception **If** students also increase denominators as they count, **then** remind them that it is fourths we are counting. Count as if you are counting apples: 1 apple, 2 apples, etc.; 1 fourth, 2 fourths, etc.

Solutions

Support Vocabulary Development

1 Review where each concept in the table appears on the previous pages. Ask students to explain what is to be displayed in each row and column. List students' suggestions for a word bank that they can access when completing *In My Own Words*.

2 Have students circle *two thirds* in the directions. Say: *Count and number the equal parts of the rectangle by writing the numbers below the rectangle.* Refer to the three as the equal parts of the whole. Say: *Count and number the shaded parts of the rectangle by writing the numbers above the rectangle.* Refer to the two as the number of equal parts shaded. Have students write the fraction in standard form. Ask: *How are $\frac{2}{3}$ and 2 similar?* [both are numbers] *How are $\frac{2}{3}$ and 2 different?* [$\frac{2}{3}$ is a fraction, and it represents a value less than 1. 2 is a whole number, and it has a value greater than 1.]

Supplemental Math Vocabulary

- *denominator*
- *fraction*
- *numerator*
- *unit fraction*

Name: _____ LESSON 20 SESSION 1

Prepare for Exploring What a Fraction Is

1 Think about what you know about fractions. Fill in each box. Use words, numbers, and pictures. Show as many ideas as you can. **Possible answers:**

Word	In My Own Words	Example
fraction	**a number that describes equal parts of a whole**	$\frac{2}{3}$
numerator	**The top number in a fraction; it tells how many equal parts are being described.**	$\frac{2}{3}$ ←
denominator	**The bottom number in a fraction; it tells how many equal parts are in the whole.**	$\frac{2}{3}$ ←

2 Shade two thirds of the model. Write the fraction for the parts you shaded.

parts shaded → 2
parts in the whole → 3

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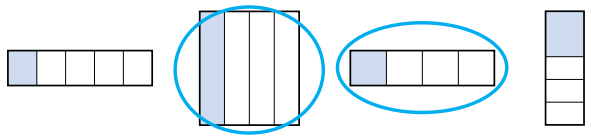
3 Students are asked to circle all the shapes with one fourth shaded. Students must ensure that the shapes they circle have four equal parts where one part is shaded.
Medium

4 Students complete the fraction and write $\frac{1}{4}$. They can find the solution by counting how many parts are shaded and write the number in the top box. They then count how many equal parts there are in all and write that number in the bottom box.
Medium

5 Students write "two fourths."
Medium

Solve.

3 Circle all the shapes that show one fourth shaded. How do you know you circled the right shapes?



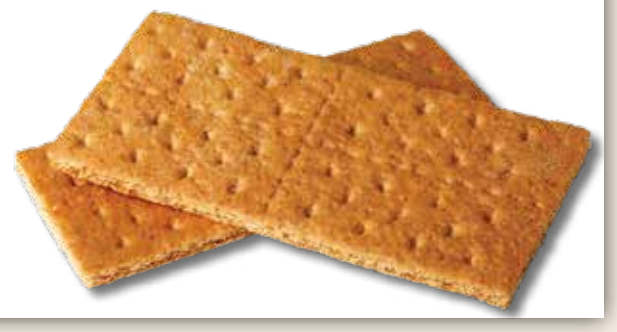
Solution Possible answer: Each shape I circled has four equal parts with one part shaded. So, they each show one fourth.

4 Write the fraction for the shaded part of the shapes you circled in problem 3.

$$\frac{\text{parts shaded} \rightarrow \boxed{1}}{\text{parts in the whole} \rightarrow \boxed{4}}$$

5 How would you write the fraction $\frac{2}{4}$ in words?

Solution two fourths



ELL English Language Learners: Differentiated Instruction

Prepare for Session 2
Use with *Model It*.

Levels 1–3

Listening/Speaking Read ***Model It*** problem 4 aloud. Display and read:

$$\frac{\text{numerator}}{\text{denominator}} = \frac{\text{parts shaded}}{\text{parts in the whole}} = \frac{2}{3}$$

Read it again and have students choral read. Ask students to circle *Draw* and *shade* in the problem. Explain the steps. Pair students. Ask: *How many parts does the square need to have?* [3] *How do you know?* [because of the denominator] *How many parts do you need to shade?* [2] *How do you know?* [because of the numerator] *Draw the square and compare your drawing with your partner's. Decide if your drawings are the same or different.*

Levels 2–4

Listening/Speaking Read ***Model It*** problem 4 aloud. Display and read:

$$\frac{\text{numerator}}{\text{parts in the whole}} = \frac{\text{parts shaded}}{\text{parts in the whole}} = \frac{2}{3}$$

Pose questions and have students provide the missing information. Have students circle *Draw* and *shade* in the problem. Pair students. Ask: *How many parts does the square need to have? How do you know?* [3, because of the denominator] *How many parts do you need to shade? How do you know?* [2, because of the numerator] *Complete and compare the drawing with your partner.*

Levels 3–5

Listening/Speaking Have pairs read ***Model It*** problem 4. Display:

$$\frac{\quad}{\text{denominator}} = \frac{\text{parts shaded}}{\quad} = \frac{2}{3}$$

Pose questions and have students provide the missing information. Have students circle *Draw* and *shade* in the problem. Ask students to explain the steps they will follow to complete the problem. Pair students. Say: *Discuss how the drawing will look. Complete the drawing. Then compare your drawings with another pair's drawings. Decide if your drawings are the same or different.* Call on pairs to share their findings.

LESSON 20

SESSION 2 Develop

Purpose In this session students write fractions for partially shaded models by comparing the numbers of equal parts and shaded parts. They draw a whole and shade a fractional part given the unit fraction part.

Start

Connect to Prior Knowledge

Materials For each student: 4 unit tiles

Why Reinforce students' understanding of fraction concepts.

How Have students use tiles to build a rectangle that shows halves and tell how many halves are in the whole.

Use tiles to build a rectangle that shows halves.

How many halves are in the whole rectangle?

Solution

2 halves

Look for a rectangle built of 2 tiles.

Develop Language

Why Clarify the multiple-meaning word *shade*.

How Say: *When it is sunny outside, you can shade your eyes with your hand.* Make the gesture of shading your eyes with your hand, and have students mimic you. Then draw a rectangle and shade one half. Say: *Shade also means to darken an area in a drawing or a shape with a pencil.* Have students draw a circle. Show them how to draw a line down the center of the circle. Then ask them to use a pencil to shade half of it.

MODEL IT: WRITE FRACTIONS FROM MODELS

1 – 3 Present the problems and have students complete. As students work, have them identify that they are being asked to write a fraction reflecting the shading in each figure and that unit fractions can help when there is more than 1 shaded part.

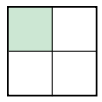
Common Misconception **If** students write part to part fractions, **then** review again the meaning and role of the denominator—it always tells all the parts in the whole, shaded and unshaded.

Develop Describing Parts of a Whole with Fractions

MODEL IT: WRITE FRACTIONS FROM MODELS

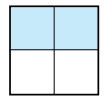
Try these problems.

1 **a.** What unit fraction is shown?



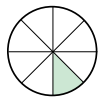
$\frac{1}{4}$

b. Shade 2 parts of the model. What fraction of the square did you shade?



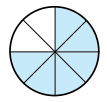
$\frac{2}{4}$

2 **a.** What unit fraction is shown?



$\frac{1}{8}$

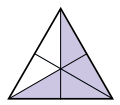
b. Shade 6 parts of the model. What fraction of the circle did you shade?



$\frac{6}{8}$

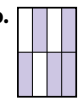
3 Write the fraction of the figure that is shaded. The parts in each model are all equal.

a.



$\frac{4}{6}$

b.



$\frac{5}{8}$

DISCUSS IT

- How did you know what fractions to write in problem 3?
- I think shading equal parts of a figure shows a fraction because . . .

DISCUSS IT

Support Partner Discussion

Encourage students to use the words *numerator* and *denominator* when describing the models and fractions.

Support as needed with questions such as:

- *How did you use unit fractions in Part a to help you write the fractions in Part b in problems 1 and 2?*
- *What do you notice about the denominators in problems 1a and 1b? In 2a and 2b?*
- *How did you and your partner approach answering problem 3?*

Support Whole Class Discussion

For each problem, have several students share their thinking process for determining the fractions.

Ask *How would a model showing $\frac{0}{3}$ look? A model showing $\frac{3}{3}$?*

Listen for For $\frac{0}{3}$, the whole is divided into 3 equal parts, but no parts are shaded. For $\frac{3}{3}$, the whole is divided into 3 equal parts, and all 3 parts are shaded.

MODEL IT: DRAW MODELS OF FRACTIONS

4 – 5 Present the problems and have students complete. As students work, have them identify that they are being asked to use one part of a whole to draw the whole and then to shade a non-unit fractional part.

DISCUSS IT

Support Partner Discussion

Encourage partners to connect the models with the fractions.

Support as needed with questions such as:

- *How many times did you use each model to make a whole?*
- *How could counting help you solve the problem?*

Support Whole Class Discussion

For each problem, have students share drawings.

Ask *Could you draw a different whole for either problem? Explain.*

Listen for In problem 4, there's only one square to draw. For problem 5, a rectangle could be drawn vertically or horizontally, or in a 2 × 2 square form.

Hands-On Activity

Find equal parts that are different shapes.

If . . . *students think equal parts must look the same,*

Then . . . *use this concrete activity.*

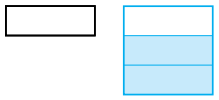
Materials For each student: 3 inch tiles, 2 crayons, Activity Sheet *1-Inch Grid Paper*

- Have each student outline two 3 × 3 squares.
- On one square, have students place the tiles in the top row and then color these 3 squares. This is one equal part. Then have students put a tile in a lower corner with a tile on either side (a right angle) and color these 3 squares a different color. Ask: *Is this also an equal part?* Explain that equal parts are the same size, but they do not have to be the same shape.
- Have students repeat this process on their own with the second 3 × 3 square to find other equal parts of different shapes. Ask: *Must the 3 tiles touch?* [No.]

MODEL IT: DRAW MODELS OF FRACTIONS

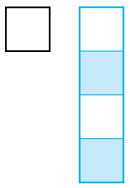
Draw the figure described.

4 The model below shows $\frac{1}{3}$ of a square. Draw to show the whole square. Then shade to show $\frac{2}{3}$.



5 The model below shows $\frac{1}{4}$ of a rectangle. Draw to show what the whole rectangle could look like. Then shade to show $\frac{2}{4}$.

Possible student work:



DISCUSS IT

- Did you and your partner draw the same figures for problems 4 and 5? Is there more than one correct answer for each problem?
- I think you need to know what the unit fraction piece of a model looks like to draw the rest of the model because . . .

CONNECT IT

Complete the problems below.

6 How can you use a shaded model to name a fraction?

Possible answer: I can count the total number of equal parts and then the number of equal parts that are shaded to name a fraction. A shape with 3 equal parts and 1 shaded is $\frac{1}{3}$.

7 Look at the rectangle.

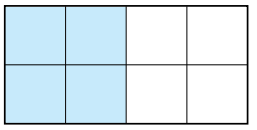
a. What unit fraction is each part?

$\frac{1}{8}$

b. Shade 4 parts of the rectangle and write the fraction you shaded.

$\frac{4}{8}$

Possible student work:



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CONNECT IT

6 Student responses show understanding of how the total number of equal parts is the denominator and the number of shaded equal parts is the numerator of the fraction.

Close: Exit Ticket

7 Look for understanding that the unit fraction is $\frac{1}{8}$ and that the 4 shaded parts represent $\frac{4}{8}$. The shaded parts do not need to be next to each other.

Common Misconception **If** students write 4 for Part b, **then** remind them that 4 is a whole number and names the number of shaded parts, but a fraction uses two numbers—a numerator and a denominator—to represent the number of shaded parts out of the total number of equal parts.

Solutions

1. equal parts: 2
shaded part: 1
fraction of the whole that is shaded: $\frac{1}{2}$
Basic

2. equal parts: 6
shaded parts: 2
fraction of the whole that is shaded: $\frac{2}{6}$
Basic

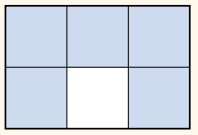
Name: _____ LESSON 20 SESSION 2

Practice Describing Parts of a Whole with Fractions

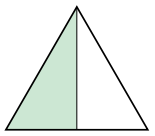
Study how the Example shows how to write a fraction for parts of a whole. Then solve problems 1–8.

EXAMPLE

- There are 6 equal parts.
- Each part is one sixth, or $\frac{1}{6}$.
- 5 parts are shaded.
- Five sixths of the whole is shaded.
- This model shows the fraction $\frac{5}{6}$.



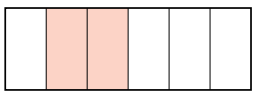
Fill in the blanks to describe each shape in problems 1 and 2.

1. 

equal parts: 2

shaded part(s): 1

fraction of the whole that is shaded: $\frac{1}{2}$

2. 

equal parts: 6

shaded part(s): 2

fraction of the whole that is shaded: $\frac{2}{6}$

Vocabulary

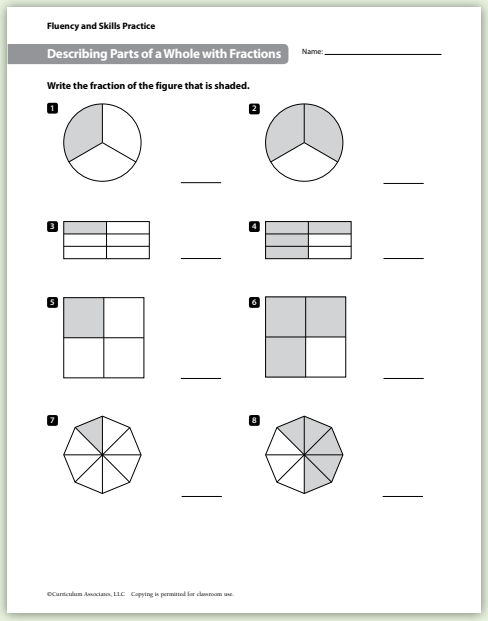
fraction a number that names equal parts of a whole.

Fluency & Skills Practice

Teacher Toolbox

Assign *Describing Parts of a Whole with Fractions*

In this activity students practice identifying the fraction that names the part of a figure that is shaded. Students may apply the same concept to describe real-world situations that involve fractions. For example, students may identify that a pizza is cut into 8 equal parts and that 2 of those parts have been eaten, so $\frac{2}{8}$ of the pizza has been eaten. Or, students may identify that a game board is made up of 6 equal parts and that 4 of those parts are red, so $\frac{4}{6}$ of the game board is red.



Fluency and Skills Practice

Describing Parts of a Whole with Fractions Name: _____

Write the fraction of the figure that is shaded.

1 _____ 2 _____

3 _____ 4 _____

5 _____ 6 _____

7 _____ 8 _____

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3 Shading on any 3 of the 4 parts
Medium

4 Shading on any 2 of the 6 parts
Medium

5 Shading on any 3 of the 8 parts; $\frac{3}{8}$
Medium

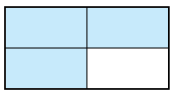
6 Shading on any 7 of the 8 parts; $\frac{7}{8}$
Medium

7 Drawing of a large rectangle made up of 4 of the given rectangles; See student page for a sample student drawing.
Challenge

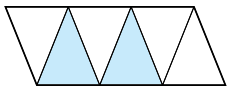
8 Drawing of a large rectangle made up of 4 of the given triangles, with 2 triangles shaded. See student page for a sample student drawing.
Challenge

Solve. **Students may shade different parts of the shapes.**

3 Shade this shape to show $\frac{3}{4}$.

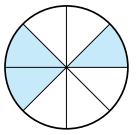


4 Shade this shape to show $\frac{2}{6}$.



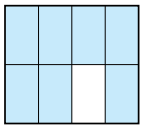
5 Shade 3 parts of this shape.

What fraction is shaded? $\frac{3}{8}$



6 Shade 7 parts of this shape.

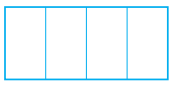
What fraction is shaded? $\frac{7}{8}$



7 ☐ is $\frac{1}{4}$ of a rectangle.

Draw the rectangle. Show the parts.

Possible answer:

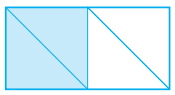


8  is $\frac{1}{4}$ of a rectangle.

Draw the rectangle. Show the parts.

Then shade $\frac{2}{4}$ of your rectangle.

Possible answer:



ELL English Language Learners: Differentiated Instruction

Prepare for Session 3
Use with *Apply It*.

Levels 1–3

Speaking/Writing Read ***Apply It*** problem 2 aloud. Arrange students in groups of four. Give each group three congruent paper squares. Model how to fold the paper to match the problem squares and have groups replicate. Display, read, and have students repeat:

$\frac{\text{numerator}}{\text{denominator}}$ $\frac{\text{parts shaded}}{\text{parts in the whole}}$

Ask a student in each group to shade a piece of each square. Say: *Decide together how to label each shaded piece.* Display, read, and have students complete the sentence frames:

_____ *is correct. Each square has* __4__ *equal pieces. Only* __1__ *piece is shaded. Each shaded piece is* __$\frac{1}{4}$__ .

Levels 2–4

Speaking/Writing Read ***Apply It*** problem 2 aloud. Arrange students in groups of four. Give each group three congruent paper squares. Model how to fold the paper to match the problem squares and have groups replicate. Display, read, and have students repeat:

$\frac{\text{numerator}}{\text{denominator}}$ $\frac{\text{parts shaded}}{\text{parts in the whole}}$

Ask a student in each group to shade a piece of each square. Say: *Decide in your group how to label the shaded part of each square.* Display, read, and have students complete the sentence frames.

_____ *is correct. Each shaded piece is* __$\frac{1}{4}$__ *because* _____. Select groups to share.

Levels 3–5

Speaking/Writing Have groups of four read ***Apply It*** problem 2 aloud. Give each group three congruent paper squares. Model how to fold the paper to match the problem squares and have groups replicate. Display, read, and have students repeat:

$\frac{\text{numerator}}{\text{denominator}}$ $\frac{\text{parts shaded}}{\text{parts in the whole}}$

Ask a student in each group to shade a piece of each square. Say: *Decide in your group how to label the shaded part of each square.*

Have group members write and review their answers. Ask a student from each group to share.

LESSON 20

SESSION 3 Refine

Purpose In this session students demonstrate their understanding of fractions and how to represent them as they talk through three problems. Then they divide and partially shade rectangles to match given fractions, and explain their process.

Start

Develop Fluency

Materials For each student: 10 color tiles (5 yellow, 5 red)

Why Support students' facility with fraction models and notation.

How Have students use red and yellow tiles to model the fraction one fifth and write the fraction.

Use red and yellow tiles to build a rectangle that shows fifths. Use only 1 yellow tile.

What fraction of your model is yellow?

Solution

$\frac{1}{5}$

Look for a rectangle made with 1 yellow and 4 red tiles

APPLY IT

Have students work independently or with a partner.

1 CREATE

Look for understanding that the unit-fraction piece is the building block for the model of the whole and that the denominator of the given unit fraction tells the number of equal parts in the whole.

Have students explain their model using fraction language. Use the following to start a discussion:

- *Could you have drawn a different rectangle? Explain.*
- *How did you know how many equal parts to draw?*
- *How is your model like other students' models? How is it different?*

Refine Ideas About What a Fraction Is

APPLY IT

Complete these problems on your own.

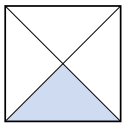
1 CREATE

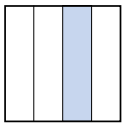
The part shown is $\frac{1}{6}$ of a rectangle. Draw a model to show what the whole rectangle might look like.

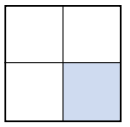
Possible drawing shown.

2 EXPLAIN

Look at these squares. Each is divided into equal parts.





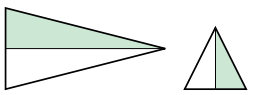


Lynn says each square has the same fraction shaded. Rose says each square has a different fraction shaded. Explain who is correct and why.

Lynn is correct; Possible explanation: Each square is the same size and is divided into 4 equal parts. 1 of the 4 parts is shaded, so each square has $\frac{1}{4}$ shaded.

3 COMPARE

Look at these triangles. Each is divided into equal parts.



What is the same about the fraction of each model that is shaded?

Possible answer: The fractions are each $\frac{1}{2}$, because each is divided into 2 equal parts and has 1 part shaded.

What is different about the fraction of each model that is shaded?

Possible answer: The wholes are different sizes, so the amounts shown by $\frac{1}{2}$ are different.

PAIR/SHARE

Discuss your solutions for these three problems with a partner.

2 EXPLAIN

Look for understanding that equal parts have the same size but do not need to have the same shape.

Prompt discussion with questions such as:

- *How do you know what fraction is shaded in each square?*
- *What does each part of a fraction represent?*
- *If the squares are the same size and are divided into the same number of equal parts, what does that mean about the fraction parts?*

Common Misconception **If** students think that Rose is correct, **then** have them use dot paper to model each figure. Count the equal parts before shading. Have students explain what unit fraction each equal part of a model represents.

3 COMPARE

Look for understanding that two different models can each show the same fraction, but they do not represent the same amount if the models are different sizes.

Have students explain what they know about a shape described as "one half is shaded." Have them explain what they do not know about the shape.

4 **Before students begin,** read through problem 4 as a class. Make sure students understand their task is to divide each rectangle, shade parts to represent the fraction of pizza each person has, and explain their work.

As students work on their own, walk around to assess their progress and understanding, to answer their questions, and to give additional support, if needed.

Have students share their models with a partner and discuss their solution process, pointing out how they represented the numerator and denominator of each fraction.

Scoring Rubrics

Part A

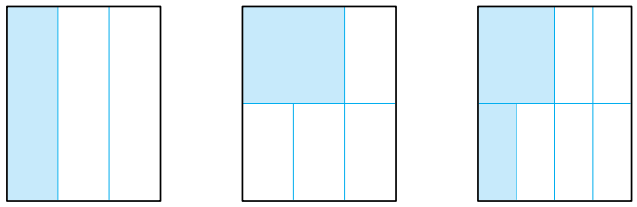
Points	Expectations
2	The student shows the correct number of parts in each pizza and shades the correct number of parts for the three pizzas.
1	The student shows the correct number of parts for each pizza but does not shade in parts correctly, or the student shows an incorrect fraction for one of the pizzas.
0	The student is unable to show the fractions for the three pizzas.

Part B

Points	Expectations
2	The student provides a clear explanation of how to figure out the number of equal parts to show in the pizza and the number of parts to shade.
1	The explanation provided is not clear about how to figure out the number of equal parts to show in the pizza or how many parts to shade.
0	No explanation is provided or the explanation is incorrect or unclear.

Use what you have learned to complete problem 4.

4 Adam has $\frac{1}{3}$ of a pizza, Hillary has $\frac{2}{6}$ of a pizza, and John has $\frac{3}{8}$ of a pizza.



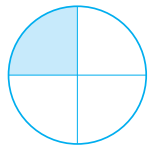
Part A Show the number of equal parts in each pizza. Then shade each pizza to show the fraction each person has.
Possible drawings shown above.

Part B Circle one of the pizzas. Explain how you knew how many equal parts to show and how many parts to shade.

Possible answer: I looked at the denominator to know how many equal parts to divide the pizza into. Then I looked at the numerator to know how many parts to shade.

5 MATH JOURNAL

Mike has a circle divided into equal parts. One part is shaded, and the other three parts are not. Mike says his circle shows the fraction $\frac{1}{3}$. Is he correct? Draw a picture to help you explain.



Mike is incorrect. Possible explanation: The circle shows the fraction $\frac{1}{4}$ because 1 part is shaded and there are 4 parts that make up the whole. Mike used the number of unshaded parts as the denominator when he should have used the total number of parts.

Close: Exit Ticket

5 MATH JOURNAL

Students should recognize that Mike's model represents $\frac{1}{4}$ because the denominator should include all the parts, shaded and unshaded. Students should also show a model divided into 4 equal parts, with 1 part shaded.

Error Alert **If** students agree that Mike is correct, **then** have them use yellow and red tiles to model the problem, using red tiles for the shaded parts.

Lesson Overview

LESSON 23

Find Equivalent Fractions

Lesson Objectives

Content Objectives

- Use fraction models and number lines to identify and create equivalent fractions, including those that are greater than or equal to one whole.
- Identify, model, and write equivalent fractions for whole numbers.

Language Objectives

- Write equivalent fractions for numbers greater than 1.
- Write whole numbers as fractions and justify, using area models or number lines.
- Write a fraction that represents a whole number.
- Tell why a fraction with a denominator of 1 is equivalent to a whole number.

Prerequisite Skills

- Understand the meaning of fractions.
- Identify fractions represented by models.
- Understand that the size of a fractional part is relative to the size of the whole.
- Understand how to use number lines to count and identify fractional parts.

Standards for Mathematical Practice (SMP)

SMPs 1, 2, 3, 4, 5, and 6 are integrated in every lesson through the *Try-Discuss-Connect* routine.*

In addition, this lesson particularly emphasizes the following SMPs:

4 Model with mathematics.

5 Use appropriate tools strategically.

6 Attend to precision.

7 Look for and make use of structure.

8 Look for and express regularity in repeated reasoning.

*See page 455i to see how every lesson includes these SMPs.

Lesson Vocabulary

There is no new vocabulary. Review the following key terms.

- **denominator** the number below the line in a fraction that tells the number of equal parts in the whole.
- **equivalent fractions** two or more different fractions that name the same part of a whole or the same point on a number line.
- **fraction** a number that names equal parts of a whole. A fraction names a point on the number line.
- **numerator** the number above the line in a fraction that tells the number of equal parts that are being described.

Learning Progression

In the previous lesson students developed a conceptual understanding of equivalent fractions by using fraction models and number lines.

In this lesson students extend their understanding to include identifying and generating equivalent fractions, including equivalent fractions for whole numbers that are equal to or greater than 1. Students continue to use fraction models and number lines to reason about fraction equivalency and to find equivalent fractions.

Students learn to write a whole number as an equivalent number of fraction parts and as a fraction with a denominator of 1.

Students' work with equivalent fractions in this lesson provides a foundation for learning to compare fractions in the next two Grade 3 lessons.

In Grade 4 students will learn to find equivalent fractions by multiplying or dividing the numerator and denominator of a fraction by the same whole number.

Lesson Pacing Guide

Whole Class Instruction

SESSION 1 **Explore** *45–60 min*	**Equivalent Fractions** • Start *5 min* • Try It *10 min* • Discuss It *10 min* • Connect It *15 min* • Close: Exit Ticket *5 min*	**Additional Practice** Lesson pages 497–498
SESSION 2 **Develop** *45–60 min*	**Finding Equivalent Fractions** • Start *5 min* • Try It *10 min* • Discuss It *10 min* • Picture It & Model It *5 min* • Connect It *10 min* • Close: Exit Ticket *5 min*	**Additional Practice** Lesson pages 503–504 **Fluency** Finding Equivalent Fractions
SESSION 3 **Develop** *45–60 min*	**Writing a Whole Number as a Fraction** • Start *5 min* • Try It *10 min* • Discuss It *10 min* • Picture It & Model It *5 min* • Connect It *10 min* • Close: Exit Ticket *5 min*	**Additional Practice** Lesson pages 509–510 **Fluency** Writing a Whole Number as a Fraction
SESSION 4 **Develop** *45–60 min*	**Writing a Whole Number as a Fraction with a Denominator of 1** • Start *5 min* • Try It *10 min* • Discuss It *10 min* • Picture It & Model It *5 min* • Connect It *10 min* • Close: Exit Ticket *5 min*	**Additional Practice** Lesson pages 515–516 **Fluency** Writing a Whole Number as a Fraction with a Denominator of 1
SESSION 5 **Refine** *45–60 min*	**Finding Equivalent Fractions** • Start *5 min* • Example & Problems 1–3 *15 min* • Practice & Small Group Differentiation *20 min* • Close: Exit Ticket *5 min*	**Lesson Quiz** or **Digital Comprehension Check**

Lesson Materials

Lesson *(Required)*	*Per student:* fraction tiles *Activity Sheet:* Number Lines**
Activities	*Per student:* scissors, tape, colored pencils; *Per pair:* fraction circles *Activity Sheets:* 1-Inch Grid Paper**, Multiplication Table
Math Toolkit	fraction circles, fraction tiles, fraction bars, number lines, grid paper, index cards, crayons
Digital Math Tools	Fraction Models, Number Line

**Used for more than one activity.

Teacher Toolbox

Small Group Differentiation

PREPARE

Ready Prerequisite Lesson

Grade 2
- Lesson 29 Understand Partitioning Shapes Into Halves, Thirds, and Fourths

RETEACH

Tools for Instruction

Grade 2
- Lesson 30 Make Equal Shares

Grade 3
- Lesson 23 Find Equivalent Fractions

REINFORCE

Math Center Activity

Grade 3
- Lesson 23 Building Equivalent Fractions

EXTEND

Enrichment Activity

Grade 3
- Lesson 23 Colorful Quilts

i-Ready

Independent Learning

PERSONALIZE

i-Ready Lesson*

Grade 3
- Find Equivalent Fractions

Learning Game
- *Bounce*

*We continually update the Interactive Tutorials. Check the Teacher Toolbox for the most up-to-date offerings for this lesson.

Connect to Family, Community, and Language Development

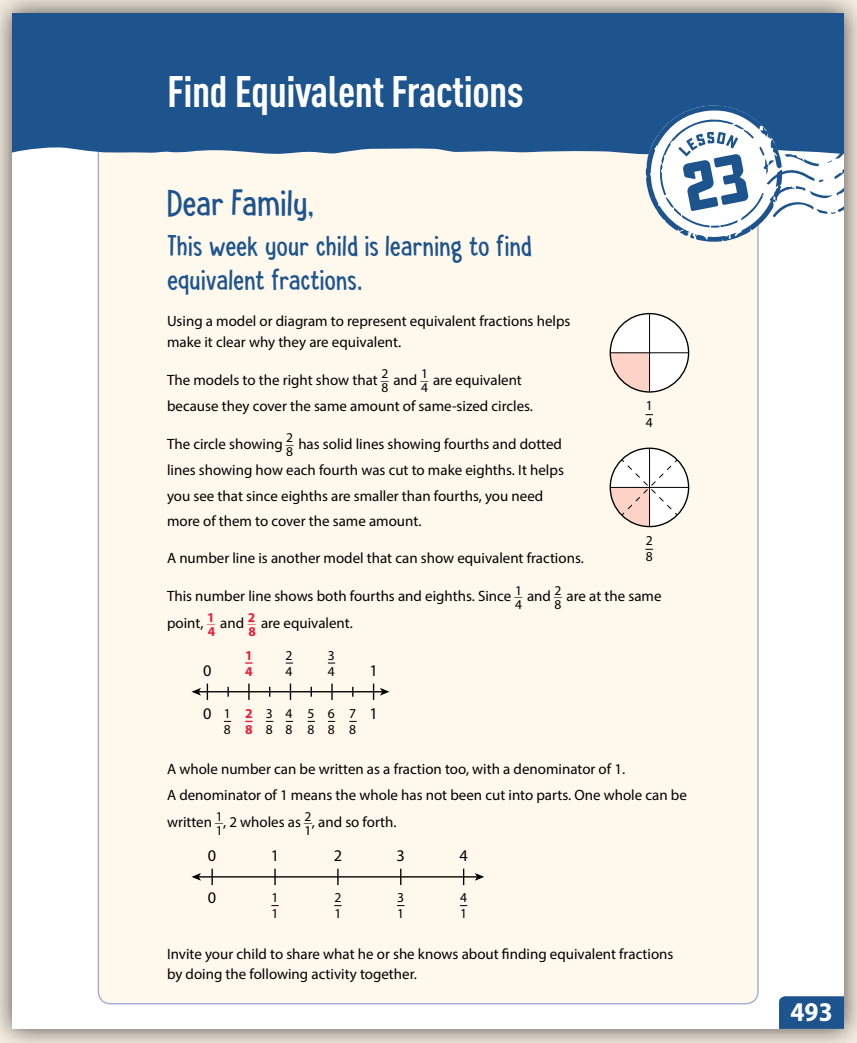
The following activities and instructional supports provide opportunities to foster school, family, and community involvement and partnerships.

Connect to Family

Use the **Family Letter**—which provides background information, math vocabulary, and an activity—to keep families apprised of what their child is learning and to encourage family involvement.

Available in Spanish

Teacher Toolbox



Find Equivalent Fractions

LESSON 23

Dear Family,

This week your child is learning to find equivalent fractions.

Using a model or diagram to represent equivalent fractions helps make it clear why they are equivalent.

The models to the right show that $\frac{2}{8}$ and $\frac{1}{4}$ are equivalent because they cover the same amount of same-sized circles.

$\frac{1}{4}$

The circle showing $\frac{2}{8}$ has solid lines showing fourths and dotted lines showing how each fourth was cut to make eighths. It helps you see that since eighths are smaller than fourths, you need more of them to cover the same amount.

$\frac{2}{8}$

A number line is another model that can show equivalent fractions.

This number line shows both fourths and eighths. Since $\frac{1}{4}$ and $\frac{2}{8}$ are at the same point, $\frac{1}{4}$ and $\frac{2}{8}$ are equivalent.

0 $\frac{1}{4}$ $\frac{2}{4}$ $\frac{3}{4}$ 1

0 $\frac{1}{8}$ $\frac{2}{8}$ $\frac{3}{8}$ $\frac{4}{8}$ $\frac{5}{8}$ $\frac{6}{8}$ $\frac{7}{8}$ 1

A whole number can be written as a fraction too, with a denominator of 1.

A denominator of 1 means the whole has not been cut into parts. One whole can be written $\frac{1}{1}$, 2 wholes as $\frac{2}{1}$, and so forth.

0 1 2 3 4

0 $\frac{1}{1}$ $\frac{2}{1}$ $\frac{3}{1}$ $\frac{4}{1}$

Invite your child to share what he or she knows about finding equivalent fractions by doing the following activity together.

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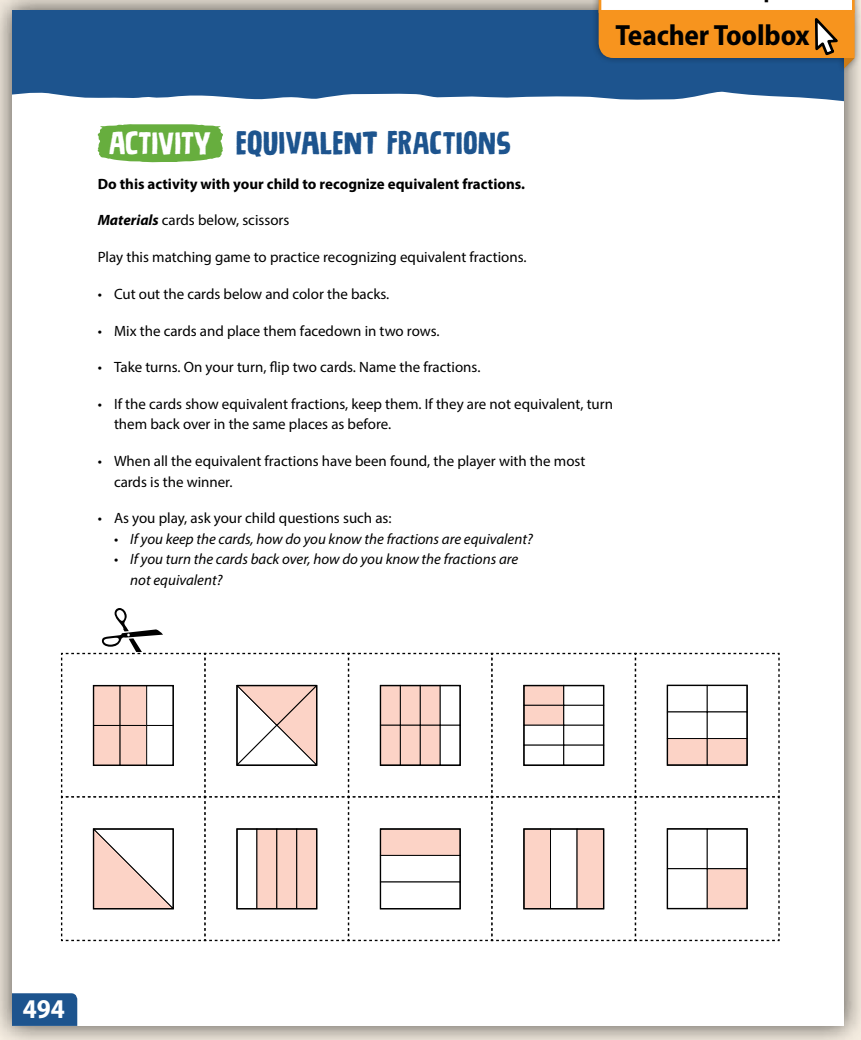

ACTIVITY EQUIVALENT FRACTIONS

Do this activity with your child to recognize equivalent fractions.

Materials cards below, scissors

Play this matching game to practice recognizing equivalent fractions.

- Cut out the cards below and color the backs.
- Mix the cards and place them facedown in two rows.
- Take turns. On your turn, flip two cards. Name the fractions.
- If the cards show equivalent fractions, keep them. If they are not equivalent, turn them back over in the same places as before.
- When all the equivalent fractions have been found, the player with the most cards is the winner.
- As you play, ask your child questions such as:
  - *If you keep the cards, how do you know the fractions are equivalent?*
  - *If you turn the cards back over, how do you know the fractions are not equivalent?*

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Goal

The goal of the Family Letter is to provide additional models of equivalent fractions. Students are expected to recognize and find equivalent fractions using area models and number lines. Whole numbers are represented as fractions.

Activity

Look at the *Equivalent Fractions* activity and adjust it if necessary to connect with your students.

Math Talk at Home

Encourage students and their family members to talk about fractions. Challenge them to find examples of fractions at home, and to use these fractions as a basis for creating equivalent fractions.

Conversation Starters Below are additional conversation starters students can write in their Family Letter or math journal to engage family members.

- *How did you learn about fractions in school? Did you use models and number lines?*
- *How do you figure out whether or not two fractions are equivalent?*
- *What does $\frac{5}{1}$ mean?*

Connect to Community and Cultural Responsiveness

Use these activities to connect with and leverage the diverse backgrounds and experiences of all students.

Session 1 **Use with *Try It*.**

- Ask students to tell their favorite snack. Suggest that many people like to snack on granola bars. Draw and label a model as you say: *When you eat a whole granola bar, the whole bar is represented as* $\frac{1}{1}$. *If you give your bar to two friends to share, the unit fraction they each get is* $\frac{1}{2}$ (display). *If you give your bar to three friends to share equally, the unit fraction they each get is* $\frac{1}{3}$ (display). *Turn to a partner and decide what unit fraction four friends receive if they share equally.* Display $\frac{1}{4}$. Point to the models: *What happens to the pieces of the granola bar as more friends share?* [the pieces get smaller] Point to the fractions. Ask: *What happens to the denominators as more friends share?* [the number gets greater] Display and have students complete the sentence frame: *The pieces get* ___*smaller*___ *as the denominators get* ___*greater*___ .

Session 2 **Use throughout the session.**

- Say: *We have focused on food items that can be divided into equal parts.* Display a dollar bill and scissors. Ask: *Why is cutting this dollar into equal parts not a good idea?* [The dollar cannot be used.] Say: *Are there things that cannot or should not be divided into fractional pieces? Turn to a partner and discuss something that you think cannot or should not be divided into fractional pieces. Be prepared to explain why you cannot or should not divide your item.* Select pairs to share.

Session 3 **Use with *Try It*.**

- Ask students if they have or have seen a birdhouse, bird feeder, or bird bath. Ask students to explain the purpose of each. Point out that these objects are often made of wood. Ask: *What are some things that can be built with wood?* (for example, a fence, a bookcase, and a tree house) Display a list of items students suggest. Ask students to share any experiences they may have had building something out of wood.

Session 4 **Use with *Apply It* problem 10.**

- Explain that breads in different cultures can vary quite a bit by ingredients, size, and shape. Invite students to tell about different types of breads that they know or like. You may also ask: *What type of bread is most common in your home?*

Session 5 **Use with the Example.**

- Display the word *melon*. Ask students to share different types of melon they have eaten. Display the words *watermelon*, *cantaloupe*, and *honeydew*.

Connect to Language Development

For ELLs, use the Differentiated Instruction chart to plan and prepare for specific activities in every session.

ELL English Language Learners: Differentiated Instruction

Prepare for Session 1
Use with *Try It*.

Levels 1–3

Listening/Speaking Read the first two sentences of the ***Try It*** problem aloud. Have students form pairs and give each pair a square sheet of paper. Say: *This is the cake.* Model how to fold the paper in half. Ask: *What unit fraction describes each part of the cake?* $\left[\frac{1}{2}\right]$ Have students label each half with a flavor. Read the rest of the problem. Model how to fold the paper in fourths. Ask: *What unit fraction describes each part of the cake now?* $\left[\frac{1}{4}\right]$ Have students label each part. Say: *Fold your square so that you can only see the half with chocolate frosting.* Display: $\frac{1}{2} = \frac{2}{4}$.

Say: *Discuss with your partner how to complete this equation.*

Levels 2–4

Speaking/Writing Read the first two sentences of the ***Try It*** problem. Have students form pairs and give each pair a square sheet of paper. Say: *This is the cake. Fold the paper in half. What unit fraction describes each part of the cake?* $\left[\frac{1}{2}\right]$ Have students label each half with a flavor. Read the rest of the problem. Ask: *How can you fold the paper to show fourths?* Display students' squares. Ask: *What unit fraction describes each part of the cake now?* $\left[\frac{1}{4}\right]$ *Label each part. Fold your square so that you see the half with chocolate frosting. With your partner, write an equation to show how many fourths of the cake are equal to one half.* Call on pairs to share their equations.

Levels 3–5

Speaking/Writing Have pairs read the ***Try It*** problem. Give each pair a square sheet of paper. Say: *This is the cake. Fold the paper in half. What unit fraction describes each part of the cake?* $\left[\frac{1}{2}\right]$ *Label each half with a flavor. Fold the paper to show fourths.* Display students' squares. Ask: *What unit fraction describes each part of the cake now?* $\left[\frac{1}{4}\right]$ *Label each part. Fold your square so that you see the half with chocolate frosting.* Have students complete the sentence frame with a partner:

One half of the cake is equal to _____ *fourths because* _____.

Call on pairs to share their explanations.

LESSON 23

SESSION 1 Explore

Purpose In this session students draw on their knowledge of area models and equivalent fractions. They compare models to explore how to rename a fraction using a different denominator. They will look ahead to think about how to find equivalent fractions using the same area model or number line.

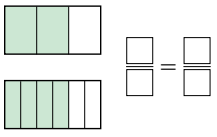
Start

Connect to Prior Knowledge

Why Support students' facility with recognizing and writing equivalent fractions.

How Have students write the equivalent fractions represented by two area models.

Write equivalent fractions for the shaded parts of the models.



$\square = \square$

Solution

$\frac{2}{3} = \frac{4}{6}$

TRY IT

Make Sense of the Problem

To support students in making sense of the problem, have them show that they understand that the cake is visually divided into halves by the kinds of frosting, but that the cake will be cut into fourths, and that each fourth must be all chocolate or all vanilla.

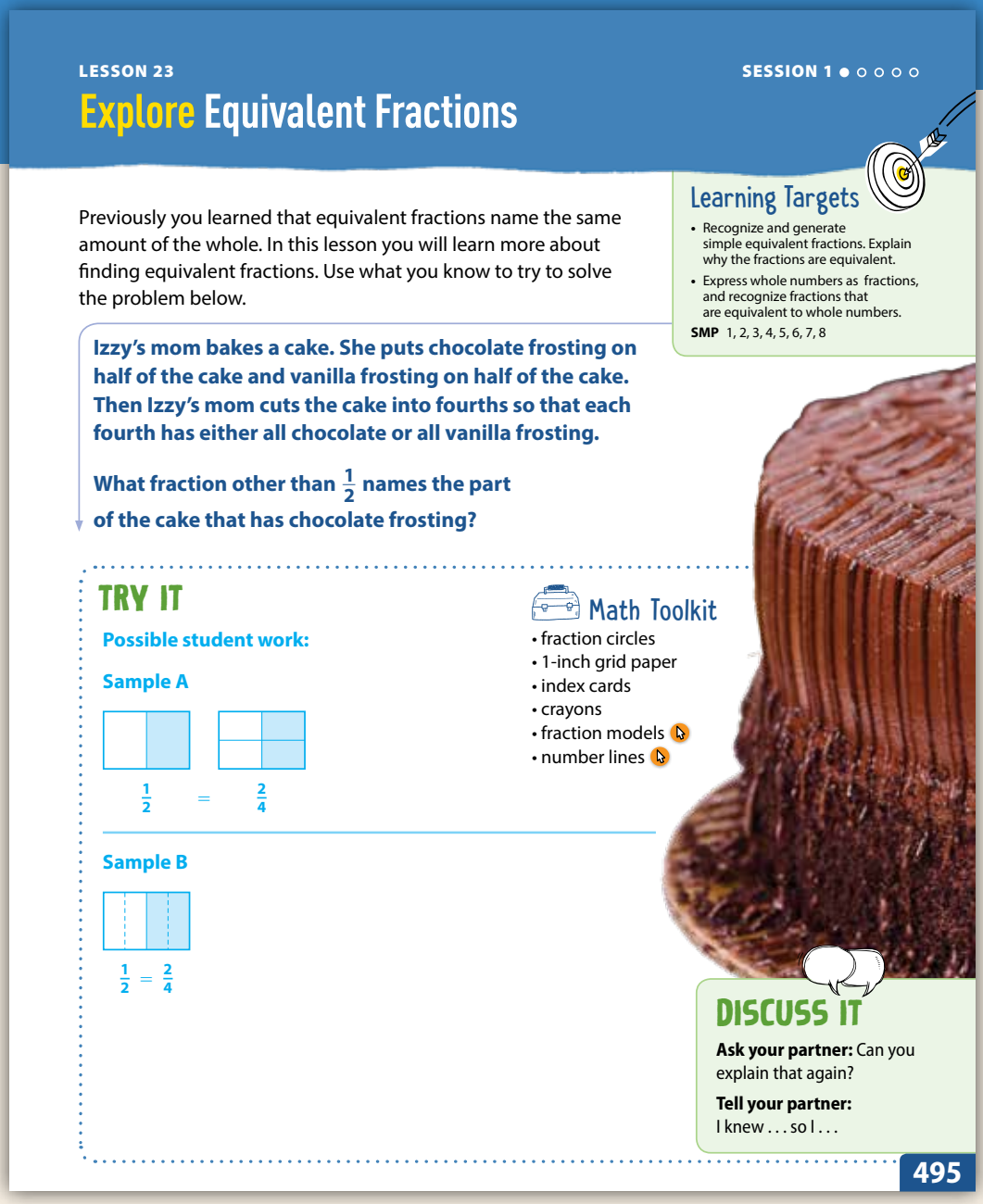
DISCUSS IT

Support Partner Discussion

To reinforce the fact that the problem is asking for another name for $\frac{1}{2}$, encourage students to use the terms *half* and *fourths* as they talk to each other.

Look for, and prompt as necessary for, understanding that:

- $\frac{1}{2}$ of the cake has chocolate frosting
- the cake is cut into fourths
- they are looking for another fraction that describes the part of the cake with chocolate frosting



LESSON 23

SESSION 1 ● ○ ○ ○ ○

Explore Equivalent Fractions

Previously you learned that equivalent fractions name the same amount of the whole. In this lesson you will learn more about finding equivalent fractions. Use what you know to try to solve the problem below.

Learning Targets

- Recognize and generate simple equivalent fractions. Explain why the fractions are equivalent.
- Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers.

SMP 1, 2, 3, 4, 5, 6, 7, 8

Izzy's mom bakes a cake. She puts chocolate frosting on half of the cake and vanilla frosting on half of the cake. Then Izzy's mom cuts the cake into fourths so that each fourth has either all chocolate or all vanilla frosting.

What fraction other than $\frac{1}{2}$ names the part of the cake that has chocolate frosting?

TRY IT

Possible student work:

Sample A

$\frac{1}{2} = \frac{2}{4}$

Sample B

$\frac{1}{2} = \frac{2}{4}$

Math Toolkit

- fraction circles
- 1-inch grid paper
- index cards
- crayons
- fraction models
- number lines

DISCUSS IT

Ask your partner: Can you explain that again?

Tell your partner: I knew . . . so I . . .

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Common Misconception Look for students who are not comfortable with visualizing how the cake is cut. As students present solutions, have them specify how they think the cake is cut.

Select and Sequence Student Solutions

One possible order for whole class discussion:

- models that have been cut apart
- freehand drawings of the cake
- drawings on grid paper
- solutions with no drawing or visual model

Support Whole Class Discussion

Prompt students to note the relationship between the numbers in each model and the numbers in the problem.

Ask *How do [student name]'s and [student name]'s models show which part(s) of the cake have chocolate frosting?*

Listen for One half, or two fourths, of the cake should be shaded or somehow marked to indicate chocolate frosting.

CONNECT IT

1 LOOK BACK

Look for understanding that the amount of cake with chocolate frosting stays the same whether it is cut into halves or fourths.

Hands-On Activity

Use grid paper to model equivalent fractions.

If . . . *students have trouble understanding that each $\frac{1}{4}$ of the cake is the same amount regardless of its shape or how the cake is cut,*

Then . . . *use this activity to have them explore different ways to cut the cake into fourths.*

Materials For each student: scissors, tape, Activity Sheet *1-Inch Grid Paper*

- Have students draw at least two models of the cake, making each one a 2-inch by 4-inch rectangle. Then have them draw lines to show each model divided into fourths in a different way.
- Ask: *In each of the models you drew, how many grid squares are used to make one fourth?* [2] For models drawn so that the fourths are triangles, have students cut the triangle along a grid line and tape the pieces back together so that any partial grid squares are combined to form full squares.

2 LOOK AHEAD

Point out that no matter how the cake is cut into fourths, there are 2 pieces with chocolate frosting. No matter how the cake is cut into eighths, there are 4 pieces with chocolate frosting.

Note that the cake diagrams and the number line are different types of fraction models. In the Additional Practice, students will spend more time learning about the different types of fraction models.

CONNECT IT

1 LOOK BACK

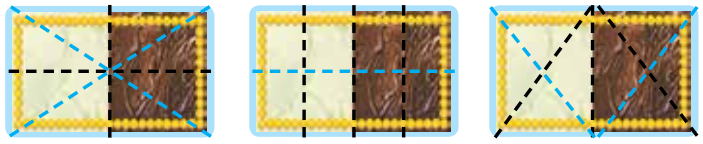
What fraction other than $\frac{1}{2}$ names the part of the cake that has chocolate frosting? $\frac{2}{4}$ How did you get your answer?

Possible answer: When you divide the cake into 4 equal parts, 2 of them have chocolate frosting.

2 LOOK AHEAD

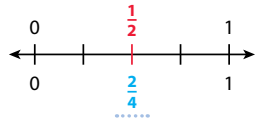
You have seen many different types of fraction models, such as area models, number lines, and fractions bars. You can find equivalent fractions by dividing the same model in different ways.

a. Each cake below shows fourths. Draw lines on one of the cakes to show eighths. **Possible answers:**



b. How many pieces of the cake have chocolate frosting now? 4

c. You can also look at different equal-sized parts on a number line to find equivalent fractions. Fill in the fraction for fourths that is equivalent to $\frac{1}{2}$.



3 REFLECT

Why does it make sense that $\frac{1}{2}$ and $\frac{2}{4}$ can name the same amount?

Possible answer: One half of a cake can be divided again to make two fourths, but the amount of cake does not change.

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Close: Exit Ticket

3 REFLECT

Look for understanding that $\frac{1}{2}$ and $\frac{2}{4}$ represent the same quantity divided into different numbers of parts.

Common Misconception **If** students are confused by the single number line, **then** draw two number lines, one labeled with 0, $\frac{1}{2}$, and 1, and the other directly beneath it labeled with 0, $\frac{1}{4}$, $\frac{2}{4}$, $\frac{3}{4}$, and 1. Explain that combining the two number lines, as in problem 2c, makes identifying equivalent fractions easier.

Real-World Connection

Use two pieces of fruit that are the same size to help students reason that they are getting a fair share or equivalent amount if they cut the same-sized fruits into halves two different ways. For example, if you have two lemons, cut one lemon in half the long way, and the other lemon in half the short way. Compare halves from each lemon. Emphasize that each half is the same amount even though the shapes are different. Then cut each half into two fourths. Hold the pieces together and then separate them again so that they can see that $\frac{2}{4}$ of the lemon is the same as $\frac{1}{2}$ of the lemon.

Solutions

Support Vocabulary Development

1 Have students point at and say the phrase *fraction models*. Point out that a model of a fraction can be a picture. Ask: *What models have you used in previous lessons to represent fractions?* [number lines, area models, fraction bars] *What shapes have you used to represent fractions?* [rectangles, squares, circles] *What fractions can you represent?* [possible responses include: unit fractions, $\frac{2}{3}$, and $\frac{3}{4}$] List students' responses to the questions as a scaffold.

2 Have students label each part of the first rectangle on the outside of the rectangle. Ask: *What unit fraction describes each piece of the first rectangle?* [$\frac{1}{3}$] *How many equal pieces must a model have to represent sixths?* [6] *With a partner, discuss how you can draw one or more lines on the rectangle to make six equal pieces.* Students may draw one vertical or three horizontal lines to create sixths. Validate both strategies. Repeat the process and questions for the next two models if students need additional support.

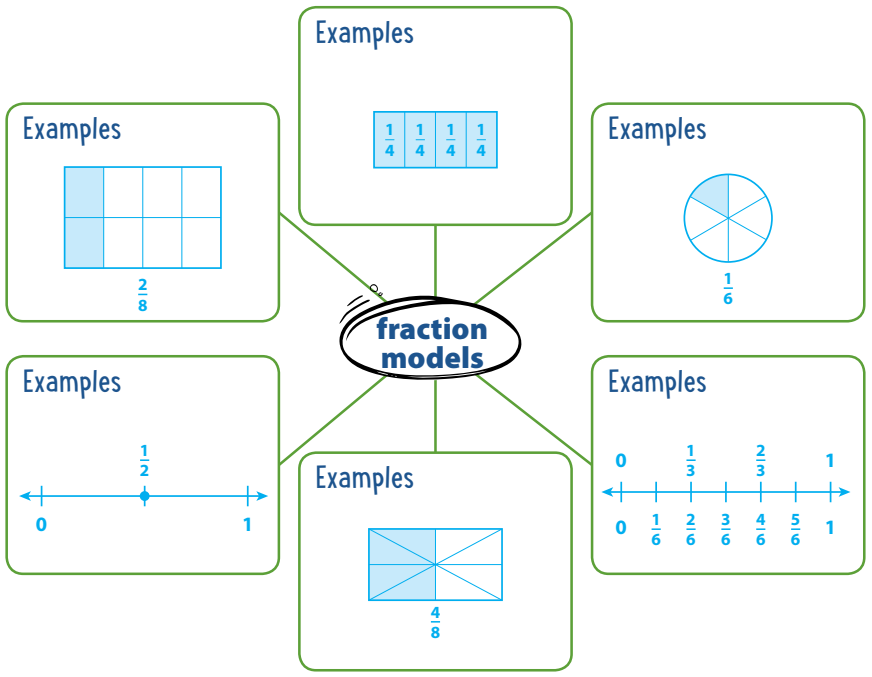
Supplemental Math Vocabulary

- *denominator*
- *equivalent fraction*
- *fraction*
- *numerator*

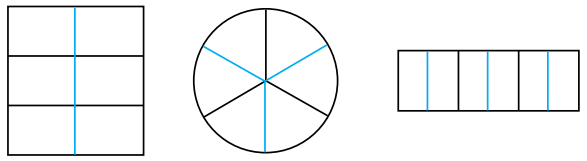
Name: _____ LESSON 23 SESSION 1

Prepare for Finding Equivalent Fractions

1 Think about what you know about fractions. Fill in each box. Use words, numbers, and pictures. Show as many ideas as you can. **Possible answers:**



2 Each fraction model below shows thirds. Draw lines on each model to show sixths. **Possible answers:**



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3 Assign problem 3 to provide another look at finding equivalent fractions.

This problem is very similar to the problem about Izzy's mom cutting a cake into equal pieces. In both problems, students are given a word problem where they must divide a shape into smaller parts to find an equivalent fraction. The question asks what fraction other than one third names the part of the rectangle that is red.

Students may want to use fraction tiles or construction paper.

Suggest that students read the problem three times, asking themselves one of the following questions each time:

- *What is this problem about?*
- *What is the question I am trying to answer?*
- *What information is important?*

Solution: $\frac{2}{6}$ of the rectangle is red.
Medium

4 Have students solve the problem a different way to check their answer.

3 Solve the problem. Show your work.

Len has 3 strips of construction paper. Each strip is the same size and a different color—red, yellow, and pink. He tapes the strips together to make a rectangle.

Then Len divides the rectangle into sixths so that each sixth is one color. What fraction other than $\frac{1}{3}$ names the part of the rectangle that is red?

Possible student work using pictures:

$\frac{1}{3} = \frac{2}{6}$

4 Check your answer. Show your work.

Possible student work:

$\frac{1}{3} = \frac{2}{6}$

I divided the rectangle differently and got the same answer.

ELL English Language Learners: Differentiated Instruction

Prepare for Session 2
Use with *Apply It*.

Levels 1–3	Levels 2–4	Levels 3–5
Listening/Speaking Give pairs two congruent circles. Read aloud ***Apply It*** problem 8. Ask: *How many equal pieces are in Lina's pizza?* [4] *How do you know?* Provide the sentence frame: *The* ___*denominator*___ *is 4.* Model how to fold one circle to create fourths. Ask: *How many slices did Lina eat?* [3] *Shade the circle to represent* $\frac{3}{4}$. Repeat the process with the second circle. Say: *Both pizzas show* $\frac{3}{4}$. *Discuss how the four slices of Adam's pizza can be made into eight equal pieces.* Validate suggestions. Model how to fold or draw lines to create eighths. Display and have students complete the sentence frame: *Adam ate* ___*6*___ *slices.*	**Listening/Speaking** Give pairs two congruent circles. Read aloud ***Apply It*** problem 8. Ask: *How many equal pieces are in Lina's pizza?* [4] *How do you know?* [The denominator is 4.] *How many slices did Lina eat?* [3] *Discuss with your partner how you can use a circle to represent Lina's pizza.* Validate suggestions. Have them fold one circle to create fourths. Say: *Shade the circle to represent* $\frac{3}{4}$. Repeat the process with the second circle. Say: *With your partner, show eighths on the second circle to represent Adam's pizza. Decide how many eighths are equal to the three slices Lina ate.* Have students complete the sentence frame: *Adam ate* ___*6*___ *pizza slices.* Have students take turns reading the sentence to their partners.	**Listening/Speaking** Give pairs two congruent circles. Read aloud ***Apply It*** problem 8. Ask: *How many equal pieces are in Lina's pizza? How do you know? How many slices did Lina eat?* Encourage students to answer in complete sentences. Say: *Discuss with your partner how you can use a circle to represent Lina's pizza.* Validate suggestions. Have them fold one circle to create fourths. Say: *Shade the circle to represent* $\frac{3}{4}$. *With your partner, show eighths on the second circle to represent Adam's pizza. Decide how many eighths are equal to the three slices Lina ate. Write a sentence that tells how many slices Adam ate.* Select pairs to share their process.

LESSON 23

SESSION 2 Develop

Purpose In this session students solve a problem that requires showing that two fractions are equivalent. Students model and compare the fractions either on paper or with manipulatives. The purpose of this session is to have students understand how to use models such as area models and number lines to show two fractions are equivalent and represent the same quantity.

Start

Connect to Prior Knowledge

Why Reinforce the concept that two different fractions can name the same part of a whole.

How Have students write two different fractions for a rectangular area model.

Write two different fractions that describe the shaded part of the area model.

Solution

$\frac{3}{4}, \frac{6}{8}$

Develop Language

Why Clarify the meaning and use of *amount*.

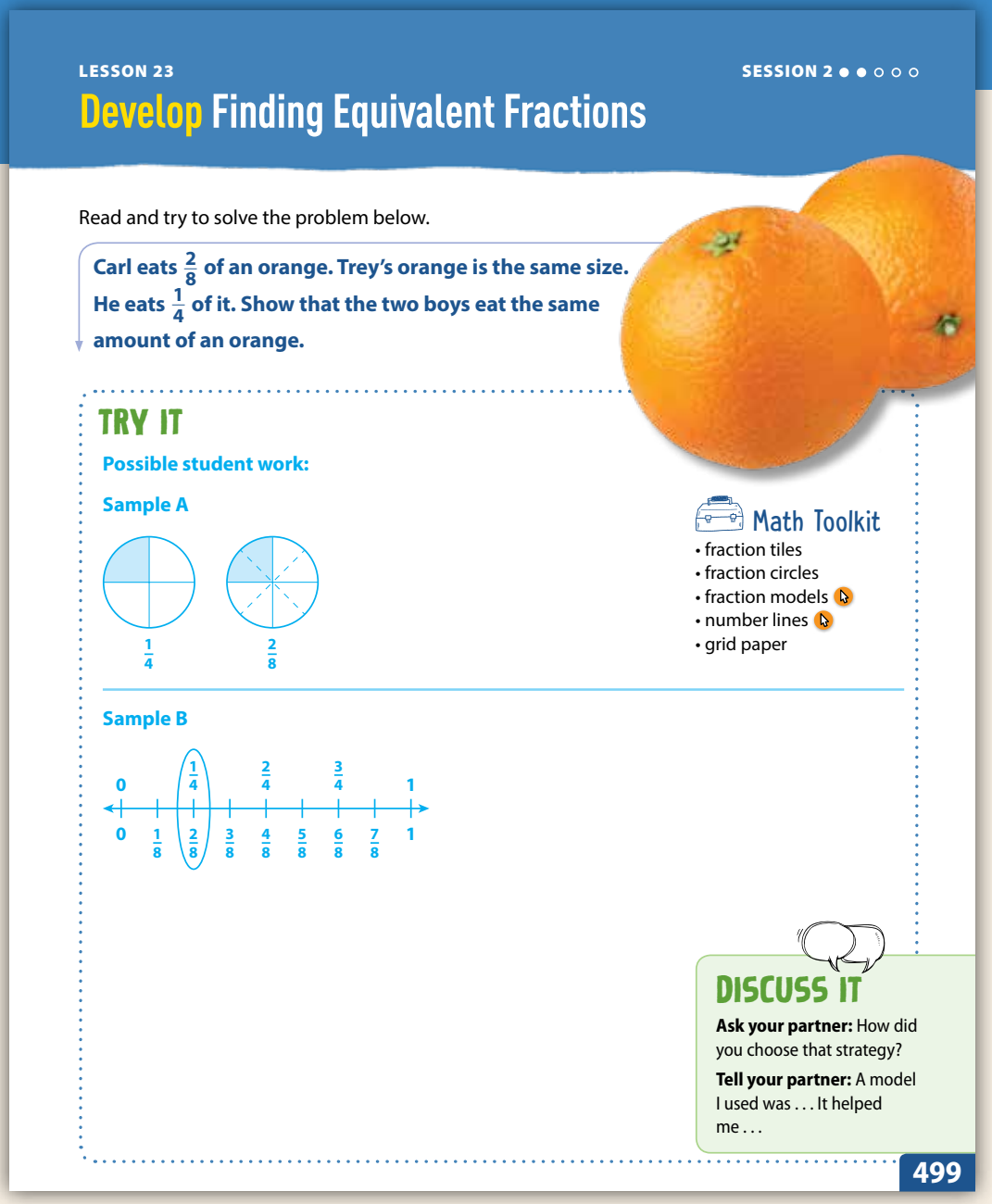
How Explain that the word *amount* refers to the quantity of something. Point out that *amount* is usually used with things that cannot be counted or that are difficult to count. Have students read the ***Try It*** problem. Explain that if the problem was about how many whole oranges Trey and Carl eat, a number could be used, for example: *Trey and Carl eat 3 oranges.* Explain that the problem refers to how much of an orange Trey eats. Since this quantity cannot be easily counted, we say: *Trey eats the same amount of orange as Carl.* Ask students whether they would use a number or the word *amount* with the following and to explain why: rice (amount), books (number), rain (amount), shoes (number)

TRY IT

Make Sense of the Problem

To support students in making sense of the problem, have them identify the fractions they are being asked to compare.

Ask *What fraction of an orange did each boy eat? Why is it important that Trey's orange is the same size as Carl's orange?*



LESSON 23 SESSION 2 ● ● ○ ○ ○

Develop Finding Equivalent Fractions

Read and try to solve the problem below.

Carl eats $\frac{2}{8}$ of an orange. Trey's orange is the same size. He eats $\frac{1}{4}$ of it. Show that the two boys eat the same amount of an orange.

TRY IT

Possible student work:

Sample A

Math Toolkit

- fraction tiles
- fraction circles
- fraction models
- number lines
- grid paper

Sample B

DISCUSS IT

Ask your partner: How did you choose that strategy?

Tell your partner: A model I used was . . . It helped me . . .

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DISCUSS IT

Support Partner Discussion

Encourage students to use the term *equivalent* as they discuss their solutions.

Support as needed with questions such as:

- *What model did you use?*
- *How did you show fourths? How did you show eighths?*

Common Misconception Look for students who use two different models that cannot be compared, such as an area model and a number line, rather than using two models that are the same kind and size.

Select and Sequence Student Solutions

One possible order for whole class discussion:

- fraction tiles or fraction circles
- area models divided into fourths and eighths
- number lines showing fourths and eighths

Support Whole Class Discussion

Compare and connect the different representations and have students identify how they are related.

Ask *How does each model show the two different denominators? Where does each model show the amount of orange each boy eats? How does each model show that the fractions are equivalent?*

Listen for Area models and number lines should be broken up into four parts and eight parts to show fourths and eighths, respectively, or fourths and eighths fraction tiles should be used. The amount eaten can be shown by shading on an area model, comparing the correct number of each tile, and by drawing points on a number line. The amounts shaded or tiles compared should be the same size and the points on the number line should coincide.

PICTURE IT & MODEL IT

If no student presented these models, connect them to the student models by pointing out the ways they each show:

- the $\frac{2}{8}$ Carl eats
- the $\frac{1}{4}$ Trey eats
- $\frac{2}{8} = \frac{1}{4}$

Ask *How can you tell which area model shows each fraction? How do the area models show that the fractions are equivalent? How does the number line show that the fractions are equivalent?*

Listen for The number of parts the model is divided into shows the denominator and the number of shaded parts shows the numerator. The area models show that the same amount is shaded, even though they are broken into different numbers of parts. The number line shows that the two fractions name the same point.

For an area model, prompt students to identify the numerator and denominator of each fraction and how they are represented in the models.

- *How many equal parts is each orange divided into?*
- *How many parts did each boy eat?*

For a number line, prompt students to explain how to represent each fraction on a number line.

- *How can you label both fourths and eighths on the same number line?*
- *How can you show the amount each boy ate?*

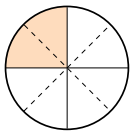
Explore different ways to understand finding equivalent fractions.

Carl eats $\frac{2}{8}$ of an orange. Trey's orange is the same size. He eats $\frac{1}{4}$ of it. Show that the two boys eat the same amount of an orange.

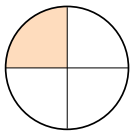
PICTURE IT

You can use models to help find equivalent fractions.

This model shows $\frac{2}{8}$.



This model shows $\frac{1}{4}$.

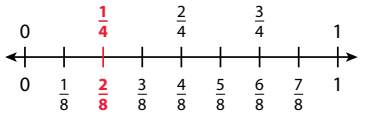


Look at the model of $\frac{2}{8}$. The solid lines divide the circle into fourths. The dashed lines divide each fourth in half to make eighths.

MODEL IT

You can also use a number line to help find equivalent fractions.

This number line shows both fourths and eighths.



Deepen Understanding

Area Models of Equivalent Fractions

SMP 4 Reason quantitatively.

When discussing the area models, prompt students to think about how they can divide each part into smaller equal parts to find other equivalent fractions.

Ask *How could you change the area model for $\frac{1}{4}$ to show $\frac{2}{8}$? Explain.*

Listen for Divide each fourth into 2 equal parts.

Ask *How do the numerator and denominator change when you divide each part into 2 parts?*

Listen for Both the numerator and denominator are doubled.

Ask *Suppose you have an area model showing halves. How can you use that model to show an equivalent fraction with a denominator of 6? How will the numerator and denominator change?*

Listen for Divide each half into 3 equal parts and count the number of shaded parts to find the numerator. Each number in the fraction will be multiplied by 3.

CONNECT IT

- Remind students that one thing that is alike about all the representations is the numbers.
- Explain that on this page they will use those numbers to explain why $\frac{2}{8} = \frac{1}{4}$.

Monitor and Confirm

1 – 2 Check for understanding that:

- The denominator tells how many parts the model is divided into.
- The numerator tells how many parts are shaded.

Support Whole Class Discussion

3 – 5 Tell students that these problems will prepare them to provide the explanation required in problem 6.

Be sure students understand that the problems are asking them to explain how the models on the previous page show that the fractions $\frac{2}{8}$ and $\frac{1}{4}$ are equivalent, using both words and numbers.

Ask *Why is it important for the two circles to be the same size? Why is it helpful to use one number line for both fractions instead of two separate number lines?*

Listen for The wholes must be the same size to compare fractions. The circles should be the same size so that the amount shaded is the same for equivalent fractions. Using one number line ensures that the wholes are the same size, and it is easier to tell whether the fractions represent the same point.

6 Look for the understanding that equivalent fractions represent the same amount in same-sized models or the same point on a number line.

7 REFLECT Have all students focus on the strategies used to solve this problem. If time allows, have students share their preferences with a partner.

CONNECT IT

Now you will use the problem from the previous page to help you understand how to find equivalent fractions.

1 Look at the models in **Picture It**. How do you know that $\frac{2}{8}$ of the first model is shaded? **There are 8 equal parts, and 2 are shaded.**

2 How do you know that $\frac{1}{4}$ of the second model is shaded?

There are 4 equal parts, and 1 is shaded.

3 Explain how the models show that the fractions $\frac{2}{8}$ and $\frac{1}{4}$ are equivalent.

The wholes are equal in size, and the amount that is shaded is the same.

4 How does the number line in **Model It** show that the fractions $\frac{2}{8}$ and $\frac{1}{4}$ are equivalent? **$\frac{1}{4}$ and $\frac{2}{8}$ are located at the same point on the number line.**

5 Complete the sentences to show that the fractions of the two oranges name the same amount.

Use words: Two eighths is equal to **one fourth**.

Use fractions: $\frac{2}{8} =$ **$\frac{1}{4}$**.

6 Describe two different ways to show two fractions are equivalent.

You can see if they show the same amount in equal-sized area models, or you can see if they are at the same point on a number line.

7 REFLECT

Look back at your **Try It**, strategies by classmates, and **Picture It** and **Model It**. Which models or strategies do you like best for finding equivalent fractions? Explain.

Some students may prefer drawing area models to represent the fractions because it just takes a simple visual check to see that the quantities are equal. Students with strong number sense may prefer using a number line.

Hands-On Activity

Explore different area models showing fourths and eighths.

If . . . *students struggle with seeing how different models can model the same fractions,*

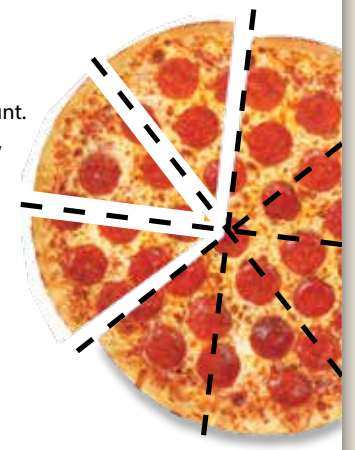
Then . . . *use this activity to let them explore different ways to divide a shape into equal parts.*

Materials For each student: colored pencils, Activity Sheet *1-Inch Grid Paper*

- Have students record on the board all the different models they drew to show $\frac{2}{8} = \frac{1}{4}$.
- Encourage them to think of additional ways they can show fourths and eighths on a single model. For example, students may have drawn a rectangle with three vertical lines to mark fourths and one dashed horizontal line to show eighths. Another way to show this is to used dashed vertical lines to show eighths, or to outline fourths with one color and outline eighths with another color.
- Have students draw a square on the grid paper and see how many ways they can divide it into fourths and then into eighths.

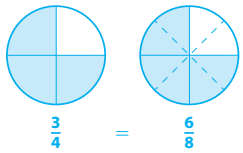
APPLY IT

Use what you just learned to solve these problems.



8 Lina and Adam each order a small pizza. They eat the same amount. Lina eats $\frac{3}{4}$ of her pizza. Adam's pizza is divided into 8 slices. How many slices of pizza did Adam eat? Show your work.

Possible student work:

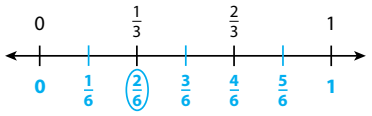


Solution **Adam ate 6 slices of pizza.**

9 Draw a model to show $\frac{2}{3} = \frac{4}{6}$.

Possible student model:

10 Use the number line to find a fraction equivalent to $\frac{1}{3}$. Show your work. **Possible student work:**



Solution **Possible answer: $\frac{2}{6}$**

APPLY IT

For all problems, encourage students to draw some kind of model to support their thinking. Allow some leeway in precision; the number of the parts of a model and the accuracy of the labels are more important than whether the parts are exactly the same size.

8 6 slices; Area models should consist of two same-sized wholes, one divided into 4 parts with 3 shaded and the other divided into 8 parts with 6 shaded to show that $\frac{3}{4} = \frac{6}{8}$. Number lines should show fourths and eighths labeled, with the point that is labeled both $\frac{3}{4}$ and $\frac{6}{8}$ highlighted.

9 See Student Worktext page; Area models should be divided into three equal parts with two parts shaded, and each of the three parts should be further divided into two equal parts.

Close: Exit Ticket

10 Possible solution: $\frac{2}{6}$; The number line should show each third divided into two (or more) equal parts, and tick marks labeled appropriately.

Students' solutions should indicate understanding that:

- equivalent fractions refer to the same point on a number line
- each third must be divided into equal parts in order to find a fraction equivalent to $\frac{1}{3}$

Error Alert **If** students' number lines show $\frac{1}{3}$ equal to $\frac{1}{4}$, $\frac{3}{8}$, or another incorrect fraction, **then** have the student redraw the number line on centimeter grid paper with either 12 or 24 grid squares between 0 and 1. Tell them how many grid squares make up each third and each of the other unit fractions so that they can label the number line appropriately and see their error.

Solutions

1. $\frac{1}{4} = \frac{2}{8}$
Basic

2. $\frac{6}{8} = \frac{3}{4}$
Basic

3. $\frac{2}{4} = \frac{4}{8}$ or $\frac{2}{4} = \frac{1}{2}$
Basic

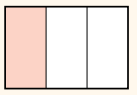
Name: _____ LESSON 23 SESSION 2

Practice Finding Equivalent Fractions

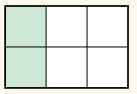
Study the Example showing how to find equivalent fractions. Then solve problems 1–8.

EXAMPLE

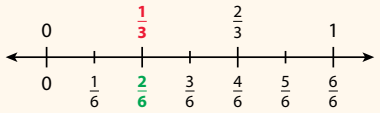
Maria colors $\frac{1}{3}$ of her art paper red. Erica colors $\frac{2}{6}$ of her art paper green. The papers are the same size. Do the two girls color the same amount of their art papers?



Maria colors $\frac{1}{3}$.



Erica colors $\frac{2}{6}$.

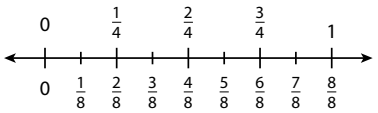


One third is equal to two sixths.

$\frac{1}{3} = \frac{2}{6}$

The girls color the same amount of their art papers.

Use the number line to complete the equivalent fractions in problems 1–3.



1. $\frac{1}{4} = \frac{2}{8}$

2. $\frac{6}{8} = \frac{3}{4}$

3. $\frac{2}{4} = \frac{4}{8}$ or $\frac{1}{2}$

Vocabulary

equivalent fractions
fractions that name the same point on a number line. $\frac{1}{2}$ and $\frac{2}{4}$ are equivalent.

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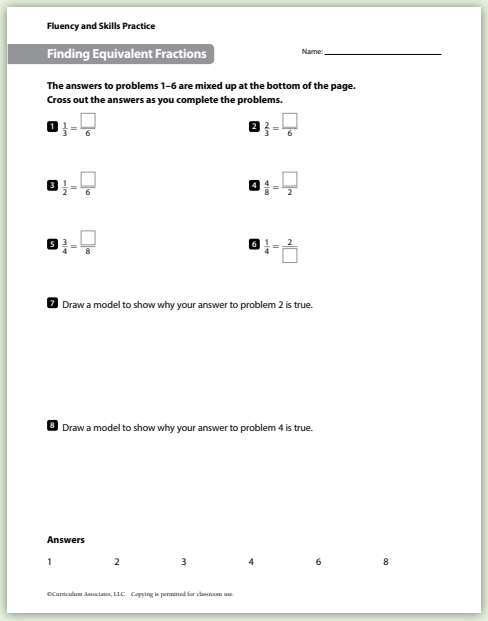
Fluency & Skills Practice

Teacher Toolbox

Assign *Finding Equivalent Fractions*

In this activity students practice finding equivalent fractions. Students may experience real-world situations that involve equivalent fractions. For example, students should understand that 1 slice of a loaf of banana bread that has been cut into 5 equal slices is the same amount as 2 slices of the same size loaf of banana bread that has been cut into 10 equal slices.



Fluency and Skills Practice

Finding Equivalent Fractions

Name: _____

The answers to problems 1–6 are mixed up at the bottom of the page. Cross out the answers as you complete the problems.

1. $\frac{1}{3} = \frac{\square}{6}$
2. $\frac{2}{3} = \frac{\square}{6}$
3. $\frac{1}{2} = \frac{\square}{6}$
4. $\frac{4}{8} = \frac{\square}{2}$
5. $\frac{3}{4} = \frac{\square}{8}$
6. $\frac{1}{4} = \frac{2}{\square}$
7. Draw a model to show why your answer to problem 2 is true.
8. Draw a model to show why your answer to problem 4 is true.

Answers

1 2 3 4 6 8

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4 See student page for shading; $\frac{1}{2} = \frac{4}{8}$
Medium

5 See student page for shading; $\frac{2}{3} = \frac{4}{6}$
Medium

6 See student page for models; $\frac{1}{2} = \frac{2}{4}$
Medium

7 See student page for models; $\frac{4}{6} = \frac{2}{3}$
Medium

8 Answers will vary. Possible answers include $\frac{2}{2}$; $\frac{3}{3}$; $\frac{6}{6}$; and $\frac{8}{8}$; Explanations show recognition that any fraction where the numerator and denominator are the same number equals 1 whole.
Challenge

Shade the models to show equivalent fractions in problems 4 and 5. Then fill in the blanks to write equivalent fractions.

4 

$\frac{1}{2} = \frac{4}{8}$

5 

$\frac{2}{3} = \frac{4}{6}$

Draw lines and shade to show equivalent fractions in problems 6 and 7. Then fill in the blanks to write equivalent fractions.

6 

$\frac{1}{2} = \frac{2}{4}$

7 

$\frac{4}{6} = \frac{2}{3}$

8 What is a fraction equivalent to $\frac{4}{4}$? Explain how you know.

Answers will vary. Possible answer: $\frac{2}{2}$; If I shaded $\frac{4}{4}$ of a shape, I would shade the whole shape. If I shaded $\frac{2}{2}$ of the same shape, I would also shade the whole shape. $\frac{4}{4}$ and $\frac{2}{2}$ are both equal to 1 whole, so $\frac{4}{4} = \frac{2}{2}$.

ELL English Language Learners: Differentiated Instruction

Prepare for Session 3
Use with *Apply It*.

Levels 1–3

Listening/Speaking Read ***Apply It*** problem 6. Display the graphic. Ask: *How many parts are in each triangle?* [2] *What unit fraction names each part?* [one half] Label $\frac{1}{2}$ and $\frac{2}{2}$ on the first triangle. Have students replicate. Point to, say, and have students repeat: *one half*, *two halves*. Point to the next triangle. Label $\frac{3}{2}$, $\frac{4}{2}$ and say: *three halves*, *four halves*. Have students replicate and repeat. Say: *Label the last triangle*. Have students chorally count the halves. Display and chorally complete:

- $\frac{2}{2}$ = *1 whole*
- $\frac{4}{2}$ = *2 wholes*
- $\frac{6}{2}$ = *3 wholes*

Levels 2–4

Listening/Speaking Read ***Apply It*** problem 6. Display the graphic. Ask: *How many parts are in each triangle?* [2] *What unit fraction names each part?* [one half] Label $\frac{1}{2}$ and $\frac{2}{2}$ on the first triangle. Have students replicate. Point to, say, and have students repeat: *one half*, *two halves*. Point to the next triangle. Label $\frac{3}{2}$ and say: *three halves*. Have students replicate and repeat with the fourth half. Say: *Finish labeling the halves*. Have students chorally count the halves. Display:

- $\frac{2}{2}$ = *1 whole*
- $\frac{4}{2}$ = *2 wholes*
- $\frac{6}{2}$ = *3 wholes*

Have pairs complete the fractions.

Levels 3–5

Listening/Speaking Have students read ***Apply It*** problem 6. Display the graphic. Ask: *How many parts are in each triangle?* [2] *What unit fraction names each part?* [one half] Label $\frac{1}{2}$ and $\frac{2}{2}$ on the first triangle. Have students replicate. Point to, say, and have students repeat: *one half*, *two halves*. Say: *Finish labeling the halves*. Have students chorally count the halves. Display:

- $\frac{2}{2}$ = *1 whole*
- $\frac{4}{2}$ = *2 wholes*
- $\frac{6}{2}$ = *3 wholes*

Have students form pairs and complete the fractions. Call on students to say each equation.

LESSON 23

SESSION 3 Develop

Purpose In this session students solve a problem that requires writing a fraction to represent a whole number. Students model the quantity with area models, fraction bars, or number lines to find the numerator and denominator of the fraction. The purpose of this session is to develop strategies for writing whole numbers as fractions.

Start

Connect to Prior Knowledge

Materials For each student: Activity Sheet *Number Lines*

Why Support students' facility with labeling a number line with fractions involving whole numbers.

How Have students label a number line with thirds from 0 to 2.

Write the missing fraction labels.

Solution

$\frac{1}{3}, \frac{2}{3}, \frac{3}{3}, \frac{4}{3}, \frac{5}{3}, \frac{6}{3}$

Develop Language

Why Clarify that *cut* and *divide* can be synonyms.

How Display the word *cut*. Say: *In the problem, Kacey cuts boards into fourths.* Have students find a synonym for *cut* in ***Picture It***. [divided] Clarify that the present tense of *divided* is *divide*. Reread the problem and substitute *divides* for *cuts*. Say: *Both* cut *and* divide *in this context mean* to separate. Provide the sentence frame: *Kacey _____ the boards into fourths.*

Have students practice the sentence using *cuts, divides* and *separates*.

TRY IT

Make Sense of the Problem

To support students in making sense of the problem, have them identify that there are 2 boards and each one is cut into fourths. They need to write 2 as a fraction in fourths.

Ask *How many whole boards did Kacey start with? How many parts did he cut each board into?*

Develop Writing a Whole Number as a Fraction

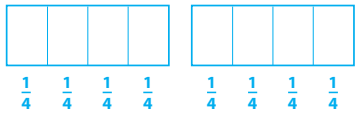
Read and try to solve the problem below.

Kacey uses 2 boards of the same size to build a birdhouse. He cuts each board into fourths. How can you write the number 2 as a fraction to find how many fourths Kacey cuts the boards into?

TRY IT

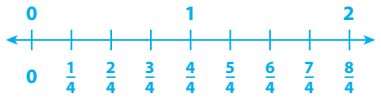
Possible student work:

Sample A



He cuts the boards into $\frac{8}{4}$.

Sample B



He cuts the boards into $\frac{8}{4}$.

Math Toolkit

- fraction tiles
- fraction circles
- fraction bars
- fraction models
- number lines
- grid paper

DISCUSS IT

Ask your partner: How did you get started?

Tell your partner: A model I used was . . . It helped me . . .

DISCUSS IT

Support Partner Discussion

Encourage students to use the term *fourths* as they discuss their solutions.

Support as needed with questions such as:

- *Why did you choose the model you did?*
- *How did you use your model to get the final answer?*

Common Misconception Look for students who do not understand that there is more than one whole being divided into fourths. Have students reread the problem and underline "2 boards of the same size" and "cuts each board into fourths."

Select and Sequence Student Solutions

One possible order for whole class discussion:

- fraction tiles or fraction circles
- drawings or area models showing fourths
- number lines showing fourths
- solutions without visual models

Support Whole Class Discussion

Compare and connect the different representations and have students identify how they are related.

Ask *How does each model show that Kacey had 2 boards? How does each model show that each board was cut into fourths?*

Listen for Area models should show two wholes, each divided into four equal parts. Number lines should go up to 2 and have tick marks at every fourth.

PICTURE IT & MODEL IT

If no student presented these models, connect them to the student models by pointing out the ways they each represent:

- two wholes
- each whole divided into fourths

Ask *How many wholes are there? How are they divided? Why are $\frac{4}{4}$ and $\frac{8}{4}$ shown in red on the number line?*

Listen for Each fraction bar represents one whole and each whole is divided into 4 equal parts, or fourths. The red labels on the number line are the whole numbers.

For the fraction bars, prompt students to identify how many wholes there are and how many parts in each.

- *How many boards did Kacey start with?*
- *How many parts did he cut each board into?*

For the number line, prompt students to describe how wholes and parts are represented.

- *How are the tick marks labeled?*
- *How could you tell which tick marks represented whole numbers if "1" and "2" were not labeled?*

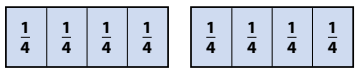
Explore different ways to understand writing a whole number as a fraction.

Kacey uses 2 boards of the same size to build a birdhouse. He cuts each board into fourths. How can you write the number 2 as a fraction to find how many fourths Kacey cuts the boards into?

PICTURE IT

You can use models to help you write a whole number as a fraction.

The fraction bars below show 2 wholes, each divided into fourths.



Each part is $\frac{1}{4}$ of a whole. There are eight $\frac{1}{4}$s in all.

MODEL IT

You can use a number line to help you write a whole number as a fraction.

This number line shows whole numbers on the top and fourths on the bottom.

Notice that each whole number has an equivalent fraction with a denominator of 4.

Deepen Understanding

Modeling Fractions Greater Than 1

SMP 6 Attend to precision.

When discussing the area model, prompt students to take note of how many fraction bars are used to model this one quantity.

Ask *Why is there more than 1 fraction bar to model just one number?*

Listen for The number modeled is 2, or 2 wholes, so there needs to be a fraction bar for each whole.

Ask *The problem states that Kacey cut the boards into fourths; however, there are more than 4 parts. Why?*

Listen for It is not the total amount that is divided into 4 parts, but each individual whole. There is more than one whole, so there are more than 4 parts.

Ask *Why is the denominator 4 instead of 8?*

Listen for The denominator is the number of parts that make up one whole, not all the wholes combined. Each whole is divided into 4 parts, so the denominator is 4.

CONNECT IT

- Remind students that one thing that is alike about all the representations is the numbers.
- Explain that on this page they will use those numbers to write a fraction equivalent to 2.

Monitor and Confirm

1 – 2 Check for understanding that:

- there are 4 parts in 1 whole
- there are 2 wholes
- there are 8 parts in all

Support Whole Class Discussion

3 Be sure students understand that the fraction represents the total number of parts (fourths) in both wholes.

Ask *Explain why each part of the two boards is labeled $\frac{1}{4}$. How did you find the numerator and denominator of the fraction you wrote in problem 3?*

Listen for Each part is $\frac{1}{4}$ because it takes 4 of them to make a whole. The numerator is the total number of parts you have (8) and the denominator is the number of parts it takes to make 1 whole (4).

4 Look for understanding that the steps for writing a whole number as a fraction are the same as the steps for writing other fractions: identify the number of parts in a whole (the denominator) and the number of parts you have (the numerator).

5 REFLECT Have all students focus on the strategies used to solve this problem. If time allows, have students share their preferences with a partner.

CONNECT IT

Now you will use the problem from the previous page to help you understand how to write a whole number as a fraction.

1 Look at the models in **Picture It**. How many equal parts are shown in 1 whole? Explain how you know. **Possible answer: There are 4 parts in 1 whole. Each fraction bar has 4 parts.**

2 How many equal parts are shown in 2 wholes? Explain how you know.

Possible answer: There are 8 parts in 2 wholes. I counted the number of parts in both fraction bars.

3 Complete the sentences to show the fraction that is equivalent to 2.

Use words: Two wholes equals **eight fourths**.

Use a fraction: 2 = **$\frac{8}{4}$**.

How many fourths does Kacey cut the boards into? **eight fourths**

4 Explain how to find a fraction equivalent to a whole number.

Possible answer: You look at how many parts are in one whole, and that is the denominator. Then you count how many parts there are in all of the wholes together, and that is the numerator.

5 REFLECT

Look back at your **Try It**, strategies by classmates, and **Picture It** and **Model It**. Which models or strategies do you like best for writing a whole number as a fraction? Explain.

Some students may prefer using a number line or an area model so that they can count the parts and visually check that they combine to make the given number of wholes. Other students may be fluent enough with their math facts to calculate the numerator by multiplying.

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Hands-On Activity

Use fraction circles to write whole numbers as fractions.

If . . . *students have trouble writing whole numbers as fractions,*

Then . . . *use this activity to let them build fractions with concrete materials.*

Materials For each pair: 4 sets of fraction circles, Activity Sheet *Multiplication Table*

- Have one student from each pair use one-fourth pieces to model 1 whole and have the other student write the fraction modeled. $\left[\frac{4}{4}\right]$ Tell them to switch roles and repeat the process for 2 wholes, 3 wholes, and 4 wholes. They should write the fractions in order. $\left[\frac{4}{4}, \frac{8}{4}, \frac{12}{4}, \frac{16}{4}\right]$
- Ask one partner to locate the column of the multiplication table under the number 4 and read the first 4 numbers in the column. [4, 8, 12, 16]
- Have the other partner read the numerators of the fractions they wrote, in order. [4, 8, 12, 16]
- Repeat the activity using another denominator, such as 2, 3, 6, or 8.

APPLY IT

For all problems, encourage students to draw some kind of model to support their thinking. Allow some leeway in precision; the number of the parts of a model and the accuracy of the labels are more important than whether the parts are exactly the same size.

6 $\frac{6}{2}$; 6 halves = 3 wholes

7 $\frac{16}{8}$; 16 eighths = 2 wholes

Close: Exit Ticket

8 See Student Worktext page; Area models should show 3 wholes each divided into 6 equal parts. Number lines should go from 0 to 3 and have a tick mark at every sixth.

Students' solutions should indicate understanding that:

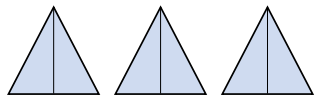
- there are 3 wholes
- each whole is divided into 6 equal parts
- there are 18 equal parts in all

Error Alert **If** students draw a model that shows 6 wholes divided into thirds, **then** ask them to point out the denominator and the whole number mentioned in the problem and discuss how each of those numbers should be used to create the model.

APPLY IT

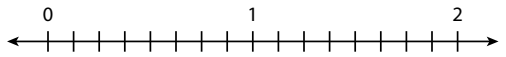
Use what you just learned to solve these problems.

6 Use the model below to write a fraction equivalent to 3.



Solution $\frac{6}{2}$

7 Louisa has 2 ribbons that are the same length. She cuts each one into eighths. Use the number line below to help you write the number 2 as a fraction to show how many eighths she cuts the ribbons into.



Solution $\frac{16}{8}$

8 Draw a model to show $3 = \frac{18}{6}$. Show your work.

Possible student model:

Solutions

1. $1 = \frac{3}{3}$
   Basic

2. $2 = \frac{6}{3}$
   Basic

3. $3 = \frac{9}{3}$
   Basic

4. $4 = \frac{12}{3}$
   Medium

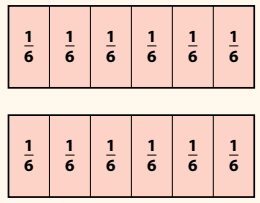
Name: _____ LESSON 23 SESSION 3

Practice Writing a Whole Number as a Fraction

Study the Example showing different ways to write whole numbers as fractions. Then solve problems 1–13.

EXAMPLE

Mrs. Clark cuts 2 same-sized pieces of colored paper into sixths to make strips for paper chains. How many strips does she make?



1 whole = six $\frac{1}{6}$s

$1 = \frac{6}{6}$

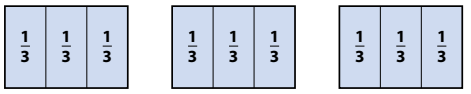
2 wholes = twelve $\frac{1}{6}$s

$2 = \frac{12}{6}$

Each strip is $\frac{1}{6}$ of a whole piece of paper.

Mrs. Clark makes 12 strips.

Write the whole numbers as fractions in problems 1–4.



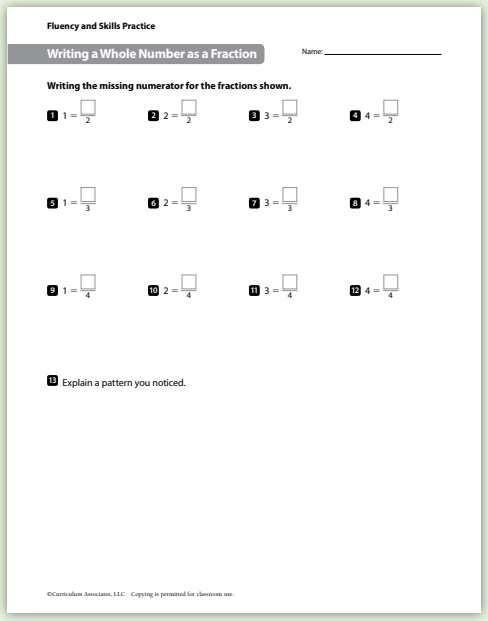
1. $1 = \frac{3}{3}$
2. $2 = \frac{6}{3}$
3. $3 = \frac{9}{3}$
4. $4 = \frac{12}{3}$

Fluency & Skills Practice

Teacher Toolbox

Assign *Writing a Whole Number as a Fraction*

In this activity students practice writing whole numbers as fractions. Students may apply this skill in real-world situations in which they are finding the total number of equal parts in more than one whole, such as finding the total number of slices of pie if two pies are each cut into eighths. This skill is useful in preparing students to learn how to write mixed numbers as fractions.


Fluency and Skills Practice

Writing a Whole Number as a Fraction Name: _____

Writing the missing numerator for the fractions shown.

1. $1 = \frac{\square}{2}$ 2. $2 = \frac{\square}{2}$ 3. $3 = \frac{\square}{2}$ 4. $4 = \frac{\square}{2}$

5. $1 = \frac{\square}{3}$ 6. $2 = \frac{\square}{3}$ 7. $3 = \frac{\square}{3}$ 8. $4 = \frac{\square}{3}$

9. $1 = \frac{\square}{4}$ 10. $2 = \frac{\square}{4}$ 11. $3 = \frac{\square}{4}$ 12. $4 = \frac{\square}{4}$

13. Explain a pattern you noticed.

5 $1 = \frac{4}{4}$
Basic

6 $2 = \frac{8}{4}$
Basic

7 $3 = \frac{12}{4}$
Basic

8 $0 = \frac{0}{4}$
Medium

9 8
Medium

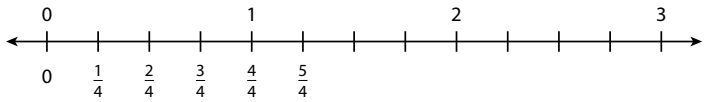
10 2
Medium

11 $3 = \frac{24}{8}$
Medium

12 $3 = \frac{12}{4}$
Medium

13 See student page. Area models and fraction bars should each show 2 wholes divided into 4 equal parts. Number lines should go from 0 to 2 with a tick mark at every fourth.
Challenge

Use this number line to solve problems 5–8.



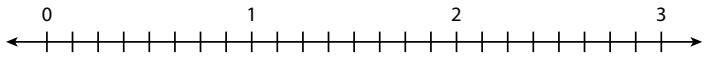
5 $1 = \frac{4}{4}$

6 $2 = \frac{8}{4}$

7 $3 = \frac{12}{4}$

8 $0 = \frac{0}{4}$

Use this number line to solve problems 9–11.



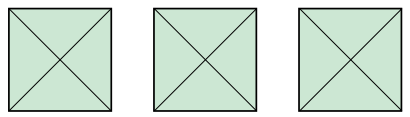
9 One whole is equal to 8 eighths.

10 16 eighths is equal to 2 wholes.

11 $3 = \frac{24}{8}$

12 Use the model below to write a fraction equivalent to 3.

$3 = \frac{12}{4}$



13 Draw a model to show $2 = \frac{8}{4}$.

Students' models will vary. They may draw number lines labeled with whole numbers and fourths. They may also draw 2 same-sized shapes divided into 4 equal pieces, with all 4 pieces in each shape shaded.

ELL English Language Learners: Differentiated Instruction

Prepare for Session 4
Use with *Apply It*.

Levels 1–3

Listening/Speaking Read ***Apply It*** problem 10 aloud. Arrange students in groups of four. Give each group a piece of yarn and four strips of paper folded in half and stapled one inch from the fold. Say: *You are going to make a number line. In your group, decide what whole numbers represent Oscar's loaves of bread.* When students share, display 0 through 3. Say: *Zero is the first whole number. Label one strip 0. The numerator for the first loaf is 1. The denominator is 1 because the loaf is whole.* Label a strip $\frac{1}{1}$. Slide the strips onto the yarn. Have students replicate. Say: *Use your strips. Discuss and say fractions to represent the other two whole loaves of bread. Make a number line with the yarn and paper strips.*

Levels 2–4

Listening/Speaking Read ***Apply It*** problem 10 aloud. Arrange students in groups of four. Say: *You are going to make a number line.* Give each group a piece of yarn and four strips of paper folded in half and stapled one inch from the fold. Ask: *What number will start your number line?* [0] *Label one strip 0.* Model how to slide the strips onto the yarn. Say: *In your group, decide what fraction represents the first loaf.* When students respond, write $\frac{1}{1}$ below the staple. Say: *Write a fraction on each strip of paper to represent each loaf. Make a number line with the yarn and paper strips.* When the number lines are complete, guide a discussion about equal placement of the strips along the piece of yarn.

Levels 3–5

Listening/Speaking Have students read ***Apply It*** problem 10. Arrange students in groups of four. Ask: *What model will you use to solve the problem?* [number line] Give each group a piece of yarn and four strips of paper folded in half and stapled one inch from the fold. Say: *What number will start your number line?* [0] *Label one strip 0.* Model how to slide the strips onto the yarn. Say: *Write a fraction on three strips of paper to represent each loaf of bread. Then make a number line with the yarn and paper strips.* When the number lines are complete, facilitate a discussion about equal placement of the strips along the piece of yarn.

LESSON 23

SESSION 4 Develop

Purpose In this session students solve a problem that requires writing a whole number as a fraction with a denominator of 1. Students model the quantity either on paper or with manipulatives. The purpose of this session is to have students develop strategies for writing whole numbers as fractions with a denominator of 1.

Start

Connect to Prior Knowledge

Materials For each student: 2 sets of fraction tiles

Why Reinforce the idea that there are different ways to write a whole number as a fraction.

How Have students model and write three different fractions that are equivalent to 2, given the denominators.

Use fraction tiles to model the whole number 2 with halves, thirds, and fourths. Then write the missing numerators.

$2 = \frac{\square}{2}$ $2 = \frac{\square}{3}$ $2 = \frac{\square}{4}$

Solutions

$\frac{4}{2}; \frac{6}{3}; \frac{8}{4}$

Look for tile models
4 one-halves,
6 one-thirds, and
8 one-fourths.

Develop Language

Why Clarify how to read a whole number represented as a fraction.

How Display $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{4}$. Say: *You have learned how to read fractions.* Have students read the fractions chorally. Display $\frac{2}{1}$, $\frac{3}{1}$, and $\frac{4}{1}$. Say: *Each of these fractions represents a whole number. These fractions are read as two over one, three over one, and four over one. The numerator is over the denominator when you write a fraction*. Point to and read each fraction. Have students repeat.

TRY IT

Make Sense of the Problem

To support students in making sense of the problem, have them identify a complete pepper as both the part and the whole.

LESSON 23 SESSION 4 ● ● ● ● ○

Develop Writing a Whole Number as a Fraction with a Denominator of 1

Read and try to solve the problem below.

Justin picks 4 green peppers from his garden. He does not cut them into pieces. How can you write the number of peppers Justin picks, 4, as a fraction?

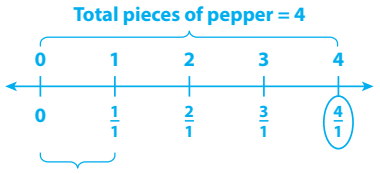
TRY IT

Possible student work:

Sample A

$\frac{\text{Pieces of pepper}}{\text{Pieces that make up a whole pepper}} = \frac{4}{1}$

Sample B



Math Toolkit

- fraction circles
- fraction tiles
- fraction bars
- fraction models
- number lines
- grid paper

DISCUSS IT

Ask your partner: Do you agree with me? Why or why not?

Tell your partner: I agree with you about . . . because . . .

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DISCUSS IT

Support Partner Discussion

Encourage students to use the term *whole* as they discuss their solutions.

Support as needed with questions such as:

- *How did you decide what the denominator should be?*
- *How did you decide what the numerator should be?*

Common Misconception Look for students who are confused by the idea that "1 part = 1 whole" or that think the pepper is divided into 0 parts since it is not divided at all. When referring to the denominator, have students use the phrase "number of parts that make up a whole" rather than "number of parts a whole is divided into." To reinforce the concept, have students use a 1-whole fraction tile to represent each pepper.

Select and Sequence Student Solutions

One possible order for whole class discussion:

- fraction tiles or fraction circles
- drawings or area models showing wholes
- number lines showing wholes

Support Whole Class Discussion

Compare and connect the different representations and have students identify how they are related.

Ask *How can you tell from each model that there are 4 peppers? How does each model show that the peppers are not cut up, but left whole?*

Listen for Drawings and fraction tiles or circles will show 4 wholes that are not divided into parts. A number line will show 0 through 4, with no tick marks between whole numbers.

PICTURE IT & MODEL IT

If no student presented these models, connect them to the student models by pointing out the ways they each represent:

- there are 4 wholes
- the wholes are not divided into parts

Ask *How many parts are there? How can you tell that one part makes a whole?*

Listen for The picture shows 4 circles and the number line goes up to 4. The circles are not divided into parts, so each circle = 1 part = 1 whole. There are no tick marks dividing the wholes on the number line, so each whole has only 1 part.

For an area model, prompt students to identify the number of wholes in the problem and that the wholes are not divided into parts.

- *How can you tell how many whole peppers Justin has?*
- *How can you tell that the peppers were left whole instead of cut into pieces?*

For a number line, prompt students to think about how far the number line goes up to and where the tick marks are.

- *How many wholes does the number line show?*
- *What quantity does the space between tick marks represent?*

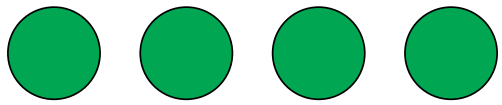
Explore different ways to understand writing a whole number as a fraction with a denominator of 1.

Justin picks 4 green peppers from his garden. He does not cut them into pieces. How can you write the number of peppers Justin picks, 4, as a fraction?

PICTURE IT

You can use models to help you write a whole number as a fraction with a denominator of 1.

Each circle stands for 1 green pepper.

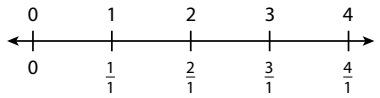


They are not divided into pieces, so each whole has one part.

MODEL IT

You can use a number line to help you write a whole number as a fraction with a denominator of 1.

This number line shows whole numbers on the top and fractions on the bottom.



Notice that each whole number has an equivalent fraction. The spaces between whole numbers are not divided into parts. Each whole number has one part, so the denominator of each equivalent fraction is 1.

Deepen Understanding

Wholes on a Number Line

SMP 8 Use repeated reasoning.

As you discuss the number-line model, prompt students to think about the space between each whole number as "a whole" just as in an area model.

Ask *What patterns do you see in the fraction labels for the whole numbers?*

Listen for The numerator is the same as the whole number.

Ask *What are the fraction labels at each whole number if the whole is divided into 2 parts?* $\left[\frac{2}{2}, \frac{4}{2}, \frac{6}{2}, \frac{8}{2}\right]$ *What patterns do you see in these labels?*

Listen for The numerator is two times the whole number.

Ask *What are the labels and patterns if the whole is divided into 3 parts?*

Listen for $\frac{3}{3}, \frac{6}{3}, \frac{9}{3}, \frac{12}{3}$; The numerator is three times the whole number.

Generalize Elicit a general pattern: When you divide the whole into a number of equal parts, the numerators of the whole numbers will be multiples of the number of equal parts the whole is divided into.

CONNECT IT

- Remind students that one thing that is alike about all the representations is the numbers.
- Explain that on this page they will use those numbers to develop a strategy for writing whole numbers as fractions with a denominator of 1.

Monitor and Confirm

1–4 Check for understanding that:

- there are 4 parts
- 1 part = 1 whole
- the numerator shows how many parts you have
- the denominator shows how many parts make a whole

Support Whole Class Discussion

5 Tell students that this problem will prepare them to provide the explanation required in problem 6.

Be sure students understand why there is only 1 part in the whole.

Ask *How many separate parts are there? How many parts make 1 whole?*

Listen for There are 4 parts. Each part is a whole pepper, so it only takes 1 part to make a whole.

6 Look for the understanding that the numerator is the whole number and the denominator is 1, because it only takes 1 part to make a whole.

7 REFLECT Have all students focus on the strategies used to solve this problem. If time allows, have students share their preferences with a partner.

CONNECT IT

Now you will use the problem from the previous page to help you understand how to write a whole number as a fraction with a denominator of 1.

1 Look at the models in **Picture It**. Explain how you know each whole has only 1 part.

The wholes are not divided into parts, so there is only 1 part.

2 How many parts do the 4 green peppers make? **4**

3 What does the numerator of a fraction show?

how many parts are being described

4 What does the denominator of a fraction show?

how many parts are in the whole

5 Write a fraction equivalent to 4. Use the fraction below to help you.

$\frac{\text{number of parts described}}{\text{number of equal parts in the whole}}$ **$\frac{4}{1}$**

6 Explain how to write a whole number as a fraction with a denominator of 1.

Possible answer: Write the whole number as the numerator and use 1 for the denominator. The whole number is the number of parts described. The denominator is the number of parts in the whole, which is 1.

7 REFLECT

Look back at your **Try It**, strategies by classmates, and **Picture It** and **Model It**. Which models or strategies do you like best for writing a whole number as a fraction with a denominator of 1? Explain.

Some students may prefer using diagrams or number lines if they are still working on the concept of having only one part in the whole. Others may be comfortable enough to simply write a fraction with the whole number as the numerator and 1 as the denominator without using a visual model.

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Visual Model

Generalize the writing of equivalent fractions for whole numbers.

If . . . *students struggle with writing fractions for undivided wholes,*

Then . . . *use this illustration to help them apply the same steps for all fractions.*

- Draw 3 same-sized circles on the board and divide each into 2 equal parts. Elicit that each part is called a *half*. Draw 3 more same-sized circles, but do not divide these. Explain that because they are not divided, each part is called a *whole*. Elicit that both models show the whole number 3.
- Together, write the fraction shown by the first model. $\left[\frac{6}{2}\right]$ Ask a volunteer to explain the process and record the steps on the board. [Count the number of equal parts in each whole to find the denominator of the fraction. Count the total number of equal parts to find the numerator.]
- Follow the same steps to write the fraction shown by the second model. $\left[\frac{3}{1}\right]$ Point to each numerator and denominator as you tell students, *6 halves equals 3 wholes.*
- Repeat the activity for $\frac{8}{4}$ and $\frac{2}{1}$.

APPLY IT

For all problems, encourage students to draw some kind of model to support their thinking. Allow some leeway in precision; the number of the parts of a model and the accuracy of the labels are more important than whether the parts are exactly the same size.

8 $\frac{6}{1}$; The wholes are not divided into more than one part, so the denominator is 1. There are 6 parts, or 6 wholes, so the numerator is 6.

9 See Student Worktext page; Area models or fraction bars should show 5 wholes that are not divided. Number lines should go from 0 to 5 and have tick marks only at whole numbers.

Close: Exit Ticket

10 $\frac{3}{1}$; The number line should go up to 3 and have tick marks only at whole numbers.

Students' solutions should indicate understanding that:

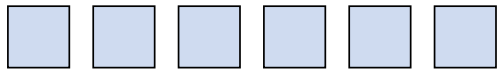
- there are 3 wholes
- the wholes are not divided into smaller parts
- 1 part = 1 whole

Error Alert **If** students write $\frac{3}{3}$, **then** review the definition of denominator. Use fraction tiles or area models to show $\frac{3}{1}$ and $\frac{3}{3}$ and discuss the difference between the two quantities.

APPLY IT

Use what you just learned to solve these problems.

8 Use the model below to write a fraction equivalent to 6.



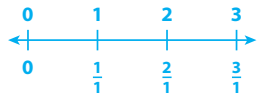
Solution $\frac{6}{1}$

9 Draw a model to show $\frac{5}{1} = 5$.

Possible student model:

10 Oscar has 3 loaves of bread that he has not sliced yet. Use a number line to write the pieces of bread Oscar has as a fraction. Show your work.

Possible student work:



Solution $\frac{3}{1}$

Solutions

1. 4
Basic

2. 2
Basic

3. 5
Basic

4. 8
Medium

5. $\frac{2}{1}$
Basic

6. $\frac{5}{1}$
Basic

7. $\frac{1}{1}$
Basic

8. $\frac{7}{1}$
Medium

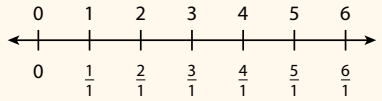
Name: _____ LESSON 23 SESSION 4

Practice Writing a Whole Number as a Fraction with a Denominator of 1

Study the Example showing how to write a whole number as a fraction with a denominator of 1. Then solve problems 1–14.

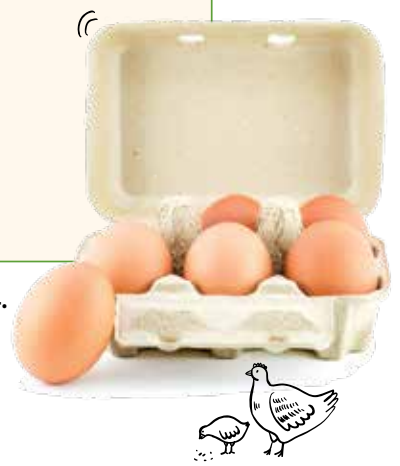
EXAMPLE

The spaces between whole numbers on this number line are not divided into smaller parts. So, each whole has only 1 part.



The number line shows that $\frac{3}{1}$ is equal to 3.

$\frac{3}{1}$ is a fraction name for 3.



Write the whole number for each fraction in problems 1–4.

1. $\frac{4}{1} =$ 4
2. $\frac{2}{1} =$ 2
3. $\frac{5}{1} =$ 5
4. $\frac{8}{1} =$ 8

Write a fraction with a denominator of 1 for each whole number in problems 5–8.

5. $2 =$ $\frac{2}{1}$
6. $5 =$ $\frac{5}{1}$
7. $1 =$ $\frac{1}{1}$
8. $7 =$ $\frac{7}{1}$

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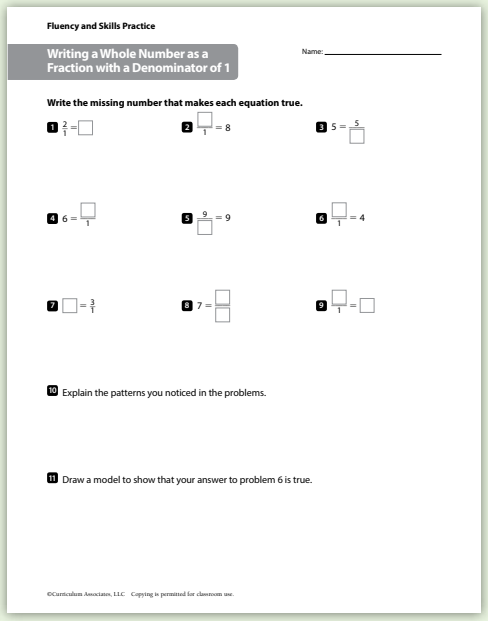
Fluency & Skills Practice

Teacher Toolbox

Assign *Writing a Whole Number as a Fraction with a Denominator of 1*

In this activity students practice writing whole numbers as fractions with a denominator of 1. When a certain number of wholes are each divided into 1 equal part, the total number of parts equals the number of wholes. Students may encounter this situation in real-world situations. For example, if a student has 3 whole apples, and each apple is divided into 1 equal part (or is still a whole apple), then there are 3 equal parts (the 3 whole apples).


Fluency and Skills Practice

Writing a Whole Number as a Fraction with a Denominator of 1

Name: _____

Write the missing number that makes each equation true.

1. $\frac{2}{1} = \square$
2. $\frac{\square}{1} = 8$
3. $5 = \frac{5}{\square}$
4. $6 = \frac{\square}{1}$
5. $\frac{9}{\square} = 9$
6. $\frac{\square}{1} = 4$
7. $\square = \frac{3}{1}$
8. $7 = \frac{\square}{\square}$
9. $\frac{\square}{1} = \square$
10. Explain the patterns you noticed in the problems.
11. Draw a model to show that your answer to problem 6 is true.

9 9
Medium

10 10
Medium

11 $\frac{12}{1}$
Medium

12 $\frac{18}{1}$
Medium

13 See student page; Use the whole number as the numerator and 1 as the denominator.
Challenge

14 See student page; Explanations should show understanding that equivalent fractions with different denominators would indicate that the wholes are divided differently but the amount would be the same.
Challenge

Write the whole number for each fraction in problems 9 and 10.

9 $\frac{9}{1} =$ 9

10 $\frac{10}{1} =$ 10

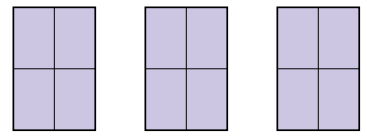
Write a fraction with a denominator of 1 for each whole number in problems 11 and 12.

11 $12 =$ $\frac{12}{1}$

12 $18 =$ $\frac{18}{1}$

13 Explain how to write a whole number as a fraction with a denominator of 1.
Possible answer: Write the whole number as the numerator of the fraction and use 1 as the denominator.

14 Bella says this model shows 3 wholes. She says it shows that if you write the whole number 3 as a fraction, you have to write $3 = \frac{12}{4}$.
How can you explain to Bella that there are other ways to write 3 as a fraction?



Possible answer: I would tell her there are different ways to write a whole number as a fraction. The denominator could be 1 instead of 4 if you don't divide the whole into pieces. So, you could write 3 as $\frac{3}{1}$. You could also divide each whole into different numbers of parts. If you divided each whole into 2 parts, you would get $3 = \frac{6}{2}$.

Vocabulary

numerator the number above the line in a fraction; it tells how many equal parts are being described.

denominator the number below the line in a fraction; it tells how many equal parts are in the whole.

ELL English Language Learners: Differentiated Instruction

Prepare for Session 5
Use with *Apply It*.

Levels 1–3

Listening/Speaking Read ***Apply It*** problem 2 and have students form pairs. Ask: *What whole number do you need to show as a fraction?* [5] Give each student a tile. Model how to trace the tile. Say: *Trace the tile five times in a row. Divide each square in half.* Point to the first square and say: *There are two halves in this square.* Display $\frac{2}{2}$. Say: *There are two halves in each square.* Label the halves. Say: *Let's count the halves.* Have students chorally count by ones or twos. Display and say: $\frac{10}{2}$. Have students trace five more tiles. Ask: *How many equal pieces do you want to make in each square?* Field students' suggestions and provide support as needed. Repeat the process.

Levels 2–4

Listening/Speaking Read ***Apply It*** problem 2. Pair students and give each pair a tile. Ask: *What whole number do you need to show as a fraction?* [5] Model and say: *Trace the tile five times in a row. Divide each square in half.* Point to each and ask: *How many halves are in this square?* [2] Label each square $\frac{2}{2}$. Ask: *How many halves are in all the squares?* [10] Display $\frac{10}{2}$. Say: *Trace five more squares. Divide the first square in fourths. What fraction represents the whole square?* $\left[\frac{4}{4}\right]$ Say: *Work with your partner. Divide the squares into fourths. Label each square. Write a fraction that shows how many fourths are in all five squares.* Call on pairs to say the fraction.

Levels 3–5

Listening/Speaking Have pairs read ***Apply It*** problem 2. Give each student a tile. Ask: *What whole number do you need to show as a fraction?* [5] Model and say: *Trace the tile five times in a row. Divide each square in half.* Point to each and ask: *How many halves are in this square?* [2] Label each square $\frac{2}{2}$. Ask: *How many total halves are in all the squares?* [10] Display: $\frac{10}{2}$. Say: *Trace five more squares. Work with your partner. Decide how you will divide the five squares equally. Label each square. Write a fraction to represent all of the squares.* Call on pairs to share their fractions and discuss their strategy.

LESSON 23

SESSION 5 Refine

Purpose In this session students solve word problems involving equivalent fractions, then discuss and confirm their answers with a partner.

Before students begin work, use their responses to the *Check for Understanding* to determine those who will benefit from additional support.

As students complete the Example and problems 1–3, observe and monitor their reasoning to identify groupings for differentiated instruction.

Start

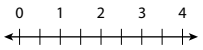
Check for Understanding

Materials For each student: Activity Sheet *Number Lines*

Why Confirm understanding of finding equivalent fractions.

How Have students find a fraction equivalent to $\frac{4}{1}$ given a number line that shows halves.

Use the number line to write a fraction that is equivalent to $\frac{4}{1}$.



Possible Solution

$\frac{8}{2}$

LESSON 23 SESSION 5 ● ● ● ● ●

Refine Finding Equivalent Fractions

Complete the Example below. Then solve problems 1–9.

EXAMPLE

Caleb and Hannah buy two melons that are the same size. Caleb cuts his melon into fourths. Hannah cuts her melon into eighths. Hannah eats $\frac{4}{8}$ of her melon. Caleb eats an equal amount of his melon. What fraction of his melon does Caleb eat?

Look at how you could show your work using a model.

Solution **Caleb eats $\frac{2}{4}$ of his melon.**

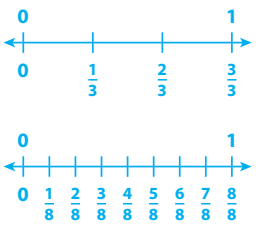
The student used solid lines to show fourths. She used dashed lines to show how to divide fourths to make eighths.

PAIR/SHARE
How could you solve this problem using a number line?

APPLY IT

1 Matt says $\frac{3}{3}$ is equivalent to 1. Elisa says $\frac{8}{8}$ is equivalent to 1. Who is correct? Show your work.

Possible student work using number lines:



How many thirds are in 1 whole? How many eighths are in 1 whole?

Solution **They are both correct.**

PAIR/SHARE
What is another fraction that is equivalent to 1?

Error Alert

If the error is . . .	*Students may . . .*	*To support understanding . . .*
students label the halves but do not clearly show that $\frac{4}{1}$ and $\frac{8}{2}$ are the same point on the number line and, therefore, are equivalent	not understand how to show that two fractions on a number line are equivalent or they may be confused as to what the fraction $\frac{4}{1}$ means.	Review with students the meaning of $\frac{4}{1}$. Be sure students understand that the denominator names the number of parts in one whole. Since the denominator is 1, there is 1 part. So, each part is one whole. The numerator tells how many parts, or wholes, there are. Ask students to write the fraction for the whole numbers on the top part of the number line $\left[\frac{1}{1}, \frac{2}{1}, \frac{3}{1}, \frac{4}{1}\right]$. Direct students to draw a point at $\frac{4}{1}$ and explain in words that $\frac{4}{1}$ and $\frac{8}{2}$ name that same point.
$\frac{8}{1}$, $\frac{8}{4}$, or $\frac{8}{8}$	not understand what the denominator of the fraction represents.	Review the meaning of *denominator*. Be sure students understand that the denominator names the number of parts in one whole. Have them identify one whole on the number line and count the number of parts it is divided into.
$\frac{2}{2}$, $\frac{4}{2}$, or $\frac{2}{8}$	not understand what the numerator of the fraction represents.	Review the meaning of *numerator*. Be sure students understand that the numerator tells how many parts there are.

EXAMPLE

Caleb eats $\frac{2}{4}$ of his melon; The model shown is one way to solve the problem. Students could also solve the problem by drawing a number line labeled with both eighths and fourths and finding that $\frac{2}{4}$ is located at the same point as $\frac{4}{8}$.

Look for If fourths and eighths are shown in the same model, then there should be some way to distinguish one from the other.

APPLY IT

1. They are both correct; Students could also solve the problem by drawing a whole divided into thirds and a whole divided into eighths and finding that both $\frac{3}{3}$ and $\frac{8}{8}$ are equal to 1 whole.
DOK 3

   Look for There are 3 thirds in 1 whole and 8 eighths in 1 whole.

2. Possible answers include $\frac{5}{1}$, $\frac{10}{2}$, $\frac{15}{3}$, $\frac{20}{4}$, $\frac{30}{6}$, and $\frac{40}{8}$; Students could also solve the problem by drawing number lines from 0 to 5 labeled with two different unit fractions.
DOK 3

   Look for Each model should show 5 wholes all divided into the same number of equal parts.

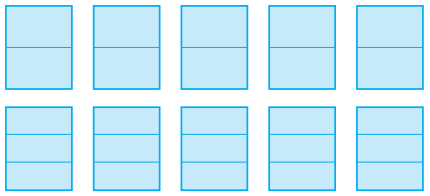
3. **D**; Students could solve the problem by identifying the fraction that shows the same amount on a fraction model, or is at the same location on a number line as $\frac{3}{6}$.

   Explain why the other two answer choices are not correct:

   B and **C** are not correct because neither fraction shows the same amount on a fraction model, nor is at the same location on a number line, as $\frac{3}{6}$.
DOK 3

2. Write two fractions that are equivalent to 5. Show your work.

Possible student work using models:



There will be 5 wholes in all. Think about how many parts will be in each whole.

Solution **Possible answers: $\frac{10}{2}$; $\frac{15}{3}$**

PAIR/SHARE
How did you decide what denominators to use in your fractions?

3. Kaia ate $\frac{3}{6}$ of a banana. Zoie ate an equivalent amount. Which fraction shows how much of a banana Zoie ate?

Ⓐ $\frac{1}{3}$

Ⓑ $\frac{2}{3}$

Ⓒ $\frac{5}{8}$

Ⓓ $\frac{1}{2}$

Find $\frac{3}{6}$ on a number line. What is another fraction that names the same location?

Landon chose Ⓐ as the correct answer. How did he get that answer?

Possible answer: He found a fraction that is equivalent to $\frac{2}{6}$, not $\frac{3}{6}$.

PAIR/SHARE
Does Landon's answer make sense?

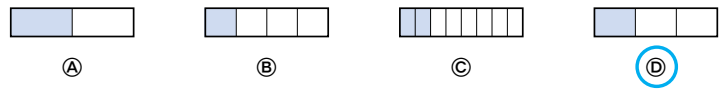
4 **D**; $\frac{1}{3} = \frac{2}{6}$
DOK 1

Error Alert Students may choose the model in choice C because there are 2 shaded parts and 6 unshaded parts, or they may choose the model in choice B because it shows a fraction equivalent to the fraction shown in choice C.

5 Possible answer: $\frac{2}{8}$; Model should show 1 whole divided into 4 equal parts with 1 part shaded, and then each of the 4 parts further divided into smaller equal parts.
DOK 3

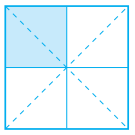
6 **A** (**Yes**);
D (**No**);
F (**No**)
DOK 3

4 Which model shows a fraction equivalent to $\frac{2}{6}$?



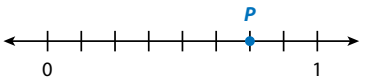
5 Draw a model to find a fraction equivalent to $\frac{1}{4}$. Show your work.

Possible student model:

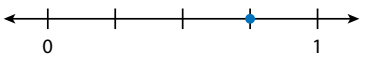
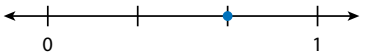
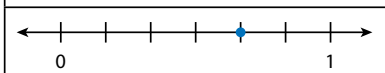


$\frac{1}{4}$ is equivalent to $\frac{2}{8}$.

6 Look at point *P* on the number line.



Does the point on each number line below represent a fraction equivalent to the fraction shown by point *P*?

	Yes	No
	Ⓐ	Ⓑ
	Ⓒ	Ⓓ
	Ⓔ	Ⓕ

Differentiated Instruction

RETEACH

Hands-On Activity
Use paper models to find equivalent fractions.

Students struggling with the concept of naming fractions greater than 1

Will benefit from additional work creating equivalent fractions.

Materials For each student: Activity Sheet *1-Inch Grid Paper*

- Have students draw three 2 inch-by-4 inch rectangles side by side so that the shorter sides lie along the same two grid lines. Identify each rectangle as a whole and have them write a fraction for the model. $\left[\frac{3}{1}\right]$
- Have students draw a line down the center of each rectangle and write a new fraction for the model. $\left[\frac{6}{2}\right]$ Then have students draw a line across the center of each rectangle and write a new fraction for the model. $\left[\frac{12}{4}\right]$
- Now have students draw a line along the remaining untraced gridlines in all 3 rectangles. Ask them to write another fraction for the model. $\left[\frac{24}{8}\right]$

EXTEND

Challenge Activity
Find equivalent unit fractions.

Students who have achieved proficiency

Will benefit from deepening understanding of finding equivalent fractions.

Materials For each student: Activity Sheet *1-Inch Grid Paper*

- Have students draw a rectangle that is 2 units long and 1, 2, 3, or 4 units wide, and shade one whole row or column of the rectangle. Have them write a fraction for the model, using the grid squares as equal parts.
- Have students divide the rectangle into equal parts so that the shaded region is exactly one part of the rectangle. Have them write a unit fraction for the shaded region.
- Repeat the activity with a different rectangle.

7 **A** (**Yes**);
D (**No**);
E (**Yes**);
G (**Yes**)
DOK 2

8 $\frac{9}{3}$; The number line should be labeled from $\frac{1}{3}$ to $\frac{9}{3}$ with $\frac{9}{3}$ at the same point as 3.
DOK 3

7 Does the point on each number line represent one whole?

	Yes	No
0, $\frac{1}{1}$, $\frac{2}{1}$, $\frac{3}{1}$, $\frac{4}{1}$	Ⓐ	Ⓑ
0, $\frac{1}{1}$, $\frac{2}{1}$, $\frac{3}{1}$, $\frac{4}{1}$	Ⓒ	Ⓓ
0, $\frac{1}{4}$, $\frac{2}{4}$, $\frac{3}{4}$, 1	Ⓔ	Ⓕ
0, $\frac{1}{4}$, $\frac{2}{4}$, $\frac{3}{4}$, $\frac{4}{4}$	Ⓖ	Ⓗ

8 Use the number line to find a fraction equivalent to 3. Show your work.

Possible student work:

3 is equivalent to $\frac{9}{3}$.

9 MATH JOURNAL

Write two fractions equivalent to 4 using the denominators 1 and 3. Use a number line to show how you found your answers.

Check that student number lines show fractions equivalent to 4 with denominators of 1 and 3.

$\frac{4}{1}$, $\frac{12}{3}$

SELF CHECK Go back to the Unit 4 Opener and see what you can check off.

REINFORCE

Problems 4–9

Find equivalent fractions.

All students will benefit from additional work with equivalent fractions by solving problems in a variety of formats.

- Have students work on their own or with a partner to solve the problems.
- Encourage students to show their work.

PERSONALIZE

i-Ready

Provide students with opportunities to work on their personalized instruction path with *i-Ready* Online Instruction to:

- fill prerequisite gaps
- build up grade-level skills

Close: Exit Ticket

9 MATH JOURNAL

Student responses should indicate understanding of writing a whole number as a fraction with a denominator of 1 by using the whole number as the numerator, and an understanding that equivalent fractions are located at the same point on a number line.

Error Alert **If** students write $\frac{4}{3}$, **then** review the definition of *numerator* with them. Check that their number lines show thirds and if so, have them count the number of thirds that correspond to the whole number 4.

SELF CHECK Have students consider whether they feel they are ready to check off any new skills on the Unit 4 Opener.

Lesson Overview

LESSON 25

Use Symbols to Compare Fractions

Lesson Objectives

Content Objectives

- Use symbols to record the results of comparing fractions with the same numerator or the same denominator.
- Read comparison statements fluently and accurately.
- Use models and number lines to explain and justify fraction comparisons.

Language Objectives

- Draw area models and number lines to justify fraction comparisons.
- Write comparison statements using the symbols $<$, $>$, and $=$ to compare fractions.
- Orally describe how to compare fractions to one another.

Prerequisite Skills

- Understand the meaning of fractions.
- Identify fractions represented by models and number lines.
- Understand that the size of a fractional part is relative to the size of the whole.
- Identify equivalent fractions and explain why they are equivalent.
- Understand how to compare two fractions by reasoning about the size of the unit fractions shown by the denominators and number of parts shown by the numerators of each fraction.

Standards for Mathematical Practice (SMP)

SMPs 1, 2, 3, 4, 5, and 6 are integrated in every lesson through the *Try-Discuss-Connect* routine.*

In addition, this lesson particularly emphasizes the following SMPs:

5 Use appropriate tools strategically.

7 Look for and make use of structure.

*See page 455i to see how every lesson includes these SMPs.

Lesson Vocabulary

There is no new vocabulary. Review the following key terms.

- **compare** to decide if numbers, amounts, or sizes are greater than, less than, or equal to each other.
- **denominator** the number below the line in a fraction that tells the number of equal parts in the whole.
- **greater than symbol (>)** a symbol used to compare two numbers when the first is greater than the second.
- **less than symbol (<)** a symbol used to compare two numbers when the first is less than the second.
- **numerator** the number above the line in a fraction that tells the number of equal parts that are being described.

Learning Progression

In previous lessons students developed an understanding of equivalent fractions and found equivalent fractions using models and number lines. Students also used models and number lines to explore the concept of comparing fractions that have the same numerator or the same denominator.

In this lesson students continue their work comparing two fractions that have the same numerator or the same denominator. Students now use symbols ($>$, $<$, or $=$) to record the comparisons. Students use models and number lines to explain the reasoning behind their comparisons.

In Grade 4 students will compare fractions with different numerators and different denominators by rewriting the fractions as equivalent fractions with the same numerator or the same denominator.

Lesson Pacing Guide

Whole Class Instruction

SESSION 1
Explore
45–60 min

Using Symbols to Compare Fractions
- Start *5 min*
- Try It *10 min*
- Discuss It *10 min*
- Connect It *15 min*
- Close: Exit Ticket *5 min*

Additional Practice
Lesson pages 537–538

SESSION 2
Develop
45–60 min

Comparing Fractions Using Symbols
- Start *5 min*
- Try It *10 min*
- Discuss It *10 min*
- Picture It & Model It *5 min*
- Connect It *10 min*
- Close: Exit Ticket *5 min*

Additional Practice
Lesson pages 543–544

Fluency
Comparing Fractions Using Symbols

SESSION 3
Refine
45–60 min

Using Symbols to Compare Fractions
- Start *5 min*
- Example & Problems 1–3 *15 min*
- Practice & Small Group Differentiation *20 min*
- Close: Exit Ticket *5 min*

Lesson Quiz or **Digital Comprehension Check**

Lesson Materials

Lesson *(Required)*	*Per student:* 12 counters *Activity Sheet:* 1-Centimeter Grid Paper
Activities	*Per pair:* 34 index cards *Per group:* 6 index cards
Math Toolkit	fraction tiles, fraction bars, fraction circles, grid paper, sticky notes
Digital Math Tools	Fraction Models, Number Line

Teacher Toolbox

Small Group Differentiation

PREPARE

Ready Prerequisite Lesson

Grade 2
- Lesson 29 Understand Partitioning Shapes into Halves, Thirds, and Fourths

RETEACH

Tools for Instruction

Grade 2
- Lesson 30 Make Equal Shares

Grade 3
- Lesson 25 Use Symbols to Compare Fractions

REINFORCE

Math Center Activity

Grade 3
- Lesson 25 Comparing Fractions

EXTEND

Enrichment Activity

Grade 3
- Lesson 25 What Fractions Go in the Box?

i-Ready

Independent Learning

PERSONALIZE

i-Ready Lesson*

Grade 3
- Understand Comparing Fractions

Learning Game
- *Bounce*

*We continually update the Interactive Tutorials. Check the Teacher Toolbox for the most up-to-date offerings for this lesson.

Connect to Family, Community, and Language Development

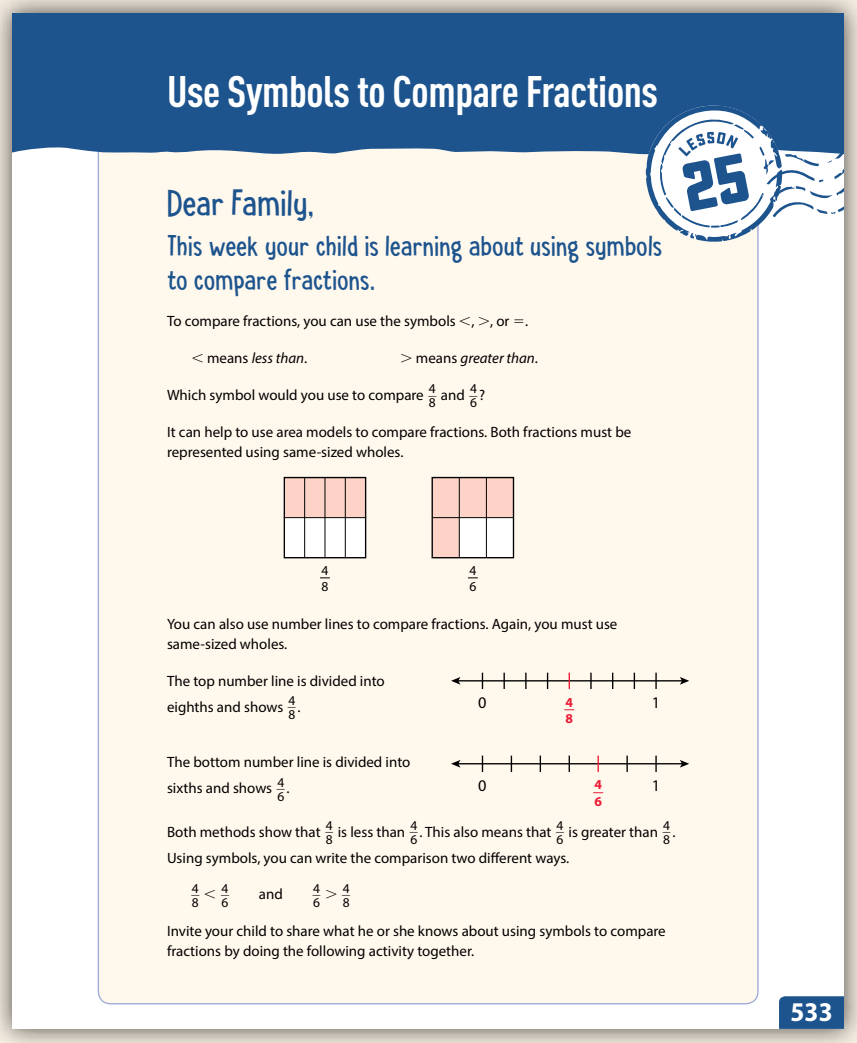
The following activities and instructional supports provide opportunities to foster school, family, and community involvement and partnerships.

Connect to Family

Use the **Family Letter**—which provides background information, math vocabulary, and an activity—to keep families apprised of what their child is learning and to encourage family involvement.

Available in Spanish

Teacher Toolbox



Use Symbols to Compare Fractions

LESSON 25

Dear Family,

This week your child is learning about using symbols to compare fractions.

To compare fractions, you can use the symbols $<$, $>$, or $=$.

$<$ means *less than*. $>$ means *greater than*.

Which symbol would you use to compare $\frac{4}{8}$ and $\frac{4}{6}$?

It can help to use area models to compare fractions. Both fractions must be represented using same-sized wholes.

$\frac{4}{8}$ $\frac{4}{6}$

You can also use number lines to compare fractions. Again, you must use same-sized wholes.

The top number line is divided into eighths and shows $\frac{4}{8}$.

The bottom number line is divided into sixths and shows $\frac{4}{6}$.

Both methods show that $\frac{4}{8}$ is less than $\frac{4}{6}$. This also means that $\frac{4}{6}$ is greater than $\frac{4}{8}$. Using symbols, you can write the comparison two different ways.

$\frac{4}{8} < \frac{4}{6}$ and $\frac{4}{6} > \frac{4}{8}$

Invite your child to share what he or she knows about using symbols to compare fractions by doing the following activity together.

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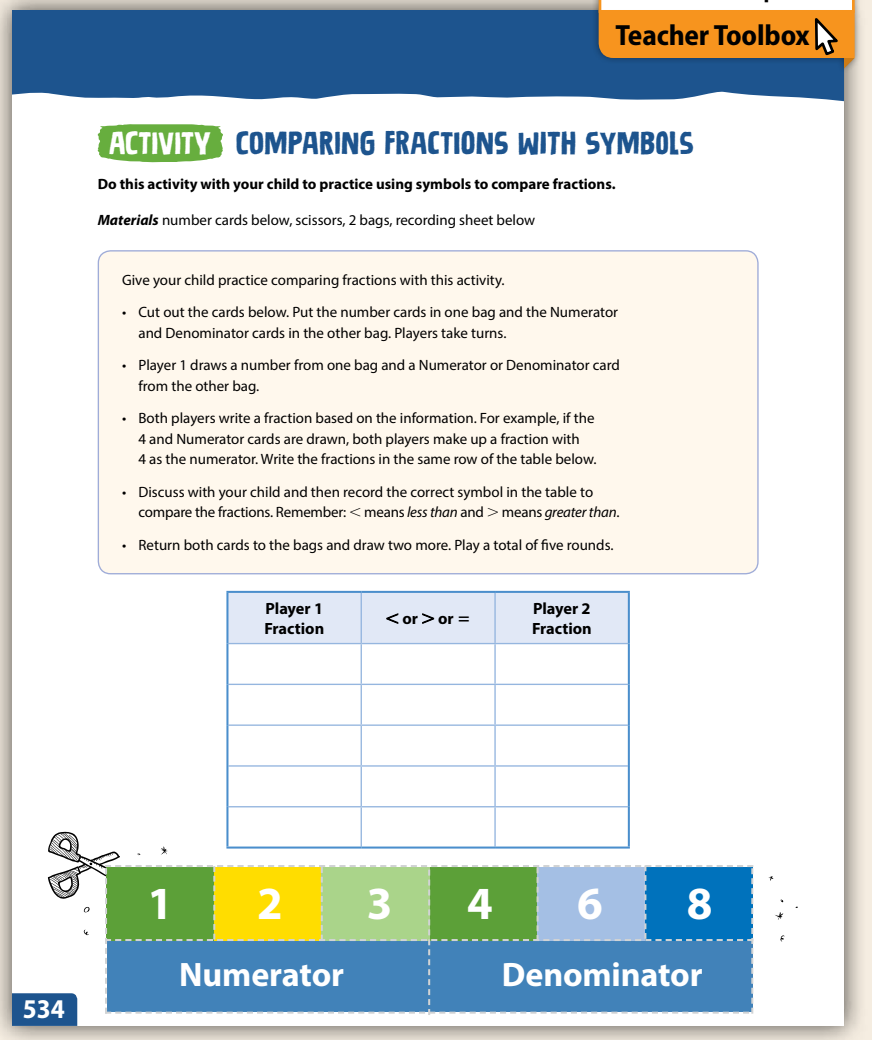

ACTIVITY COMPARING FRACTIONS WITH SYMBOLS

Do this activity with your child to practice using symbols to compare fractions.

Materials number cards below, scissors, 2 bags, recording sheet below

Give your child practice comparing fractions with this activity.

- Cut out the cards below. Put the number cards in one bag and the Numerator and Denominator cards in the other bag. Players take turns.
- Player 1 draws a number from one bag and a Numerator or Denominator card from the other bag.
- Both players write a fraction based on the information. For example, if the 4 and Numerator cards are drawn, both players make up a fraction with 4 as the numerator. Write the fractions in the same row of the table below.
- Discuss with your child and then record the correct symbol in the table to compare the fractions. Remember: $<$ means *less than* and $>$ means *greater than*.
- Return both cards to the bags and draw two more. Play a total of five rounds.

Player 1 Fraction	< or > or =	Player 2 Fraction

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Goal

The goal of the Family Letter is to compare fractions using symbols. Fractions are represented with area models and number lines. Comparisons can be written three ways.

- When the lesser fraction is written first, the less than symbol ($<$) is used. When the greater fraction is written first, the greater than symbol ($>$) is used. When the fractions are equal, $=$ is used.

Activity

Look at the *Comparing Fractions with Symbols* activity and adjust it if necessary to connect with your students. Students can cut out the playing pieces prior to taking the Family Letter home. Place the pieces in an envelope or plastic snack bag and staple the envelope or bag to the letter.

Share that as they play each round, students can draw models to compare the two fractions and help determine the correct symbol.

Math Talk at Home

Encourage students to talk with their family members about fractions. Have students think about professions or jobs that may use fractions on a regular basis.

Conversation Starters Below are additional conversation starters students can write in their Family Letter or math journal to engage family members:

- *What jobs require workers to use fractions? How are they used?*
- *How do you remember the meaning of the symbols $<$ and $>$?*

Connect to Community and Cultural Responsiveness

Use these activities to connect with and leverage the diverse backgrounds and experiences of all students.

Sessions 1–3 **Use anytime during these sessions.**

- Display the symbols ☑ and ⦸. Ask: *What do you think of when you see these symbols?* [yes/done; no/don't] *What other symbols have similar meanings?* [Answers will vary.] Say: *People use symbols to communicate. Meanings can sometimes be understood more quickly with symbols than with words.* Explain that many different symbols are used in the world. Invite students to share symbols they know and identify their meanings.
- Say: *Mathematics uses many different symbols to communicate ideas.* Display $+$, $-$, $\times$, $\div$, and $=$. Have students identify each symbol and its meaning. Display the symbols $<$ and $>$. Ask: *What are these symbols?* [less than and greater than] Say: *Talk with a partner about how you can remember what each symbol means.* Select students to share their ideas. Display in a table the different strategies students use to remember the symbols.
- Remind students that the open or larger end of the symbol is always next to the greater value. When the open or larger end of the symbol appears first, the symbol means *greater than*. When the pointed or smaller end of the symbol appears first, the symbol means *less than*.

Connect to Language Development

For ELLs, use the Differentiated Instruction chart to plan and prepare for specific activities in every session.

ELL English Language Learners: Differentiated Instruction

Prepare for Session 1
Use with *Try It*.

Levels 1–3

Listening/Speaking Read ***Try It*** aloud. Draw and label two tall, same-sized rectangles. Divide them in sixths. Have students replicate. Ask: *How many parts does each glass have?* [six] *Shade the rectangles to represent* $\frac{4}{6}$ *and* $\frac{5}{6}$. *Does the first glass have more or less juice than the second glass?* [less] Display, read, and have students chorally repeat:

- $\frac{4}{6}$ *is less than* $\frac{5}{6}$.

Levels 2–4

Listening/Speaking Read ***Try It*** aloud. Draw and label two tall, equal-sized rectangles for students to replicate. Ask: *How is each glass divided?* [in sixths] *Divide and shade the figures to represent* $\frac{4}{6}$ *and* $\frac{5}{6}$. *Is* $\frac{4}{6}$ *more or less than* $\frac{5}{6}$*?* Display:

- $\frac{4}{6}$ *is* less than $\frac{5}{6}$.

Say: *Decide with your partner which comparison is correct.* Come to a consensus.

Levels 3–5

Listening/Speaking Have pairs read ***Try It***. Draw and label two tall, equal-sized rectangles and have students replicate. Say: *Divide and shade the figures to represent* $\frac{4}{6}$ *and* $\frac{5}{6}$. Display:

- $\frac{4}{6}$ *is* less than $\frac{5}{6}$.
- $\frac{4}{6}$ < $\frac{5}{6}$.

Say: *Compare the fractions with words and with a symbol.* Select students to share their answers.

LESSON 25

SESSION 1 Explore

Purpose In this session students draw on their knowledge of comparison symbols to compare fractions. They reason about fraction size to explore how to compare two fractions with the same denominator. They will look ahead to think about using comparison symbols to write comparison statements about fractions with the same numerator.

Start

Connect to Prior Knowledge

Materials For each student: 12 counters

Why Support students' facility with using comparison symbols.

How Have students use comparison symbols to compare pairs of whole numbers.

Use counters to compare each pair of numbers using $<$, $>$, or $=$.

7 5

3 3

4 8

Solutions

$>$;

$=$;

$<$

TRY IT

Make Sense of the Problem

To support students in making sense of the problem, have them show that they understand that the problem is asking them to tell who has more juice by comparing two fractions using $<$, $>$, or $=$.

DISCUSS IT

Support Partner Discussion

To reinforce comparing fractions with the same denominator, encourage students to use the terms *numerator* and *denominator* as they talk to each other.

Look for, and prompt as necessary for, understanding of:

- the wholes are the same size
- the denominators are the same, so compare numerators
- $<$ means *less than* and $>$ means *greater than*

Explore Using Symbols to Compare Fractions

Previously you learned how to compare fractions. In this lesson you will use the symbols $<$, $>$, and $=$ to show how fractions compare. Use what you know to try to solve the problem below.

Erica and Ethan have same-sized glasses. Erica's glass is $\frac{4}{6}$ full of juice. Ethan's glass is $\frac{5}{6}$ full of juice. Compare $\frac{4}{6}$ and $\frac{5}{6}$ using $<$, $>$, or $=$. Who has more juice?

Learning Target

- Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual fraction model.

SMP 1, 2, 3, 4, 5, 6, 7

TRY IT

Possible student work:

Sample A

Erica's glass $\frac{4}{6}$ $<$ Ethan's glass $\frac{5}{6}$

Ethan has more juice.

Sample B

It takes four $\frac{1}{6}$s to make $\frac{4}{6}$. It takes five $\frac{1}{6}$s to make $\frac{5}{6}$.

$\frac{4}{6} < \frac{5}{6}$, so Ethan has more juice.

Math Toolkit

- fraction tiles
- fraction bars
- fraction models
- number lines
- grid paper
- sticky notes

DISCUSS IT

Ask your partner: How did you get started?

Tell your partner: I started by . . .

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Common Misconception Look for students who confuse how to compare fractions with the same denominator with how to compare fractions with the same numerator. As students present solutions, have them specify what model or reasoning they used to determine which fraction was greater.

Select and Sequence Student Solutions

One possible order for whole class discussion:

- using fraction tiles or fraction bars
- drawing area models
- drawing one or two number lines
- reasoning about unit fractions

Support Whole Class Discussion

Prompt students to note the relationship between the numbers in each model and the numbers in the problem.

Ask *How do [student name]'s and [student name]'s models show the amount in Erica's glass and Ethan's glass?*

Listen for The model is divided into 6 equal parts for each glass. The model for Erica has 4 parts shaded or marked and the model for Ethan has 5 parts shaded or marked.

CONNECT IT

1 LOOK BACK

Look for understanding that $\frac{5}{6}$ is greater than $\frac{4}{6}$ because five $\frac{1}{6}$ parts is more than four $\frac{1}{6}$ parts.

Hands-On Activity

Act out fraction comparisons.

If . . . *students are unsure about the concept of using comparison symbols with fractions,*

Then . . . *use this activity to give concrete experience with the symbols.*

Materials For each group of 4: 6 index cards

- Have students write one of each of these fractions on an index card: $\frac{1}{6}, \frac{2}{6}, \frac{3}{6}, \frac{4}{6}, \frac{5}{6}, \frac{6}{6}$.
- Shuffle the index cards and place them facedown. Have three students stand side-by-side. The two outer students each pick and hold up a fraction card. The middle student compares the fractions and imitates $<$ or $>$ by opening his or her arms toward the greater fraction.
- The fourth student writes down the fraction statement made by the others, states whether it is true or not, and explains why.
- Shuffle the cards and repeat the activity with two new fractions and a new arrangement of students. Have students swap roles so each has at least one chance to be a fraction, the symbol, or the judge.
- Repeat with the fractions $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}$ if time allows.

2 LOOK AHEAD

Remind students that the pointed comparison symbols open toward the greater fraction and point to the lesser fraction.

Students should be able to accurately state the comparison both in words and using symbols.

Ask *When might you decide to use an equal sign when comparing two different-looking fractions?*

Listen for Equivalent fractions can look different and still show the same amount, and so they are equal to each other.

CONNECT IT

1 LOOK BACK

Who has more juice? How did you compare $\frac{4}{6}$ and $\frac{5}{6}$ to find out?

Ethan; Possible answer: There are more $\frac{1}{6}$s in $\frac{5}{6}$ than in $\frac{4}{6}$, so $\frac{4}{6} < \frac{5}{6}$.

2 LOOK AHEAD

You can use the symbols $<$, $>$, or $=$ to compare fractions just as you did to compare whole numbers. Remember that the symbol opens to the greater fraction and points to the lesser fraction.

greater fraction $>$ lesser fraction and lesser fraction $<$ greater fraction

You can use words or a symbol to compare fractions.

$<$ means *less than.* $>$ means *greater than.* $=$ means *equal to.*

a. Use words and a symbol to compare $\frac{1}{2}$ and $\frac{1}{3}$.

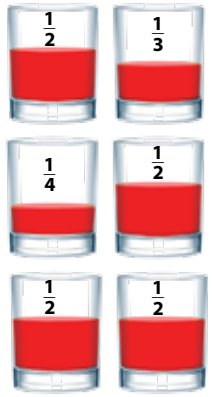
$\frac{1}{2}$ is **greater than** $\frac{1}{3}$. (words) $\frac{1}{2} > \frac{1}{3}$ (symbol)

b. Use words and a symbol to compare $\frac{1}{4}$ and $\frac{1}{2}$.

$\frac{1}{4}$ is **less than** $\frac{1}{2}$. (words) $\frac{1}{4} < \frac{1}{2}$ (symbol)

c. Use words and a symbol to compare $\frac{1}{2}$ and $\frac{1}{2}$.

$\frac{1}{2}$ is **equal to** $\frac{1}{2}$. (words) $\frac{1}{2} = \frac{1}{2}$ (symbol)



3 REFLECT

What helps you remember what the symbols $>$ and $<$ mean when comparing two numbers?

Answers will vary. Possible answer: I think $<$ looks a little like an L, and this helps me remember that it means "less than."

Close: Exit Ticket

3 REFLECT

Look for understanding of the meaning of each comparison symbol.

Common Misconception **If** students still confuse the meaning of the comparison symbols, **then** list on the board the different strategies students describe for knowing how to use the symbols $<$ and $>$. Have volunteers use the list to make a poster. For example, another strategy is connecting the *larger* open end of the symbol with *greater than*, and the *smaller* pointed end of the symbol with *less than*. Remind them to always read a comparison statement from left to right. So if they see the open end of the symbol first when reading a comparison, they know to say *greater than*, but if they see the point of the symbol first, they know to say *less than*.

Real-World Connection

Encourage students to think about everyday places or situations where people might need to compare two fractions. Have volunteers share ideas. Examples: A bowl holds $\frac{1}{2}$ cup of milk. You need $\frac{1}{4}$ cup of milk to make pancakes. Is the amount of milk in the bowl greater than or less than the amount you need? Carol ran $\frac{3}{5}$ of a mile on Monday and $\frac{2}{5}$ of a mile on Tuesday. On which day did she run a greater distance?

Solutions

Support Vocabulary Development

1 Ask students to read the first column of the table. Say: *Compare the words and symbols in the first column. What do you notice about them?* [The words in the first three rows have the same meaning as the symbols in the last three rows.] *All of the words and symbols in the first column can be used to make comparisons between two numbers.* To help students complete the first row, guide them to look for other places the words *greater than* appear in the lesson. Invite students to share their findings with the class. Say: *Remember that* greater than *and* less than *compare two number that are different, while* equal to *compares two numbers that are the same.*

2 In pairs, have students compare the fractions $\frac{1}{4}$ and $\frac{1}{3}$. Explain that the words and symbol used to complete the comparisons can be found in the table. After completing the problem, have students practice reading the word form and telling the meaning of the symbol.

Supplemental Math Vocabulary

- *compare*
- *denominator*
- *numerator*

Name: _____ LESSON 25 SESSION 1

Prepare for Using Symbols to Compare Fractions

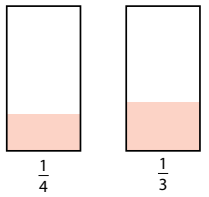
1 Think about what you know about fractions. Fill in each box. Use words, numbers, and pictures. Show as many ideas as you can. **Possible answers:**

Word	In My Own Words	Example
greater than	A number is greater than another number if it has a greater value than the other number.	$\frac{1}{2}$ is greater than $\frac{1}{6}$.
less than	A number is less than another number if it has a lesser value than the other number.	$\frac{1}{4}$ is less than $\frac{1}{2}$.
equal to	A number is equal to another number if the numbers have the same value.	$\frac{1}{3}$ is equal to $\frac{1}{3}$.
$>$	a symbol used to show that one number is greater than another number	$\frac{1}{2} > \frac{1}{6}$
$<$	a symbol used to show that one number is less than another number	$\frac{1}{4} < \frac{1}{2}$
$=$	a symbol used to show that one number is equal to another number	$\frac{1}{3} = \frac{1}{3}$

2 Use words and a symbol to compare $\frac{1}{4}$ and $\frac{1}{3}$.

$\frac{1}{4}$ is less than $\frac{1}{3}$.
words

$\frac{1}{4} < \frac{1}{3}$
symbol



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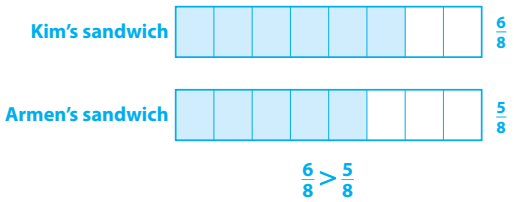
3 Solve the problem. Show your work.

Kim and Armen each buy same-sized sandwiches. Kim ate $\frac{6}{8}$ of her sandwich. Armen ate $\frac{5}{8}$ of his sandwich. Compare $\frac{6}{8}$ and $\frac{5}{8}$ using <, >, or =. Who ate more?

Possible student work using pictures:



$\frac{6}{8} > \frac{5}{8}$

Solution **Kim ate more.**

4 Check your answer. Show your work.

Possible student work:

It takes six $\frac{1}{8}$s to make $\frac{6}{8}$. It takes five $\frac{1}{8}$s to make $\frac{5}{8}$.

The parts are the same size. So, 6 parts is more than 5.

$\frac{6}{8} > \frac{5}{8}$, so Kim ate more.

3 Assign problem 3 to provide another look at using symbols to compare fractions.

This problem is very similar to the problem about comparing the amount of juice Erica and Ethan have. In both problems, students are given a word problem where they must compare two fractions using <, >, or =. The question asks students to compare $\frac{6}{8}$ and $\frac{5}{8}$ to find out whether Kim or Armen ate more sandwich.

Students may want to use fraction bars, fraction circles, or draw diagrams with pencil and paper.

Suggest that students read the problem three times, asking themselves one of the following questions each time:

- *What is this problem about?*
- *What is the question I am trying to answer?*
- *What information is important?*

Solution: $\frac{6}{8} > \frac{5}{8}$. Kim ate more.
Medium

4 Have students solve the problem a different way to check their answer.

ELL English Language Learners: Differentiated Instruction

Prepare for Session 2
Use with *Apply It*.

Levels 1–3

Listening/Speaking Read ***Apply It*** problem 9. Display $\frac{5}{8}$ and $\frac{5}{6}$. Say: *The denominator tells how many parts one whole in a number line should have.* Draw a number line from 0 to 1 for each fraction. Have students replicate. Ask: *What is the denominator of $\frac{5}{8}$? How many parts does the number line have between 0 and 1? Where does $\frac{5}{8}$ fall?* Repeat for $\frac{5}{6}$. Say: *Manny has read $\frac{5}{8}$ of the book and Sarah has read $\frac{5}{6}$. Who has read more?* Display, read, and have students chorally repeat: *$\frac{5}{8} < \frac{5}{6}$. $\frac{5}{6}$ is the greater fraction. Sarah has read more than Manny.*

Levels 2–4

Speaking/Writing Read ***Apply It*** problem 9. Display $\frac{5}{8}$ and $\frac{5}{6}$. Ask: *Do numerators or denominators tell how many parts one whole in a number line should have?* Draw two same-sized number lines labeled 0 to 1. Divide them and label $\frac{5}{8}$ and $\frac{5}{6}$. Have students replicate. Say: *Compare the fractions.* Check students' answers. Say: *Decide with your partner who has read more of the book and why.* Display, read, and have students complete: *I know* ___Sarah___ *has read more of the book because $\frac{5}{6}$ is* ___greater___ *than $\frac{5}{8}$.*

Levels 3–5

Speaking/Writing Have students read ***Apply It*** problem 9. Display $\frac{5}{8}$ and $\frac{5}{6}$. Ask: *What tells how many parts one whole in a number line should have?* [the denominator of the fraction] Model how to draw two same-sized number lines labeled 0 to 1. Divide one into eighths and the other into sixths. Label $\frac{5}{8}$ and $\frac{5}{6}$. Have students replicate. Say: *Compare the fractions.* Check students' answers. Say: *Write a sentence that explains how you know whether Manny or Sarah has read more of the book.* Select students to read their sentence aloud.

LESSON 25

SESSION 2 Develop

Purpose In this session students solve a problem in which they use comparison symbols to compare fractions with the same numerator. Students model the fractions, either on paper or with manipulatives. The purpose of this session is to develop and strengthen strategies for using symbols to compare fractions with the same numerator or the same denominator.

Start

Connect to Prior Knowledge

Materials For each student: Activity Sheet *1-Centimeter Grid Paper*

Why Support students' facility with comparing fractions with the same numerator.

How Have students draw a model or number line to tell which is greater, $\frac{3}{4}$ or $\frac{3}{5}$.

Draw a model or number line to help you compare $\frac{3}{4}$ and $\frac{3}{5}$.

Which fraction is greater?

Solution

$\frac{3}{4}$

Look for accurate models.

Develop Language

Why Clarify the meaning of the homophones *whole* and *hole*.

How Display the words *whole* and *hole*. Say: *These words sound the same but have different meanings.* Read *Picture It* on the next page. Say: *Circle the word* whole. *A whole is something full or complete. These two models are complete. They are wholes. The full models are the same size.* Have students complete: *The models are same-sized* ___*wholes*___. Contrast the meaning of *hole*. Guide students to draw models and identify the parts and wholes.

TRY IT

Make Sense of the Problem

To support students in making sense of the problem, have them identify that the problem is asking them to compare two fractions using symbols.

Ask *What does the $<$ symbol indicate? the $>$ symbol? the $=$ symbol?*

LESSON 25 SESSION 2 ● ● ○

Develop Comparing Fractions Using Symbols

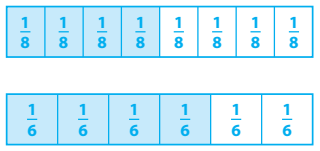
Read and try to solve the problem below.

Compare $\frac{4}{8}$ and $\frac{4}{6}$ using $<$, $>$, or $=$.

TRY IT

Possible student work:

Sample A



$\frac{4}{8} < \frac{4}{6}$

Sample B

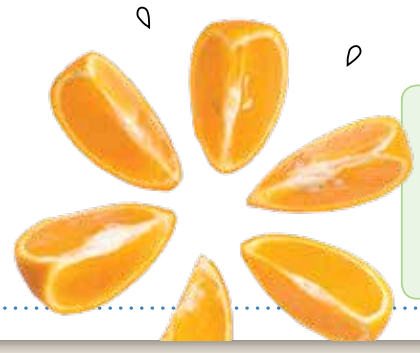
$\frac{1}{8}$s are smaller than $\frac{1}{6}$s because it takes more of them to make a whole.

If $\frac{1}{8}$ is less than $\frac{1}{6}$, then four $\frac{1}{8}$s are less than four $\frac{1}{6}$s.

$\frac{4}{8} < \frac{4}{6}$

Math Toolkit

- fraction tiles
- fraction bars
- fraction circles
- fraction models
- number lines
- grid paper
- sticky notes



DISCUSS IT

Ask your partner: Why did you choose that strategy?

Tell your partner: The strategy I used to find the answer was . . .

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DISCUSS IT

Support Partner Discussion

Encourage students to use the terms *numerator*, *denominator*, *greater than*, and *less than* as they discuss their solutions.

Support as needed with questions such as:

- *Why did you choose the model you used?*
- *What was your first step?*
- *How can you label your model to show that your comparison is correct?*

Common Misconception Look for students who believe that fractions with the same numerator are equivalent and thus write $\frac{4}{8} = \frac{4}{6}$.

Select and Sequence Student Solutions

One possible order for whole class discussion:

- modeling with fraction tiles or fraction bars
- drawing area models
- drawing number lines
- using unit fractions to reason through the comparison

Support Whole Class Discussion

Compare and connect the different representations and have students identify how they relate.

Ask *How does each model show the unit fraction? How can thinking about unit fractions help?*

Listen for Students should realize that the equal parts an accurate model is divided into show the unit fraction. You can think about the size of the unit fractions each fraction is built from if the numerators are the same, or the number of unit fractions in each fraction if the denominators are the same.

PICTURE IT & MODEL IT

If no student presented these models, connect them to the student models by pointing out the ways they each represent:

- a whole divided into eighths and a whole divided into sixths
- how 4 is represented in each model

Ask *How is the unit fraction for each model shown?*

Listen for The whole is divided into equal parts, each of which represents the unit fraction of the original fraction.

For area models, prompt students to identify the way each fraction is shown and compared.

- *For $\frac{4}{8}$, how many equal parts is the model divided into? How many parts are shaded?*
- *For $\frac{4}{6}$, how many equal parts is the model divided into? How many parts are shaded?*
- *Which sized parts are smaller?*

For number lines, prompt students to identify how they compared the fractions.

- *For $\frac{4}{8}$, how many equal parts is the line divided into? Where is the point located?*
- *For $\frac{4}{6}$, how many equal parts is the line divided into? Where is the point located?*

Explore different ways to understand comparing fractions.

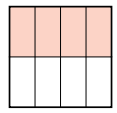
Compare $\frac{4}{8}$ and $\frac{4}{6}$ using <, >, or =.

PICTURE IT

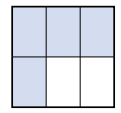
You can use area models to help you compare fractions.

The models show same-sized wholes.

This model shows $\frac{4}{8}$.



This model shows $\frac{4}{6}$.

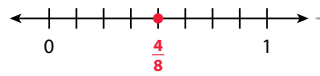


MODEL IT

You can also use number lines to help you compare fractions.

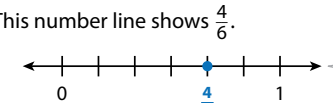
The number lines also show same-sized wholes.

This number line shows $\frac{4}{8}$.

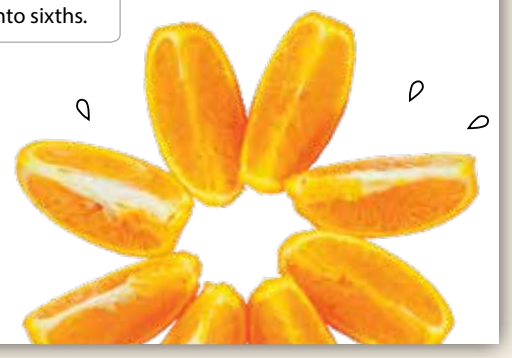


This number line is divided into eighths.

This number line shows $\frac{4}{6}$.



This number line is divided into sixths.



Deepen Understanding

Comparing Fractions on Number Lines

SMP 7 Use structure.

When discussing the number lines, prompt students to consider the best way to set up the number lines for comparison.

Ask *What is similar about these two number lines? What is different?*

Listen for They both start at 0 and end at 1, and the distance between 0 and 1 is the same for both number lines. I can tell this because both same numbers line up. The top number line is divided into eighths, and the bottom number line is divided into sixths.

Ask *Why are there two number lines for this comparison? For what kind(s) of comparisons might you use only one number line?*

Listen for The denominators are different, so it is easier to use two number lines since there are a different number of equal parts in each whole. If you are comparing two fractions with the same denominator, you might use just one number line since each whole is divided into the same number of equal parts.

CONNECT IT

- Remind students that one thing that is alike about all the representations is the fractions.
- Explain that on this page they will see how the models they have been using to compare fractions also allow them to use symbols to compare fractions.

Monitor and Confirm

1 – 2 Check for understanding that:

- the wholes for each model are the same size
- the wholes are divided into eighths and sixths, and eighths are smaller
- you compare the amount shaded in the area models
- you compare which fraction is farther to the right or left on the number line models

Support Whole Class Discussion

3 – 4 Tell students that these problems will prepare them to provide the explanation required in problem 5.

Be sure students understand that the problems are asking them to use words and a symbol to compare each pair of fractions.

Ask *How do you know which symbol to use?*

Listen for The point of the symbol always points to the lesser fraction, so after I decide which fraction is lesser, I pick the symbol that points that way.

5 Look for the idea that first the fractions are compared using any strategy or model of their choice. Then the process for selecting the correct comparison symbol is described.

6 REFLECT Have all students focus on the strategies used to solve this problem. If time allows, have students share their preferences with a partner.

CONNECT IT

Now you will use the problem from the previous page to help you understand how to compare fractions using symbols.

1 Look at the models in **Picture It**. How can you use them to compare $\frac{4}{8}$ and $\frac{4}{6}$?

Possible answer: The models are the same size. The area shaded on the $\frac{4}{8}$ model is less than the area shaded on the $\frac{4}{6}$ model.

2 Look at the number lines in **Model It**. How can you use them to compare the two fractions?

Possible answer: $\frac{4}{8}$ is to the left of $\frac{4}{6}$, so it is less.

3 Compare with words: 4 eighths is **less** than 4 sixths.

Compare with a symbol: $\frac{4}{8}$ **<** $\frac{4}{6}$?

4 Now switch the order of the fractions.

Compare with words: 4 sixths is **greater** than 4 eighths.

Compare with a symbol: $\frac{4}{6}$ **>** $\frac{4}{8}$?

5 Explain how to use symbols to compare two fractions.

Possible answer: You can use a model to find which fraction is greater. If you write the greater fraction first, use the > symbol. If you write the greater fraction last, use the < symbol.

6 REFLECT

Look back at your **Try It**, strategies by classmates, and **Picture It** and **Model It**. Which models or strategies do you like best for using symbols to compare fractions? Explain.

Some students may be most comfortable using area models because they are concrete representations of fractions. Students who fully grasp the concept of number lines may prefer that model. Students with good number sense may not need models to compare fractions that share a numerator or denominator.

Hands-On Activity

Read comparison statements and state the related statement.

If . . . *students are unsure about reading comparison statements or knowing how to restate using the opposite symbol,*

Then . . . *use this activity to give more concrete practice.*

Materials For each pair: 8 index cards

- Have pairs write the following comparisons on index cards, one per card: $\frac{5}{6} > \frac{2}{6}, \frac{2}{6} < \frac{2}{3}, \frac{3}{4} > \frac{3}{8}, \frac{1}{4} < \frac{3}{4}, \frac{7}{8} > \frac{3}{8}, \frac{2}{4} < \frac{2}{2}, \frac{3}{3} > \frac{1}{3}, \frac{1}{6} < \frac{1}{4}$. Shuffle all the cards together and place them facedown in a pile.
- One student takes a card and reads the comparison out loud, such as "five sixths is greater than two sixths." The partner then states the related statement using the opposite symbol, such as "two sixths is less than five sixths."
- Students switch roles and repeat with a new card until all cards are gone through.
- If time allows, you could also have students make a new card with the related comparison statement as they state it, i.e., $\frac{2}{6} < \frac{5}{6}$.

APPLY IT

For all problems, encourage students to use models (given or drawn) to support their thinking. Allow some leeway in precision; the number of the parts of a model and the accuracy of the labels are more important than whether the parts are exactly the same size.

7 $\frac{4}{6} > \frac{2}{6}; \frac{2}{8} < \frac{2}{2}; \frac{1}{2} = \frac{1}{2}$; See Student Worktext page for model shading.

8 $\frac{3}{4} = \frac{3}{4}; \frac{2}{4} < \frac{2}{3}; \frac{2}{3} > \frac{1}{3}$

Close: Exit Ticket

9 $\frac{5}{8} < \frac{5}{6}$ or $\frac{5}{6} > \frac{5}{8}$; Sarah has read more than Manny. See Student Worktext page for possible student work. Students may also solve using area models.

Students' solutions should indicate understanding of:

- how to model the fractions $\frac{5}{8}$ and $\frac{5}{6}$
- when fractions have the same numerator, the fraction with the greater denominator is the lesser fraction
- the symbol $<$ means "less than" and the symbol $>$ means "greater than"
- the person who has read the greater fraction of the book has read more

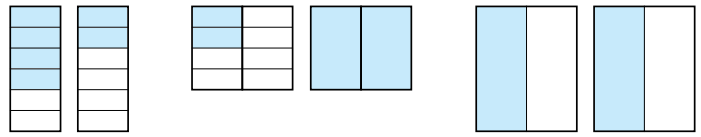
Error Alert **If** students think $\frac{5}{8}$ is greater because 8 is greater than 6, **then** have them compare $\frac{1}{8}$ and $\frac{1}{6}$ fraction tiles or pieces to see which is greater. Then ask if five $\frac{1}{8}$s would be greater than five $\frac{1}{6}$s.

APPLY IT

Use what you just learned to solve these problems.

7 Compare each pair of fractions using $<$, $>$, or $=$. Shade the models to help.

Possible student work:



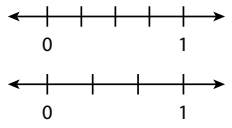
$\frac{4}{6} > \frac{2}{6}$ $\quad \frac{2}{8} < \frac{2}{2}$ $\quad \frac{1}{2} = \frac{1}{2}$

8 Compare each pair of fractions using $<$, $>$, or $=$. Use the number lines to help.

$\frac{3}{4} = \frac{3}{4}$

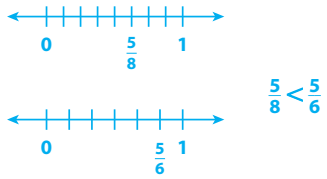
$\frac{2}{4} < \frac{2}{3}$

$\frac{2}{3} > \frac{1}{3}$



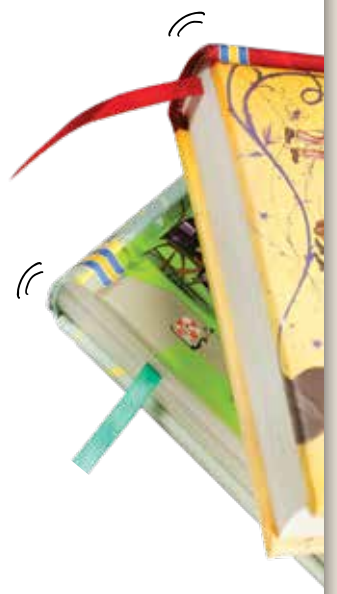
9 Manny and Sarah are reading the same book. Manny has read $\frac{5}{8}$ of the book. Sarah has read $\frac{5}{6}$ of the book. Compare $\frac{5}{8}$ and $\frac{5}{6}$ using $<$, $>$, or $=$. Who has read more? Show your work.

Possible student work:



$\frac{5}{8} < \frac{5}{6}$

Solution $\frac{5}{8} < \frac{5}{6}$; Sarah has read more than Manny.



Solutions

1. $\frac{3}{8} < \frac{6}{8}$
Basic

2. $\frac{1}{3} < \frac{1}{2}$
Basic

3. $\frac{3}{8} < \frac{5}{8}$
Medium

4. $\frac{4}{6} > \frac{1}{6}$
Medium

5. $\frac{5}{8} < \frac{5}{6}$
Medium

Name: _____ LESSON 25 SESSION 2

Practice Comparing Fractions Using Symbols

**Study the Example showing how to use symbols to compare fractions.
Then solve problems 1–16.**

EXAMPLE

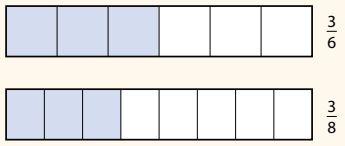
Compare the fractions $\frac{3}{6}$ and $\frac{3}{8}$.

$\frac{3}{6}$ is greater than $\frac{3}{8}$.

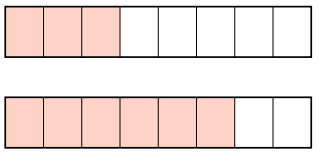
$\frac{3}{6} > \frac{3}{8}$

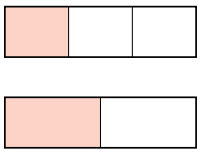
$\frac{3}{8}$ is less than $\frac{3}{6}$.

$\frac{3}{8} < \frac{3}{6}$



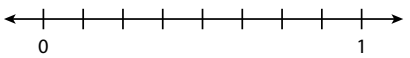
Use the models to compare the fractions in problems 1 and 2. Write <, >, or =.

1. 
$\frac{3}{8} < \frac{6}{8}$

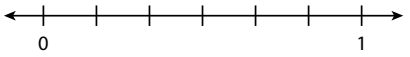
2. 
$\frac{1}{3} < \frac{1}{2}$

Use the number lines to compare the fractions in problems 3–5. Write <, >, or =.

3. $\frac{3}{8} < \frac{5}{8}$



4. $\frac{4}{6} > \frac{1}{6}$



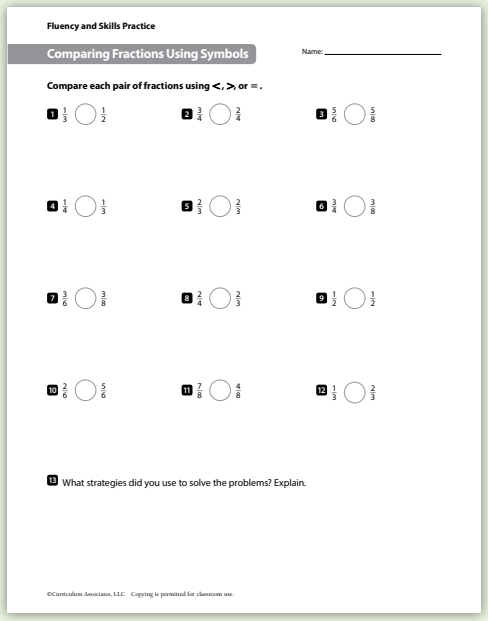
5. $\frac{5}{8} < \frac{5}{6}$

Fluency & Skills Practice

Teacher Toolbox

Assign *Comparing Fractions Using Symbols*

In this activity students practice comparing fractions with the same numerator or same denominator. Students may experience real-world situations that involve comparing fractions, such as comparing fractional lengths or distances.



Fluency and Skills Practice

Comparing Fractions Using Symbols

Name: _____

Compare each pair of fractions using <, >, or =.

1. $\frac{1}{3}$ ◯ $\frac{1}{2}$
2. $\frac{3}{4}$ ◯ $\frac{2}{4}$
3. $\frac{5}{6}$ ◯ $\frac{5}{8}$
4. $\frac{1}{4}$ ◯ $\frac{1}{3}$
5. $\frac{2}{3}$ ◯ $\frac{2}{3}$
6. $\frac{3}{4}$ ◯ $\frac{3}{8}$
7. $\frac{3}{6}$ ◯ $\frac{3}{8}$
8. $\frac{2}{4}$ ◯ $\frac{2}{3}$
9. $\frac{1}{2}$ ◯ $\frac{1}{2}$
10. $\frac{2}{6}$ ◯ $\frac{5}{6}$
11. $\frac{7}{8}$ ◯ $\frac{4}{8}$
12. $\frac{1}{3}$ ◯ $\frac{2}{3}$
13. What strategies did you use to solve the problems? Explain.

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6 $\frac{3}{8}$
Medium

7 $\frac{2}{4}$
Medium

8 $\frac{2}{3}$
Medium

9 $\frac{3}{4}$
Medium

10 $\frac{2}{6}$
Medium

11 $\frac{2}{4} > \frac{2}{6}; \frac{2}{6} < \frac{2}{4}$
Medium

12 $\frac{2}{3} > \frac{2}{6}; \frac{2}{6} < \frac{2}{3}$
Medium

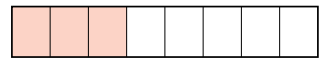
13 $\frac{3}{4} > \frac{3}{8}; \frac{3}{8} < \frac{3}{4}$
Medium

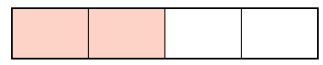
14 $\frac{2}{4} < \frac{3}{4}; \frac{3}{4} > \frac{2}{4}$
Medium

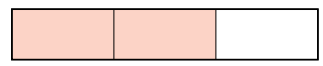
15 Possible answer: $\frac{1}{8}$
Challenge

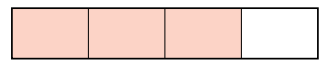
16 Possible answer: $\frac{1}{6}$
Challenge

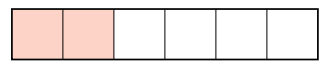
Write the fraction shown in problems 6–10.

6  $\frac{3}{8}$

7  $\frac{2}{4}$

8  $\frac{2}{3}$

9  $\frac{3}{4}$

10  $\frac{2}{6}$

Compare the fractions in problems 11–14. You can use the models above to help you. Write <, >, or =.

11 $\frac{2}{4} > \frac{2}{6}$
$\frac{2}{6} < \frac{2}{4}$

12 $\frac{2}{3} > \frac{2}{6}$
$\frac{2}{6} < \frac{2}{3}$

13 $\frac{3}{4} > \frac{3}{8}$
$\frac{3}{8} < \frac{3}{4}$

14 $\frac{2}{4} < \frac{3}{4}$
$\frac{3}{4} > \frac{2}{4}$

Write a fraction to make the statement true in problems 15 and 16.

15 $\frac{6}{8} >$ Possible answer: $\frac{1}{8}$

16 $\frac{1}{4} >$ Possible answer: $\frac{1}{6}$

ELL English Language Learners: Differentiated Instruction

Prepare for Session 3
Use with *Apply It*.

Levels 1–3

Listening/Speaking Pair students. Read ***Apply It*** problem 7 aloud. Have pairs draw two same-sized number lines or area models. Ask: *Does a numerator or denominator show the number of parts in a whole?* [denominator] Display: _____ $< \frac{3}{4}$. Have pairs represent $\frac{3}{4}$ in one of their models. Check their work. Guide students to identify the known information in the problem. Display: $\frac{3}{-}$. Say: *The numerator is 3. Write a denominator so the fraction is less than* $\frac{3}{4}$. Have pairs represent the fraction in their other model. Check their work. Help revise if necessary.

Levels 2–4

Listening/Speaking Pair students. Read ***Apply It*** problem 7 aloud. Have pairs draw two same-sized number lines or area models. Ask: *What shows the number of parts in a whole?* [denominator] Display: _____ $< \frac{3}{4}$. Have pairs represent $\frac{3}{4}$ in one of their models. Check their work. Guide students to identify the known and unknown information in the problem. Say: *Write a fraction less than* $\frac{3}{4}$ *with numerator 3.* Have them represent it in their other model. Ask: *Is your fraction less than or greater than* $\frac{3}{4}$*? How do you know?* If necessary, help pairs revise their work.

Levels 3–5

Listening/Speaking Have pairs read ***Apply It*** problem 7. Guide them to identify the known and unknown information. Display: _____ $< \frac{3}{4}$. Say: *Write a fraction to complete the comparison. What does a denominator show?* [parts in a whole] Have pairs draw two same-sized number lines or area models to compare the fractions. Have students compare their work. Display: *A fraction with a numerator of 3 is less than* $\frac{3}{4}$ *when* _____. Have students complete the sentence. Ask: *Is there more than one answer? Explain how you know.* Select students to share.

LESSON 25

SESSION 3 Refine

Purpose In this session students solve word and math problems involving using a symbol to compare fractions with either the same numerator or the same denominator and then discuss and confirm their answers with a partner.

Before students begin work, use their responses to the *Check for Understanding* to determine those who will benefit from additional support.

As students complete the Example and problems 1–3, observe and monitor their reasoning to identify groupings for differentiated instruction.

Start

Check for Understanding

Materials For remediation: fraction tiles

Why Confirm understanding of using symbols to compare fractions.

How Present a word problem and have students use symbols to compare fractions using any strategy they want.

Ty and Luke have same-sized juice boxes. Ty drinks $\frac{3}{8}$ of his juice and Luke drinks $\frac{3}{6}$ of his juice. Use symbols to compare fractions. Who drinks more juice?

Possible Solutions

$\frac{3}{6} > \frac{3}{8}$ or $\frac{3}{8} < \frac{3}{6}$; Luke

LESSON 25 SESSION 3

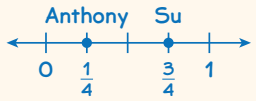
Refine Using Symbols to Compare Fractions

Complete the Example below. Then solve problems 1–8.

EXAMPLE

Su and Anthony live the same distance from school. Su bikes $\frac{3}{4}$ of the way to school in five minutes. Anthony walks $\frac{1}{4}$ of the way to school in five minutes. Compare the fractions using <, >, or =. Who travels the greater distance in these five minutes?

Look at how you could show your work using a number line.



Solution **$\frac{3}{4} > \frac{1}{4}$; Su travels the greater distance.**

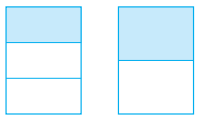
The fractions have the same denominator, so they are easy to compare on the same number line.

PAIR/SHARE
How do you find the greater number on a number line?

APPLY IT

1 Julia and Mackenzie have the same number of homework problems. Julia has done $\frac{1}{3}$ of her homework. Mackenzie has done $\frac{1}{2}$ of her homework. Compare the fractions using <, >, or =. Which student has done less of her homework? Show your work.

Possible student work using area models:



Solution **$\frac{1}{3} < \frac{1}{2}$; Julia has done less homework.**

What do you need to think about when you compare fractions that have different denominators?

PAIR/SHARE
How did you know which fraction was less?

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Error Alert

If the error is . . .	*Students may . . .*	*To support understanding . . .*
students say Luke drinks more juice but write $\frac{3}{8} > \frac{3}{6}$ or $\frac{3}{6} < \frac{3}{8}$	be confused about the meaning of the > and < symbols.	Have students describe the strategy they use for remembering what each symbol means, or help them choose one. Guide students in applying the strategy to the answer and to other fraction comparisons.
students say Ty drinks more juice and write $\frac{3}{8} > \frac{3}{6}$ or $\frac{3}{6} < \frac{3}{8}$	believe that a greater denominator means a greater fraction.	Have students use fraction tiles or make a quick drawing of each fraction of juice the boys drink. Students should also understand that the wholes need to be the same to compare fractions.
students say Ty drinks more juice and write $\frac{3}{6} > \frac{3}{8}$ or $\frac{3}{8} < \frac{3}{6}$	be confused about what the question is asking.	Have students make a table for what they are given and what they need to find. Encourage students to draw a model or number line to represent the amounts of juice that Ty and Luke drink and ask students to describe how they will decide which boy drinks more.

EXAMPLE

$\frac{3}{4} > \frac{1}{4}$; Su travels the greater distance; The number line shown is one way to solve the problem. Students could also solve the problem by using area models or reasoning about unit fractions.

Look for One number line can be used since the denominators are the same.

APPLY IT

1. $\frac{1}{3} < \frac{1}{2}$; Julia has done less homework; Students could solve the problem by drawing area models or number lines. See Student Worktext page for a sample area model.
DOK 3

   Look for Different denominators mean that the whole is divided into different numbers of equal parts.

2. $\frac{3}{4} > \frac{3}{6}$; Rob eats more; Students could solve the problem by drawing area models or number lines. See Student Worktext page for a sample area model.
DOK 3

   Look for The wholes need to be the same size and divided into equal parts based on each denominator.

3. **C**; Students could solve the problem by drawing a number line divided into eighths and locating each fraction on the number line.

   Explain why the other two answer choices are not correct:

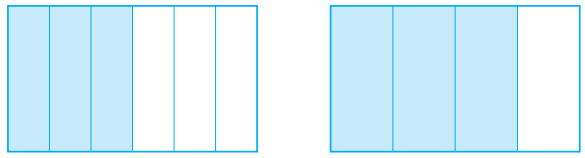
   B is not correct because $\frac{4}{8}$ is less than $\frac{5}{8}$. The question asks for a fraction that is greater than $\frac{5}{8}$.

   D is not correct because $\frac{1}{8}$ is less than $\frac{5}{8}$. The question asks for a fraction that is greater than $\frac{5}{8}$.
DOK 3

2 Deon and Rob each get same-sized packs of crackers. Deon eats $\frac{3}{6}$ of his crackers. Rob eats $\frac{3}{4}$ of his crackers. Compare the fractions using $<$, $>$, or $=$. Who eats more of his crackers?

Show your work.

Possible student work using area models:



Solution **$\frac{3}{4} > \frac{3}{6}$; Rob eats more.**

I think drawing a model might help. Be sure the wholes are the same size.

PAIR/SHARE
Which fraction is made of greater unit fractions? How do you know?

3 Which fraction goes in the blank to make the comparison true?

$\frac{5}{8} < \_\_\_\_$

Ⓐ $\frac{5}{8}$

Ⓑ $\frac{4}{8}$

Ⓒ $\frac{6}{8}$

Ⓓ $\frac{1}{8}$

Is $\frac{5}{8}$ less than or greater than the fraction that goes in the blank?

Blake chose Ⓐ as the correct answer. How did he get that answer?

Possible answer: He chose the one that was equal to $\frac{5}{8}$ instead of looking for one that was greater than $\frac{5}{8}$.

PAIR/SHARE
Does Blake's answer make sense?

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4 **C**; Students need to identify the fraction that is less than $\frac{2}{8}$.
DOK 2

5 **B**; Students need to find the models that correctly compare the fractions $\frac{1}{3}$ and $\frac{1}{6}$.
DOK 3

Error Alert Students who choose A or D may not remember that the wholes need to be the same size in order to compare the fractions.

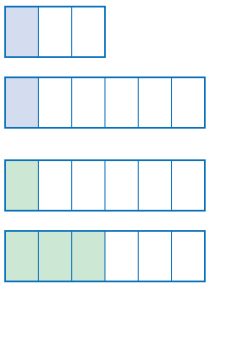
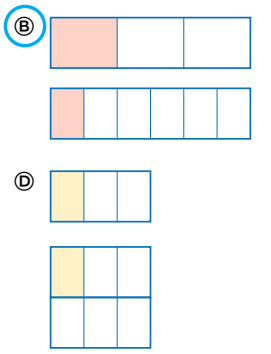
6 Answers may vary; See Student Worktext page for a sample answer. The first box must show a number that is less than the number in the second box.
DOK 3

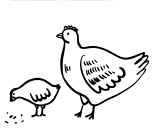
4 Which fraction goes in the blank to make the comparison true?

$\_\_\_\_ < \frac{2}{8}$

Ⓐ $\frac{2}{4}$
Ⓑ $\frac{4}{8}$
Ⓒ $\frac{1}{8}$
Ⓓ $\frac{2}{6}$

5 Which model can you use to compare the fractions $\frac{1}{3}$ and $\frac{1}{6}$?

Ⓐ 
Ⓑ 
Ⓒ
Ⓓ



6 Write a number from the list below in each box to make the statement true.

6 8 1 3 4

$\frac{3}{8} < \frac{6}{8}$ Sample answer shown.

Differentiated Instruction

RETEACH

Hands-On Activity
Practice using symbols to compare fractions.

Students struggling with using symbols to compare fractions

Will benefit from additional work with a partner.

Materials For each pair: 13 index cards

- Have students write the following fractions and symbols on cards, one per card: $\frac{2}{4}, \frac{3}{4}, \frac{4}{4}, \frac{3}{3}, \frac{2}{6}, \frac{3}{6}, \frac{4}{6}, \frac{2}{8}, \frac{3}{8}, \frac{4}{8}, <, >, =$
- Have pairs shuffle the cards and place them facedown in two equal piles. Each player takes the top card from one pile and places the cards faceup side by side. Together they compare the fractions and choose the symbol card to make a true statement. They record the comparison statement and read it aloud. Then they switch the order of the fraction cards, choose the correct symbol card, and record and read the new true statement.
- Repeat the steps with the remaining cards in the piles.

EXTEND

Challenge Activity
Compare fractions greater than or equal to 1.

Students who have achieved proficiency

Will benefit from deepening understanding of comparing fractions greater than or equal to 1.

Materials For each pair: 13 index cards

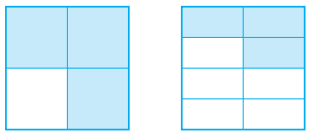
- Have students write the following fractions and symbols on cards, one per card: $\frac{2}{1}, \frac{4}{1}, \frac{2}{2}, \frac{4}{2}, \frac{8}{2}, \frac{2}{4}, \frac{4}{4}, \frac{8}{4}, \frac{2}{8}, \frac{8}{8}, <, >, =$
- Pairs should follow the same process as the Hands-On Activity game: take two cards, choose a symbol to make a true comparison statement, record the comparison, and read it aloud. Make new comparisons after switching the order of the fraction cards.
- Repeat for all remaining pairs of cards.

7 Look at the comparison below.

$\_\_\_\_ < \frac{3}{4}$

Tyrone writes a fraction in the blank to make the comparison true. His fraction has a 3 in the numerator. What fraction could Tyrone have written? Show your work.

Possible student work using area models:



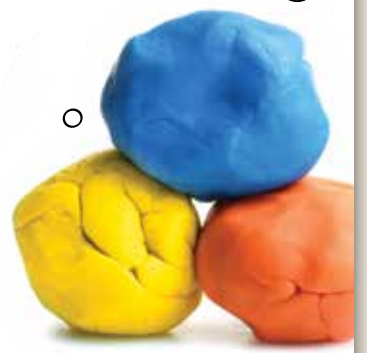
$\frac{3}{6}$ and $\frac{3}{8}$ are both possible answers. Note that any fraction with 3 in the numerator and a number greater than 4 in the denominator is correct.

Solution **$\frac{3}{6}$ or $\frac{3}{8}$**

8 MATH JOURNAL

Tran and Noah are each given the same amount of clay in art class. Tran divides his clay into 3 equal pieces. He uses 2 pieces to make a bowl. Noah divides his clay into 4 equal pieces. He also uses 2 pieces to make a bowl. Tran says that he has more clay left over than Noah. Is Tran correct? Explain.

No, Tran is not correct. Possible explanation: Tran uses $\frac{2}{3}$ of his clay, and Noah uses $\frac{2}{4}$ of his clay. Tran divides his clay into fewer pieces, so his 2 pieces are bigger than Noah's 2 pieces. That means that he uses more clay than Noah, so he has less clay left over.



SELF CHECK Go back to the Unit 4 Opener and see what you can check off.

7 $\frac{3}{6}$ and $\frac{3}{8}$ are both possible answers; Note that any fraction with a 3 in the numerator and a number greater than 4 in the denominator is correct. See Student Worktext page for possible student work.
DOK 3

REINFORCE

Problems 4–8
Compare fractions.

All students will benefit from additional work with comparing fractions by solving problems in a variety of formats.

- Have students work on their own or with a partner to solve the problems.
- Encourage students to show their work.

PERSONALIZE

i-Ready

Provide students with opportunities to work on their personalized instruction path with *i-Ready* Online Instruction to:

- fill prerequisite gaps
- build up grade-level skills

Close: Exit Ticket

8 MATH JOURNAL

Student responses should indicate understanding of comparing fractions with the same numerator.

Error Alert **If** students believe that Tran is correct, **then** have students use fraction circles or a quick drawing to model the problem. Have students describe the size and number of leftover pieces for each boy. Remind students that the question is asking about the amount of clay each has left.

SELF CHECK Have students consider whether they feel they are ready to check off any new skills on the Unit 4 Opener.