Lesson Overview

LESSON 5

Multiply with 0, 1, 2, 5, and 10

Lesson Objectives

Content Objectives

- Use strategies such as repeated addition and skip-counting by twos, fives, and tens to solve multiplication problems involving multiplying with 0, 1, 2, 5, and 10.
- Use models such as arrays and equal groups to solve multiplication problems involving multiplying with 0, 1, 2, 5, and 10.
- Interpret a multiplication problem situation using pictures, objects, words, numbers, and equations.

Language Objectives

- Write multiplication facts for 0, 1, 2, 5, and 10.
- Understand and represent "groups of 0."
- Make general statements about multiplying with 0 and 1.

Prerequisite Skills

- Skip-count by twos, fives, and tens.
- Understand multiplication of whole numbers as finding the total number of objects in some number of equal groups.
- Draw equal groups or an array to represent a given multiplication equation.
- Write an equation to represent equal groups or an array using the × symbol.

Standards for Mathematical Practice (SMP)

SMPs 1, 2, 3, 4, 5, and 6 are integrated in every lesson through the *Try-Discuss-Connect* routine.*

In addition, this lesson particularly emphasizes the following SMPs:

5 Use appropriate tools strategically.

6 Attend to precision.

8 Look for and express regularity in repeated reasoning.

*See page 1i to see how every lesson includes these SMPs.

Lesson Vocabulary

There is no new vocabulary. Review the following key terms.

- **array** a set of objects arranged in equal rows and equal columns.
- **factor** a number that is multiplied.
- **multiplication** an operation used to find the total number of items in a given number of equal-sized groups.
- **multiply** to repeatedly add the same number a certain number of times. Used to find the total number of items in equal-sized groups.
- **product** the result of multiplication.

Learning Progression

In Grade 2 students used skip-counting to find the total number of items in an array and to solve repeated addition problems.

In this lesson students use skip-counting, equal-group models, and arrays to find products with factors of 2, 5, and 10. They explore the special cases of multiplying with 0 and 1 and generalize their findings. They start to develop fluency with the multiplication facts for 0, 1, 2, 5, and 10.

In the next lessons students will learn how to break apart factors so that they can use their knowledge of multiplying with 0, 1, 2, 5, and 10 to multiply with 3, 4, 6, 7, 8, and 9.

In later grades fluency with multiplication facts up to 9 × 9 becomes a critical skill. For this reason, it is important for students to develop a deep understanding of multiplication and division and use this understanding to create meaningful strategies to help them master multiplication and division facts.

Lesson Pacing Guide

Whole Class Instruction

SESSION 1
Explore
45–60 min

Interactive Tutorial* *(Optional)*
Prerequisite Review:
Understand Multiplication, Part 1

Multiplying with 0, 1, 2, 5, and 10
- Start *5 min*
- Try It *10 min*
- Discuss It *10 min*
- Connect It *15 min*
- Close: Exit Ticket *5 min*

Additional Practice
Lesson pages 107–108

SESSION 2
Develop
45–60 min

Multiplying with 2, 5, and 10
- Start *5 min*
- Try It *10 min*
- Discuss It *10 min*
- Model Its *5 min*
- Connect It *10 min*
- Close: Exit Ticket *5 min*

Additional Practice
Lesson pages 113–114

Fluency
Multiplying with 2, 5, and 10

SESSION 3
Develop
45–60 min

Multiplying with 0 and 1
- Start *5 min*
- Try It *10 min*
- Discuss It *10 min*
- Model Its *5 min*
- Connect It *10 min*
- Close: Exit Ticket *5 min*

Additional Practice
Lesson pages 119–120

Fluency
Multiplying with 0 and 1

SESSION 4
Refine
45–60 min

Multiplying with 0, 1, 2, 5, and 10
- Start *5 min*
- Example & Problems 1–3 *15 min*
- Practice & Small Group Differentiation *20 min*
- Close: Exit Ticket *5 min*

Lesson Quiz or **Digital Comprehension Check**

Lesson Materials

Lesson *(Required)*	none
Activities	*Per student:* 10 counters, 10 cups, base-ten blocks (10 tens rods) *Per group:* 10 index cards, 75 paper clips
Math Toolkit	base-ten blocks, counters, cups, hundred charts, 1-centimeter grid paper, number lines
Digital Math Tools	Number Line, Multiplication Models

*We continually update the Interactive Tutorials. Check the Teacher Toolbox for the most up-to-date offerings for this lesson.

Teacher Toolbox

Small Group Differentiation

PREPARE

Ready Prerequisite Lessons
Grade 2
- Lesson 31 Add Using Arrays
- Lesson 32 Even and Odd Numbers

RETEACH

Tools for Instruction
Grade 2
- Lesson 31 Adding Using Arrays
- Lesson 32 Identifying Even and Odd Numbers

Grade 3
- Lesson 5 Multiplication Facts with 0, 1, 2, 5, and 10

REINFORCE

Math Center Activity
Grade 3
- Lesson 5 Multiplication Race 1

EXTEND

Enrichment Activity
Grade 3
- Lesson 5 Shopping Spree

i-Ready

Independent Learning

PERSONALIZE

i-Ready Lessons*
Grade 3
- Practice: Multiplying by 0 and 1
- Practice: Multiples of 2
- Practice: Multiplying by 10
- Practice: Multiplying by 5

Learning Games
- *Match*
- *Cupcake*
- *Pizza*

Connect to Family, Community, and Language Development

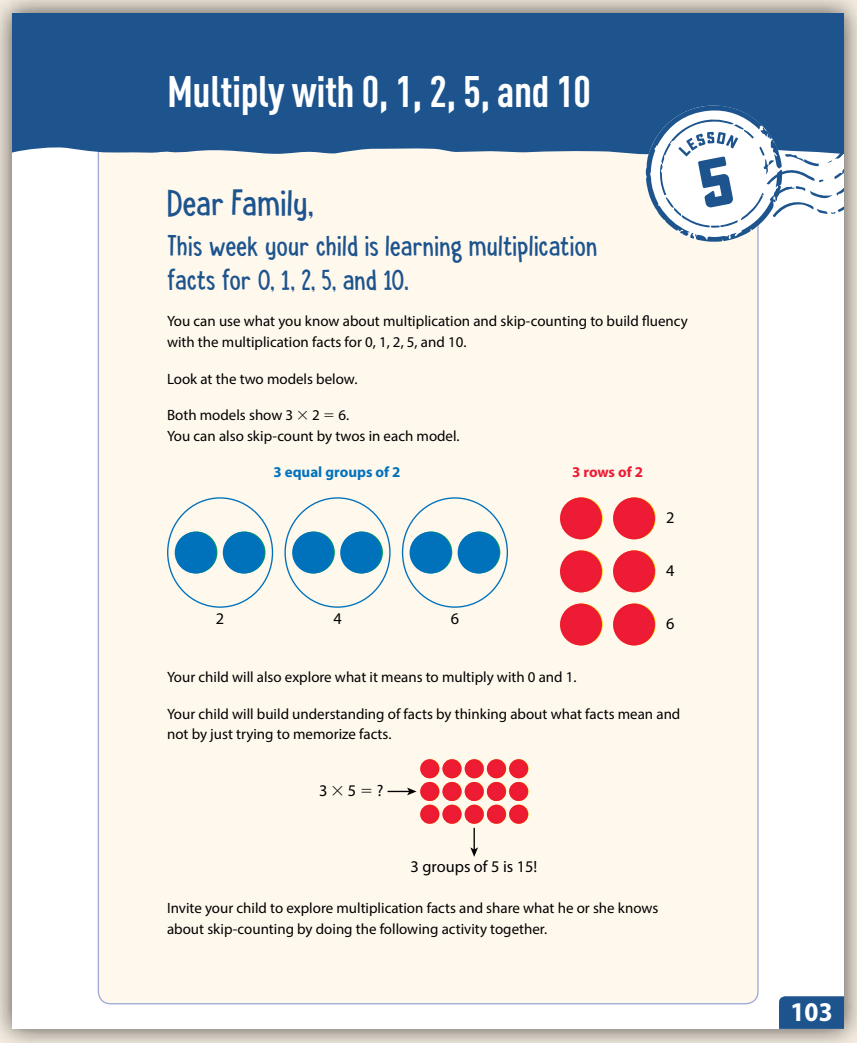
The following activities and instructional supports provide opportunities to foster school, family, and community involvement and partnerships.

Connect to Family

Use the **Family Letter**—which provides background information, math vocabulary, and an activity—to keep families apprised of what their child is learning and to encourage family involvement.

Available in Spanish

Teacher Toolbox



Multiply with 0, 1, 2, 5, and 10

LESSON 5

Dear Family,

This week your child is learning multiplication facts for 0, 1, 2, 5, and 10.

You can use what you know about multiplication and skip-counting to build fluency with the multiplication facts for 0, 1, 2, 5, and 10.

Look at the two models below.

Both models show $3 \times 2 = 6$.
You can also skip-count by twos in each model.

3 equal groups of 2

3 rows of 2

Your child will also explore what it means to multiply with 0 and 1.

Your child will build understanding of facts by thinking about what facts mean and not by just trying to memorize facts.

$3 \times 5 = ?$ →

3 groups of 5 is 15!

Invite your child to explore multiplication facts and share what he or she knows about skip-counting by doing the following activity together.

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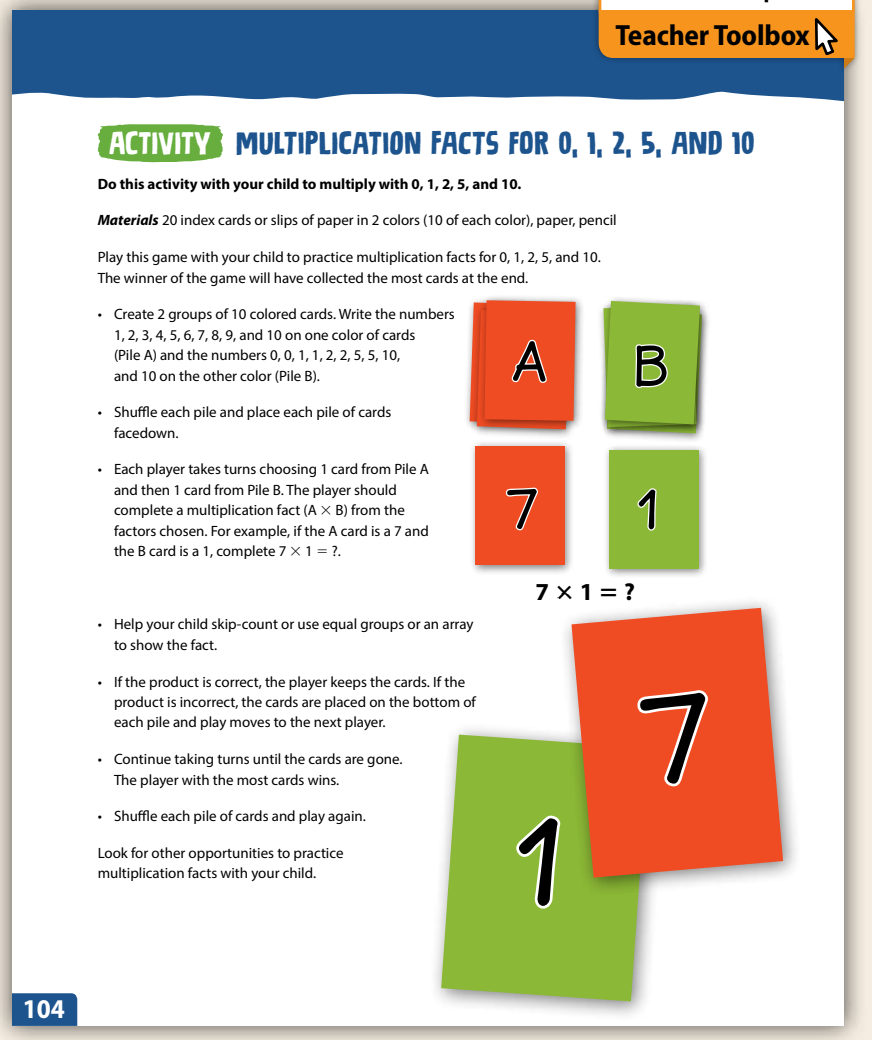

ACTIVITY MULTIPLICATION FACTS FOR 0, 1, 2, 5, AND 10

Do this activity with your child to multiply with 0, 1, 2, 5, and 10.

Materials 20 index cards or slips of paper in 2 colors (10 of each color), paper, pencil

Play this game with your child to practice multiplication facts for 0, 1, 2, 5, and 10. The winner of the game will have collected the most cards at the end.

- Create 2 groups of 10 colored cards. Write the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10 on one color of cards (Pile A) and the numbers 0, 0, 1, 1, 2, 2, 5, 5, 10, and 10 on the other color (Pile B).
- Shuffle each pile and place each pile of cards facedown.
- Each player takes turns choosing 1 card from Pile A and then 1 card from Pile B. The player should complete a multiplication fact (A × B) from the factors chosen. For example, if the A card is a 7 and the B card is a 1, complete $7 \times 1 = ?$.

- Help your child skip-count or use equal groups or an array to show the fact.
- If the product is correct, the player keeps the cards. If the product is incorrect, the cards are placed on the bottom of each pile and play moves to the next player.
- Continue taking turns until the cards are gone. The player with the most cards wins.
- Shuffle each pile of cards and play again.

Look for other opportunities to practice multiplication facts with your child.

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Goal

The goal of the Family Letter is to practice multiplying with 0, 1, 2, 5, and 10 using different representations. Multiplication models include skip-counting, equal groups, arrays, and equations.

Activity

Look at the *Multiplication Facts for 0, 1, 2, 5, and 10* activity and adjust it if necessary to connect with your students.

Math Talk at Home

Encourage students to talk with their family members about situations when multiplying with 0, 1, 2, 5, and 10 can be used, such as finding the total number of socks in a given number of pairs or the total value of eight dimes.

Conversation Starters Below are additional conversation starters students can write in their Family Letter or math journal to engage family members:

- *What do you multiply with when you find the value of a group of pennies, a group of nickels, or a group of dimes?*
- *What are some situations when you need to multiply with two, five, and ten?*

Connect to Community and Cultural Responsiveness

Use these activities to connect with and leverage the diverse backgrounds and experiences of all students.

Session 1 **Use any time during the session.**

- Ask: *How can we skip-count or multiply with two to determine how many legs are in the classroom?* Record an equation to illustrate the point. Ask students to share what types of bugs they know about or have seen. List in a table the ones that students identify (such as bees, ants, spiders, and flies). Next to each type of bug, list the number of legs each has. Ask: *How can we determine the number of legs for 0, 1, 2, 5, and 10 of each kind of bug?* Lead students to generalize about zero and one. Reinforce the connection between skip-counting and multiplying with 2, 5, and 10.

Session 2 **Use with *Try It*.**

- Remind students of the conversation in Session 1 about bugs and the number of legs they have. Share that most bugs have antennas on their heads that serve many different functions. Some antennas are called "feelers" and help the bug feel by touch. Other antennas help the bug with other senses such as sound (by sensing vibrations), taste, and smell. Ask students to discuss with a partner what the function of antennas on a robot would be. Accept reasonable suggestions. Share that robots, like televisions, can be controlled remotely. The antennas receive the messages from the remote control and tell the robot what to do.

Session 3 **Use with *Try It*.**

- Explain that the number zero is very old, as is our number system. The number zero was developed after the other counting numbers because people did not think they needed a number or a symbol to represent "nothing." When positional numbering systems were developed, a placeholder was needed to show the idea of nothing in a given place. Display different words that are used in the English language to express the idea of nothing: *nil*, *zip*, *zilch*, *nada*, and *love* (in tennis).

Session 4 **Use with *Apply It* problem 2.**

- Ask students to say what their favorite fruit is. Substitute students' selections for the apples, oranges, and peaches in the problem to reflect any cultural influences.

Connect to Language Development

For ELLs, use the Differentiated Instruction chart to plan and prepare for specific activities in every session.

ELL English Language Learners: Differentiated Instruction

Prepare for Session 1
Use with *Try It*.

Levels 1–3

Listening/Speaking Read the ***Try It*** problem aloud. Display the table shown. Say: *We have represented multiplication with groups, items in each group, rows, and columns. Let's review these models by filling in this table.* Work with students to complete the first two rows of the table.

groups	×	**items in a group**	=	total
rows	×	columns	=	total
bugs	×	legs on a bug	=	**total legs**

Say: *This problem mentions cartoon bugs and legs.* Have students discuss with a partner how to complete the last row of the table. Guide students to share their thinking aloud.

Levels 2–4

Listening/Speaking Read the ***Try It*** problem aloud. Display the table shown. Say: *We have represented multiplication with different models. Let's review these models by filling in this table.* Question students until the first two rows of the table are complete.

groups	×	**items in a group**	=	total
rows	×	columns	=	total
bugs	×	legs on a bug	=	total legs

Say: *This problem mentions cartoon bugs and legs.* Have students form pairs and discuss with a partner how to complete the last row of the table. Select students to share their thinking aloud.

Levels 3–5

Listening/Speaking Have students form pairs and read the ***Try It*** problem to each other. Say: *We have represented multiplication with different models. This problem mentions cartoon bugs and legs. How can we draw:*

- *equal groups to represent the bugs and their legs?*
- *an array to represent the bugs and their legs?*
- *an equation to represent the bugs and their legs?*

Assign a different model for each pair to create. Select pairs to share and explain their models.

LESSON 5

SESSION 1 Explore

Purpose In this session students draw on their knowledge of multiplication as combining equal groups and on skip-counting to find products. They share models to explore how to find the total number of items in several equal groups. They will look ahead to see how skip-counting by twos, fives, and tens is connected to multiplying with 2, 5, and 10.

Start

Connect to Prior Knowledge

Why Reinforce that equal groups are needed in order to multiply.

How Have students solve a word problem about 3 equal groups of 5.

You have 3 equal groups of 5 pencils.

How many pencils do you have in all?

Solution
15 pencils

TRY IT

Make Sense of the Problem

To support students in making sense of the problem, have them identify how many bugs Jenny drew and how many legs each bug has.

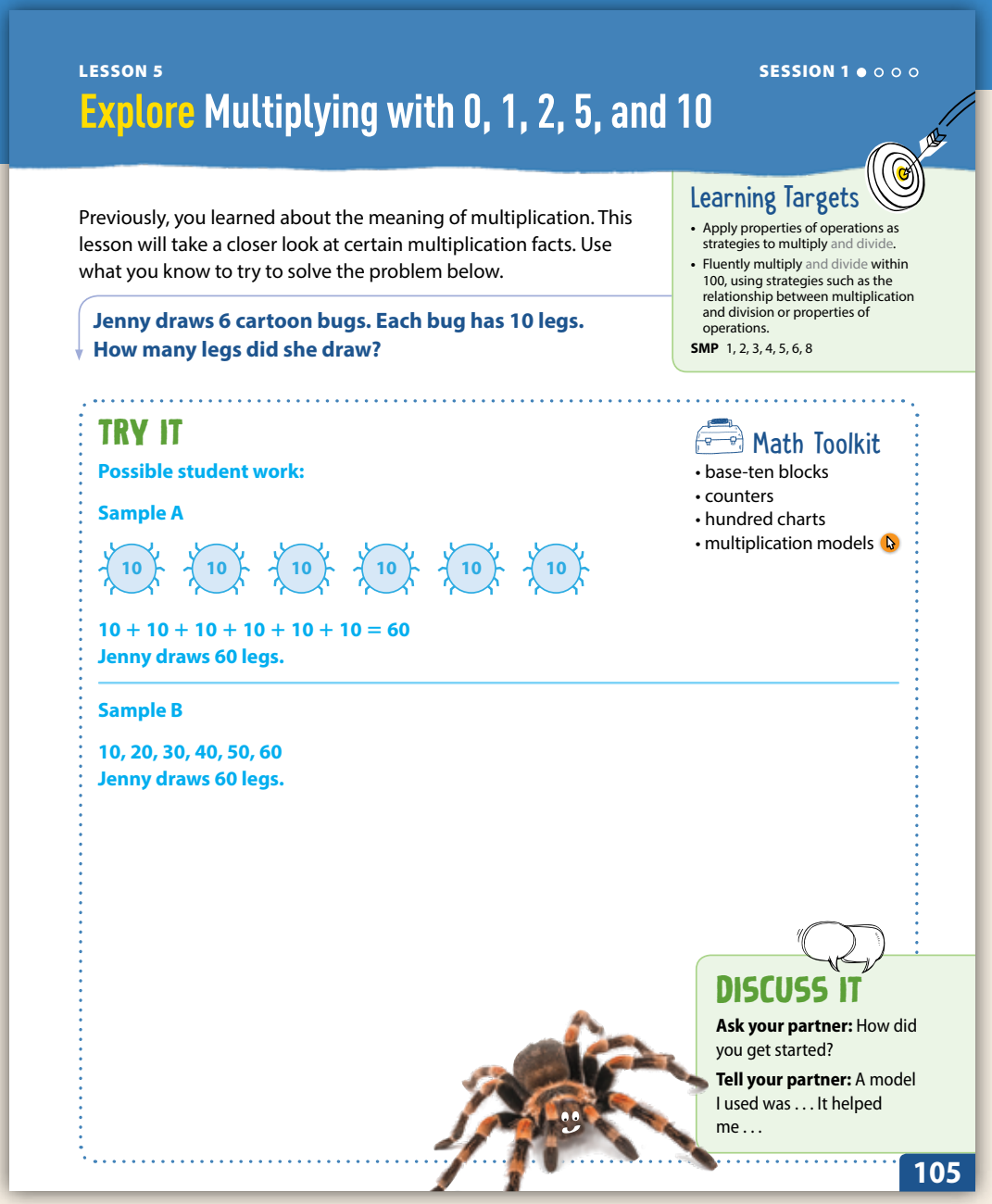
DISCUSS IT

Support Partner Discussion

To reinforce the concept of multiplication, encourage students to use the terms *times* and *equal groups of* as they talk to each other.

Look for, and prompt as necessary for, understanding of:

- there are 6 bugs
- 10 legs on each bug
- 6 bugs with 10 legs or 6 equal groups of 10



LESSON 5 SESSION 1 ● ○ ○ ○

Explore Multiplying with 0, 1, 2, 5, and 10

Previously, you learned about the meaning of multiplication. This lesson will take a closer look at certain multiplication facts. Use what you know to try to solve the problem below.

Jenny draws 6 cartoon bugs. Each bug has 10 legs. How many legs did she draw?

Learning Targets

- Apply properties of operations as strategies to multiply and divide.
- Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division or properties of operations.

SMP 1, 2, 3, 4, 5, 6, 8

TRY IT

Possible student work:

Sample A

10 10 10 10 10 10

10 + 10 + 10 + 10 + 10 + 10 = 60
Jenny draws 60 legs.

Sample B

10, 20, 30, 40, 50, 60
Jenny draws 60 legs.

Math Toolkit

- base-ten blocks
- counters
- hundred charts
- multiplication models

DISCUSS IT

Ask your partner: How did you get started?

Tell your partner: A model I used was . . . It helped me . . .

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Common Misconception Look for students who do not recognize that the legs can be thought of as 6 equal groups of 10. As students present solutions, point out that each bug has an *equal* number of legs and that each bug can be thought of as one *group*.

Select and Sequence Student Solutions

One possible order for whole class discussion:

- base-ten blocks or hundred charts that are used to count by tens
- drawings that show 6 groups of 10 that could be counted by ones
- equations that show repeated addition of 10
- multiplication equations with factors of 6 and 10

Support Whole Class Discussion

Prompt students to note the relationship between the numbers in each model and the numbers in the problem.

Ask *How do [student name]'s and [student name]'s models each show the number of bugs and the number of legs on each bug?*

Listen for Drawings and models show 6 groups of 10. Addition equations show six 10s added together. Multiplication equations have factors of 6 and 10.

CONNECT IT

1 LOOK BACK

Look for understanding that there were 6 groups of 10 legs, with a total of 60 legs.

Hands-On Activity

Use models to create equal groups.

If . . . *students are unsure about the concept of equal groups and multiplication in this context,*

Then . . . *use this activity to have them model the problem.*

Materials For each group of 3: 10 index cards, 75 paper clips

- Tell students that they are going to create index card "bugs" with paper clip "legs."
- Ask: *How many index cards should you use to model Jenny's bugs?* [6] *How many paper clips should you attach to each index card?* [10]
- Have students attach 10 paper clips to each index card until each group has six "bugs" with ten "legs" each.
- Have a group member remove and count the paper clips from one card. Have the student write the total on the card and put the paper clips in a pile on top of the card.
- Have group members take turns doing the same with the rest of the cards, one at a time, each counting on from where they left off with the card before. Each student should write the aggregate total on his or her card.
- Ask: *How many groups of paper clips do you have?* [6] *How many are in each group?* [10] *How many paper clips do you have in all?* [60]
- Have each group line up their index cards in order and read the numbers. Prompt them to recognize the set of numbers they say as counting by tens.
- Repeat with other numbers of bugs or legs.

2 LOOK AHEAD

Point out that skip-counting is a quick method of repeated addition, which is also called multiplication.

Students should be able to use skip-counting to count equal groups of items and then model the groups and total as a multiplication equation.

CONNECT IT

1 LOOK BACK

Explain how you found the number of legs Jenny drew for the 6 cartoon bugs.

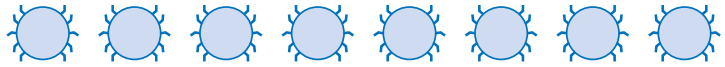
Possible answer: I drew 6 groups of 10 legs. Jenny drew 60 legs in all.

2 LOOK AHEAD

You can show and solve multiplication problems in different ways, such as using arrays or equal groups.

One way to find products when multiplying with 2, 5, or 10 is to use skip-counting.

Suppose Jenny draws 8 cartoon bugs with 10 legs each.



a. Show how you could use skip-counting to find the number of legs Jenny drew.

10, 20, **30, 40, 50, 60, 70, 80**

b. Write a multiplication fact to find the number of legs.

number of bugs	×	legs on each bug	=	total number of legs
8	×	10	=	80

3 REFLECT

Suppose you have 8 bugs with 8 legs each. What other method besides skip-counting can you use to find the total number of legs?

Possible answer: I can draw 8 equal groups of 8 or an array with 8 rows and 8 columns.

Close: Exit Ticket

3 REFLECT

Look for understanding that the situation can be solved by skip-counting by eights, but that modeling the situation with 8 groups of 8 or with an array of 8 rows and 8 columns may be easier.

Common Misconception **If** students lose track while skip-counting and continue beyond the answer to the problem, **then** have them refer to the drawing and count again, pointing to each bug as they count another ten. Suggest that they use their fingers when no picture is available.

Real-World Connection

Ask students to think of real-world situations that use or model skip-counting by twos, fives, and tens. Some examples include the even house numbers on one side of a street, reading the number of minutes when telling time on an analog clock, or counting the value of several dimes.

Solutions

Support Vocabulary Development

1 Ask students to look at the graphic organizer. Tell students that a fact is a statement that is true all of the time. Say: *Multiplication facts are true all of the time.* Ask students to explain how the graphic organizer can be completed. Encourage students to reflect on the ways numbers have been represented in the session. Ask: *What might be a way to show and explain a multiplication fact with words, numbers, and pictures?* Record the suggestions offered by students as a scaffold.

2 Have students count and label the number of groups and the number of items in each group. Ask students to record a multiplication fact to represent the model. Say: *Explain what each number in your multiplication fact means.*

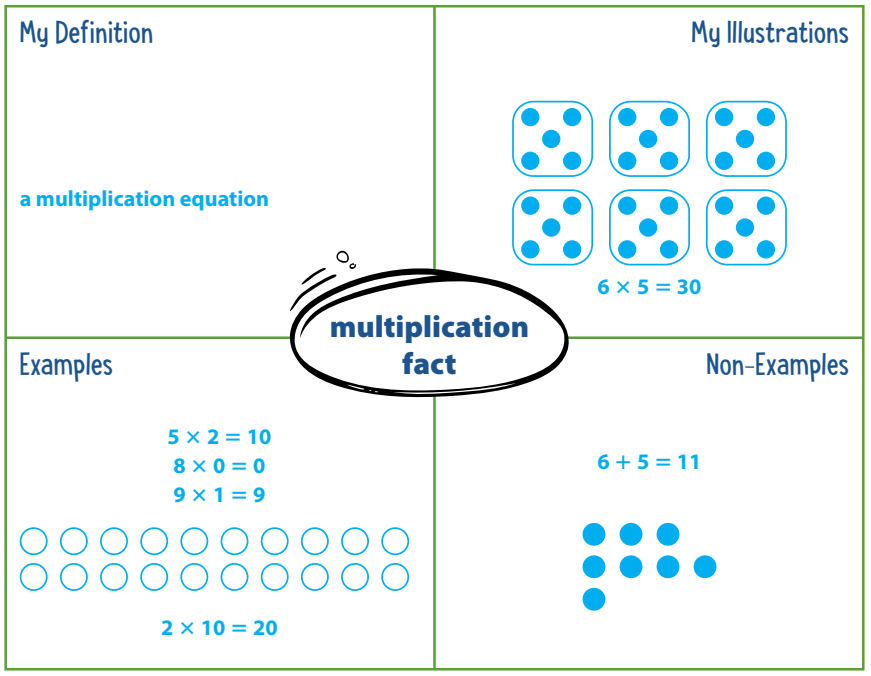
Supplemental Math Vocabulary

- *array*
- *factor*
- *multiplication*
- *multiply*
- *product*

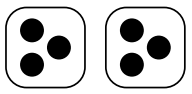
Name: _____ LESSON 5 SESSION 1

Prepare for Multiplying with 0, 1, 2, 5, and 10

1 Think about what you know about multiplication. Fill in each box. Use words, numbers, and pictures. Show as many ideas as you can. **Possible answers:**



2 What multiplication fact is shown by the model?



2 × 3 = 6

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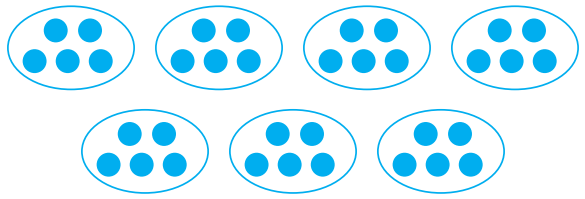
3 Solve the problem. Show your work.

Julio makes 7 cookies. Each cookie has 5 chocolate chips. How many chocolate chips did he use?

Possible student work using a picture:



5 + 5 + 5 + 5 + 5 + 5 + 5 = 35

Solution **Julio used 35 chocolate chips.**

4 Check your answer. Show your work.

Possible student work:

5, 10, 15, 20, 25, 30, 35
Julio used 35 chocolate chips.

3 Assign problem 3 to provide another look at solving a problem by multiplying with 0, 1, 2, 5, or 10.

This problem is very similar to the problem about finding the total number of legs the bugs in Jenny's drawing have. In both problems, students are given a word problem that requires multiplying by 5 or 10 to solve. The question asks for the total number of chocolate chips in Julio's cookies.

Students may want to use counters.

Suggest that students read the problem three times, asking themselves one of the following questions each time:

- *What is this problem about?*
- *What is the question I am trying to answer?*
- *What information is important?*

Solution: Students may skip-count by fives 7 times. 5, 10, 15, 20, 25, 30, 35. Julio used 35 chocolate chips.
Medium

4 Have students solve the problem another way to check their answer.

ELL English Language Learners: Differentiated Instruction

Prepare for Session 2
Use with *Connect It*.

Levels 1–3

Listening/Speaking Read ***Connect It*** problem 3 aloud. Give each student 12 counters. Pair students. Have one partner make an array of 6 rows of 2 counters each as pictured in the problem. Ask: *How many rows of counters are there? How many columns are there? How many counters are there in all? What multiplication fact does this array represent?*

Record $6 \times 2 = 12$. Have the other partner make an array of 2 rows of 6 counters each. Repeat the same series of questions. Record $2 \times 6 = 12$ under $6 \times 2 = 12$. Ask: *How are these arrays/equations the same? How are the arrays/equations different?*

Levels 2–4

Listening/Speaking Read ***Connect It*** problem 3 aloud. Pair students. Give each student pair 12 counters. Ask pairs to create the arrays as pictured in the problem. Ask a series of questions for each array: *How many rows of counters are there? How many columns are there? How many counters are there in all? What multiplication fact does this array represent?* Record:

$$6 \times 2 = 12$$
$$2 \times 6 = 12$$

Ask: *How are these arrays/equations the same? How are the arrays/equations different?* Repeat the same process for other facts, such as 3×4 and 4×3.

Levels 3–5

Listening/Speaking Have student pairs read ***Connect It*** problem 3. Give each pair 12 counters. Ask pairs to create the arrays as shown in the problem. Ask a series of questions for each array: *How many rows of counters are there? How many columns are there? How many counters are there in all? What multiplication fact does this array represent?* Record:

$$6 \times 2 = 12$$
$$2 \times 6 = 12$$

Ask: *How are these arrays/equations the same? How are the arrays/equations different?* Have student pairs use the counters to make another pair of arrays that model the commutative property for another multiplication fact.

LESSON 5

SESSION 2 Develop

Purpose In this session students solve a problem that requires finding both 6×2 and 6×5. Students model equal groups either on paper or with manipulatives to represent the two products. The purpose of this session is to have students apply repeated addition strategies to solve multiplication problems involving the multiplication facts for 2, 5, and 10.

Start

Connect to Prior Knowledge

Why Reinforce skip-counting skills for use as a multiplication strategy.

How Skip-count aloud by fives and tens.

Skip-count aloud by tens to 100.

Skip-count aloud by fives to 100.

Solutions

Listen for

10, 20, 30, 40, 50, 60, 70, 80, 90, 100,
5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, 90, 95, 100

Develop Language

Why Clarify the meaning of the term *equal* and the equal sign (=).

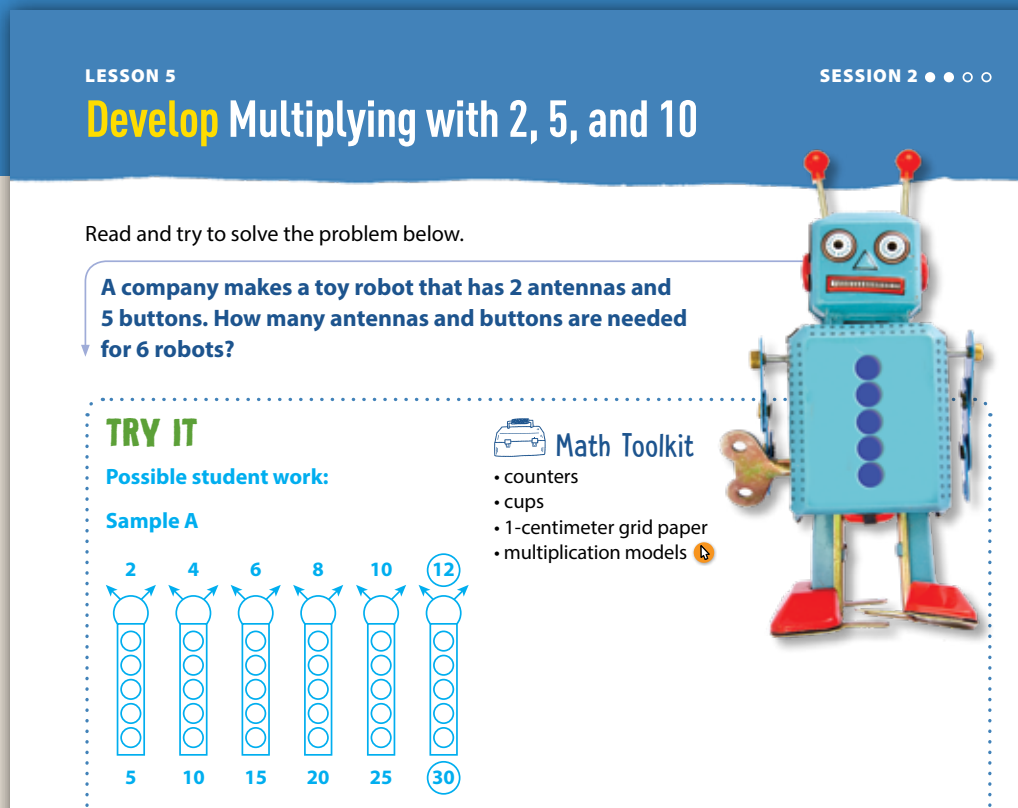
How When we say that two things are *equal*, we mean that they are the same or that they have the same value. Display the equation $2 \times 5 = 10$ and circle the equal sign. State and underline the expression on either side of the equal sign. Say: *In an equation, the equal symbol means that the two sides of the equation have the same value. 2×5 has the same value as 10.*

TRY IT

Make Sense of the Problem

To support students in making sense of the problem, have them demonstrate that they are familiar with the meaning of *antennas*.

Ask *How many robots are there? How many antennas does each robot have? How many buttons? What are you asked to find?*



LESSON 5 SESSION 2 ● ● ○ ○

Develop Multiplying with 2, 5, and 10

Read and try to solve the problem below.

A company makes a toy robot that has 2 antennas and 5 buttons. How many antennas and buttons are needed for 6 robots?

TRY IT

Possible student work:

Sample A

2 4 6 8 10 12

5 10 15 20 25 30

12 antennas and 30 buttons

Sample B

2 ◯◯ 5 ◯◯◯◯◯

4 ◯◯ 10 ◯◯◯◯◯

6 ◯◯ 15 ◯◯◯◯◯

8 ◯◯ 20 ◯◯◯◯◯

10 ◯◯ 25 ◯◯◯◯◯

12 ◯◯ 30 ◯◯◯◯◯

2 + 2 + 2 + 2 + 2 + 2 = 12 antennas
5 + 5 + 5 + 5 + 5 + 5 = 30 buttons

Math Toolkit

- counters
- cups
- 1-centimeter grid paper
- multiplication models

DISCUSS IT

Ask your partner: Why did you choose this strategy?

Tell your partner: I started by . . .

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DISCUSS IT

Support Partner Discussion

Encourage students to use the term *equal groups* as they discuss solutions.

Support as needed with questions such as:

- *Which numbers did you use to find each total?*
- *Did you use the same method to find the number of antennas that you used to find the number of buttons? Explain.*

Common Misconception Look for students who had trouble deciding what information to use to find each total.

Select and Sequence Student Solutions

One possible order for whole class discussion:

- counters showing equal groups and totals for antennas and for buttons
- separate drawings for each total to be found: one for antennas and one for buttons
- one drawing used for both totals
- equations that show repeated addition
- multiplication equations

Support Whole Class Discussion

Compare and connect the different representations and have students identify how they are related.

Ask *How does each model show that there are six robots? How does each model show that each robot has two antennas and five buttons?*

Listen for Drawings show 6 groups of 2 and 6 groups of 5, or 6 groups each with 2 of one object and 5 of another object. Equations show six 2s added together and, separately, six 5s added together. Lists of numbers that indicate skip-counting by twos and fives show 6 numbers each.

MODEL ITS

If no student presented these models, connect them to the student models by pointing out the ways they each represent:

- 6 equal groups of 2
- 6 equal groups of 5
- the total number of antennas
- the total number of buttons

Ask *How are equal groups shown in the drawings? In the arrays? How does each model show skip-counting? How do you find the totals in each model?*

Listen for Each equal group is a circle in the drawings and a row in the arrays. Both models list the numbers 2, 4, 6, 8, 10, 12 and 5, 10, 15, 20, 25, 30. The totals are the last numbers in each list.

For drawings of equal groups, prompt students to describe how each number is represented in the problem.

- *How can you tell how many robots there are?*
- *How can you tell how many antennas there are for each robot?*
- *How can you tell how many buttons there are for each robot?*

For arrays, prompt students to explain what the rows and columns represent.

- *How are the two arrays alike? What do the rows represent in this situation?*
- *How are the two arrays different? What do the columns represent in this situation?*

Explore different ways to understand multiplying with 2, 5, and 10.

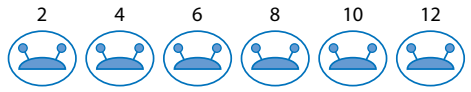
A company makes a toy robot that has 2 antennas and 5 buttons. How many antennas and buttons are needed for 6 robots?

MODEL IT

You can use equal groups and skip-count.

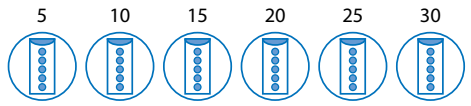
The drawings show the antennas and buttons of **6** robots.

You can skip-count by **twos** to find the number of antennas.



6 groups of **2** antennas

You can skip-count by **fives** to find the number of buttons.

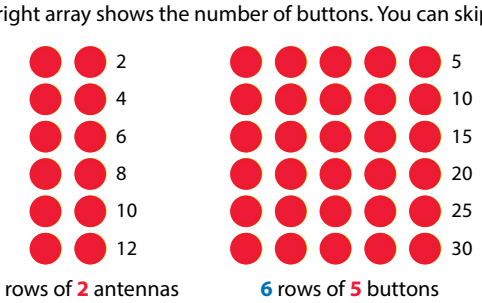


6 groups of **5** buttons

MODEL IT

You can use arrays and skip-count.

The left array shows the number of antennas. You can skip-count by **twos**.
The right array shows the number of buttons. You can skip-count by **fives**.



6 rows of **2** antennas

6 rows of **5** buttons

Deepen Understanding

Multiples

SMP 8 Use repeated reasoning.

When discussing the array model, prompt students to look for patterns among the multiples of 2 and the multiples of 5.

Ask *Look at the antenna array. What do you notice about the ones digits when counting by twos? If you continue, will you say 18? 23? 31? 44?*

Listen for The ones digits are all even, and they will repeat. You will say 18 and 44, but not 23 or 31.

Ask *Look at the button array. What do you notice about the ones digits when counting by fives? If you continue, will you say 35? 47? 50? 61?*

Listen for The ones digits are either 5 or 0. You will say 35 and 50, but not 47 or 61.

Generalize *Write rules about the ones digits when counting by twos and by fives.*
By twos: The ones digit of each number you say will be even (0, 2, 4, 6, or 8).
By fives: The ones digits of the numbers you say will be either 0 or 5.

CONNECT IT

- Remind students that one thing that is alike about all the representations is the numbers and the quantities in the models.
- Explain that on this page they will use those numbers to write multiplication equations that represent the problems and solutions from the previous page.

Monitor and Confirm

1 – 2 Check for understanding that:

- $6 \times 2 = 12$ means "six groups of two is twelve"
- $6 \times 5 = 30$ means "six groups of five is thirty"
- the totals are found by skip-counting by twos and by fives

Support Whole Class Discussion

3 – 4 Tell students that these problems will show them different ways to think about and represent the number of antennas.

Be sure students understand that the two arrays represent the same number grouped in different ways.

Ask *How is the number of rows and columns of the first array related to the number of rows and columns of the second array? How are the two equations related?*

Listen for The number of rows of the first array is the number of columns of the second array, and the number of columns of the first array is the number of rows of the second array. The equations are the same except that the factors are in reverse order. In other words, in the arrays the rows and columns are switched, and in the equations the factors are switched.

5 Look for the understanding that multiplying a number by two and doubling a number both mean combining two equal groups of that number, or adding the number to itself.

6 REFLECT Have all students focus on the strategies used to solve this problem. If time allows, have students share their preferences with a partner.

CONNECT IT

Now you will use the problem from the previous page to help you understand how to multiply with 2, 5, and 10.

1 Look at both **Model Its**. What multiplication equations can you write for the number of antennas and number of buttons?

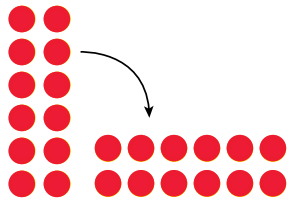
$6 \times 2 = 12$ and $6 \times 5 = 30$

2 How do both models use skip-counting?

Possible answer: You skip-count by 2s for the number of antennas and skip-count by 5s for the number of buttons.

3 If you take the antenna array in the second **Model It** and turn it, what would the equation be for each array?

$6 \times 2 = 12$ and $2 \times 6 = 12$



4 Did the order of the factors in problem 3 change the product? Explain why or why not.

No; Possible explanation: The product does not change because you still have the same number of antennas.

5 What addition doubles fact can you write for the turned array in problem 3? Why can you use a doubles fact when you multiply with 2?

$6 + 6 = 12$; Possible explanation: Multiplying with 2 means you have 2 equal groups, and that is what doubles are.

6 REFLECT

Look back at your **Try It**, strategies by classmates, and **Model Its**. Which models or strategies do you like best for multiplying with 2 and 5? Explain.

Some students may be more comfortable skip-counting with 2 and 5 to find products with these factors. Others may choose to draw arrays or equal groups and add the numbers in the groups.

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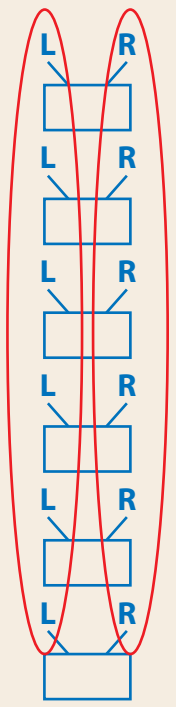
Visual Model

Illustrate reordering factors in the robot problem.

If . . . *students have trouble understanding why $6 \times 2 = 2 \times 6$,*

Then . . . *use drawings to group the antennas differently.*

- Draw six rectangle "robots" in a vertical column as shown. Draw two antennas on each robot.
- Explain that if we group the antennas by which robot they belong to, there are six groups of two with twelve in all. Point to the drawing as you explain: *These two antennas belong to this robot, these antennas belong to this robot*, and so on.
- Explain that another way to organize all the antennas is to group the left antennas together and the right antennas together. Label each of the antennas "L" or "R" as shown. Then draw a circle around each of the two groups.
- Elicit that the drawing represents $2 \times 6 = 12$. Ask: *How many groups are there now? How many antennas in each group? How many antennas are there in all?*



APPLY IT

For all problems, encourage students to draw some kind of model to support their thinking.

7 $5 \times 10 = 50$; Students may show five groups of ten or an array with five rows and ten columns.

8 $10 \times 5 = 50$; Students may show ten groups of five or an array with ten rows and five columns.

Close: Exit Ticket

9 20 beds; Students may show four groups of five or an array with four rows and five columns.

Students' solutions should indicate understanding of:

- there are four equal groups of five
- appropriate use of groups, arrays, repeated addition, or skip-counting to represent and solve the problem

Error Alert **If** students begin counting by fives and mistakenly switch to tens (i.e., count "5, 10, 20, 30"), **then** have them write out the numbers and check that they increase by 5 at a time or use a hundred chart to support accuracy.

APPLY IT

Use what you just learned to solve these problems.

7 How much is 5 groups of 10? Write a multiplication equation. Show your work.

Possible student work:

10	10	10	10	10
10	20	30	40	50

Solution $5 \times 10 = 50$

8 How much is 10 groups of 5? Write a multiplication equation. Show your work.

Possible student work:

5	5	5	5	5
5	10	15	20	25
5	5	5	5	5
30	35	40	45	50

Solution $10 \times 5 = 50$

9 Each cabin at camp has 5 beds. There are 4 cabins. How many beds are there at camp? Show your work.

Possible student work:

5, 10, 15, 20

Solution 20 beds

Solutions

1 Students should circle six equal groups of two; 6 groups of 2 is 12, so $6 \times 2 = 12$.
Basic

2 **a.** $8 \times 10 = 80$ pencils;
b. $9 \times 10 = 90$ pencils;
c. $10 \times 10 = 100$ pencils
Medium

Name: _____ LESSON 5 SESSION 2

Practice Multiplying with 2, 5, and 10

Study the Example showing how to multiply with 5. Then solve problems 1–5.

EXAMPLE

Wes has 3 rows of tomato plants. There are 5 plants in each row. How many tomato plants does Wes have in all?

$3 \times 5 = 15$. Wes has 15 tomato plants.

1 Circle equal groups of 2. Then fill in the blanks to show the multiplication fact.

Possible student work:

6 groups of 2 is 12, so 6 × 2 = 12.

2 Each box of pencils has 10 pencils. Write a multiplication fact for 8, 9, and 10 boxes of pencils.

a. 8 boxes: 8 × 10 = 80 pencils

b. 9 boxes: 9 × 10 = 90 pencils

c. 10 boxes: 10 × 10 = 100 pencils

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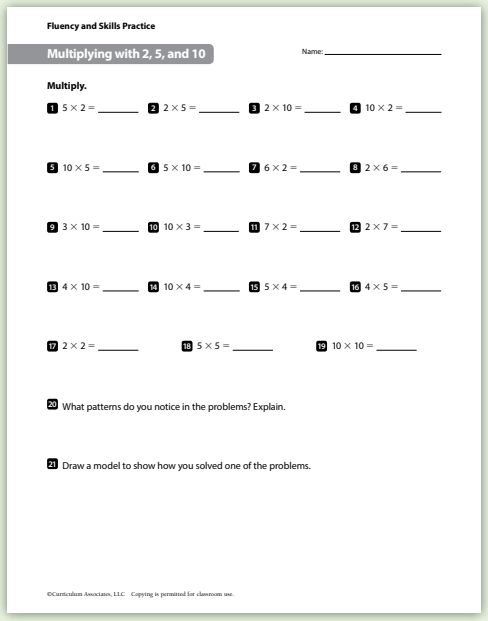
Fluency & Skills Practice Teacher Toolbox

Assign *Multiplying with 2, 5, and 10*

In this activity students practice multiplying numbers by 2, 5, and 10. Students will multiply by 2 when they need to double a value, such as when purchasing two of the same item, doubling a recipe, or finding a round-trip distance. Students will multiply by 5 or 10 when finding the total value of a group of nickels or dimes. Many items are also sold in packages of 2, 5, and 10, so students can multiply by these numbers to find the total number of items in multiple packages.



Fluency and Skills Practice

Multiplying with 2, 5, and 10 Name: _____

Multiply.

1 5 × 2 = _____ 2 2 × 5 = _____ 3 2 × 10 = _____ 4 10 × 2 = _____

5 10 × 5 = _____ 6 5 × 10 = _____ 7 6 × 2 = _____ 8 2 × 6 = _____

9 3 × 10 = _____ 10 10 × 3 = _____ 11 7 × 2 = _____ 12 2 × 7 = _____

13 4 × 10 = _____ 14 10 × 4 = _____ 15 5 × 4 = _____ 16 4 × 5 = _____

17 2 × 2 = _____ 18 5 × 5 = _____ 19 10 × 10 = _____

20 What patterns do you notice in the problems? Explain.

21 Draw a model to show how you solved one of the problems.

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3 **a.** $2 \times 10 = 20$
b. $4 \times 5 = 20$
Medium

4 See Student Worktext page.
Medium

5 See Student Worktext page.
Medium

3 Cole arranges his blueberries into different arrays before he eats them. Write a multiplication fact for each array.

a. 

2 × 10 = 20

b. 

4 × 5 = 20

4 Fill in the blanks to complete the multiplication facts for 2.

$0 \times 2 =$ 0
$1 \times 2 =$ 2
$2 \times 2 =$ 4
$3 \times 2 =$ 6
$4 \times 2 =$ 8
$5 \times 2 =$ 10
$6 \times 2 =$ 12
$7 \times 2 =$ 14
$8 \times 2 =$ 16
$9 \times 2 =$ 18
$10 \times 2 =$ 20

5 Fill in the blanks to complete the multiplication facts for 5.

$0 \times 5 =$ 0
$1 \times 5 =$ 5
$2 \times 5 =$ 10
$3 \times 5 =$ 15
$4 \times 5 =$ 20
$5 \times 5 =$ 25
$6 \times 5 =$ 30
$7 \times 5 =$ 35
$8 \times 5 =$ 40
$9 \times 5 =$ 45
$10 \times 5 =$ 50

ELL English Language Learners: Differentiated Instruction

Prepare for Session 3
Use with *Try It*.

Levels 1–3

Listening/Writing To prepare for the ***Try It*** problem, place on the floor a 0–9 number line, using a separate sheet of paper for each digit. Stand on 0, step, and say: *I am going across six sheets at one time. I stop on the number 6.* Display $1 \times 6 = 6$. Repeat the activity, stepping from 0 to 1, from 1 to 2, and so on until you get to 6. Display $6 \times 1 = 6$. Select students to replicate the activity for facts with 1 as a factor. Display the sentence frame: *When 1 is a factor, the product is the other* ___*factor*___ . Repeat the same activity for facts with 0 as a factor. Provide the sentence frame: *When 0 is a factor, the product is* ___*zero*___ . Have students copy and complete the sentence frames.

Levels 2–4

Listening/Writing To prepare for the ***Try It*** problem, place on the floor a 0–9 number line, using a separate sheet of paper for each digit. Stand on 0, step, and say: *I am going across six sheets at one time. I stop on the number 6.* Display $1 \times 6 = 6$. Repeat the activity, stepping from 0 to 1, from 1 to 2, and so on until you get to 6. Display $6 \times 1 = 6$. Select students to model the actions for facts with 1 as a factor. Repeat the same modeling and guidance for facts with 0 as a factor. Provide the sentence frame: *When* _____ *is a factor, the product is* _____.

Have students form pairs. Ask them to discuss, copy, and complete in their math journal the sentence frames for the factors of 1 and 0.

Levels 3–5

Listening/Writing To prepare for the ***Try It*** problem, place on the floor a 0–9 number line, using a separate sheet of paper for each digit. Select students to model going across six sheets at one time and stepping from sheet to sheet six times. Display $1 \times 6 = 6$ and $6 \times 1 = 6$. Have students model the process for other facts with 1 as a factor. Repeat the same modeling and guidance for facts with 0 as a factor. Provide the sentence frame:

When _____ *is a factor, the product is* _____.

Have students form pairs. Ask them to discuss, copy, and complete in their math journal the sentence frame for the factors of 1 and 0.

LESSON 5

SESSION 3 Develop

Purpose In this session students solve a problem that requires thinking about what it means to multiply with 0 and 1. Students model the given multiplication facts either on paper or with manipulatives to identify true and false statements. The purpose of this session is to have students identify patterns in multiplying with 0 and 1.

Start

Connect to Prior Knowledge

Why Prepare students for multiplying with 0 and 1.

How Practice repeated addition with 0 and 1.

Find each sum.

$0 + 0 + 0 + 0 + 0 + 0 =$

$1 + 1 + 1 + 1 =$

Solutions

0

4

Develop Language

Why Clarify the meaning of the multiple-meaning word *right*.

How Display and read:

Right is the opposite of left.

Right is the opposite of wrong.

Ask students to read the question in ***Try It***. Have partners consider the displayed sentences and discuss which meaning for *right* is being used in the problem. Ask students to explain their reasoning.

TRY IT

Make Sense of the Problem

To support students in making sense of the problem, have them understand that they are trying to find out which of the given equations is correct.

LESSON 5 SESSION 3 ● ● ● ○

Develop Multiplying with 0 and 1

Read and try to solve the problem below.

Jon says 6 × 1 = 6. Jeff says 6 × 0 = 6. Who is right? Explain how you know.

TRY IT

Possible student work:

Sample A

1 1 1 1 1 1

6 groups of 1 = 6

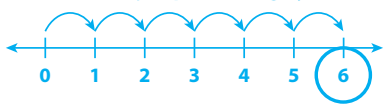
0 0 0 0 0 0

6 groups of 0 = 0

Jon is right. 6 groups of 0 is 0 and not 6.

Sample B

6 × 1 is 6 jumps counting by 1s.



6 × 0 is 6 jumps counting by 0.

When I count by 0, I stay in the same place. It does not matter how many times I jump, I am not going anywhere. Jon is right. 6 × 1 = 6.

Math Toolkit

- counters
- cups
- number lines
- 1-centimeter grid paper

DISCUSS IT

Ask your partner: Can you explain that again?

Tell your partner: I do not understand how . . .

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DISCUSS IT

Support Partner Discussion

Encourage students to use the term *equal groups* as they discuss their solutions.

Support as needed with questions such as:

- *Did you use the same model to check both equations?*
- *Did you consider other types of models?*

Common Misconception Look for students who have trouble modeling the quantity 0. Have them draw 6 circles on a piece of paper and write zero Xs in each circle. Then ask them how many Xs there are in all.

Select and Sequence Student Solutions

One possible order for whole class discussion:

- drawing an equal-group model
- drawing an array model
- drawing a number line model
- reasoning without a visual model

Support Whole Class Discussion

Compare and connect the different representations and have students identify how they are related.

Ask *What does the factor 6 represent in each equation? How do the models show that? What do the 1 and 0 represent? How are they shown in the models?*

Listen for The factor 6 represents the number of equal groups (or rows in an array). The 1 and 0 represent the number in each group (or row).

MODEL ITS

If no student presented these models, connect them to the student models by pointing out the ways they each represent:

- six groups
- equal groups of 1
- equal groups of 0

Ask *How many groups are there in the model for 6 × 1? How many are in each group? How many groups are there in the model for 6 × 0? How many are in each group? How are groups of zero shown?*

Listen for The model for 6 × 1 shows six groups of 1. The model for 6 × 0 shows six groups of 0, or six groups with nothing in them.

For a model of 6 × 1, prompt students to explain how each factor is represented by a model showing equal groups.

- *What does the "6" tell you? What does the "1" tell you?*
- *How will you use the model to find the product?*

For a model of 6 × 0, prompt students to explain how they can model groups of 0.

- *How many groups are there? How many in each group?*
- *How do you show that there are 0 in each group?*

Explore different ways to understand multiplying with 0 and 1.

Jon says 6 × 1 = 6. Jeff says 6 × 0 = 6. Who is right? Explain how you know.

MODEL IT

You can use equal groups to understand multiplying with 1.

6 × 1 means there are 6 groups with 1 in each group.

MODEL IT

You can use equal groups to understand multiplying with 0.

6 × 0 means there are 6 groups with 0 in each group. A group of 0 is empty.

Deepen Understanding

The Number 0

SMP 6 Attend to precision.

When discussing the model for 6 × 0, prompt students to think about 0.

Ask *How many is 0? Hold up 0 fingers. Take 0 steps. Draw 0 stars.*

Listen for Zero means "not any." Students do nothing for each command.

Draw 5 stars on the board.

Ask *If I draw 0 more stars, how many will there be? If I erase 0 stars, how many will there be? Does adding or subtracting 0 change a quantity?*

Listen for Still 5 stars. Adding or subtracting 0 does not change a quantity.

Write the number 5 on the board.

Ask *If I write a 0 to the right of the 5, does that change the number? Explain.*

Listen for 5 becomes 50 because the place value of the 5 changes.

Generalize *Does including a 0 change a value? Always? Never? Sometimes?*
Including 0 can sometimes change a value, but not when you add or subtract 0.

CONNECT IT

- Remind students that one thing that is alike about all the representations is the numbers.
- Explain that on this page, students will further explore multiplying with 0 and 1.

Monitor and Confirm

1–3 Check for understanding that:

- $6 \times 1 = 6$, so Jon is correct
- when there is only one in each of a certain number of groups, the total is the same as the number of groups

Support Whole Class Discussion

4–5 Be sure students understand that there are different results when multiplying with 0 than when multiplying with 1.

Ask *What do groups of 0 look like? Do you have to count anything to find the product? Why or why not? Does it matter how many groups of 0 there are?*

Listen for Groups of 0 have nothing in them. There is nothing there to count, so you can see that the product is 0. It doesn't matter how many groups there are.

6 Look for the understanding that a number multiplied by 1 equals itself and that a number multiplied by 0 equals 0.

7 REFLECT Have all students focus on the strategies used to solve this problem. If time allows, have students share their preferences with a partner.

CONNECT IT

Now you will use the problem from the previous page to help you understand how to multiply with 1 and 0.

1 Look at the first **Model It** for 6×1. There are 6 groups of 1, so $6 \times 1 =$ 6. Is Jon right? Yes.

2 Draw the first **Model It** and add another group of 1.

Now there are 7 groups of 1, so 7 $\times 1 =$ 7.

3 What do you notice about the number of groups and the product when you multiply 6×1? 7×1?

They are the same number.

4 Look at the second **Model It** for 6×0. There are 6 groups of 0, so $6 \times 0 =$ 0. Is Jeff right? No.

5 Explain what would happen if more groups of 0 were added.

Possible answer: 0 will always be the product because adding 0 to 0 will still be 0.

6 What do you think is true about the product of any number multiplied by 1? Multiplied by 0?

Possible answers: If a number is multiplied by 1, the product is always the number you started with. If a number is multiplied by 0, the product is always 0.

7 REFLECT

Look back at your **Try It**, strategies by classmates, and **Model Its**. Which models or strategies do you like best for multiplying by 1 and 0? Explain.

Some students may prefer to use the properties to multiply now that they understand multiplying by 1 will not change a number and multiplying by 0 is always 0. Others may still prefer to think of multiplication as equal groups to find a product.

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Hands-On Activity

Model multiplication with 0 and 1 using cups and counters.

If . . . *students are unsure about the rules for multiplying with 0 and 1,*

Then . . . *have them use manipulatives to test each rule.*

Materials For each student: 10 counters, 10 cups

- Explain to students that they will model equal groups by putting the same number of counters into several cups.
- Have them model 6×1 and find the product by counting all the counters. Ask: *How many cups do you need? How many counters in each cup? How many counters in all?*
- Tell them to repeat the process for 7×1, 8×1, and 9×1.
- Have students model 6×0. Ask: *How many cups do you need? How many counters in each cup? How many counters in all?* Be sure they understand why there are no counters in the cups.
- Tell them to repeat the process for 7×0, 8×0, and 9×0.
- Ask students if their work supports the answer they gave in problem 6. If not, have them discuss with a partner to resolve.

APPLY IT

Encourage students to answer problems 8 and 9 using mental math and reasoning about 0 and 1.

8 **a.** 0; **b.** 5; **c.** 0; **d.** 3

9 **A**, **B**, **E**; Each fact with a factor of 0 has a product of 0.

Close: Exit Ticket

10 0; Students should draw four groups of 0.

Students' solutions should indicate understanding of:

- multiplication can be modeled with equal groups
- any number of groups of 0 equals 0

Error Alert **If** students draw an accurate model but write "4" as the product, **then** review how the model represents each factor and the product. They may have counted the groups instead of the (0) items in the groups, or they may have ignored their model and answered according to the rule for multiplying with 1 instead of multiplying with 0.

APPLY IT

Use what you just learned to solve these problems.

8 Fill in the missing numbers to complete each fact.

a. $5 \times 0 =$ 0

b. 5 $\times 1 = 5$

c. $3 \times$ 0 $= 0$

d. $3 \times 1 =$ 3

9 Which of the following facts have a product of 0?

Ⓐ $1 \times 0 = ?$

Ⓑ $0 \times 1 = ?$

Ⓒ $10 \times 1 = ?$

Ⓓ $5 \times 1 = ?$

Ⓔ $5 \times 0 = ?$

10 Draw a model to show 4×0. Then find the product.

Possible student work:

$4 \times 0 =$ 0

Solutions

1 See Student Worktext page.
Medium

Name: _____ LESSON 5 SESSION 3

Practice Multiplying with 0 and 1

Study the Example showing how to multiply with 1. Then solve problems 1–4.

EXAMPLE

Steve uses a model to create a list of multiplication facts for 1. He starts with 0 equal groups of 1 and then keeps adding a group of 1 for each fact as shown. Describe a pattern he can use to find the 1s facts for 6, 7, 8, 9, and 10.

$0 \times 1 = 0$	
$1 \times 1 = 1$	★
$2 \times 1 = 2$	★★
$3 \times 1 = 3$	★★★
$4 \times 1 = 4$	★★★★
$5 \times 1 = 5$	★★★★★

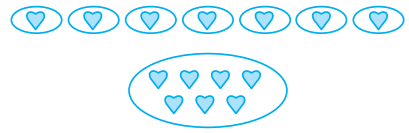
Steve can see that any number times 1 equals that number.

$6 \times 1 = 6$
$7 \times 1 = 7$
$8 \times 1 = 8$
$9 \times 1 = 9$
$10 \times 1 = 10$

The number of groups of 1 is the same as the product.

1 Create a model of 7×1 and 1×7. How are they different? How are they the same?

Possible student work:



Solution The first model shows 7 groups of 1, and the second shows 1 group of 7. They both have a total of 7 but are grouped differently.

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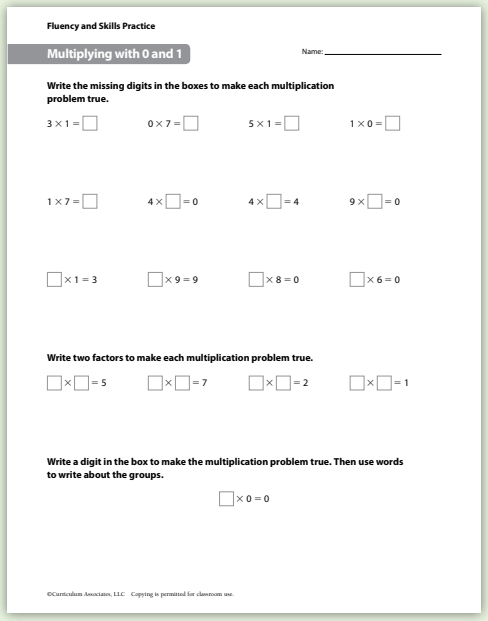
Fluency & Skills Practice

Teacher Toolbox

Assign *Multiplying with 0 and 1*

In this activity students practice multiplying numbers by 0 and 1. Students may encounter groups of 0 when something is empty. For example, if there are 0 animals in each reptile exhibit at the zoo, then there are 0 reptiles altogether. A single group (a group of 1) is also common. For example, if there is 1 animal in each reptile exhibit at the zoo, then the total number of reptiles equals the number of exhibits.



Fluency and Skills Practice

Multiplying with 0 and 1 Name: _____

Write the missing digits in the boxes to make each multiplication problem true.

$3 \times 1 = \square$ $0 \times 7 = \square$ $5 \times 1 = \square$ $1 \times 0 = \square$

$1 \times 7 = \square$ $4 \times \square = 0$ $4 \times \square = 4$ $9 \times \square = 0$

$\square \times 1 = 3$ $\square \times 9 = 9$ $\square \times 8 = 0$ $\square \times 6 = 0$

Write two factors to make each multiplication problem true.

$\square \times \square = 5$ $\square \times \square = 7$ $\square \times \square = 2$ $\square \times \square = 1$

Write a digit in the box to make the multiplication problem true. Then use words to write about the groups.

$\square \times 0 = 0$

2 $0 \times 8 = 0$;
$1 \times 10 = 10$;
$1 \times 5 = 5$;
$0 \times 6 = 0$;
Challenge

3 **B** (**No**);
D (**No**);
E (**Yes**);
H (**No**);
Basic

4 See Student Worktext page.
Challenge

2 Jenna makes a table to show the school supplies she has. Write a multiplication fact to show how many of each school item Jenna has.

Materials	Number of Boxes	Multiplication Fact
Box of 8 crayons	0	**0 × 8 = 0**
Box of 10 pencils	1	**1 × 10 = 10**
Box of 5 erasers	1	**1 × 5 = 5**
Box of 6 markers	0	**0 × 6 = 0**

3 Is each multiplication fact correct?

	Yes	No
$1 \times 0 = 1$	Ⓐ	Ⓑ
$9 \times 1 = 0$	Ⓒ	Ⓓ
$0 \times 5 = 0$	Ⓔ	Ⓕ
$6 \times 0 = 6$	Ⓖ	Ⓗ

4 Xavier starts to create a list of multiplication facts for 1. Explain the mistake he is making. What will make his facts correct?

$1 \times 1 = 2$
$2 \times 1 = 3$
$3 \times 1 = 4$

Possible answer: Xavier is adding the factors instead of multiplying. To correct these facts, Xavier must add the given number of groups of 1, not add the factors.

ELL English Language Learners: Differentiated Instruction

Prepare for Session 4
Use with *Apply It*.

Levels 1–3

Listening/Speaking To work with ***Apply It*** problem 3, list and say the terms: *factor*, *product*, *rows*, *columns*, and *array*. Have students repeat each word after you. Give each student 10 counters. Display 1 row of 10 counters. Say: *This array is the model for $1 \times 10 = 10$.* Ask students to explain the connection between each number in the equation and the model. Repeat the steps for $10 \times 1 = 10$. Ask: *How can we use 10 counters to make another array that has a product of 10?* Have students form pairs to discuss and work. If needed, repeat the steps for $5 \times 2 = 10$ and $2 \times 5 = 10$. List all of the equations. Say: *Multiplication facts can have the same product, but the arrays may be different.*

Levels 2–4

Listening/Speaking To work with ***Apply It*** problem 3, list the terms: *factor*, *product*, *rows*, *columns*, and *array*. Have students read the list chorally. Give each student 10 counters. Display and have students replicate 1 row of 10. Say: *This array is the model for $1 \times 10 = 10$.* Have students explain the connection between each number in the equation and the model. Ask: *How can we use 10 counters to make other arrays that have a product of 10?* Have students form pairs to discuss and work. Repeat the same process for $10 \times 1 = 10$, $5 \times 2 = 10$, and $2 \times 5 = 10$, as the arrays are suggested. List all of the equations. Ask: *What is the same about all of the arrays and equations? What is different?*

Levels 3–5

Listening/Speaking Pair students for ***Apply It*** problem 3. Give each pair 10 counters. List the terms: *factor*, *product*, *rows*, *columns*, and *array*. Ask: *How can we use 10 counters and make arrays that have a product of 10?* Provide time for pairs to discuss and work. List the equations $1 \times 10 = 10$, $10 \times 1 = 10$, $5 \times 2 = 10$, and $2 \times 5 = 10$, as the arrays are suggested. Have students explain the connection between each number in the equation and the corresponding model. Ask: *What is the same about all of the arrays and equations? What is different?* If time permits, repeat the activity with eight counters.

LESSON 5

SESSION 4 Refine

Purpose In this session students solve word problems involving multiplying with 0, 1, 2, 5, and 10 and then discuss and confirm their answers with a partner.

Before students begin work, use their responses to the *Check for Understanding* to determine those who will benefit from additional support.

As students complete the Example and problems 1–3, observe and monitor their reasoning to identify groupings for differentiated instruction.

Start

Check for Understanding

Materials For remediation: counters

Why Confirm understanding of multiplying with 0, 1, 2, 5, and 10.

How Have students find 3×0, 9×1, and 7×5 using any strategy they want.

Find each product.

$3 \times 0 =$

$9 \times 1 =$

$7 \times 5 =$

Solutions

0

9

35

LESSON 5 SESSION 4 ● ● ● ●

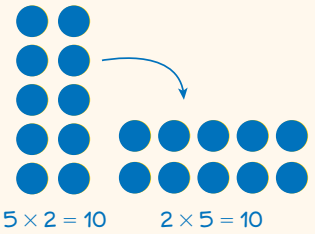
Refine Multiplying with 0, 1, 2, 5, and 10

Complete the Example below. Then solve problems 1–9.

EXAMPLE

Liam says 2×5 has the same product as 5×2. Do you agree?

Look at how you could show your work using an array.



$5 \times 2 = 10$ $2 \times 5 = 10$

Solution **Yes, 2×5 and 5×2 both equal 10.**

Liam created 5 rows of 2 and 2 rows of 5.

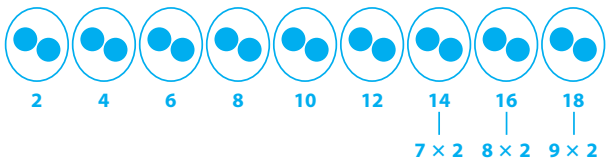
PAIR/SHARE

How can you find the total once you create an array?

APPLY IT

1 Find 7×2. Then find 8×2 and 9×2 using the same model. Explain the pattern you see in the products. Show your work.

Possible student work:



Solution **$7 \times 2 = 14$, $8 \times 2 = 16$, and $9 \times 2 = 18$. Each product is 2 more than the previous product.**

How many are in each group?

PAIR/SHARE

How did finding all three facts with the same model help you see the pattern?

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Error Alert

If the error is . . .	*Students may . . .*	*To support understanding . . .*
$3 \times 0 = 3$ and $9 \times 1 = 10$ and $7 \times 5 = 12$	have added instead of multiplying.	Remind students that $\times$ is the multiplication symbol and can be read as *groups of*. Have students model each problem with drawings or counters.
$3 \times 0 = 3$ and $9 \times 1 = 1$	have confused the patterns for multiplying with 0 and 1.	Have students draw three groups of 0 and nine groups of 1 to check their answers. Repeat with other numbers if necessary to guide them to realize that the product of any number and 0 is 0, and the product of any number and 1 is the original number.
$7 \times 5 = 60$ (or some other multiple of 10)	have switched from counting by fives to counting by tens.	Review skip-counting by fives with students. Remind them to slow down while counting so that they can pay attention to the numbers.
$7 \times 5 = 40$ (or some other higher multiple of 5)	have forgotten when to stop counting.	Remind students to slow down and pay attention to the numbers. Suggest that they use their fingers or pencil and paper to keep track of how many 5s they should count.

EXAMPLE

Yes; the arrays shown are one way to solve the problem. Students could also solve the problem by drawing five equal groups of 2 and two equal groups of 5 to check that they both equal 10.

Look for Students understand that both arrays represent the same quantity.

APPLY IT

1. $7 \times 2 = 14$; $8 \times 2 = 16$; $9 \times 2 = 18$; See Student Worktext page for possible explanation. Students could solve the problem with seven, eight, and nine groups of 2 or with arrays that have seven, eight, and nine rows of 2.
   DOK 2

   Look for All the groups are groups of 2.

2. 7 apples, 0 oranges, 30 peaches; Students could solve the problem using arrays or equal groups.
   DOK 2

   Look for Students apply the rules for multiplying with 1 and 0.

3. **A**; Students could solve the problem by skip-counting or by drawing equal groups or an array.

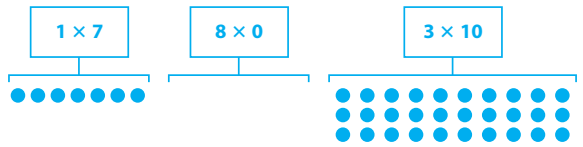
   Explain why the other two answer choices are not correct:

   B is not correct because $5 \times 5 = 25$. The numbers should be multiplied, not added.

   D is not correct because $1 \times 9 = 9$. The numbers should be multiplied, not added.
   DOK 2

2 Rami has 1 bag with 7 apples, 8 bags with 0 oranges, and 3 bags with 10 peaches. How many apples, oranges, and peaches does Rami have? Show your work.

Possible student work:



1 × 7 = 7, 8 × 0 = 0, and 3 × 10 = 30

Solution **7 apples, 0 oranges, and 30 peaches**

Think about what you know about multiplying with 0 and 1.

PAIR/SHARE
How are your models the same as your partner's models? How are they different?

3 Which of the following equals 10?

Ⓐ 2×5

Ⓑ 5×5

Ⓒ 10×0

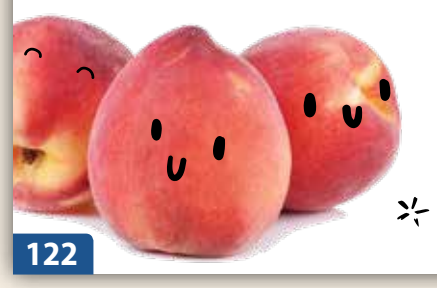
Ⓓ 1×9

Rey chose Ⓒ as the correct answer. How did he get that answer?

Rey may have confused adding with multiplying.

Find the product of each choice first.

PAIR/SHARE
What strategy for multiplying do you like best?



4 **B**; Each product matches the factor given, so the other factor in each equation is 1.
DOK 1

5 See Student Worktext page.
DOK 1

6 **A** (**True**);
D (**False**);
E (**True**);
H (**False**);
I (**True**);
K (**True**)
DOK 1

Error Alert Students may choose C (True) and G (True) if they add instead of multiply or if they confuse the reasoning for multiplying with 0 with the reasoning for multiplying with 1.

4 Which factor will correctly complete all of the following facts:

$1 \times \_\_\_\_ = 1$ $2 \times \_\_\_\_ = 2$ $3 \times \_\_\_\_ = 3$ $4 \times \_\_\_\_ = 4$

Ⓐ 0
Ⓑ 1
Ⓒ 2
Ⓓ 10

5 Fill in the blanks to complete the multiplication facts for 10.

$0 \times 10 =$ 0 $6 \times 10 =$ 60
$1 \times 10 =$ 10 $7 \times 10 =$ 70
$2 \times 10 =$ 20 $8 \times 10 =$ 80
$3 \times 10 =$ 30 $9 \times 10 =$ 90
$4 \times 10 =$ 40 $10 \times 10 =$ 100
$5 \times 10 =$ 50

6 Is each multiplication fact *True* or *False*?

	True	False
$7 \times 2 = 14$	Ⓐ	Ⓑ
$10 \times 0 = 10$	Ⓒ	Ⓓ
$1 \times 10 = 10$	Ⓔ	Ⓕ
$5 \times 0 = 5$	Ⓖ	Ⓗ
$2 \times 1 = 2$	Ⓘ	Ⓙ
$3 \times 10 = 30$	Ⓚ	Ⓛ

Differentiated Instruction

RETEACH

Hands-On Activity
Use base-ten blocks to skip-count by tens.

Students struggling with skip-counting to multiply with 10

Will benefit from additional work with concrete representations of groups of ten.

Materials For each student: base-ten blocks (10 tens rods)

- Ask: *How is a tens rod like a group of ten? Why can you think of all tens rods as equal groups?*
- Tell students to model 7×10 with the tens rods. Have them skip-count to find the product.
- Ask: *What number did you count up to? What number is seven 10s equal to?*
- Repeat for other multiplication facts for 10.

EXTEND

Challenge Activity
Write a true multiplication fact using just one number.

Students who have achieved proficiency

Will benefit from deepening understanding of multiplying with 0 and 1.

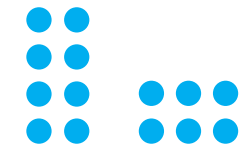
- Write the following on the board:

  _____ × _____ = _____
- Ask: *What same number can you write in all three blanks to make a true equation? Is there more than one number that will work?*

  [Answers: 0, 1]

7 Emile has 4 packs of shirts. Each pack has 2 shirts. He also has 2 packs of shorts. Each pack has 3 shorts. Does he have more shirts or shorts? Show your work.

Possible student work:



Solution **Emile has more shirts; 8 > 6.**

8 Principal Green talks to 5 different students every school day. How many students does she talk to in 10 school days?

Noa says this is a 10 groups of 5 problem and can be solved by multiplying 10 × 5 or skip-counting by fives 10 times. Sara says this problem can be solved by skip-counting by tens 5 times or finding 5 × 10. Who is correct? Explain and provide the answer.

Both are correct. 10 days of 5 students can be represented by finding 10 × 5 or skip-counting by fives 10 times, which is 50. But 5 × 10 or skip-counting by tens 5 times is also 50. Principal Green talks to 50 students.

9 MATH JOURNAL

Explain how you would solve the problem below. What multiplication fact could you use?

Lauren paints 8 paintings. She puts 2 trees in each painting. How many trees does Lauren paint?

Possible answer: This is an 8 groups of 2 problem. I can skip-count by twos 8 times: 2, 4, 6, 8, 10, 12, 14, 16. I could also use the multiplication fact 8 × 2 = 16. Lauren paints 16 trees.

SELF CHECK Go back to the Unit 2 Opener and see what you can check off.

7 Emile has more shirts than shorts; He has $4 \times 2 = 8$ shirts and $2 \times 3 = 6$ shorts; $8 > 6$
DOK 2

8 Both are correct. See Student Worktext page for possible explanation.
DOK 3

REINFORCE

Problems 4–9

Multiply with 0, 1, 2, 5, and 10.

All students will benefit from additional work with multiplying with 0, 1, 2, 5, and 10 by solving problems in a variety of formats.

- Have students work on their own or with a partner to solve the problems.
- Encourage students to show their work.

PERSONALIZE

i-Ready

Provide students with opportunities to work on their personalized instruction path with *i-Ready* Online Instruction to:

- fill prerequisite gaps
- build up grade-level skills

Close: Exit Ticket

9 MATH JOURNAL Student responses should indicate understanding that a problem involving equal groups can be represented by a multiplication equation and solved with skip-counting or by making a model such as an array or a drawing of equal groups.

Error Alert **If** students use the multiplication fact $2 \times 4 = 8$, **then** have students reread the problem and identify the number of groups and the number in each group, drawing models if necessary.

SELF CHECK Have students consider whether they feel they are ready to check off any new skills on the Unit 2 Opener.

Lesson Overview

LESSON 11

Understand How Multiplication and Division Are Connected

Lesson Objectives

Content Objectives

- Understand the relationship between multiplication and division.
- Demonstrate informally that related multiplication and division equations form fact families.
- Find the unknown number in a whole-number multiplication or division equation.

Language Objectives

- Describe the relationship between multiplication and division using words or diagrams.
- Correctly use the terms *array*, *divide*, *divided by*, *times*, *factor*, *product*, and *quotient* when discussing multiplication and division.

Prerequisite Skills

- Understand the meaning of multiplication and division.
- Use an equation to represent and solve a multiplication or division word problem involving equal groups, arrays, and equal measurement quantities.

Standards for Mathematical Practice (SMP)

SMPs 1, 2, 3, 4, 5, and 6 are integrated in every lesson through the *Try-Discuss-Connect* routine.*

In addition, this lesson particularly emphasizes the following SMPs:

2 Reason abstractly and quantitatively.

6 Attend to precision.

7 Look for and make use of structure.

*See page 1i to see how every lesson includes these SMPs.

Lesson Vocabulary

- **quotient** the result of division.

Review the following key terms.

- **divide** to separate into equal groups and find the number in each group or the number of groups.
- **division equation** an equation with a division symbol and an equal sign. For example, $15 \div 3 = 5$.
- **factor** a number that is multiplied.
- **multiplication** an operation used to find the total number of items in a given number of equal-sized groups.
- **multiplication equation** an equation with a multiplication symbol and an equal sign. For example, $3 \times 5 = 15$.
- **multiply** to repeatedly add the same number a certain number of times. Used to find the total number of items in equal-sized groups.
- **product** the result of multiplication.

Learning Progression

In the previous lesson students explored the meaning of division.

In this lesson students extend their understanding of both division and multiplication by recognizing how the two operations are related. In multiplication, a number of equal-sized groups are combined to find the total (or product). In division, a known total is separated into a certain number of equal-sized groups.

Students are informally introduced to multiplication and division fact families and explore the relationships among fact family numbers.

Knowing how multiplication and division are related helps students work toward fluency in single-digit multiplication and division, a key goal by the end of Grade 3.

In the next lesson students formally learn multiplication and division fact families and use a multiplication table to find related multiplication and division facts.

In Grade 4 students use their knowledge of multiplication and division facts to represent and solve more complex multiplication and division problems, including those that involve multiplicative comparisons.

Lesson Pacing Guide

Whole Class Instruction

SESSION 1
Explore
45–60 min

Interactive Tutorial* *(Optional)*
Prerequisite Review:
Understand Division, Part 1

How Multiplication and Division Are Connected
- Start *5 min*
- Model It *10 min*
- Discuss It *5 min*
- Model It *10 min*
- Discuss It *10 min*
- Close: Exit Ticket *5 min*

Additional Practice
Lesson pages 241–242

SESSION 2
Develop
45–60 min

Understanding of How Multiplication and Division Are Connected
- Start *5 min*
- Model It: Multiplication and Division Situations *5 min*
- Discuss It *5 min*
- Model It: Multiplication and Division Facts *5 min*
- Discuss It *5 min*
- Connect It *15 min*
- Close: Exit Ticket *5 min*

Additional Practice
Lesson pages 245–246

Fluency
Understanding of How Multiplication and Division Are Connected

SESSION 3
Refine
45–60 min

Ideas About How Multiplication and Division Are Connected
- Start *5 min*
- Apply It *35 min*
- Close: Exit Ticket *5 min*

Lesson Quiz or **Digital Comprehension Check**

Lesson Materials

Lesson *(Required)*	*Per student:* 12 counters
Activities	*Per pair:* 20 stickers, 5 index cards, 1 number cube, 2 markers *Activity Sheet:* Multiplication Triangles
Digital Math Tools	Number Line, Multiplication Models

*We continually update the Interactive Tutorials. Check the Teacher Toolbox for the most up-to-date offerings for this lesson.

Teacher Toolbox

Small Group Differentiation

PREPARE

Ready Prerequisite Lesson

Grade 2
- Lesson 31 Add Using Arrays

RETEACH

Tools for Instruction

Grade 2
- Lesson 31 Adding Using Arrays

Grade 3
- Lesson 11 Relate Multiplication and Division

REINFORCE

Math Center Activities

Grade 3
- Lesson 11 Use a Related Fact
- Lesson 11 Find the Missing Number
- Lesson 11 Use Multiplication to Solve Division

EXTEND

Enrichment Activity

Grade 3
- Lesson 11 Arranging Desks

i-Ready

Independent Learning

PERSONALIZE

i-Ready Lessons*

Grade 3
- Understand Division, Part 2
- Practice: Understand Division

Connect to Family, Community, and Language Development

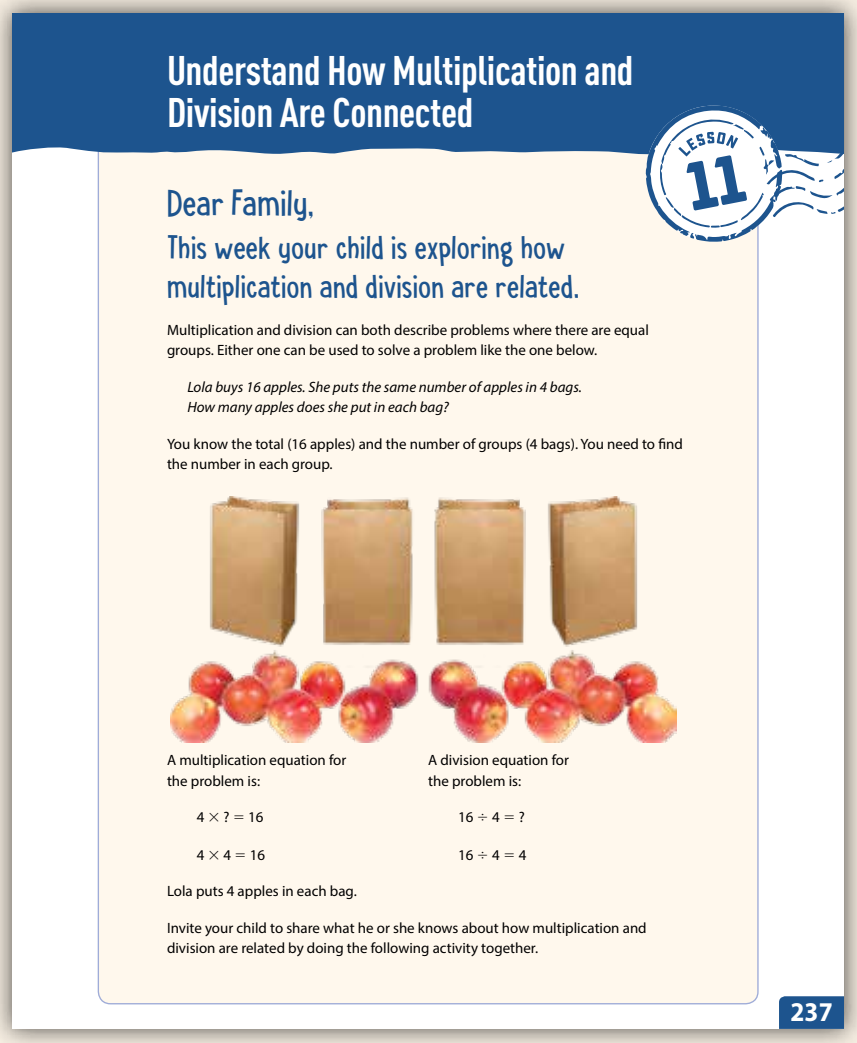
The following activities and instructional supports provide opportunities to foster school, family, and community involvement and partnerships.

Connect to Family

Use the **Family Letter**—which provides background information, math vocabulary, and an activity—to keep families apprised of what their child is learning and to encourage family involvement.

Available in Spanish

Teacher Toolbox



Understand How Multiplication and Division Are Connected

LESSON 11

Dear Family,

This week your child is exploring how multiplication and division are related.

Multiplication and division can both describe problems where there are equal groups. Either one can be used to solve a problem like the one below.

Lola buys 16 apples. She puts the same number of apples in 4 bags. How many apples does she put in each bag?

You know the total (16 apples) and the number of groups (4 bags). You need to find the number in each group.

A multiplication equation for the problem is:

$4 \times ? = 16$

$4 \times 4 = 16$

A division equation for the problem is:

$16 \div 4 = ?$

$16 \div 4 = 4$

Lola puts 4 apples in each bag.

Invite your child to share what he or she knows about how multiplication and division are related by doing the following activity together.

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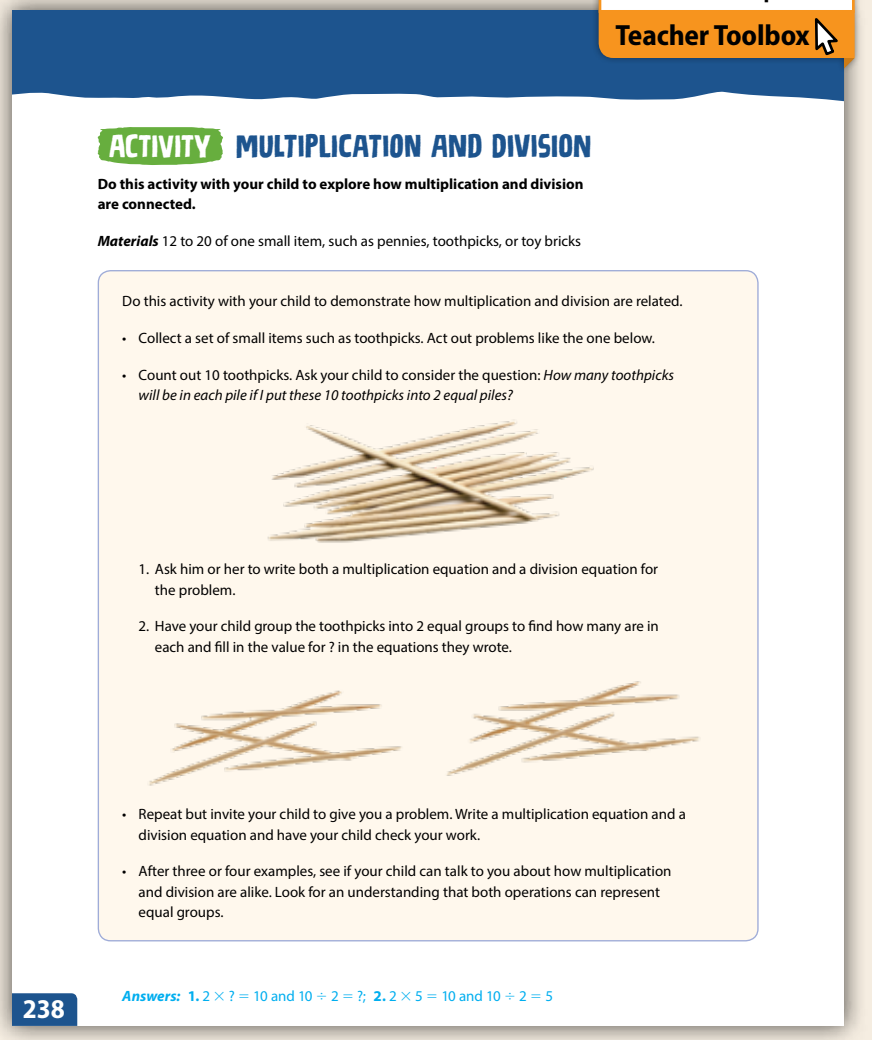

ACTIVITY MULTIPLICATION AND DIVISION

Do this activity with your child to explore how multiplication and division are connected.

Materials 12 to 20 of one small item, such as pennies, toothpicks, or toy bricks

Do this activity with your child to demonstrate how multiplication and division are related.

- Collect a set of small items such as toothpicks. Act out problems like the one below.
- Count out 10 toothpicks. Ask your child to consider the question: *How many toothpicks will be in each pile if I put these 10 toothpicks into 2 equal piles?*

1. Ask him or her to write both a multiplication equation and a division equation for the problem.
2. Have your child group the toothpicks into 2 equal groups to find how many are in each and fill in the value for ? in the equations they wrote.

- Repeat but invite your child to give you a problem. Write a multiplication equation and a division equation and have your child check your work.
- After three or four examples, see if your child can talk to you about how multiplication and division are alike. Look for an understanding that both operations can represent equal groups.

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Answers: **1.** $2 \times ? = 10$ and $10 \div 2 = ?$; **2.** $2 \times 5 = 10$ and $10 \div 2 = 5$

Goal

The goal of the Family Letter is to explain the inverse relationship of multiplication and division. Both operations can be used to find solutions to problems that involve equal groups.

Activity

Understanding the inverse relationship of multiplication and division supports the development of numerical fluency. Look at the *Multiplication and Division* activity and adjust it if necessary to connect with your students.

Math Talk at Home

Encourage students to talk with their family members about how multiplication and division are related. Deciding how to equally share grapes or equally share the cost of a gift can be determined using multiplication or division.

Conversation Starters Below are additional conversation starters students can write in their Family Letter or math journal to engage family members.

- *How can you organize 18 books in equal groups of 3?*
- *How many groups do you get?*
- *How can you check your answer?*

Connect to Community and Cultural Responsiveness

Use these activities to connect with and leverage the diverse backgrounds and experiences of all students.

Session 1 **Use anytime during this session.**

- Ask students why people are taught how to share. Ask students to tell how they feel when things such as books, toys, or treats are not shared fairly with them. Explain the difference between sharing and sharing fairly. Act out a situation of sharing something, like pencils, unfairly. Contrast sharing unfairly with sharing the same item fairly. Stress that sharing fairly means items are shared equally so that everyone receives the same number of items. Repeat the role-play with a different item to emphasize the concept. Reiterate that sharing something does not always guarantee that it will be shared fairly. Explain that by using division it is possible to determine how to share an item equally.

Connect to Language Development

For ELLs, use the Differentiated Instruction chart to plan and prepare for specific activities in every session.

English Language Learners: Differentiated Instruction

Prepare for Session 1
Use with *Model It*.

Levels 1–3	Levels 2–4	Levels 3–5
Listening/Speaking Read ***Model It*** problem 3 aloud. Give partners 20 counters. Ask: *What do the counters represent?* Have partners draw five pages or provide five "pages." Ask: *Do the pages represent the groups or the number in each group?* Have the partners distribute the counters between the five pages. Record and say: *A division equation begins with the total number of items. What number will you use to begin the division equation? What number represents how the stickers/counters were divided? How many stickers/counters are in each group? Look at the equal groups you have made. What multiplication fact represents the equal groups?* Record: $20 \div 5 = 4$, $5 \times 4 = 20$. Review the meaning of each number.	**Listening/Speaking** Read ***Model It*** problem 3 aloud. Give partners 20 counters. Ask: *What do the counters represent?* Have partners draw five pages or provide five "pages." Have the partners distribute the counters between the five pages. Record and say: *What number will you use to begin the division equation? Why? What represents the groups? What number represents the groups? How many stickers/counters are in each group? Write a division equation. Look at the equal groups you made. Write a multiplication fact to represent the equal groups.* Select partners to share their equations. Say: *Discuss with your partner how the two equations are related. Be prepared to share your thinking.*	**Listening/Speaking** Have students read ***Model It*** problem 3. Give partners 20 counters. Ask: *What do the counters represent?* Have partners draw five pages or provide five "pages." Have the partners distribute the counters between the five pages. Say: *Write a division equation to represent the process you just completed. Look at the equal groups you made. Write a multiplication fact to represent the equal groups.* Select partners to share their equations. Say: *Discuss with your partner how the two equations are related. Be prepared to share your thinking.*

LESSON 11

SESSION 1 Explore

Purpose In this session students explore the idea that multiplication and division are inverse operations. Students see that in multiplication they combine equal groups to find the total number and that in division they start with the total number and divide into equal groups. Students realize that there are related multiplication and division equations involving the same three numbers.

Start

Connect to Prior Knowledge

Why Prepare students for connecting multiplication and division by practicing representing a multiplication equation with equal groups and an array.

How Draw equal groups and an array to represent 3×6.

Draw equal groups and an array to represent 3×6.

Possible Solutions

equal groups:
3 groups of 6

array: 3 rows of 6

MODEL IT

Read the problem at the top of the Student Worktext page. Remind students that arrays show equal groups using equal rows of objects and are used to represent both multiplication and division equations.

1 – 2 Tell students that they are going to use what they know about multiplication and division to connect the two operations. Then clarify the task and have students complete the problems.

Common Misconception **If** students have trouble writing the division problem in 1c, **then** remind them that the total comes right before the division symbol.

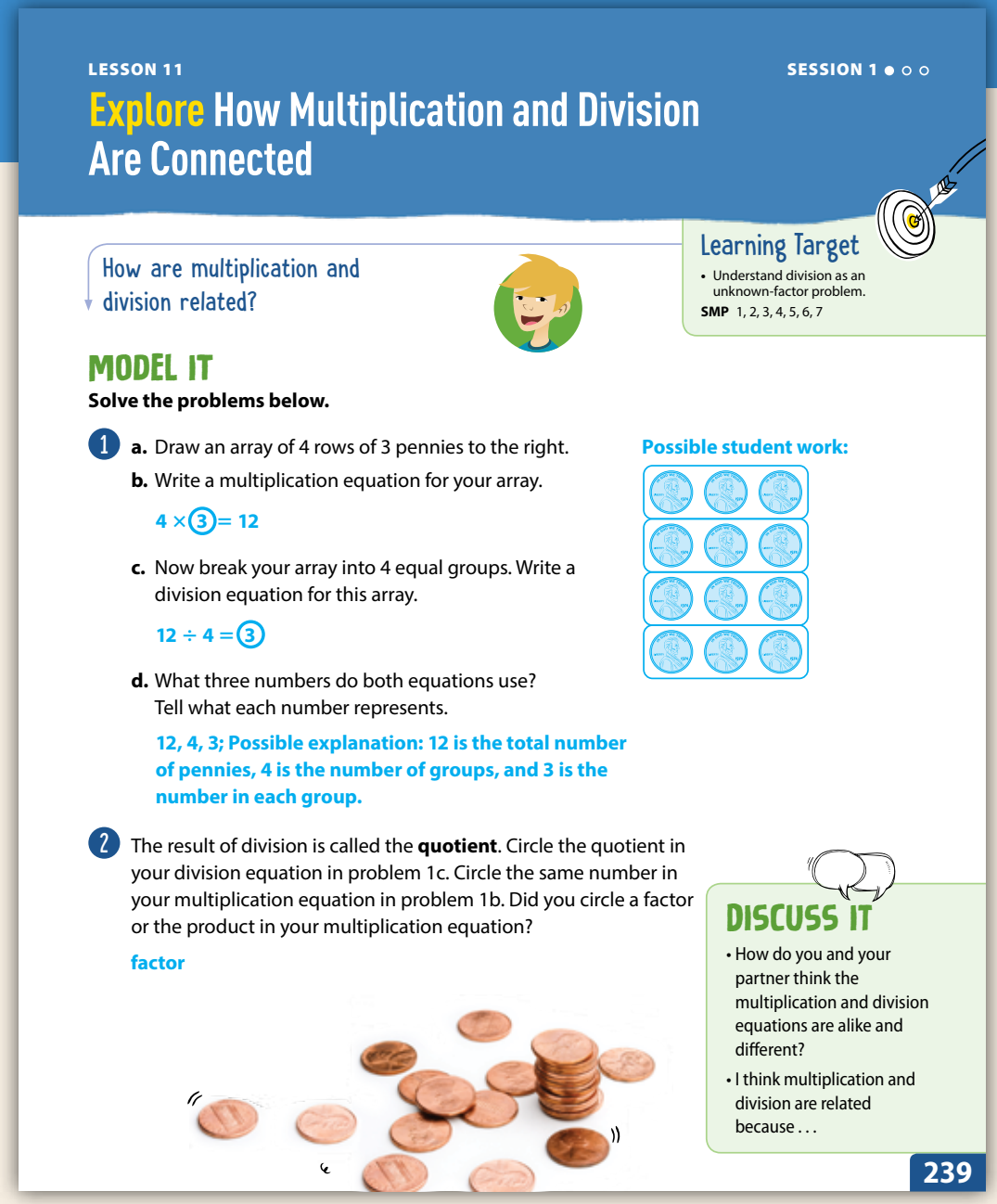
DISCUSS IT

Support Partner Discussion

Encourage students to use math vocabulary (*factor*, *product*, *quotient*) to describe the numbers in the problems.

Support as needed with questions such as:

- *How did your models show each factor? The total?*
- *How did you find the product? The quotient?*



LESSON 11 SESSION 1 ● ○ ○

Explore How Multiplication and Division Are Connected

How are multiplication and division related?

Learning Target

- Understand division as an unknown-factor problem.

SMP 1, 2, 3, 4, 5, 6, 7

MODEL IT

Solve the problems below.

1 **a.** Draw an array of 4 rows of 3 pennies to the right.

b. Write a multiplication equation for your array.

$4 \times 3 = 12$

c. Now break your array into 4 equal groups. Write a division equation for this array.

$12 \div 4 = 3$

d. What three numbers do both equations use? Tell what each number represents.

12, 4, 3; Possible explanation: 12 is the total number of pennies, 4 is the number of groups, and 3 is the number in each group.

Possible student work:

2 The result of division is called the **quotient**. Circle the quotient in your division equation in problem 1c. Circle the same number in your multiplication equation in problem 1b. Did you circle a factor or the product in your multiplication equation?

factor

DISCUSS IT

- How do you and your partner think the multiplication and division equations are alike and different?
- I think multiplication and division are related because . . .

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Support Whole Class Discussion

For each problem, have several students share their models and equations. Encourage the use of math vocabulary.

Ask *How did you know how to write the equations?*

Listen for For multiplication, the factors are the number of rows and columns, and the product is the total number of pennies in the array. For division, the total is the number before the division symbol, the number after the division symbol in this case is the number of equal groups, and the quotient is the number in each group.

Ask *How can 3 be both a factor and a quotient for this array?*

Listen for There are 4 equal groups with 3 pennies in each group, so 3 is a factor in the multiplication equation. In the division equation, you are told the number of equal groups and asked to find the number in each group, so 3 is the answer, or quotient, of the division equation.

MODEL IT

3 – 4 Tell students that they will now think about how to use a related multiplication equation to solve a division problem. Clarify the task and have students complete the problems.

Common Misconception **If** students have trouble explaining in problem 4, **then** ask them what roles the numbers before and after the division symbol play in a multiplication equation. [the product and a factor] Ask them how knowing these two values can help them find the other factor.

DISCUSS IT

Support Partner Discussion

Encourage students to refer to their models and equations as they discuss.

Look for representations showing:

- 20 total stickers on 5 pages with 4 stickers per page
- 20, 5, and 4 have the same meaning in both equations

Hands-On Activity

Act out the problem.

If . . . *students are unsure about how the model relates division and multiplication,*

Then . . . *use this activity to provide a more concrete experience.*

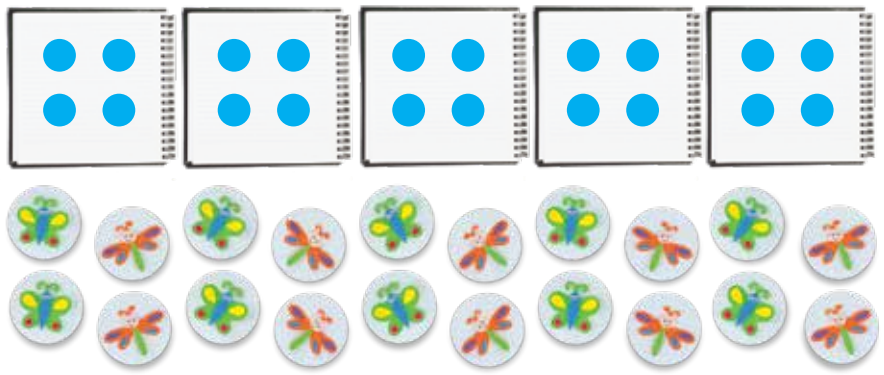
Materials For each pair: 20 stickers, 5 index cards

- Tell students they will act out the sticker problem.
- Elicit from students how many total stickers [20] and scrapbook pages (index cards) [5] they need.
- Have student pairs place the 20 stickers on the 5 cards one at a time. Guide students in placing one sticker on the first card, one on the next card, and so on, returning to the first card after placing a sticker on the last card until all stickers are placed.
- Together, count to find the number of stickers on each card [4].
- Write 20 ÷ 5 = ___ and 5 × ___ = 20 on the board. Ask students to tell what each equation means for this situation.
- Ask students if they find one equation easier to solve than the other and explain why.

MODEL IT

Complete the problems below.

3 Nick buys 20 stickers. He puts the same number of stickers on each of 5 pages in his scrapbook. Draw the stickers Nick puts on the pages. Write a division equation and a multiplication equation for this problem.



Division equation: 20 ÷ 5 = 4

Multiplication equation: 5 × 4 = 20

4 Explain how you could use a multiplication equation to find 20 ÷ 5.

Possible answer: You can think "5 times what number gives me 20?" 5 × 4 = 20, so 20 ÷ 5 = 4.

5 REFLECT

How is division doing the reverse of multiplication?

Possible answer: In multiplication, you start with a number of equal groups and put them together to find the total. In division, you start with the total and separate it into a number of equal groups.

DISCUSS IT

- What numbers are used in both the multiplication and division equations in problem 3? What do the numbers mean?
- I think I can use multiplication to help me divide because . . .

Support Whole Class Discussion

Prompt students to compare the two equations to identify how multiplication could help you solve a division problem.

Ask *How can you use multiplication to help you divide?*

Listen for You can write a multiplication equation related to the division equation. Since the two equations involve the same three numbers, if you know two of the numbers, then finding the unknown number for the multiplication equation will give you the same unknown number for the division equation.

Close: Exit Ticket

5 REFLECT

Look for understanding of separating into equal groups for division and putting together equal groups for multiplication.

Common Misconception **If** students mention subtracting or "taking away" as part of the process of division, **then** remind them that subtracting involves taking away some from a total, while division involves separating a total into equal groups. Draw an example of taking away, such as 20 − 5 = 15, as compared to the scrapbook division problem 20 ÷ 5 = 4.

Solutions

Support Vocabulary Development

1 Ask students to look at the graphic organizer. Display a multiplication equation and label the product. Display the inverse of the multiplication equation as a division equation. Label the solution as the quotient. Remind students that examples of multiplication and division both involve equal groups. Ask: *In what boxes of the graphic organizer will equal groups be included in words, pictures, or numbers?* Isolate each box and determine whether including references to equal groups is appropriate. Guide students to say how equal groups can be represented.

2 Have students label the number of rows and columns in the array. Ask: *When you use multiplication, are you trying to find a part of the array or the total?* Have students write a multiplication equation. Ask: *When you write a division equation, do you begin with the total of the array or a part of the array? How can you show the array is separated in equal groups?* Ask students to use the multiplication equation to write a corresponding division equation.

Supplemental Math Vocabulary

- *divide*
- *division*
- *equation*
- *factor*
- *multiplication*
- *multiply*
- *product*
- *quotient*

Name: _____ LESSON 11 SESSION 1

Prepare for Exploring How Multiplication and Division Are Connected

1 Think about what you know about division. Fill in each box. Use words, numbers, and pictures. Show as many ideas as you can. **Possible answers:**

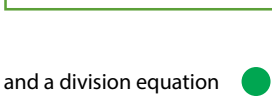
What Is It?
the result of a division problem

What I Know About It
The quotient can be the number of groups or the number of objects in each group.

quotient

Examples
$8 \div 2 = 4$

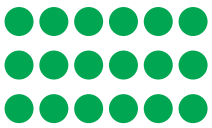
Examples
$6 \div 3 = ?$



Quotient: 2

Examples
10 items, 2 groups
How many items in each group?
5

2 Write a multiplication equation and a division equation for the array.



Possible answer: $3 \times 6 = 18$ and $18 \div 3 = 6$

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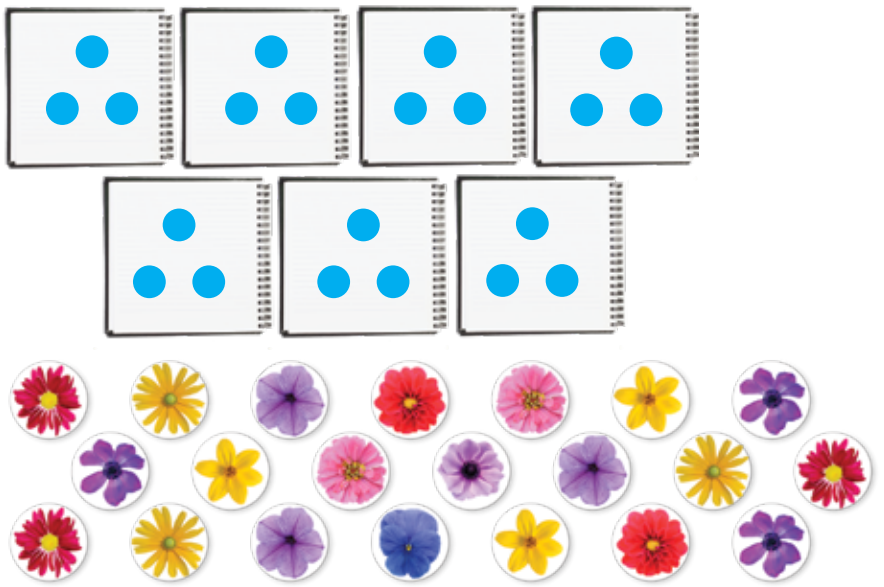
3 $21 \div 7 = 3$ and $7 \times 3 = 21$
Basic

4 Students can think "7 times what number gives me 21?" $7 \times 3 = 21$, so $21 \div 7 = 3$.
Medium

5 Student responses should include the understanding that in multiplication, you start with a number of equal groups and put them together to find the total but that in division, you start with the total and break it into a number of equal groups.
Medium

Solve.

3 Yuki has 21 flowers. She puts the same number of flowers on each of 7 pages in her scrapbook to dry them. Draw the flowers Yuki puts on the pages. Write a division equation and a multiplication equation for this problem.



Division equation: 21 ÷ 7 = 3

Multiplication equation: 7 × 3 = 21

4 Explain how you could use a multiplication equation to find 21 ÷ 7.

I can think "7 times what number gives me 21?" 7 × 3 = 21, so 21 ÷ 7 = 3.

5 How is multiplication doing the reverse of division?

Possible answer: In division, you start with a total and separate it into equal groups. In multiplication, you can take those same equal groups and put them together to find the total.

ELL English Language Learners: Differentiated Instruction

Prepare for Session 2
Use with *Model It*.

Levels 1–3	Levels 2–4	Levels 3–5
Listening/Speaking Read ***Model It*** problem 1 aloud. Say: *Multiplication and division involve equal groups: the number of groups, the number of items in each group, and the total number of items. Look at problem 1 again. What does 18 represent?* [the total number of items or hamsters] *What does 3 represent?* [the number of items in each group or cage] *What are you finding?* [the number of groups or cages] Encourage students to share their thinking with partners.	**Speaking/Writing** Read ***Model It*** problem 1 aloud. Say: *Multiplication and division involve equal groups: the number of groups, the number of items in each group, and the total number of items. Look at problem 1 again. What does 18 represent?* [the total number of items or hamsters] *What does 3 represent?* [the number of items in each group or cage] *What are you finding?* [the number of groups or cages] *Have partners write and complete the division equation:* *total hamsters* ÷ *hamsters in each cage* = *number of cages* 18 ÷ 3 = 6	**Speaking/Writing** Have students read ***Model It*** problem 1 aloud. Say: *Multiplication and division involve equal groups: the number of groups, the number of items in each group, and the total number of items. Look at problem 1 again.* Have students tell what each number in the problem represents and explain what is being asked for. Have students write and solve a division equation to represent the problem. Select partners to share their process and their division equations.

LESSON 11

SESSION 2 Develop

Purpose In this session students further explore the connection between multiplication and division by using models involving equal groups, arrays, and related equations to solve problems. Students then write all possible related multiplication and division equations from the same three numbers.

Start

Connect to Prior Knowledge

Materials For each student: 12 counters

Why Reinforce ways to model a division equation.

How Students use counters to model and solve a division situation (12 ÷ 2).

Use counters to model and solve this problem.

Mrs. Brown has 12 markers. She puts 2 markers in each cup. How many cups are there?

Solution

6 cups; Students may make 6 groups of 2 or an array with 6 rows of 2

Develop Language

Why Understand the prefix *un-* to clarify the meaning of the word *unknown*.

How List the following words: *unhappy*, *undo*, *unknown*. Explain that *un-* is a prefix that is put at the beginning of a word to make it mean its opposite. Rewrite each word with *not* in place of the prefix *un-*. Have students chorally repeat the lists of words. Circle *unknown* and *not known*. Point out that in mathematics, when a number is not known or missing, a symbol is used to represent the missing number. Ask students to identify the symbol that represents an unknown number in ***Model It*** problems 2 and 3.

MODEL IT: MULTIPLICATION AND DIVISION SITUATIONS

1–3 Present the problems and have students complete. As students work, have them identify that they are being asked to model the problem using both multiplication and division.

Common Misconception **If** students struggle with writing a division equation, **then** have them write the multiplication equation first. Present a completed (different) related division and multiplication equation set for them to use as a model to write the related division equation for this problem.

Develop Understanding of How Multiplication and Division Are Connected

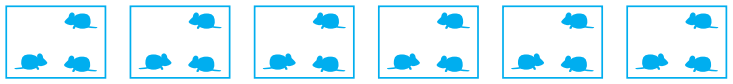
MODEL IT: MULTIPLICATION AND DIVISION SITUATIONS

Read the following problem. Then try problems 1–3.

A pet store has 18 hamsters. The shop owner wants to put 3 hamsters in each cage. How many cages does the shop owner need for all the hamsters?

1 Draw a model using equal groups or an array to show the problem.

Possible student work:



2 **a.** Write a division equation for the problem. Use a ? for the unknown number.

18 ÷ 3 = ?

b. Write a multiplication equation for the problem. Use a ? for the unknown number.

? × 3 = 18 or 3 × ? = 18

3 The shop owner needs **6** cages.

DISCUSS IT

- Did you and your partner write the same equations for problem 2? How did you know which equations to write?
- I think word problems that can be solved using division can also be solved using multiplication because . . .

DISCUSS IT

Support Partner Discussion

Encourage students to compare their models and equations to the meaning of the numbers in the problem.

Support as needed with questions such as:

- *How did you know whether you needed to find the number of groups or the number in each group?*
- *How did your model show the total and the number of hamsters in each cage? How did your equations show this?*

Support Whole Class Discussion

For each problem, have several students share their models and equations.

Ask *How many different related multiplication problems can we write?*

Listen for There are two different multiplication problems: ? × 3 = 18 and 3 × ? = 18.

MODEL IT: MULTIPLICATION AND DIVISION FACTS

4 – 5 Present the problems and have students complete. As students work, have them identify that they are being asked to complete related multiplication and division facts.

DISCUSS IT

Support Partner Discussion

Encourage partners to connect the numbers in the related division and multiplication equations.

Support as needed with questions such as:

- *Did you find it easier to find the missing number in the division or the multiplication equation first?*
- *Why might you want to use a multiplication fact to help you solve a division equation?*

Support Whole Class Discussion

For each problem, have several students share their strategies for finding the missing numbers.

Ask *How many different strategies could you use to find the unknown number in a division problem?*

Listen for You can model with equal groups, an array, a division equation (and complete with a known fact), or a related multiplication equation.

Hands-On Activity

Make fact cards.

If . . . *students struggle to see the connection between related multiplication and division facts,*

Then . . . *use this activity to make multiplication and division fact cards.*

Materials For each pair: 1 number cube, 2 markers, 2 triangles from Activity Sheet *Multiplication Triangles*

- Have students roll a number cube twice to generate two different-number factors and write each on a different corner of their card. Then write the product on the third corner.
- Have partners exchange cards and then write the four related multiplication and division facts using the numbers on the card.
- Have pairs create additional fact cards as time allows.

MODEL IT: MULTIPLICATION AND DIVISION FACTS

Find the value of ? to complete each fact.

4 $24 \div 3 = ?$

$3 \times ? = 24$

$? =$ 8

5 $? \times 9 = 54$

$54 \div ? = 9$

$? =$ 6

DISCUSS IT

- How did you find the missing number in each fact?
- I can think of a division fact as a multiplication problem because . . .

CONNECT IT

Complete the problems below.

6 How can you use the three numbers in a division equation to write the related multiplication equations?

Possible answer: Use the total number you divide as the product in the multiplication equations. Then the number you divide by and the quotient are the factors written in either order.

7 Use the numbers 7, 8, and 56 to write two multiplication equations and two division equations.

$7 \times 8 = 56$, $8 \times 7 = 56$, $56 \div 8 = 7$, $56 \div 7 = 8$

CONNECT IT

6 Student responses show understanding that related facts use the same numbers in a different order. Use the total of a division problem for the product of the related multiplication problem. Use the number right after the division symbol and the quotient as the factors in the related multiplication equations.

Close: Exit Ticket

7 Look for recognition that the four equations use the same three numbers in different positions. The multiplication problems use 56 as the product, while the division problems use 56 as the number right before the division symbol. The factors 7 and 8 switch positions in each pair of multiplication or division equations.

Common Misconception **If** students write division equations with 56 in a position other than right before the division symbol, **then** have students read their equation out loud and explain the role of each number, especially as compared to the role of each number in their multiplication equations.

Solutions

1. See Student Worktext page.
Basic

2. 5
Basic

3. 15 ÷ 3 = 5
Medium

4. 6; 6
Medium

5. 2; 2
Medium

Name: _____ LESSON 11 SESSION 2

Practice How Multiplication and Division Are Connected

Study how the Example shows one way to relate multiplication and division. Then solve problems 1–12.

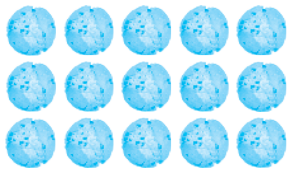
EXAMPLE

Marta bakes **15** muffins. She puts an equal number of muffins in **3** baskets.

She thinks, **3** times what number equals **15**?
3 × ? = **15**
3 × **5** = **15**

So, Marta puts **5** muffins in each basket.

1. Draw an array of 15 muffins in 3 rows.



2. How many muffins did you put in each row? 5

3. Fill in the blanks to write a division equation for the array you drew.
15 ÷ 3 = 5

Use the array to complete the equations.

4. 2 × 6 = 12 and 12 ÷ 2 = 6

5. 6 × 2 = 12 and 12 ÷ 6 = 2

Vocabulary

divide to separate into equal groups and find the number in each group or the number of groups.

array a set of objects arranged in equal rows and equal columns.

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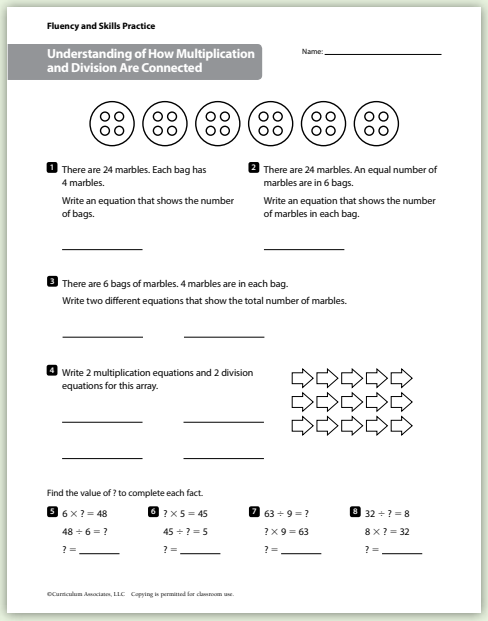
Fluency & Skills Practice

Teacher Toolbox

Assign *Understanding of How Multiplication and Division Are Connected*

In this activity students practice writing multiplication and division facts for the same model. Students should understand that many real-world problems can be solved using multiplication or division. For example, suppose a student with a pack of strawberries wants to put the same number of berries in her lunch each day. She can find how many berries to pack each day using division: total berries ÷ number of days = ? berries. Or, she can think of it as a multiplication problem: number of days × ? berries = total berries.



Fluency and Skills Practice

Understanding of How Multiplication and Division Are Connected

Name: _____

1. There are 24 marbles. Each bag has 4 marbles.
Write an equation that shows the number of bags.
_____

2. There are 24 marbles. An equal number of marbles are in 6 bags.
Write an equation that shows the number of marbles in each bag.
_____

3. There are 6 bags of marbles. 4 marbles are in each bag.
Write two different equations that show the total number of marbles.
_____ _____

4. Write 2 multiplication equations and 2 division equations for this array.
_____ _____
_____ _____

Find the value of ? to complete each fact.

5. 6 × ? = 48
48 ÷ 6 = ?
? = _____

6. ? × 5 = 45
45 ÷ ? = 5
? = _____

7. 63 ÷ 9 = ?
? × 9 = 63
? = _____

8. 32 ÷ ? = 8
8 × ? = 32
? = _____

6 $18 \div 6 = 3$
Medium

7 $18 \div 3 = 6$
Medium

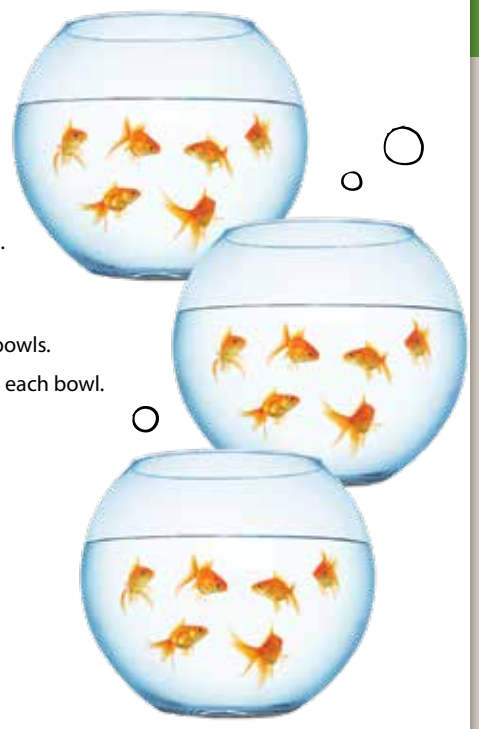
8 $3 \times 6 = 18$;
$6 \times 3 = 18$
Medium

9 42; 42
Basic

10 7; 7
Medium

11 7; 7
Medium

12 6; 6
Medium



Use the numbers 3, 6, and 18 to write the equations for problems 6–8.

6 There are 18 fish. Each bowl holds 6 fish.
Write an equation that shows the number of bowls.
18 ÷ 6 = 3

7 There are 18 fish. An equal number of fish are in 3 bowls.
Write an equation that shows the number of fish in each bowl.
18 ÷ 3 = 6

8 There are 3 bowls. 6 fish are in each bowl.
Write two different equations that show the total number of fish.
3 × 6 = 18
6 × 3 = 18

This array shows that 6 × 7 = 42. Use this fact to complete the equations in problems 9–12.

9 $6 \times 7 =$ 42 $7 \times 6 =$ 42

10 $6 \times$ 7 $= 42$ $42 =$ 7 $\times 6$

11 $42 \div 6 =$ 7 $42 \div$ 7 $= 6$

12 6 $= 42 \div 7$ $7 = 42 \div$ 6

ELL English Language Learners: Differentiated Instruction

Prepare for Session 3
Use with *Apply It*.

Levels 1–3	Levels 2–4	Levels 3–5
Listening/Speaking Read ***Apply It*** problem 1 aloud. Ask: *What multiplication equation could represent the picture?* Display: $4 \times 3 = 12$ and $12 \div 4 = 3$. Say: *You can read this division equation to mean: If you put 12 flowers into four pots equally, there will be three flowers in each pot.* Give partners 12 counters. Say: *Use your counters to represent three pots with four flowers each. What multiplication equation could represent the counters?* Display: $3 \times 4 = 12$ Say: *Tell your partner one possible division equation for this problem; for example: You have 12 flowers. You put the flowers in three pots. There are four flowers in each pot.* Record $12 \div 3 = 4$. Repeat the interpretation of each equation.	**Listening/Speaking** Read ***Apply It*** problem 1 aloud. Ask: *What multiplication equation could represent the picture?* Display: $4 \times 3 = 12$, $12 \div 4 = 3$. Say: *You can read this division equation to mean: If you put 12 flowers into four pots equally, there will be three flowers in each pot.* Give partners 12 counters. Say: *Use your counters to represent three pots with four flowers each. Work with your partner to write a multiplication and division equation to represent your counters. Be prepared to share your thinking.* Call on pairs to share their equations. Repeat the interpretation of each equation.	**Listening/Speaking** Have partners read ***Apply It*** problem 1 aloud. Say: *Write a multiplication equation to represent the picture.* Display: $4 \times 3 = 12$, $12 \div 4 = 3$. Say: *Discuss with your partner how we can explain the picture using a division equation.* Select students to share their thinking. Give partners 12 counters. Say: *Use your counters to represent three pots with four flowers each. Work with your partner to write a multiplication and division equation to represent your counters. Be prepared to share your thinking.* Select students to share their equations.

LESSON 11

SESSION 3 Refine

Purpose In this session students demonstrate their understanding of how multiplication and division are related as they talk through three problems. Then they use a related multiplication equation and a model to solve a division equation.

Start

Connect to Prior Knowledge

Why Reinforce the idea that equal groups can be represented with a division equation.

How Students write a division equation to represent an equal-groups model.

Write a division equation to represent the groups of apples.

Possible Solutions

$15 \div 3 = 5$

or

$15 \div 5 = 3$

APPLY IT

Have students work independently or with a partner.

1 IDENTIFY

Look for understanding that all equations should use the numbers 3, 4, and 12.

Have students share their equations. Choose several students to explain how their equations represent the situation. Use the following to start a discussion:

- *Explain your process for writing the four equations.*
- *Why can this situation be represented by both multiplication and division equations?*
- *How do you know what each number represents?*

LESSON 11 SESSION 3 ● ● ●

Refine Ideas About How Multiplication and Division Are Connected

APPLY IT

Complete these problems on your own.

1 IDENTIFY

Ed plants the same number of flowers in each pot at the right. Write two multiplication equations and two division equations that this picture shows.

$4 \times 3 = 12$; $3 \times 4 = 12$; $12 \div 3 = 4$; $12 \div 4 = 3$

2 EXPLAIN

Yasmin sees $63 \div ? = 7$ and thinks, "There are 63 things in all that are divided into groups. There are 7 in each group." Explain how Yasmin can use multiplication to help her find the number of groups.

Possible answer: Yasmin can think "How many groups of 7 make 63? What number times 7 equals 63?" She can use the fact $9 \times 7 = 63$ to find the answer.

3 ANALYZE

Marissa has 4 boxes of markers with 6 markers in each box. She wrote the following equations:

$4 \times 6 = 24$ $\quad 6 \times 4 = 24$ $\quad 24 \div 4 = 6$ $\quad 24 \div 6 = 4$

Circle the number in each equation that shows the total number of markers. Put a box around the number in each equation that shows the number of groups. Underline the number in each equation that shows the number in each group.

PAIR/SHARE

Discuss your solutions for these three problems with a partner.

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2 EXPLAIN

Look for understanding that a multiplication problem involving the same three numbers could be used to solve the division problem.

Prompt discussion with questions such as:

- *How can you rewrite $63 \div ? = 7$ using multiplication?*
- *How do you know the multiplication fact Yasmin could use to help solve the division problem?*

Common Misconception **If** students use addition terminology (*add*, *join*, *sum*, etc.), **then** remind them that addition is the reverse of subtraction and multiplication is the reverse of division. Have students model and compare each set of equations: $2 + 4 = 6$ and $6 - 2 = 4$; $2 \times 3 = 6$ and $6 \div 2 = 3$. Encourage students to use multiplication words (*factor*, *times*, *product*, *equal groups*, etc.) as they describe using multiplication.

3 ANALYZE

Look for understanding that 24 is the total, 4 is the number of groups, and 6 is the number in each group.

Have students explain how they knew which number represented the total, the number of groups, and the number in each group.

4 **Before students begin,** read through problem 4 as a class. Make sure students understand their task is to write a related multiplication equation and draw a model to solve the given division equation.

As students work on their own, walk around to assess their progress and understanding, to answer their questions, and to give additional support, if needed.

Have students share their equations and models with a partner and explain how they found the quotient.

Scoring Rubrics

Part A

Points	Expectations
2	The student correctly writes a multiplication equation that can be used to solve the division equation.
1	The student writes a multiplication equation, but not one that can be used to solve the division equation.
0	The student does not write a multiplication equation.

Part B

Points	Expectations
2	The student draws a model that represents the problem and correctly solves the problem.
1	The student draws a model, but the model does not represent the problem and/or the problem is not correctly solved.
0	The student makes no attempt to draw a model, draws a model that is unrelated to the problem, and/or does not solve or incorrectly solves the problem.

Use what you have learned to complete problem 4.

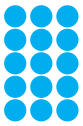
4 Look at the division equation 15 ÷ 5 = ?.

Part A Write a multiplication equation you can use to solve this division problem. Use a ? for the unknown number.

5 × ? = 15 or ? × 5 = 15

Part B Draw a model that could help you solve the division problem. Then solve the problem.

Possible student work using an array:



15 ÷ 5 = 3

5 MATH JOURNAL

Write a story problem that can be modeled by the equation 35 ÷ 7 = ?. Explain how you can use multiplication to solve this problem. Then solve the problem.

Possible answer: Brylee wants to give 35 stickers to her friends. If she gives 7 friends the same number of stickers, how many will each friend get? I can solve by thinking "7 times what number equals 35?" 7 × 5 = 35, so each friend gets 5 stickers.

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Close: Exit Ticket

5 MATH JOURNAL

Students may write a story problem in which the quotient is the number of equal groups or the number in each group. Their responses should indicate that 7 times an unknown number equals 35, so the quotient is 5. Answers should be given in the context of the story problem.

Error Alert **If** students attempt to multiply 35 × 7, **then** remind students that the number that comes right before the division symbol is the total amount. Have them explain what the 7 could mean and connect this to a multiplication situation. Then have them find the other factor.

Lesson Overview

LESSON 12

Multiplication and Division Facts

Lesson Objectives

Content Objectives

- Fluently multiply and divide within 100.
- Use fact families and the relationship between multiplication and division to find unknown whole numbers in multiplication and division equations.
- Solve word problems using equations with the unknown whole number in different places in the equations.

Language Objectives

- Write multiplication and division fact families.
- Write related facts to find the unknown number in a multiplication or division equation.
- Tell which multiplication or division facts can represent a particular word problem.

Prerequisite Skills

- Understand the meaning of multiplication.
- Know basic multiplication facts for 0 through 10.
- Understand the meaning of division.
- Connect multiplication and division.

Standards for Mathematical Practice (SMP)

SMPs 1, 2, 3, 4, 5, and 6 are integrated in every lesson through the *Try-Discuss-Connect* routine.*

In addition, this lesson particularly emphasizes the following SMPs:

2 Reason abstractly and quantitatively.

5 Use appropriate tools strategically.

7 Look for and make use of structure.

8 Look for and express regularity in repeated reasoning.

*See page 1i to see how every lesson includes these SMPs.

Lesson Vocabulary

- **fact family** a group of related equations that use the same numbers, but in a different order, and two different operation symbols. A fact family can show the relationship between addition and subtraction or between multiplication and division.
- **multiplication table** a table showing multiplication facts.

Review the following key terms.

- **division equation** an equation with a division symbol and an equal sign. For example, $15 \div 3 = 5$.
- **factor** a number that is multiplied.
- **multiplication equation** an equation with a multiplication symbol and an equal sign. For example, $3 \times 5 = 15$.
- **product** the result of multiplication.
- **quotient** the result of division.

Learning Progression

In Grade 3 multiplication is a major focus. Students also build a foundational understanding of division and learn how the two operations are related. Students have learned these concepts in previous Grade 3 lessons.

In this lesson students use fact families and a multiplication table to solve multiplication and division problems and to write related multiplication and division facts. Students learn how to use the rows and columns in a multiplication table to find missing numbers in multiplication and division facts. Students then apply that understanding to finding the third number in fact families.

In later grades fluency with multiplication facts up to 9×9 and the related division facts becomes a critical skill. For this reason, it is important for students to develop a deep understanding of multiplication and division and use this understanding to make meaningful strategies to help them master multiplication and division facts.

Lesson Pacing Guide

Whole Class Instruction

SESSION 1
Explore
45–60 min

Interactive Tutorial* *(Optional)*
Prerequisite Review:
Understand Division, Part 2

Multiplication and Division Facts
- Start *5 min*
- Try It *10 min*
- Discuss It *10 min*
- Connect It *15 min*
- Close: Exit Ticket *5 min*

Additional Practice
Lesson pages 253–254

SESSION 2
Develop
45–60 min

Working with Division Facts
- Start *5 min*
- Try It *10 min*
- Discuss It *10 min*
- Model Its *5 min*
- Connect It *10 min*
- Close: Exit Ticket *5 min*

Additional Practice
Lesson pages 259–260

Fluency
Working with Division Facts

SESSION 3
Develop
45–60 min

Using a Multiplication Table
- Start *5 min*
- Try It *10 min*
- Discuss It *10 min*
- Picture It & Model It *5 min*
- Connect It *10 min*
- Close: Exit Ticket *5 min*

Additional Practice
Lesson pages 265–266

Fluency
Using a Multiplication Table

SESSION 4
Refine
45–60 min

Working with Multiplication and Division Facts
- Start *5 min*
- Example & Problems 1–3 *15 min*
- Practice & Small Group Differentiation *20 min*
- Close: Exit Ticket *5 min*

Lesson Quiz or **Digital Comprehension Check**

Lesson Materials

Lesson *(Required)*	*Per student:* 30 counters
Activities	*Per student:* red crayon, yellow crayon *Per pair:* 30 counters or connecting cubes, ruler *Activity Sheets:* Multiplication Triangles, Multiplication Table**, Blank Multiplication Table
Math Toolkit	connecting cubes, counters, buttons, cups, 1-centimeter grid paper, multiplication tables
Digital Math Tools	Number Line, Multiplication Models

**Used for more than one activity.

*We continually update the Interactive Tutorials. Check the Teacher Toolbox for the most up-to-date offerings for this lesson.

Teacher Toolbox

Small Group Differentiation

PREPARE

Ready Prerequisite Lessons
Grade 2
- Lesson 31 Add Using Arrays
- Lesson 32 Even and Odd Numbers

RETEACH

Tools for Instruction
Grade 2
- Lesson 31 Adding Using Arrays
- Lesson 32 Identifying Even and Odd Numbers

Grade 3
- Lesson 12 Write Multiplication and Division Facts

REINFORCE

Math Center Activities
Grade 3
- Lesson 12 Place Missing Numbers
- Lesson 12 Complete a Fact Family

EXTEND

Enrichment Activity
Grade 3
- Lesson 12 Display of Cans

i-Ready

Independent Learning

PERSONALIZE

i-Ready Lessons*
Grade 3
- Practice: Multiply Within 100
- Practice: Multiply and Divide Within 100
- Practice: Divide and Multiply (Within 100)

Learning Games
- *Match*
- *Pizza*

Connect to Family, Community, and Language Development

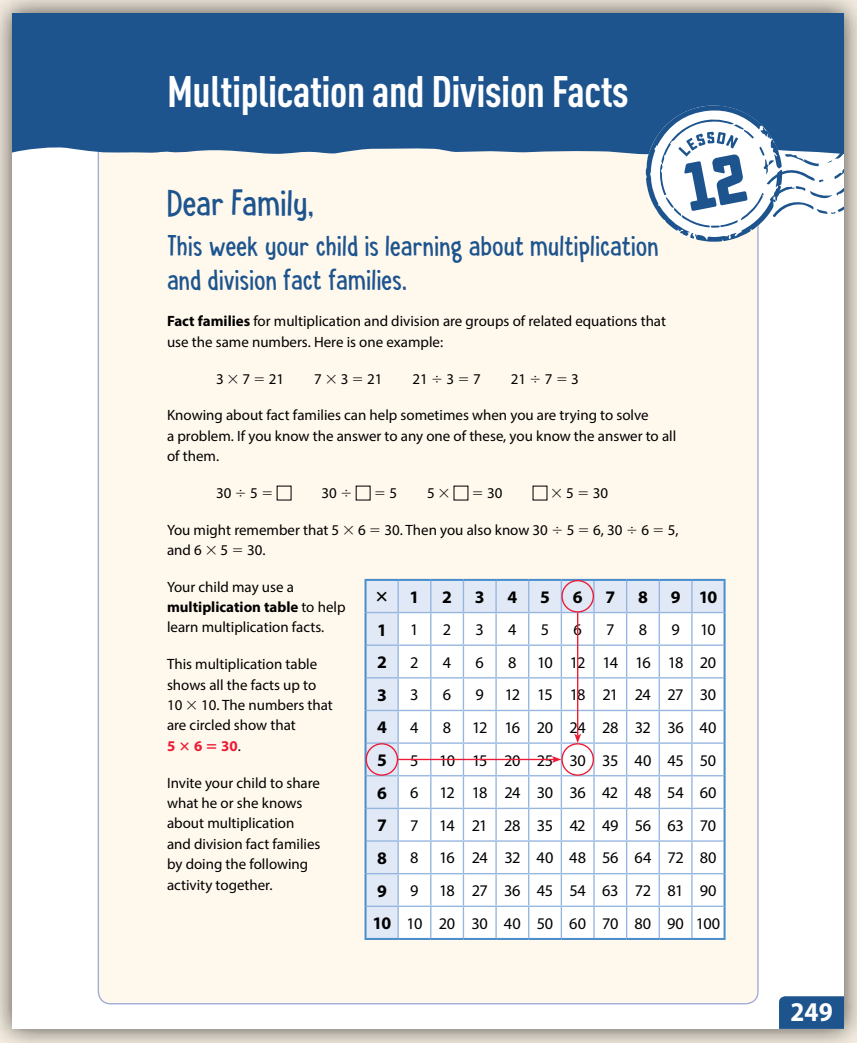
The following activities and instructional supports provide opportunities to foster school, family, and community involvement and partnerships.

Connect to Family

Use the **Family Letter**—which provides background information, math vocabulary, and an activity—to keep families apprised of what their child is learning and to encourage family involvement.

Available in Spanish

Teacher Toolbox



Multiplication and Division Facts

LESSON 12

Dear Family,

This week your child is learning about multiplication and division fact families.

Fact families for multiplication and division are groups of related equations that use the same numbers. Here is one example:

$3 \times 7 = 21$ $7 \times 3 = 21$ $21 \div 3 = 7$ $21 \div 7 = 3$

Knowing about fact families can help sometimes when you are trying to solve a problem. If you know the answer to any one of these, you know the answer to all of them.

$30 \div 5 = \square$ $30 \div \square = 5$ $5 \times \square = 30$ $\square \times 5 = 30$

You might remember that $5 \times 6 = 30$. Then you also know $30 \div 5 = 6$, $30 \div 6 = 5$, and $6 \times 5 = 30$.

Your child may use a **multiplication table** to help learn multiplication facts.

This multiplication table shows all the facts up to 10×10. The numbers that are circled show that $5 \times 6 = 30$.

Invite your child to share what he or she knows about multiplication and division fact families by doing the following activity together.

×	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

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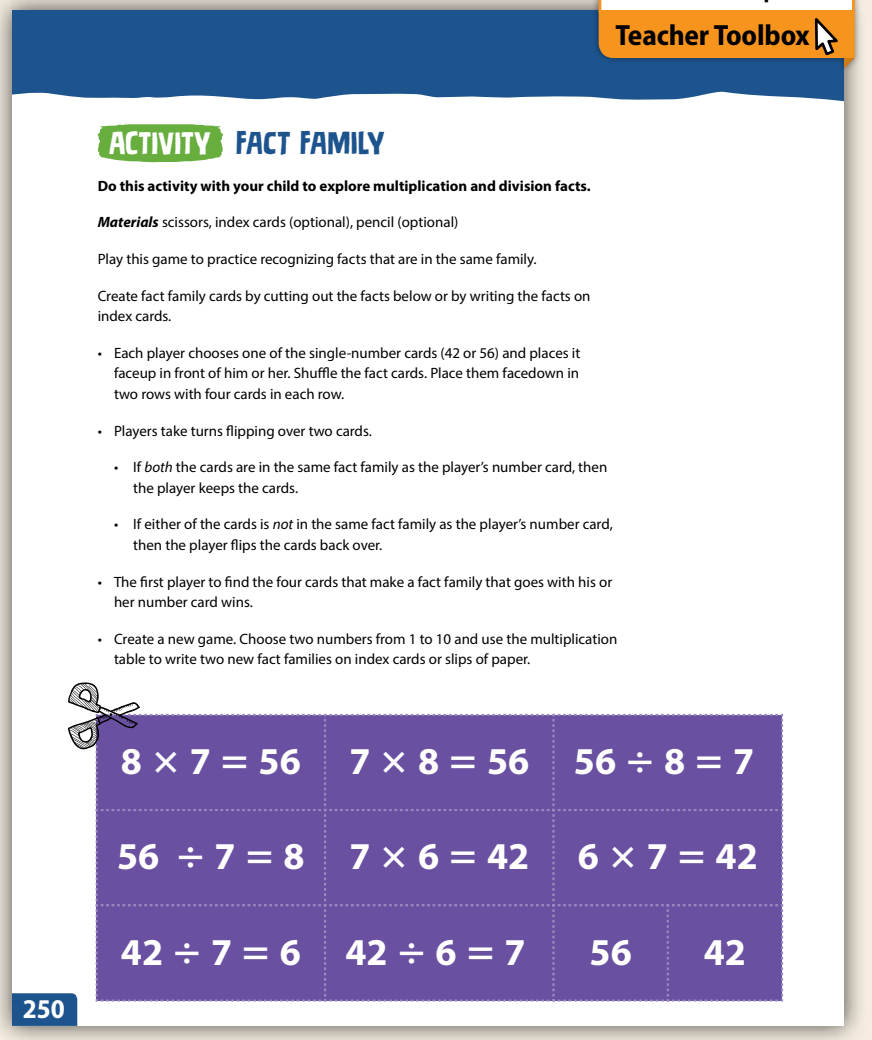

ACTIVITY FACT FAMILY

Do this activity with your child to explore multiplication and division facts.

Materials scissors, index cards (optional), pencil (optional)

Play this game to practice recognizing facts that are in the same family.

Create fact family cards by cutting out the facts below or by writing the facts on index cards.

- Each player chooses one of the single-number cards (42 or 56) and places it faceup in front of him or her. Shuffle the fact cards. Place them facedown in two rows with four cards in each row.
- Players take turns flipping over two cards.
  - If *both* the cards are in the same fact family as the player's number card, then the player keeps the cards.
  - If either of the cards is *not* in the same fact family as the player's number card, then the player flips the cards back over.
- The first player to find the four cards that make a fact family that goes with his or her number card wins.
- Create a new game. Choose two numbers from 1 to 10 and use the multiplication table to write two new fact families on index cards or slips of paper.

$8 \times 7 = 56$	$7 \times 8 = 56$	$56 \div 8 = 7$	
$56 \div 7 = 8$	$7 \times 6 = 42$	$6 \times 7 = 42$	
$42 \div 7 = 6$	$42 \div 6 = 7$	56	42

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Goal

The goal of the Family Letter is to highlight how related multiplication and division equations make up a fact family.

- Students and family members create fact families of multiplication and division facts.

Activity

Understanding the relationship between multiplication and division facts within a family can be used by students during problem solving to check the accuracy of their computation. Look at the *Fact Family* activity and adjust it if necessary to connect with your students.

Math Talk at Home

Encourage students to discuss with family members division situations with a related multiplication fact. Explain that $9 \div 3 = 3$ can be used to determine how to fairly share 9 grapes among 3 people. The related multiplication fact, $3 \times 3 = 9$, can also be used to determine how many grapes each person receives.

Conversation Starters Below are additional conversation starters students can write in their Family Letter or math journal to engage family members:

- *How are $24 \div 6 = 4$ and $6 \times 4 = 24$ related?*
- *What multiplication and division facts can you use to figure out how to fairly share 12 pieces of pizza among six people?*

Connect to Community and Cultural Responsiveness

Use these activities to connect with and leverage the diverse backgrounds and experiences of all students.

Session 1 **Use with *Try It*.**

- Ask students if they are familiar with marbles and the games played with marbles. Explain a marbles game or have a volunteer with experience explain one. Ask students to share other games they have played. Broaden students' thinking by mentioning and discussing categories such as sports, video games, board games, card games, and physical games such as tag. Ask students how the playing pieces or equipment are stored for each game or sport named.

Session 2 **Use with *Try It*.**

- Ask students if they have had experience with money used by other cultures or countries. Select students to share if applicable. Share that Canada, the country that borders the United States to the north, has coins with the same values as those in the United States. Explain that the coins have the same value but do not look the same. Point out that in Mexico, the country that borders the United States to the south, the coins and paper money are called *pesos*. There are 1-, 2-, 5-, 10-, and 20-*peso* coins in Mexico. Explain that countries decorate their coins with pictures of people, animals, places, and other symbols that have special meaning to the citizens of the country. Display and name the people and symbols pictured on the penny, nickel, dime, and quarter. Have students design the front and back of a coin to show what has special meaning to them. Ask students to assign a value to their coin.

Session 3 **Use anytime during the session.**

- Ask volunteers to share how many people they have in their immediate families. Display *immediate* and *extended*. Explain that *immediate family* includes their closest relatives, such as parents/guardians and brothers and sisters. Share that *extended family* includes other relatives, such as aunts, uncles, and grandparents. Devote time to volunteers sharing their personal situations. Validate immediate and extended family situations that may reflect your students' cultural backgrounds.

Session 4 **Use with *Apply It* problem 8.**

- Ask students how their parents/guardians save their schoolwork, awards, pictures, and report cards. They may be displayed on the refrigerator, placed in a drawer, or kept in a special place such as a scrapbook. Share that scrapbooking is a very popular hobby. Explain that scrapbooks are books with blank pages that people fill with items that remind them of a special event, place, or person. Explain that scrapbooks often contain photographs, but can also contain items such as a napkin from a favorite restaurant, a special card from a friend, or a postcard of a city recently visited. Ask students if they know anyone who enjoys scrapbooking. Segue into asking students what their favorite hobby is. Explain that hobbies are things people like to do for enjoyment or entertainment.

Connect to Language Development

For ELLs, use the Differentiated Instruction chart to plan and prepare for specific activities in every session.

ELL English Language Learners: Differentiated Instruction

Prepare for Session 1
Use with *Try It*.

Levels 1–3	Levels 2–4	Levels 3–5
Listening/Speaking Pair students. Read the ***Try It*** problem aloud. Ask: *What do you want to find out?* [how many marbles are in each bag] Ask: *Which words help you understand what to do? Talk with your partner.* Mark when "*the same number of marbles into each of 3 bags*" is suggested. Connect *3 bags* to three groups and *same number* to equal groups. Give counters to partners. Say: *Act out the problem. Think about how to explain the strategy you used.* Display: *first*, *then*, *next*, and *last*. Say: *Use these words to organize your thoughts.*	**Listening/Speaking** Pair students. Read the ***Try It*** problem aloud. Ask: *What do you want to find out?* [how many marbles are in each bag] *What do you know for sure? Discuss your ideas with your partner.* Ask additional questions: *What do the three bags represent?* [the groups] *What does "same number" mean?* [They have to be equal.] Give counters to partners. Say: *Use the counters to solve the problem.* Display: *first*, *then*, *next*, *last*. Say: *Think about how to explain the strategy you used. Use these words to organize your thoughts.* Select partners to share their solution process.	**Speaking/Writing** Have students read the ***Try It*** problem in pairs. Ask: *What do you want to find out? What do you know for sure? Discuss your ideas with your partner.* Facilitate a discussion by asking additional questions: *What do the three bags represent?* [the groups] *What does "same number" mean?* [They have to be equal.] Give counters to partners. Say: *Use the counters to solve the problem.* Display: *first*, *then*, *next*, *last*. Say: *Write two to four sentences to explain the strategy you used, using these words to organize your writing.* Select partners to share their solution process.

LESSON 12

SESSION 1 Explore

Purpose In this session students draw on what they know about how multiplication and division are related to solve a division problem. They will look ahead to think about using fact families to find an unknown number in a multiplication or division equation.

Start

Connect to Prior Knowledge

Materials For each student: 30 counters

Why Support students' understanding of the meaning of division as separating into equal groups to prepare them for working with multiplication and division fact families.

How Have students show 28 ÷ 7 using equal groups to find the quotient.

Use counters to show equal groups and find the quotient.

28 ÷ 7 = ...

Solution

4

Look for 28 counters in 7 groups of 4 or 4 groups of 7.

TRY IT

Make Sense of the Problem

To support students in making sense of the problem, have them describe what Kenny decides to do with his 24 marbles.

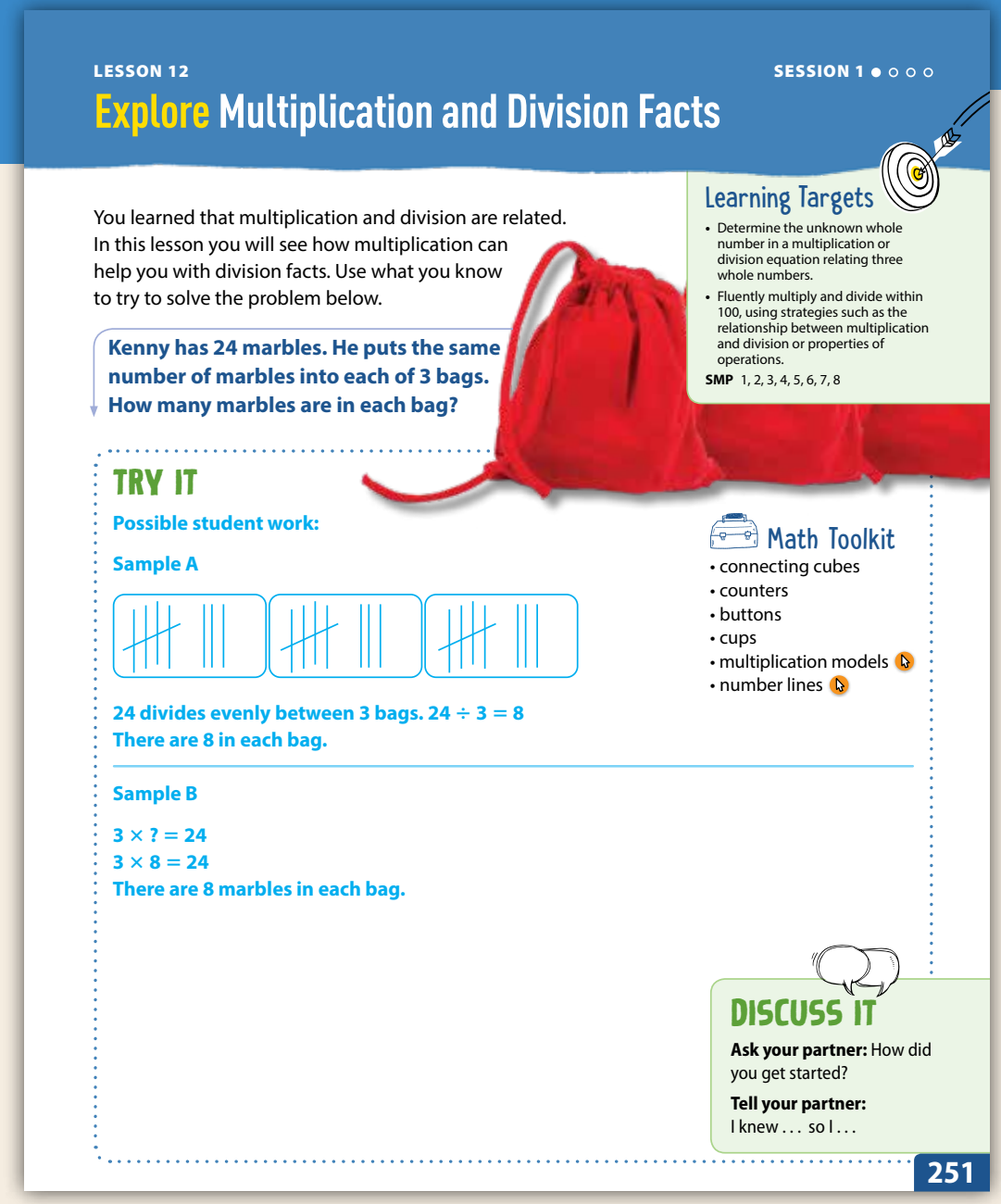
DISCUSS IT

Support Partner Discussion

To reinforce that there are 3 equal groups, encourage students to draw 3 circles or use 3 cups to represent the 3 bags.

Look for, and prompt as necessary for, understanding of:

- 24 total marbles
- 3 equal groups
- division or multiplication can be used to solve the problem



LESSON 12 SESSION 1 ● ○ ○ ○

Explore Multiplication and Division Facts

You learned that multiplication and division are related. In this lesson you will see how multiplication can help you with division facts. Use what you know to try to solve the problem below.

Kenny has 24 marbles. He puts the same number of marbles into each of 3 bags. How many marbles are in each bag?

Learning Targets

- Determine the unknown whole number in a multiplication or division equation relating three whole numbers.
- Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division or properties of operations.

SMP 1, 2, 3, 4, 5, 6, 7, 8

TRY IT

Possible student work:

Sample A

24 divides evenly between 3 bags. 24 ÷ 3 = 8
There are 8 in each bag.

Sample B

3 × ? = 24
3 × 8 = 24
There are 8 marbles in each bag.

Math Toolkit

- connecting cubes
- counters
- buttons
- cups
- multiplication models
- number lines

DISCUSS IT

Ask your partner: How did you get started?

Tell your partner: I knew . . . so I . . .

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Common Misconception Look for students who are not comfortable with division situations and add [27], subtract [21], or multiply (using repeated addition) [72] the numbers given in the problem. As students present solutions, have them specify how they know which operation to use.

Select and Sequence Student Solutions

One possible order for whole class discussion:

- counting 3 groups of 8 objects
- using a 3 × 8 array
- writing and solving the multiplication equation 3 × ☐ = 24 or the division equation 24 ÷ 3 = ☐

Support Whole Class Discussion

Prompt students to note the relationship between the numbers in each model and the numbers in the problem.

Ask *How do [student name]'s and [student name]'s models show the total, the number of equal groups, and the number in each group?*

Listen for There are 24 total objects to represent Kenny's 24 marbles. There are 3 equal groups for the 3 bags, and there are 8 marbles in each group (or bag).

CONNECT IT

1 LOOK BACK

Look for understanding that there are 8 marbles in each bag because 3 groups of 8 make 24.

Hands-On Activity

Interpret division situations.

If . . . *students are unsure about the meaning of division and solving a division equation,*

Then . . . *use this activity to have them interpret and solve a division problem.*

Materials For each pair: 30 counters or connecting cubes

- Write 27 ÷ 3 = ☐ on the board.
- Have each group come up with a scenario that can be solved using the division equation. For example, if you share 27 pretzels equally between 3 friends, how many pretzels does each friend get?
- Have each group use the counters or cubes to act out or model the division problem they made in the previous step. For example, have them pass out "pretzels" in equal shares to 3 "friends."
- Then have each group solve the problem and find the quotient.
- Repeat activity for 27 ÷ 9 = ☐.

2 LOOK AHEAD

Ask a volunteer to describe a multiplication and division fact family in his or her own words.

Students should be able to write four unique related facts, two multiplication and two division, for each fact family (if no repeated factors). Only multiplication and division operations should be shown. Students will spend more time learning about fact families in the Additional Practice.

Ask *Will you write a subtraction equation for either of these fact families? An addition equation? Explain.*

Listen for No, a multiplication and division fact family involves only multiplication and division.

Ask *Will every fact family have four different facts? Explain.*

Listen for No, if both factors are the same, then you will have only two facts. For example, 5 × 5 = 25 and 25 ÷ 5 = 5.

CONNECT IT

1 LOOK BACK

How many marbles are in each bag? Explain how you can prove you are right.

8; Possible explanation: If you make 3 groups of 8 and count them all, there are 24.

2 LOOK AHEAD

Fact families for multiplication and division are groups of related equations. All the equations, or facts, use the same numbers.

If you know one fact in a family, you can find all the others.

a. Say you need to solve ☐ ÷ 9 = 6. You can write the facts in this family to find one that you might know. Use the array to help you complete this fact family.

6 × 9 = **54**

9 × 6 = **54**

54 ÷ 6 = 9

54 ÷ 9 = 6

b. Look back at the problem on the previous page. Write the complete fact family using the three numbers for this situation. **Accept facts in any order.**

3 × 8 = 24 **8 × 3 = 24**

24 ÷ 3 = 8 **24 ÷ 8 = 3**

3 REFLECT

How are the multiplication facts in the fact families above alike? How are they different? How are the division facts alike and different?

Possible answer: Multiplication facts use the same three numbers, but the factors are in a different order. Division facts also use the same three numbers, but the order of the numbers on either side of the equal sign is switched.

Close: Exit Ticket

3 REFLECT

Look for understanding of how the multiplication facts in a fact family use the same three numbers but the order of the factors is different. The division facts also use the same three numbers, but the numbers that represent the number of groups and the number in each group change order.

Common Misconception **If** students are confused about how to write four different facts for a fact family, **then** use counters to make an equal-groups model or array and talk through each equation in 2b. Have students describe the models by the number in each group, number of groups, and the total and then relate to each equation.

Real-World Connection

Encourage students to think about everyday places or situations where people might use a fact family to help solve division or multiplication problems. Have volunteers share ideas and ask them to tell which fact they would use and why. Examples: finding the number of grapes each camper gets if a bag of 48 grapes is shared equally among 8 campers, finding the number of balloons you need if you put 5 balloons each in 8 party favor bags, and finding the number of players on each team if 36 players sign up for 4 teams.

Solutions

Support Vocabulary Development

1 Ask students to explain how to complete the graphic organizer. Focus on the difference between the boxes across the top and those on the bottom. Ask: *What might be a way to show and explain—with words, numbers, and pictures—examples of a fact family?* Record the suggestions offered by students as a scaffold.

2 Have students circle 40 ÷ 8. Say: *Think about the fact family that has the numbers 40 and 8. What number is missing? What multiplication fact can help you complete the division fact? What do you know about the product of the multiplication fact?*

Supplemental Math Vocabulary

- *division equation*
- *factor*
- *multiplication equation*
- *multiplication table*
- *product*
- *quotient*

Name: _____ LESSON 12 SESSION 1

Prepare for Multiplication and Division Facts

1 Think about what you know about multiplication and division. Fill in each box. Use words, numbers, and pictures. Show as many ideas as you can. **Possible answers:**

What Is It?

in multiplication, a group of related multiplication and division equations or facts

What I Know About It

I can use one fact I know in the family to find all the others.

fact family

Examples

2 × 3 = 6
3 × 2 = 6
6 ÷ 2 = 3
6 ÷ 3 = 2

Examples

3 × □ = 21
□ × 3 = 21
21 ÷ 3 = □
21 ÷ □ = 3

Examples

I can use the multiplication fact 5 × 4 = 20 to find 20 ÷ 4.
20 ÷ 4 = 5

2 How can you use fact families to help find 40 ÷ 8? Explain.

Possible answer: I know the multiplication fact 8 × 5 = 40. The division fact 40 ÷ 8 = □ is in the same fact family, so 40 ÷ 8 = 5.

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3 Assign problem 3 to provide another look at solving a problem with multiplication and division facts.

This problem is very similar to the problem about Kenny putting the same number of marbles in each bag. In both problems, students are given a word problem that requires them to find the unknown number in a multiplication or division equation. The question asks students to solve a problem about putting 28 muffins into 4 equal groups.

Students may want to use connecting cubes or counters.

Suggest that students read the problem three times, asking themselves one of the following questions each time:

- *What is this problem about?*
- *What is the question I am trying to answer?*
- *What information is important?*

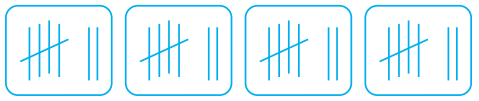
Solution: 28 ÷ 4 = 7; There are 7 muffins in each box.
Medium

4 Have students solve the problem another way to check their answer.

3 Solve the problem. Show your work.

Jada has 28 muffins. She puts the same number of muffins into each of 4 boxes. How many muffins are in each box?

Possible student work using a picture:



28 divides evenly between 4 boxes. 28 ÷ 4 = 7

Solution **There are 7 muffins in each box.**

4 Check your answer. Show your work.

Possible student work:

4 × □ = 28

4 × 7 = 28

There are 7 muffins in each box.

ELL English Language Learners: Differentiated Instruction

Prepare for Session 2
Use with *Apply It*.

Levels 1–3	Levels 2–4	Levels 3–5
Listening/Speaking Read ***Apply It*** problem 6 aloud. Ask: *What are you going to divide?* [24] Say: *Look at the number line. Discuss it with a partner. Are you going to make four equal groups?* [no] *Is the size of the group four?* [yes] Ask: *What is the next number on the number line?* [8] *How do you know?* [4 + 4 = 8] *How many groups do you have now?* [2] Repeat the questions until 24 is displayed. Ask: *How many groups did you make?* [6] *What is 24 ÷ 4?* [6] *What are the other facts related to 24 ÷ 4 = 6?*	**Listening/Speaking** Read ***Apply It*** problem 6 aloud. Say: *Discuss this question with a partner: What does 24 ÷ 4 mean?* Say: *Look at the number line. Are you going to make four equal groups, or is the size of the group four?* [The size is four.] Ask: *What is the next number on the number line?* [8] *How do you know?* [4 + 4 = 8] *How many groups do you have now?* [2] *What will you do next?* [Add/jump four more.] *What will be the last number?* [24] *With a partner, find the quotient of 24 ÷ 4.* Ask: *How many groups did you make?* [6] *Write the fact family for 24 ÷ 4 = 6.* Select students to share facts until the fact family is displayed.	**Listening/Speaking** Have students form pairs and read ***Apply It*** problem 6. Say: *Discuss this question with your partner: What can 24 ÷ 4 mean?* Say: *Look at the number line. Does the four represent equal groups, or the size of the group?* [the size of the group] *How do you know? What will be the next number you will write on the number line?* [8] *How do you know?* [4 + 4 = 8] *How many groups of four do you have now?* [2] *What will you do next?* [Add/jump four more.] Ask: *What will be the last number?* [24] *With a partner, find the quotient of 24 ÷ 4. Then write the fact family for 24 ÷ 4.* Select partners to share different facts until the fact family is displayed.

LESSON 12

SESSION 2 Develop

Purpose In this session students solve a problem that requires finding the quotient of 40 ÷ 5. The purpose of this problem is to have students model a division situation and develop a strategy for finding an unknown number in a division fact.

Start

Connect to Prior Knowledge

Materials For each student: 20 counters

Why Support students' facility with modeling a multiplication problem and then writing related multiplication and division facts.

How Have students make a 4 × 5 array and write a multiplication and division equation that represents the array.

Use counters to make a 4 × 5 array.

Then write a multiplication and division equation to represent the array.

Possible Solutions

4 × 5 = 20 or
5 × 4 = 20;
20 ÷ 4 = 5 or
20 ÷ 5 = 4

Look for a 4 × 5 array.

Develop Language

Why Clarify the meaning of the term *worth*.

How Explain that what something is *worth* is the value the item has, especially in money. Write and display the following:

- *Nickels are worth 5 cents.*
- *Nickels have a value of 5 cents.*

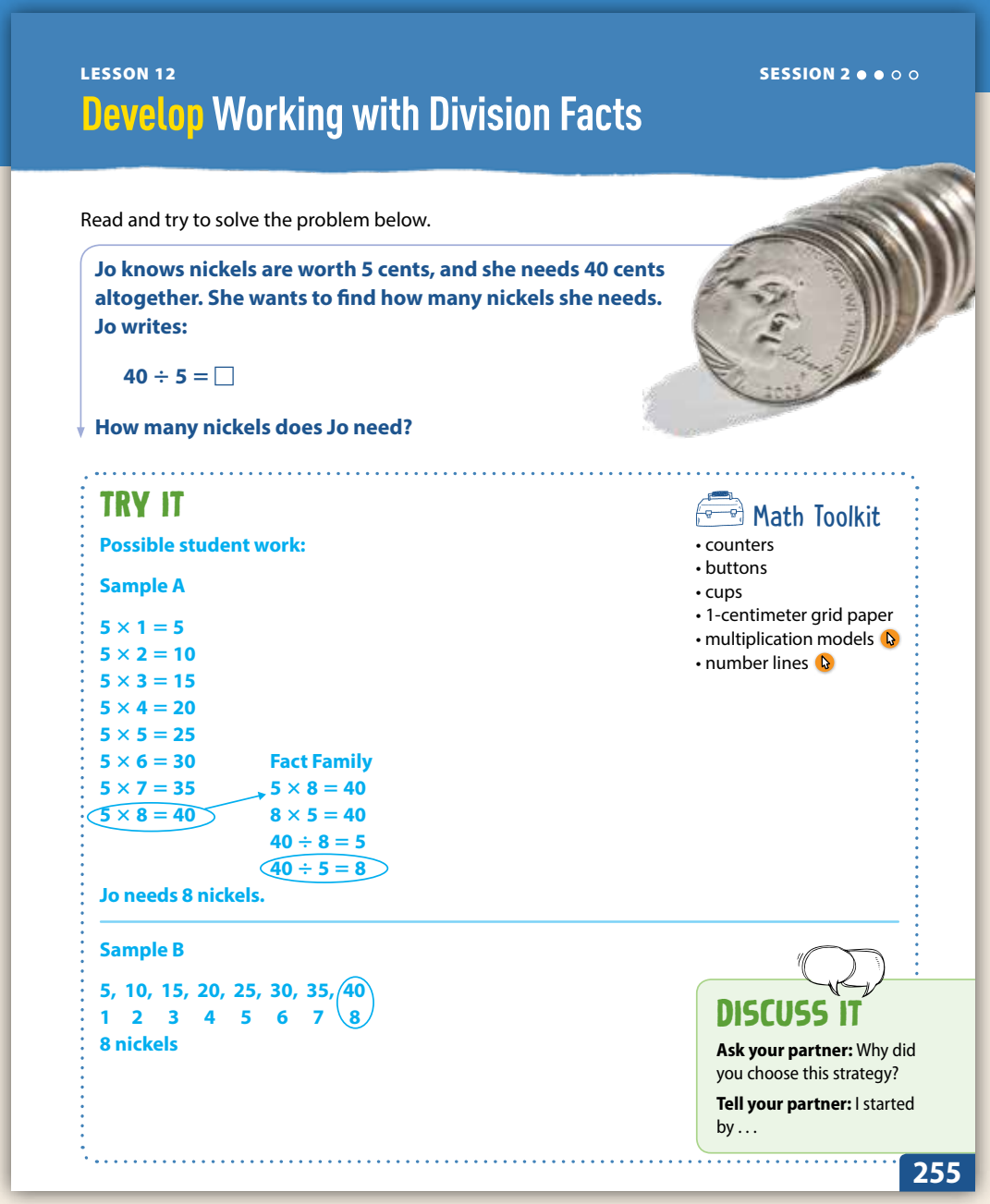
Substitute *dimes* for *nickels* and *10 cents* for *5 cents* and have students read. Ask students to provide other coins and values to substitute.

TRY IT

Make Sense of the Problem

To support students in making sense of the problem, have them identify the number of cents Jo needs.

Ask *What coin does Jo know about? How many cents is that coin worth?*



LESSON 12 SESSION 2 ● ● ○ ○

Develop Working with Division Facts

Read and try to solve the problem below.

Jo knows nickels are worth 5 cents, and she needs 40 cents altogether. She wants to find how many nickels she needs. Jo writes:

40 ÷ 5 = □

How many nickels does Jo need?

TRY IT

Possible student work:

Sample A

5 × 1 = 5
5 × 2 = 10
5 × 3 = 15
5 × 4 = 20
5 × 5 = 25
5 × 6 = 30
5 × 7 = 35
5 × 8 = 40

Fact Family
5 × 8 = 40
8 × 5 = 40
40 ÷ 8 = 5
40 ÷ 5 = 8

Jo needs 8 nickels.

Sample B

5, 10, 15, 20, 25, 30, 35, 40
1 2 3 4 5 6 7 8
8 nickels

Math Toolkit
- counters
- buttons
- cups
- 1-centimeter grid paper
- multiplication models
- number lines

DISCUSS IT

Ask your partner: Why did you choose this strategy?

Tell your partner: I started by . . .

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DISCUSS IT

Support Partner Discussion

Encourage students to use the terms *equal groups* and *quotient* as they discuss.

Support as needed with questions such as:

- *What did you notice about your partner's strategy that is different than yours?*
- *Is there another way of looking at that?*
- *How could you use a fact family to find the number of nickels Jo needs?*

Common Misconception Look for students who try to multiply 40 × 5 instead of dividing or using the multiplication facts 5 × 8 = 40 or 8 × 5 = 40. Ask students to explain their reasoning as to whether the answer will be greater than or less than 40.

Select and Sequence Student Solutions

One possible order for whole class discussion:

- drawing or modeling equal groups
- drawing a number line to skip-count
- skip-counting by fives without a model
- using a fact from the related fact family

Support Whole Class Discussion

Compare and connect the different representations and have students identify how they relate.

Ask *Where does each model show the total cents Jo needs? The amount each nickel is worth? The total number of nickels Jo needs?*

Listen for Students should recognize that accurate representations show that the total, 40, is divided by 5 into 8 equal groups. Responses may include an equal-groups model, array, number line, skip-counting, and/or a fact family.

MODEL ITS

If no student presented these models, connect them to the student models by pointing out the ways they each represent:

- the total number of cents Jo needs
- the number of cents in each nickel
- the total number of nickels Jo needs
- either division or multiplication

Ask *What number tells the number of equal groups in the number line? In the fact family? What does that number represent?*

Listen for 8 is the number of jumps or equal groups in the number line and the fact family. It represents the number of nickels Jo needs.

For a number line model, prompt students to identify the ending number on the number line and the length of the jumps used.

- *How is the number line set up?*
- *Why is each jump by 5?*
- *Why is the number of jumps counted?*

For a fact family, prompt students to identify how knowing their multiplication facts can help them complete a fact family.

- *How does knowing the multiplication facts for 5 help you to complete the fact family?*
- *What other multiplication facts could you use to help you complete the fact family?*

Explore different ways to find the unknown number in a division fact.

Jo knows nickels are worth 5 cents, and she needs 40 cents altogether. She wants to find how many nickels she needs. Jo writes:

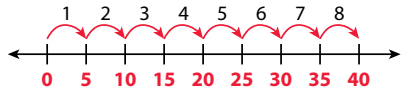
40 ÷ 5 = □

How many nickels does Jo need?

MODEL IT

You can use a number line to help you understand the problem.

Skip-count by **fives** to find the answer. Start at 0 and jump by fives until you get to 40.



MODEL IT

You can use fact families and multiplication facts you know.

Here are the facts in this family:

5 × 8 = 40 　 8 × 5 = 40 　 40 ÷ 8 = 5 　 40 ÷ 5 = 8

Write the multiplication facts for 5:

1 × 5 = 5	2 × 5 = 10	3 × 5 = 15	4 × 5 = 20	5 × 5 = 25
6 × 5 = 30	7 × 5 = 35	8 × 5 = 40	9 × 5 = 45	10 × 5 = 50

Look for the fact that has the numbers you know from the fact family, 5 and 40. Use that fact to fill in the unknown numbers above.

Deepen Understanding

Relate Multiplication and Division Facts

SMP 2 Reason abstractly and quantitatively.

To support discussion of the fact family model, prompt students to consider the relationship between the numbers in a fact family.

Have students discuss how to know which fact family works for this problem. Write the equation □ × 5 = 40 on the board.

Ask *Where did this equation come from?*

Listen for The missing number tells how many times you add 5 to get to 40. It uses the same numbers as the equation in the problem, 40 ÷ 5 = □.

Write the equation 5 × □ = 40 on the board.

Ask *Where did this equation come from?*

Listen for It is the same as □ × 5 = 40, with the order of the factors switched.

Generalize Prompt students to see that you can write related multiplication facts for a division problem using the same numbers as the division facts. Both the division and multiplication facts tell the total of equal groups.

CONNECT IT

- Remind students that one thing that is alike about all the representations is the numbers.
- Explain that on this page, students will use fact families to complete a division fact with an unknown in any position.

Monitor and Confirm

1 – 2 Check for understanding that:

- the other division fact in the same family includes 45, 5, and ☐, but 5 and ☐ switch places
- multiplication facts in the same family include 45, 5, and ☐, but the factors 5 and ☐ change order

Support Whole Class Discussion

3 Tell students that this problem will prepare them to provide the explanation required in problem 4.

Be sure students understand that the problem is asking them to select a multiplication fact from the table of multiplication facts for 5 in the second *Model It* on the previous page.

Ask *How do you know which numbers to look for in the multiplication facts table?*

Listen for A fact family uses the same three numbers in four related multiplication and division facts. So, a related multiplication fact will include 5 and 45. Only $9 \times 5 = 45$ includes 5 and 45.

4 Look for the idea that you need a fact in the same fact family, so the multiplication fact must use the same numbers as in the division fact.

5 REFLECT Have all students focus on the strategies used to solve this problem. If time allows, have students share their preferences with a partner.

CONNECT IT

Now you will use the problem from the previous page to help you understand how to use fact families to find an unknown number in a division fact.

1 Mo wants to know how many nickels he needs to make 45 cents. He writes 45 ÷ ☐ = 5. What other division fact can he write to model this problem?

45 ÷ 5 = ☐

2 Write the two multiplication facts that are in the same fact family. Use ☐ for the unknown number.

5 × ☐ = 45 and ☐ × 5 = 45

3 Look at the list of multiplication facts for 5 on the previous page. Which fact will help Mo answer his division problem? How many nickels does Mo need?

9 × 5 = 45; Mo needs 9 nickels.

4 Explain how you know which multiplication fact you can use to help you find the unknown number in a division fact.

Possible answer: You need to find a multiplication fact that has the same numbers that are given in the division problem.

5 REFLECT

Look back at your **Try It**, strategies by classmates, and **Model Its**. Which models or strategies do you like best for finding unknown numbers in multiplication and division facts? Explain.

Some students may like skip-counting because they are comfortable with it and can do it quickly. Others may prefer using fact families because they know their multiplication facts and can easily find the missing third number in a fact.

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Hands-On Activity

Make fact triangle cards.

If . . . *students have trouble writing the division facts of a fact family,*

Then . . . *use this activity to give a more concrete experience and practice writing division facts.*

Materials For each pair: 10 triangles from Activity Sheet *Multiplication Triangles*

- Have students write out the multiplication facts for 6 (or other facts you wish students to practice). Instruct them to circle the three numbers in each equation.
- Ask each pair to make a set of 10 triangle fact cards, one for each fact. For each card, have the students write the product in the top corner of the triangle and one of the factors in one of the other triangle corners.
- Have students practice their division facts by going through the cards, naming the missing factor, and then stating the related division fact with the missing factor, now the quotient of the division fact. The missing factor can be written on the back of each card.
- Repeat with facts for other numbers as time allows.

APPLY IT

For all problems, encourage students to draw some kind of model to support their thinking.

6 6; See possible student work on the Student Worktext page.

7 $2 \times 3 = 6$; $3 \times 2 = 6$;
$6 \div 2 = 3$; $6 \div 3 = 2$
The last three facts may be written in any order.

Close: Exit Ticket

8 $7 \times 3 = 21$; $3 \times 7 = 21$
Facts may be written in any order.

Students' solutions should indicate understanding of:

- how to write related multiplication and division facts from the same fact family
- how multiplication facts can be used to solve a division problem

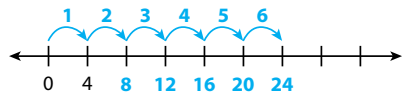
Error Alert **If** students write multiplication facts that include 3 or 7 but are not in the same fact family, **then** have them make a multiplication facts for 3 table and then select the multiplication fact that includes both numbers used in the division problem.

APPLY IT

Use what you just learned to solve these problems.

6 Use the number line to solve $24 \div 4 = \square$.
Show your work.

Possible student work:



Solution 6

7 Write the unknown product. Then complete this fact family.

$2 \times 3 =$ 6

$3 \times 2 = 6$

$6 \div 2 = 3$

$6 \div 3 = 2$

8 Write two multiplication facts Brice can use to solve $\square \div 3 = 7$.

$7 \times 3 = 21$ and $3 \times 7 = 21$

Solutions

1. $3 \times 5 = 15$ or $5 \times 3 = 15$
   Basic

2. $15 \div 3 = 5$
   Basic

3. $15 \div 5 = 3$
   Basic

4. $4 \times 9 = 36$;
   $9 \times 4 = 36$;
   $36 \div 4 = 9$;
   $36 \div 9 = 4$
   Pairs of equations may be written in any order.
   Medium

Name: _____ LESSON 12 SESSION 2

Practice Working with Division Facts

Study the Example showing how a drawing can help you understand division facts. Then solve problems 1–9.

EXAMPLE

Here is an array of 15 fish.

There are 3 rows with 5 fish in each row.

The fact family describes the array in different ways.

$3 \times 5 = 15$ $15 \div 5 = 3$

$5 \times 3 = 15$ $15 \div 3 = 5$

Write one of the facts from the list above that can help you solve problems 1–3.

1. How many fish are there altogether?
   3 × 5 = 15 or 5 × 3 = 15

2. 15 fish swim in 3 equal rows. How many fish are in each row?
   15 ÷ 3 = 5

3. 15 fish swim in rows of 5 fish. How many rows of fish are there?
   15 ÷ 5 = 3

4. You know that $4 \times 9 = 36$. Write the whole fact family. Use the numbers 4, 9, and 36.

   4 × 9 = 36
   9 × 4 = 36
   36 ÷ 4 = 9
   36 ÷ 9 = 4

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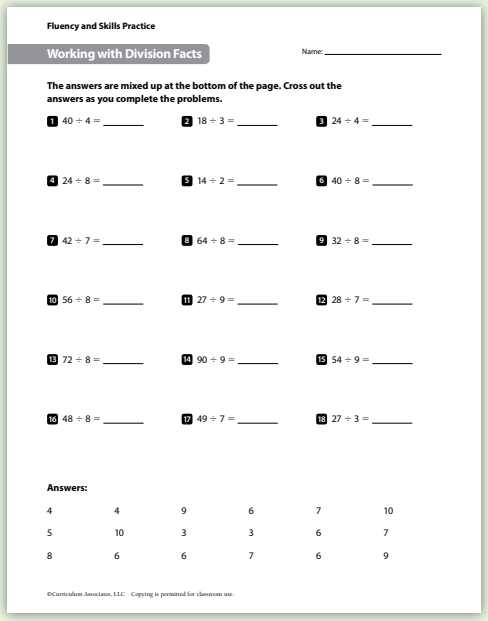
Fluency & Skills Practice

Teacher Toolbox

Assign *Working with Division Facts*

In this activity students practice solving division facts. Students will use division anytime they need to split a quantity into equal groups. Such situations could include sharing a group of marbles equally among friends, determining how many pages they need to read each night to finish a book, or cutting a length of ribbon into a certain number of pieces all the same length.



Fluency and Skills Practice

Working with Division Facts

Name: _____

The answers are mixed up at the bottom of the page. Cross out the answers as you complete the problems.

1. 40 ÷ 4 = _____
2. 18 ÷ 3 = _____
3. 24 ÷ 4 = _____
4. 24 ÷ 8 = _____
5. 14 ÷ 2 = _____
6. 40 ÷ 8 = _____
7. 42 ÷ 7 = _____
8. 64 ÷ 8 = _____
9. 32 ÷ 8 = _____
10. 56 ÷ 8 = _____
11. 27 ÷ 9 = _____
12. 28 ÷ 7 = _____
13. 72 ÷ 8 = _____
14. 90 ÷ 9 = _____
15. 54 ÷ 9 = _____
16. 48 ÷ 8 = _____
17. 49 ÷ 7 = _____
18. 27 ÷ 3 = _____

Answers:

4	4	9	6	7	10
5	10	3	3	6	7
8	6	6	7	6	9

5 **C**; 18 ÷ 2 = 9
Medium

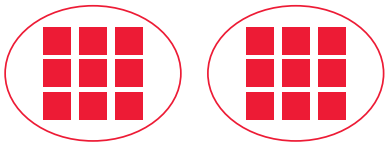
6 18 ÷ 6 = 3; 18 ÷ 3 = 6
Equations may be written in any order.
Medium

7 3 friends; See Student Worktext page for possible student work.
Medium

8 24 ÷ 8 = 3; 24 ÷ 3 = 8
Equations may be written in any order.
Medium

9 8 × 3 = 24; 3 × 8 = 24
Equations may be written in any order.
Medium

5 Sienna draws 18 squares in two equal groups of 9. Which division equation does her drawing show?



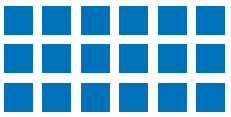
Ⓐ 9 ÷ 3 = 3

Ⓑ 18 ÷ 6 = 3

Ⓒ 18 ÷ 2 = 9

Ⓓ 6 ÷ 2 = 3

6 Write two different division equations about the array.



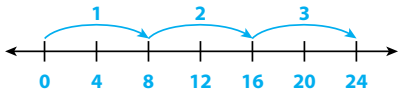
18 ÷ 6 = 3

18 ÷ 3 = 6

Chee has 24 trading cards. He gives away all his cards to friends. He gives 8 cards to each friend.

7 Use the number line to show how you can find how many friends Chee gave cards to.

Possible student work:



Solution 3 friends

8 Write two different division facts for the story.

24 ÷ 8 = 3 and 24 ÷ 3 = 8

9 Write the multiplication facts that belong to the same fact family.

8 × 3 = 24 and 3 × 8 = 24

ELL English Language Learners: Differentiated Instruction

Prepare for Session 3
Use with *Apply It*.

Levels 1–3	Levels 2–4	Levels 3–5
Listening/Speaking Read ***Apply It*** problem 8 aloud. Ask: *What do you want to know?* [how many apples Jan and Jon each get] *Talk with a partner. Do you think Jan and Jon will each get 18 apples? Say 'yes' if you agree and 'no' if you disagree. How do you know?* [No; they only have 18 apples.] *How many groups of apples will Jan and Jon make?* [2] *What equation with 18 and 2 can you use?* Display the first multiplication or division equation suggested. Say: *Work with a partner to write the other equations that complete the fact family.* Call on pairs to share their responses until all equations are listed.	**Listening/Speaking** Read ***Apply It***, problem 8 aloud. Ask: *What do you want to know?* [how many apples Jan and Jon each get] *Talk with your partner. I think Jan and Jon will each get 18 apples. Do you agree or disagree? How do you know?* [Disagree; they only have 18 apples.] *What do Jan and Jon represent?* [They are the groups.] *What equation can you write to find the number of apples each gets?* Display the first multiplication or division equation suggested. Say: *Work with your partner to write the other equations that complete the fact family.* Call on pairs to share equations until all equations are listed.	**Listening/Speaking** Pair students and have them read ***Apply It*** problem 8. Ask: *What are you trying to find?* [how many apples Jan and Jon each get] *Talk with your partner. Do you agree or disagree that Jan and Jon will each get 18 apples? How do you know?* [Disagree; they only have 18 apples.] *Do you know the number of groups to make, or the size of the groups?* [the number of groups] *How do you know?* [Jan and Jon are the groups.] *Work with your partner to write the fact family that can be used to find the number of apples Jan and Jon each get.* Call on pairs to share until all equations are listed.

LESSON 12

SESSION 3 Develop

Purpose In this session students find the unknown number in several multiplication and division equations. The purpose of this problem is to have students discover how they can use a multiplication table to complete multiplication and division facts.

Start

Connect to Prior Knowledge

Materials For each student: 10 counters

Why Support students' facility in modeling a division problem and writing a related multiplication fact.

How Have students model the division fact □ ÷ 4 = 2 and write a related multiplication fact.

Use counters to model the division fact below. Then write a related multiplication fact.

□ ÷ 4 = 2

Possible Solutions

4 × 2 = 8 or
2 × 4 = 8

Look for 8 counters in 4 equal groups of 2 or 2 equal groups of 4 or in a 2 × 4 or 4 × 2 array.

Develop Language

Why Clarify the multiple-meaning word *table*.

How Explain that information organized in rows and columns is also called a *table*. Have students refer to the multiplication table and identify the rows and columns. Point out that the information in a table does not have to be numbers.

TRY IT

Make Sense of the Problem

To support students in making sense of the problem, have them show that they understand that the problem involves finding the unknown number in four multiplication and division equations.

Develop Using a Multiplication Table

Read and try to solve the problem below.

Complete the facts.

$2 \times \square = 10$ $24 \div 6 = \square$ $\square \times 6 = 48$ $\square \div 1 = 8$

TRY IT

Possible student work:

Sample A

2s: 2, 4, 6, 8, 10 $2 \times (5) = 10$

6s: 6, 12, 18, 24 $24 \div 6 = (4)$

6s: 6, 12, 18, 24, 30, 36, 42, 48 $(8) \times 6 = 48$

1s: 1, 2, 3, 4, 5, 6, 7, 8 $(8) \div 1 = 8$

Sample B

(2)(2)(2)(2)(2) = 10, so 2 × 5 = 10.

(||||||) (||||||) 24 split into 6 groups; 24 ÷ 6 = 4

6, 12, 18, 24, 30, 36, 42, 48 8 × 6 = 48

(1)(1)(1)(1)(1)(1)(1)(1) = 8 groups of 1 is 8, so 8 ÷ 1 = 8.

Math Toolkit

- connecting cubes
- counters
- multiplication tables
- 1-centimeter grid paper
- multiplication models
- number lines

DISCUSS IT

Ask your partner: Do you agree with me? Why or why not?

Tell your partner: The strategy I used to find the answer is . . .

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DISCUSS IT

Support Partner Discussion

Encourage students to use the terms *factor*, *product*, and *quotient* as they discuss their solutions.

Support as needed with questions such as:

- *Can you explain why you solved it that way?*
- *Did you draw a picture or a model? Why or why not?*
- *Could you use fact families to find the unknown numbers?*

Common Misconception Look for students who stop after solving only one or two of the problems because they don't understand that they are supposed to solve all four problems, they don't know some of the multiplication or division facts, or they think the four problems are part of the same fact family. Have them divide the work space into 4 sections and write an equation in each of the sections.

Select and Sequence Student Solutions

One possible order for whole class discussion:

- modeling equal groups or arrays
- using a number line
- using repeated addition or skip-counting
- using fact families

Support Whole Class Discussion

Compare and connect the different representations and have students identify how they relate.

Ask *Where does each model show whether you are finding a factor, a quotient, or a total?*

Listen for Students should recognize that accurate representations show whether you are modeling and finding a factor or quotient (by finding the number of groups or the number in each group) or finding the total.

PICTURE IT & MODEL IT

If no student presented these models, connect them to the student models by pointing out the ways they each represent:

- the factors in a multiplication problem
- the total in each problem
- the unknown number in each problem

Ask *What do the numbers in the shaded top row and first column of the multiplication table represent? What do the other numbers represent?*

Listen for The shaded numbers represent the factors in a multiplication problem. Every other number represents the product of two factors.

For the multiplication table, prompt students to identify what the rows and columns mean.

- *How does each number in the shaded first column relate to the other numbers in the same row?*
- *How does each number in the shaded top row relate to the other numbers in the same column?*

For the fact family, prompt students to explain how they can use the multiplication table to find the missing factor in $2 \times \square = 10$.

- *Find 2 in the first column of the table and then the 10 in this row. How can you now find the other factor?*
- *What 3 numbers make up the fact family that this fact belongs to? How do these numbers relate to the numbers you just found in the table?*

Explore different ways to use a multiplication table to complete multiplication and division facts.

Complete the facts.

$2 \times \square = 10$ $24 \div 6 = \square$ $\square \times 6 = 48$ $\square \div 1 = 8$

PICTURE IT

You can use a multiplication table to find the numbers in multiplication and division fact families.

A **multiplication table** shows both multiplication and division fact families.

×	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

MODEL IT

Use the table above to complete the fact family.

The multiplication table shows the three numbers that belong in the fact family for $2 \times \square = 10$. Look at the row for 2. Go across to find 10. Then look up to the top of that column to find the third number in the fact family. Fill in the blanks below.

$2 \times$ 5 $= 10$ $10 \div 2 =$ 5

5 $\times 2 = 10$ $10 \div$ 5 $= 2$

Deepen Understanding

Multiplication Table as a Strategic Tool

SMP 5 Use tools.

When discussing the multiplication table, prompt students to consider how it is an appropriate tool to use when completing fact families.

Materials For each student: crayon, Activity Sheet *Multiplication Table*

Have students write the multiplication facts for 5 . Have them circle the products and let them know that the circled numbers are also called the *multiples of 5*. Ask them to color all the multiples of 5 on their table.

Ask *What did you color on the table? Why should you not be surprised?*

Listen for I colored the 5 column and 5 row. Both contain the products of the facts for 5 (or multiples of 5) since each number in the column (or row) has 5 as a factor.

Generalize Explain how each product on a multiplication table gives you enough information to make a fact family. [The numbers at the start of the row and the column that the product is in are the factors of the product; these factors and the product give you the three numbers in the fact family.]

CONNECT IT

- Remind students that one thing that is alike about all the representations is the numbers.
- Explain that on this page, students will use the multiplication table to find the missing numbers in each multiplication and division problem.

Monitor and Confirm

1 – 3 Check for understanding that:

- the non-shaded table numbers represent the total and the shaded numbers represent the number of groups and the number in each group
- the three numbers in a fact family will include two shaded numbers (1 from the first column and 1 from the top row) and 1 non-shaded number where the row and column intersect

Deepen Understanding

Multiplication Table Organization

SMP 7 Use structure.

When discussing the multiplication table, prompt students to identify the *factors* and *product*.

Materials For each student: red crayon, yellow crayon, Activity Sheet *Multiplication Table*

- Write the problem $2 \times 4 = 8$ on the board.
- Have students color the 2 row yellow. Have them color the 4 column red. Identify the orange box where the two colored rows intersect [8].
- Have students identify and connect the factors and product in the multiplication fact and in the table.
- Repeat for other facts as time allows.

Support Whole Class Discussion

4 Look for understanding of how the shaded numbers (factors) and non-shaded numbers (products) on a multiplication table can be used to find an unknown number in any position in a multiplication or division problem.

5 REFLECT Have all students focus on the strategies used to solve this problem. If time allows, have students share their preferences with a partner.

CONNECT IT

Now you will use the problem from the previous page to help you understand how to use a multiplication table to complete a multiplication or division fact.

1 Look at the multiplication table. What are the three numbers in the fact family for $24 \div 6 = \square$?

4, 6, and 24

Now fill in the blank: $24 \div 6 =$ **4**

2 What are the three numbers in the fact family for $\square \times 6 = 48$?

6, 8, and 48

Now fill in the blank: **8** $\times 6 = 48$

3 What are the three numbers in the fact family for $\square \div 1 = 8$?

1, 8, and 8

Fill in the blank: **8** $\div 1 = 8$

4 Explain how you can use a multiplication table to find the three numbers in any fact family.

Possible answer: To find the product, find the row with one factor and the column with the other factor. Follow the row and column until they meet to find the product. To find an unknown factor, first find the known factor in one of the rows. Follow that row across until you find the product. Follow that column up to find the second factor.

5 REFLECT

Look back at your **Try It**, strategies by classmates, and **Picture It** and **Model It**. Which models or strategies do you like best for completing multiplication and division facts? Explain.

Some students may prefer using the multiplication table because of its organization and ease in locating factors and products. Other students, who are fluent in their multiplication facts, may be able to quickly recognize the missing number in a fact family and prefer to use related facts to solve a multiplication or division problem.

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Hands-On Activity

Build a Multiplication Table

If . . . *students are having trouble learning and remembering multiplication and division facts,*

Then . . . *use this activity to deepen their understanding of the relationships among the numbers in fact families.*

Materials For each student: Activity Sheet *Blank Multiplication Table*

- Instruct students to first fill in the shaded column and row. You may want to fill in the facts for 1, 2, and 5 next or have students suggest an order for completing the table.
- As students start to fill in the table, they begin to realize how many facts they already know. Be sure to point out that if they fill in one whole row, they also know the products to write in the corresponding column. You may wish to have students complete the table in one sitting or over several sessions.

APPLY IT

For all problems, encourage students to draw or use a model to support their thinking.

6 $6 \times 7 = 42, 7 \times 6 = 42, 42 \div 6 = 7, 42 \div 7 = 6$
Facts may be written in any order.
See Student Worktext page for possible student work.

7 7

Close: Exit Ticket

8 **B, C, E**; Since the apples are equally shared between two people, the fact family includes the numbers 18, 2, and 9. Only multiplication and division facts with these three numbers could be used to find the number of apples each person gets.

Students' solutions should indicate understanding of:

- the number 2 is an understood factor since the apples are shared equally between two people
- a fact family involves the same three numbers
- a fact family has both multiplication and division facts

Error Alert **If** students choose A and D, **then** have students read the problem aloud. Ask how many people are sharing the apples.

APPLY IT

Use what you just learned to solve these problems.

6 Use the multiplication table to write the equations in the fact family that includes 42 and 6. Show your work.

Possible student work:

I found 6 in the first column and went across the row until I found 42. Then I went up that column to see that the third number in this fact family is 7.

×	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

Solution $6 \times 7 = 42, 7 \times 6 = 42, 42 \div 6 = 7, 42 \div 7 = 6$

7 Fill in the blank.

$56 \div$ 7 $= 8$

8 Jan and Jon pick 18 apples. They share them equally. Which facts could be used to find the number of apples each person gets?

Ⓐ $6 \times 3 = 18$
Ⓑ $2 \times 9 = 18$
Ⓒ $18 \div 2 = 9$
Ⓓ $18 \div 3 = 6$
Ⓔ $18 \div 9 = 2$

Solutions

1. $6 \times 4 = 24$; $24 \div 4 = 6$
   $4 \times 6 = 24$; $24 \div 6 = 4$
   Pairs of equations may be written in any order.
   Basic

2. 6; 4
   6; 24
   Medium

3. 3; 7
   3; 7
   Medium

Name: _____ LESSON 12 SESSION 3

Practice Using a Multiplication Table

Study the Example showing how a multiplication table can help you solve multiplication and division problems. Then solve problems 1–6.

EXAMPLE

You can use the multiplication table to multiply or divide.

Look at the green row of products for 4.
Look at the green column of products for 6.

You can see how $4 \times 6 = 24$ is related to $24 \div 4 = 6$ and related to $24 \div 6 = 4$.

×	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

1. Write the fact family for the three numbers 6, 4, and 24.

   6 × 4 = 24 24 ÷ 4 = 6
   4 × 6 = 24 24 ÷ 6 = 4

2. Use the table or your fact family in problem 1 to fill in the unknown numbers.

   4 × 6 = 24 24 ÷ 6 = 4
   6 × 4 = 24 24 ÷ 4 = 6

3. Find 21 on the table above. Use the table to fill in the unknown numbers in this fact family.

   7 × 3 = 21 21 ÷ 3 = 7
   3 × 7 = 21 21 ÷ 7 = 3

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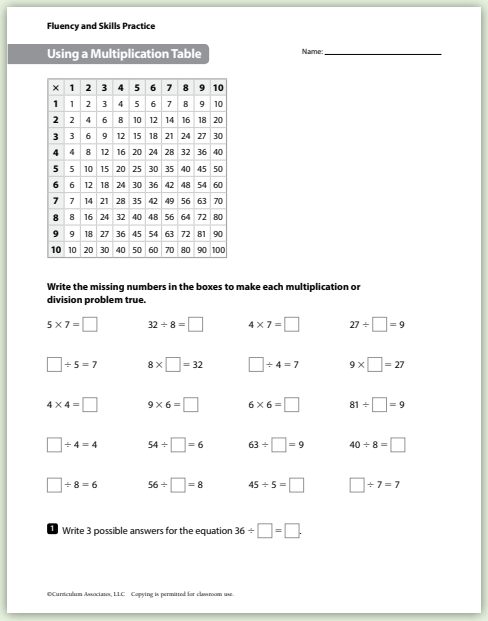
Fluency & Skills Practice

Teacher Toolbox

Assign *Using a Multiplication Table*

In this activity students practice using a multiplication table to find the missing number in a multiplication or division fact. This is one tool students may utilize in real-world situations that involve multiplication or division. For example, suppose students are setting up chairs in equal rows in the school auditorium. The students can find where the total number of chairs appears in the table and use the corresponding factors to determine possible arrangements.



Fluency and Skills Practice

Using a Multiplication Table

Name: _____

×	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

Write the missing numbers in the boxes to make each multiplication or division problem true.

5 × 7 = ☐ 32 ÷ 8 = ☐ 4 × 7 = ☐ 27 ÷ ☐ = 9

☐ ÷ 5 = 7 8 × ☐ = 32 ☐ ÷ 4 = 7 9 × ☐ = 27

4 × 4 = ☐ 9 × 6 = ☐ 6 × 6 = ☐ 81 ÷ ☐ = 9

☐ ÷ 4 = 4 54 ÷ ☐ = 6 63 ÷ ☐ = 9 40 ÷ 8 = ☐

☐ ÷ 8 = 6 56 ÷ ☐ = 8 45 ÷ 5 = ☐ ☐ ÷ 7 = 7

1 Write 3 possible answers for the equation 36 ÷ ☐ = ☐.

4 4, 7, 28; Numbers may be written in any order.
$7 \times 4 = 28$; $28 \div 4 = 7$
$4 \times 7 = 28$; $28 \div 7 = 4$
Pairs of equations may be written in any order.
Medium

5 6, 7, 42; Numbers may be written in any order.
$7 \times 6 = 42$; $42 \div 6 = 7$
$6 \times 7 = 42$; $42 \div 7 = 6$
Pairs of equations may be written in any order.
Medium

6 6, 8, 48; Numbers may be written in any order.
$8 \times 6 = 48$; $48 \div 6 = 8$
$6 \times 8 = 48$; $48 \div 8 = 6$
Pairs of equations may be written in any order.
Medium

Use the multiplication table to solve problems 4–6.

×	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

4 What are the three numbers in the fact family for $28 \div 4 = \square$? 4, 7, 28

Write the fact family.

7 × 4 = 28 28 ÷ 4 = 7

4 × 7 = 28 28 ÷ 7 = 4

5 What are the three numbers in the fact family for $6 \times \square = 42$? 6, 7, 42

Write the fact family.

7 × 6 = 42 42 ÷ 6 = 7

6 × 7 = 42 42 ÷ 7 = 6

6 What are the three numbers in the fact family for $\square \div 6 = 8$? 6, 8, 48

Write the fact family.

8 × 6 = 48 48 ÷ 6 = 8

6 × 8 = 48 48 ÷ 8 = 6

ELL English Language Learners: Differentiated Instruction

Prepare for Session 4
Use with *Apply It*.

Levels 1–3

Listening/Speaking Read ***Apply It*** problem 3 and have students follow along. Help students identify and understand relevant information in the problem. Ask: *How many juice boxes does Mrs. Tobin need? How many juice boxes are there in one package?* Have students work with partners to solve the problem. Ask: *What do you do to find the answer? How many packages does Mrs. Tobin need?* Allow students to point to their answers or respond using single words or phrases.

Levels 2–4

Listening/Speaking Choral read ***Apply It*** problem 3. Help students identify and understand relevant information in the problem. Ask: *How many juice boxes does Mrs. Tobin need? How many juice boxes are there in one package?* Provide sentence frames to help them answer:

Mrs. Tobin needs _____ juice boxes.
One package has _____ juice boxes.

Have students work with partners to solve the problem. Ask: *What did you do to find the answer? How many packages does Mrs. Tobin need?* Provide sentence frames:

My partner and I _____ to find the answer.
Mrs. Tobin needs _____ packages.

Levels 3–5

Listening/Speaking Have students read ***Apply It*** problem 3 with partners. Make sure they identify and understand relevant information. Ask: *What do you know? What do you need to find out? How will you find that out?* Have partners answer the questions and solve the problem. Ask: *How many packages does Mrs. Tobin need?* Encourage students to answer in complete sentences.

LESSON 12

SESSION 4 Refine

Purpose In this session students solve multiplication and division problems using related equations within fact families and then discuss and confirm their answers with a partner.

Before students begin work, use their responses to the *Check for Understanding* to determine those who will benefit from additional support.

As students complete the Example and problems 1–3, observe and monitor their reasoning to identify groupings for differentiated instruction.

Start

Check for Understanding

Materials For remediation: Activity Sheet *Multiplication Table*

Why Confirm understanding of multiplication and division facts.

How Have students find the multiplication fact Lena can use to complete 56 ÷ 7 = ___ and then find the quotient using any strategy they want.

Write a multiplication fact Lena can use to complete the fact below. Then write the quotient.

56 ÷ 7 = ...

Solutions

7 × 8 = 56 or 8 × 7 = 56;
8

LESSON 12 SESSION 4 ● ● ● ●

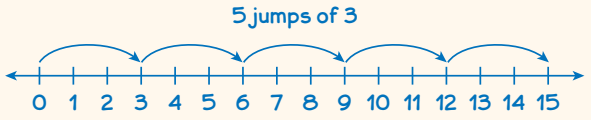
Refine Working with Multiplication and Division Facts

Complete the Example below. Then solve problems 1–9.

EXAMPLE

Today some students will give an oral report. The teacher is planning to have all the reports done in 15 minutes. Each student will get 3 minutes. How many students will give reports? Solve 3 × □ = 15.

Look at how you could show your work using a number line.



Solution 5 students

Each jump on the number line shows the amount of time a student gets.

PAIR/SHARE
What other equations can be used to solve this problem?

APPLY IT

1 Solve 35 ÷ □ = 5. Show your work.

Possible student work:

5 × 7 = 35
so
35 ÷ 7 = 5

How can you find the third number in this fact family?

PAIR/SHARE
What are the other facts that belong to this fact family?

Solution 7

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Error Alert

If the error is . . .	*Students may . . .*	*To support understanding . . .*
students write a division fact instead of a multiplication fact	not understand how knowing a multiplication fact can help you find the missing number in a division fact.	Write a simple division problem, such as 12 ÷ 3 = ___. Have students circle the total [12] and underline the known factor [3]. Point out they are looking for the factor that multiplied with 3 equals the product 12. Help them rewrite the equation as 3 × ___ = 12 and then identify the missing factor and complete the division fact: 12 ÷ 3 = ___ [4].
students write 7 or 9 for the quotient	not know how to use a multiplication table to find the missing number in a division fact.	Ask students to circle the total in the equation and underline the factor. On a multiplication table, have students find the 7 row on the left side of the table, follow the row until they reach 56, and look to see the column number for the missing factor.
students write 7 or 9 for the quotient	not know the multiplication facts for 7.	Review the structure of the multiplication table with students. Then ask students to identify where they would find the multiplication facts for 7 on the table.

2 Solve 4 × 9 = ☐. Show your work.

Possible student work:

4 × 9 = 36

Are you looking for a factor or a product?

Solution **36**

PAIR/SHARE
Explain how you solved this problem.

3 Mrs. Tobin needs 30 juice boxes for her class. The juice boxes come in packages of 6. How many packages does she need? Solve 30 ÷ 6 = ☐.

Ⓐ 4

Ⓑ 5

Ⓒ 6

Ⓓ 36

Do you know a multiplication fact that can help you solve this problem?

Pia chose Ⓓ as the correct answer. How did she get that answer?

Possible answer: Pia added 30 + 6 = 36 instead of dividing.

PAIR/SHARE
Does Pia's answer make sense?

EXAMPLE

5 students; The number line shown is one way to solve the problem. Students could also solve the problem by writing a fact family.

Look for The number in each group is given, so you need to find the number of groups.

APPLY IT

1 7; Students could solve the problem by using a related multiplication fact from the same fact family.
DOK 1

Look for The numbers in this fact family are 5, 7, and 35.

2 36; Students could solve the problem by drawing equal groups or writing a known multiplication fact.
DOK 1

Look for The missing number is a product or total.

3 **B**; Students could solve the problem by writing the fact family for 5, 6, and 30.

Explain why the other two answer choices are not correct:

A is not correct because 4 × 6 = 24, not 30. Students may have used a multiplication table incorrectly.

C is not correct because 6 × 6 = 36, not 30. Students may have used a multiplication table incorrectly.
DOK 2

4 **B**; The fact family includes 12, 4, and 3, so 2 does not belong in this fact family.
DOK 1

5 **B**; The fact family includes 4, 5, and 20, so you could use $4 \times 5 = 20$ to solve the division problem.
DOK 1

6 **B** (**No**); $9 \times 8 \neq 64$
C (**Yes**); $6 \times 8 = 48$
F (**No**); $56 \div 8 \neq 8$
G (**Yes**); $32 \div 8 = 4$
DOK 1

Error Alert Students with any incorrect answers may be misreading the multiplication table or not know how to use a fact family to help solve a multiplication or division problem.

4 Which equation does NOT belong to the same fact family as $12 \div \square = 4$?

Ⓐ $\square \times 4 = 12$
Ⓑ $\square \times 2 = 12$
Ⓒ $4 \times \square = 12$
Ⓓ $12 \div 4 = \square$

5 Which fact can you use to solve $\square \div 5 = 4$?

Ⓐ $5 \times 5 = 25$
Ⓑ $4 \times 5 = 20$
Ⓒ $5 + 4 = 9$
Ⓓ $6 \times 4 = 24$

6 Does putting the number 8 in the box make each equation true?

	Yes	No
$9 \times \square = 64$	Ⓐ	Ⓑ
$6 \times \square = 48$	Ⓒ	Ⓓ
$56 \div \square = 8$	Ⓔ	Ⓕ
$32 \div \square = 4$	Ⓖ	Ⓗ

Differentiated Instruction

RETEACH

Hands-On Activity
Use patterns to learn facts for 9.

Students struggling with remembering their 9 facts

Will benefit from finding patterns to help learn and remember.

Materials For each student: crayon, Activity Sheet *Multiplication Table*

- Have students color the multiples of 9 on their table.
- Have them describe any patterns they see. [e.g., the product digits add to 9; the ones digit decreases by 1 as the tens digit increases by 1]
- Illustrate how patterns can be helpful in learning facts using the *sum of 9* pattern. Help students reason that the product of 4×9 must be in the 30s because $4 \times 10 = 40$ and 4×9 is 4 less. Ask: *What number added to 3 equals 9?* [6] Write the completed fact. [$4 \times 9 = 36$]
- Repeat these steps for other 9 facts, such as 8×9. (The product will be in 70s, and $7 + 2 = 9$, so the product is 72.)

EXTEND

Challenge Activity
Use patterns to extend the multiplication table.

Students who have achieved proficiency

Will benefit from using patterns to find facts to 15×15.

Materials For each pair: ruler, Activity Sheet *Multiplication Table*

- Help students to add rows and columns to the multiplication table until it has 15 rows and 15 columns.
- Ask students to use patterns to extend the table, such as *add 2* to extend the 2 column. Advise them to work on completing columns rather than rows.
- Work the problem 15×5 together. Have students find row 15 and column 5 to find the product 75.
- Have students use their tables to find other products, such as 13×7, 12×8, and 15×15.

7 See Student Worktext page for a possible answer. Any multiplication equation with two identical factors will work.
DOK 2

8 $4 \times 8 = 32, 8 \times 4 = 32$; 4 stickers
DOK 2

7 Some fact families have only one multiplication equation and one division equation. Fill in the blanks to show an example. **Possible answer:**

5 × 5 = 25

25 ÷ 5 = 5

8 Sasha has 32 stickers to use in her scrapbook. The scrapbook has 8 pages, and she wants to put the same number of stickers on each page. Write two multiplication facts Sasha can use to find how many stickers to put on each page. How many stickers can Sasha put on each page?

Solution **4 × 8 = 32, 8 × 4 = 32; 4 stickers**

9 MATH JOURNAL

Draw a picture to show a fact family. Then write the fact family.

Possible answer:

3 × 5 = 15
5 × 3 = 15
15 ÷ 3 = 5
15 ÷ 5 = 3
Check that students' drawings match their fact family.

SELF CHECK Go back to the Unit 2 Opener and see what you can check off.

REINFORCE

Problems 4–9
Use multiplication and division facts.

All students will benefit from additional work with multiplication and division facts by solving problems in a variety of formats.

- Have students work on their own or with a partner to solve the problems.
- Encourage students to show their work.

PERSONALIZE

i-Ready

Provide students with opportunities to work on their personalized instruction path with *i-Ready* Online Instruction to:

- fill prerequisite gaps
- build up grade-level skills

Close: Exit Ticket

9 MATH JOURNAL

Student responses should indicate understanding that a fact family uses the same three numbers in two multiplication and two division equations; the student drawing should reflect the product and factors of the multiplication problems by the total number organized into equal groups.

Error Alert **If** students write only 1 multiplication and 1 division fact (and the two factors are not the same number), **then** use an array to review how four different equations can represent the array.

SELF CHECK Have students consider whether they feel they are ready to check off any new skills on the Unit 2 Opener.